

The relationship between relative and absolute mobility – theory and empirics

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Abstract

Chetty et al. (2014a) proposed a new measure of absolute intergenerational income mobility: the fraction of children with greater real-terms income than their parents. Chetty et al. (2017) studied this empirically using United States income data. They found that this measure of absolute income mobility has decreased over the last four decades. They explained this as a consequence of unequal income growth. Here we establish an analytical relationship between their absolute mobility measure and traditional measures of relative mobility. We find that the absolute mobility measure is, *ceteris paribus*, inversely related to traditional relative mobility measures. Additionally, our model suggests mechanisms by which changes in the marginal distributions of parent and child incomes influence absolute mobility. Our findings also offer a way to estimate absolute intergenerational mobility based on existing data on relative intergenerational mobility and on income distributions without the need for high-quality panel data sets unavailable in many countries.

Keywords: Mobility, inequality, bivariate income distributions, copula modeling

JEL Codes: E0, H0, J0, R0

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1 Introduction

The “growing public perception that intergenerational income mobility [...] is declining in the United States” (Chetty et al., 2014*b*, p. 141) has led scholars to seek quantifiable measures of it. Typically such measures are divided into two categories, “which capture different normative concepts” (Chetty et al., 2014*a*, p. 1560), relative and absolute: relative measures gauge children’s propensity to occupy a different position in the income distribution than their parents; absolute measures gauge their propensity to have higher or lower incomes than their parents in money terms. A hypothetical economy in which all children have exactly twice the real incomes of their parents would exhibit minimal relative mobility and maximal absolute mobility. Therefore, the definitions of quoted mobility measures are important.

The canonical measures in each category yield different and contradictory interpretations of ostensibly the same concept, which may mislead the unaware. In addition, “attaching a precise normative significance to ‘income mobility’ is difficult because of the multidimensionality of this concept.” (Fields and Ok, 1999, p. 588)

While relative intergenerational mobility has been studied for decades (Becker and Tomes, 1979; Borjas, 1992; Piketty, 2000; Mazumder, 2005; Aaronson and Mazumder, 2008; Lee and Solon, 2009; Hauser, 2010; Corak, 2013; Chetty et al., 2014*b*; Berman, 2016), investigations of absolute intergenerational mobility remain “scarce, mainly because of the lack of large, high-quality panel data sets linking children to their parents” (Chetty et al., 2017, p. 398). In a recent paper, Chetty et al. (2017) considered trends in absolute mobility in income in the United States since 1940. They define the rate of absolute mobility as the fraction of children earning more than their parents in real terms and at the same age, we denote as A . They show that the rate of absolute mobility has fallen from approximately 90% for children born in 1940 to 50% for children born in the 1980s (see Fig. 1). Based on the Panel Study of Income Dynamics (2017), Isaacs (2007) found that “67% percent of Americans had higher levels of family incomes than their own parents” (for children born during the 1950s and 1960s), agreeing with the values reported by Chetty et al. (2017).

