The relationship between relative and absolute mobility – theory and empirics

Yonatan Berman* and Alexander Adamou†‡

Original version: June 20, 2017 Last updated: October 2, 2017

Abstract

Chetty et al. (2014*a*) proposed a new measure of absolute intergenerational income mobility: the fraction of children with greater real-terms income than their parents. Chetty et al. (2017) studied this empirically using United States income data. They found that this measure of absolute income mobility has decreased over the last four decades. They explained this as a consequence of unequal income growth. Here we establish an analytical relationship between their absolute mobility measure and traditional measures of relative mobility. We find that the absolute mobility measure is, ceteris paribus, inversely related to traditional relative mobility measures. Additionally, our models suggest mechanisms by which changes in the marginal distributions of parent and child incomes influence absolute mobility.

Keywords: Mobility, inequality, bivariate income distributions

JEL Codes: E0, H0, J0, R0

*Paris School of Economics, Paris, France, yonatanb@post.tau.ac.il

†London Mathematical Laboratory, London, UK, a.adamou@lml.org.uk

‡We wish to thank Tslil Aloni, Mikkel Høst Gandil, Ole Peters, Yoash Shapira and Alex Teytelboym for fruitful discussions and for their feedback on the manuscript.

1 Introduction

The "growing public perception that intergenerational income mobility [...] is declining in the United States" (Chetty et al., 2014*b*, p. 141) has led scholars to seek quantifiable measures of it. Typically such measures are divided into two categories, "which capture different normative concepts" (Chetty et al., 2014*a*, p. 1560), relative and absolute: relative measures gauge children's propensity to occupy a different position in the income distribution than their parents; absolute measures gauge their propensity to have higher income than their parents in money terms. A hypothetical economy in which all children have exactly twice the real incomes of their parents would exhibit minimal relative mobility and maximal absolute mobility. Therefore, the definitions of quoted mobility measures are important.

The canonical measures in each category yield different and contradictory interpretations of ostensibly the same concept, which may mislead the unaware. In addition, "attaching a precise normative significance to "income mobility" is difficult because of the multidimensionality of this concept." (Fields and Ok, 1999, p. 588)

While relative intergenerational mobility has been studied for decades (Mazumder, 2005; Aaronson and Mazumder, 2008; Lee and Solon, 2009; Hauser, 2010; Corak, 2013; Chetty et al., 2014*b*; Berman, 2016), investigations of absolute intergenerational mobility remain "scarce, mainly because of the lack of large, high-quality panel data sets linking children to their parents in the United States" (Chetty et al., 2017, p. 398). In a recent paper, Chetty et al. (2017) considered trends in absolute mobility in income in the United States since 1940. They define the rate of absolute mobility as the fraction of children earning more than their parents in real terms and at the same age. They show that the rate of absolute mobility has fallen from approximately 90% for children born in 1940 to 50% for children born in the 1980s (see Fig. 1).

The canonical measure of relative intergenerational mobility is the elasticity of child income with respect to parent income, known as the intergenerational earnings elasticity (IGE) (Mulligan, 1997; Lee and Solon, 2009; Chetty et al., 2014*a*). IGE is a measure of

