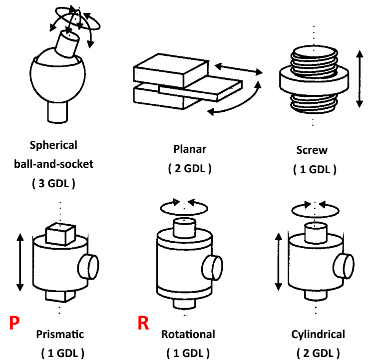
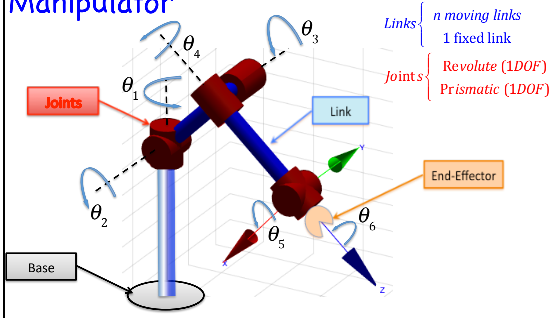
Kinematics

Index

Kind of joints

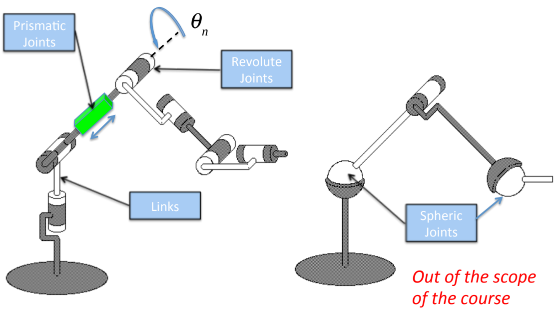


Manipulator



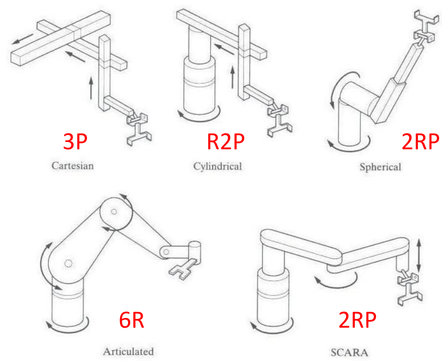
6R Robot Manipulator

Manipulators

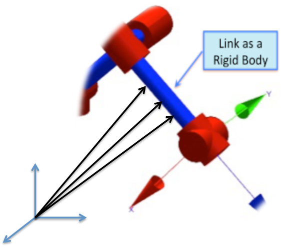


2RP4R Robot Manipulator *2S Robot Manipulator*

Robot configuration



Configuration parameters



Configuration parametes:

A set of position parameters that describe the full configutation of the system:

First choise: for n moving links $\equiv$ *joints*

$\therefore$ 9n parameters(3*points x* 3*vector*)

Generalized coordinates:

A set of independent configuration *parameters.*

Second *choise: for n moving links*

$\therefore$ 6n parameters: 3position+3orientation.

Degree of fredoom:

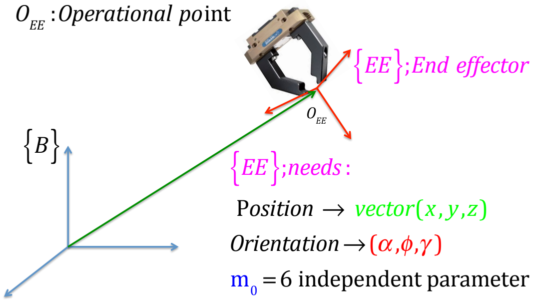
Number of generalized coordinates.

Degree of fredoome of the system:

$\therefore$ 6n-5n=n(each joint has 5 constrains)

Operational Coordinates

O_{EE} *:Operational point*



$\{EE\}$;*needs:*

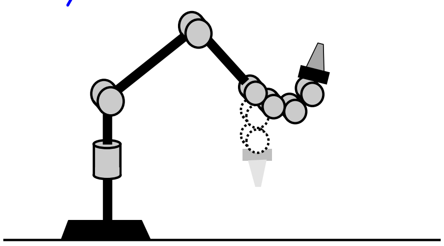
Position $\rightarrow$ *vector*(x,y,z)