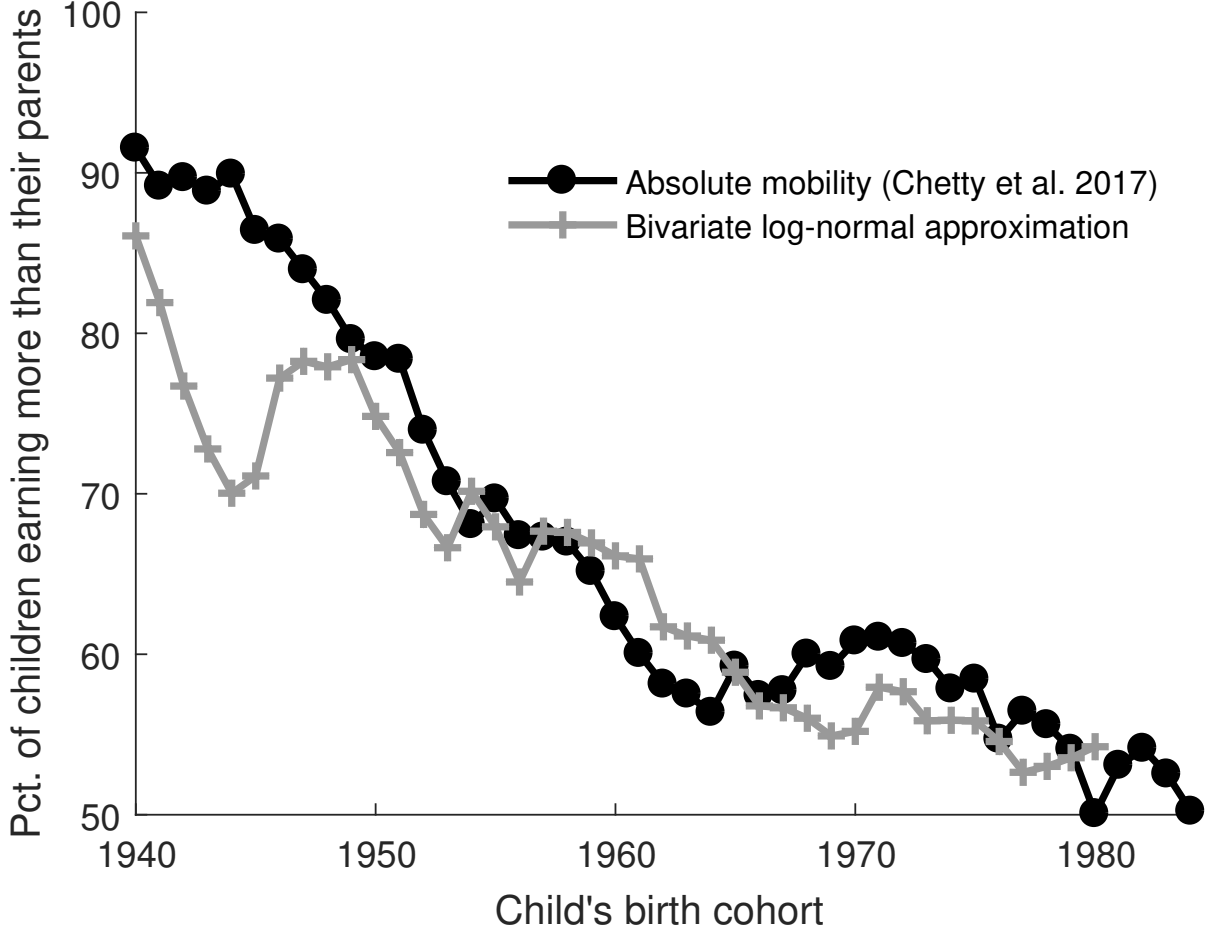


Figure 1: A comparison between the measured (black) and approximated absolute mobility (gray). A bivariate normal distribution reproduces the historical trend of the rate of absolute mobility (the calculations are based on the pre-tax national income per adult and pre-tax income distribution reported in The World Wealth and Income Database (2017) assuming fixed correlation of $\rho = 0.313$). The correlation coefficient between the measured and approximated time series was 0.91.

The canonical measure of relative intergenerational mobility is the elasticity of child income with respect to parent income, known as the intergenerational earnings elasticity (IGE) (Mulligan, 1997; Lee and Solon, 2009; Chetty et al., 2014a). IGE is a measure of immobility rather than of mobility: the larger it is, the stronger the relationship between parent and child income. Therefore, $R_1 \equiv 1 - \beta$ is used as a measure of relative mobility. Unlike absolute mobility, most empirical studies of IGE and other relative mobility measures in the United States show them holding stable over recent decades (Lee and Solon, 2009; Hauser, 2010; Chetty et al., 2014a,b).

An additional important measure of relative mobility is the rank-rank slope (RRS),

defined as the regression coefficient when “regressing the child’s rank on his parents’ rank” (Chetty et al., 2014a, p. 1561). Unlike the IGE, the RRS depends entirely on the copula of the joint income distribution of parents and children. The IGE also depends on the standard deviation of the log-income distributions. Therefore, it is argued that rank-rank slopes “prove to be much more robust across specifications and are thus more suitable for comparisons across areas from a statistical perspective” (Chetty et al., 2014a, p. 1561). Similarly to the IGE, the RRS is also a measure of immobility, and we use $R_2 \equiv 1 - \text{RRS}$ as the corresponding measure of relative mobility.

Co-observations of declining absolute mobility and stable relative mobility require careful interpretation. The explanation of Chetty et al. (2017) for what seems to be a contrast is that income growth has been unequally distributed – positive for high earners and stagnant for the rest – meaning that aggregate income growth has contributed little to absolute mobility. This finding is consistent with their data but may not describe the only mechanism at work.

Here we present a theoretical study of the absolute mobility measure A , particularly focusing on the relationship between A and the two measures of relative mobility mentioned above. We find that under a very general model of the joint log-income distribution, absolute mobility is inversely related to both relative mobility measures – R_1 and R_2 . We find this relationship to be robust for different copula families and marginal distributions.

Although it is argued that “the income distribution is not well approximated by a bivariate log-normal distribution” (Chetty et al., 2014a, p. 1574), we find that the falling absolute mobility in the United States (Chetty et al., 2017) can be well described by a bivariate normal model for the joint log-income distribution. No other choice of copula and marginal distribution resulted in a better description of this trend.

Using the bivariate normal model, it is possible to obtain a closed-form expression for the dependencies of A and R_1 and A and R_2 . This enables us to study mechanisms by which changes in the marginal distributions of parent and child incomes influence absolute mobility, which were not fully described in the past.

Our contribution is fourfold. Firstly, we theoretically study the relationship between distinct measures and interpretations of intergenerational mobility for the first time, suggesting that their co-movement should not, in general, be expected.

Secondly, from an empirical perspective, we are able to show that for describing the long run dynamics of absolute mobility, a model as simple as a bivariate log-normal distribution is satisfactory. This enables using powerful theoretical and empirical tools when addressing the dynamics of absolute mobility.

Thirdly, our findings suggest that using available data on the marginal income distributions and on *relative* intergenerational mobility, one can produce estimates of *absolute* intergenerational mobility without the need for additional high-quality panel data sets, which remain unavailable in most countries.

Fourthly, our study extends a body of theory of economic systems without assuming ergodicity. *YB: Alex - please complete. I am not sure of how to write this paragraph, if at all. YB*

2 Model

Our starting point is a population of N parent-child pairs. We denote by Y_p^i and Y_c^i the incomes of the parent and the child (at the same age), respectively, in family $i = 1 \dots N$. We assume the incomes are all positive and define the log-incomes $X_p^i = \log Y_p^i$ and $X_c^i = \log Y_c^i$.

The intergenerational earnings elasticity (IGE) is defined as the slope (β) of the linear regression

$$X_c = \alpha + \beta X_p + \epsilon, \quad (2.1)$$

where α is the regression intercept and ϵ is the error term. We define $R_1 \equiv 1 - \beta$ as the corresponding measure of intergenerational mobility.

The rank-rank slope (sometimes called the rank-rank correlation or Spearman's ρ) is defined as the correlation coefficient between the child percentile rank in the income

distribution to that of parents'. We denote it by ρ_S and define $R_2 \equiv 1 - \rho_S$ as the corresponding measure of intergenerational mobility.

