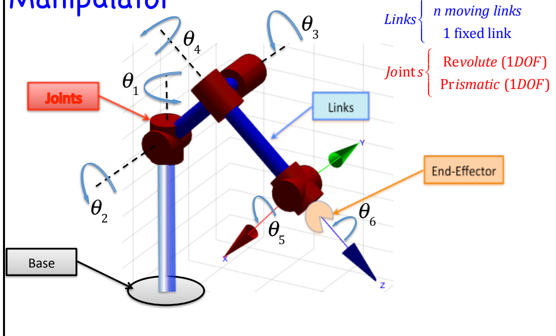
Manipulator Kinematics

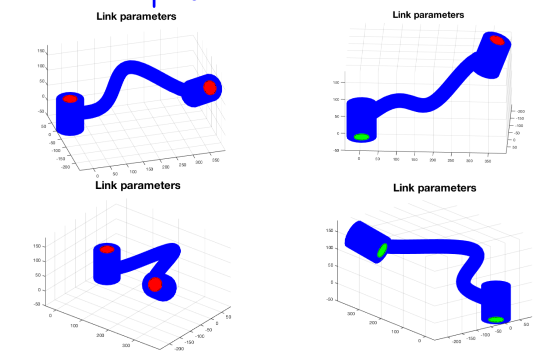
- Link Description
- Denavit-Hartenberg Notation
- Frame Attachment
- Forward Kinematics

Manipulator



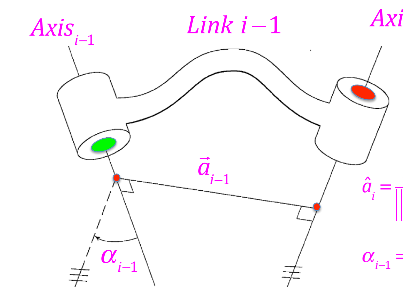
6R Robot Manipulator

Link description



Top view Bottom view

Link description



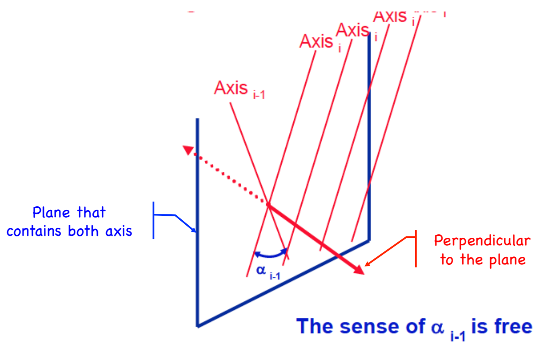
$$\hat{a}_i = \frac{\hat{Z}_i \times \hat{Z}_{i-1}}{\left\|\hat{Z}_i \times \hat{Z}_{i-1}\right\|}$$

$$\alpha_{i-1} = a\cos\frac{\hat{Z}_i \cdot \hat{Z}_{i-1}}{\left\|\hat{Z}_i\right\|\left\|\hat{Z}_{i-1}\right\|}$$

a_{i-1} : Link Length - shortes distance in: Mutual perpendicular

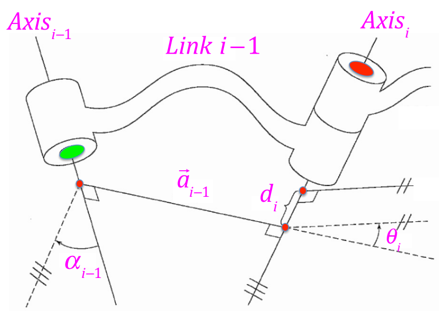
α_{i-1} : Link twist - measured in the right hand sense about $\vec{a}_{i-1}$

