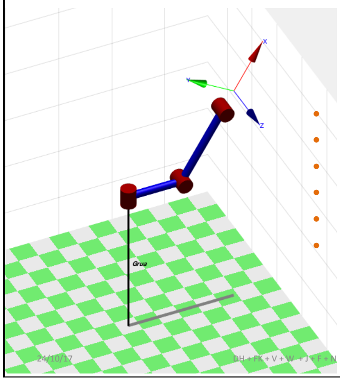
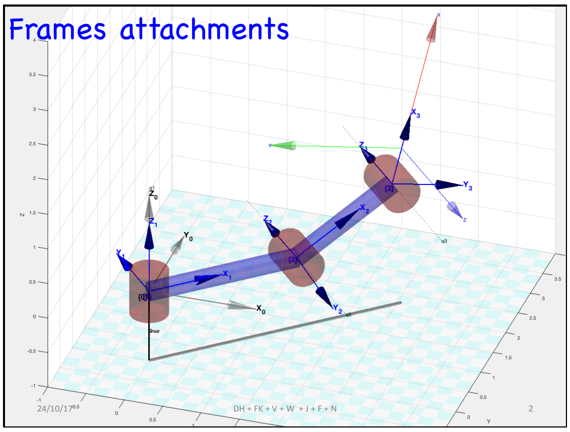
A complete DH & Jacobian exercise

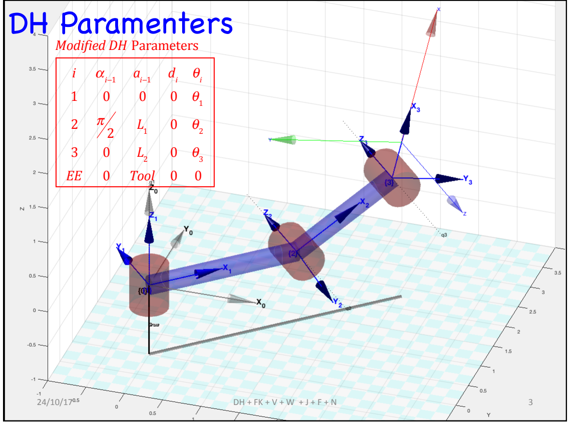


- Frame attachment
- DH Parameters
- Forward Kinematics
- Velocity Propagation.
- Jacobians
- Forces&Torques

Frames attachments



DH Paramenters



Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	L_1	0	θ_2
3	0	L_2	0	θ_3
EE	0	$Tool$	0	0

Forward Kinematics

Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$\pi/2$	L_1	0	θ_2
3	0	L_2	0	θ_3
EE	0	L_3	0	0

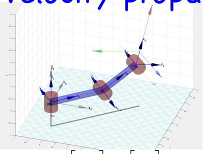
$$; \ {}^{i-1}_{i}T(\alpha_{i-1},a_{i-1},\theta_i,d_i)=\begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1}\\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i\\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i\\ 0&0&0&1\end{pmatrix}$$

$${}^0_1T=\begin{pmatrix} c_1 & -s_1 & 0 & 0\\ s_1 & c_1 & 0 & 0\\ 0&0&1&0\\0&0&0&1\end{pmatrix}; {}^1_2T=\begin{pmatrix} c_2 & -s_2 & 0 & L_1\\ 0&0&-1&0\\ s_2 & c_2 & 0 & 0\\ 0&0&0&1\end{pmatrix}; {}^2_3T=\begin{pmatrix} c_3 & -s_3 & 0 & L_2\\ s_3 & c_3 & 0 & 0\\ 0&0&1&0\\ 0&0&0&1\end{pmatrix}; {}^3_{Tool}T=\begin{pmatrix}1&0&0&L_3\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}$$