The rate of absolute mobility, denoted by A , as defined and measured by Chetty et al. (2017) is the fraction of children earning more than their parents, equal to the probability $P(X_c - X_p > 0)$.

One hypothetical sample of the joint parents and children log-income distribution is presented in Fig. 2. Fig. 2 also depicts how A and β are defined – the blue line is $y = x$, hence the rate of absolute mobility is defined as the fraction of parent-child pairs which are above it. The red line is the linear regression $y = \alpha + \beta x$, for which β is the IGE.

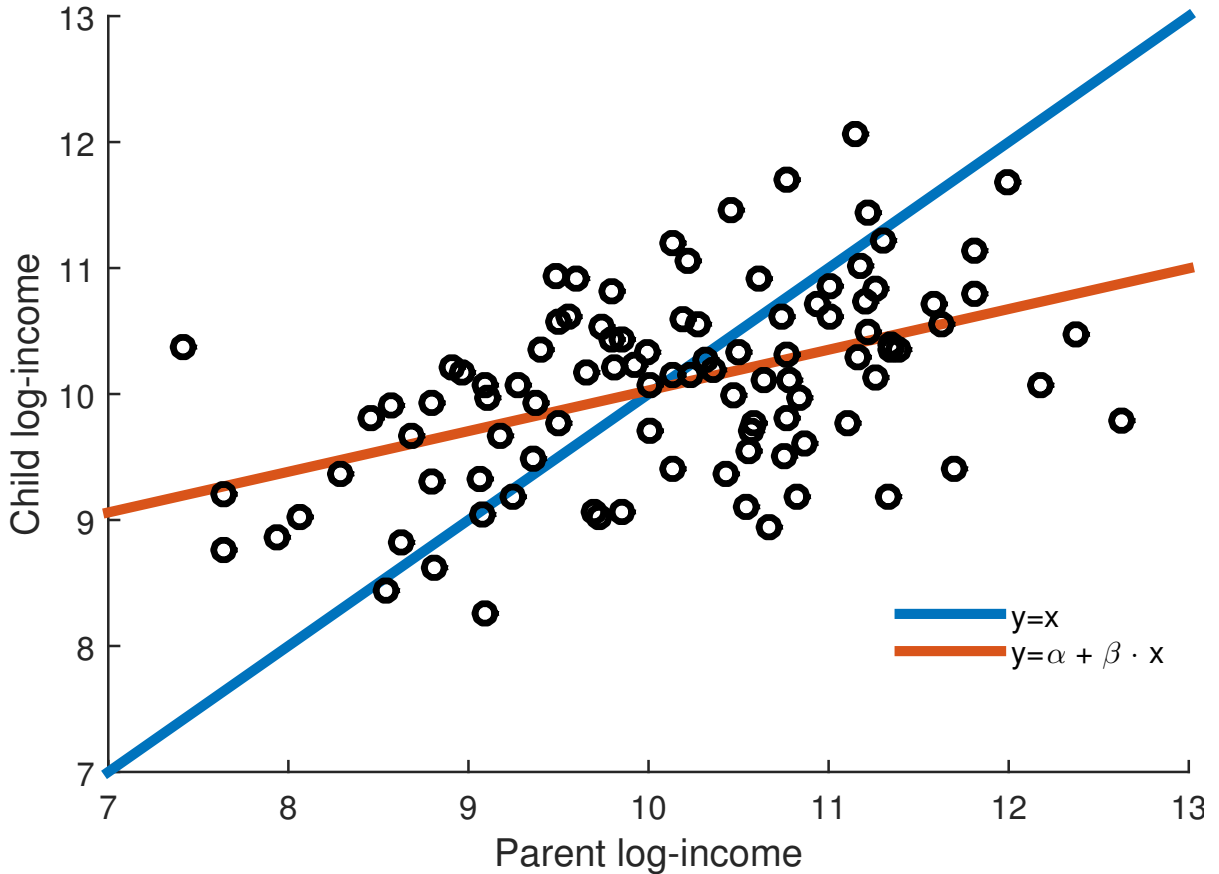


Figure 2: An illustration of the absolute and relative mobility measures. The black circles are a randomly chosen sample of 100 parent-child log-income pairs, assuming a bivariate normal distribution. The parameters used were $\mu_p = 10.1$, $\sigma_p = 0.78$ (for the parents marginal distribution) and $\mu_c = 10.25$, $\sigma_c = 1.15$ (for the children marginal distribution) with correlation of $\rho = 0.6$. The resulting α and β were 1.8 and 0.84, respectively.

Since income distributions are known to be well approximated by the log-normal distribution (Pinkovskiy and Sala-i-Martin, 2009), a simple plausible model for the joint

distribution of parent and child log-incomes is the bivariate normal distribution. Under this assumption, the marginal income distributions of both parents and children are log-normal and the correlation between their log-incomes is defined by a single parameter ρ . The marginal distributions of the parents and the children follow $\mathcal{N}(\mu_p, \sigma_p^2)$ and $\mathcal{N}(\mu_c, \sigma_c^2)$, respectively. Hence the joint distribution is fully characterized by 5 parameters: μ_p , σ_p , μ_c , σ_c and ρ .