Orientation $\rightarrow (\alpha,\phi,\gamma)$

$m_0 = 6$ independent parameter

m_0=6DOF *of the End Effector*$_{max}$

Redundancy



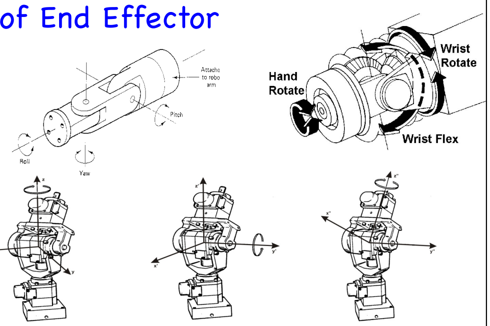
Degree of Redundancy =n- m_0

if $n > m_0$ The robot is redundant

Re*dundancy increase accesibility*

The course does not deal with redundant manipulators

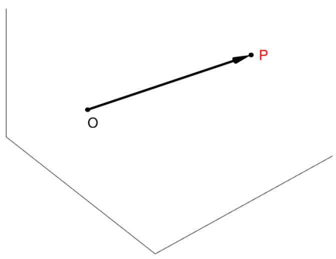
DOF of End Effector



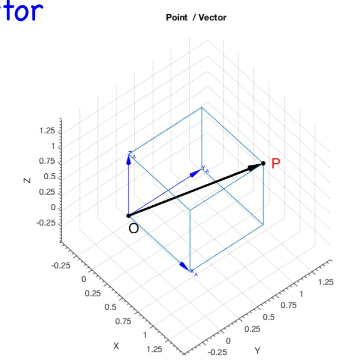
$\{EE\}$; m_0=6DOF *of the End Effector*

Orientation:roll, pitch and yaw angles and position (x,y,z)

Point / Vector



Point / Vector

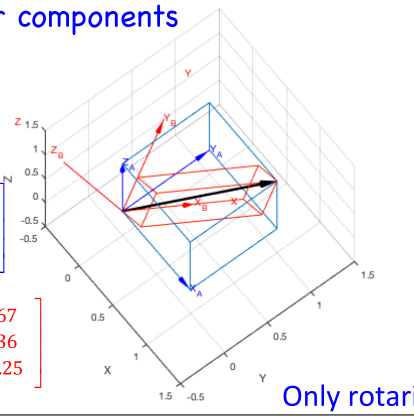


$${}^{A}P = \begin{bmatrix} {}^{A}P_x \\ {}^{A}P_y \\ {}^{A}P_z \end{bmatrix}$$

Vector components

$$ {}^{A}P=\begin{bmatrix}1\\1\\1\end{bmatrix} $$

$$ {}^{B}P=\begin{bmatrix}1.67\\0.36\\-0.25\end{bmatrix} $$

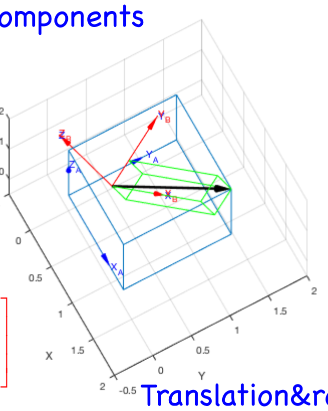


Only rotarions

Vector components

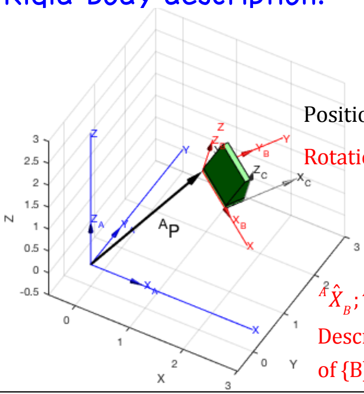
$$ {}^{A}P=\begin{bmatrix}1\\1\\1\end{bmatrix} $$

$$ {}^{B}P=\begin{bmatrix}1.67\\0.36\\-0.25\end{bmatrix} $$



Translation&rotarions

Rigid Body description:



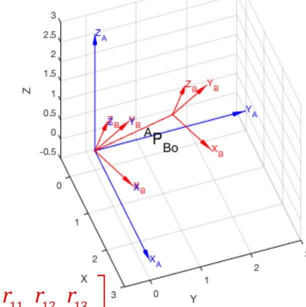
Position: ${}^{A}P$

Rotation: $\left\{ {}^{A}\hat{X}_{B} \quad {}^{A}\hat{Y}_{B} \quad {}^{A}\hat{Z}_{B} \right\}$

${}^{A}\hat{X}_{B};{}^{A}\hat{Y}_{B};{}^{A}\hat{Z}_{B}$

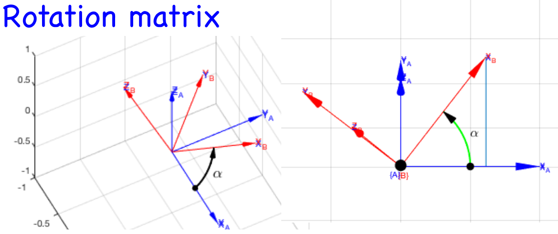
Describe unitary vector of {B} with respect to {A}

Rotation matrix



$$ {}^{A}_{B}R=\begin{bmatrix} {}^{A}\hat{X}_{B} & {}^{A}\hat{Y}_{B} & {}^{A}\hat{Z}_{B} \end{bmatrix}=\begin{bmatrix} r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3x3} $$