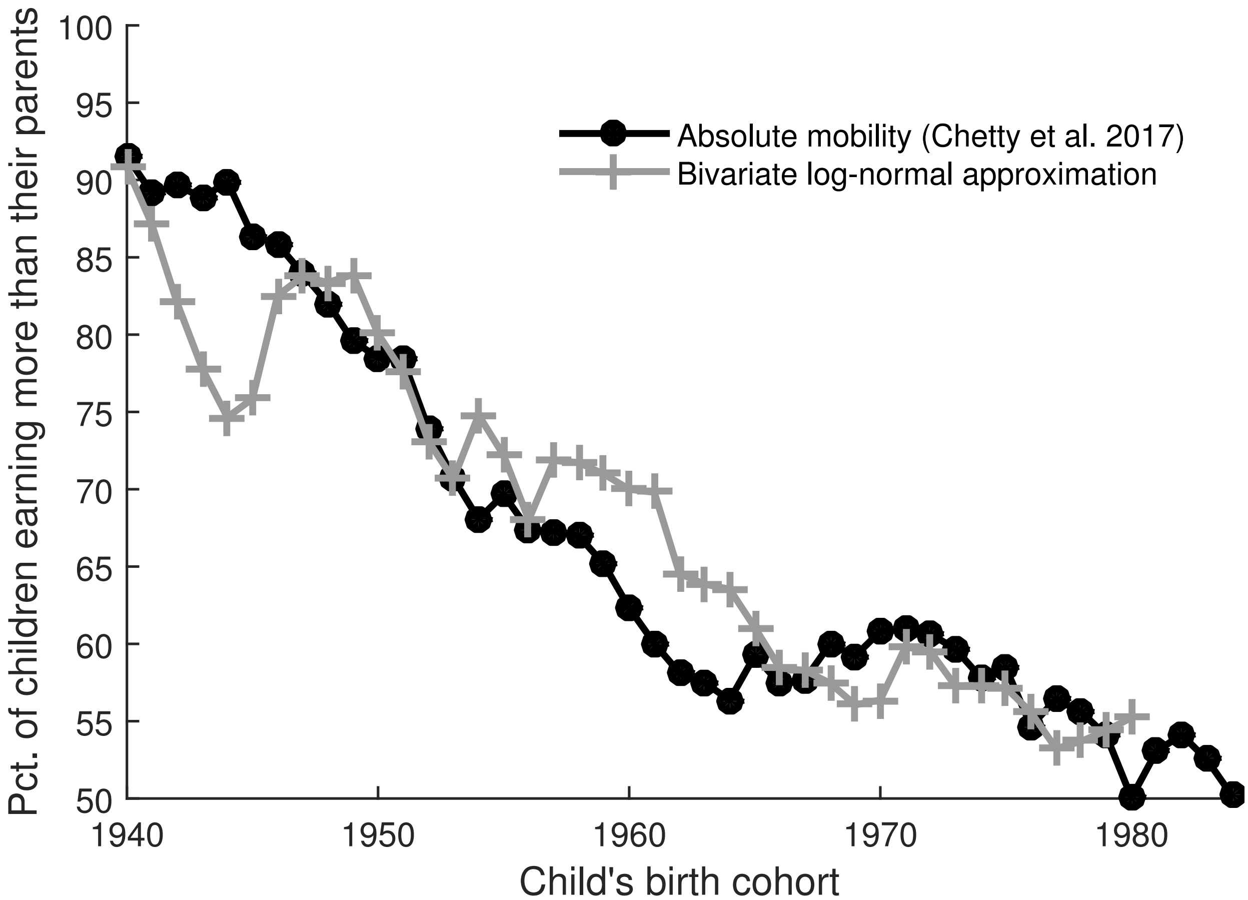

Figure 1: A comparison between the measured (black) and approximated absolute mobility (gray). A bivariate normal distribution reproduces the historical trend of the rate of absolute mobility (the calculations are based on the pre-tax income per capita and pre-tax income distribution reported in Piketty and Zucman (2014) and The World Wealth and Income Database (2016) assuming fixed correlation of $\rho = 0.57$).

immobility rather than of mobility: the larger it is, the stronger the relationship between parent and child income. Therefore, $R_1 \equiv 1 - \beta$ is used as a measure of relative mobility. Unlike absolute mobility, most empirical studies of IGE and other relative mobility measures in the United States show them holding stable over recent decades (Lee and Solon, 2009; Hauser, 2010; Chetty et al., 2014*a*,*b*).

An additional important measure of relative mobility is the rank-rank slope (RRS), defined as the regression coefficient when "regressing the child's rank on his parents' rank" (Chetty et al., 2014*a*, p. 1561). Unlike the IGE, the RRS depends entirely on the copula of the joint income distribution of parents and children. The IGE also depends on the standard deviation of the log-income distributions. Therefore, it is argued that rank-rank slopes "prove to be much more robust across specifications and are thus more suitable for comparisons across areas from a statistical perspective" (Chetty et al., 2014*a*, p. 1561). Similarly to the IGE, the RRS is also a measure of immobility, and we use $R_2 \equiv 1 - \text{RRS}$ as a measure of relative mobility.

