Anonymous and Non-Anonymous Growth Incidence Curves in the United States, 1968–2016

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Abstract

This paper combines cross-sectional and longitudinal labor income data to present a comparison between anonymous and non-anonymous growth incidence curves in the United States during the past 50 years. If anonymous growth incidence tend to be upward sloping because of increasing inequality during that period, the same is not true of non-anonymous curves. The latter prove to be flat or non-significantly downward sloping, suggesting some neutrality of growth when initial income positions are accounted for. This is true when using either panel data or synthetic panels based on CPS data and one-parameter functional representations of income mobility. Flat non-anonymous curves are observed even in periods of increasing cross-sectional income inequality. Differences between anonymous and non-anonymous curves thus matter for the interpretation of inequality changes, social welfare and policy.

Keywords: Mobility, inequality, income growth
JEL Codes: D3, H0, J6

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1 Introduction

Is income growth benefiting primarily the rich or the poor? Are inequality changes informative of individual experience of income growth? How do income inequality and income mobility interact? How can we compare the social welfare associated with different growth spells? These are all crucial questions for our understanding of economic growth, for social welfare analysis and for policy. This paper aims to provide a simple framework for such an evaluation, implemented using panel and cross-sectional data for the United States over the last 50 years.

There are two ways of evaluating income distribution changes in a given population over a period of time. In one case, the new distribution is compared to the original one without considering the identity of income earners and their rank in the initial income scale. The comparison is thus 'anonymous'. Poorest (richest) incomes in the initial period are compared to poorest (richest) incomes in the final period. This is the common form of evaluating the distributional impact of growth.

In the other case, the comparison is made between the two distributions conditional on initial incomes. The comparison is thus 'non-anonymous'. It now matters how individual incomes changed depending on their value in the initial distribution.

It has become common to represent the anonymous distributional change brought by economic growth using the Growth Incidence Curve (GIC) (Ravallion and Chen, 2003). This convenient tool shows the rate of income growth of successive fractiles of the distribution in the initial and final periods. The slope of that curve is related to conventional inequality comparisons of two income distributions. In particular, an upward sloping GIC implies that the Lorenz curve of the new distribution is everywhere below that of the old one. Growth is then unambiguously associated with more inequality. The opposite holds for downward sloping GICs.

Non-Anonymous Growth Incidence Curves (NAGICs) have been introduced independently in the economic literature by Grimm (2007), Van Kerm (2009) and Bourguignon (2011).[1] They have been used extensively since, given the increasing availability of income panel data, *e.g.*, Palmisano and Peragine (2015); Jenkins and Van Kerm (2016); Splinter (2019); Hammar and Waldenström (2020). NAGICs show the mean income growth rate in successive fractiles of the initial distribution. An important question is then how comparable NAGICs and GICs are? Are they likely to have the same shape or are they expected to differ in some

[1] In the income mobility literature, tables showing mean or median growth rates of income by initial decile were not uncommon. See, *e.g.*, Hungerford (1993).

radical way?

Answers to these questions are relevant for the interpretation of inequality changes. A downward sloping GIC is associated with less inequality. If mean income growth is positive, then social welfare is unambiguously increasing. Likewise, a downward sloping NAGIC means that the income of initially poor people grew faster than rich people, which might also be considered as a socially favorable development. Yet, is it possible that the two curves slope in different directions? If so, how should it be interpreted?

It turns out that GICs and NAGICs that correspond to the same panel data (the panel dimension of which is ignored in GICs) may have substantially different shapes. This is due to the fact that NAGICs incorporate two sources of change:

1. changes in the shape of the income distribution between the beginning and the end of the studied period;

2. rank mobility of income earners between these two dates.

GICs only account for the first of these two sources, and in the absence of mobility NAGICs and GICs would coincide. Yet, with high mobility their shape could be very different. It is possible that a GIC would slope upward, showing an increase in inequality, whereas the corresponding NAGIC would slope downward leading perhaps to the opposite conclusion. This can happen if poorer people in the initial distribution move strongly upward in the income ranking, and the opposite for richer people. This demonstrates how differences between GICs and NAGICs may capture important aspects regarding the ways in which growth spells and changes in the income distribution are interpreted.

This paper analyzes GICs and NAGICs in the United States over the last 50 years. The main goal is to detect possible regularities in the shape of the NAGICs and/or in their relation with the GICs. Interesting regularities, which have not been thoroughly studied so far, indeed exist. We provide explanations for them. Most notably, NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality.

The paper begins with a short section devoted to the formal definitions of GICs and NAGICs. It presents their properties in the simplifying case where the joint log-income distribution in the beginning and the end of a time period is bivariate normal, *i.e.* the marginal distributions are log-normal. The rest of the paper is divided into three additional sections.

A first section (Section 3) undertakes a comparison of the shape of GICs and NAGICs in the United States using panel data. We initially use the Panel Study of Income Dynamics

(PSID), an income panel that now spans 50 years. The NAGIC tends to be systematically downward sloping, a property commonly found with income panel data (Fields et al., 2003; Jenkins and Van Kerm, 2016; Hammar and Waldenström, 2020; Splinter, 2019). We show that this property weakens when considering narrow age cohorts, thus eliminating life-cycle effects, and even flatter when restricting samples to full-time workers. We also provide a decomposition of the NAGIC into two components corresponding respectively to the effect of rank mobility and that of the change in the cross-sectional income distribution. Rank mobility is then shown to impart a strong downward sloping shape onto the NAGIC. First, pure reranking, for a given shape of the income distribution, necessarily contributes to NAGICs being downward sloping. Second, rank mobility attenuates the effect changes in the distribution shape may have on the shape of NAGICs. The decomposition thus demonstrates that rising mobility may lead to flatter or even negative NAGIC slope. This helps understand why, overall, even in the presence of a strong increase in cross-sectional income inequality the NAGIC may remain downward sloping.

In the next section (Section 5) we first discuss various limitations of panel data for analyzing NAGICs. In particular, small sample sizes do not allow focusing on narrow age groups with enough statistical precision. Another important issue concerns measurement errors. Dealing with GICs and NAGICs make this problem more serious since measurement errors cumulate over the initial and terminal years. As we show, this creates a bias that leads NAGICs to slope downward, as if poorer individuals in the initial distribution moved strongly upward in the income ranking and richer moved downward. To overcome these limitations we show that synthetic panels based on larger cross-sectional datasets and one-parameter functional representations of rank mobility made consistent with panel data evidence allow for a finer analysis of NAGICs and their relationship with GICs. This is shown on US synthetic panels that combine information on rank mobility from PSID with cross-sectional earnings distribution from the Current Population Survey (CPS).[2] We find that when switching to synthetic panels, the fundamental characteristic of NAGICs being downward sloping and then flattening when focusing on narrow age groups and full-time workers, remains valid.

We also reflect on the meaning of a 'downward sloping' NAGIC. Clearly, individual income growth is unlikely to be neither monotonically decreasing nor increasing with initial income rank. Cases of an individual moving upward and the one ranked just above her moving downward, or vice versa, are frequent, so that monotonicity is ruled out. As a matter of fact,

[2]There is a short literature on synthetic panels in developing countries as an attempt at compensating for the lack of true panel data (Dang et al., 2014; Dang and Lanjouw, 2015; Bourguignon and Moreno M., 2018). Somewhat surprisingly, however, only a few of them make use of copulas. A recent exception is Bourguignon and Dang (2019).

pure rank mobility effect in the previous decomposition does not imply monotonically decreasing NAGICs. It leads instead to a property we call 'non-monotonically mean decreasing' – in which the mean growth rate of incomes below some threshold must be greater than the growth rate of incomes above the threshold, whatever the level of the latter. This criterion is close to the 'growth pro-poorness' developed in a recent literature – *e.g.*, Palmisano and Peragine (2015); Lo Bue and Palmisano (2019).[3]

The paper contributes to different threads of the inequality and mobility literatures. First, from a methodological point of view, the paper contributes to the intragenerational mobility literature by its use of synthetic panels, illustrating and explaining possible pitfalls when using panel data. The study of NAGICs also conceptually contributes to the way growth spells are evaluated. This has a clear significance for social welfare, and for a major thread of the inequality literature on how growth spells are analyzed (Palmisano and Peragine, 2015; Piketty, Saez and Zucman, 2018; Lo Bue and Palmisano, 2019). From an empirical perspective, we provide a systematic description of NAGICs in the United States over the last 50 years and their juxtaposition with the respective anonymous GICs. We highlight that NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality, a distributional feature rarely emphasized.

2 Anonymous and Non-Anonymous Growth Incidence Curves: A Primer

We begin by defining anonymous and non-anonymous growth incidence curves and their basic properties.

Consider a two-period $(t; t')$ panel income dataset, with $t < t'$. Let the joint distribution of incomes in the two periods be $f(y_t, y_{t'})$ with marginal cdfs $F_t(p)$ and $F_{t'}(p)$, respectively. Let the conditional distribution of terminal income on initial income be $\Phi_{t'}(. \mid y_t)$. Use will be made below of the copula of that distribution defined by:

$$R_f(p, p') = f\left(F_t^{-1}(p), F_{t'}^{-1}(p')\right), \tag{2.1}$$

where $p, p' \in (0, 1)$ are the unit-normalized income ranks at t and t', respectively, and the notation $^{-1}$ stands for inverse functions. The joint distribution is thus fully described by the copula R_f and the two marginal distributions F_t and $F_{t'}$. With these notations, *Non-*

[3] Pro-poorness criteria were initially proposed by Ravallion (2004); Son and Kakwani (2008) and others.

Anonymous Growth Incidence Curves (NAGICs) are defined as follows:

Definition 1 (Non-Anonymous Growth Incidence Curve)**.** *The Non-Anonymous Growth Incidence Curve (NAGIC), $G_f(p)$, associated with the joint distribution f, shows the income growth rate between times t and t' of people at rank p at time t:*

$$G_f(p) = \frac{\int_0^1 R_f(p, p') F_{t'}^{-1}(p')\, dp' - F_t^{-1}(p)}{F_t^{-1}(p)}. \tag{2.2}$$

In comparison, the standard, anonymous growth incidence curve is defined as

Definition 2 (Anonymous Growth Incidence Curve)**.** *The Anonymous Growth Incidence Curve (GIC), $G_f^a(p)$, associated with the joint distribution f, shows the income growth rate between incomes at rank p in both periods t and t':*

$$G_f^a(p) = \frac{F_{t'}^{-1}(p) - F_t^{-1}(p)}{F_t^{-1}(p)}. \tag{2.3}$$

The NAGIC and the GIC thus differ through the former incorporating personal income mobility, *i.e.* the copula R_f. Indeed, the GIC and the NAGIC coincide when no reranking occurs between the initial and terminal dates (*i.e.* if the copula is the identity copula).[4] The potential differences are illustrated in Figure 1. It presents both curves using the same data from the Panel Study of Income Dynamics (PSID, 2018), for the period 1980–1990, a period of both fast income growth and rise in income inequality.[5]

The slopes of GICs and NAGICs encode relevant information for the interpretation of the distributional impact of growth. They have intuitive social welfare implications. For example, a downward sloping GIC curves means that cross-sectional inequality is decreasing, implying that poor or rich people in the terminal year are less distant from the average income than poor or rich people in the initial year (*i.e.* the Lorenz curve of the terminal year is above that of the initial year). Likewise, downward sloping NAGIC curves show that initially poor people gain relatively more from growth than people initially rich. Yet, they convey different meanings of social welfare, and it is necessary to understand whether these two views are consistent with each other.