Rotation matrix



$$ {}^{A}_{B}R=\begin{bmatrix} {}^{A}\hat{X}_{B} & {}^{A}\hat{Y}_{B} & {}^{A}\hat{Z}_{B} \end{bmatrix}=\begin{bmatrix} r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33} \end{bmatrix}_{3x3} \quad ;Cosinus\ Matrix $$

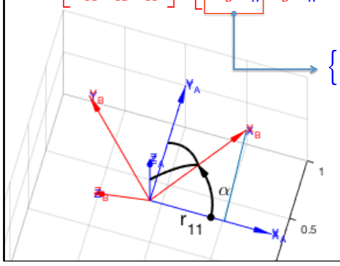
$r_{11} = Projection\ of\ \hat{X}_{B}\ onto\ \hat{X}_{A} \equiv dot\ produt$

Rotation matrix

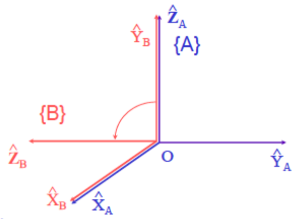
$$ {}^{A}_{B}R=\begin{bmatrix} r_{11} & r_{12} & r_{13}\\ r_{21} & r_{22} & r_{23}\\ r_{31} & r_{32} & r_{33} \end{bmatrix}=\begin{bmatrix} \hat{X}_{B}.\hat{X}_{A} & \hat{Y}_{B}.\hat{X}_{A} & \hat{Z}_{B}.\hat{X}_{A}\\ \hat{X}_{B}.\hat{Y}_{A} & \hat{Y}_{B}.\hat{Y}_{A} & \hat{Z}_{B}.\hat{Y}_{A}\\ \hat{X}_{B}.\hat{Z}_{A} & \hat{Y}_{B}.\hat{Z}_{A} & \hat{Z}_{B}.\hat{Z}_{A} \end{bmatrix} $$

$\left\{ {}^{B}\hat{X}_{A} \right.$

$\left\{ {}^{A}\hat{X}_{B} \right.$



Rotation matrix: 2D Example 1

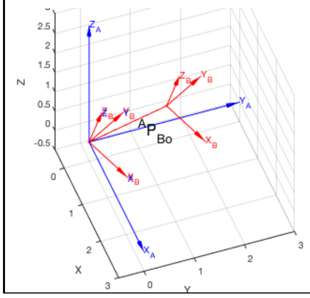


$$\begin{array}{ccc} {}^A_BR = & \begin{pmatrix}1&0&0\\0&0&-1\\0&1&0\end{pmatrix} & \begin{array}{l}\leftarrow {}^B\hat X_A^T\\ \leftarrow {}^B\hat Y_A^T\\ \leftarrow {}^B\hat Z_A^T\end{array} \\ & \begin{array}{ccc}\uparrow&\uparrow&\uparrow\\ {}^A\hat X_B & {}^A\hat Y_B & {}^A\hat Z_B\end{array} & \end{array}$$

Rotation matrix

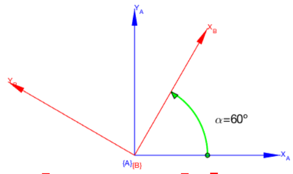
$$\left[[{}^A\hat X_B]_{3x1} \; [{}^A\hat Y_B]_{3x1} \; [{}^A\hat Z_B]_{3x1} \right]_{3x3} = {}^A_BR_{3x3} \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$$\{{}^B\hat X_B$$



$${}^A_BR = \left[{}^A\hat X_B \quad {}^A\hat Y_B \quad {}^A\hat Z_B \right]$$

Rotation matrix: 2D example 2

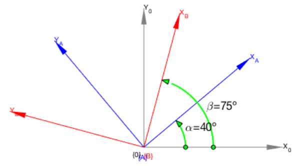


Angle	cos	sin
60	0.5	0.866

$${}^A_BR = \begin{bmatrix}\hat X_B\cdot\hat X_A & \hat Y_B\cdot\hat X_A\\ \hat X_B\cdot\hat Y_A & \hat Y_B\cdot\hat Y_A\end{bmatrix} = \begin{bmatrix}\hat X_B^T\hat X_A & \hat Y_B^T\hat X_A\\ \hat X_B^T\hat Y_A & \hat Y_B^T\hat Y_A\end{bmatrix}$$

$${}^A_BR = \begin{bmatrix} [0.5\ \ 0.866]\begin{bmatrix}1\\0\end{bmatrix} & [-0.866\ \ 0.5]\begin{bmatrix}1\\0\end{bmatrix} \\ [0.5\ \ 0.866]\begin{bmatrix}0\\1\end{bmatrix} & [-0.866\ \ 0.5]\begin{bmatrix}0\\1\end{bmatrix}\end{bmatrix} = \begin{bmatrix}0.500 & -0.866\\0.866 & 0.500\end{bmatrix}$$