Forward Kinematics: Tan*slational of* ${}^0_{Tool}T={}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_{Tool}T\equiv{}^0P_{Tool\_ORG}$

$${}^0P_{Tool\_ORG}=\begin{pmatrix} L_1c_1+L_2c_1c_2+L_3c_1c_{23}\\ L_1s_1+L_2s_1c_2+L_3s_1c_{23}\\ L_2s_2+L_3s_{23}\end{pmatrix}; {}^0_4R=\begin{pmatrix} c_1c_{23} & -c_1s_{23} & s_1\\ s_1c_{23} & s_1s_{23} & -c_1\\ s_{23} & c_{23} & 0\end{pmatrix}; {}^4_0R=\begin{pmatrix} c_1c_{23} & s_1c_{23} & s_{23}\\ -c_1s_{23} & s_1s_{23} & c_{23}\\ s_1 & -c_1 & 0\end{pmatrix}$$

Velocity propagation



Use:

$${}^{i+1}v_{i+1}={}^{i+1}_{i}R\left({}^iv_i+{}^iw_i\times{}^iP_{i+1}\right)$$

$${}^{i+1}w_{i+1}={}^{i+1}_{i}R\,{}^iw_i+\dot\theta_{i+1}\,{}^{i+1}\hat Z_{1+1}$$

$${}^1w_1=\begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix}; v_1=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$${}^2w_2={}^2_1R\,{}^1w_1+\dot\theta_2\,{}^2\hat Z_2={}^2_1R\,{}^1w_1+\begin{bmatrix}0\\0\\\dot\theta_2\end{bmatrix}=\begin{pmatrix}c_2&-s_2&0\\0&0&-1\\s_2&c_2&1\end{pmatrix}\begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix}+\begin{bmatrix}0\\0\\\dot\theta_2\end{bmatrix}=\begin{bmatrix}s_2\dot\theta_1\\c_2\dot\theta_1\\\dot\theta_2\end{bmatrix}$$

$${}^2v_2={}^2_1R\left({}^1v_1+{}^1w_1\times{}^1P_2\right)=\begin{pmatrix}c_2&-s_2&0\\0&0&-1\\s_2&c_2&1\end{pmatrix}\left(\begin{bmatrix}0\\0\\0\end{bmatrix}+\underbrace{\begin{pmatrix}0&-\dot\theta_1&0\\\dot\theta_1&0&0\\0&0&0\end{pmatrix}\begin{pmatrix}L_1\\0\\0\end{pmatrix}}_{\hat wP}\right)=\begin{bmatrix}0\\0\\-L_1\dot\theta_1\end{bmatrix}$$

Velocity propagation

$${}^3w_3={}^3_2R\,{}^2w_2+\begin{bmatrix}0\\0\\\dot\theta_3\end{bmatrix}=\begin{pmatrix}c_3&s_3&0\\-s_3&c_3&0\\0&0&1\end{pmatrix}\begin{bmatrix}s_2\dot\theta_1\\c_2\dot\theta_1\\\dot\theta_2\end{bmatrix}+\begin{bmatrix}0\\0\\\dot\theta_3\end{bmatrix}=\begin{bmatrix}s_2\dot\theta_1\\c_2\dot\theta_1\\\dot\theta_2\end{bmatrix}=\begin{bmatrix}s_{23}\dot\theta_1\\c_{23}\dot\theta_1\\\dot\theta_2+\dot\theta_3\end{bmatrix}$$

$${}^3v_3={}^3_2R\left({}^2v_2+{}^2w_2\times{}^2P_3\right)=\begin{pmatrix}c_3&s_3&0\\-s_3&c_3&0\\0&0&1\end{pmatrix}\left(\begin{bmatrix}0\\0\\-L_1\dot\theta_1\end{bmatrix}+\underbrace{\begin{pmatrix}0&-\dot\theta_2&c_2\dot\theta_1\\\dot\theta_2&0&s_2\dot\theta_1\\-c_2\dot\theta_1&s_2\dot\theta_1&0\end{pmatrix}\begin{pmatrix}L_2\\0\\0\end{pmatrix}}_{{}^2\hat w_2P}\right)$$