To gain some intuition of the relationship between GICs and NAGICs, it is useful to con-

[4] Reranking has been analyzed in depth in the income redistribution literature but mostly in a descriptive way, as one component of the change in the distribution, and therefore of inequality. See, in particular, Lambert (2001) and Urban and Lambert (2008).

[5] In the next sections we will clarify the limitations of using the PSID for producing NAGICs, while the example in Figure 1 is mainly illustrative for the potentially large differences between GICs and NAGICs.

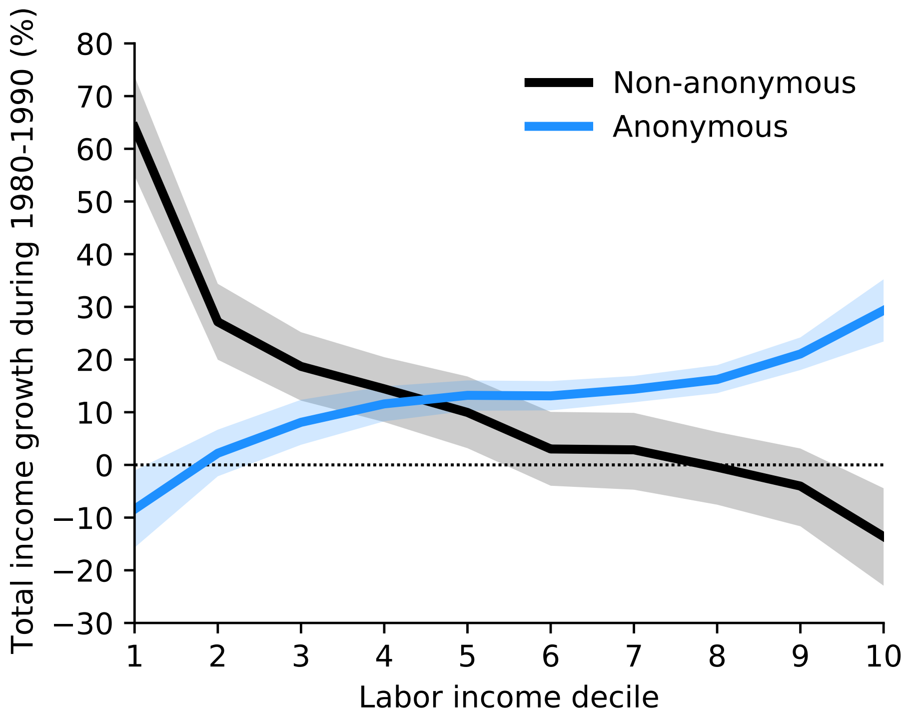

Figure 1: Growth incidence curves for the period 1980–1990 in the United States. The curves are based on individual annual labor income for all 20–55 year-old workers (in 1980) included in the PSID in both 1980 and 1990. The shaded areas represent 95% confidence intervals produced by bootstrapping.

sider the particular case in which the joint distribution of log-incomes is bivariate normal. Assume thus that the marginal distributions are $Log\mathcal{N}\left(\mu_t, \sigma_t^2\right)$ and $Log\mathcal{N}\left(\mu_{t'}, \sigma_{t'}^2\right)$ at t and t', respectively. In this case, for simplicity, we consider the NAGIC under the approximation that the difference in log-incomes is the same as the growth rate (*i.e.* that $g = x_{t+1}/x_t - 1 \approx \log\left(x_{t+1}\right) - \log\left(x_t\right)$). It may be shown that the NAGIC is given by (see Appendix A for the proof):

$$G_f\left(p\right) = \left(\mu_{t'} - \mu_t\right) + \left(\rho\sigma_{t'} - \sigma_t\right)\Phi^{-1}\left(p\right) \ , \tag{2.4}$$

where μ_t is the mean log-income at time t, σ_t the standard deviation, ρ the coefficient of correlation of log-incomes in the two periods, and Φ^{-1} is the inverse standard normal cumulative distribution function. Here too, the fact that the NAGIC combines mobility and the difference in marginal distribution is readily apparent.

This has implications for the slope of the NAGIC, the sign of which is given by:

$$\operatorname{sgn}\left(\frac{dG_f}{dp}\right) = \operatorname{sgn}\left(\rho\sigma_{t'} - \sigma_t\right) . \tag{2.5}$$

Therefore, if the NAGIC is upward sloping then it must be the case that $\rho\sigma_{t'} - \sigma_t > 0$, implying $\sigma_{t'} > \sigma_t$, since $\rho \leq 1$. This means that an upward sloping NAGIC is necessarily associated with increasing inequality. Yet, the reciprocal does not hold: $\sigma_{t'} > \sigma_t$ does not imply that the NAGIC is upward sloping since the inequality may be inverted if ρ is small enough, *i.e.* there is enough mobility. Conversely, $\sigma_{t'} \leq \sigma_t$ unambiguously implies a downward sloping NAGIC, so if inequality decreases, the NAGIC is necessarily downward sloping. We note that the anonymous GIC corresponds to Eq. (2.4) with $\rho = 1$, for which the slope is positive if $\sigma_{t'} > \sigma_t$, *i.e.* if inequality increases cross-sectionally, and negative otherwise.

The relationship between the slope of the anonymous GIC (or the change in inequality) and the slope of the NAGIC may thus be summarized as follows:

- GIC $\searrow \Rightarrow$ NAGIC $\searrow$ – if the GIC is downward sloping so is the corresponding NAGIC.
- NAGIC $\nearrow \Rightarrow$ GIC $\nearrow$ – if the NAGIC is upward sloping so is the corresponding GIC.
- GIC $\nearrow \not\Rightarrow$ NAGIC $\nearrow$ – if the GIC is upward sloping the corresponding NAGIC is not necessarily upward sloping.

This logic holds in a more general sense and not only under the log-normal approximation, although not in all cases. This is discussed in greater detail in Section 3.1 and in Section 6.

3 Anonymous and Non-Anonymous in the United States using Panel Data

Following the acquired understanding of GICs and NAGICs, we proceed to estimate them for the United States in practice. We focus on the differences between GICs and NAGICs and the role of mobility. We are particularly interested to find to what extent GICs are informative of the individual experience of labor income growth, *i.e.* NAGICs. This is important for the interpretation of changes in inequality over a time period. Clearly, income has become more unequal since the 1970s, which has a profound importance in and of itself. Yet, as discussed, it also matters how growth was experienced by individuals at different levels of the income distribution, especially in periods of growing inequality.

To study these GICs and NAGICs we first use the Panel Study of Income Dynamics (PSID, 2018), or PSID. This is a longitudinal panel survey of families conducted annually or biennially from 1967 to 2017 in the United States. We use the labor income of family heads and their spouses in the survey. The sample sizes differ in each wave due to methodology changes in the survey and the survey tracking the descendants of past surveyed individuals. 4,500–10,000 families were surveyed in each wave. Individual labor income is available from 1967 onward; it is adjusted for inflation using the national income price index. The focus on labor income is motivated by its lesser sensitivity to measurement errors in household surveys compared to capital income. Also, note that the analysis bears on pre-tax income. This avoids the present analysis to combine the issue of income mobility and that of the distributional incidence of reforms in the tax-benefit system. In any case, individual labor income in PSID data is only pre-tax.

Estimating the intragenerational copulas using PSID data has clear limitations. Specifically, the small sample sizes, the limited coverage of the top of the distribution and the measurement errors may all lead to an overestimation of relative mobility. We try to correct for this bias through methods discussed in detail below.

Figure 2 presents GICs and NAGICs for the period 2006–2016. It compares GICs and NAGICs considering all workers (aged 20–55 in the initial year) and when limiting the sample to full-time workers who were 35–40 years-old in 2006 only. The reason to include only full-time workers is to exclude the mechanical effect of moving from part-time to full-time work, and vice versa. In addition, including only 35–40 years-old workers eliminates life-cycle effects. The figure also shows the effect of averaging incomes over several years, a method used to attenuate the impact of measurement errors, which is also discussed in detail in the next section.

Figure 2 illustrates a typical feature in the PSID data. NAGICs are generally downward sloping, while GICs are generally upward sloping. This remains so in periods of increase in inequality. Even after partially controlling for age and when looking only at full-time workers, NAGICs still have a very different shape from GICs. This is rather surprising, especially given the rise in inequality in all GICs.

3.1 A decomposition of NAGICs

The difference between the GICs and NAGICs is that the former describe the change in the shape of the marginal income distribution at two points of time whereas the latter adds to it the mobility of people from one position to another in the distribution. We can formally

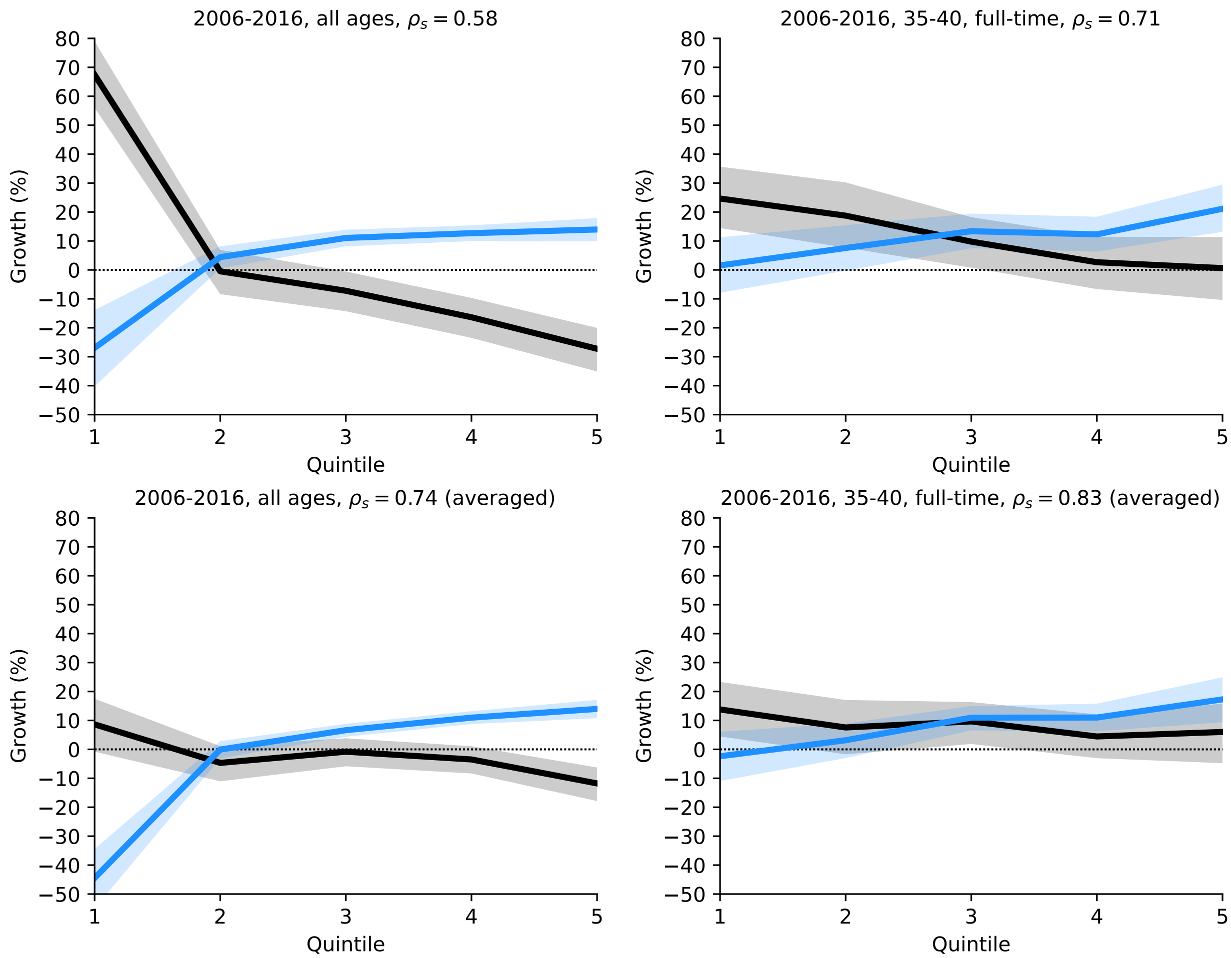

Figure 2: Anonymous (blue) and Non-anonymous (black) growth incidence curves 2006–2016. We consider all workers (left) and full-time workers aged 35–40 in 2006 (right). We also consider the curves without (top) and with (bottom) averaging over several years of data. Averaging is used to attenuate measurement error (see next section). The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. ρ_s is the estimated rank correlation in each case.

decompose NAGICs into these 'distribution' and 'mobility' components to further study what drives the different shapes of NAGICs and GICs.