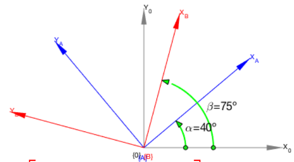
Rotation matrix: 2D example 3



Angle	cos	sin
40º	0.766	0.642
75º	0.258	0.965

$${}^A_BR = \begin{bmatrix}\hat X_B\cdot\hat X_A & \hat Y_B\cdot\hat X_A & \hat Z_B\cdot\hat X_A\\ \hat X_B\cdot\hat Y_A & \hat Y_B\cdot\hat Y_A & \hat Z_B\cdot\hat Y_A\\ \hat X_B\cdot\hat Z_A & \hat Y_B\cdot\hat Z_A & \hat Z_B\cdot\hat Z_A\end{bmatrix} = \begin{bmatrix}\hat X_B^T\hat X_A & \hat Y_B^T\hat X_A & \hat Z_B^T\hat X_A\\ \hat X_B^T\hat Y_A & \hat Y_B^T\hat Y_A & \hat Z_B^T\hat Y_A\\ \hat X_B^T\hat Z_A & \hat Y_B^T\hat Z_A & \hat Z_B^T\hat Z_A\end{bmatrix}$$

Rotation matrix: 2D example 3

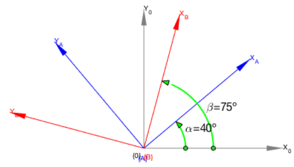


Angle	cos	sin
40º	0.766	0.642
75º	0.258	0.965

$${}^A_BR = \begin{bmatrix}\hat X_B^T\hat X_A & \hat Y_B^T\hat X_A & \hat Z_B^T\hat X_A\\ \hat X_B^T\hat Y_A & \hat Y_B^T\hat Y_A & \hat Z_B^T\hat Y_A\\ \hat X_B^T\hat Z_A & \hat Y_B^T\hat Z_A & \hat Z_B^T\hat Z_A\end{bmatrix}$$

$${}^A_BR = \begin{bmatrix} [0.258\ \ 0.965]\begin{bmatrix}0.766\\0.642\end{bmatrix} & [-0.965\ \ 0.258]\begin{bmatrix}0.766\\0.642\end{bmatrix} \\ [0.258\ \ 0.965]\begin{bmatrix}-0.642\\0.766\end{bmatrix} & [-0.965\ \ 0.258]\begin{bmatrix}-0.642\\0.766\end{bmatrix}\end{bmatrix} = \begin{bmatrix}0.819 & -0.573\\0.573 & 0.819\end{bmatrix}$$

Rotation matrix: 2D example 3

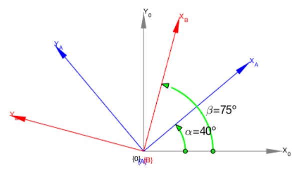


Angle	cos	sin
40º	0.766	0.642
75º	0.258	0.965

$${}^A_BR = \begin{bmatrix}\hat X_B^T\hat X_A & \hat Y_B^T\hat X_A\\ \hat X_B^T\hat Y_A & \hat Y_B^T\hat Y_A\end{bmatrix} = \begin{bmatrix}\hat X_A^T\hat X_B & \hat X_A^T\hat Y_B\\ \hat Y_A^T\hat X_B & \hat Y_A^T\hat Y_B\end{bmatrix} = \begin{bmatrix}\hat X_A^T\\ \hat Y_A^T\end{bmatrix}_{2x2} \left[\hat X_B \quad \hat Y_B\right]_{2x2}$$

$${}^A_BR = \begin{bmatrix}0.766 & 0.642\\ -0.642 & 0.766\end{bmatrix}\begin{bmatrix}0.258 & -0.965\\ 0.965 & 0.258\end{bmatrix} = \begin{bmatrix}0.819 & -0.573\\ 0.573 & 0.819\end{bmatrix}$$

Rotation matrix: 2D example 3



Angle	cos	sin
40º	0.766	0.642
75º	0.258	0.965

$${}^{A}_{B}R_{2x2} = \begin{bmatrix} \hat{X}_A^T \\ \hat{Y}_A^T \end{bmatrix}_{2x2} \begin{bmatrix} \hat{X}_B & \hat{Y}_B \end{bmatrix}_{2x2} = [\,\hat{X}_A \;\; \hat{Y}_A\,]^T_{2x2} \begin{bmatrix} \hat{X}_B & \hat{Y}_B \end{bmatrix}_{2x2}$$

$${}^{A}_{B}R_{2x2} = \begin{bmatrix} \hat{X}_A^T \\ \hat{Y}_A^T \end{bmatrix}_{2x2} \begin{bmatrix} \hat{X}_B & \hat{Y}_B \end{bmatrix}_{2x2} = \underbrace{{}^{0}RF_A^T}_{\substack{\text{Frame A} \\ \text{Description} \\ \text{wrt } RF_0}} \; \underbrace{{}^{0}RF_B}_{\substack{\text{Frame B} \\ \text{Description} \\ \text{wrt } RF_0}}$$