Intersecting joint anxes



The sense of α i-1 is free

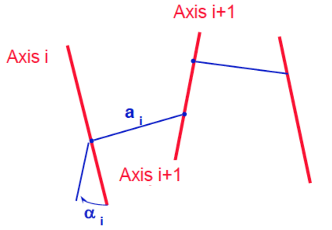
Link connections



d_i : Link offset - Variable if joit is prismatic

θ_i : Joint angle - Variable if the joint is revolute

First & last links



a_i and α_i depend on joint axes i and i+1

Axes 1 to n: determined

➡ a_1 , a_2 ... a_{n-1} and α_1, α_2 ... α_{n-1}

Convention: $a_0 = a_n = 0$ and $\alpha_0 = \alpha_n = 0$

Denavit-Hartenberg Parameters

4 D-H parameters $(\alpha_i\,,\; a_i\,,\; d_i\,,\; \theta_i)$

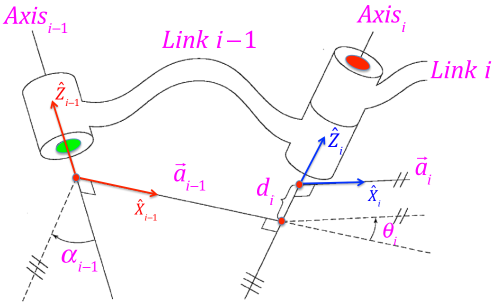
3 fixed link parameters

1 joint variable $\begin{cases} \theta_i \text{ revolute joint} \\ d_i \text{ prismatic joint} \end{cases}$

α_i and a_i : describe the Link i

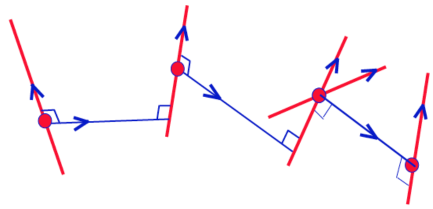
d_i and θ_i : describe the Link's connection

Link connections



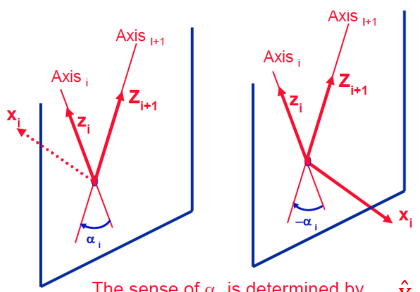
$\hat{Y}$ – *unitary vector* : *complete* right-hand frames

Summary – Frame Attachment



1. Normals
2. Origins
3. Z-axes
4. X-axes

Intersecting joint anxes



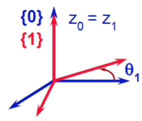
The sense of α_i is determined by the direction of x

$$\hat{X}_i = \frac{\hat{Z}_i \times \hat{Z}_i}{\left\|\hat{Z}_i \times \hat{Z}_i\right\|}$$

Lets put it in practice

First Link

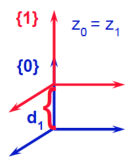
Revolute



$a_0 = 0$
$\alpha_0 = 0$
$d_1 = 0$

$\theta_1 = 0 \longrightarrow \{0\}\equiv\{1\}$

Prismatic

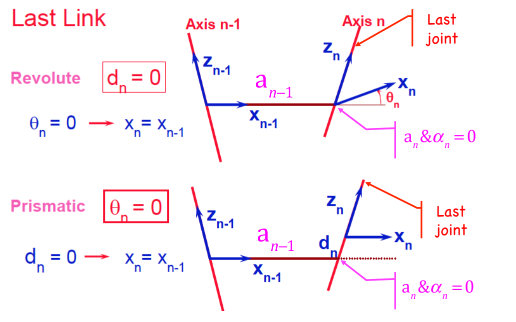


$a_0 = 0$
$\alpha_0 = 0$
$\theta_1 = 0$

$d_1 = 0 \longrightarrow \{0\}\equiv\{1\}$

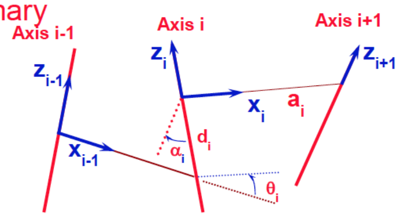
Convention: $a_0 = a_n = 0$ and $\alpha_0 = \alpha_n = 0$