In the simplified case, where the joint distribution of log-incomes is bivariate normal, the approximated NAGIC (Eq. (2.4)) can be rewritten as

$$G_f(p) = \underbrace{(\mu_{t'} - \mu_t)}_{\text{growth}} + \underbrace{\sigma_t(\rho - 1)\,\Phi^{-1}(p)}_{\text{mobility}} + \underbrace{\rho(\sigma_{t'} - \sigma_t)\,\Phi^{-1}(p)}_{\text{shape}}, \tag{3.1}$$

where the NAGIC is decomposed into a growth term, a pure mobility term and a distribution term. If there is no mobility, or $\rho = 1$, the mobility term vanishes. If the distribution does not change between t and t' then $\sigma_{t'} = \sigma_t$ and the shape term vanishes. It also follows immediately that the mobility term is monotonically decreasing in the rank p, since $\rho < 1$ (and $\Phi^{-1}(p)$ is increasing in p). Similarly, the shape term is increasing in p if inequality increases, and decreasing in p if it decreases.

A generalization, which follows the same intuition, can be directly derived from the NAGIC definition Eq. (2.2):

$$G_f(p) = \frac{\int_0^1 R(p,p') \left[F_{t'}^{-1}(p') - \widetilde{F}_t^{-1}(p')\right] dp'}{F_t^{-1}(p)} + \frac{\int_0^1 R(p,p') \widetilde{F}_t^{-1}(p')\, dp' - F_t^{-1}(p)}{F_t^{-1}(p)}, \quad (3.2)$$

where $\widetilde{F}_t$ is the cdf of the initial distribution after scaling it up, so that its mean is the same as in the final distribution, *i.e.* $\widetilde{F}_t^{-1}(p) = (1+g) F_t^{-1}(p)$, where g is the growth rate of the mean income between t and t'.

To simplify, denote $y_t(p)$ the quantile $F_t^{-1}(p)$ and

$$y^R(p;x) = \int_0^1 R(p,p')\, x(p')\, dp' \quad (3.3)$$

the expected final income of the individuals initially at quantile p, when the marginal distribution of final income is the quantile function $x(p)$. The preceding decomposition may be now written as

$$\begin{aligned} G_f(p) &= \frac{y^R(p;y_{t'}) - (1+g)\, y^R(p;y_t)}{y_t(p)} + (1+g) \frac{y^R(p;y_t) - y_t(p)}{y_t(p)} + g & (3.4)\\ &= D(p) + M(p) + g\,. & (3.5) \end{aligned}$$

The NAGIC is thus decomposed into three terms. Starting from the end:

- The mean income growth rate, g.
- A pure mobility (or reranking) effect, $M(p)$ – the hypothetical NAGIC corresponding to no change in the marginal distribution of income, the final distribution being the same as the initial one.
- A 'distribution' (or shape) effect, $D(p)$, due to the change in the marginal distributions. This corresponds to the difference in expected final income of people initially at rank p, when the final distribution is the terminal one and the initial one is scaled up by $(1+g)$, *i.e.* as it would result from uniform growth.

The decomposition is demonstrated in Figure 3 for full-time workers who were 35 years old in the initial year in three 10-year periods: 1968–1978, 1980–1990 and 2006–2016, based on CPS data (see Section 5 for details on the estimation of these NAGICs). It shows that in all cases the contribution of the mobility term to the slope of the NAGIC is strongly negative, although uniformly so. This is as expected: mobility only entails that bottom ranks gain and top ranks lose, but says nothing about the sign of income change in between, which moreover depends on the initial marginal income distribution. The shape term has a flat U-shape in 1968–1978 and 2006–2016 and is strongly upward sloping in 1980–1990. These shapes differ from the GIC in Figure 2 for 2006–2016, and they are close to being flat rather than moderately upward sloping. The difference with the GIC is that the shape component of the NAGIC is defined not on the difference between observed income at some rank p in the initial and terminal year. Instead, it is the hypothetical difference in the *expected* income of people initially at rank p when the terminal distribution is the observed one and when it would be the same as the initial distribution after scaling it up to the terminal income mean. Such expectation operation necessarily reduces income changes in comparison with the GIC.

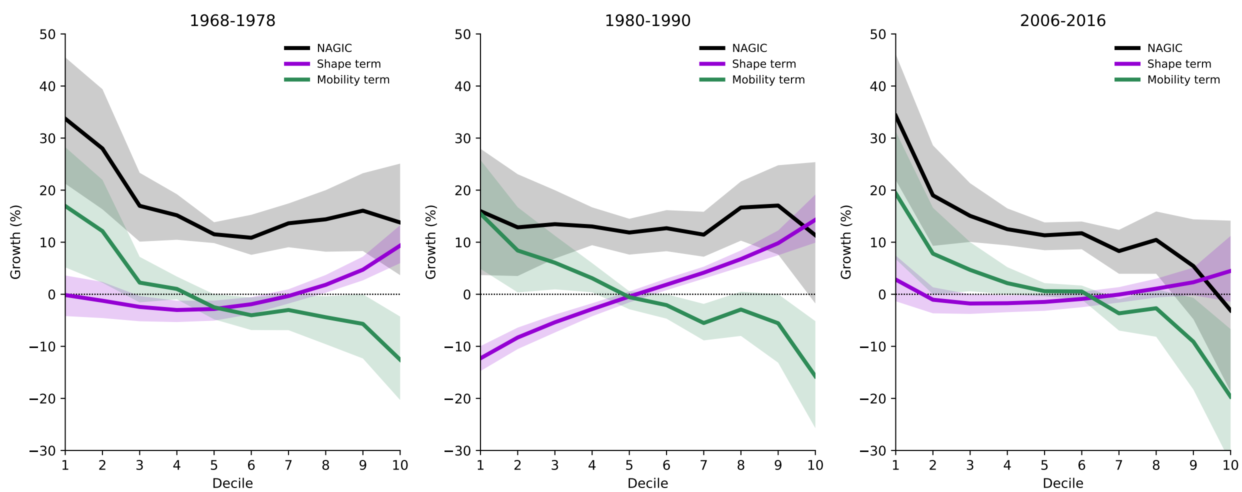

Figure 3: The shape term (magenta) and the mobility term (green) for CPS NAGICs (black), based on the NAGIC decomposition (Eq. (3.5)). The terms are based on synthetic NAGICs combining CPS data with PSID-based rank correlations using full-time workers aged 35 in the initial year in each period. The shaded gray areas represent 95% confidence intervals for the estimates produced by bootstrapping (see Section 5 and Figure 7).

The shape component of the NAGIC in 1980–1990 is much steeper than in the other periods because of the surge of cross-sectional inequality during this period, but it would be less steep than the GIC (see Section 5). Overall, it turns out that the shape term contributes very little to the shape of the NAGIC, compared to the mobility term in 1968–1978 and in

2006–2016, whereas it plays a bigger role during 1980–1990. The confidence bounds in the mobility term are also much wider than in the shape term. Note that the two terms do not sum up to the NAGIC, since they exclude the average income growth.

The main observation from Figure 3 is that as long as the change inequality is not extreme as in periods such as 1980–1990, the mobility term seems to be the dominant contributor to the NAGIC shape. This clarifies the differences obtained between GICs and NAGICs in Figure 2 and Figure 7 below.

This also clarifies the interplay between mobility and inequality described in Section 2. A downward sloping GIC necessarily corresponds to a downward sloping NAGIC. An upward sloping NAGIC necessarily corresponds to an upward sloping GIC. Yet, an upward sloping GIC does not necessarily imply an upward sloping NAGIC, and this would depend on the balance between the shape term and the mobility term.

4 NAGICs and Measurement Error

The typical negative slope of NAGICs found above may be the outcome of measurement errors. Income measurement necessarily involves some measurement errors and when a panel is considered this may have a crucial impact on NAGICs. In the intergenerational context, measurement error creates attenuation bias, *i.e.* estimated copulas represent mobility that is higher than real mobility. Since income measurement is noisy, income persistence estimates are attenuated (Bound and Kruger, 1991; Solon, 1992; Bound et al., 1994; Fields et al., 2003).

In the intragenerational context, *i.e.* for NAGICs, measurement errors mechanically attach higher growth rates to individuals falsely identified as occupying lower income ranks at the beginning of time periods, and vice versa for individuals falsely identified as belonging to higher ranks. This may have a large impact on the mobility term (less so on the shape term), and thus, to NAGICs whose slopes are biased downward. Appendix B shows how a systematic bias on reported income and some autocorrelation of measurement error lead to a negative bias in the estimation of the covariance between the observed growth rate and the observed initial income. Sampling error is also important, and may lead to inaccurate estimates of the shape term. In the case of panel data, such as the PSID, this may be due to significant attrition over time, and a plethora other possible biases that are typical to households surveys (Blanchet, Flores and Morgan, 2018; Yonzan et al., 2019).

Dealing with measurement error is therefore essential. In the intergenerational context it is many times done by averaging over several years of data (Solon, 1992; Chetty et al., 2014).

Another option is based on an error model presented by Fields et al. (2003), assuming that the measurement error is negatively correlated with true incomes, *i.e.* high incomes tend to under-report and the low incomes to over-report, that the ratio of the variance of measurement error to true variance in true incomes is known, but ignores serial autocorrelation of the measurement error over periods of several years. This model is based on the observation of measurement errors obtained by comparing survey to social security data (Duncan and Hill, 1985). In Fields et al. (2003), it predicts the underestimation of rank correlations based on surveys by 5%–20%. The derivation of a correction factor to estimate rank correlation based on this model is given in Appendix C. As will be explained in the next section, our use of synthetic panels, for which the copula can be corrected to take measurement error into account, will help overcoming this crucial issue and to obtain more reliable NAGICs.

The bottom panels in Figure 2 present GICs and NAGICs for the same time periods as the top panels, but with averaged income data and without the preceding correction of rank mobility since the full survey, rather than a rank correlation coefficient, is being used. Averaging was done so that each sample consisted of individual log-incomes at a given year averaged with the same individual log-incomes one wave before and after the given year. This naturally leads to higher rank correlations, due to lower impact of measurement errors and transitory shocks. In most cases the GICs and NAGICs still qualitatively resemble the results without averaging. Yet, shapes do change in some cases, and these more realistic NAGICs have slopes that are higher than in the top panels of Figure 2 (*i.e.* lower in absolute value since slopes generally are negative).

4.1 Measurement errors and synthetic panels

The measurement errors described above may be important even for the NAGICs presented in the bottom panels of Figure 2. They may still generate an unreliable picture. Synthetic NAGICs not relying on the surveyed copula but only on some adjusted estimate of the rank correlation permit to avoid these sources of irregularities in comparisons across time periods.

