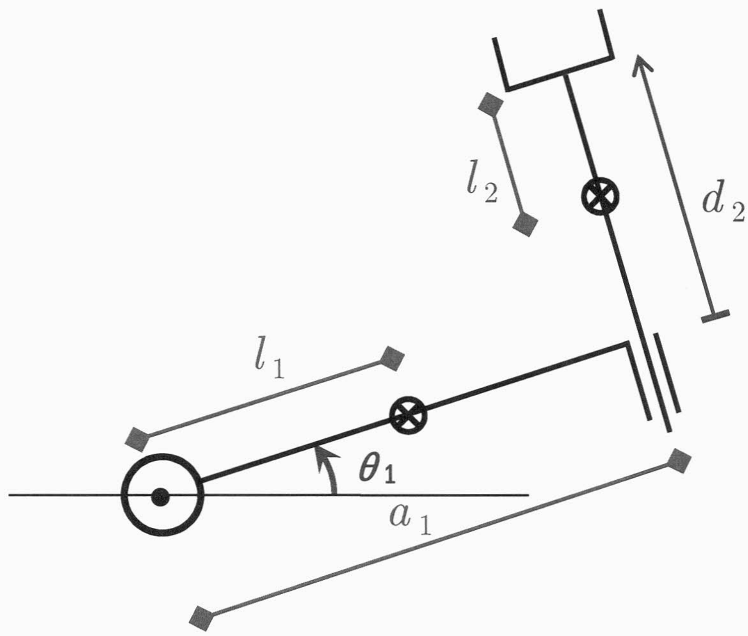
ROBOT KINEMATICS, DYNAMICS, AND CONTROL

Academic year 2012-13 Final Exam 17/01/2013

Part II. PROBLEMS
(130 minutes)

Note: Each problem must be solve on separated sheets

Consider the two-link planar arm with a revolute joint and a prismatic joint in the figure,



with the following parameters:

m_{ℓ_1}, m_{ℓ_2}	masses of the two links
ℓ_1, ℓ_2	distances of the centres of mass from the joint axis and from the end effector, respectively
I_{ℓ_1}, I_{ℓ_2}	inertia tensor of the links about the respective joint axes
a_1	length of link 1

1) For this robot,
   a) Solve the direct kinematics.
   b) Given a position of the end effector described by the transformation $T_2^0 = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$ solve the inverse kinematics of the arm.

2) It is desired to move the above given robot from the initial point P_i = [-6.11, 12] to the final point P_f = [2.13, 12] in 5 seconds.

a) Write a path generator system which calculates a trajectory in joint space based on cubic polynomials. The system must accomplish the desired motion with parabolic velocity profile and brings the arm to rest at the goal. In order to solve the problem assume $a_1 = 10$.

b) If you want to buy two equal electric motors, in order to allow the robot to accomplish the previous movement, which must be the motor velocity requirements ? Justify the obtained solution.

3) Assuming that, neglecting the contributions of the motors, the inertia matrix of the robot manipulator is:

$$B(\theta_1, d_2) = \begin{bmatrix} m_{\ell_1}\ell_1^2 + I_{\ell_1} + m_{\ell_2}(d_2 - \ell_2)^2 + I_{\ell_2} & 0 \\ 0 & m_{\ell_2} \end{bmatrix},$$

and the link Jacobian matrices (linear velocity) are:

$$J_P^{(\ell_1)} = \begin{bmatrix} -\ell_1 s_1 & (d_2 - \ell_2)c_1 \\ \ell_1 c_1 & -(d_2 - \ell_2)s_1 \\ 0 & 0 \end{bmatrix} \qquad J_P^{(\ell_2)} = \begin{bmatrix} 0 & s_1 \\ 0 & c_1 \\ 0 & 0 \end{bmatrix}$$

when s_1 and c_1 stand for sin θ_1 and cos θ_1, respectively, determine the matrix $\boldsymbol{C}$ (using the *Christoffel symbols of the first type*) and the gravitational term $\boldsymbol{g}$ of the dynamic model.

4) Design an *inverse dynamics joint control law* for the two-link planar arm, for tracking the trajectory $\boldsymbol{q}_d^T = [\theta_1 \; d_2]^T = [1+\cos t \;\; 2+\sin 2t]^T$.