Anonymous and Non-Anonymous Growth Incidence Curves: United States, 1968–2016

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Abstract

This paper compares anonymous and non-anonymous growth incidence curves in the United States during the past 50 years. The former show income growth by distribution quantiles irrespectively of initial individual incomes, whereas the latter is conditional on original income ranks. While anonymous curves tend to be upward sloping due to increasing inequality, the same is not true of non-anonymous curves, which are generally flat or non-significantly downward sloping, suggesting some neutrality of growth when initial income positions are accounted for. The paper proposes a decomposition of non-anonymous curves into a mobility and a shape, or distributional change components. The former is always downward sloping, whereas the latter is upward sloping in periods of increasing inequality, so that flat non-anonymous curves can be observed even when income inequality is increasing. The paper then exploits that decomposition to show that the slope of non-anonymous incidence curves in the United States is mostly determined by the evolution of cross-sectional income distributions. It also proposes a generalized pro-poorness criterion to interpret the shape of non-anonymous incidence curves in social welfare terms. A simple approximate test is suggested, which permits inferring whether non-anonymous growth incidence curves exhibit this property.

Keywords: mobility, inequality, income growth

JEL Codes: D3, H0, J6

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1 Introduction

Is income growth benefiting primarily the rich or the poor? Are inequality changes informative of the individual experience of income growth? How do income inequality and income mobility interact? How can we compare the social welfare associated with different growth spells? These are all crucial questions for our understanding of economic growth, for social welfare analysis and for policy. This paper aims to provide a simple framework for such an evaluation and applies it to US panel and cross-sectional data over the last 50 years.

There are two ways of evaluating income distribution changes in a given population over a period of time. In one case, the new distribution is compared to the original one without considering the identity of income earners and their rank in the initial income scale. The comparison is thus 'anonymous'. Poorest (richest) incomes in the initial period are compared to poorest (richest) incomes in the final period. This is the common form of evaluating the distributional impact of growth.

In the other case, the comparison is made between the two distributions conditional on initial incomes. The comparison is thus 'non-anonymous'. It now matters how individual incomes changed over time, that is whether income grew faster among the poor or among the rich, and whether some people fell into poverty whereas some poor people got out of it.

It has become common to represent the anonymous distributional change brought by economic growth using the Growth Incidence Curve (GIC) (Ravallion and Chen, 2003). This convenient tool shows the rate of income growth of successive increasing fractiles of the distribution in the initial and final periods. The slope of that curve is particularly important as it relates GICs to conventional inequality comparisons of two income distributions. An upward sloping GIC necessarily implies that the Lorenz curve of the new distribution is everywhere below that of the old one (although the reciprocal is not true). Growth is then unambiguously associated with more inequality, and social welfare is negatively affected in comparison to the case where income growth is uniform, in which the GIC would be horizontal. The opposite holds for downward sloping GICs.

Non-Anonymous Growth Incidence Curves (NAGICs) have been introduced independently in the economic literature by Grimm (2007), Van Kerm (2009) and Bourguignon (2011).[1] They have been used extensively since, given the increasing availability of income panel data, *e.g.*, Palmisano and Peragine (2015); Jenkins and Van Kerm (2016); Hammar and Waldenström (2020); Splinter (2021). NAGICs show the mean income growth rate in successive fractiles of

[1]In the income mobility literature, tables showing mean or median growth rates of income by initial decile were not uncommon even earlier. See, *e.g.*, Hungerford (1993).

the initial distribution rather than the growth rate of the mean income of the same fractiles in the initial and terminal distributions. Given this analogy, the question then arises of how comparable NAGICs and GICs are. In particular, are they likely to have the same shape with the same social welfare interpretation, or are they expected to differ in some radical way?

Answers to these questions are relevant for the interpretation of inequality changes (Palmisano and Peragine, 2015; Hendren, 2020). A downward sloping GIC is associated with less inequality. If mean income growth is positive, then social welfare is unambiguously increasing. Likewise, a downward sloping NAGIC means that the income of initially poor people grew faster than initially rich people, which might also be considered as a socially favorable development. Yet, is it possible that the two curves slope in different directions? If so, how should it be interpreted?

It turns out that GICs and NAGICs that correspond to the same panel data (the panel dimension of which is ignored in GICs) may have substantially different shapes. This is due to the fact that NAGICs incorporate two sources of change:

- changes in the shape of the income distribution between the beginning and the end of the studied period;
- rank mobility of income earners between these two dates.

GICs only account for the first of these two sources, and in the absence of mobility NAGICs and GICs would coincide. Yet, when including mobility their shape could be very different. It is possible that a GIC would slope upward, showing an increase in inequality, whereas the corresponding NAGIC would slope downward with presumably a favorable effect on social welfare. This can happen if poorer people in the initial distribution move upward in the income ranking, and the opposite for richer people. This demonstrates how differences between GICs and NAGICs may capture important aspects regarding the ways in which growth spells and changes in the income distribution are interpreted and evaluated.

This paper analyzes decennial GICs and NAGICs in the United States over the last 50 years as derived from panel (PSID) and cross-sectional (CPS) data. The main goal is to detect possible regularities in the shape of the NAGICs and/or in their relation with the GICs. Such regularities, which have not been thoroughly studied so far, indeed exist. Most notably, NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality (and thus upward sloping GICs) over the last three or four decades. An explanation of this difference is provided by a copula-based decomposition of a NAGIC, into mobility and shape components, the latter accounting for the difference in the initial and terminal income distributions. This decomposition generalizes for the whole distribution an approach proposed by Van Kerm (2004) for mobility measures (see also Seth and Yalonetzky (2021)). Using it, we find, seemingly paradoxically, that the shape of NAGICs in the United States is predominantly determined by the evolution of the cross-sectional distribution of income. This observation also relies on the observed parametric stability of the copula, *i.e.*, the rank transition matrix, over time in the PSID data, evidence for which also appears in social security data (Kopczuk, Saez and Song, 2010). Given that stability, it turns out that synthetic panels based on the observed distribution of the rank correlation coefficient and purely cross-sectional observations, may yield reasonably close approximations of NAGICs and income dynamics.

The paper is organized as follows. It begins with a short section (Section 2) devoted to the formal definitions of GICs and NAGICs, analyzing the relationship between them in the simple, yet informative, case where the joint log-income distribution in the beginning and the end of a time period is bivariate normal, *i.e.*, the marginal distributions are log-normal.

The next section (Section 3) undertakes a comparison of the shape of decennial GICs and NAGICs in the United States using PSID panel data that now spans 50 years. The NAGIC tends to be downward sloping, a property commonly found with income panel data (Fields et al., 2003; Jenkins and Van Kerm, 2016; Hammar and Waldenström, 2020; Splinter, 2021). Yet, this property weakens when considering narrow age cohorts, thus eliminating life-cycle effects, and even flatter when restricting samples to full-time workers. Unsurprisingly, the flattening is still stronger when incomes are averaged over adjacent years to correct for measurement errors.

Section 4 provides a copula-based decomposition of the NAGIC into two components corresponding respectively to the effect of rank mobility and that of the change in the cross-sectional income distribution. When applied to various 10-year periods in PSID data, rank mobility is then shown to impart a strong downward sloping shape onto the NAGIC, which helps understand why, overall, even in the presence of a strong increase in cross-sectional income inequality that causes the GIC to slope upward, the NAGIC may remain downward sloping. A somewhat surprising result, however, is the apparent little sensitivity of the decomposition to the empirical copula being used, which suggests either that rank mobility does not change much over time or that its influence on NAGICs is limited.

Section 5 delves deeper into this apparent stability of rank correlation in income panel data. It turns out that, on the one hand, empirical copulas in PSID data are satisfactorily approximated by the Plackett one-parameter functional specification and, on the other hand, the distribution of that parameter – the rank correlation – is rather stable over time.

Given the decomposition of NAGICs into mobility and shape components, the stability of the copula suggests that differences over time in the slope of NAGICs come from the latter. Section 6 builds upon that observation. It studies synthetic panels based on pure repeated cross-sections of individual incomes and a distribution of rank correlations based on that observed in PSID. The synthetic panels are then used to approximate NAGICs. Synthetic panels relying on the Current Population Survey (CPS) cross-sectional data lead to NAGICs that are consistent with those obtained from the original PSID panel data, but prove to be considerably flatter and not always downward sloping when defined on narrow age groups, which, after all, fits intuition better than the strong downward slope initially observed (Splinter, 2021). It is important to stress that the synthetic panels used in this paper rely on income mobility parameters obtained from true panel data, unlike in the recent literature on synthetic panels based exclusively on repeated cross-sections and virtual matching techniques between them (see the survey paper by Hérault and Jenkins (2019)).

A final section questions the social welfare interpretation of the downward slope of NAGICs. It is shown that monotonicity is unlikely to hold at a more granular level than quintiles or deciles. A Non-Monotonically Mean Decreasing property is proposed instead, which may be interpreted as a generalized pro-poorness criterion. The synthetic panel instrument is then used to elaborate an approximate test of whether NAGICs consistent with given cross-sectional marginal distribution data at both ends of a period are likely to be NMMD or not, thus bypassing true panel data.

The main findings are then summarized in a conclusion.

The paper contributes to different threads of the inequality and mobility literatures. First, from an empirical perspective, the main contribution is the description of NAGICs in the United States over the last 50 years and their juxtaposition with the respective anonymous GICs. We highlight that NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality, a distributional feature rarely emphasized. From a methodological point of view, the paper contributes to the intragenerational mobility literature by its use of synthetic panels, illustrating and explaining possible pitfalls when using panel data, as well as by offering a decomposition of NAGICs into a mobility and shape terms. The study of NAGICs also conceptually contributes to the way growth spells are evaluated. This has a clear significance to a major thread of the inequality literature on how changes in inequality matter for social welfare (Palmisano and Peragine, 2015; Piketty, Saez and Zucman, 2018; Lo Bue and Palmisano, 2019; Hendren, 2020).

2 Anonymous and Non-Anonymous Growth Incidence Curves: A Primer

We begin by defining anonymous and non-anonymous growth incidence curves and their basic properties.

Consider a two-period $(t;t')$ panel income dataset, with $t < t'$. Let the joint distribution of incomes in the two periods be $f(y_t, y_{t'})$ with marginal cdfs $F_t(p)$ and $F_{t'}(p)$, respectively. Use will be made below of the copula of that distribution, or the joint rank distribution, $R_f(p, p')$ (where $p, p' \in (0,1)$ are the unit-normalized income ranks at t and t', respectively).