As seen above in Eq. (2.2), a NAGIC is defined by:

$$G_f(p) = \frac{\int_0^1 R(p, p')\, F_{t'}^{-1}(p')\, dp' - F_t^{-1}(p)}{F_t^{-1}(p)}. \tag{4.1}$$

This expression shows that the panel dimensions of a NAGIC essentially consists of the copula $(R(p, p'))$ of the joint distribution of initial and terminal incomes, as long as the marginal income distributions (F_t and $F_{t'}$) are known. Thus, it is practically possible to construct NAGICs via synthetic panels where cross-sectional income distributions are taken from one source, and the copula is taken from another source, or modeled.[6]

This method has clear advantages. It allows using richer data sets that lack panel components, such as the Current Population Survey (CPS, 2018), for the marginal distributions. This improves the inaccuracy of NAGICs due to sampling errors. It also allows to control for empirical uncertainties in NAGICs. In particular, using synthetic panels allows overcoming the limitations and biases in the panel-based copulas discussed above.

Synthetic NAGICs require using copulas consistent with the information delivered by panel data but less sensitive to the specific draw of measurement errors in observed copulas. Parametric functional specifications are most useful from that point of view. Adjusting their parameters so as to fit their empirical counterpart, they allow to consider variations of parameters around these values that are representative of the effect of measurement errors and to easily generate the corresponding variations in the whole copula. As already noted by Bonhomme and Robin (2009) for France, the Plackett copula (Plackett, 1965) has been found to work rather well on US data from PSID, as demonstrated in Figure 4.

Plackett copulas are especially convenient since they are defined using a single parameter that uniquely corresponds to the rank correlation. It is then simple to select a (Spearman's) rank correlation ρ_s and construct a Plackett copula accordingly.

To demonstrate that synthetic panels can be used to create NAGICs similar to standard panels (while enjoying the advantages mentioned above), we compare NAGICs estimated via PSID for the United States to NAGICs that were reproduced using modeled Plackett copulas. This is presented in Figure 5. For each period, we use the marginal distributions from the panel and create a joint income distribution by fitting them to a Plackett copula with its parameter consistent with the panel rank correlation. This produces NAGICs that are within the statistical uncertainty range of those estimated directly from the panel.

5 Synthetic Panels: Growth Incidence Curves in the United States, 1968–2016

Based on the above we can turn to our main goal – describing anonymous and non-anonymous growth incidence curves in the United States from 1968 onwards. We will use marginal income data from the current population survey (CPS) with synthetic panels as described.

[6]For similar uses of synthetic panels created by combining marginal distribution and copulas separately see Bonhomme and Robin (2009); Chetty et al. (2017); Berman (2019b,a).

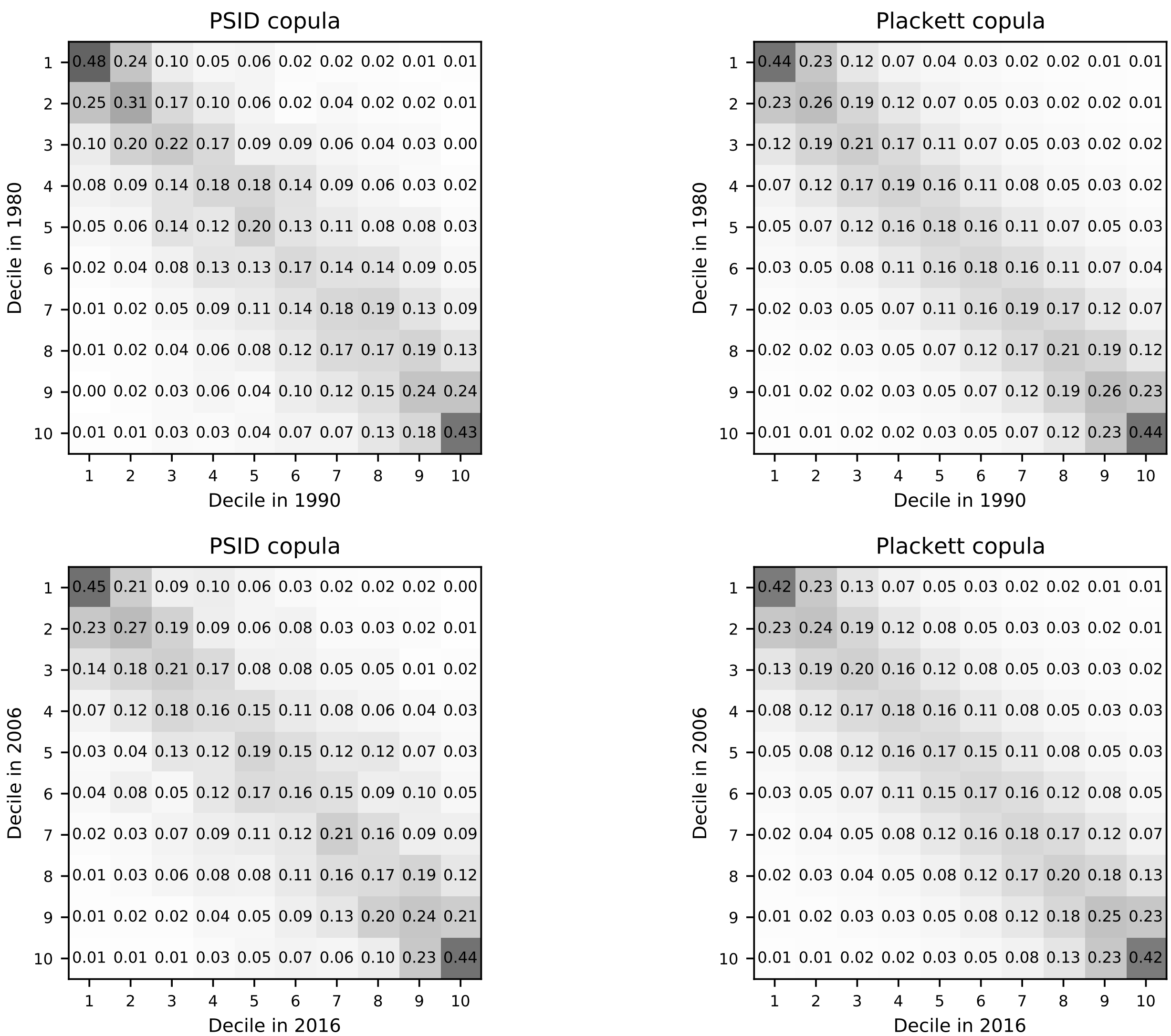

Figure 4: Observed (left) and fitted Plackett (right) labor income copulas for the United States based on PSID data. The periods considered were 1980–1990 (top) and 2006–2016 (bottom).

To prepare for the use of such synthetic NAGICs, it is necessary to provide adequate estimates of 10-year rank correlations. Year-by-year PSID-based estimates are shown in Figure 6.

The rank correlation estimates in Figure 6 raise several observations, which are important when constructing synthetic NAGICs:

- There is no clear trend in rank correlations over time. In all specifications they seem to fluctuate over time within relatively narrow bands: either 0.6–0.8 for the 35–40 year-olds without averaging and 0.75–0.85 after averaging, or 0.5–0.6 and 0.65–0.75 for the entire population of workers, before and after averaging, respectively (the same

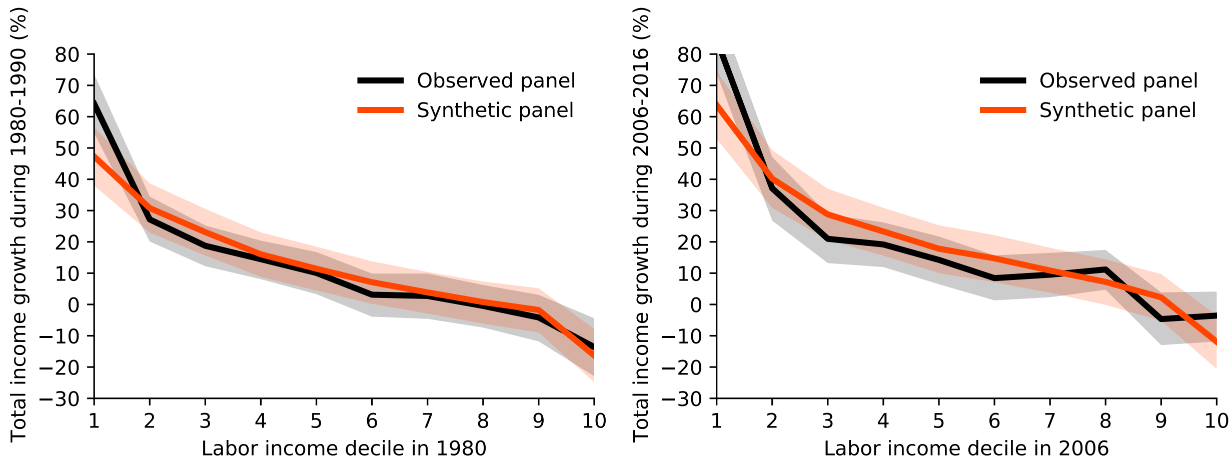

Figure 5: Non-anonymous growth incidence curves for the period 1980–1990 (left) and 2006–2016 (right) in the United States. The curves are based on individual annual labor income for all 20–55 year-old workers (in the earlier year of each period) included in the PSID in both 1980–1990 (left) and both 2006–2016 (right) periods. The black curve is the observed NAGIC in the panel data. The red curve is estimated as a synthetic panel assuming a Plackett copula. The shaded areas represent 95% confidence intervals produced by bootstrapping.

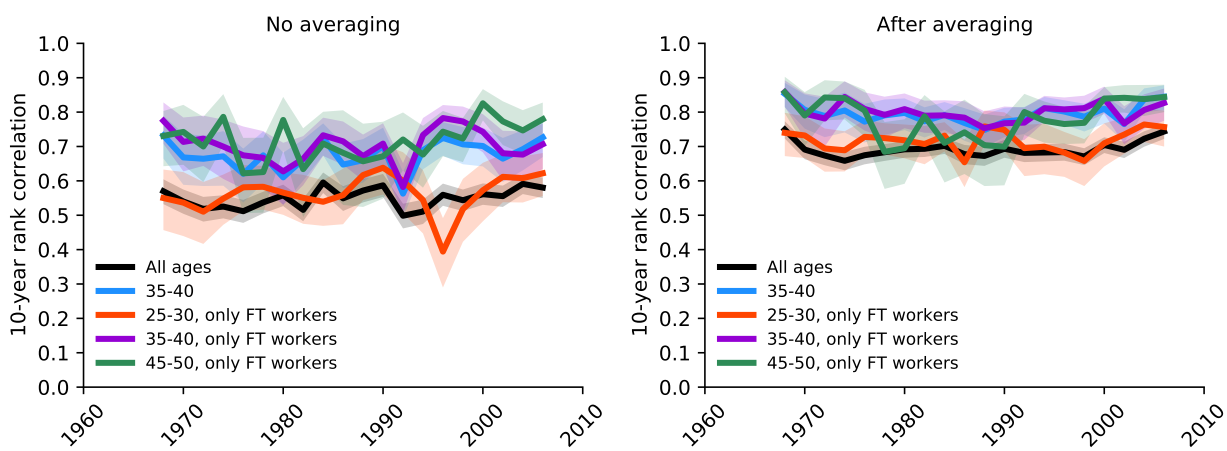

Figure 6: 10-year rank correlations in the United States based on PSID data from 1968 onward. The value on the x-axis specifies the initial year of each 10-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. Several groups are considered: all ages, full-time workers + non-full-time workers; 35–40 year-olds, full-time workers + non-full-time workers; 35–40 year-olds, only full-time workers; 25–30 year-olds, only full-time workers; 45–50 year-olds, only full-time workers. We present estimates before (left) and after (right) income averaging (done as explained above).

stability is observed for a shorter time span as well, see Appendix D).