Co-observations of declining absolute mobility and stable relative mobility require careful interpretation. The explanation of Chetty et al. (2017) for the contrast is that income growth has been unequally distributed – positive for high earners and stagnant for the rest – meaning that aggregate income growth has contributed little to absolute mobility. This finding is consistent with their data but may not describe the only mechanism at work.

Here we present a theoretical study of the absolute mobility measure A, particularly focusing on the relationship between A and the two measures of relative mobility mentioned above. We find that under a very general model of the joint log-income distribution, A is inversely related to both relative mobility measures – R_1 and R_2. We find this relationship to be robust for different copula families and marginal distributions.

Although it is argued that "the income distribution is not well approximated by a bivariate log-normal distribution" (Chetty et al., 2014*a*, p. 1574), we find that the falling absolute mobility presented in Chetty et al. (2017) can be well described by a bivariate normal model for the joint log-income distribution. Under this model we obtain a closed-form expression for the dependence of A and R_1. This enables us to study mechanisms by which changes in the marginal distributions of parent and child incomes influence absolute mobility, which were not fully described previously.

Our contribution is threefold. Firstly, we theoretically study the relationship between distinct measures and interpretations of intergenerational mobility for the first time, suggesting that their co-movement should not, in general, be expected.

Secondly, from an empirical perspective, we are able to show that for describing the long run dynamics of absolute mobility, a model as simple as a bivariate log-normal distribution is satisfactory. This enables using powerful theoretical and empirical tools when addressing the dynamics of absolute mobility.

Thirdly, our study extends a body of theory of economic systems without assuming ergodicity. *YB: Alex - please complete. I am not sure of how to write this paragraph, if at all. YB*

2 Model

Our starting point is a population of N parent-child pairs. We denote by Y_p^i and Y_c^i the incomes of the parent and the child (at the same age), respectively, in family $i = 1 \ldots N$. We assume the incomes are all positive and define the log-incomes $X_p^i = \log Y_p^i$ and $X_c^i = \log Y_c^i$.

The intergenerational earnings elasticity (IGE) is defined as the slope (β) of the linear regression

$$X_c = \alpha + \beta X_p + \epsilon \,, \tag{2.1}$$

where α is the regression intercept and ϵ is the error term.

The rate of absolute mobility, denoted by A, as defined and measured by Chetty et al. (2017) is the fraction of children earning more than their parents, equal to the probability $P\left(X_c - X_p > 0\right)$.

One hypothetical sample of the joint parents and children log-income distribution is presented in Fig. 2. Fig. 2 also depicts how A and β are defined – the blue line is $y = x$, hence the rate of absolute mobility is defined as the fraction of parent-child pairs which are above it. The red line is the linear regression $y = \alpha + \beta x$, for which β is the IGE.

Since income distributions are known to be well approximated by the log-normal distribution (Pinkovskiy and Sala-i-Martin, 2009), a simple plausible model for the joint distribution of parent and child log-incomes is the bivariate normal distribution. Under this assumption, the marginal income distributions of both parents and children are log-normal and