Rotation Matrix

$${}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A & {}^{B}\hat{Y}_A & {}^{B}\hat{Z}_A \end{bmatrix}^T = {}^{B}_{A}R^T$$

$${}^{A}_{B}R = {}^{B}_{A}R^T$$

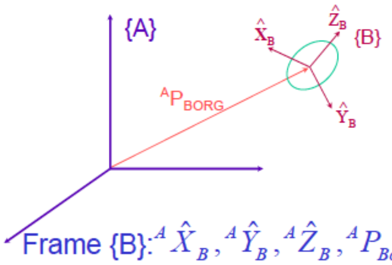
Inverse of Rotation Matrices

$${}^{A}_{B}R^{-1} = {}^{B}_{A}R = {}^{A}_{B}R^T$$

$$\boxed{{}^{A}_{B}R^{-1} = {}^{A}_{B}R^T}$$

Orthonormal Matrix

Frame description



Frame {B}: ${}^{A}\hat{X}_B,\ {}^{A}\hat{Y}_B,\ {}^{A}\hat{Z}_B,\ {}^{A}P_{Borg}$

$$\{B\} = \left\{ {}^{A}_{B}R \quad {}^{A}P_{Borg} \right\}$$

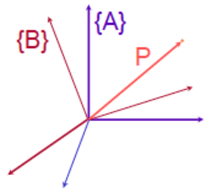
Mapping

Changing description from frame to frame

Rotations

$$\left.\begin{aligned} {}^{A}p_x &= {}^{B}\hat{X}_A \cdot {}^{B}P \\ {}^{A}p_y &= {}^{B}\hat{Y}_A \cdot {}^{B}P \\ {}^{A}p_z &= {}^{B}\hat{Z}_A \cdot {}^{B}P \end{aligned}\right\}$$

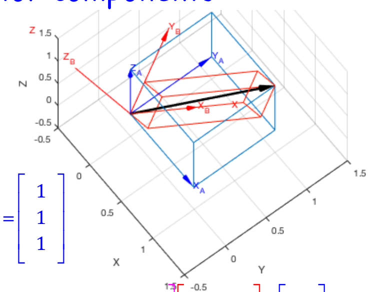
${}^{B}P$ projected onto unit vector of its frame



$${}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B & {}^{A}\hat{Y}_B & {}^{A}\hat{Z}_B \end{bmatrix} = \begin{bmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{bmatrix}$$

$$\left.\begin{aligned} {}^{A}p_x &= {}^{B}\hat{X}_A \cdot {}^{B}P = {}^{B}\hat{X}_A^T\,{}^{B}P \\ {}^{A}p_y &= {}^{B}\hat{Y}_A \cdot {}^{B}P = {}^{B}\hat{Y}_A^T\,{}^{B}P \\ {}^{A}p_z &= {}^{B}\hat{Z}_A \cdot {}^{B}P = {}^{B}\hat{Z}_A^T\,{}^{B}P \end{aligned}\right\} {}^{A}P = {}^{A}_{B}R\,{}^{B}P$$

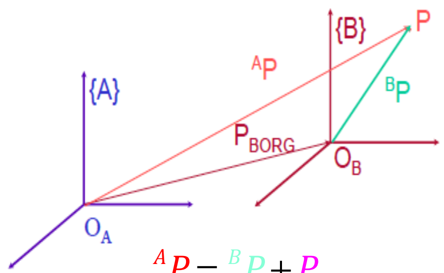
Mapping: Vector components



$${}^{B}P = \begin{bmatrix} 1.67 \\ 0.36 \\ -0.25 \end{bmatrix} \rightarrow {}^{A}P = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$${}^{A}P = {}^{A}_{B}R\,{}^{B}P = \begin{bmatrix} 0.6124 & -0.5 & -0.6124 \\ 0.3536 & 0.866 & -0.3536 \\ 0.7071 & 0 & 0.7071 \end{bmatrix} \begin{bmatrix} 1.67 \\ 0.36 \\ -0.25 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