$${}^3v_3=\begin{pmatrix}c_3&s_3&0\\-s_3&c_3&0\\0&0&1\end{pmatrix}\left(\begin{bmatrix}0\\0\\-L_1\dot\theta_1\end{bmatrix}+\begin{bmatrix}0\\L_2\dot\theta_2\\-L_2c_2\dot\theta_1\end{bmatrix}\right)=\begin{pmatrix}L_2s_3\dot\theta_2\\L_2c_3\dot\theta_2\\-L_1\dot\theta_1-L_2c_2\dot\theta_1\end{pmatrix}$$

$${}^4w_4={}^3w_3$$

$${}^4v_4={}^4_3R\left({}^3v_3+{}^3w_3\times{}^3P_4\right)=\begin{pmatrix}L_2s_3\dot\theta_2\\L_2c_3\dot\theta_2\\-L_1\dot\theta_1-L_2c_2\dot\theta_1\end{pmatrix}+\begin{pmatrix}0&-(\dot\theta_2+\dot\theta_3)&c_{23}\dot\theta_1\\(\dot\theta_2+\dot\theta_3)&0&-s_{23}\dot\theta_1\\-c_{23}\dot\theta_1&s_{23}\dot\theta_1&0\end{pmatrix}\begin{pmatrix}L_3\\0\\0\end{pmatrix}$$

$${}^4v_4=\begin{pmatrix}L_2s_3\dot\theta_2\\L_2c_3\dot\theta_2\\-L_1\dot\theta_1-L_2c_2\dot\theta_1\end{pmatrix}+\begin{pmatrix}0\\L_3(\dot\theta_2+\dot\theta_3)\\-L_3c_{23}\dot\theta_1\end{pmatrix}=\begin{pmatrix}L_2s_3\dot\theta_2\\L_2c_3\dot\theta_2+L_3(\dot\theta_2+\dot\theta_3)\\-L_1\dot\theta_1-L_2c_2\dot\theta_1-L_3c_{23}\dot\theta_1\end{pmatrix}$$

Jacobians: from velocity propagation

$$^{4}v_4=\begin{pmatrix} L_2 s_3\dot\theta_2 \\ L_2c_3\dot\theta_2+L_3\left(\dot\theta_2+\dot\theta_3\right) \\ -L_1\dot\theta_1-L_2c_2\dot\theta_1-L_3c_{23}\dot\theta_1\end{pmatrix}\rightarrow {}^{4}v_4=\underbrace{\begin{pmatrix} 0 & L_2s_3 & 0 \\ 0 & L_2c_3+L_3 & L_3 \\ -L_1-L_2c_2-L_3c_{23} & 0 & 0\end{pmatrix}}_{^{4}J(\theta)}\begin{pmatrix}\dot\theta \\ \dot\theta_2 \\ \dot\theta_3\end{pmatrix}$$

$$^{4}v_4={}^{4}J(\theta)\left(\dot\theta's\right)$$

$$^{0}v_4={}^{0}_{4}R\,{}^{4}v_4=\underbrace{{}^{0}_{4}R\,{}^{4}J(\theta)}_{^{0}J(\theta)}\left(\dot\theta's\right)={}^{0}J(\theta)\left(\dot\theta's\right)$$

$$^{0}J(\theta)=\begin{pmatrix} c_1c_{23} & -c_1s_{23} & s_1 \\ s_1c_{23} & s_1s_{23} & -c_1 \\ s_{23} & c_{23} & 0\end{pmatrix}\begin{pmatrix} 0 & L_2s_3 & 0 \\ 0 & L_2c_3+L_3 & L_3 \\ -L_1-L_2c_2-L_3c_{23} & 0 & 0\end{pmatrix}$$

$$^{0}J(\theta)=\begin{pmatrix} -L_1s_1-L_2s_1c_2-L_3s_1c_{23} & L_2c_1s_2+L_3c_1s_{23} & -L_3c_1s_{23} \\ L_1c_1+L_2c_1c_2+L_3c_1c_{23} & L_2s_1s_2+L_3s_1s_{23} & L_3s_1s_{23} \\ 0 & L_2c_2-L_3c_{23} & L_3c_{23}\end{pmatrix}$$