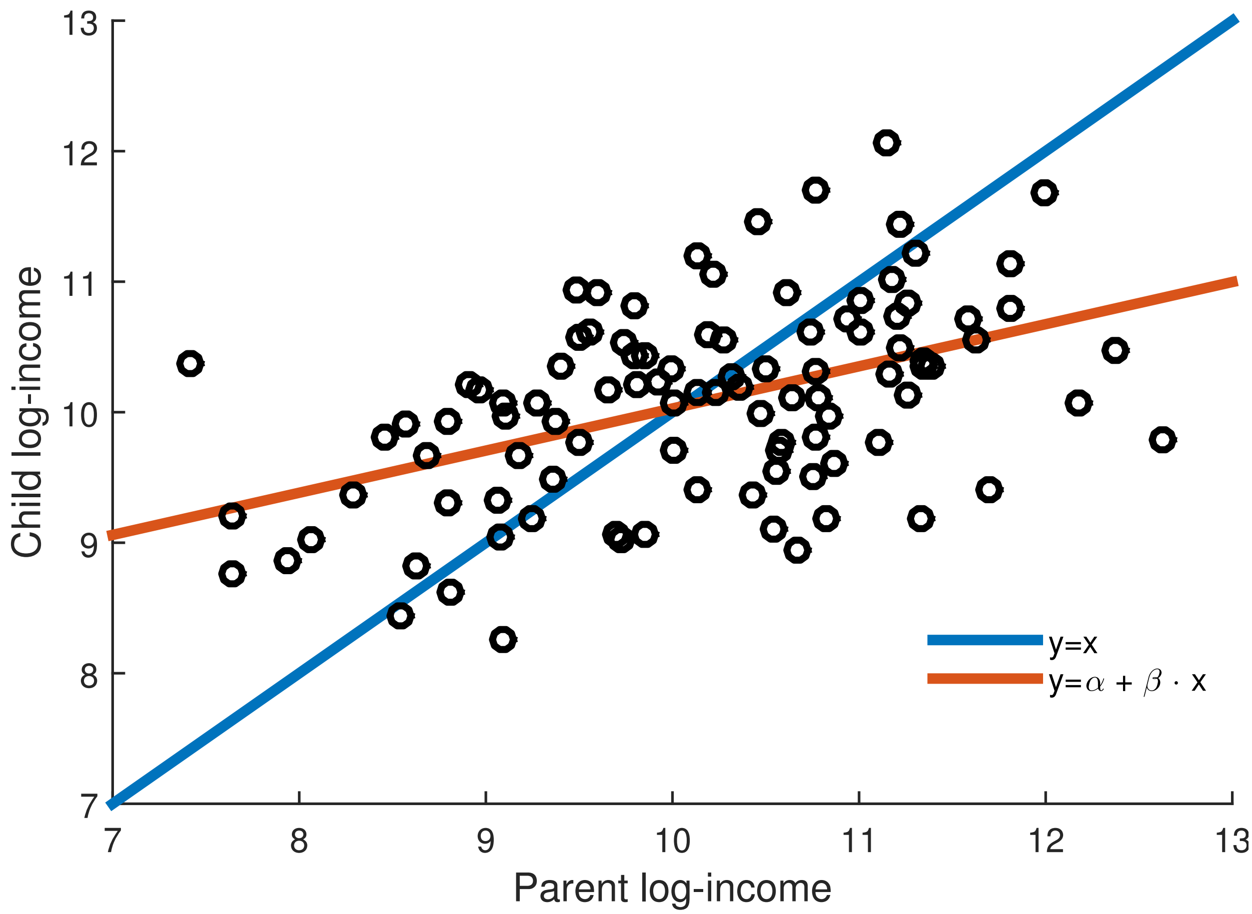

Figure 2: An illustration of the absolute and relative mobility measures. The black circles are a randomly chosen sample of 100 parent-child log-income pairs, assuming a bivariate normal distribution. The parameters used were $\mu_p = 10.1$, $\sigma_p = 0.78$ (for the parents marginal distribution) and $\mu_c = 10.25$, $\sigma_c = 1.15$ (for the children marginal distribution) with correlation of $\rho = 0.57$. The resulting α and β were 1.8 and 0.84, respectively.

the correlation between their log-incomes is defined by a single parameter ρ. The marginal distributions of the parents and the children follow $\mathcal{N}\left(\mu_p, \sigma_p^2\right)$ and $\mathcal{N}\left(\mu_c, \sigma_c^2\right)$, respectively. Hence the joint distribution is fully characterized by 5 parameters: μ_p, σ_p, μ_c, σ_c and ρ.

3 Results

We first address the properties of the bivariate normal approximation. In particular, we derive close form expressions for both measures of mobility, A and R, in terms of the distribution parameters:

Proposition 1 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution) and μ_c, σ_c (for the children marginal distribution) assuming correlation ρ, the relative mobility R is*

$$R = 1 - \frac{\sigma_c}{\sigma_p}\rho\,. \tag{3.1}$$

Following Prop. 1 it is also possible to derive the rate of absolute mobility as a function of the distribution parameters and the IGE:

Proposition 2 *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and $\rho = \sigma_p\beta/\sigma_c$ (where β is the IGE), the rate of absolute mobility is*

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right)\,, \tag{3.2}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Our next step is to test whether the bivariate normal model for the joint distribution of the log-incomes is empirically sound. For that purpose we compare the model prediction for the historical rate of absolute mobility in the United States with the historical rate reported by Chetty et al. (2017).