- Statistical uncertainty is roughly ±5 percentage points for the 35–40 age group, and

±2 percentage points for the entire population of workers.

- Limiting the sample to full-time workers has an insignificant effect on the rank correlation.
- Limiting the sample to 35–40 year-olds significantly increases the rank correlation when compared to the entire population of workers, by 3 to 10 percentage points.
- The 35–40 and 45–50 age groups are quite similar, whereas the younger age group of 25–30 year-olds shows higher rank mobility.

The above demonstrates the relevance of using synthetic panels. Especially the fact that there is no clear trend in rank correlations and that the confidence intervals are relatively narrow and do not become wide or narrow over time. That there is no trend in labor income rank mobility echoes the findings of Kopczuk, Saez and Song (2010) using social security data.

5.1 Non-anonymous growth incidence curves in the Current Population Survey

We now use data from the Current Population Survey (CPS) to estimate GICs and NAGICs in the United States. We use the IPUMS-CPS microdata available from 1968 onward, sampling about 60000 households in every wave. We also use additional relevant covariates from both surveys such as age, number of hours worked, *etc.* . The CPS data are then used for obtaining marginal (cross-sectional) labor income distributions.

Using the error model proposed by Fields et al. (2003) and detailed in Appendix C, we also estimate corrected rank correlations in the non-averaged income case, as measurement errors are presumably fully taken into account by the correction procedure. Based on the error measurement parameters reported in Bound and Kruger (1991); Bound et al. (1994); Fields et al. (2003) the relevant correction factors turn out to be in the 1.05–1.2 range. Thus, on the basis of Figure 6, the true 10-year rank correlation values for 35 year-olds, for example, are most likely within the range 0.7–0.9. This range can be further justified based on additional considerations. On the one hand, the 10-year rank correlations for 35 year-olds after income averaging, and thus partly correcting for measurement errors, are within the range 0.75–0.85, so it is unlikely that the true rank correlations are lower than 0.7 even after taking remaining measurement errors into account. On the other hand, the year-to-year earnings rank correlation for all workers estimated in (Kopczuk, Saez and Song, 2010) is found to be stable over time and close to 0.9. It is therefore unlikely that for a period of 10 years the rank correlation would be higher than 0.9, even when restricted to small age groups.

In practice, for each period being analyzed, we use bootstrapping to obtain a distribution of plausible rank correlation values using the PSID. This distribution is then combined with bootstrapped marginal distributions to obtain a distribution of GICs and NAGICs. The resulting NAGICs are presented in Figure 7, for full-time workers in three different 10-year periods and for different 1-year age groups, that is with much more precision than what PSID sample size allowed for (in practice, looking at the age group 35–40 as before leads to similar results to the 1-year 35 year-olds case).

Figure 7 shows that the differences found in Figure 6 between the shapes of GICs and NAGICs in the PSID, are robust to using large CPS samples for cross-sectionsl distributions, to variations in the rank mobility and to narrow age groups. Most notably, the NAGICs are mostly neither significantly upward or downward sloping.

Although no simple statistical test is available to ground the preceding statement, a heuristic argument based on properties of the log-normal approximation discussed in at Section 2 can be used. Since the slope of NAGICs approximately depends on the sign of $\rho\sigma_{t'} - \sigma_t$, $\rho_s^{crit} \approx \frac{\sigma_t}{\sigma_{t'}}$ is the critical value of ρ such that a NAGIC is upward (downward) sloping above (below) it.[7]

Figure 8 exhibits ρ_s^{crit} based on the CPS data. It shows that ρ_s^{crit} is usually between 0.8 and 1. As discussed, the range 0.7–0.9 is a realistic plausible range of values for the rank correlation. The direct implication is that whether the slope of NAGICs for 35 year-old full-time workers over 10 years is upward or downward sloping, is case-specific. Yet, ρ_s^{crit} is in many cases above the 0.7–0.9 range or in the upper part of it, suggesting a downward sloping NAGIC. Taking into account the uncertainty about the true rank correlation, NAGICs are approximately flat, but essentially never upward sloping. The results are similar for older workers. However, for younger workers ρ_s^{crit} is lower. Hence, higher mobility is required to counteract the effect of increasing inequality on the NAGICs' slopes.

[7]It is an approximation since in practice the bivariate log-normal distribution is only an approximation. We also note the difference between ρ, the correlation between log-incomes, and ρ_s, the rank correlation, which are usually very similar but not identical.

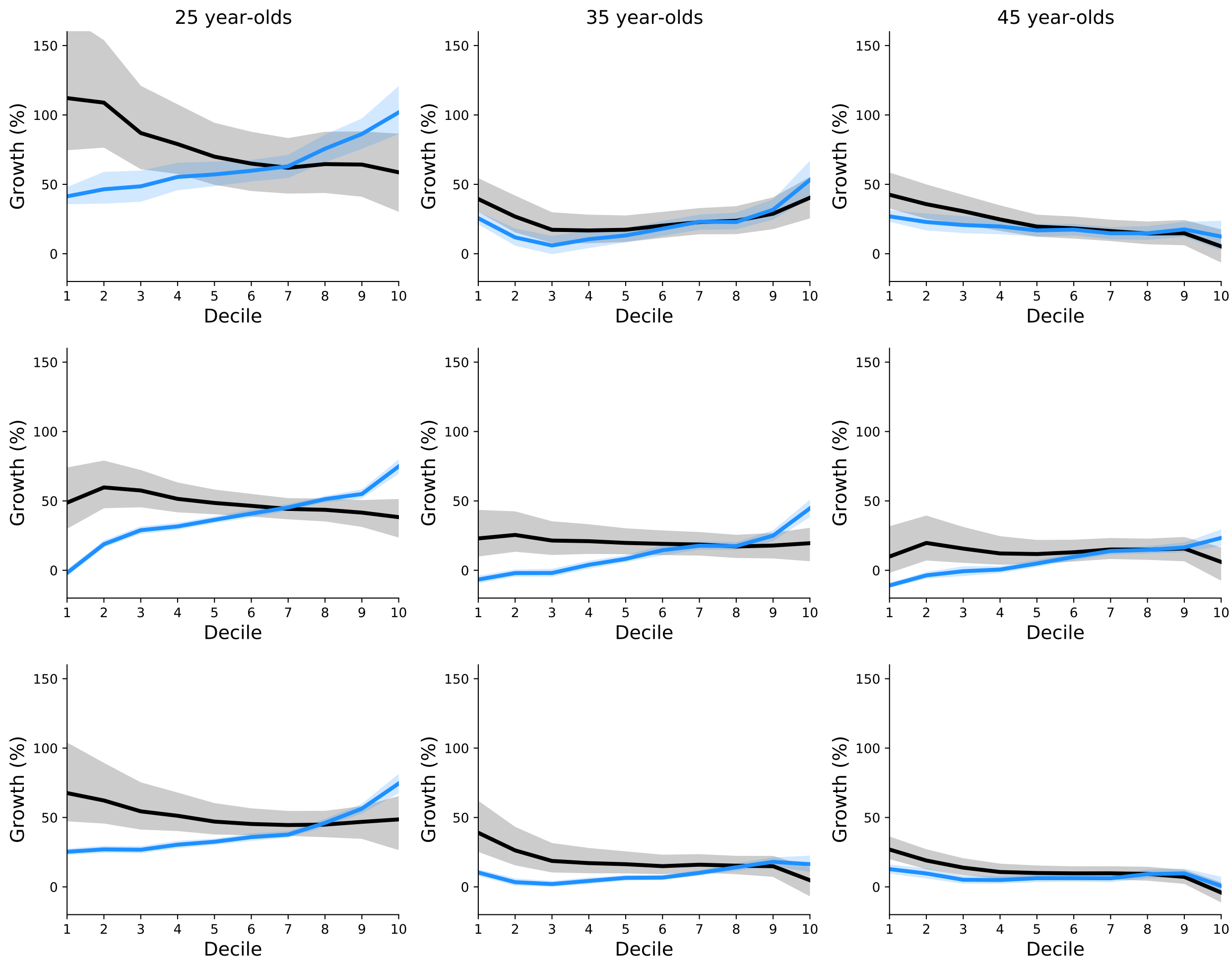

Figure 7: Anonymous (blue) and Non-anonymous (black) growth incidence curves based on CPS labor income data for 1968–1978 (top), 1980–1990 (middle), and 2006–2016 (bottom). For each period we workers who were 25 years-old in the initial year (left); 35 years-old (middle); 45 years-old (right). Only workers who were full-time employed in both initial and final years were considered. The shaded gray areas represent 95% confidence intervals for the estimates produced by bootstrapping – we create 10,000 NAGICs, each time randomly leaving out 20% of the sample and randomly drawing the rank correlation from its empirical distribution (see Figure 6).

6 A Rigorous Characterization of the NAGIC Slope

The characterization of NAGICs as downward sloping is ambiguous. First, NAGICs are generally non-monotonic when considering income ranks that are narrow enough, as both positive and negative growth of individual incomes are usually observed. The downward sloping property should thus be understood in a rough way as meaning that the curve is on average higher for low initial income ranks than for high ranks. This is because mobility is most likely to be upward at the bottom of the initial income scale and downward at the top.

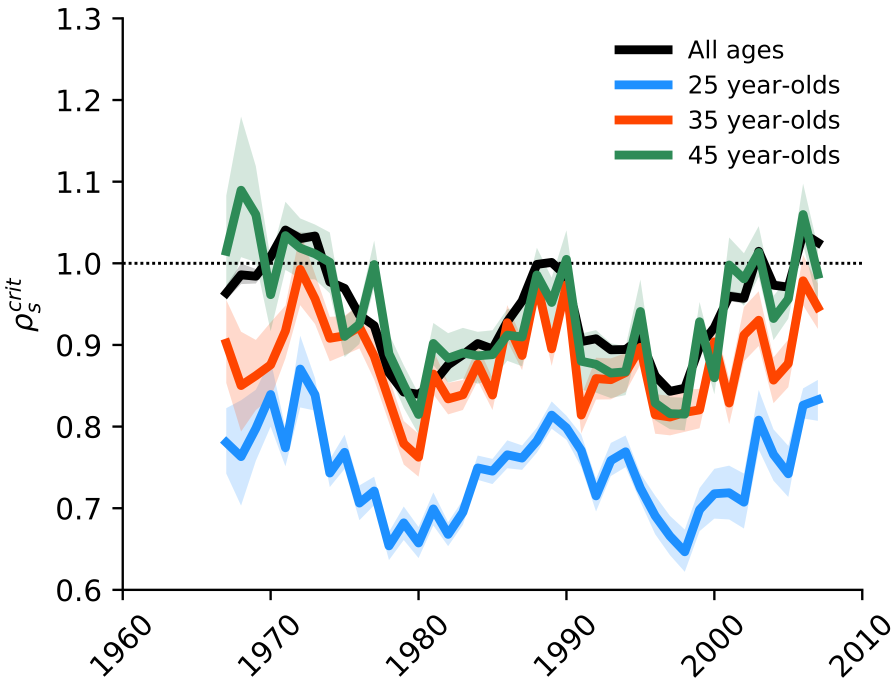

Figure 8: ρ_s^{crit} for 10-year rank correlations in the United States based on CPS data from 1967 onward. The value on the x-axis specifies the initial year of each 10-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. The groups considered were full-time workers in all ages (black); 25 year-olds (blue); 35 real-olds (red); 45 year-olds (green).