The joint distribution is thus fully described by the copula R_f and the two marginal distributions F_t and $F_{t'}$ (this follows immediately from Sklar's theorem (Sklar, 1959)). With these notations, *Non-Anonymous Growth Incidence Curves* (NAGICs) are defined as follows:

Definition 1 (Non-Anonymous Growth Incidence Curve). *The Non-Anonymous Growth Incidence Curve (NAGIC), $G_f(p)$, associated with the joint distribution f, shows the growth rate of the average income between times t and t' of people at rank p at time t (the notation $^{-1}$ stands for inverse functions):*

$$G_f(p) = \frac{\int_0^1 R_f(p, p') F_{t'}^{-1}(p')\, dp' - F_t^{-1}(p)}{F_t^{-1}(p)}. \tag{2.1}$$

In comparison, the standard, anonymous growth incidence curve is defined as

Definition 2 (Anonymous Growth Incidence Curve). *The Anonymous Growth Incidence Curve (GIC), $G_f^a(p)$, associated with the joint distribution f, shows the growth rate of incomes at rank p in both periods t and t':*

$$G_f^a(p) = \frac{F_{t'}^{-1}(p) - F_t^{-1}(p)}{F_t^{-1}(p)}. \tag{2.2}$$

The NAGIC and the GIC thus differ through the former incorporating personal income mobility, *i.e.*, the copula R_f. Indeed, the GIC and the NAGIC coincide when no reranking occurs between the initial and terminal dates (*i.e.*, if the copula is the identity copula).[2]

The slopes of GICs and NAGICs encode relevant information for the interpretation of the distributional impact of growth. They have intuitive social welfare implications. For exam-

[2] Reranking has been analyzed in depth in the income redistribution literature but mostly in a descriptive way, as one component of the change in the distribution, and therefore of inequality. See, in particular, Lambert (2001) and Urban and Lambert (2008).

ple, a downward sloping GIC means that cross-sectional inequality is decreasing, implying that poor or rich people in the terminal year are less distant from the average income than poor or rich people in the initial year (*i.e.*, the Lorenz curve of the terminal year is above that of the initial year).[3]

Likewise, a downward sloping NAGIC shows that initially poor people gain relatively more from growth than people initially rich, which may be considered as a desirable social outcome (Ray and Genicot, 2022). Yet, GICs and NAGICs convey different meanings of social welfare, so it is necessary to understand the relationship between their respective slopes. To gain some intuition of that relationship, it is useful to consider the particular case in which the joint distribution of log-incomes is bivariate normal. Assume thus that the marginal distributions are $Log\mathcal{N}\left(\mu_t, \sigma_t^2\right)$ and $Log\mathcal{N}\left(\mu_{t'}, \sigma_{t'}^2\right)$ at t and t', respectively. In this case, for simplicity, we consider the NAGIC under the approximation that the difference in log-incomes is the same as the growth rate (*i.e.*, that $g = x_{t+1}/x_t - 1 \approx \log\left(x_{t+1}\right) - \log\left(x_t\right)$). It may be shown that the NAGIC is given by (see Appendix A for the proof):

$$G_f\left(p\right) = \left(\mu_{t'} - \mu_t\right) + \left(\rho\sigma_{t'} - \sigma_t\right)\Phi^{-1}\left(p\right) \ , \tag{2.3}$$

where μ_t is the mean log-income at time t, σ_t the standard deviation, ρ the correlation coefficient of log-incomes in the two periods, and Φ^{-1} is the inverse standard normal cumulative distribution function. Here, the fact that the NAGIC combines mobility and the difference in marginal distribution is readily apparent.

This has implications for the slope of the NAGIC, the sign of which is given by:

$$\operatorname{sgn}\left(\frac{dG_f}{dp}\right) = \operatorname{sgn}\left(\rho\sigma_{t'} - \sigma_t\right) \ . \tag{2.4}$$

Therefore, if the NAGIC is upward sloping then it must be the case that $\rho\sigma_{t'} - \sigma_t > 0$, implying $\sigma_{t'} > \sigma_t$, since $\rho \leq 1$. This means that an upward sloping NAGIC is necessarily associated with increasing inequality. Yet, the reciprocal does not hold: $\sigma_{t'} > \sigma_t$ does not imply that the NAGIC is upward sloping since the inequality may be inverted if ρ is small enough, *i.e.*, there is enough mobility. Conversely, $\sigma_{t'} \leq \sigma_t$ unambiguously implies a downward sloping NAGIC, so if inequality decreases, the NAGIC is necessarily downward sloping. We note that the anonymous GIC corresponds to Eq. (2.3) with $\rho = 1$, for which the slope is positive if $\sigma_{t'} > \sigma_t$, *i.e.*, if inequality increases cross-sectionally, and negative otherwise.

[3]Note that the reciprocal is not true. Lorenz dominance does not necessarily imply a downward sloping GIC.

The relationship between the slope of the anonymous GIC (or the change in inequality) and the slope of the NAGIC may thus be summarized as follows:

- GIC $\searrow \Rightarrow$ NAGIC $\searrow$ – if the GIC is downward sloping so is the corresponding NAGIC.
- NAGIC $\nearrow \Rightarrow$ GIC $\nearrow$ – if the NAGIC is upward sloping so is the corresponding GIC.
- GIC $\nearrow \not\Rightarrow$ NAGIC $\nearrow$ – if the GIC is upward sloping the corresponding NAGIC is not necessarily upward sloping.

This logic holds in a more general sense and not only under the log-normal approximation, although not in all cases. This is discussed in greater detail in Section 4 and in Section 7.

3 Anonymous and Non-Anonymous Growth Incidence Curves in the Panel Survey Of Income Dynamics

Following the acquired understanding of GICs and NAGICs, we proceed to estimate them for the United States in practice. We focus on the differences between GICs and NAGICs and the role of mobility. We are particularly interested to find to what extent GICs are informative of the individual experience of income growth. Clearly, income has become more unequal since the 1970s, which has a profound importance in and of itself. Yet, as discussed, it also matters how growth was experienced by individuals at different levels of the income distribution, especially in periods of growing inequality.

To study these GICs and NAGICs we use the Panel Study of Income Dynamics (PSID, 2018), or PSID. This is a longitudinal panel survey of families conducted annually or biennially since 1967 in the United States. We use the labor income of family heads and their spouses in the survey. The sample sizes differ in each wave due to methodology changes in the survey and the survey tracking the descendants of past surveyed individuals. 4,500–10,000 families were surveyed in each wave. Individual labor income is available from 1967 onward; it is adjusted for inflation using the GDP deflator. The focus on labor income is motivated by its lesser sensitivity to measurement errors in household surveys compared to capital income. Also, note that the analysis bears on pre-tax income. This avoids the present analysis to combine the issue of income mobility and that of the distributional incidence of reforms in the tax-benefit system. In any case, individual labor income in PSID data is only pre-tax.

Figure 1 uses these data to present GICs and NAGICs in four 10-year periods: 1968–1978, 1980–1990, 1992–2002, 2006–2016, which together cover about 50 years. Figure 1 juxtaposes the GIC and the NAGIC in each period and illustrates the large differences between them. Specifically, in all the periods the GICs are mildly sloping upwards, whereas the NAGICs are generally sloping downwards.

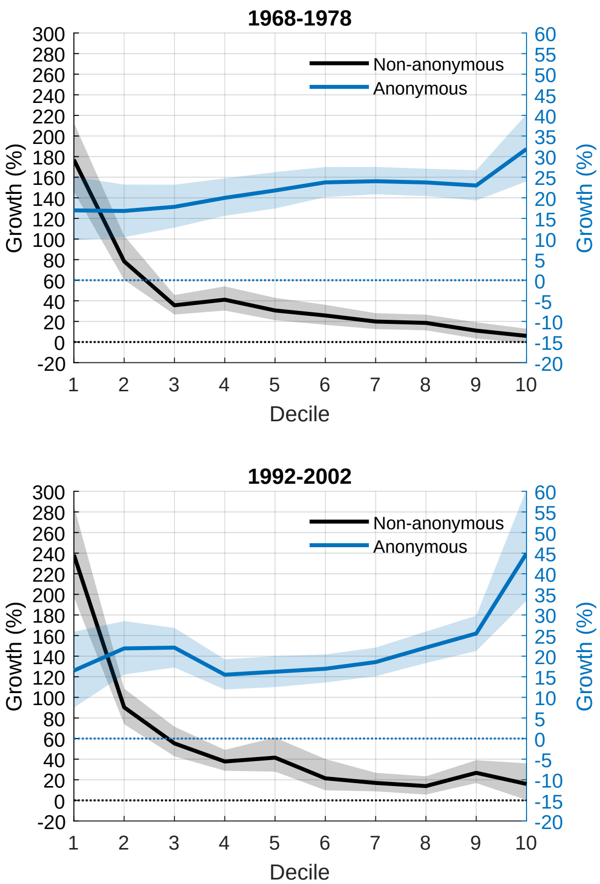

Figure 1: Growth incidence curves for the periods 1968–1978, 1980–1990, 1992–2002, 2006–2016 in the United States. The curves are based on individual annual labor income for all 20–55 year-old workers (in the early year in each period) included in the PSID in both the beginning and the end of each period. The shaded areas represent 95% confidence intervals produced by bootstrapping.

A naïve comparison of GICs and NAGICs would indeed lead to big differences. Yet, estimating these curves, and especially NAGICs, using PSID data has clear limitations. Specifically, the small sample sizes, the limited coverage of the top of the distribution and the measurement errors may all lead to an overestimation of relative mobility, which in turn, leads to more negative slopes in NAGICs. We address this bias using methods discussed in detail below.

Figure 2 presents GICs and NAGICs for the period 1980–1990. It compares GICs and NAGICs considering all workers (aged 20–55 in the initial year), and when limiting the sample to those who were 35–40 years-old in 1980 only, either all workers or full-time workers only.