Lets put it in practice



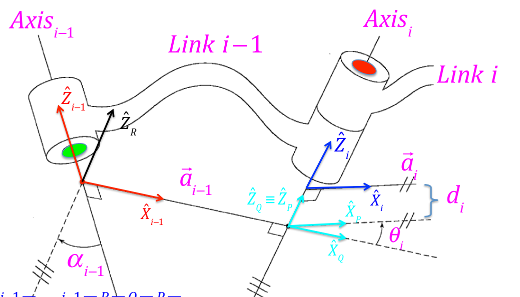
$$Convention: \; a_0 = a_n = 0 \text{ and } \alpha_0 = \alpha_n = 0$$

Summary



- a_i : distance (z_i , z_{i+1}) along x_i
- α_i : angle (z_i , z_{i+1}) about x_i
- d_i : distance (x_{i-1} , x_i) along z_i
- θ_i : angle (x_{i-1} , x_i) about z_i

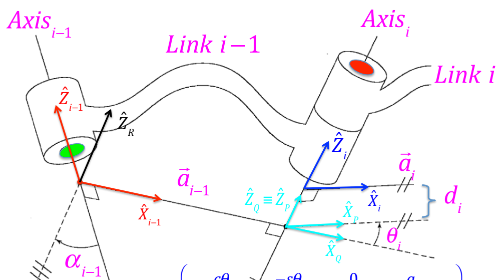
Forward Kinematics



$${}^{i-1}_{i}T = {}^{i-1}_{R}T \, {}^{R}_{Q}T \, {}^{Q}_{P}T \, {}^{P}_{i}T$$

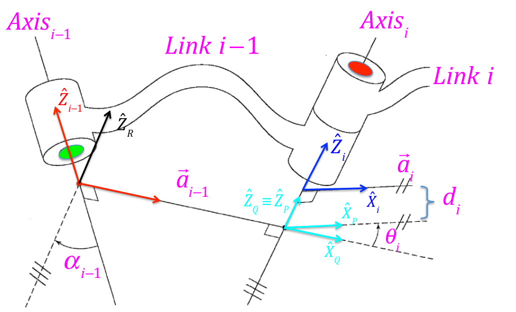
$${}^{i-1}_{i}T(\alpha_i, a_{i-1}, \theta_i, d_i) = R_X(\alpha_i) D_X(a_{i-1}) R_Z(\theta_i) D(d_i)$$

Forward Kinematics: for a link



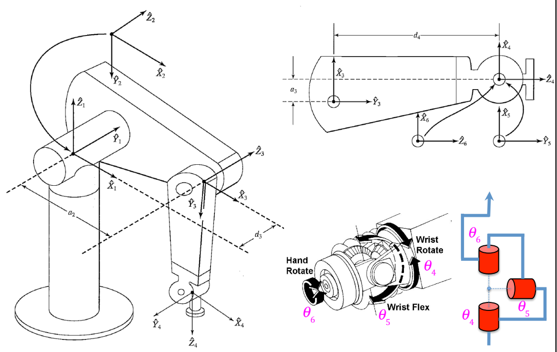
$${}^{i-1}_{i}T(\alpha_{i-1}, a_{i-1}, \theta_i, d_i) = \begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Forward Kinematics: for a chain