This section investigates a more rigorous characterization of this 'rough downward sloping property' and applies it to the shape and mobility components of the NAGIC, as defined in the preceding section. Doing so permits generalizing properties initially derived under the assumption of a log-normal bivariate distribution of income (see Section 2). While NAGICs are quite intuitive, we cannot make welfare statements out of them and it is necessary to go beyond the property downward or upward sloping curve. For that purpose we would need to define more properties. Yet, the results that we would obtain are in agreement with what saw before about NAGICs, and inline with the intuition developed above.

In what follows, the downward sloping property of the curve describing a function is approximated by the following property:

Definition 3 (Non-Monotonically Mean Decreasing (NMMD) Function)**.** *A function* φ *on the interval* $[0, 1]$ *is said to be non-monotonically mean decreasing if*

$$\frac{1}{p}\int_0^p \varphi(v)\, dv \geq \frac{1}{1-p}\int_p^1 \varphi(v)\, dv \;\; \forall x \in [0,1] \tag{6.1}$$

This property generalizes the 'rough' downward sloping property requiring simply that a function be higher at the bottom than the top of its interval of definition. It states that this should be true of the mean of the function for all partitions of the interval into a lower and an upper sub-interval. Thus, the incomplete mean of the function from below must not be lower than its incomplete mean from above, for all bipartitions of the $[0, 1]$ interval. Note that this definition differs from the practice of averaging a function on successive disjoint intervals – as with deciles in the preceding sections. It also differs from considering moving averages on overlapping intervals.

The preceding definition bears a relationship with the generalized pro-poorness criterion developed by Palmisano and Peragine (2015) and further analyzed by Lo Bue and Palmisano (2019). Thus the characterization of the shape of the NAGIC through the NMMD property is instilling a social welfare in the analysis.

Using the definition above, we first focus on the mobility term of the decomposition of the NAGIC, which corresponds to the hypothetical case where the final and initial income distribution were the same.

Proposition 1. *The pure mobility component of the NAGIC ($M(p)$) is NMMD when mobility is defined on the difference of income logarithms. It is not necessarily so if mobility is defined in growth rates, but it is the case that the incomplete mean of $M(p)$ is non-negative over* $[0, 1]$.

Proof. See Appendix A.

□

Proposition 1 states that for the mobility component of the NAGIC, $M(p)$, to be NMMD, income mobility must be expressed in log differences rather than growth rates, as it is usually the case. Such a substitution is justified if mobility is not too high so that growth rates are low enough. In other words, the probability of large differences between initial and terminal ranks in $R(p, q)$, or outside the area along the main diagonal, if the copula is represented by a matrix, must be limited. In practice, this requirement is satisfied due to the typical shape of intragenerational copulas, as discussed above. This will also be demonstrated below.

We now consider possible properties of the shape of the 'distribution' component, $D(p)$, of the NAGIC. To do so requires assuming some regularity property of copulas, namely that the probability density of reaching a low final income rank is higher than that of reaching a high rank if the initial rank is low and vice-versa. This property is formalized as follows:

Definition 4 (Regular Copula)**.** *A copula $R(q,p)$ is regular if for all $i,j \in [0,1]$ such that $i < j$, there exists $p^* \in (0,1)$ such that $R(q,i) \geq R(q,j)$ if $p \leq p^*$ and vice-versa.*

It can be seen that such a property is equivalent to assuming that $R(i,p) - R(i,q)$ is a decreasing function of i for all pairs $p < q$. A particular case would be for instance the copula being a decreasing function of the distance between the initial and terminal rank, *i.e.* $R(p,q) = g(\mid p - q \mid)$. Restricting the analysis to such regular copulas, the following property then holds.

Proposition 2. *If the final marginal distribution of income is unambiguously less unequal than (i.e. Lorenz-dominates) the initial marginal distribution, and if the copula is regular, then the incomplete mean of the shape term is everywhere positive on* $(0,1)$.

Proof. See Appendix A.

□

This proposition may be seen of mild interest because it does not say much about the shape of the distribution component of the NAGIC, except that it is initially positive and can never be negative over such an interval and to an extent that would make its incomplete mean negative. Yet, it turns out that a function closely associated to the incomplete mean of $D(p)$ is NMMD.

Consider the function:

$$\overline{D}^{RM}(p) = \frac{\overline{y}^R(p;y_{t'}) - (1+g)\overline{y}^R(p;y_t)}{\overline{y}_t(p)} = \frac{\overline{D}^A(p)}{\overline{y}_t(p)}, \tag{6.2}$$

where $\overline{y}^R(p;x)$ is the incomplete mean, up to p, of $y^R(p;x)$ defined above. It is a kind of dual of $\overline{D}(p)$, the incomplete mean of the $D(p)$ shape term of the NAGIC. $\overline{D}^{RM}(p)$ is the ratio of the incomplete mean difference of expected incomes to the incomplete mean initial income, whereas $\overline{D}(p)$ is the incomplete mean of these ratios, *i.e.* the ratio of the means vs. the mean of the ratios. $\overline{D}(p)$ and $\overline{D}^{RM}(p)$ are not identical, but given their definition, they similarly depend on p. In particular, being everywhere positive and being 0 at $p = 1$, $\overline{D}^{RM}(p)$ is NMMD.

6.1 Implications for the shape of the NAGIC

Going back to the shape of the NAGIC, we may now generalize the properties found in the bivariate log-normal case in Section 2:

- Lorenz improvement of marginal distribution ⇒ Downward sloping GIC ⇒ Incomplete mean of $D(p)$ positive and approximately NMMD ⇒ Same property for NAGIC
- Lorenz worsening of marginal distribution ⇒ Upward sloping GIC ⇒ Incomplete mean of $D(p)$ non-positive ⇒ No particular shape of the NAGIC
- Incomplete NAGIC mean with some negative value or non NMMD ⇒ non-downward sloping GIC (under some restrictions, as discussed)

The fact that the incomplete mean of the mobility term in the decomposition is non-negative and approximately NMMD explains the final implication for the shape of the NAGIC.

We also note that the influence of the shape of the GIC on the shape of the NAGIC is filtered by the mobility characteristics of the panel income data. A high mobility is generating both a more strongly NMMD $M(p)$ curve, but also less impact of the change in marginal distributions represented by the GIC on the NAGIC. For this reason NAGIC often look downward sloping with an incomplete mean curve that is NMMD.

6.2 Application to the synthetic panel in the United States

We now use the CPS data to test whether NAGICs and their corresponding $M(p)$ (mobility) components are NMMD. To show this we consider the expressions $\frac{1}{p}\int_0^p \varphi(v)\,dv$ and $\frac{1}{1-p}\int_p^1 \varphi(v)\,dv$ (φ being either the NAGIC or its mobility component). For the curves to be NMMD, we require the former expression to be above the latter for all p. This means that when dividing the distribution into two, from quantile 0 to quantile p and from p to 1, the poorer part (*i.e.* from 0 to p) enjoys higher average growth, for any choice of $p \in [0,1]$. The results are presented in Figure 9.

Figure 9 shows that indeed, the mobility component is always NMMD, as expected. It is always positive and always above the counterpart expressions in the NMMD definition. At the same time, as already observed, NAGICs are not necessarily NMMD. This property is important since the shape of NAGICs, *i.e.* their slope, is not informative from a welfare point of view and, as a matter of fact, is most likely to vary a lot in the $[0,1]$ range. Although its true welfare signification is to be investigated in more depth, incomplete means show more regularity and are easily interpreted as a generalized pro-poorness criteria. A NAGIC that is NMMD is such that poor people's average growth rate is above that of non-poor, whatever the poverty threshold. For this reason we stress the importance of the NMMD property, especially for formal analysis of NAGICs and for social welfare interpretations.

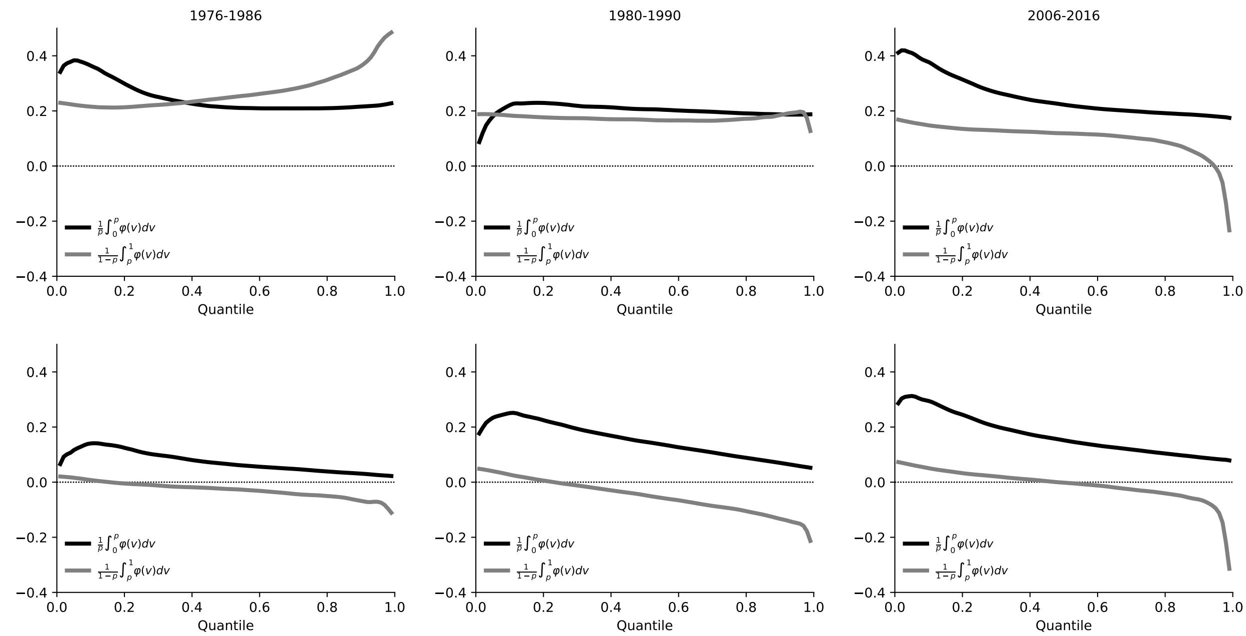

Figure 9: Non-monotonically mean decreasing in NAGICs. In all figures the black line corresponds to the two terms in the definition of NMMD: $\frac{1}{p}\int_0^p \varphi(v)\,dv$ and the gray line to $\frac{1}{1-p}\int_p^1 \varphi(v)\,dv$. We consider the same three periods considered before (1968–1978 (left); 1980–1990 (middle); 2006–2016 (right)) and for each period we consider the NAGIC (top) and its mobility component (bottom). The results are based on CPS labor income data for 35 year-old full-time workers in the initial year of each period.

7 Discussion

In this paper we studied anonymous and non-anonymous growth incidence curves (GICs and NAGICs, respectively) in the United States since 1968. We use these tools to highlight the importance of combining mobility and inequality to accurately describe the individual experience of growth. In particular, we ask to what extent the shape of GICs and NAGICs differ for a given time period, and under what conditions they might have substantially different shapes. Most notably, we find that NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality.

The paper undertakes a systematic comparison of the shape of GICs and NAGICs in the United States. We use the Panel Study of Income Dynamics (PSID) and find that NAGICs tend to be systematically downward sloping, a property commonly found with income panel data. We show that this property weakens when correcting for income measurement errors and when eliminating life-cycle effects and restricting samples to full-time workers.