Jacobians: from FK

$$^{0}P_{Tool\_ORG}=P=\begin{pmatrix} L_1c_1+L_2c_1c_2+L_3c_1c_{23} \\ L_1s_1+L_2s_1c_2+L_3s_1c_{23} \\ L_2s_2+L_3s_{23}\end{pmatrix}$$

$$^{0}J(\theta)=\begin{pmatrix} \frac{\delta P_x}{\delta\theta_1} & \frac{\delta P_x}{\delta\theta_2} & \frac{\delta P_x}{\delta\theta_3} \\ \frac{\delta P_y}{\delta\theta_2} & \frac{\delta P_y}{\delta\theta_2} & \frac{\delta P_y}{\delta\theta_2} \\ \frac{\delta P_z}{\delta\theta_3} & \frac{\delta P_z}{\delta\theta_3} & \frac{\delta P_z}{\delta\theta_3}\end{pmatrix}=\begin{pmatrix} -L_1s_1-L_2s_1c_2-L_3s_1c_{23} & L_2c_1s_2+L_3c_1s_{23} & -L_3c_1s_{23} \\ L_1c_1+L_2c_1c_2+L_3c_1c_{23} & L_2s_1s_2+L_3s_1s_{23} & L_3s_1s_{23} \\ 0 & L_2c_2-L_3c_{23} & L_3c_{23}\end{pmatrix}$$

Same result!!

Changing Jacobian's Reference Frames

Remember:

$$^{0}J_v(\theta)={}^{0}_{4}R\,{}^{4}J_v(\theta)\rightarrow {}^{4}J_v(\theta)={}^{4}_{0}R\,{}^{0}J_v(\theta)$$

$$^{4}J_v(\theta)=\begin{pmatrix} c_1c_{23} & s_1c_{23} & s_{23} \\ -c_1s_{23} & s_1s_{23} & c_{23} \\ s_1 & -c_1 & 0\end{pmatrix}\begin{pmatrix} -L_1s_1-L_2s_1c_2-L_3s_1c_{23} & L_2c_1s_2+L_3c_1s_{23} & -L_3c_1s_{23} \\ L_1c_1+L_2c_1c_2+L_3c_1c_{23} & L_2s_1s_2+L_3s_1s_{23} & L_3s_1s_{23} \\ 0 & L_2c_2-L_3c_{23} & L_3c_{23}\end{pmatrix}$$

Use:

$c_{12}=c_1c_2-s_1s_2; s_{12}=c_1s_2+s_1c_2;$ *It is tedious but same result*

$$^{4}J_v(\theta)=\begin{pmatrix} 0 & L_2s_3 & 0 \\ 0 & L_2c_3+L_3 & L_3 \\ -L_1-L_2c_2-L_3c_{23} & 0 & 0\end{pmatrix}$$

Static Forces&Torques

Use:

$$f_i={}^{i}_{i+1}R\,{}^{i+1}f_{i+1}$$

$$^{i}n_i={}^{i+1}_{i}R\,{}^{i+1}n_{i+1}+{}^{i}P_{i+1}\times{}^{i}f_i$$

Remember:

$$^{0}_{1}T=\begin{pmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{pmatrix};\ {}^{1}_{2}T=\begin{pmatrix} c_2 & -s_2 & 0 & L_1 \\ 0 & 0 & -1 & 0 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 0 & 1\end{pmatrix};\ {}^{2}_{3}T=\begin{pmatrix} c_3 & -s_3 & 0 & L_2 \\ s_3 & c_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{pmatrix};\quad {}^{3}_{Tool}T=I_{3x3}$$

$$^{4}F_4=\begin{bmatrix} F_x \\ F_y \\ F_z\end{bmatrix};\ {}^{4}N_4=\begin{bmatrix} 0 \\ 0 \\ 0\end{bmatrix};$$

$$^{3}F_3={}^{3}_{4}R\,{}^{4}F_4={}^{4}F_4$$

$$^{3}N_3={}^{3}_{4}R\,{}^{4}N_4+{}^{3}P_4\times{}^{3}F_3=\begin{pmatrix} L_3 \\ 0 \\ 0\end{pmatrix}\times\begin{bmatrix} F_x \\ F_y \\ F_z\end{bmatrix}=\begin{bmatrix} 0 \\ -L_3F_z \\ L_3F_y\end{bmatrix}$$