In practice, our choice of model may affect the analysis of the absolute and relative mobility measures. Other possible models for the joint income distribution of parents and children can include marginal distributions which are not log-normal, such as beta and gamma distributions, as well as other types of copula. In the bivariate log-normal model the copula is Gaussian, while other types found useful for various empirical applications (Trivedi and Zimmer, 2007), such as the Clayton, the Gumbel and the Plackett copula families (Bonhomme and Robin, 2009), may prove to be a better description of the relationship between the marginal income distributions. In their study of intergenerational mobility in France, Bonhomme and Robin (2009) argue that the Gaussian copula “tends to underestimate the dependence in the middle of the distribution, that is, the probabilities of remaining in the second, third, and fourth quintiles” and show that the empirical copula is best estimated by the Plackett copula. In Appendix B we show that the bivariate log-normal model with Gaussian copula reproduces the measured historical trend of absolute mobility in the United States as good as the other models tested.

3 Results

We first address the properties of the bivariate log-normal approximation. In particular, we derive close form expressions for the measures of mobility – A , R_1 and R_2 , in terms of the distribution parameters:

Proposition 1 *For a bivariate normal distribution with parameters μ_p , σ_p (for the parents marginal distribution) and μ_c , σ_c (for the children marginal distribution) assuming correlation ρ , the relative mobility R_1 is*

$$R_1 = 1 - \frac{\sigma_c}{\sigma_p} \rho. \quad (3.1)$$

Following Prop. 1 it is also possible to derive the rate of absolute mobility as a function of the distribution parameters and the IGE:

Proposition 2 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and $\rho = \sigma_p \beta / \sigma_c$ (where β is the IGE), the rate of absolute mobility is*

$$A = \Phi \left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} \right), \quad (3.2)$$

where Φ is the cumulative distribution function of the standard normal distribution.

Equation (3.2) enables the calculation of A assuming the knowledge of the marginal income distributions and β (or R_1). It is therefore also possible to calculate A in terms of R_2 :

Corollary 1 *Under Prop. 2 notations, the rate of absolute mobility A is*

$$A = \Phi \left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - \frac{4\sigma_c}{\sigma_p} \sin \left(\frac{\pi(1-R_2)}{6} \right) \right) + \sigma_c^2}} \right). \quad (3.3)$$

where Φ is the cumulative distribution function of the standard normal distribution.

Our next step is to test whether the bivariate log-normal model for the joint income distribution is empirically sound. For that purpose we compare the model prediction for the historical rate of absolute mobility in the United States with the historical rate reported by Chetty et al. (2017).

We use data for the pre-tax national income per adult in the United States and the top 10% income share data (The World Wealth and Income Database, 2017) to obtain μ_p, σ_p, μ_c and σ_c every year. Since in the bivariate log-normal model, the marginal log-income distributions are normal, these parameters can be obtained directly and no fit is required.

The Lorenz curve of the log-normal distribution $\log \mathcal{N}(\mu, \sigma^2)$ is $\Phi(\Phi^{-1}(z) - \sigma)$ (Cowell, 2011). Therefore, top 10% income share s corresponds to

$$\sigma = \Phi^{-1}(0.9) - \Phi^{-1}(1 - s) . \quad (3.4)$$

If we denote the per-capita pre-tax income as m , it follows that the parameter μ is

$$\mu = \log(m) - \frac{\sigma^2}{2} . \quad (3.5)$$

Using Eqs. (3.1–3.3) in order to calculate the historical value of A also requires knowing the correlation ρ . Chetty et al. (2014b) report a consistent RRS of 0.3, which corresponds to a fixed value of $\rho = 2 \sin(0.05\pi) \approx 0.313$ (see the proof of corollary 1 or Trivedi and Zimmer (2007) for the relationship between ρ and the RRS).

The gray curve in Fig. 1 shows that the model reproduces faithfully the evolution of absolute mobility reported by Chetty et al. (2017), despite its comparative methodological naïvety. The correlation coefficient between the measured and approximated time series was 0.91. The model only fails in reproducing the reported absolute mobility for the 1940s birth cohorts. A possible explanation is that for these cohorts the value of ρ , and hence the IGE and the RRS values, were substantially different from the reported RRS of 0.3, since this estimation was based only on the 1971–1986 birth cohorts (Chetty et al., 2014b). However, this cannot be the only explanation, since the quoted absolute mobility values are also based on a time-constant RRS (Chetty et al., 2017).

In Appendix B we show that our results are robust under different choices of copula and when the gamma distribution approximation for the marginal income distributions is used instead of the log-normal approximation. The choice of copula is of no importance to the values of the absolute mobility. In addition, the log-normal approximation for the marginal distributions provides consistent better results than the gamma distribution approximation.

3.1 Absolute mobility and median incomes

Chetty et al. (2017) also find that the share of children earning more than the median parent declined from 92% in the 1940 birth cohort to 45% in the 1984 cohort (Katz and Krueger, 2017). This alternative definition of absolute mobility move almost identically to A across cohorts in the United States (Katz and Krueger, 2017). Defining the share of children earning more than the median parent as \tilde{A} , it simply follows that in the bivariate log-normal model \tilde{A} is defined as

$$\tilde{A} \equiv \Phi \left(\frac{\mu_c - \mu_p}{\sigma_c} \right), \quad (3.6)$$

where Φ is the cumulative distribution function of the standard normal distribution.

