Measuring the "American Dream": The relationship between relative and absolute mobility

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Abstract

We model the joint log-income distribution of parents and children and derive analytic expressions for canonical relative and absolute intergenerational mobility measures. We find that both types of mobility measures are inversely related theoretically.

Keywords: mobility, inequality, bivariate income distributions
JEL Codes: E0, H0, J0, R0

1. Introduction

The "growing public perception that intergenerational income mobility [...] is declining in the United States" (Chetty et al., 2014a, p. 141) has led scholars to seek quantifiable measures of it. Typically such measures are divided into two categories, relative and absolute: relative measures gauge children's propensity to occupy a higher position in the income distribution than their parents; absolute measures gauge their propensity to have higher income than their parents in money terms. A hypothetical economy in which all children have exactly twice the real incomes of their parents would exhibit minimal relative mobility and maximal absolute mobility. Therefore, the definitions of quoted mobility measures are important. The canonical measures in each category yield different and contradictory interpretations of ostensibly the same concept, which may mislead the unaware.

While relative intergenerational mobility has been studied for decades (Mazumder, 2005; Aaronson and Mazumder, 2008; Lee and Solon, 2009; Hauser, 2010; Corak, 2013; Chetty et al., 2014a; Berman, 2016), investigations of absolute intergenerational mobility remain "scarce, mainly because of the lack of large, high-quality panel data sets linking children to their parents in the United States" (Chetty et al., 2017, p. 398). In a recent paper, Chetty et al. (2017) considered trends in absolute mobility in income in the United States since 1940. They define the rate of absolute mobility as the fraction of children earning more than their parents. They show that the rate of absolute mobility has fallen from approximately 90% for children born in 1940 to 50% for children born in the 1980s (see Fig. 1).

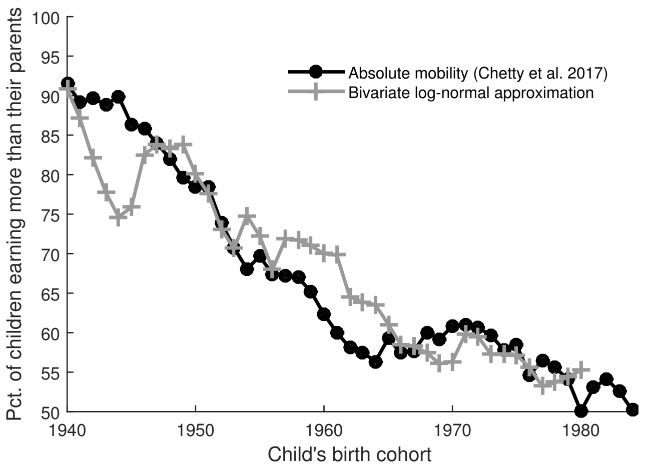

Figure 1: A comparison between the measured (black) and approximated absolute mobility (gray). A bivariate normal distribution reproduces the historical trend of the rate of absolute mobility (the calculations are based on the pre-tax income per capita and pre-tax income distribution reported in Piketty and Zucman (2014) and The World Wealth and Income Database (2016) assuming fixed correlation of $\rho = 0.57$).

The canonical measure of relative intergenerational mobility is the elasticity of child income with respect to parent income, known as the intergenerational earnings elasticity (IGE) (Mulligan, 1997; Lee and Solon, 2009; Chetty et al., 2014b). IGE is a measure of immobility rather than of mobility: the larger it is, the stronger the relationship between parent and child income. Therefore, $1 - \beta$ is used as a measure of mobility. Unlike absolute mobility, empirical studies of IGE and other relative mobility measures in the United States show them holding stable over recent decades (Lee and Solon, 2009; Hauser, 2010; Chetty et al., 2014b,a).

Co-observations of declining absolute mobility and sta-

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ble relative mobility are seemingly contradictory and require careful interpretation. The explanation of Chetty et al. (2017) for the contrast is that income growth has been unequally distributed – positive for high earners and stagnant for the rest – meaning that aggregate income growth has contributed little to absolute mobility. This finding is consistent with their data but may not describe the only mechanism at work. Here we note a theoretical relationship between relative and absolute mobility, which suggests that their co-movement should not, in general, be expected.