Static Forces&Torques

Use:

$$f_i={}^{i}_{i+1}R\,{}^{i+1}f_{i+1}$$

$$^{i}n_i={}^{i+1}_{i}R\,{}^{i+1}n_{i+1}+{}^{i}P_{i+1}\times{}^{i}f_i$$

$$^{2}F_2={}^{2}_{3}R\,{}^{3}F_3=\begin{pmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1\end{pmatrix}\begin{bmatrix} F_x \\ F_y \\ F_z\end{bmatrix}=\begin{bmatrix} c_3F_x-s_3F_y \\ s_3F_x-c_3F_y \\ F_{yz}\end{bmatrix}$$

$$^{2}N_2={}^{2}_{3}R\,{}^{3}N_3+{}^{2}P_3\times{}^{2}F_2=\begin{pmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1\end{pmatrix}\begin{bmatrix} 0 \\ -L_3F_z \\ L_3F_y\end{bmatrix}+\begin{pmatrix} L_2 \\ 0 \\ 0\end{pmatrix}\times\begin{bmatrix} c_3F_x-s_3F_y \\ s_3F_x-c_3F_y \\ F_z\end{bmatrix}$$

$$^{2}N_2=\begin{bmatrix} s_3L_3F_z \\ -c_3L_3F_z \\ L_3F_y\end{bmatrix}+\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -L_2 \\ 0 & L_2 & 0\end{pmatrix}\begin{bmatrix} c_3F_x-s_3F_y \\ s_3F_x-c_3F_y \\ F_z\end{bmatrix}=\begin{bmatrix} s_3L_3F_z \\ -c_3L_3F_z-L_2F_z \\ L_2\left(s_3F_x+c_3F_y\right)+L_3F_y\end{bmatrix}$$

Static Forces&Torques

Use:

$$f_i={}^{i}_{i+1}R\,{}^{i+1}f_{i+1}$$

$$^{i}n_i={}^{i+1}_{i}R\,{}^{i+1}n_{i+1}+{}^{i}P_{i+1}\times{}^{i}f_i$$

$$^{1}F_1={}^{1}_{2}R\,{}^{2}F_2=\begin{pmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0\end{pmatrix}\begin{bmatrix} c_3F_x-s_3F_y \\ s_3F_x-c_3F_y \\ F_{yz}\end{bmatrix}=\begin{bmatrix} c_2\left(c_3F_x-s_3F_y\right)-s_2\left(s_3F_x-c_3F_y\right) \\ F_z \\ s_2\left(c_3F_x-s_3F_y\right)-c_2\left(s_3F_x-c_3F_y\right)\end{bmatrix}$$

$$^{1}N_1={}^{1}_{2}R\,{}^{2}N_2+{}^{1}P_2\times{}^{1}F_1=\begin{pmatrix} c_2 & -s_2 & 0 \\ 0 & 0 & -1 \\ s_2 & c_2 & 0\end{pmatrix}\begin{bmatrix} s_3L_3F_z \\ -c_3L_3F_z-L_2F_z \\ L_2\left(s_3F_x+c_3F_y\right)+L_3F_y\end{bmatrix}+\begin{pmatrix} L_1 \\ 0 \\ 0\end{pmatrix}\times{}^{1}F_1$$

$$^{1}N_1=\begin{bmatrix} c_2s_3L_3F_z+s_2c_3L_3F_z+s_2L_2F_z \\ -L_2\left(s_3F_x+c_3F_y\right)-L_3F_y \\ s_2s_3L_3F_z-c_2c_3L_3F_z-c_2L_2F_z\end{bmatrix}+\begin{bmatrix} 0 \\ -L_2\left(s_3F_x+c_3F_y\right)-L_3F \\ -L_1F_z\end{bmatrix}$$