We use data for the per capita pre-tax income in the United States (Piketty and Zucman, 2014) and the income share data (The World Wealth and Income Database, 2016) to obtain μ_p, σ_p, μ_c and σ_c every year. Since in the log bivariate normal model, the marginal log-income distributions are normal, these parameters can be obtained directly and no fit is required. The Lorenz curve of the log-normal distribution $\log\mathcal{N}\left(\mu, \sigma^2\right)$ is $\Phi\left(\Phi^{-1}\left(z\right) - \sigma\right)$ (Cowell, 2011). Therefore, top 1% income share s corresponds to

$$\sigma = \Phi^{-1}\left(0.99\right) - \Phi^{-1}\left(s\right) \tag{3.3}$$

If we denote the per-capita pre-tax income as m, it follows that the parameter μ is

$$\mu = \log(m) - \frac{\sigma^2}{2} \tag{3.4}$$

We then use Eq. (3.1) and Eq. (3.2) to calculate the historical value of A, while fitting the correlation ρ to a fixed value which minimizes the sum of squared errors from the values reported by Chetty et al. (2017).

The gray curve in Fig. 1 shows that the model reproduces faithfully the evolution of absolute mobility reported by Chetty et al. (2017), despite its comparative methodological naïvety. Although it was also argued that "the income distribution is not well approximated by a bivariate log-normal distribution" (Chetty et al., 2014*a*, p. 1574), the choice in this particular model do not change the overall analysis presented, as demonstrated in Appendix B for a bivariate gamma distribution.

Having established the model's empirical soundness, we can use its properties to further study the measures of mobility – A and R. Prop. 2 demonstrates that the rate of absolute mobility can be explicitly described as a function of the relative mobility. Fig. 3 shows A as a function of R for different birth cohorts in the United States. It shows that the bivariate normal model – with positive income growth and inequality changes consistent with data, but absent other effects – predicts an *inverse* relationship between absolute and relative mobility.

Proposition 2 illustrates that the rate of absolute mobility increases with increasing income growth and decreases with increasing income inequality, as described by Chetty et al. (2017). However, it also demonstrates that an additional mechanism can be at play, since the rate of absolute mobility decreases with increasing relative mobility.