The reason to include only full-time workers is to exclude the mechanical effect of moving from part-time to full-time work, and vice versa. In addition, including only 35–40 years-old workers attenuates life-cycle effects.[4]

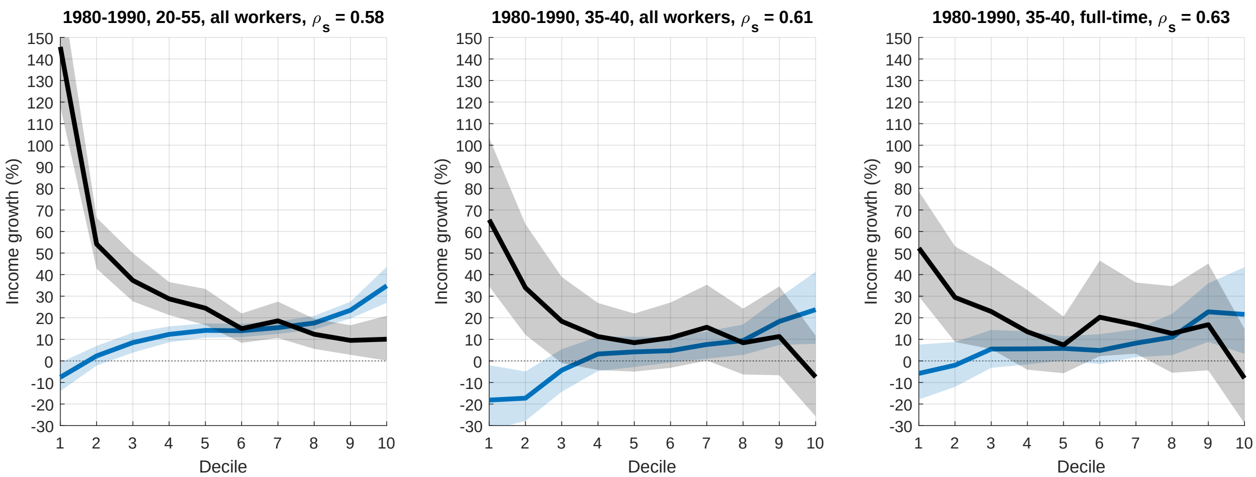

Figure 2: Anonymous (blue) and Non-anonymous (black) growth incidence curves 1980–1990. We consider all workers aged 20–50 in 1980 (left); all workers aged 35–40 in 1980 (middle); full-time workers aged 35–40 in 1980 (right). The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. ρ_s is the estimated rank correlation in each case.

Figure 2 repeats the typical feature in the PSID data: NAGICs are generally downward sloping, while GICs are generally upward sloping. This remains so in periods of increase in inequality.

Figure 3 presents a similar comparison as in Figure 2, between a GIC and a NAGIC, while showing the effect of averaging incomes over several years (over periods of four years, covered by three waves of the survey), a method used to attenuate the impact of measurement errors, discussed in detail later on. It shows that even after partially controlling for age, after restricting the sample to full-time workers and averaging initial and terminal earnings with values for adjacent years, NAGICs still have a very different shape from GICs. The NAGIC is flatter than before, as expected, while the GIC is still upward sloping. This seems rather surprising, especially given the apparent rise in inequality in all GICs that could have been thought as likely to influence both GICs and NAGICs.

[4]An additional reason for considering full-time workers only is focusing on earning potential rather than earnings (there is, by design, a selection bias as part-time workers or those who move out of the labor force could also have a decrease in their earning potential).

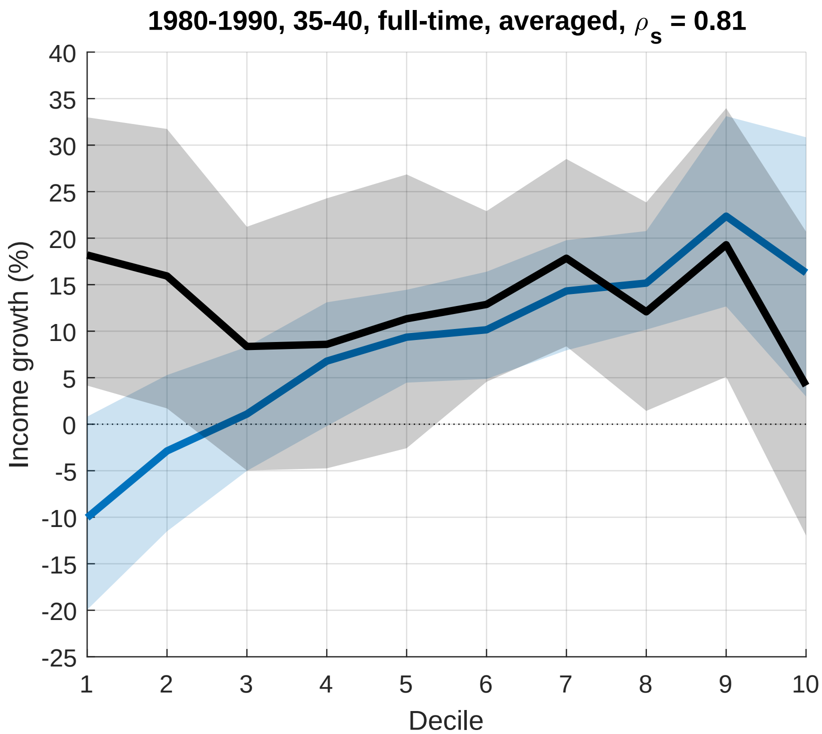

Figure 3: Anonymous (blue) and Non-anonymous (black) growth incidence curves 1980–1990 for full-time workers aged 35–40 in 1980 after averaging incomes over several years of data. Averaging is used to attenuate measurement error. Incomes are averaged over periods of four years, covered by three waves of the survey (*i.e.*, averaging incomes at years $x-2$, x and $x+2$ for every year x). The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. ρ_s is the estimated rank correlation.

4 Decomposition of the NAGIC into a Growth, Mobility and Shape Effects

Figures 1–3 show that indeed, GICs and NAGICs tend to be very different for the same data. The difference between the GICs and NAGICs is that the former describe the change in the shape of the marginal income distribution at two points of time whereas the latter adds to it the mobility of people from one position to another in the distribution. We can formally decompose NAGICs into these 'distribution', or 'shape', and 'mobility' components to further study what drives the different shapes of NAGICs and GICs. This decomposition generalizes for the whole distribution an approach proposed by Van Kerm (2004) for mobility measures, later extended by Seth and Yalonetzky (2021).

In the simplified case, where the joint distribution of log-incomes is bivariate normal, the approximated NAGIC (Eq. (2.3)) can be rewritten as

$$G_f(p) = \underbrace{(\mu_{t'} - \mu_t)}_{\text{growth}} + \underbrace{\sigma_t(\rho - 1)\,\Phi^{-1}(p)}_{\text{mobility}} + \underbrace{\rho(\sigma_{t'} - \sigma_t)\,\Phi^{-1}(p)}_{\text{shape}}, \tag{4.1}$$

where the NAGIC is decomposed into a growth term, a pure mobility term and a distribution term. If there is no mobility, or $\rho = 1$, the mobility term vanishes. If the distribution does not change (up to changes in μ) between t and t' then $\sigma_{t'} = \sigma_t$ and the shape term vanishes. It also follows immediately that the mobility term is monotonically decreasing in the rank p, since $\rho < 1$ (and $\Phi^{-1}(p)$ is increasing in p). Similarly, the shape term is increasing in p if inequality increases, and decreasing in p if it decreases.

A generalization, which follows the same intuition, can be directly derived from the NAGIC definition Eq. (2.1):

$$G_f(p) = \frac{\int_0^1 R(p,p')\left[F_{t'}^{-1}(p') - \widetilde{F}_t^{-1}(p')\right]dp'}{F_t^{-1}(p)} + \frac{\int_0^1 R(p,p')\,\widetilde{F}_t^{-1}(p')\,dp' - F_t^{-1}(p)}{F_t^{-1}(p)}, \tag{4.2}$$

where $\widetilde{F}_t$ is the cdf of the initial distribution after scaling it up, so that its mean is the same as in the final distribution, *i.e.*, $\widetilde{F}_t^{-1}(p) = (1+g)\,F_t^{-1}(p)$, where g is the growth rate of the mean income between t and t'.

To simplify, denote $y_t(p)$ the quantile $F_t^{-1}(p)$ and

$$y^R(p;x) = \int_0^1 R(p,p')\,x(p')\,dp' \tag{4.3}$$

the expected final income of the individuals initially at quantile p, when the marginal distribution of final income is the quantile function $x(p)$. The preceding decomposition may be now written as

$$\begin{aligned} G_f(p) &= \frac{y^R(p;y_{t'}) - (1+g)\,y^R(p;y_t)}{y_t(p)} + (1+g)\,\frac{y^R(p;y_t) - y_t(p)}{y_t(p)} + g && (4.4)\\ &= D(p) + M(p) + g\,. && (4.5)\end{aligned}$$

The NAGIC is thus decomposed into three terms. Starting from the end:

- The mean income growth rate, g.

- A pure mobility (or reranking) effect, $M(p)$ – the hypothetical NAGIC corresponding to no change in the marginal distribution of income, the final distribution being the same as the initial one, up to a scaling factor.

- A 'distribution' (or shape) effect, $D(p)$, due to the change in the marginal distributions. This corresponds to the difference in expected final income of people initially at rank p, when the final distribution is the terminal one and when it is the initial one scaled up by $(1+g)$, *i.e.*, as it would result from uniform growth.

The decomposition is shown in Figure 4 for all workers who were 20–55 years old in the initial year in four 10-year periods: 1968–1978, 1980–1990, 1992–2002 and 2006–2016 (the same as in Figure 1), based on the PSID data. It shows that in all cases the contribution of the mobility term to the slope of the NAGIC is strongly negative, and uniformly so. This is as expected: mobility only entails that bottom ranks gain and top ranks lose, but says nothing about the sign of income change in between, which moreover depends on the initial marginal income distribution. The shape term in most periods is upward sloping as it is expected from inequality in the final year being higher than in the initial year.

The shape term differs from the GIC, as it is not defined on the difference between observed income at some rank p in the initial and terminal year. Instead, it is the hypothetical difference in the *expected* income of people initially at rank p when the terminal distribution is the observed one and when it would be the same as the initial distribution after scaling it up to the terminal income mean. Such expectation operation necessarily reduces income changes in comparison with the GIC.

The decomposition also clarifies the interplay between mobility and inequality described in Section 2. A downward sloping GIC necessarily corresponds to a downward sloping NAGIC. An upward sloping NAGIC necessarily corresponds to an upward sloping GIC. Yet, an upward sloping GIC does not necessarily imply an upward sloping NAGIC, and this would depend on the balance between the shape term and the mobility term.

Equation (4.5) shows that both shape and mobility terms depend on the marginal distributions and the copulas. In practice, however, the decomposition is such that the shape term depends mostly on the marginal distributions and the mobility term mostly on the copula. This empirical regularity clearly appears in Figure 5, which compares shape and mobility terms where the marginal distributions remain unchanged, but the copula is replaced by a copula for a different time period. This is done for 1968–1978, 1980–1990 and 1992–2002. We find that in all the cases the shape terms effectively depend on the marginal distributions only, and not on the copula. This could be either because copulas are stable, which will be discussed in the next section, or because they are 'filtered' by the distribution. Either way we get quasi-insensitivity of the shape term to the period over which the copula is evaluated.