Using \tilde{A} has obvious advantages over A . In particular, they can be “directly computed from standard public-use cross-sectional household survey data and do not require data that longitudinally link children to parents” (Katz and Krueger, 2017, p. 382). However, \tilde{A} would be close to A only if the IGE is close to 0.5:

Proposition 3 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and assuming IGE of β , then*

$$A = \tilde{A} \iff \beta = \frac{1}{2}. \quad (3.7)$$

It is therefore no surprise that for the United States A and \tilde{A} are relatively similar – Aaronson and Mazumder (2008) estimate the IGE for the 1950–1970 birth cohorts at 0.46–0.58. In countries such as Denmark or Finland, in which the IGE is substantially lower than 0.5 (Corak, 2013), using \tilde{A} neglects the part high relative mobility plays in determining A . This will lead to significant overestimation of absolute intergenerational mobility. For example, in Denmark, the average estimated A for 1950–1980 birth cohorts is 78.7% (see Sec. 3.2). If \tilde{A} is considered, the average value is 87%. For the United States this difference would be considerably smaller – 70.7% and 72.8%, respectively. Setting aside the normative question on which measure of absolute intergenerational mobility is

of particular interest, we emphasize that \tilde{A} cannot be used as a proxy for A , unless the IGE is close to 0.5.

3.2 Estimating absolute mobility globally

Without directly estimating absolute mobility the way done by Chetty et al. (2017), it is possible to use the bivariate log-normal model for describing the dynamics of absolute mobility in many countries under the assumption that their RRS is fixed and is known. Although this seems as a very limiting assumption, in practice, in most cases, the effect of the RRS value on the estimated absolute mobility is relatively mild. A 10% error in the estimation of the RRS yields an error of about 1% in the average estimated absolute mobility for the United States. We note, however, that for specific birth cohorts the effect of changing the RRS on the absolute mobility may be dramatic, as described below and illustrated in Fig. 4.

Using available data on the marginal income distributions and on relative intergenerational mobility, we demonstrate how one can produce estimates of absolute intergenerational mobility without the need for high-quality panel data sets. Such data sets remain unavailable in most countries. This is depicted in Fig. 3 for France, Sweden and Denmark, in addition to the reproduced absolute mobility in the United States presented above. The calculations are based on the pre-tax national income per adult and the income share data for France, Sweden and Denmark as reported in The World Wealth and Income Database (2017) and on measurements of the IGE as reported by Lefranc and Trannoy (2005) for France, by Björklund and Jäntti (1997) for Sweden and by Landersø and Heckman (2017) for Denmark.

The results indicate that the decrease in absolute mobility rates found in the United States also occurred in France, Sweden and Denmark, however in these countries, this decrease is mainly due to decreasing income growth rates rather than rising inequality. The results also echo stylized facts of relative mobility – Denmark and Sweden are found as the most mobile of the countries analyzed, and the United States as the least.

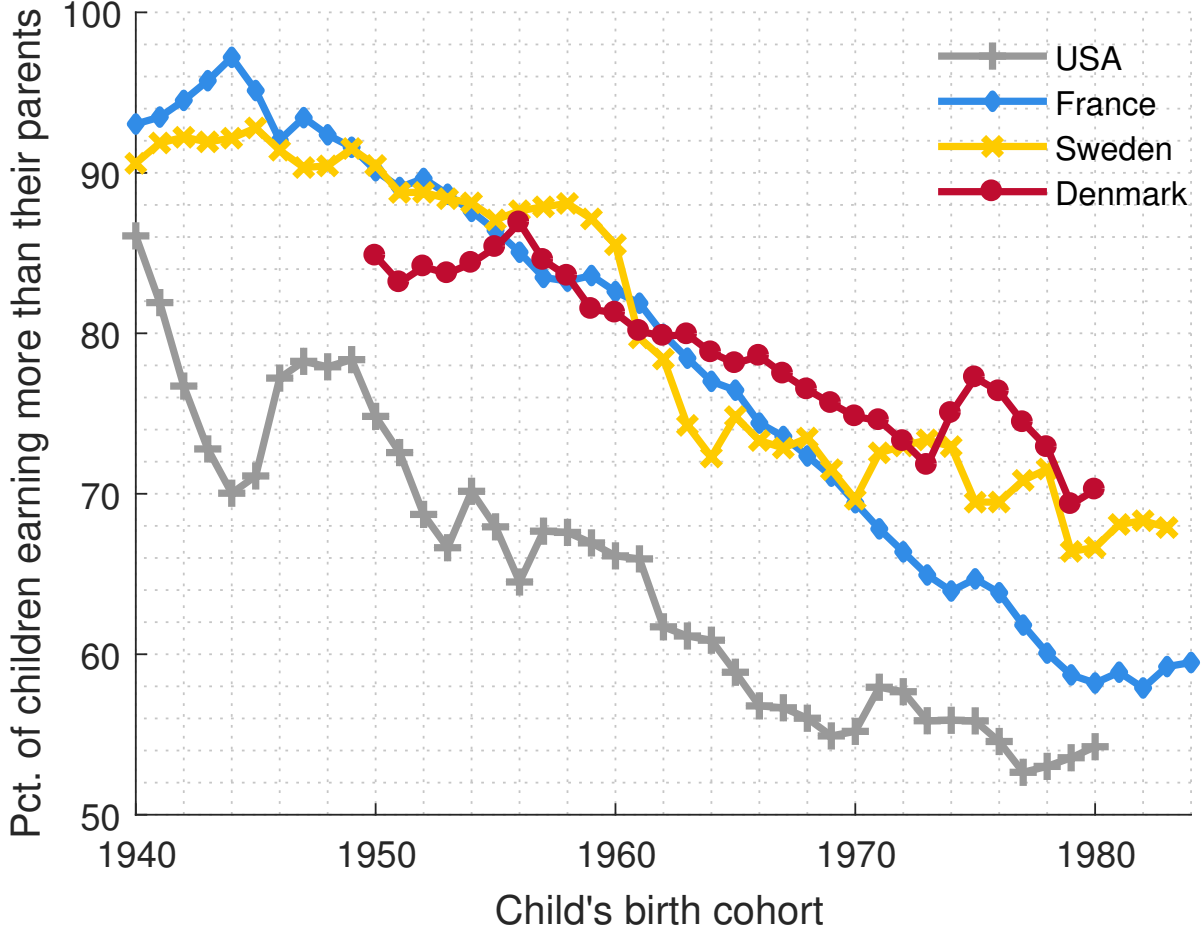


Figure 3: Estimated rate of absolute mobility in the United States, France, Sweden and Denmark assuming the bivariate log-normal model and assuming fixed RRS of 0.3 (Chetty et al., 2014a), 0.42 (Lefranc and Trannoy, 2005), 0.3 (Björklund and Jäntti, 1997) and 0.19 (Landersø and Heckman, 2017), respectively.