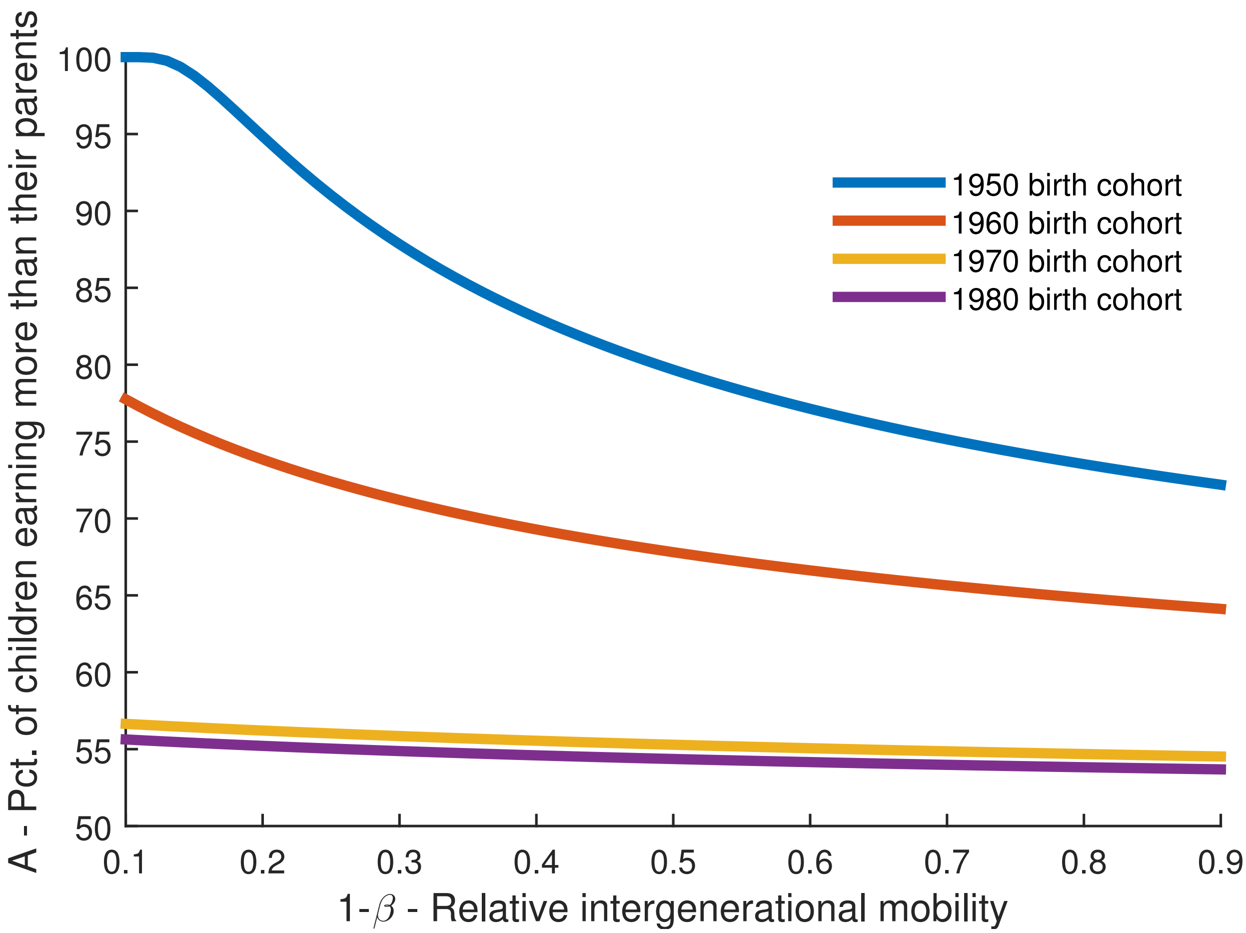

Figure 3: The theoretical relationship between the rate of absolute mobility and the complement of the intergenerational earnings elasticity, assuming the bivariate normal log-incomes model for different birth cohorts in the United States. This demonstrates the counterintuitive result that the two mobility measures are inversely related.

4 Discussion

The counterintuitive inverse theoretical relationship found between absolute and relative mobility stems from a fundamental conceptual difference between the two categories of mobility. It exposes the problems that can arise if both are treated as measuring similarly the same phenomenon. In particular, absolute mobility is very sensitive to across-the-board economic growth. For example, during the Middle Ages – when relative mobility rates were low because social class and profession were predominantly inherited (Goldthorpe, 1987; Clark, 2014), even the slightest positive or negative income growth would result in very high or low absolute mobility. A misleading picture of intergenerational mobility may arise if the basic properties of these measures are overlooked. Therefore, empirically addressing concepts like the "American Dream" (Corak, 2009; Chetty et al., 2017) and equality of opportunity (Roemer, 2000; Chetty et al., 2014*a*) requires careful delineation of the phenomena of interest and the manner in which quoted measures reflect them.

References

Aaronson, Daniel, and Bhashkar Mazumder. 2008. “Intergenerational economic mobility in the United States, 1940 to 2000.” *Journal of Human Resources*, 43(1): 139–172.

Berman, Yonatan. 2016. “Understanding the Mechanical Relationship between Inequality and Intergenerational Mobility.” *SSRN*. `http://ssrn.com/abstract=2796563`.

Chetty, Raj, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang. 2017. “The fading American dream: Trends in absolute income mobility since 1940.” *Science*, 356(6336): 398–406.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. 2014*a*. “Where is the Land of Opportunity? The Geography of Intergenerational Mobility in the United States.” *The Quarterly Journal of Economics*, 129(4): 1553–1623.