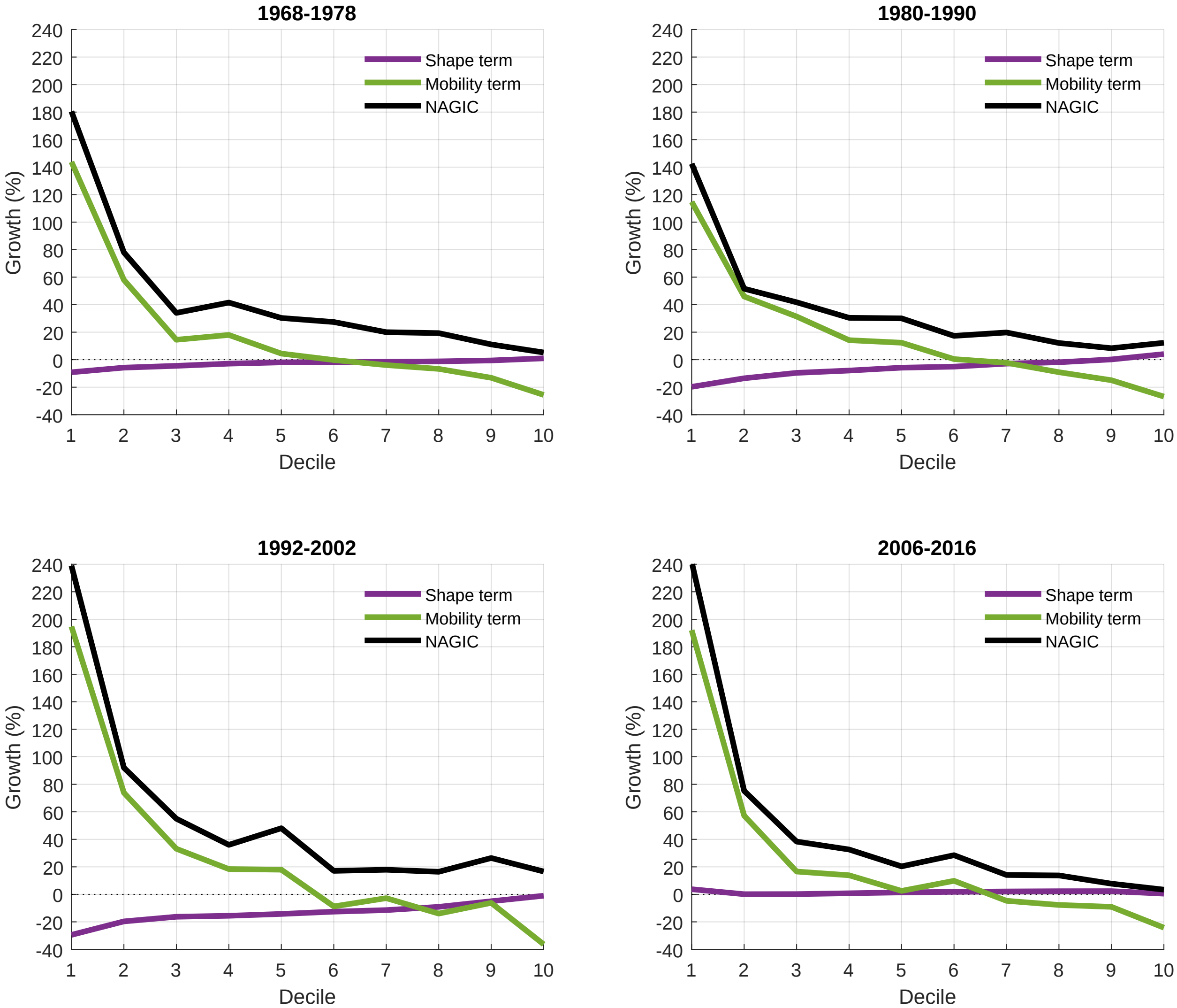

Figure 4: The shape term (magenta) and the mobility term (green) for PSID-based NAGICs (black), based on the NAGIC decomposition (Eq. (4.5)). The curves are based on individual annual labor income for all 20–55 year-old workers (in the early year in each period) included in the PSID in both the beginning and the end of each period (same data as in Figure 1).

We also compare mobility terms in the same manner. This time the replacement of copulas makes a much bigger impact, as expected, and the mobility term strongly depends on the copula, although not always. Figure 5 shows little change when the copula of one period is replaced by that of another in the earlier 10-year period.

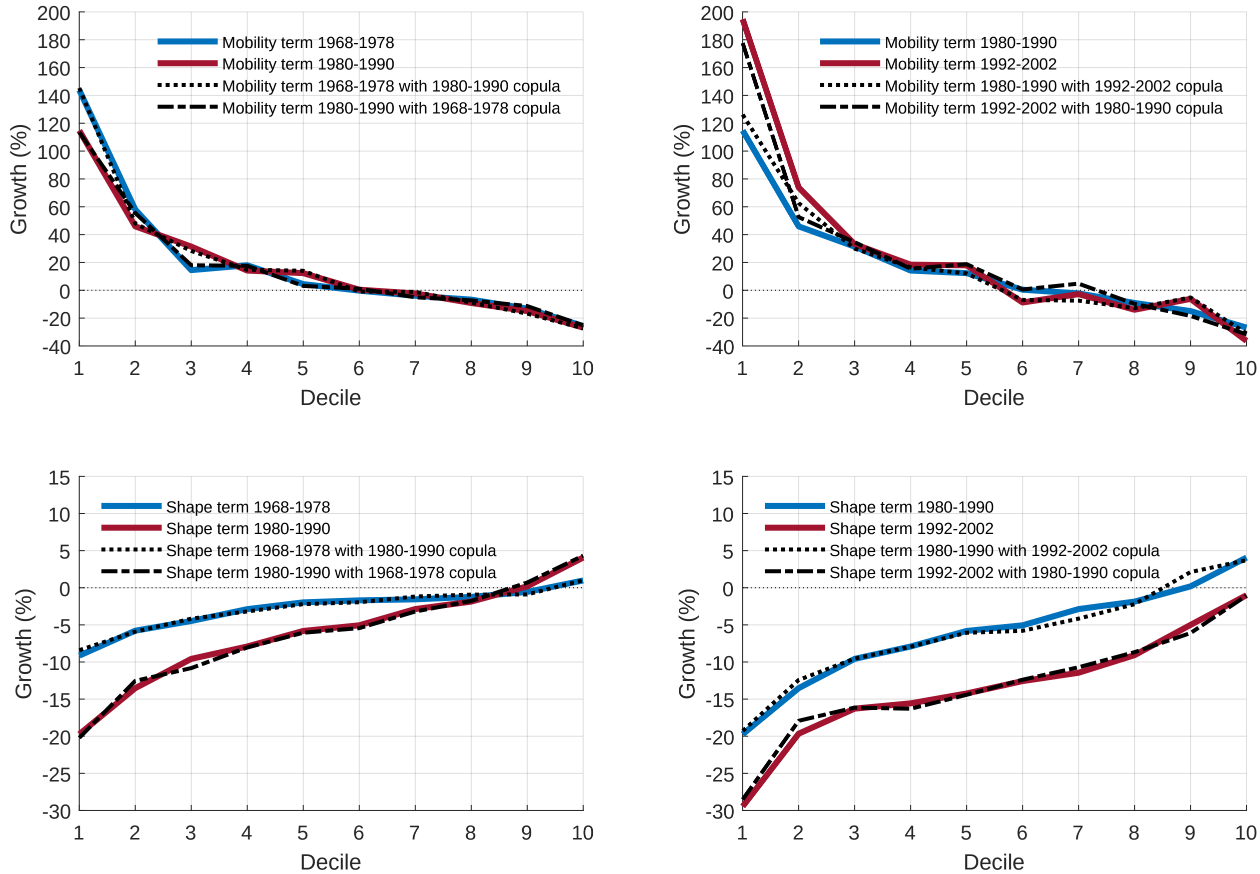

Figure 5: The mobility term (top) and the shape term (bottom) based on the NAGIC decomposition (Eq. (4.5)) using PSID data for 1968–1978, 1980–1990 and 1992–2002. In each case we consider the shape and mobility terms with either the original copula (red and blue lines) and with an alternative copula (black lines), taken from a different 10-year period.

5 The Relative Stability of Copulas

Figure 5 shows that the shape term in the NAGICs decomposition is relatively insensitive to the date of the empirical copula being used to evaluate it. Is it simply because copulas for different periods are close to each other? To test this hypothesis, empirical copulas have been approximated by well-known functional specifications that make the whole copula depend only a few parameters. Then the goodness of the fit was analyzed and compared across specifications while the stability of the key parameters was tested.

As already noted by Bonhomme and Robin (2009) for France, the Plackett copula (Plackett, 1965) turns out to work rather well on US data from PSID, as demonstrated in Appendix B (also in Berman (2021a)). Plackett copulas are especially convenient since they are defined using a single parameter that uniquely corresponds to the rank correlation between initial and terminal incomes. Therefore, to model copulas resembling those observed on 10-year periods in the PSID database, it is only necessary to compute the (Spearman's) rank correlation ρ_s in the data and to apply the Plackett formula. Then, to test whether copulas are indeed stable over time boils down to study how the rank correlations changed over time in the PSID data. This is shown in Figure 6.

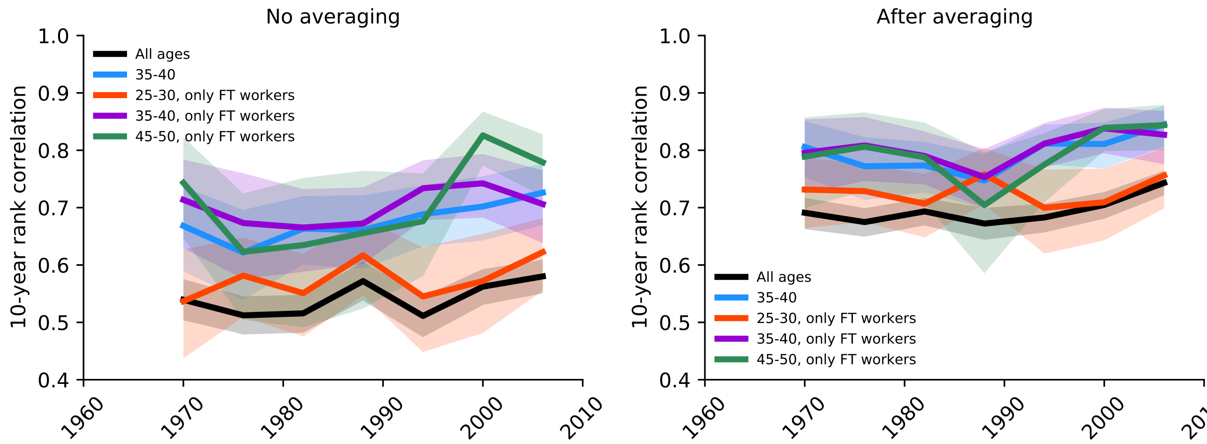

Figure 6: 10-year rank correlations in the United States based on PSID data from 1970 onward. The value on the x-axis specifies the initial year of each 10-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. Several groups are considered: all ages, full-time workers + non-full-time workers; 35–40 year-olds, full-time workers + non-full-time workers; 35–40 year-olds, only full-time workers; 25–30 year-olds, only full-time workers; 45–50 year-olds, only full-time workers. We present estimates before (left) and after (right) income averaging (as explained above).