3.3 Absolute and relative intergenerational mobility

Having established the model's empirical soundness, we can use its properties to further study the measures of mobility – A , R_1 and R_2 . Eqs. (3.2–3.3) demonstrate that the rate of absolute mobility can be explicitly described as a function of the relative mobility measures R_1 and R_2 . Fig. 4 shows A as a function of R_1 and R_2 for different birth cohorts in the United States. It shows that the bivariate normal model – with positive income growth and inequality changes consistent with data, but absent other effects – predicts an *inverse* relationship between absolute and relative mobility.

Proposition 2 illustrates that the rate of absolute mobility increases with increasing income growth and decreases with increasing income inequality, as described by Chetty

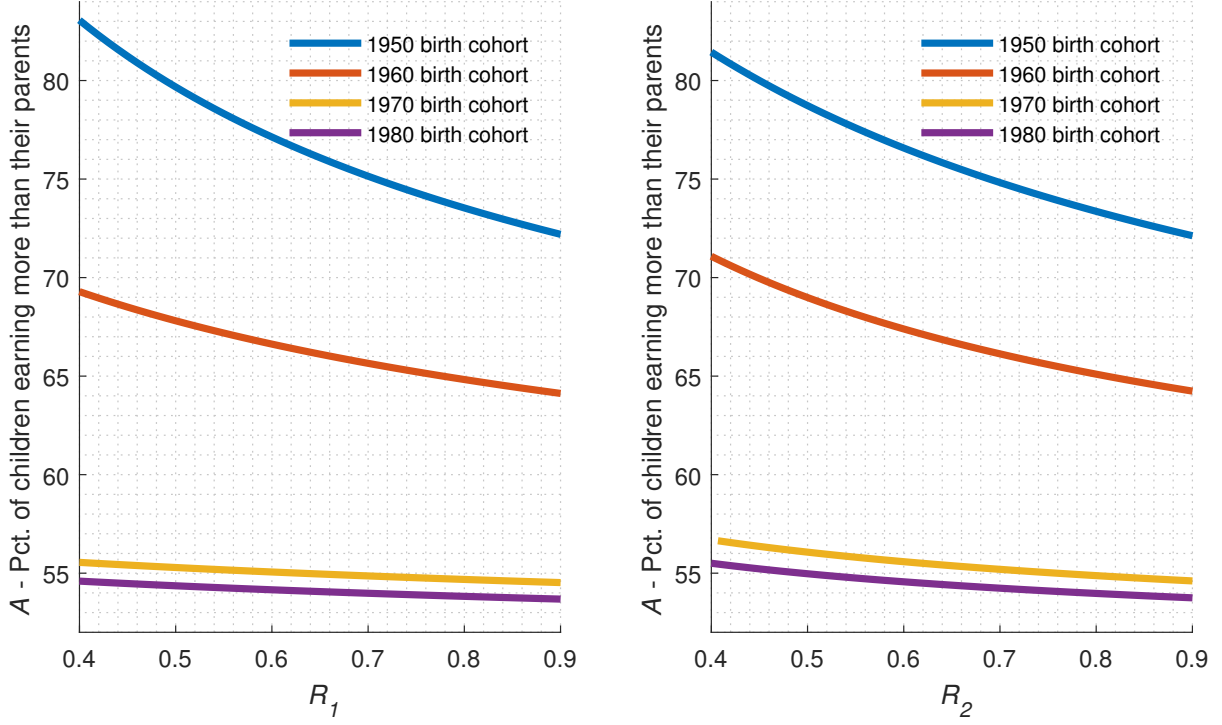


Figure 4: The theoretical relationship between the rate of absolute mobility, R_1 (left) and R_2 (right), assuming the bivariate normal log-incomes model for different birth cohorts in the United States. This demonstrates the inverse relationship between absolute and relative mobility measures.

et al. (2017). However, it also demonstrates that an additional mechanism can be at play, since the rate of absolute mobility decreases with increasing relative mobility. The direct implication is that if the relative mobility in the United States had been decreasing during the past few decades, as some argue (Aaronson and Mazumder, 2008; Putnam, Frederick and Snellman, 2012), the decrease in absolute mobility is less significant than reported by Chetty et al. (2017).

Figure 4 seemingly stands in contrast to the findings about absolute intergenerational mobility in the United States, France, Sweden and Denmark (see Fig. 3), since the absolute and relative mobility in those countries were found to be ordered similarly. However, in practice, had the relative mobility in the United States been higher, the difference in absolute mobility rates with respect to the other countries would have become even larger.

4 Discussion

The seemingly counterintuitive inverse theoretical relationship found between absolute and relative mobility stems from a fundamental conceptual difference between the two categories of mobility. It exposes the problems that can arise if both are treated as measuring similarly the same phenomenon. In particular, absolute mobility is very sensitive to across-the-board economic growth. For example, during the Middle Ages – when relative mobility rates were low because social class and profession were predominantly inherited (Goldthorpe, 1987; Clark, 2014), even the slightest positive or negative income growth would result in very high or low absolute mobility. A misleading picture of intergenerational mobility may arise if the basic properties of these measures are overlooked. Therefore, empirically addressing this measure of intergenerational mobility requires careful delineation of the phenomena of interest and the manner in which quoted measures reflect them. In this study we exposed this particular property of the absolute mobility measure.