Computing Torques

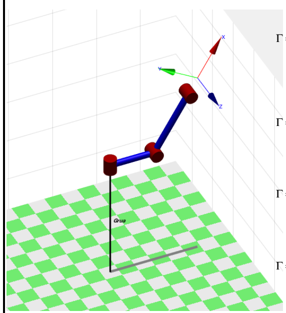
Consider only the ${}^{i}\hat{Z}_i$ of ${}^{i}N_i$

$$
{}^{3}N_3=\begin{bmatrix}0\\-L_3F_z\\L_3F_y\end{bmatrix};\ {}^{2}N_2=\begin{bmatrix}s_3L_3F_z\\-c_3L_3F_z-L_2F_z\\L_2\left(s_3F_x+c_3F_y\right)+L_3F_y\end{bmatrix};\ {}^{1}N_1=\begin{bmatrix}c_2s_3L_3F_z+s_2c_3L_3F_z+s_2L_2F_z\\-L_2\left(s_3F_x+c_3F_y\right)-L_3F_y-L_2\left(s_3F_x+c_3F_y\right)-L_3F\\s_2s_3L_3F_z-c_2c_3L_3F_z-c_2L_2F_z-L_1F_z\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}s_2s_3L_3F_z-c_2c_3L_3F_z-c_2L_2F_z-L_1F_z\\L_2\left(s_3F_x+c_3F_y\right)+L_3F_y\\L_3F_y\end{bmatrix}\rightarrow\Gamma=\underbrace{\begin{pmatrix}0&0&-L_1-L_2c_2-L_3c_{23}\\L_2s_3&L_2c_3+L_3&0\\0&L_3&0\end{pmatrix}}_{{}^{4}J_N^T(\theta)}{}^{4}\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix}
$$

Computing Torques

Consider only the ${}^{i}\hat{Z}_i$ of ${}^{i}N_i$



$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}=\underbrace{\begin{pmatrix}0&0&-L_1-L_2c_2-L_3c_{23}\\L_2s_3&L_2c_3+L_3&0\\0&L_3&0\end{pmatrix}}_{{}^{4}J_N^T(\theta)}{}^{4}\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix}
$$

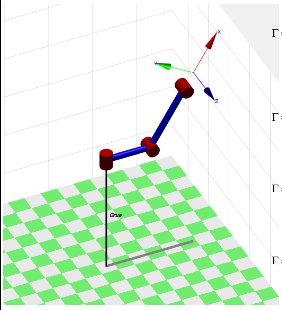
$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}F_x&0&0\end{bmatrix}}=\begin{bmatrix}0\\L_2s_3F_x\\0\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}0&F_y&0\end{bmatrix}}=\begin{bmatrix}0\\\left(L_2c_3+L_3\right)F_y\\L_3F_y\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}0&0&F_z\end{bmatrix}}=\begin{bmatrix}\left(-L_1-L_2c_2-L_3c_{23}\right)F_z\\0\\0\end{bmatrix}
$$

Computing Torques: Using J at {EE}

Consider only the ${}^{i}\hat{Z}_i$ of ${}^{i}N_i$



$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}=\underbrace{\begin{pmatrix}0&0&-L_1-L_2c_2-L_3c_{23}\\L_2s_3&L_2c_3+L_3&0\\0&L_3&0\end{pmatrix}}_{{}^{4}J_N^T(\theta)}{}^{4}\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}F_x&0&0\end{bmatrix}}=\begin{bmatrix}0\\L_2s_3F_x\\0\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}0&F_y&0\end{bmatrix}}=\begin{bmatrix}0\\\left(L_2c_3+L_3\right)F_y\\L_3F_y\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}0&0&F_z\end{bmatrix}}=\begin{bmatrix}\left(-L_1-L_2c_2-L_3c_{23}\right)F_z\\0\\0\end{bmatrix}
$$