The rank correlation estimates in Figure 6 raise several important observations:

- There is no clear trend in rank correlations over time. In all specifications they seem to fluctuate over time within relatively narrow bands: either 0.6–0.8 for the 35–40 year-olds without averaging and 0.75–0.85 after averaging, or 0.5–0.6 and 0.65–0.75 for the entire population of workers, before and after averaging, respectively (the same stability is observed for a shorter time span as well, see Appendix C).

- Statistical uncertainty is roughly ±5 percentage points for the 35–40 age group, and ±2 percentage points for the entire population of workers.

- Limiting the sample to full-time workers has an insignificant effect on the rank correlation.

- Limiting the sample to 35–40 year-olds significantly increases the rank correlation when compared to the entire population of workers, by 3 to 10 percentage points.

- The 35–40 and 45–50 age groups are quite similar, whereas the younger age group of 25–30 year-olds shows higher rank mobility.

Given the decomposition of NAGICs into mobility and shape components, the stability of the copula suggests that differences over time in the slope of the NAGICs come mainly from the latter. This is a crucial observation since it allows us to incorporate more accurate information on the marginal distributions than the information used in Figure 1 to study NAGICs. It motivates using synthetic panels, as further explored in the next section. We note that the stability of labor income rank correlations over periods of several years in the United States is a robust result, previously studied in detail by Kopczuk, Saez and Song (2010).

6 Synthetic Panels Based on Cross-Sectional Income Data

Given the relative stability of empirical copulas, or rank correlation, across periods for various population groups, and in view of the decomposition (Eq. (4.5)) of NAGICs, it turns out that differences in the shape of the latter should come mostly from changes in the initial and terminal earnings distribution. This also implies that synthetic panels could be constructed using alternative data sources for these marginal distributions and some estimates of the likely distribution of the rank correlation in panel data.

Namely, the definition of a NAGIC (Eq. (2.1)) , shows that it is the result of two different sets of information: i) the copula that fully describes the panel dimension of the NAGIC, and ii) the marginal distributions F_t and $F_{t'}$. Crucially, the two sets of information may come from different sources, especially when the copula is known to exhibit some stability over time and possibly across population groups. Then, the copula can be combined with any cross-sectional source about income distribution (see similar applications in Chetty et al. (2017) and Berman (2021b)). This section is dedicated to the analysis of such synthetic panels when the cross-sectional source for the United States is the CPS.

Before getting into the construction of synthetic panels, some preliminary remarks are necessary about the measurement error likely to affect initial or terminal earnings – summarily corrected above by averaging. Clearly, the error also affects rank correlation estimates, if no precaution is taken, as well as the marginal distributions of income, which now may come from another source than panel data.

6.1 Measurement error issues

Income measurement necessarily involves some measurement errors and when a panel is considered this may have a crucial impact on NAGICs. In the intergenerational context it is well-known that measurement error creates an attenuation bias, *i.e.*, estimated copulas represent mobility that is higher than real mobility. Since income measurement is noisy, income persistence estimates are attenuated (Bound and Kruger, 1991; Solon, 1992; Bound et al., 1994; Fields et al., 2003).

In the intragenerational context, measurement errors mechanically attach higher growth rates to individuals falsely identified as occupying lower income ranks at the beginning of time periods, and vice versa for individuals falsely identified as belonging to higher ranks. This may have a large impact on the shape of NAGICs whose slopes are clearly biased downward. Appendix D shows how a systematic bias on reported income and some autocorrelation of measurement error lead to a negative bias in the estimation of the covariance between the observed growth rate and the observed initial income.

Sampling error in panel data is also important, and may lead to inaccurate estimates of NAGICs due to significant attrition over time and other possible biases of panel surveys (Blanchet, Flores and Morgan, 2022; Yonzan et al., 2022). This sampling error is of much smaller magnitude in purely cross-sectional income sources, like the CPS in the United States. However, measurement error on incomes increases the level of inequality and distorts the cdf.

Dealing with measurement error is therefore essential. In the intergenerational context it is most often done by averaging over several years of data (Solon, 1992; Chetty et al., 2014). This was also done above in Figure 3 and Figure 6. Another option is based on an error model presented by Fields et al. (2003). This model assumes that the measurement error is negatively correlated with true incomes, *i.e.*, high incomes tend to under-report and the low incomes to over-report, and that the ratio of the variance of measurement error to true variance in true incomes is known, but that there is no serial autocorrelation of the measurement error over periods of several years. Fields et al. (2003) predict the underestimation of rank correlations based on panel surveys by 5%–20%. The derivation of a correction factor to estimate rank correlation based on this model is given in Appendix E.

Another measurement error issue concerns the use of cross-sectional data in synthetic data, as they differ from the panel data used to estimate the rank correlation and thus the copula. Cross-sectional income distribution data include measurement error, or possibly, deviations from usual income which may introduce a bias in the estimation of the synthetic

NAGIC, especially its mobility component in the decomposition (Eq. (4.2)) since it may be expected that measurement errors in the initial and terminal years tend to compensate each other in the shape term. This intuition is confirmed when using the log-normal approximation (Eq. (4.1)) and assuming that the observed standard deviation of log-income is over-estimated by ω_τ because of measurement errors, the NAGIC writes:

$$G_f\left(p\right) = \mu_{t'} - \mu_t + \left(\sigma_t - \omega_t\right)\left(\rho - 1\right)\Phi^{-1}\left(p\right) + \rho\left(\sigma_t - \omega_t - \sigma_{t'} + \omega_{t'}\right)\Phi^{-1}\left(p\right) \,. \qquad (6.1)$$

If the error term ω_τ is comparable across the two periods, the shape component of the NAGIC is little affected. However, this is not true of the mobility term. Ignoring the measurement error would reinforce the mobility terms and therefore the negative slope of the NAGIC, even though the correlation coefficient that is estimated on averaged panel data is immune of the error.

In the general case, a way of correcting the observed cross-sectional cdf – and consequently the associated quantile function – is to apply a kind of kernel correction to the observed density or cdf that depends on a priori judgment on the size of the measurement error.[5]

6.2 Synthetic panels

The measurement errors described above may be important even for the NAGIC presented in Figure 3. They may still generate an unreliable picture. Synthetic NAGICs not relying on the surveyed copula but only on some adjusted estimate of the rank correlation permit to avoid these sources of irregularities in comparisons across time periods. In addition, the stability of the copula (Figure 6) as well as Figure 5 suggest that differences over time in the slope of the NAGICs come mainly from the marginal income distributions. This is a key observation, which allows incorporating more accurate information on the marginal distributions than the information typically available in panel data to study NAGICs, by creating synthetic panels.

To test the validity of this approach, Figure 7 combines the marginal distributions observed in PSID data with a Plackett copula whose parameter is distributed like the empirical estimates of the panel rank correlation in Figure 6. This produces NAGICs that are within the statistical uncertainty range of those estimated directly from the panel.

Figure 7 also shows for some periods the synthetic NAGIC obtained from replacing the PSID cross-sectional distributions of averaged incomes by the appropriate marginal distributions

[5] The literature on these so-called 'deconvolution' methods is rich. For a recent contribution and survey see Hall and Meister (2007).

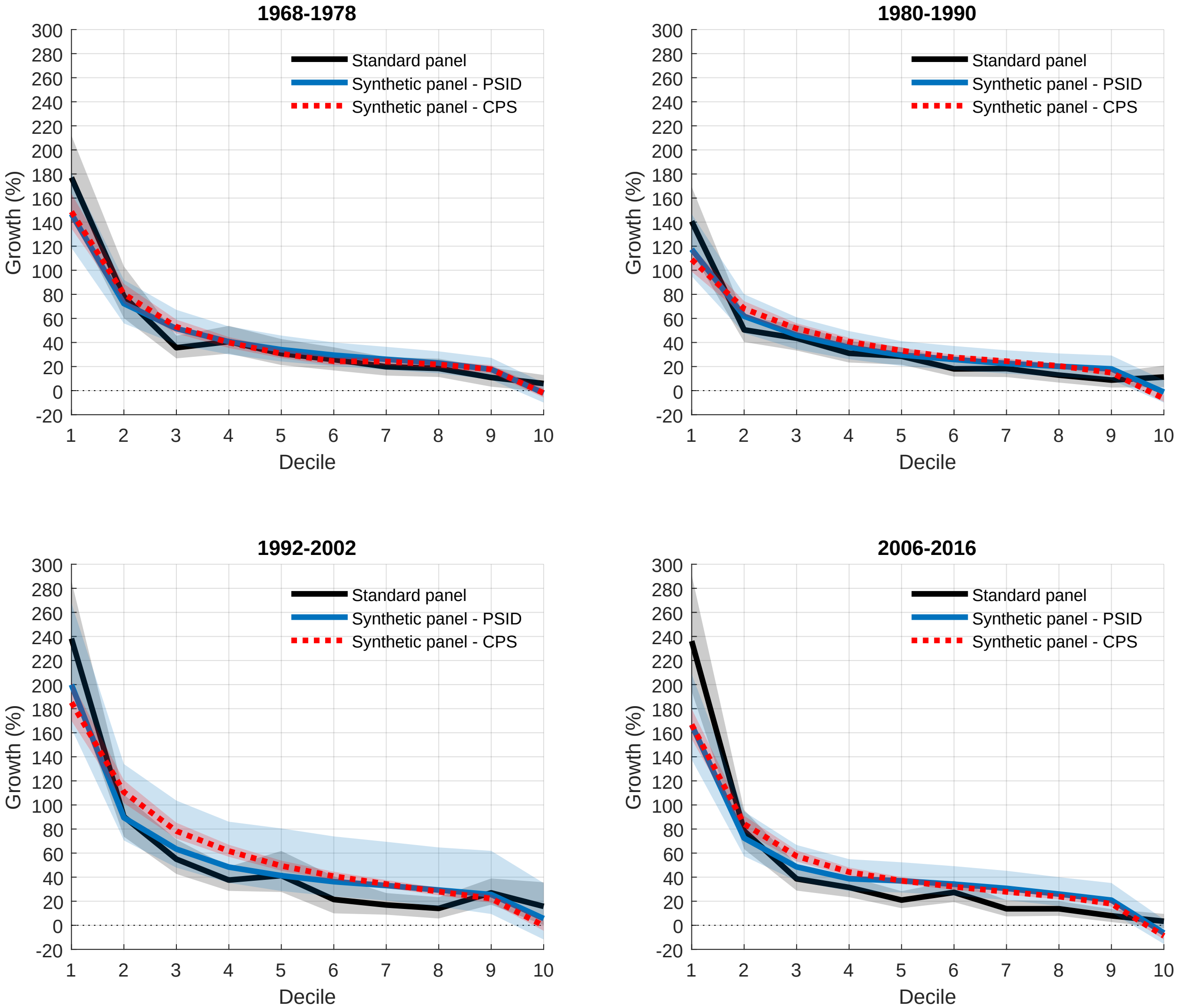

Figure 7: Non-anonymous growth incidence curves for the periods 1968–1978, 1980–1990, 1992–2002, 2006–2016 in the United States. The curves are based on individual annual labor income for all 20–55 year-old workers (in the earlier year of each period) included in the PSID in both the beginning and the end of each period. The black curve is the observed NAGIC in the panel data. The blue curve is estimated as a synthetic panel assuming a Plackett copula, with a parameter that corresponds to the rank correlation estimated in the panel. The dotted red curve is estimated as a synthetic panel obtained from replacing the PSID cross-sectional distributions by the appropriate CPS cross-sectional distributions. The average income growth was normalized to the same level in the PSID-based panels. The shaded areas represent 95% confidence intervals produced by bootstrapping.

based on CPS data.