From an empirical perspective, our findings imply that a model as simple as a bivariate log-normal distribution is satisfactory for describing the dynamics of absolute mobility. Thus, using available data on the marginal income distributions and on relative intergenerational mobility, one can produce estimates of absolute intergenerational mobility without the need for high-quality panel data sets, which remain unavailable in most countries.

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A Proofs

A.1 Proof of proposition 1

First, by definition, the correlation ρ , between X_p and X_c equals to their covariance, divided by $\sigma_p\sigma_c$

$$\rho = \frac{\text{Cov}[X_p, X_c]}{\sigma_p\sigma_c}. \quad (\text{A.1})$$

β can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^N (X_p^i - \bar{X}_p) (X_c^i - \bar{X}_c)}{\sum_{i=1}^N (X_p^i - \bar{X}_p)^2}, \quad (\text{A.2})$$

where \bar{X}_p and \bar{X}_c are the average parents and children log-incomes, respectively.

It follows that

$$\beta = \frac{\text{Cov}[X_p, X_c]}{\sigma_p^2}. \quad (\text{A.3})$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p} \rho \quad (\text{A.4})$$

and therefore

$$1 - \beta = R_1 = 1 - \frac{\sigma_c}{\sigma_p} \rho \quad (\text{A.5})$$

■

A.2 Proof of proposition 2

We start by defining a new random variable $Z = X_c - X_p$. It follows that calculating A is equivalent to calculating the probability $P(Z > 0)$.

Subtracting two dependent normal distributions yields that

$$Z \sim \mathcal{N}(\mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2\text{Cov}[X_p, X_c]), \quad (\text{A.6})$$

so according to Prop. 1

$$Z \sim \mathcal{N}(\mu_c - \mu_p, \sigma_p^2(1 - 2\beta) + \sigma_c^2) . \quad (\text{A.7})$$

It follows that

$$\frac{Z - (\mu_c - \mu_p)}{\sqrt{\sigma_p^2(1 - 2\beta) + \sigma_c^2}} \sim \mathcal{N}(0, 1) , \quad (\text{A.8})$$

so we can now write

$$\begin{aligned} P(Z > 0) &= \\ P\left(\frac{Z - (\mu_c - \mu_p)}{\sqrt{\sigma_p^2(1 - 2\beta) + \sigma_c^2}} > -\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2(1 - 2\beta) + \sigma_c^2}}\right) &= \\ \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2(1 - 2\beta) + \sigma_c^2}}\right) , \end{aligned} \quad (\text{A.9})$$

where Φ is the cumulative distribution function of the standard normal distribution. ■

A.3 Proof of corollary 1

For the bivariate log-normal model it is known that (Trivedi and Zimmer, 2007)

$$1 - R_2 = \frac{6 \arcsin \frac{\rho}{2}}{\pi} . \quad (\text{A.10})$$

hence, using Eq. (3.1)

$$R_2 = 1 - \frac{6 \arcsin \frac{\sigma_p \beta}{2\sigma_c}}{\pi} . \quad (\text{A.11})$$

Substituting into Eq. (3.2) we obtain

$$A = \Phi \left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 \left(1 - \frac{4\sigma_c}{\sigma_p} \sin \left(\frac{\pi(1-R_2)}{6} \right) \right) + \sigma_c^2}} \right). \quad (\text{A.12})$$

■

A.4 Proof of proposition 3

Following Eq. (3.2)

$$A = \Phi \left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} \right). \quad (\text{A.13})$$

Following Eq. (3.6)

$$\tilde{A} = \Phi \left(\frac{\mu_c - \mu_p}{\sigma_c} \right) \quad (\text{A.14})$$

Therefore

$$\tilde{A} = A \iff \frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} = \pm \frac{\mu_c - \mu_p}{\sigma_c}. \quad (\text{A.15})$$

We then obtain

$$\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2}} = \pm \frac{\mu_c - \mu_p}{\sigma_c} \iff \sigma_c = \pm \sqrt{\sigma_p^2 (1 - 2\beta) + \sigma_c^2} \iff \beta = \frac{1}{2}. \quad (\text{A.16})$$

■

B Model robustness check

Our analysis was done for a bivariate log-normal model, assuming the marginal log-income distributions are normal and that their copula is Gaussian. This enables the derivation of Eq. (3.2), useful for investigating the relationship between absolute and

relative mobility. This model for the joint income distributions of parents and children was found as unsatisfactory for some applications (Bonhomme and Robin, 2009). Therefore, we wish to check how robust is this model when compared to other possible models, specifically applying it for calculating absolute mobility.

For that purpose we assume two possible shapes for the marginal income distributions – log-normal and gamma, both used in numerous studies as approximations of empirical income distributions (Salem and Mount, 1974; Pinkovskiy and Sala-i-Martin, 2009). The historical parameters for the marginal distributions were calculated using data for the pre-tax national income per adult in the United States and the income share data (The World Wealth and Income Database, 2017), as described in 3.

In addition, we assume four possible copula forms – Gaussian, Clayton, Gumbel and Plackett. The reader is referred to Trivedi and Zimmer (2007) and Bonhomme and Robin (2009) for a discussion in different uses of the different copula forms and their mathematical properties.