Chetty, Raj, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. 2014*b*. “Is the United States Still a Land of Opportunity? Recent Trends in Intergenerational Mobility.” *The American Economic Review*, 104(5): 141–147.

Clark, Gregory. 2014. *The Son Also Rises: Surnames and the History of Social Mobility.* Princeton University Press.

Corak, Miles. 2009. “Chasing the same dream, climbing different ladders: Economic mobility in the United States and Canada.” Economic Mobility Project, Pew Charitable Trusts Paper.

Corak, Miles. 2013. “Income Inequality, Equality of Opportunity, and Intergenerational Mobility.” *The Journal of Economic Perspectives*, 27(3): 79–102.

Cowell, Frank. 2011. *Measuring Inequality.* Oxford, UK:OUP Oxford.

Fields, Gary S., and Efe A. Ok. 1999. "The Measurement of Income Mobility: An Introduction to the Literature." In *Handbook of Income Inequality Measurement.* , ed. Jacques Silber, 557–598. Springer.

Goldthorpe, John H. 1987. *Social Mobility and Class Structure in Modern Britain.* Oxford, UK:Oxford University Press.

Hauser, Robert M. 2010. "What Do We Know So Far about Multigenerational Mobility?" UW-Madison Center for Demography and Ecology 98-12.

Lee, Chul-In, and Gary Solon. 2009. "Trends in intergenerational income mobility." *The Review of Economics and Statistics*, 91(4): 766–772.

Mazumder, Bhashkar. 2005. "Fortunate sons: New estimates of intergenerational mobility in the United States using social security earnings data." *Review of Economics and Statistics*, 87(2): 235–255.

Mulligan, Casey B. 1997. *Parental priorities and economic inequality.* University of Chicago Press.

Piketty, Thomas, and Gabriel Zucman. 2014. "Capital is Back: Wealth-Income Ratios in Rich Countries, 1700–2010." *The Quarterly Journal of Economics*, 129(3): 1255–1310.

Pinkovskiy, Maxim, and Xavier Sala-i-Martin. 2009. "Parametric Estimations of the World Distribution of Income." Working Paper 15433, NBER.

Roemer, John E. 2000. *Equality of Opportunity.* Cambridge, MA:Harvard University Press.

The World Wealth and Income Database. 2016. "USA Top 10% and 1% and Bottom 50% Wealth Shares, 1913–2014." *http://wid.world/data/*, Accessed: 12/26/2016.

A Proofs

A.1 Proof of proposition 1

First, by definition, the correlation ρ, between X_p and X_c equals to their covariance, divided by $\sigma_p \sigma_c$

$$\rho = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p \sigma_c}. \tag{A.1}$$

β can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)\left(X_c^i - \bar{X}_c\right)}{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)}, \tag{A.2}$$

where $\bar{X}_p$ and $\bar{X}_c$ are the average parents and children log-incomes, respectively.

It follows that

$$\beta = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p^2}. \tag{A.3}$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p}\rho \tag{A.4}$$

and therefore

$$1 - \beta = R = 1 - \frac{\sigma_c}{\sigma_p}\rho \tag{A.5}$$

$\blacksquare$

A.2 Proof of proposition 2

We start by defining a new random variable $Z = X_c - X_p$. It follows that calculating A is equivalent to calculating the probability $P\left(Z > 0\right)$.

Subtracting two dependent normal distributions yields that

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2\mathrm{Cov}\left[X_p, X_c\right]\right) \ , \tag{A.6}$$

so according to Prop. 1

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2\right) \ . \tag{A.7}$$

If follows that

$$\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} \sim \mathcal{N}\left(0, 1\right) \ , \tag{A.8}$$

so we can now write

$$\begin{aligned} &P\left(Z > 0\right) = \\ &P\left(\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} > -\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right) = \\ &\Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right) \ , \end{aligned} \tag{A.9}$$

where Φ is the cumulative distribution function of the standard normal distribution.

■

B Bivariate gamma distribution model

Log-incomes are commonly approximated by a normal distribution