7 An Indirect Use of Synthetic Panels: Critical Rank CorrelationThreshold

The previous sections discussed the major differences between GICs and NAGICs, also showing that NAGICs can be accurately described using synthetic panels. In this section we go a step forward, aiming to address a social welfare interpretation of NAGICs.

The basic question we analyze in this section is as follows. Suppose you only have cross-sectional income distribution data available by age cohorts of earners. Is it possible to identify the rank correlation coefficient that, combined with the cross-sectional data according to the preceding synthetic panel methodology, will lead to NAGICs with some specific, social welfare related, property? If this rank correlation coefficient falls in the known interval of variation of rank correlations observed in panel data – *i.e.*, Figure 6 – then it is possible to conclude, that this property would hold in NAGICs based on panel data. In other words, the availability of only cross-sectional data allows the analyst to draw conclusions on some properties of the NAGIC that would be derived from panel data on the same period and the same group of people, without such data being available.

The property we are interested in is essentially the downward sloping shape of the NAGIC, except that such a property is ill-defined since there is no reason for a NAGIC to be strictly monotonic. In fact, NAGICs are generally non-monotonic when considering income ranks that are narrow enough, as still a considerable variability of individual incomes is usually observed within such ranks. The downward sloping property considered until now in this paper should thus be understood in a rough way as meaning that the curve is on average higher for low initial income ranks than for high ranks.

It is possible to characterize more rigorously such a 'rough downward sloping property'. Doing so permits generalizing properties initially derived under the assumption of a log-normal bivariate distribution of income (see Section 2). At the same time, this more rigorous characterization of the downward sloping property may be given a clearer social welfare interpretation.

7.1 The non-monotonically mean decreasing (NMMD) criterion

We define the downward sloping property of a function as follows:

Definition 3 (Non-Monotonically Mean Decreasing (NMMD) Function)**.** *An integrable* *function φ on the interval $[0,1]$ is said to be non-monotonically mean decreasing if*

$$\frac{1}{p}\int_0^p \varphi(v)\,dv \geq \frac{1}{1-p}\int_p^1 \varphi(v)\,dv \;\; \forall x \in [0,1] \tag{7.1}$$

This property generalizes the 'rough' downward sloping property requiring simply that a function be higher at the bottom than the top of its interval of definition, whatever the partitions of the income or rank interval into lower and upper sub-intervals. Thus, the incomplete mean of the function from below must not be lower than its incomplete mean from above, for all bipartitions of the $[0,1]$ interval. Note that this definition differs from the practice of averaging a function on successive disjoint intervals – as with deciles in the preceding sections. It also differs from considering moving averages on overlapping intervals.

The NMMD criterion may be given a simple welfare interpretation by seeing it as essentially a generalized 'pro-poorness' criterion, where pro-poorness is defined as the (arithmetic) average growth rate being higher among the poor than among the rich. It is 'generalized' pro-poorness because it is meant to hold for all partitions of the population into lower and higher incomes rather than the population initially below some exogenous poverty line as in Palmisano and Peragine (2015) and further analyzed in Lo Bue and Palmisano (2019).[6] The NMMD criterion is also related to the concept of 'cumulative income growth profile dominance' introduced by Jenkins and Van Kerm (2016), their income growth profile being essentially the NAGIC. Thus, the characterization of the shape of the NAGIC through the NMMD property is instilling a social welfare in the analysis.

The NMMD criterion has a few other social welfare implications or interpretations that are worth mentioning. Denoting $\bar{g}$ the mean individual growth rate in the whole population (which differs from the growth rate of mean income, g, in Eq. (4.5)), it is easily shown that the property that the NAGIC is NMMD is equivalent to:

$$\frac{1}{p}\int_0^p G_f(q)\,dq \geq \bar{g} \;\; \forall p \in [0,1] \; . \tag{7.2}$$

In the words of Jenkins and Van Kerm (2016), this is equivalent to saying that the growth spell being studied welfare-dominates the homothetic spell where all incomes would have grown at rate $\bar{g}$. The opposite to such a statement would be that the observed growth spell

[6] Note that Lo Bue and Palmisano (2019) combine this criterion with the average income growth rate of people who are terminally below the poverty line.

is welfare-dominated by the homothetic spell iff:

$$\frac{1}{1-p}\int_{p}^{1} G_f\left(q\right) dq \geq \bar{g} \;\; \forall p \in [0,1] \; . \tag{7.3}$$

With this NMMD citerion, interpreted as a generalized pro-poorness criterion, we now investigate under what conditions on the rank correlation coefficient a synthetic panel based on two cross-sectional distributions satisfies the NMMD criterion.

7.2 Critical level of the rank correlation coefficient

Any monotonically decreasing function is NMMD. The NAGIC associated with a log-normal bivariate approximation is monotonic. It is thus trivial that this NAGIC is NMMD if and only if it is decreasing. A heuristic argument based on properties of the log-normal approximation discussed in Section 2 can be used to approximate whether a NAGIC is NMMD or not. Since the slope of NAGICs under this approximation depends on the sign of $\rho\sigma_{t'} - \sigma_t$, $\rho_s^{crit} \approx \frac{\sigma_t}{\sigma_{t'}}$ is the critical value of ρ. Below ρ_s^{crit} synthetic NAGICs based on that rank correlation and the two observed initial and terminal cross-sectional distributions are NMMD, or 'generalized pro-poor'.[7]

Synthetic panels allow using pure cross-sectional information on marginal distributions to evaluate whether they likely point to a NAGIC that is NMMD or not. In the United States, we can use data from the CPS at the beginning and end of a given period to estimate ρ_s^{crit} by checking at which value of ρ_s a NAGIC becomes NMMD. If ρ_s^{crit} falls above the normal interval of variation of panel-based rank correlation coefficients, then the NAGIC associated to that period is most likely to be NMMD. No conclusion can be reached without resorting to panel data if ρ_s^{crit} happens to be within the normal range of variation of empirical rank correlation coefficient. Finally, the NAGIC is almost certainly not downward sloping if ρ_s^{crit} is below that range.

We use the IPUMS-CPS microdata from 1967 onward, sampling about 60000 households in every wave. These data are used to obtain the marginal (cross-sectional) labor income distributions. We also use additional relevant covariates from both surveys such as age, number of hours worked, *etc.*.

Figure 8 exhibits ρ_s^{crit} based on these data. It shows that the value of ρ_s^{crit} obtained through bootstrapping is usually between 0.8 and 1 (for 10-year periods), when full-time workers in

[7] We note the difference between ρ, the correlation between log-incomes, and ρ_s, the rank correlation, which are usually very similar but not identical.

all ages are considered. The actual range of variation of the rank correlation for all full-time workers in all ages is significantly lower than that as can be seen in Figure 1. A priori, one could thus say that the NAGICs for this population group are most likely to be NMMD, which they actually are when using PSID data.

For specific age groups this is slightly more ambiguous. For example, for 35 year-old full-time workers, the range of ρ_s^{crit} values is generally between 0.65 and 0.85, with relatively wide confidence bounds. Given the results in Figure 6 it is likely that NAGICs for this specific age group are not categorically NMMD. The results are similar for older workers. For 25 year-old full-time workers ρ_s^{crit} is lower, but the actual rank correlations are also lower than in other age groups.

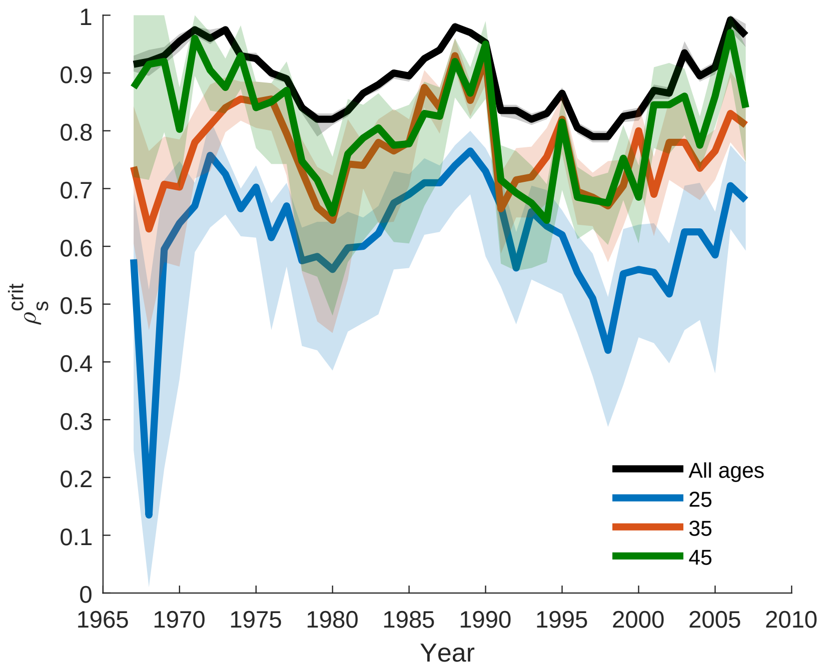

Figure 8: ρ_s^{crit} for 10-year rank correlations in the United States based on CPS data from 1967 onward. The value on the x-axis specifies the initial year of each 10-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. The groups considered were full-time workers in all ages (black); 25 year-olds (blue); 35 real-olds (red); 45 year-olds (green).

8 Discussion

In this paper we studied anonymous and non-anonymous growth incidence curves (GICs and NAGICs, respectively) in the United States since 1968. GICs show the rate of income growth of successive increasing fractiles of the distribution in the initial and final periods. It has become common to represent the anonymous distributional change brought by economic growth using such curves. NAGICs show the mean income growth rate in successive fractiles of the initial distribution rather than the growth rate of the mean income of the same fractiles in the initial and terminal distributions.