We also provide a decomposition of the NAGIC into two components corresponding to income rank mobility and to the change in the cross-sectional income distribution. We show that the rank mobility effect imparts a strong downward sloping shape onto the NAGIC. This effect is key in both components of the decomposition. First, mobility contributes to a pure reranking effect, irrespective of the change in the shape of the income distribution. Second, mobility also appears in the component that includes the change in the income distribution shape. It attenuates the effect of the change in the distribution shape on the NAGIC. The decomposition thus demonstrates that rising mobility may lead to flatter or roughly downward sloping NAGICs. This effect has two facets – higher mobility increases the effect of reranking on one hand, and it reduces the contribution of inequality changes to the NAGIC slope, on the other. This helps understanding why, overall, even in presence of a strong increase in cross-sectional income inequality the NAGIC may remain downward sloping.

To overcome problems arising from measurement errors in panel data, we move to analyze synthetic panels that are based on combining cross-sectional data from the Current Population Survey (CPS) with modeled copulas consistent with empirical observations of income rank correlation. The synthetic NAGICs are then validated against benchmarks. We find that the fundamental characteristic of NAGICs, being downward sloping and then flattening when focusing on narrow age groups and full-time workers, remains valid for synthetic panels.

We finally use the preceding decomposition to describe more accurately the shape of NAGICs and their components with the concept of a Non-Monotonically Mean Decreasing function, a property that characterizes the mobility component of a NAGIC, and that may impart to the NAGIC itself. A NAGIC that is Non-Monotonically Mean Decreasing is such that the average growth rate of the poor is above that of non-poor, whatever the poverty threshold. For this reason we stress the importance of this property, which may be interpreted as generalized 'pro-poorness', but note that a more rigorous social welfare basis will be provided in a future work.

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A Proofs

A.1 Proof of NAGIC in bivariate log-normal

Proof. We consider a bivariate distribution of log-incomes, in which the log-incomes are normally distributed with a Gaussian copula. We also consider the NAGIC as

$$G_f(p) = \int_0^1 R_f(p, p') F_{t'}^{-1}(p') \, dp' - F_t^{-1}(p) \,, \tag{A.1}$$

in which we make the assumption that the difference in log-incomes is the same as the growth rate (*i.e.* that $g = x_{t+1}/x_t - 1 \approx \log(x_{t+1}) - \log(x_t)$).

$R_f(p, p')$ can be written explicitly as

$$\frac{1}{\sqrt{1-\rho^2}} \exp\left(-\frac{(x^2+y^2)\rho^2 - 2xy\rho}{2(1-\rho^2)}\right) \,, \tag{A.2}$$

where ρ is the correlation coefficient between the log-incomes, $x = \Phi^{-1}(p')$ and $x = \Phi^{-1}(p)$. The inverse CDF $F_t^{-1}(p)$ can be written as

$$F_t^{-1}(p) = \mu_t + \sigma_t \Phi^{-1}(p) \,, \tag{A.3}$$

and we can then write

$$\begin{aligned} G_f(p) &= \int_0^1 R_f(p, p') \left(\mu_{t'} + \sigma_{t'}\Phi^{-1}(p')\right) dp' - \left(\mu_t + \sigma_t \Phi^{-1}(p)\right) & \text{(A.4)} \\ &= (\mu_{t'} - \mu_t) + \sigma_{t'} \int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp' - \sigma_t \Phi^{-1}(p) \,. & \text{(A.5)} \end{aligned}$$

To evaluate the integral $\int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp'$ we replace $\Phi^{-1}(p')$ by x and the integral limits to $-\infty$ to ∞, and we get

$$\int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp' = \frac{1}{\sqrt{1-\rho^2}} \int_{-\infty}^{\infty} x \cdot \exp\left(-\frac{(x^2+y^2)\rho^2 - 2xy\rho}{2(1-\rho^2)}\right) dx = \rho y \,, \tag{A.6}$$

where the last equality requires numerical integration.

We finally obtain

$$G_f(p) = (\mu_{t'} - \mu_t) + (\rho\sigma_{t'} - \sigma_t) \, \Phi^{-1}(p) \,. \tag{A.7}$$

□

A.2 Proof of Proposition 1

Proof. The pure mobility component of a joint distribution may be considered as a set of permutations of two individual incomes. Consider such permutations from individuals belonging to a small interval $(j-h, j+h)$ around rank j to individuals belonging to a (non-overlapping) interval of $(i-h, i+h)$ such that $i < j$ Replacing growth rates by the difference in income logarithms, the NAGIC $N_{ij}(p)$ is given by:

$$\begin{aligned}
N_{ij}(p) &= 0 \; if \;\; p \in [0, i-h] && \text{(A.8)}\\
N_{ij}(p) &= log(y_t(u)) - log(y_t(v)) > 0 \; if \;\; p \in [i-h, i+h] && \text{(A.9)}\\
N_{ij}(p) &= 0 \; if \;\; p \in [i+h, j-h] && \text{(A.10)}\\
N_{ij}(p) &= log(y_t(v)) - log(y_t(u)) > 0 \; if \;\; p \in [j-h, j+h] && \text{(A.11)}\\
N_{ij}(p) &= 0 \; if \;\; p \in [j+h, 1]\,, && \text{(A.12)}
\end{aligned}$$

where (u, v) are pairs of ranks such that $u \in [i-h, i+h]$ and $v \in [j-h, j+h]$. It follows that the incomplete mean of the corresponding NAGIC, $\overline{N}_{ij}(p)$ is such that:

$$\begin{aligned}
\overline{N}_{ij}(p) &= \;\; 0 \; if \;\; p \in [0, i-h] && \text{(A.13)}\\
\overline{N}_{ij}(p) &> 0 \quad if \;\; p \in [i+h, j-h] && \text{(A.14)}\\
\overline{N}_{ij}(p) &= \;\; 0 \; if \;\; p \in [j+h, 1]\,. && \text{(A.15)}
\end{aligned}$$

Allowing h to tend towards 0 permits to represent permutation between two single individuals, the preceding property of $\overline{N}_{ij}(p)$ then applies to the whole interval $[0, 1]$ rather than to segments in that interval. Being strictly positive on $[i, j]$ and zero over $[j, 1]$ it is NMMD. As the pure mobility component of a NAGIC is essentially a collection of such single permutations (i, j), the incomplete mean of $M(p)$ is the sum of single permutation $\overline{N}_{ij}(p)$ and is thus NMMD.

The same argument does not hold when defining NAGICs on income growth rates rather than differences in income logarithms. The incomplete mean of single permutation NAGIC is still everywhere non-negative but it is strictly positive for $p = 1$ ans thus non-NMMD. This is easily seen in the limit case of a single permutation (i, j), as the final value of the incomplete mean, $\overline{N}_{ij}(1)$, of the NAGIC is proportional to:

$$\frac{y_t(j) - y_t(i)}{y_t(j)} + \frac{y_t(i) - y_t(j)}{y_t(j)}\,, \tag{A.16}$$

which is positive.

$\square$

A.3 Proof of Proposition 2

Proof. We first recall a basic theorem in income inequality measurement, namely that a constant-mean Lorenz-dominant change in an income distribution can always be decomposed into a sequence of so-called Pigou-Dalton income transfers from an individual at some level j of the income scale to an individual at rank i below j[8] The proof then follows that of the preceding proposition.

After scaling up the initial incomes to ensure the same mean income as in the final distribution, assume that the latter differs from the former only through a one-to-one transfer of income, ε, from some individuals in the income range $(j-h, j+h)$ to an equal number individuals in the interval $(i-h, i+h)$. Letting h tend to zero, it comes that

$$yR\left(p; yt+dy\right) - \left(1+g\right) yR\left(p; yt\right) = \left[R(p,i) - R(p,j)\right] 2\varepsilon h\,, \tag{A.17}$$

where the notation $y_t + dy$ stands for the terminal distribution that differs from the original one only through the preceding infinitesimal change. Integrating over $[0,p]$ yields

$$\int_0^p \left[yR\left(q; yt+dy\right) - \left(1+g\right) yR\left(q; yt\right)\right] dq = \left[\int_0^p R\left(q,i\right) dq - \int_0^p R\left(q,j\right) dq\right] 2\varepsilon h\,. \tag{A.18}$$

The regularity of the copula, together with its bi-stochasticity ($\int_0^1 R\left(q,i\right) dq = 1$) ensure that the preceding expressions are non-negative for all $p \in [0,1]$. Indeed, it is non-negative for $p \leq p^*\left(i,j\right)$. If it were negative for some $p^0 > p^*\left(i,j\right)$, then $R\left(p,i\right) - R\left(p,j\right)$ would need to be positive at some $p \in \left(p^0, 1\right)$ to ensure $\int_0^1 \left[R\left(p,i\right) - R\left(p,j\right)\right] dp = 0$. But this is in contradiction with the regularity property.

If the terminal distribution Lorenz-dominates the initial one, it is obtained from the latter through a succession of transfers like the preceding one. It follows that

$$\overline{D}A\left(p\right) = \frac{1}{p}\int_0^p DA\left(q\right) dq = \frac{1}{p}\int_0^p \left[yR\left(q; yt'\right) - \left(1+g\right) yR\left(q; yt\right)\right] dq \geq 0 \;\forall p \,\epsilon\, \left(0,1\right)\,. \tag{A.19}$$

$\overline{D}^A\left(p\right)$ is the incomplete mean of the difference, $D^A\left(p\right) = y^R\left(q; y_{t'}\right) - \left(1+g\right) y^R\left(q; y_t\right)$ of

[8] This is central theorem of the measurement of inequality relies on the well-known Hardy-Littlewood-Polya theorem. It was first formulated by Dasgupta, Sen and Starrett (1973).

expected final income when the final distribution is the observed distribution and when it is the initial distribution after scaling it up to equalize income means.

□

A.4 Proof of corollary from Proposition 2

Proof. With the notations of Eq. (A.19) we can write down the shape component of the NAGIC as follows:

$$D\left(p\right)=\int_0^p\frac{DA\left(q\right)}{yt\left(q\right)}dq=\int_0^p DA\left(q\right)\xi\left(q\right)dq\,,\tag{A.20}$$

where $\xi\left(q\right)=1/y\left(q\right)$, assuming $y\left(q\right)$ is continuous. To get the incomplete mean of $D\left(p\right)$, we integrate by parts the RHS of the preceding definition and divide by p. It follows that

$$\overline{D}\left(p\right)=\frac{\overline{D}A\left(p\right)}{y\left(p\right)}-\frac{1}{p}\int_0^p\overline{D}A\left(q\right)\xi'\left(q\right)dq\,.\tag{A.21}$$

As $\overline{D}^A\left(p\right)$ is everywhere non-negative and $\xi'\left(q\right)$ is negative, it follows that $\overline{D}\left(p\right)$ is non-negative.

