Robotics, Kinematics, Dynamics and Control: HW #1

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1 Exercise 1

In order to be a rotation matrix, a matrix R must follow the conditions:

- $Det(R) = 1$
- $R^T * R = R * R^T = I$

. We check this for: R= $\begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}$

- $R * R^T = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix} * \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$
- $Det(R) = \frac{1}{\sqrt{2}} * (\frac{1}{2} * \frac{1}{\sqrt{2}} - (-\frac{1}{\sqrt{2}}) * \frac{1}{2}) + \frac{1}{\sqrt{2}} * ((-\frac{1}{2}) * (-\frac{1}{\sqrt{2}}) - \frac{1}{\sqrt{2}} * (-\frac{1}{2})) = 1$

Therefore R is a rotation matrix.

2 Exercise 2

The columns of ${}^A_B R_{3x3}$ are the vectors representing the frame B when observed from the frame A. The meaning of ${}^A M$ is the vector of the origin of the frame B observed from the frame A.

3 Exercise 3

It means that, being ${}^B P$ the point P viewed from the frame B, ${}^A P = {}^A_B T {}^B P$ represents that same point P viewed from the frame A.

4 Exercise 4

Even though the result is the same as the one obtained for 3), conceptually this case is different. In this case we consider the point ${}^A P_1$ observed from the A frame (the only frame existing in this case) and move that same point by applying a translation transl([1,2,4]) followed by a rotation R.

5 Exercise 5

$${}^B_A T = rot(R^T) * transl(-[1, 2, 3]) = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{2} & -\frac{1}{2} & \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & -R^T P_o \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} & \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6 Exercise 6

$$rot_{XYZ} = rot_Z(\alpha)*rot_Y(\beta)*rot_X(\gamma) = \begin{bmatrix} c(\beta)c(\alpha) & c(\alpha)s(\beta)s(\gamma) - s(\alpha)c(\gamma) & c(\alpha)s(\beta)c(\gamma) + s(\alpha)s(\gamma) \\ c(\beta)s(\alpha) & s(\alpha)s(\beta)s(\gamma) + c(\alpha)c(\gamma) & s(\alpha)s(\beta)c(\gamma) - c(\alpha)s(\gamma) \\ -s(\beta) & c(\beta)s(\gamma) & c(\beta)c(\gamma) \end{bmatrix}$$

We then can isolate first β and use this result in order to get the values of α and γ: $s(\beta) = \frac{1}{2}$. From here we get that $\beta = \frac{\pi}{6}rad$, which results in the values $s(\alpha) = -\frac{1}{\sqrt{3}}, c(\alpha) = \frac{2}{\sqrt{6}}$ and $s(\gamma) = -\frac{2}{\sqrt{6}}, c(\gamma) = \frac{1}{\sqrt{3}}$.

Therefore, we have $\alpha = -0.6155rad$ and $\gamma = -0.9553rad$.

7 Exercise 7

Following the Rodrigues Formula, we know that the rotation matrix R can be represented as:

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} v(\theta)k_x^2 + c(\theta) & v(\theta)k_x k_y - k_z s(\theta) & v(\theta)k_x k_y + k_y s(\theta) \\ v(\theta)k_x k_y + k_z s(\theta) & v(\theta)k_y^2 + c(\theta) & v(\theta)k_y k_z - k_x s(\theta) \\ v(\theta)k_z k_x - k_y s(\theta) & v(\theta)k_y k_z + k_x s(\theta) & v(\theta)k_z^2 + c(\theta) \end{bmatrix}$$

We also know that $tr(R) = 1 + 2cos(\theta)$, from this we get that $\theta = 1.0961rad$. Result which allows us to calculate the rotation vector associated to R and the positive value of θ: $k_x = \frac{r_{32}-r_{23}}{2s(\theta)} = -0.6786$, $k_y = \frac{r_{13}-r_{31}}{2s(\theta)} = 0.6786$, $k_z = \frac{r_{21}-r_{12}}{2s(\theta)} = -0.2811$.

8 Exercise 8

The euler parameters are the quaternion representation of the rotation matrix R. Therefore we know that it has a form of $(sin(\theta/2) * k, cos(\theta/2))$, which means that the euler parameters are:

- $e_1 = w_x * sin(\theta/2) = -0.3536$
- $e_2 = w_y * sin(\theta/2) = 0.3536$
- $e_3 = w_z * sin(\theta/2) = -0.1464$
- $e_4 = cos(\theta/2) = 0.8536rad$