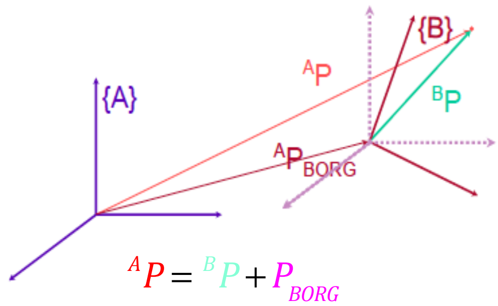
Translation



$${}^{A}P = {}^{B}P + P_{BORG}$$

Change position description

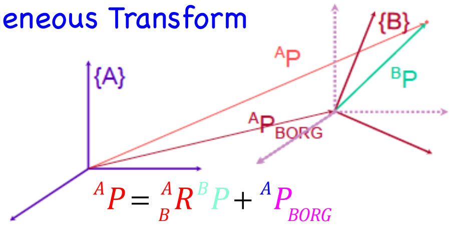
General Transform



$$^{A}P = {}^{B}P + P_{BORG}$$

$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

Homogeneous Transform

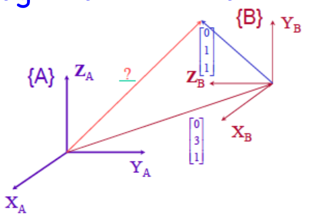


$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{BORG}$$

$$\begin{bmatrix} ^{A}P \\ 1 \end{bmatrix} = \begin{bmatrix} ^{A}_{B}R_{3x3} & ^{A}P_{BORG_{3x1}} \\ 0\ 0\ 0_{\ 1x3} & 1 \end{bmatrix}_{4x4} \begin{bmatrix} ^{B}P \\ 1 \end{bmatrix}$$

$$^{A}P_{4x1} = {}^{A}_{B}T_{4x4}\,{}^{B}P_{4x1}$$

Homogeneous Transform: example



Homogeneous Transform $\quad {}^{A}_{B}T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad {}^{B}P = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

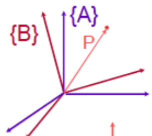
$$^{A}P = {}^{A}_{B}T\,.\,{}^{B}P \Rightarrow {}^{A}P = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

Operators

Mapping: changing descriptions from frame to frame

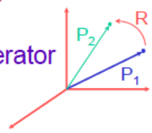
Operators: moving points (within the same frame)

Mapping



$^{A}P = {}^{A}_{B}R\ {}^{B}P$

Rotational Operator



R: $P_1 \longrightarrow P_2$

$\boxed{P_2 = R\ P_1}$

Rotational operators

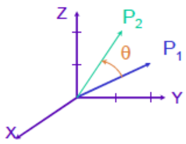
$R_K(\theta)$: $P_1 \longrightarrow P_2$

$P_2 = R_K(\theta)\ P_1$

Angle (θ)	cos	sin
36.87	0.8	0.6

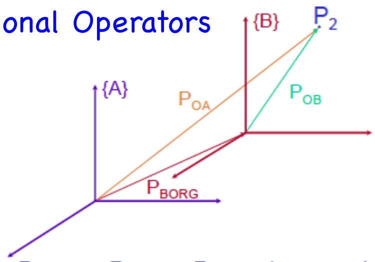
Example

$$R_X(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$



$$P_2 = R_X(\theta)P_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.8 & -0.6 \\ 0 & 0.6 & 0.8 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

Translational Operators

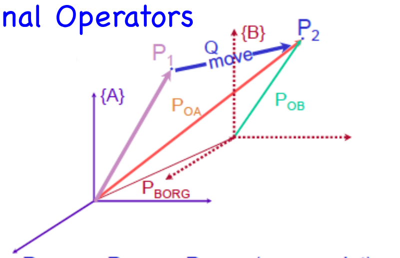


Mapping: P_{BORG}: $P_{OB} \longrightarrow P_{OA}$ (same point)

2 diff. vectors

$P_{OA} = P_{OB} + P_{BORG}$

Translational Operators



Mapping: P_{BORG} : $P_{OB} \longrightarrow P_{OA}$ (same point)
2 diff. vectors

$P_{OA} = P_{OB} + P_{BORG}$

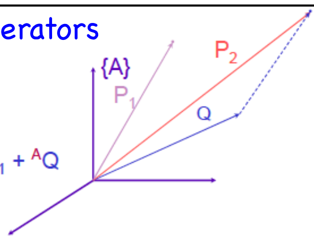
Translational Operator:

Q : $P_1 \longrightarrow P_2$ (2 points, 2 diff vectors)

$P_2 = P_1 + Q$

Translational Operators

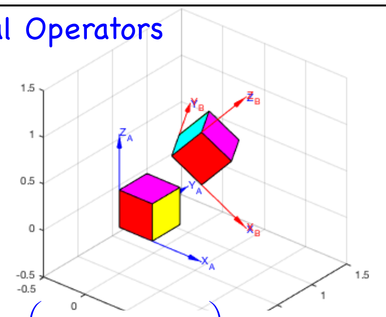
Operator: ${}^{A}P_2 = {}^{A}P_1 + {}^{A}Q$



Homogeneous Transform:

$$D_Q = \begin{bmatrix} 1 & 0 & 0 & q_x \\ 0 & 1 & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow {}^{A}P_2 = {}^{A}D_Q {}^{A}P_1$$