2. Model

Our starting point is a population of N parent-child pairs. We denote by Y_p^i and Y_c^i the incomes of the parent and the child (at the same age), respectively, for family $i = 1 \dots N$. We assume the incomes are all positive and move to define the log-incomes $X_p^i = \log Y_p^i$ and $X_c^i = \log Y_c^i$.

The intergenerational earnings elasticity (IGE) is defined as the slope (β) of the linear regression

$$X_c = \alpha + \beta X_p + \epsilon\,, \tag{2.1}$$

where α is the regression intercept and ϵ is the error term.

The rate of absolute mobility, denoted by A, as defined and measured by Chetty et al. (2017) is the fraction of children earning more than their parents, or simply, the probability $P\left(X_c - X_p > 0\right)$.

One hypothetical sample of such distribution is presented in Fig. 2. It also depicts how A and β are defined. The blue line is $y = x$, hence the rate of absolute mobility is defined as the fraction of parent-child pairs which are above it. The red line is the linear regression $y = \alpha + \beta x$, for which β is the IGE.

Since income distributions are known to be well approximated by the log-normal distribution (Pinkovskiy and Sala-i-Martin, 2009), a simple plausible model for the joint distribution of parent and child log-incomes is the bivariate normal distribution. Under this assumption, the marginal income distributions of both parents and children are log-normal and the correlation between their log-incomes is defined by a single parameter ρ. The marginal log-income distribution of the parents (children) follows $\mathcal{N}\left(\mu_p, \sigma_p^2\right)$ ($\mathcal{N}\left(\mu_c, \sigma_c^2\right)$), hence the joint distribution is fully characterized by 5 parameters: μ_p, σ_p, μ_c, σ_c and ρ.

Assuming the bivariate normal approximation for the joint distribution enables theoretically studying its properties. In particular, both measures of mobility, A and $1-\beta$, can be derived directly from the model and, notably, can both be expressed analytically as functions of the other. We first derive the IGE in terms of the distribution parameters:

Proposition 1. *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution) and μ_c, σ_c (for the children marginal distribution) assuming correlation ρ, the IGE is*

$$1 - \beta = 1 - \frac{\sigma_c}{\sigma_p}\rho\,. \tag{2.2}$$

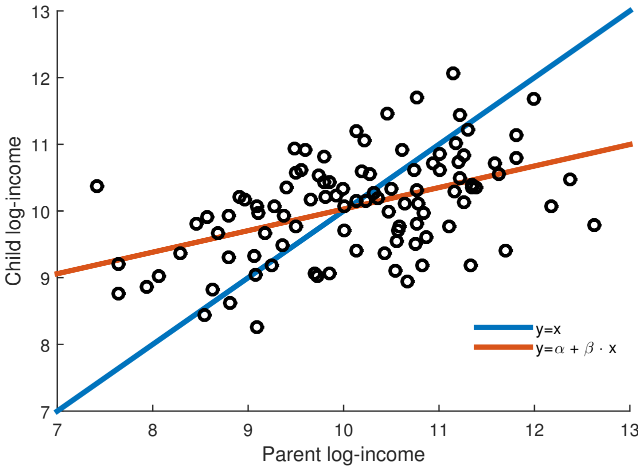

Figure 2: An illustration of the absolute and relative mobility measures. The black circles are a randomly chosen sample of 100 parent-child log-income pairs. The sample was created assuming a bivariate normal distribution and the parameters used were $\mu_p = 10.1$, $\sigma_p = 0.78$ (for the parents marginal distribution) and $\mu_c = 10.25$, $\sigma_c = 1.15$ (for the children marginal distribution) with correlation of $\rho = 0.57$. The resulting α and β were 1.8 and 0.84, respectively.

Following Prop. 1 it is also possible to derive the rate of absolute mobility as a function of the distribution parameters and the IGE:

Proposition 2. *For a bivariate normal distribution with parameters μ_p, σ_p (for the parents marginal distribution), μ_c, σ_c (for the children marginal distribution) and $\rho = \sigma_p\beta/\sigma_c$ (where β is the IGE), the rate of absolute mobility is*

$$A = \Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right)\,, \tag{2.3}$$

where Φ is the cumulative distribution function of the standard normal distribution.