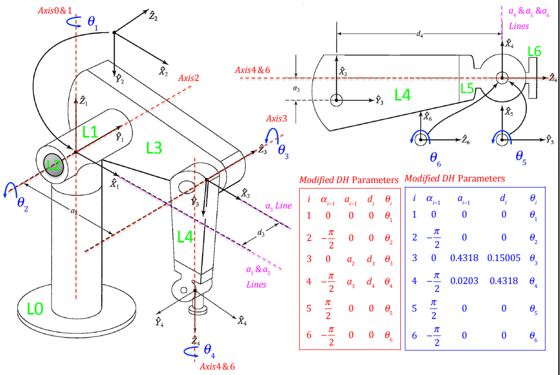


$${}^{0}_{N}T = {}^{0}_{1}T \, {}^{1}_{2}T \, {}^{2}_{3}T \ldots {}^{N-1}_{N}T$$

Forward Kinematics of Puma 560 Robot



Forward Kinematics of Puma 560 Robot



Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\frac{\pi}{2}$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-\frac{\pi}{2}$	a_3	d_4	θ_4
5	$\frac{\pi}{2}$	0	0	θ_5
6	$-\frac{\pi}{2}$	0	0	θ_6

Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\frac{\pi}{2}$	0	0	θ_2
3	0	0.4318	0.15005	θ_3
4	$-\frac{\pi}{2}$	0.0203	0.4318	θ_4
5	$\frac{\pi}{2}$	0	0	θ_5
6	$-\frac{\pi}{2}$	0	0	θ_6

Forward Kinematics of Puma 560 Robot

$$
{}^{i-1}_{\ i}T(\alpha_{i-1},a_{i-1},\theta_i,d_i)=\begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d \\ 0 & 0 & 0 & 1 \end{pmatrix}
$$

PUMA560 DH parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\frac{\pi}{2}$	0	0	θ_2
3	0	a_2	d_3	θ_3
4	$-\frac{\pi}{2}$	a_3	d_4	θ_4
5	$\frac{\pi}{2}$	0	0	θ_5
6	$-\frac{\pi}{2}$	0	0	θ_6

$$
\left\{
\begin{array}{ll}
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} &
{}^1_2T = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^2_3T = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} &
{}^3_4T = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -\sin\theta_4 & -\cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
{}^4_5T = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} &
{}^5_6T = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{array}
\right\}
$$

Forward Kinematics of Puma 560 Robot

Applying the concepts for each link:

- **First Link**, only $R_Z(\theta_1)$:

$$
{}^0_1T = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- **Second Link**, $R_X(-\frac{\pi}{2})$ and $R_Z(\theta_2)$:

$$
{}^1_2T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos-\frac{\pi}{2} & -\sin-\frac{\pi}{2} & 0 \\ 0 & \sin-\frac{\pi}{2} & \cos-\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- **Tird Link**, $Trans_X(a_2)$, $Trans_Z(d_3)$, and $R_Z(\theta_3)$:

$$
{}^2_3T = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_2 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics of Puma 560 Robot

- **Fourth Link**, $Trans_X(a_3)$, $Trans_Y(d_4)$, $R_X(-\frac{\pi}{2})$, $R_Z(\theta_4)$:

$$
{}^3_4T = \begin{bmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos-\frac{\pi}{2} & -\sin-\frac{\pi}{2} & 0 \\ 0 & \sin-\frac{\pi}{2} & \cos-\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & 0 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -\sin\theta_4 & -\cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- **Fifth Link**, $R_X(\frac{\pi}{2})$, $R_Z(\theta_5)$:

$$
{}^4_5T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\frac{\pi}{2} & -\sin\frac{\pi}{2} & 0 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} : \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

- **Sixth Link**, $R_X(-\frac{\pi}{2})$, $R_Z(\theta_6)$:

$$
{}^5_6T = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos-\frac{\pi}{2} & -\sin-\frac{\pi}{2} & 0 \\ 0 & \sin-\frac{\pi}{2} & \cos-\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} : \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Forward Kinematics of Puma 560 Robot