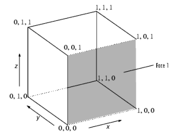
General Operators

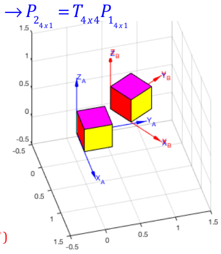


$$P_2 = \begin{pmatrix} R_{k_{3x3}} & Q_{3x1} \\ 0\ 0\ 0 & 1 \end{pmatrix} P_1 \rightarrow P_2 = TP_1$$

General Operators: example

$$P_2 = \begin{pmatrix} R_{k_{3x3}} & Q_{3x1} \\ 0\ 0\ 0_{\ 3x3} & 1 \end{pmatrix}_{4x4} \quad P_{1_{4x1}} \rightarrow P_{2_{4x1}} = T_{4x4} P_{1_{4x1}}$$

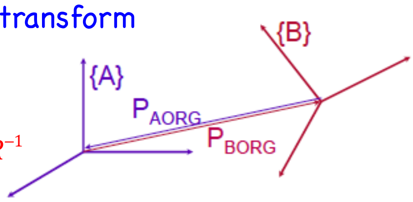




Move a 0.4 cube: $Q = tranlation(0.3,0.4,0.6)$;$R_K = Rot_Z(\alpha = 30^\circ)$

$$P_2 = \begin{pmatrix} c\alpha & -s\alpha & 0 & q_x \\ s\alpha & c\alpha & 0 & q_y \\ 0 & 0 & 1 & q_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0.4 & 0.4 & 0 & 0 & 0.4 & 0.4 & 0 \\ 0 & 0 & 0.4 & 0.4 & 0 & 0 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 & 0.4 & 0.4 & 0.4 & 0.4 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0.3 & 0.64 & 0.44 & 0.1 & 0.3 & 0.64 & 0.44 & 0.1 \\ 0.4 & 0.60 & 0.94 & 0.74 & 0.4 & 0.6 & 0.94 & 0.74 \\ 0.6 & 0.60 & 0 & 0.60 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Inverse transform



$${}^{A}_{B}R^{T} = {}^{A}_{B}R^{-1}$$

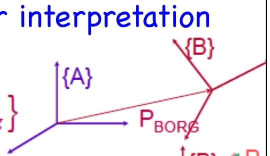
$${}^{A}_{B}T \neq {}^{B}_{A}T$$

$${}^{A}_{B}T^{-1} = \begin{bmatrix} {}^{A}_{B}R^{T} & \underbrace{\boxed{-{}^{A}_{B}R^{T}\,{}^{A}P_{BORG}}}_{{}^{B}P_{AORG}} \\ 0\ 0\ 0 & 1 \end{bmatrix}$$

Homogeneus transfor interpretation

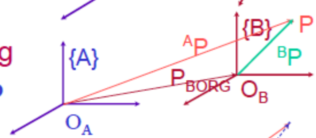
Description of a frame

${}^{A}_{B}T$: $\{B\} = \{{}^{A}_{B}R \quad {}^{A}P_{Borg}\}$



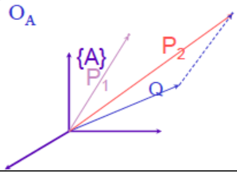
Transform mapping

${}^{A}_{B}T$: ${}^{B}P \rightarrow {}^{A}P$

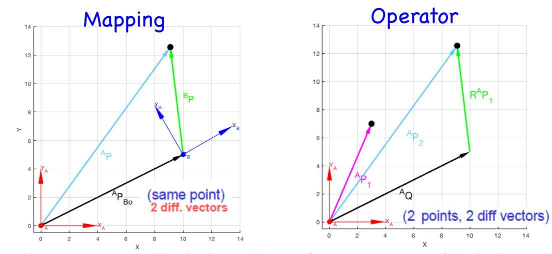


Transform operator

T : $P_1 \rightarrow P_2$



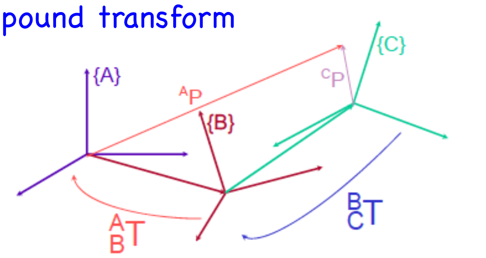
Homogeneus transform interpretation



$$
{}^{A}P=\underbrace{\begin{pmatrix}0.866 & -0.5 & 0 & 10\\ 0.5 & 0.866 & 0 & 5\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}}_{{}^{A}_{B}T}\underbrace{\begin{pmatrix}3\\7\\0\\1\end{pmatrix}}_{{}^{B}P}=\begin{pmatrix}9.09\\12.56\\0\\1\end{pmatrix}
\qquad
{}^{A}P_2=\underbrace{\begin{pmatrix}0.866 & -0.5 & 0 & 10\\ 0.5 & 0.866 & 0 & 5\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}}_{T}\underbrace{\begin{pmatrix}3\\7\\0\\1\end{pmatrix}}_{{}^{A}P_1}=\begin{pmatrix}9.09\\12.56\\0\\1\end{pmatrix}
$$