Our next step is to test whether the bivariate normal model for the joint distribution of the log-incomes is empirically sound. For that purpose we compare the model prediction for the historical rate of absolute mobility in the United States with the historical rate reported by Chetty et al. (2017). We use data for the per capita pre-tax income in the United States (Piketty and Zucman, 2014) and the income share data (The World Wealth and Income Database, 2016) to obtain μ_p, σ_p, μ_c and σ_c every year. Since in the bivariate normal model, the marginal distributions are log-normal, these parameters can be obtained directly and no fit is required. We then use Eq. (2.2) and Eq. (2.3) to calculate the historical value of A, while fitting the correlation ρ to a fixed value which minimizes the sum of squared errors from the values reported by Chetty et al. (2017).

The gray curve in Fig. 1 shows that fitting this model's parameters to data reproduces Chetty et al. (2017)'s evolution of absolute mobility faithfully, despite its comparative methodological naivety.

Having established its empirical soundness, we can use the model's properties to further study the measures of mobility – A and $1-\beta$. Prop. 2 demonstrates that the rate of absolute mobility can be explicitly described as a function of the relative mobility. Fig. 3 shows A as a function of $1-\beta$ for different birth cohorts in the United States. It shows that the bivariate normal model – with positive income growth and inequality changes consistent with data, but absent other effects – predicts an *inverse* relationship between absolute and relative mobility.

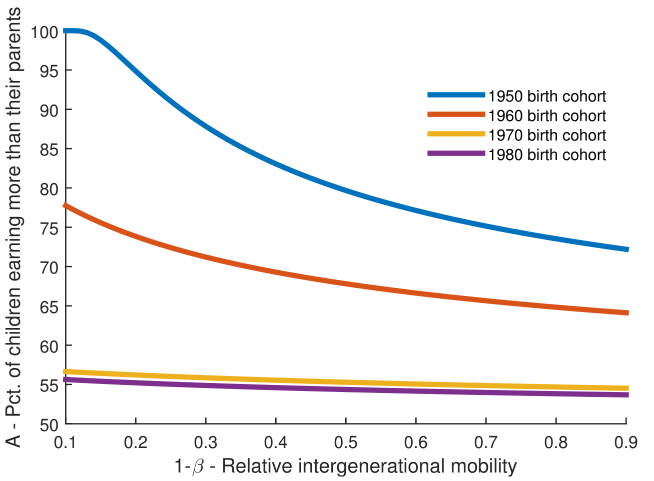

Figure 3: The theoretical relationship between the rate of absolute mobility and the complement of the intergenerational earnings elasticity, assuming the bivariate normal log-incomes model for different birth cohorts in the United States. This demonstrates the counter-intuitive result that the two mobility measures are inversely related.

3. Discussion

This counterintuitive finding stems from a fundamental conceptual difference between relative and absolute mobility. It exposes the problems that can arise if both are treated as measuring similarly the same phenomenon. In particular, absolute mobility is very sensitive to across-the-board economic growth. For example, during the Middle Ages – when relative mobility rates were low because social class and profession were predominantly inherited (Goldthorpe, 1987; Clark, 2014), even the slightest positive or negative income growth would result in very high or low absolute mobility. A misleading picture of intergenerational mobility may arise if the basic properties of these measures are overlooked. Therefore, empirically addressing concepts like the "American Dream" (Corak, 2009; Chetty et al., 2017) and equality of opportunity (Roemer, 2000; Chetty et al., 2014b) requires careful delineation of the phenomena of interest and the manner in which quoted measures reflect them.

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References

Raj Chetty, Nathaniel Hendren, Patrick Kline, Emmanuel Saez, and Nicholas Turner. Is the united states still a land of opportunity? recent trends in intergenerational mobility. *The American Economic Review*, 104(5):141–147, 2014a.

Bhashkar Mazumder. Fortunate sons: New estimates of intergenerational mobility in the united states using social security earnings data. *Review of Economics and Statistics*, 87(2):235–255, 2005.

Daniel Aaronson and Bhashkar Mazumder. Intergenerational economic mobility in the united states, 1940 to 2000. *Journal of Human Resources*, 43(1):139–172, 2008.

Chul-In Lee and Gary Solon. Trends in intergenerational income mobility. *The Review of Economics and Statistics*, 91(4):766–772, 2009.