□

B The Effect of Measurement Error on the Average Slope of NAGICs

Let X_t be observed log-income at time t, and X_t^* is true value. There is a systematic downward bias, δ, in reporting income and some random measurement error, u_t. There may be some autocorrelation, with coefficient ρ in the latter. Hence

$$\begin{aligned} X_t &= (1-\delta) X_t^* + u_t & \text{(B.1)} \\ X_{t-1} &= (1-\delta) X_{t-1}^* + u_{t-1} & \text{(B.2)} \\ u_t &= \rho u_{t-1} + \epsilon_t \,. & \text{(B.3)} \end{aligned}$$

The expected value of u_t and that of ϵ_t is zero. Both are orthogonal to X_{t-1}, X_t^*, X_{t-1}^* and ϵ_t is orthogonal to u_{t-1}. The observed growth rate in income is

$$g_t = X_t - X_{t-1} \,,$$

whereas the true growth rate is:

$$g_t^* = X_t^* - X_{t-1}^* \,.$$

We look for the sign of $\mathrm{Cov}\left(g_t, X_{t-1}\right)$ as a function of the true covariance $\mathrm{Cov}\left(g_t^*, X_{t-1}^*\right)$ and try to show how the difference is affected by the measurement error, *i.e.* δ and u_t. From the definition of g_t it comes that

$$g_t = (1-\delta)\, g_t^* + (\rho - 1)\, u_{t-1} + \epsilon_t \,.$$

It follows that:

$$\begin{aligned} \mathrm{Cov}\left(g_t, X_{t-1}\right) &= \mathrm{E}\left(g_t X_{t-1}\right) - \mathrm{E}\left(g_t\right) \mathrm{E}\left(g_t X_{t-1}\right) \\ &= (1-\delta)^2\, \mathrm{E}\left(g_t^* X_{t-1}^*\right) - (1-\rho)\, \mathrm{Var}\left(u_{t-1}\right) - (1-\delta)^2\, \mathrm{E}\left(g_t^*\right) \mathrm{E}\left(X_{t-1}^*\right) \,, \end{aligned}$$

or

$$\mathrm{Cov}\left(g_t, X_{t-1}\right) = (1-\delta)^2\, \mathrm{Cov}\left(g_t^*, X_{t-1}^*\right) - (1-\rho)\, \mathrm{Var}\left(u_{t-1}\right) \,.$$

There are two negative biases on the estimation of the covariance between growth rates. One is coming from the systematic under-reporting of income and the second from the measurement error, the latter being attenuated by the autocorrelation in measurement errors. This negative bias translates into an underestimation of the average slope of the NAGIC.

C Correcting the Rank Correlation Bias

Measurement errors in survey data are well-documented. They are known to lead to a downward bias of the correlation between log-incomes at two points in time. We follow Fields et al. (2003) who state that:

> [...] two validation studies of U.S. earnings data compared the Current Population Survey and the Panel Study of Income Dynamics to Social Security or firm records (Bound and Kruger, 1991; Bound et al., 1994). These two studies found the following three regularities: first, the ratio of the variance of measurement error to the variance of true log annual earnings, ranged from 7 to 25 percent. Second, measurement error was negatively correlated with true log earnings and this correlation, was between -0.15 and -0.03.

Accordingly, it is possible to model the measurement error, using the ratio of the variance of measurement error to the variance of true log-income, and the correlation between measurement error and true log-income, as parameters, and correct the correlation between measured log-income at two points time.

The starting point is the log-income (y_{it}) of an individual i at two points in time – t and t'. We measure y_{it}, but the true values (y^*_{it}) are unknown and there is some measurement error:

$$
\begin{aligned}
y_{it} &= y^*_{it} + u_{it} && \text{(C.1)}\\
y_{it'} &= y^*_{it'} + u_{it'}\,, && \text{(C.2)}
\end{aligned}
$$

and we assume that the error terms have zero expectation.

We estimate

$$
\hat{\rho} = \frac{\mathrm{Cov}\left[y_{it}, y_{it'}\right]}{\sigma_{y_{it}}\sigma_{y_{it'}}}\,. \qquad \text{(C.3)}
$$

According to the regularity mentioned above, the error is negatively correlated with the true income, so we can assume

$$
\begin{aligned}
u_{it} &= \alpha_t y^*_{it} + v_{it} && \text{(C.4)}\\
u_{it'} &= \alpha_{t'} y^*_{it'} + v_{it'}\,, && \text{(C.5)}
\end{aligned}
$$

where v_{it} are independent from the true incomes and α_t is a negative parameter. The measurement errors are also found to be autocorrelated in time (Fields et al., 2003), but these are usually small correlations, and since we are interested in periods of 10 years, they will be negligible.

We also assume that the ratios between the variance of the measurement error and of true income are known parameters:

$$\beta_t = \frac{\mathrm{Var}\left[u_{it}\right]}{\mathrm{Var}\left[y^*_{it}\right]} \tag{C.6}$$

$$\beta_{t'} = \frac{\mathrm{Var}\left[u_{it'}\right]}{\mathrm{Var}\left[y^*_{it'}\right]} . \tag{C.7}$$

Note that the correlation between measurement error and true log-income is

$$\rho_{uy^*} = \frac{\mathrm{E}\left[u_{it}y^*_{it}\right]}{\sqrt{\mathrm{Var}\left[u_{it}\right]\mathrm{Var}\left[y^*_{it}\right]}} = \frac{\alpha_t \mathrm{Var}\left[y^*_{it}\right]}{\sqrt{\beta_t \mathrm{Var}\left[y^*_{it}\right]2}} = \frac{\alpha_t}{\sqrt{\beta_t}} , \tag{C.8}$$

so given the range of parameters assumed for β_t and ρ_{uy^*}, we can also find α_t, and the model becomes fully specified.

We denote the true correlation between log-incomes as $\rho = \frac{\mathrm{Cov}\left[y^*_{it}, y^*_{it'}\right]}{\sigma^{y^*_{it'}}\sigma^{y^*_{it'}}}$. We would like to get ρ as a function of $\hat{\rho}$, α_t, $\alpha_{t'}$, β_t, and $\beta_{t'}$.

First, it is clear that the variance of v_{it} is $(\beta_t - \alpha_t^2)\,\mathrm{Var}\left[y^*_{it}\right]$, and the expectation of v_{it} is $-\alpha_t \mathrm{E}\left[y^*_{it}\right]$. Therefore,

$$\mathrm{Var}\left[y_{it}\right] = \mathrm{Var}\left[y^*_{it}\left(1+\alpha_t\right) + v_{it}\right] = \left(1+\alpha_t\right)2\mathrm{Var}\left[y^*_{it}\right] + \left(\beta_t - \alpha_t^2\right)\mathrm{Var}\left[y^*_{it}\right] \tag{C.9}$$

$$= \left(1 + 2\alpha_t + \beta_t\right)\mathrm{Var}\left[y^*_{it}\right] . \tag{C.10}$$

The same holds for t'. Now, we can explicitly evaluate the covariance of log-incomes:

$$\mathrm{Cov}\left[y_{it}, y_{it'}\right] = \mathrm{E}\left[y_{it}\cdot y_{it'}\right] - \mathrm{E}\left[y_{it}\right]\mathrm{E}\left[y_{it'}\right] = \mathrm{E}\left[y_{it}\cdot y_{it'}\right] - \mathrm{E}\left[y^*_{it}\right]\mathrm{E}\left[y^*_{it'}\right] , \tag{C.11}$$

and

$$\mathrm{E}\left[y_{it}\cdot y_{it'}\right] = \mathrm{E}\left[y^*_{it}y^*_{it'}\right]\left(1+\alpha_t+\alpha_{t'}+\alpha_t\alpha_{t'}\right) - \mathrm{E}\left[y^*_{it}\right]\mathrm{E}\left[y^*_{it'}\right]\left(\alpha_t+\alpha_{t'}+\alpha_t\alpha_{t'}\right) . \tag{C.12}$$

We get

$$\mathrm{Cov}\left[y_{it}, y_{it'}\right] = \mathrm{E}\left[y^*_{it}y^*_{it'}\right]\left(1+\alpha_t+\alpha_{t'}+\alpha_t\alpha_{t'}\right) \tag{C.13}$$

$$-\mathrm{E}\left[y^*_{it}\right]\mathrm{E}\left[y^*_{it'}\right]\left(1+\alpha_t+\alpha_{t'}+\alpha_t\alpha_{t'}\right) \tag{C.14}$$

$$= \mathrm{Cov}\left[y^*_{it}, y^*_{it'}\right]\left(1+\alpha_t+\alpha_{t'}+\alpha_t\alpha_{t'}\right) . \tag{C.15}$$

Now can write

$$\hat{\rho} = \frac{\mathrm{Cov}\left[y_{it}, y_{it'}\right]}{\sigma_{y_{it}}\sigma_{y_{it'}}} = \rho \times \frac{1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}}{\sqrt{\left(1 + 2\alpha_t + \beta_t\right)\left(1 + 2\alpha_{t'} + \beta_{t'}\right)}}\,. \tag{C.16}$$

Therefore, given some assumptions on α and β, we get correct the estimated correlation by some factor.

For example, if $\alpha = \alpha_t = \alpha_{t'}$ and $\beta = \beta_t = \beta_{t'}$, the correction factor is $\kappa = \frac{(1+\alpha)^2}{1+2\alpha+\beta}$. If $\beta = 0.25$ and $\rho_{uy^*} = -0.1$, both lie within the range of values reported by Bound and Kruger (1991); Bound et al. (1994), then $\alpha = -0.05$. The correction factor is then $\kappa \approx 0.785$. So the estimated correlation is downward biased by 21.5%, which can then be corrected.

It is important to note that this correction does not eliminate the large statistical uncertainty in the estimates.

D Stability of Rank Correlations in 4-year Time Periods

Figure 6 showed that over periods of 10 years the rank correlation of different age groups and of the entire working population has been generally stable over time. It also changes within a narrow interval. These observations are important when constructing synthetic NAGICs. To demonstrate the same stability for shorter time spans, we repeat the calculation done for Figure 6 with periods of 4 years. The results are presented in Figure 10, showing no clear trend over time and narrow intervals, in particular after averaging.

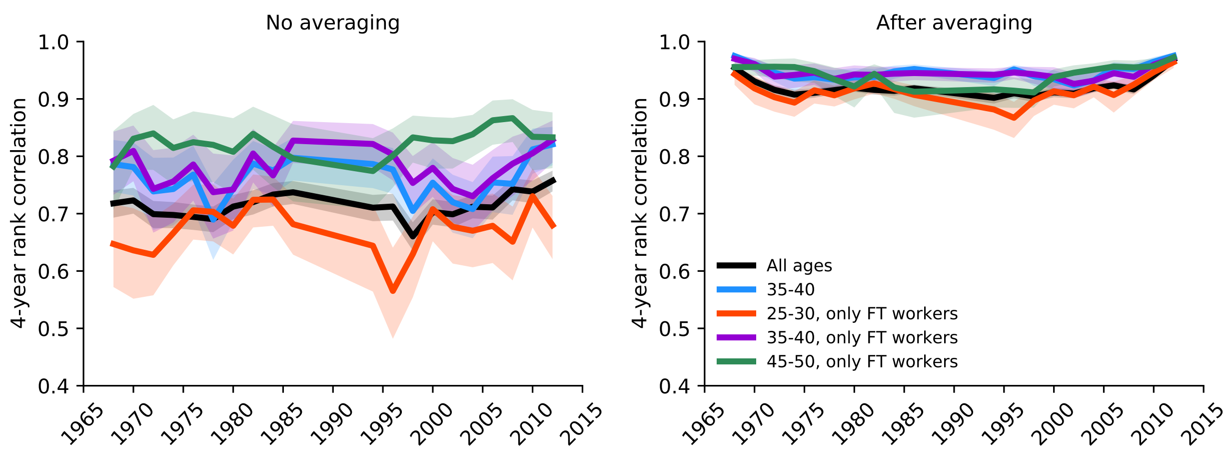

Figure 10: 4-year rank correlations in the United States based on PSID data from 1968 onward. The value on the x-axis specifies the initial year of each 4-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. Several groups are considered: all ages, full-time workers + non-full-time workers; 35–40 year-olds, full-time workers + non-full-time workers; 35–40 year-olds, only full-time workers; 25–30 year-olds, only full-time workers; 45–50 year-olds, only full-time workers. We present estimates before (left) and after (right) income averaging.