We test eight cases – assuming each copula form and assuming log-normal and gamma marginal distributions. For each copula choice we also calculate the copula dependence parameter θ (Trivedi and Zimmer, 2007), so it produces a rank-rank slope of 0.3 (Chetty et al., 2014a). For each case we calculate its root-mean-square deviation (RMSD) from the data and the relative absolute error (RAE).

The results of the robustness check are presented in Tab. 1. They show that using the log-normal marginal distributions yields consistently better results than when using gamma marginal distributions. In addition, the result were essentially independent of the copula form choice. None of the eight possible combinations of marginal distribution and copula stood out in comparison to the others.

Figure 5 depicts the absolute intergenerational mobility resulting in each of the eight combinations in comparison to data. It illustrates the results presented in Tab. 1 – particularly that the difference between the various copula forms is very small and that the log-normal approximation provides a much better description of the absolute mobility dynamics than the gamma distribution approximation.

Table 1: Model robustness check results

Marginal distribution	Copula	RMSD	RAE
Log-normal	Gaussian	0.06	0.42
	Clayton	0.06	0.43
	Gumbel	0.06	0.39
	Plackett	0.06	0.42
Gamma	Gaussian	0.10	0.76
	Clayton	0.10	0.80
	Gumbel	0.09	0.67
	Plackett	0.09	0.72

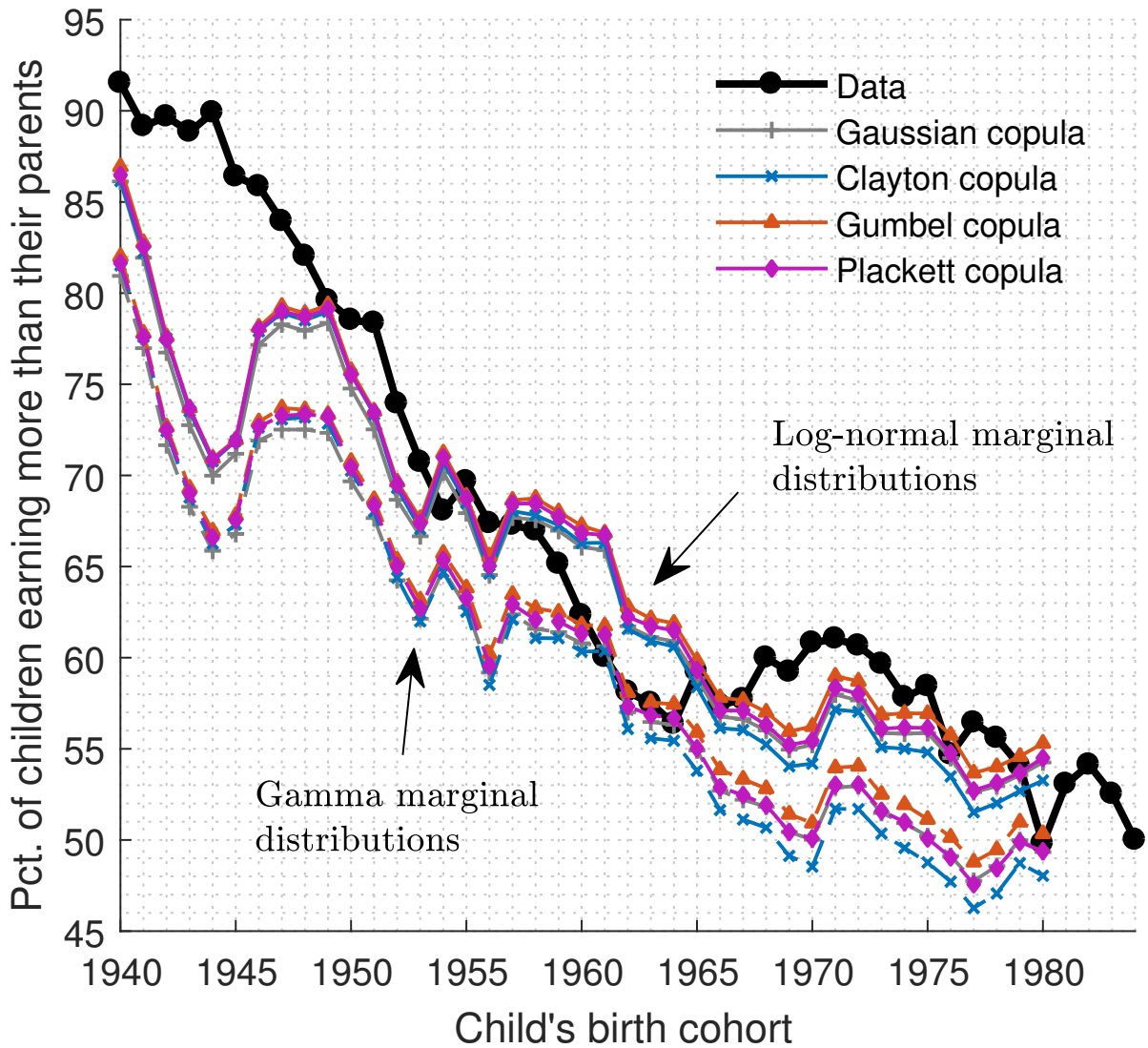


Figure 5: A comparison between the measured (black) and calculated absolute mobility for different copula forms using log-normal marginal distributions (solid lines) and using gamma marginal distributions (dashed lines).

The robustness check shows that the Gaussian copula with log-normal marginal distributions, *i.e.* the bivariate log-normal model, provides results which are no worse than any other combination. We therefore conclude that for the purpose of modeling the dynamics of absolute mobility, the bivariate log-normal model is indeed a good model.