Computing Torques: Using J at {0}

Changing Jacobians Re *ferences*

$$
{}^{4}v_4={}^{4}J(\theta)\left(\dot{\theta}'s\right)
$$

$$
{}^{0}v_4={}^{0}_{4}R\,{}^{4}v_4=\underbrace{{}^{0}_{4}R\,{}^{4}J(\theta)}_{{}^{0}J(\theta)}\left(\dot{\theta}'s\right)={}^{0}J(\theta)\left(\dot{\theta}'s\right)\rightarrow\left\{\begin{array}{l}{}^{0}J(\theta)={}^{0}_{4}R\,{}^{4}J(\theta)\\{}^{4}J(\theta)={}^{4}_{0}R\,{}^{0}J(\theta)\end{array}\right.
$$

$$
{}^{0}_{4}R=\begin{pmatrix}c_1c_{23}&-c_1s_{23}&s_1\\s_1c_{23}&s_1s_{23}&-c_1\\s_{23}&c_{23}&0\end{pmatrix};
$$

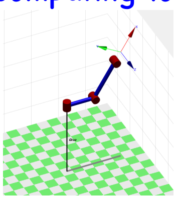
$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}=\underbrace{\begin{pmatrix}0&0&-L_1-L_2c_2-L_3c_{23}\\L_2s_3&L_2c_3+L_3&0\\0&L_3&0\end{pmatrix}}_{{}^{4}J_N^T(\theta)}{}^{4}\begin{bmatrix}F_x\\F_y\\F_z\end{bmatrix}
$$

$$
\Gamma={}^{4}J_N^T(\theta)\,{}^{4}\left(F's\right)
$$

$$
\Gamma={}^{0}J_N^T(\theta)\,{}^{0}\left(F's\right)\rightarrow\left\{\begin{array}{c}{}^{0}J(\theta)={}^{0}_{4}R\,{}^{4}J(\theta)\\{}^{0}F={}^{0}_{4}R\,{}^{4}F\end{array}\right\}\rightarrow\Gamma=\left({}^{0}_{4}R\,{}^{4}J(\theta)\right)^T\underbrace{{}^{0}_{4}R\,{}^{4}F}_{{}^{0}F}={}^{4}J^T(\theta)\,{}^{0}_{4}R^T\,{}^{0}_{4}R\,{}^{4}F={}^{4}J^T(\theta)\,{}^{4}F
$$

$$
{}^{0}J(\theta)=\begin{pmatrix}-L_1s_1-L_2s_1c_2-L_3s_1c_{23}&L_2c_1s_2+L_3c_1s_{23}&-L_3c_1s_{23}\\L_1c_1+L_2c_1c_2+L_3c_1c_{23}&L_2s_1s_2+L_3s_1s_{23}&L_3s_1s_{23}\\0&L_2c_2-L_3c_{23}&L_3c_{23}\end{pmatrix}\rightarrow{}^{0}J^T(\theta)=\begin{pmatrix}-L_1s_1-L_2s_1c_2-L_3s_1c_{23}&L_1c_1+L_2c_1c_2+L_3c_1c_{23}&0\\L_2c_1s_2+L_3c_1s_{23}&L_2s_1s_2+L_3s_1s_{23}&L_2c_2-L_3c_{23}\\-L_3c_1s_{23}&L_3s_1s_{23}&L_3c_{23}\end{pmatrix}
$$

Computing Torques: Using J at {0}

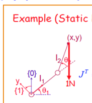


$$
\Gamma={}^{0}J^T(\theta)\underbrace{{}^{0}_{4}R\,{}^{4}F}_{{}^{0}F}=\begin{pmatrix}0&L_1+L_2c_2+L_3c_{23}&0\\L_2s_2+L_3s_{23}&0&L_2c_2-L_3c_{23}\\-L_3s_{23}&0&L_3c_{23}\end{pmatrix}\begin{pmatrix}c_{23}&-s_{23}&0\\0&0&-1\\s_{23}&c_{23}&0\end{pmatrix}{}^{4}\begin{bmatrix}F_x\\0\\0\end{bmatrix}
$$

$$
\Gamma=\begin{bmatrix}\tau_1\\\tau_2\\\tau_3\end{bmatrix}_{\begin{bmatrix}F_x&0&0\end{bmatrix}}=\begin{pmatrix}0&L_1+L_2c_2+L_3c_{23}&0\\L_2s_2+L_3s_{23}&0&L_2c_2-L_3c_{23}\\-L_3s_{23}&0&L_3c_{23}\end{pmatrix}{}^{0}\begin{bmatrix}F_xc_{23}\\0\\F_xs_{23}\end{bmatrix}=\begin{bmatrix}0\\L_2s_3F_x\\0\end{bmatrix};\textit{Same result!!}
$$