We use these tools to highlight the importance of combining mobility and inequality to accurately describe the individual experience of growth. In particular, we ask to what extent the shape of GICs and NAGICs differ for a given time period, and under what conditions they might have substantially different shapes. Most notably, we find that NAGICs in different contexts and in various periods tend to be downward sloping, even in times of increasing cross-sectional inequality.

The paper undertakes a systematic comparison of the shape of GICs and NAGICs in the United States. We use the Panel Study of Income Dynamics (PSID) and find that NAGICs tend to be systematically downward sloping, a property commonly found with income panel data. We show that this property weakens when correcting for income measurement errors and when eliminating life-cycle effects and restricting samples to full-time workers.

We also provide a decomposition of the NAGIC into components corresponding to income rank mobility and to the change in the cross-sectional income distribution. We show that the rank mobility effect imparts a strong downward sloping shape onto the NAGIC. This effect is key in both components of the decomposition. First, mobility contributes to a pure reranking effect, irrespective of the change in the shape of the income distribution. Second, mobility also appears in the component that includes the change in the income distribution shape. It attenuates the effect of the change in the distribution shape on the NAGIC. The decomposition thus demonstrates that rising mobility may lead to flatter or roughly downward sloping NAGICs. This is because higher mobility increases the effect of reranking on one hand, and reduces the contribution of inequality changes to the NAGIC slope, on the other. This helps understanding why, overall, even in presence of a strong increase in cross-sectional income inequality the NAGIC may remain downward sloping.

Taking advantage of the observed stability of rank correlation coefficients, we move to analyze synthetic panels that are based on combining cross-sectional data from the Current Population Survey (CPS) with modeled copulas consistent with empirical observations of income rank correlation. The synthetic NAGICs are then validated against benchmarks. We find that the fundamental characteristic of NAGICs, being downward sloping and then flattening when focusing on narrow age groups and full-time workers, remains valid for synthetic panels.

We finally describe more accurately the shape of NAGICs and their components with the concept of a Non-Monotonically Mean Decreasing function. A NAGIC that is Non-Monotonically Mean Decreasing is such that the average growth rate of the poor is above that of non-poor, whatever the poverty threshold. We stress the importance of the social welfare characterization of NAGICs, which may be interpreted as generalized 'pro-poorness'. If NAGICs for the whole US population of earners are roughly NMMD it turns out this is not the case for specific age groups (after eliminating life-cycle effects). Corresponding NAGICs are not unequivocally NMMD, nor the opposite. This is also in line with the observation that in practice, after taking into account measurement error and life-cycle effects, NAGICs tend to be approximately flat, growth being then distributionally neutral with the respect to initial income ranks, even in times of increasing cross-sectional inequality.

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A The NAGIC of a Bivariate Log-normal Distribution

We consider a bivariate distribution of log-incomes, in which the log-incomes are normally distributed with a Gaussian copula. We also consider the NAGIC as

$$G_f(p) = \int_0^1 R_f(p, p') F_{t'}^{-1}(p') \, dp' - F_t^{-1}(p) \,, \tag{A.1}$$

in which we make the assumption that the difference in log-incomes is the same as the growth rate (*i.e.*, that $g = x_{t+1}/x_t - 1 \approx \log(x_{t+1}) - \log(x_t)$).

The joint probability density function for a Gaussian copula is

$$f(x, y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{x^2 - 2\rho xy + y^2}{2(1-\rho^2)}\right) \,, \tag{A.2}$$

where ρ is the correlation coefficient between the log-incomes, $x = \Phi^{-1}(p')$ and $y = \Phi^{-1}(p)$. It follows that $R_f(p, p') = f(x, y) / \int_{-\infty}^{\infty} f(x, y) \, dy$ can be written explicitly as

$$\frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\left(-\frac{(x - \rho y)^2}{2(1-\rho^2)}\right) \,, \tag{A.3}$$

and the inverse CDF $F_t^{-1}(p)$ can be written as

$$F_t^{-1}(p) = \mu_t + \sigma_t \Phi^{-1}(p) \,, \tag{A.4}$$

and we can then write

$$\begin{aligned} G_f(p) &= \int_0^1 R_f(p, p') \left(\mu_{t'} + \sigma_{t'} \Phi^{-1}(p')\right) dp' - \left(\mu_t + \sigma_t \Phi^{-1}(p)\right) & \text{(A.5)} \\ &= (\mu_{t'} - \mu_t) + \sigma_{t'} \int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp' - \sigma_t \Phi^{-1}(p) \,. & \text{(A.6)} \end{aligned}$$

To evaluate the integral $\int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp'$ we replace $\Phi^{-1}(p')$ by x and the integral limits to $-\infty$ to ∞, and we get

$$\int_0^1 R_f(p, p') \, \Phi^{-1}(p') \, dp' = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}} \exp\left(-\frac{(x - \rho y)^2}{2(1-\rho^2)}\right) dx = \rho y \,. \tag{A.7}$$

We finally obtain

$$G_f(p) = (\mu_{t'} - \mu_t) + (\rho \sigma_{t'} - \sigma_t) \, \Phi^{-1}(p) \,. \tag{A.8}$$

B PSID-Estimated and Plackett-Modeled Copulas

In our analysis of synthetic panels we make use of the property that real copulas, *i.e.*, the joint labor income rank distributions across a period of time, are similar to Plackett copulas. In other words, we assume that the structure of realistic intragenerational copulas can be well approximated by a Plackett copula (Plackett, 1965).[8] This result was found for earnings data in France (Bonhomme and Robin, 2009) and in the United States (Berman, 2021a). Berman (2021a) also showed that the Plackett model provides a better approximation than other commonly used copula models. Figure 9 demonstrates the high similarity between a real copula, based on the PSID data, and a Plackett copula for 1980–1990. It shows that the copulas are statistically indistinguishable.

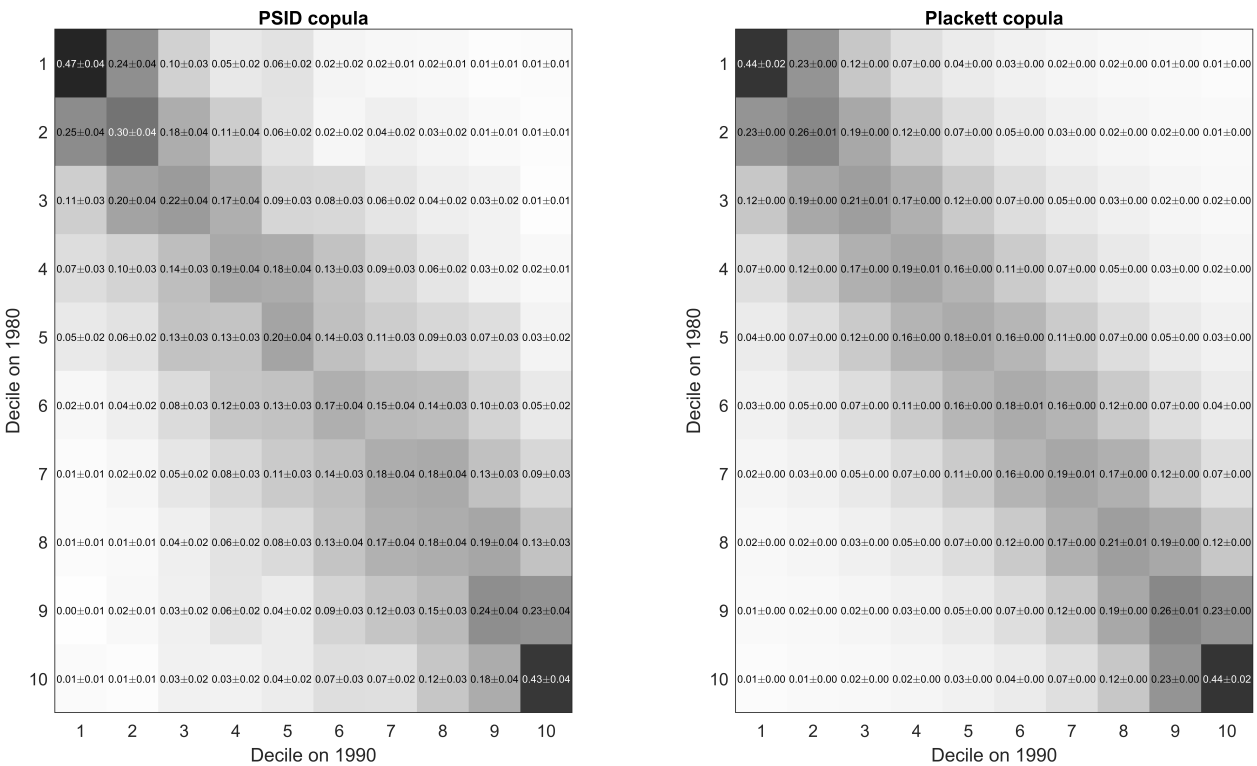

Figure 9: A comparison between a PSID-estimated copula (left) to a Plackett copula with the same rank correlation (right). The calculations are based on PSID data for 1980 and 1990. The errors represent two standard deviations of the estimated value in each cell and produced by bootstrapping.

[8]For income ranks u and v, and given a parameter θ, the Plackett copula is

$$C(u,v) = \frac{1}{2}\theta^{-1}\left(1+\theta(u+v)-\left[(1+\theta(u+v))^2-4\theta(\theta+1)uv\right]^{1/2}\right). \tag{B.1}$$

C Stability of Rank Correlations in 4-year Time Periods

Figure 6 showed that over periods of 10 years the rank correlation of different age groups and of the entire working population has been generally stable over time. It also changes within a narrow interval. These observations are important when constructing synthetic NAGICs. To demonstrate the same stability for shorter time spans, we repeat the calculation done for Figure 6 with periods of 4 years. The results are presented in Figure 10, showing no clear trend over time and narrow intervals, in particular after averaging.