Same result different interpretation

Compound transform



$$ {}^{B}P = {}^{B}_{C}T\ {}^{C}P $$

$$ {}^{A}P = {}^{A}_{B}T\ {}^{B}P $$

$$ {}^{A}P = {}^{A}_{B}T\ {}^{B}_{C}T\ {}^{C}P \ \Rightarrow\ {}^{A}_{C}T = {}^{A}_{B}T\ {}^{B}_{C}T $$

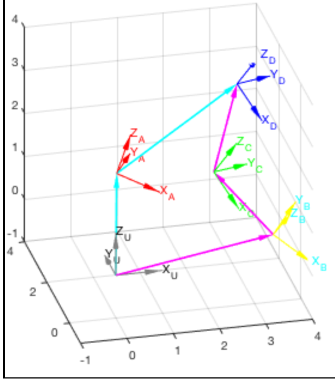
Compound transform

$$ {}^{A}_{C}T = {}^{A}_{B}T\ {}^{B}_{C}T $$

$$ {}^{A}_{C}T = \begin{bmatrix} {}^{A}_{B}R & {}^{A}P_{BORG} \\ 0_{1x3} & 1 \end{bmatrix}\begin{bmatrix} {}^{B}_{C}R & {}^{B}P_{CORG} \\ 0_{1x3} & 1 \end{bmatrix} $$

$$ {}^{A}_{C}T = \begin{bmatrix} \boxed{{}^{A}_{B}R\,{}^{B}_{C}R}_{3x3} & \boxed{{}^{A}_{B}R\,{}^{A}P_{BORG} + {}^{A}P_{BORG}}_{3x1} \\ \boxed{0}_{1x3} & \boxed{1}_{1x1} \end{bmatrix}_{4x4} $$

Transform equation



$CCW;$

$$ {}^{B}_{C}T\,{}^{C}_{D}T\,{}^{D}_{A}T^{-1}\,{}^{A}_{U}T^{-1}\,{}^{U}_{B}T = I $$

$$ {}^{B}_{C}T = {}^{U}_{B}T^{-1}\,{}^{U}_{A}T\,{}^{D}_{A}T\,{}^{C}_{D}T^{-1} $$

$CW;$

$$ {}^{B}_{C}T^{-1}\,{}^{U}_{B}T^{-1}\,{}^{U}_{A}T\,{}^{D}_{A}T\,{}^{C}_{D}T^{-1} = I $$

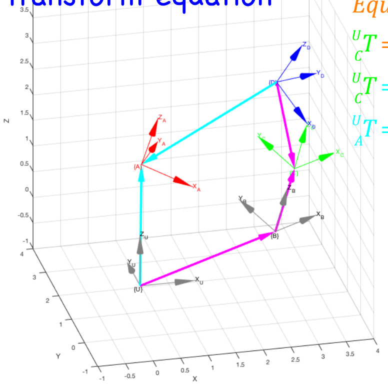
$$ {}^{B}_{C}T = {}^{U}_{B}T^{-1}\,{}^{U}_{A}T\,{}^{D}_{A}T\,{}^{C}_{D}T^{-1} $$

Equating

$$ {}^{U}_{A}T\,{}^{A}_{D}T = {}^{U}_{B}T\,{}^{B}_{C}T\,{}^{C}_{D}T $$

$$ {}^{B}_{C}T = {}^{U}_{B}T^{-1}\,{}^{U}_{A}T\,{}^{D}_{A}T\,{}^{C}_{D}T^{-1} $$

Transform equation



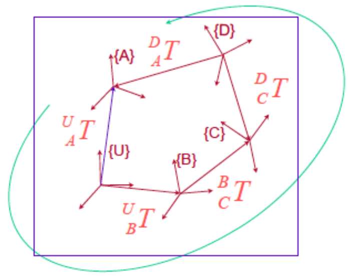
Equating

$$ {}^{U}_{C}T = {}^{U}_{A}T\,{}^{D}_{A}T^{-1}\,{}^{D}_{C}T $$

$$ {}^{U}_{C}T = {}^{U}_{B}T\,{}^{B}_{C}T $$

$$ {}^{U}_{A}T = {}^{U}_{B}T\,{}^{B}_{C}T\,{}^{D}_{C}T^{-1}\,{}^{D}_{A}T $$

Transform equation



$$ {}^{D}_{A}T^{-1}\cdot{}^{D}_{C}T\cdot{}^{B}_{C}T^{-1}\cdot{}^{U}_{B}T^{-1}\cdot{}^{U}_{A}T \equiv I $$

$$ {}^{U}_{A}T = {}^{U}_{B}T\cdot{}^{B}_{C}T\cdot{}^{D}_{C}T^{-1}\cdot{}^{D}_{A}T $$