Robert M. Hauser. What do we know so far about multigenerational mobility? Technical Report 98-12, UW-Madison Center for Demography and Ecology, 2010.

Miles Corak. Income inequality, equality of opportunity, and intergenerational mobility. *The Journal of Economic Perspectives*, 27 (3):79–102, 2013.

Yonatan Berman. Understanding the mechanical relationship between inequality and intergenerational mobility. *SSRN*, 2016. `http://ssrn.com/abstract=2796563`.

Raj Chetty, David Grusky, Maximilian Hell, Nathaniel Hendren, Robert Manduca, and Jimmy Narang. The fading american dream: Trends in absolute income mobility since 1940. *Science*, 356(6336):398–406, 2017.

Casey B. Mulligan. *Parental priorities and economic inequality*. University of Chicago Press, 1997.

Raj Chetty, Nathaniel Hendren, Patrick Kline, and Emmanuel Saez. Where is the land of opportunity? the geography of intergenerational mobility in the united states. *The Quarterly Journal of Economics*, 129(4):1553–1623, 2014b.

Maxim Pinkovskiy and Xavier Sala-i-Martin. Parametric estimations of the world distribution of income. 2009. Working Paper 15433, NBER.

Thomas Piketty and Gabriel Zucman. Capital is back: Wealth-income ratios in rich countries, 1700–2010. *The Quarterly Journal of Economics*, 129(3):1255–1310, 2014. doi: 10.1093/qje/qju018.

The World Wealth and Income Database. Usa top 10% and 1% and bottom 50% wealth shares, 1913–2014. `http://wid.world/data/`, 2016. Accessed: 12/26/2016.

John H. Goldthorpe. *Social Mobility and Class Structure in Modern Britain*. Oxford University Press, Oxford, UK, 1987.

Gregory Clark. *The Son Also Rises: Surnames and the History of Social Mobility*. Princeton University Press, 2014.

Miles Corak. Chasing the same dream, climbing different ladders: Economic mobility in the united states and canada. 2009. Economic Mobility Project, Pew Charitable Trusts Paper.

John E. Roemer. *Equality of Opportunity*. Harvard University Press, Cambridge, MA, 2000.

Appendix A. Proofs

Appendix A.1. Proof of proposition 1

First, by definition, the correlation ρ, between X_p and X_c equals to their covariance, divided by $\sigma_p\sigma_c$

$$\rho = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p\sigma_c} . \tag{A.1}$$

β can be directly calculated as follows, by the linear regression slope definition:

$$\beta = \frac{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)\left(X_c^i - \bar{X}_c\right)}{\sum_{i=1}^{N}\left(X_p^i - \bar{X}_p\right)} , \tag{A.2}$$

where $\bar{X}_p$ and $\bar{X}_c$ are the average parents and children log-incomes, respectively.

It follows that

$$\beta = \frac{\mathrm{Cov}\left[X_p, X_c\right]}{\sigma_p^2} . \tag{A.3}$$

We immediately obtain

$$\beta = \frac{\sigma_c}{\sigma_p}\rho \tag{A.4}$$

and therefore

$$1 - \beta = 1 - \frac{\sigma_c}{\sigma_p}\rho \tag{A.5}$$

□

Appendix A.2. Proof of proposition 2

We start by defining a new random variable $Z = X_c - X_p$. It follows that calculating A is equivalent to calculating the probability $P(Z > 0)$.

Subtracting two dependent normal distributions yields that

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2 + \sigma_c^2 - 2\mathrm{Cov}\left[X_p, X_c\right]\right) , \tag{A.6}$$

so according to Prop. 1

$$Z \sim \mathcal{N}\left(\mu_c - \mu_p, \sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2\right) . \tag{A.7}$$

If follows that

$$\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} \sim \mathcal{N}\left(0, 1\right) , \tag{A.8}$$

so we can now write

$$\begin{aligned} &P\left(Z > 0\right) = \\ &P\left(\frac{Z - \left(\mu_c - \mu_p\right)}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}} > -\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right) = \\ &\Phi\left(\frac{\mu_c - \mu_p}{\sqrt{\sigma_p^2\left(1 - 2\beta\right) + \sigma_c^2}}\right) , \end{aligned} \tag{A.9}$$

where Φ is the cumulative distribution function of the standard normal distribution.

□