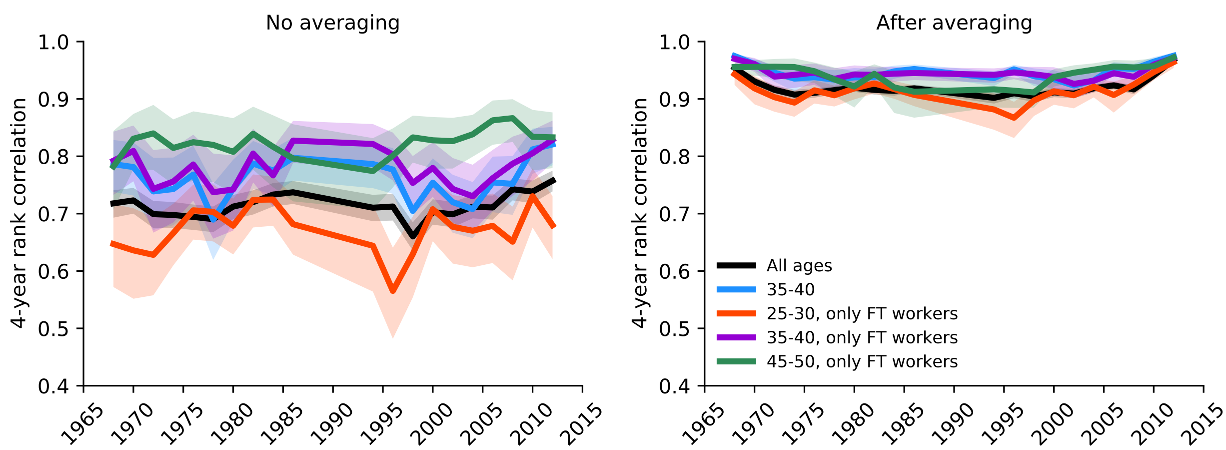

Figure 10: 4-year rank correlations in the United States based on PSID data from 1968 onward. The value on the x-axis specifies the initial year of each 4-year period. The shaded areas represent 95% confidence intervals for the estimates produced by bootstrapping. Several groups are considered: all ages, full-time workers + non-full-time workers; 35–40 year-olds, full-time workers + non-full-time workers; 35–40 year-olds, only full-time workers; 25–30 year-olds, only full-time workers; 45–50 year-olds, only full-time workers. We present estimates before (left) and after (right) income averaging.

D The Effect of Measurement Error on the Average Slope of NAGICs

We would like to assess the impact of measurement error on the slope of the NAGICs. Let X_t be observed log-income at time t, and X_t^* is the true value. We assume a systematic downward bias, δ, in reporting income and some random measurement error, u_t. There may be some autocorrelation, with coefficient ρ in the latter. Hence

$$\begin{aligned} X_t &= (1-\delta)\, X_t^* + u_t && \text{(D.1)} \\ X_{t-1} &= (1-\delta)\, X_{t-1}^* + u_{t-1} && \text{(D.2)} \\ u_t &= \rho u_{t-1} + \epsilon_t\,. && \text{(D.3)} \end{aligned}$$

The expected value of u_t and that of ϵ_t is zero. Both are orthogonal to X_{t-1}, X_t^*, X_{t-1}^* and ϵ_t is orthogonal to u_{t-1}. The observed growth rate in income is (we assume that the difference in log-incomes is the same as the growth rate, *i.e.*, that $g = x_{t+1}/x_t - 1 \approx \log(x_{t+1}) - \log(x_t)$):

$$g_t = X_t - X_{t-1}\,,$$

whereas the true growth rate is:

$$g_t^* = X_t^* - X_{t-1}^*\,.$$

We look for the sign of $\operatorname{Cov}(g_t, X_{t-1})$ as a function of the true covariance $\operatorname{Cov}\left(g_t^*, X_{t-1}^*\right)$ and try to show how the difference is affected by the measurement error, *i.e.*, δ and u_t. From the definition of g_t it comes that

$$g_t = (1-\delta)\, g_t^* + (\rho - 1)\, u_{t-1} + \epsilon_t\,.$$

It follows that:

$$\begin{aligned} \operatorname{Cov}(g_t, X_{t-1}) &= \mathrm{E}(g_t X_{t-1}) - \mathrm{E}(g_t)\, \mathrm{E}(g_t X_{t-1}) \\ &= (1-\delta)^2\, \mathrm{E}\left(g_t^* X_{t-1}^*\right) - (1-\rho)\operatorname{Var}(u_{t-1}) - (1-\delta)^2\, \mathrm{E}(g_t^*)\, \mathrm{E}\left(X_{t-1}^*\right)\,, \end{aligned}$$

or

$$\operatorname{Cov}(g_t, X_{t-1}) = (1-\delta)^2 \operatorname{Cov}\left(g_t^*, X_{t-1}^*\right) - (1-\rho)\operatorname{Var}(u_{t-1})\,.$$

There are two negative biases on the estimation of the covariance between growth rates. One is coming from the systematic under-reporting of income and the second from the measurement error, the latter being attenuated by the autocorrelation in measurement errors. This negative bias translates into an underestimation of the average slope of the NAGIC.

E Correcting the Rank Correlation Bias

Measurement errors in survey data are well-documented. They are known to lead to a downward bias of the correlation between log-incomes at two points in time. We follow Fields et al. (2003) who state that:

> [...] two validation studies of U.S. earnings data compared the Current Population Survey and the Panel Study of Income Dynamics to Social Security or firm records (Bound and Kruger, 1991; Bound et al., 1994). These two studies found the following three regularities: first, the ratio of the variance of measurement error to the variance of true log annual earnings, ranged from 7 to 25 percent. Second, measurement error was negatively correlated with true log earnings and this correlation, was between -0.15 and -0.03.

Accordingly, it is possible to model the measurement error, using the ratio of the variance of measurement error to the variance of true log-income, and the correlation between measurement error and true log-income, as parameters, and correct the correlation between measured log-income at two points time.

The starting point is the log-income (y_{it}) of an individual i at two points in time – t and t'. We measure y_{it}, but the true values (y^*_{it}) are unknown and there is some measurement error:

$$y_{it} = y^*_{it} + u_{it} \tag{E.1}$$

$$y_{it'} = y^*_{it'} + u_{it'}\ , \tag{E.2}$$

and we assume that the error terms have zero expectation.

We estimate

$$\hat{\rho} = \frac{\text{Cov}\left[y_{it}, y_{it'}\right]}{\sigma_{y_{it}}\sigma_{y_{it'}}}\ . \tag{E.3}$$

According to the regularity mentioned above, the error is negatively correlated with the true income, so we can assume

$$u_{it} = \alpha_t y^*_{it} + v_{it} \tag{E.4}$$

$$u_{it'} = \alpha_{t'} y^*_{it'} + v_{it'}\ , \tag{E.5}$$

where v_{it} are independent from the true incomes and α_t is a negative parameter. The measurement errors are also found to be autocorrelated in time (Fields et al., 2003), but these are usually small correlations, and since we are interested in periods of 10 years, they will be negligible.

We also assume that the ratios between the variance of the measurement error and of true income are known parameters:

$$\beta_t = \frac{\mathrm{Var}\left[u_{it}\right]}{\mathrm{Var}\left[y_{it}^*\right]} \tag{E.6}$$

$$\beta_{t'} = \frac{\mathrm{Var}\left[u_{it'}\right]}{\mathrm{Var}\left[y_{it'}^*\right]} . \tag{E.7}$$

Note that the correlation between measurement error and true log-income is

$$\rho_{uy^*} = \frac{\mathrm{E}\left[u_{it} y_{it}^*\right]}{\sqrt{\mathrm{Var}\left[u_{it}\right] \mathrm{Var}\left[y_{it}^*\right]}} = \frac{\alpha_t \mathrm{Var}\left[y_{it}^*\right]}{\sqrt{\beta_t \mathrm{Var}\left[y_{it}^*\right] 2}} = \frac{\alpha_t}{\sqrt{\beta_t}} , \tag{E.8}$$

so given the range of parameters assumed for β_t and ρ_{uy^*}, we can also find α_t, and the model becomes fully specified.

We denote the true correlation between log-incomes as $\rho = \frac{\mathrm{Cov}\left[y_{it}^*, y_{it'}^*\right]}{\sigma^{y_{it'}^*} \sigma^{y_{it'}^*}}$. We would like to get ρ as a function of $\hat{\rho}$, α_t, $\alpha_{t'}$, β_t, and $\beta_{t'}$.

First, it is clear that the variance of v_{it} is $\left(\beta_t - \alpha_t^2\right) \mathrm{Var}\left[y_{it}^*\right]$, and the expectation of v_{it} is $-\alpha_t \mathrm{E}\left[y_{it}^*\right]$. Therefore,

$$\mathrm{Var}\left[y_{it}\right] = \mathrm{Var}\left[y_{it}^* \left(1+\alpha_t\right) + v_{it}\right] = \left(1+\alpha_t\right) 2\mathrm{Var}\left[y_{it}^*\right] + \left(\beta_t - \alpha_t^2\right) \mathrm{Var}\left[y_{it}^*\right] \tag{E.9}$$

$$= \left(1 + 2\alpha_t + \beta_t\right) \mathrm{Var}\left[y_{it}^*\right] . \tag{E.10}$$

The same holds for t'. Now, we can explicitly evaluate the covariance of log-incomes:

$$\mathrm{Cov}\left[y_{it}, y_{it'}\right] = \mathrm{E}\left[y_{it} \cdot y_{it'}\right] - \mathrm{E}\left[y_{it}\right] \mathrm{E}\left[y_{it'}\right] = \mathrm{E}\left[y_{it} \cdot y_{it'}\right] - \mathrm{E}\left[y_{it}^*\right] \mathrm{E}\left[y_{it'}^*\right] , \tag{E.11}$$

and

$$\mathrm{E}\left[y_{it} \cdot y_{it'}\right] = \mathrm{E}\left[y_{it}^* y_{it'}^*\right] \left(1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}\right) - \mathrm{E}\left[y_{it}^*\right] \mathrm{E}\left[y_{it'}^*\right] \left(\alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}\right) . \tag{E.12}$$

We get

$$\mathrm{Cov}\left[y_{it}, y_{it'}\right] = \mathrm{E}\left[y_{it}^* y_{it'}^*\right] \left(1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}\right) \tag{E.13}$$

$$-\mathrm{E}\left[y_{it}^*\right] \mathrm{E}\left[y_{it'}^*\right] \left(1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}\right) \tag{E.14}$$

$$= \mathrm{Cov}\left[y_{it}^*, y_{it'}^*\right] \left(1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}\right) . \tag{E.15}$$

Now can write

$$\hat{\rho} = \frac{\operatorname{Cov}\left[y_{it}, y_{it'}\right]}{\sigma_{y_{it}}\sigma_{y_{it'}}} = \rho \times \frac{1 + \alpha_t + \alpha_{t'} + \alpha_t \alpha_{t'}}{\sqrt{\left(1 + 2\alpha_t + \beta_t\right)\left(1 + 2\alpha_{t'} + \beta_{t'}\right)}}. \tag{E.16}$$

Therefore, given some assumptions on α and β, we get correct the estimated correlation by some factor.

For example, if $\alpha = \alpha_t = \alpha_{t'}$ and $\beta = \beta_t = \beta_{t'}$, the correction factor is $\kappa = \frac{(1+\alpha)^2}{1+2\alpha+\beta}$. If $\beta = 0.25$ and $\rho_{uy^*} = -0.1$, both lie within the range of values reported by Bound and Kruger (1991); Bound et al. (1994), then $\alpha = -0.05$. The correction factor is then $\kappa \approx 0.785$. So the estimated correlation is downward biased by 21.5%, which can then be corrected.

It is important to note that this correction does not eliminate the large statistical uncertainty in the estimates, but only a bias.