Matrix Multiplication

$$
{}^0_6T = {}^0_1T\ {}^1_2T\ {}^2_3T\ {}^3_4T\ {}^4_5T\ {}^5_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Note: \left\{ \begin{array}{c} c_{23} = \cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3 \\ \text{and} \\ s_{23} = \cos\theta_2\sin\theta_3 - \sin\theta_2\cos\theta_3 \end{array} \right\}
$$

$$
\begin{aligned}
r_{11} &= \cos\theta_1[c_{23}(\cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\sin\theta_6) - s_{23}\sin\theta_5\cos\theta_6] + \sin\theta_1(\sin\theta_4\cos\theta_5\cos\theta_6 + \cos\theta_4\sin\theta_6) \\
r_{21} &= \sin\theta_1[c_{23}(\cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\sin\theta_6) - s_{23}\sin\theta_5\cos\theta_6] - \cos\theta_1(\sin\theta_4\cos\theta_5\cos\theta_6 + \cos\theta_4\sin\theta_6) \\
r_{31} &= -s_{23}(\cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\sin\theta_6) - c_{23}\sin\theta_5\cos\theta_6
\end{aligned}
$$

$$
\begin{aligned}
r_{12} &= \cos\theta_1[c_{23}(-\cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\cos\theta_6) - s_{23}\sin\theta_5\sin\theta_6] + \sin\theta_1(\cos\theta_4\cos\theta_6 - \sin\theta_4\cos\theta_5\sin\theta_6) \\
r_{22} &= \sin\theta_1[c_{23}(-\cos\theta_4\cos\theta_5\cos\theta_6 - \sin\theta_4\cos\theta_6) + s_{23}\sin\theta_5\sin\theta_6] - \cos\theta_1(\cos\theta_4\cos\theta_6 - \sin\theta_4\cos\theta_5\sin\theta_6) \\
r_{32} &= -s_{23}(-\cos\theta_4\cos\theta_5\sin\theta_6 - \sin\theta_4\cos\theta_6) + c_{23}\sin\theta_5\sin\theta_6
\end{aligned}
$$

$$
\begin{aligned}
r_{13} &= -\cos\theta_1(c_{23}\cos\theta_4\sin\theta_5 + s_{23}\cos\theta_5) - \sin\theta_1\sin\theta_4\sin\theta_5 \\
r_{23} &= -\sin\theta_1(c_{23}\cos\theta_4\sin\theta_5 + s_{23}\cos\theta_5) - \cos\theta_1\sin\theta_4\sin\theta_5 \\
r_{33} &= s_{23}\cos\theta_4\sin\theta_5 - c_{23}\cos\theta_5
\end{aligned}
$$

$$
\begin{aligned}
p_x &= \cos\theta_1[a_2\cos\theta_2 + a_3c_{23} - d_4s_{23}] - d_3\sin\theta_1 \\
p_y &= \sin\theta_1[a_2\cos\theta_2 + a_3c_{23} - d_4s_{23}] + d_3\cos\theta_1 \\
p_z &= -a_3s_{23} - a_2\sin\theta_2 - d_4c_{23}
\end{aligned}
$$

Forward Kinematics of Puma 560 Robot

Testing the consistency of the transformation

PUMA560 DH parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	$-\frac{\pi}{2}$	0	0	θ_2
3	0	0.4318	0.15005	θ_3
4	$-\frac{\pi}{2}$	0.0203	0.4318	θ_4
5	$\frac{\pi}{2}$	0	0	θ_5
6	$-\frac{\pi}{2}$	0	0	θ_6