RTB: Force-moment. Example

Example (Static Forces)



$$
J=\begin{pmatrix}-\left(l_1S1+l_2S12\right)&-l_2S12\\l_1C1+l_2C12&l_2C12\end{pmatrix}
$$

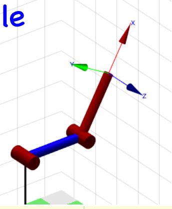
$$
J^T=\begin{pmatrix}-\left(l_1S1+l_2S12\right)&l_1C1+l_2C12\\-l_2S12&l_2C12\end{pmatrix}
$$

$$
\boxed{\tau=J^TF}
$$

$l_1=l_2=1;\quad\theta_1=0;\ \theta_2=60^\circ$

$$
\tau=\begin{pmatrix}-\left(l_1S1+l_2S12\right)&l_1C1+l_2C12\\-l_2S12&l_2C12\end{pmatrix}\begin{bmatrix}0\\-1\end{bmatrix}=-\begin{bmatrix}l_1C1+l_2C12\\l_2C12\end{bmatrix}=-\begin{bmatrix}3/2\\1/2\end{bmatrix}
$$

$$
\tau=\begin{pmatrix}0.866&1.5\\-0.86&0.5\end{pmatrix}\begin{pmatrix}0\\-1\end{pmatrix}=-\begin{pmatrix}3/2\\1/2\end{pmatrix}
$$

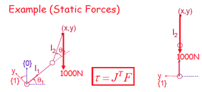



```
%% Using the RTB to compute Torques
mdl_twolink_mdh % Invoque the twolink (2R) manipulator used in the Craig
twolink.plot ([0 60*pi/180]) % ploting
J0=twolink.jacob0 ([0 60*pi/180]) % Using the method Jacobian to compute it at that config
JO_T=J0' % see the values of the Jacobian transpose
tau = J0'*[0 0 -1 0 0 0]' %Compute the torque
```

```
J0_T =

   -0.8660    0.0000    1.5000         0   -1.0000    0.0000
   -0.8660    0.0000    0.5000         0   -1.0000    0.0000


tau =

   -1.5000
   -0.5000
```

RTB: Force-moment. Example

Example (Static Forces)

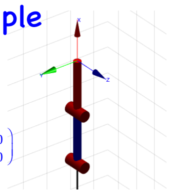


$$\boxed{\tau = J^T F}$$

$$\tau = \begin{pmatrix} -(l_1S1 + l_2S12) & l_1C1 + l_2C12 \\ -l_2S12 & l_2C12 \end{pmatrix} \begin{bmatrix} 0 \\ -1K \end{bmatrix} = \begin{bmatrix} l_1C1 + l_2C12 \\ l_2C12 \end{bmatrix} (-1K) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$l_1 = l_2 = 1; \quad \theta_1 = 90; \theta_2 = 0^\circ$$

$$\tau = \begin{pmatrix} -2 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$




```
%% Using the RTB to compute Torques
mdl_twolink_mdh % Invoque the twolink (2R) manipulator used in the Craig
twolink.plot ([90*pi/180 0]) % ploting
J0=twolink.jacob0 ([90*pi/180 0]) % Using the method Jacobian to compute it at that configuration
JO_T=J0' % see the values of the Jacobian transpose
tau = J0'*[0 0 -1 0 0 0]' %Compute the torque
```

```
JO_T =

   -2.0000    0.0000    0.0000         0   -1.0000    0.0000
   -1.0000    0.0000    0.0000         0   -1.0000    0.0000


tau =

   1.0e-15 *

   -0.1225
   -0.0612
```