Home position: match with the picture of the robot

$\theta_1 = \theta_2 = \theta_3 = \theta_4 = \theta_5 = \theta_6 = 0,$

Replacing

$$
\begin{aligned}
c_{23} &= [\cos\theta_2\cos\theta_3 - \sin\theta_2\sin\theta_3]_{\theta_1=0,\theta_2=0,\theta_3=0,\theta_4=0,\theta_5=0,\theta_6=0,a_2=0.4318,a_3=0.0203,d_3=0.15005,d_4=0.4318} = 1.0 \\
s_{23} &= [\cos\theta_2\sin\theta_3 - \sin\theta_2\cos\theta_3]_{\theta_1=0,\theta_2=0,\theta_3=0,\theta_4=0,\theta_5=0,\theta_6=0,a_2=0.4318,a_3=0.0203,d_3=0.15005,d_4=0.4318} = 0.0 \\
p_x &= [(a_2\cos\theta_2 + a_3c_{23} - d_4s_{23})\cos\theta_1 - d_3\sin\theta_1]_{\theta_1=0,\theta_2=0,\theta_3=0,\theta_4=0,\theta_5=0,\theta_6=0,a_2=0.4318,a_3=0.0203,d_3=0.15005,d_4=0.4318,c_{23}=1,s_{23}=0} = 0.4521 \\
p_y &= [(a_2\cos\theta_2 + a_3c_{23} - d_4s_{23})\sin\theta_1 + d_3\cos\theta_1]_{\theta_1=0,\theta_2=0,\theta_3=0,\theta_4=0,\theta_5=0,\theta_6=0,a_2=0.4318,a_3=0.0203,d_3=0.15005,d_4=0.4318,c_{23}=1,s_{23}=0} = 0.15005 \\
p_z &= [-a_3s_{23} - a_2\sin\theta_2 - d_4c_{23}]_{\theta_1=0,\theta_2=0,\theta_3=0,\theta_4=0,\theta_5=0,\theta_6=0,a_2=0.4318,a_3=0.0203,d_3=0.15005,d_4=0.4318,c_{23}=1,s_{23}=0} = -0.4318
\end{aligned}
$$

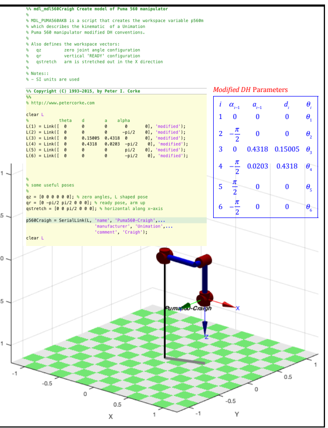
Using the RTB

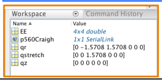
RTB setences at command window
clear;mdl_puma560_Craigh
p560Craigh.plot(qz)
EE=p560Craigh.fkine(qz)
P560-Craigh

EE =

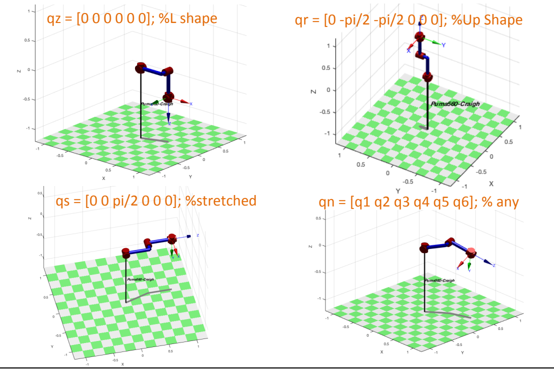
$$\begin{pmatrix} 1.0000 & 0 & 0 & 0.4521 \\ 0 & -1.0000 & 0.0000 & 0.1501 \\ 0 & -0.0000 & -1.0000 & -0.4318 \\ 0 & 0 & 0 & 1.0000 \end{pmatrix}$$

```
p560Craigh =

Puma560-Craigh (6 axis, RRRRRR, modDH, fastRNE)
 AK&B;
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|          0|          0|          0|          0|
|  2|         q2|          0|          0|     -1.571|          0|
|  3|         q3|       0.15|     0.4318|          0|          0|
|  4|         q4|     0.4318|     0.0203|     -1.571|          0|
|  5|         q5|          0|          0|      1.571|          0|
|  6|         q6|          0|          0|     -1.571|          0|
+---+-----------+-----------+-----------+-----------+-----------+

grav =     0  base = 1  0  0  0   tool =  1  0  0  0
           0         0  1  0  0           0  1  0  0
        9.81         0  0  1  0           0  0  1  0
```

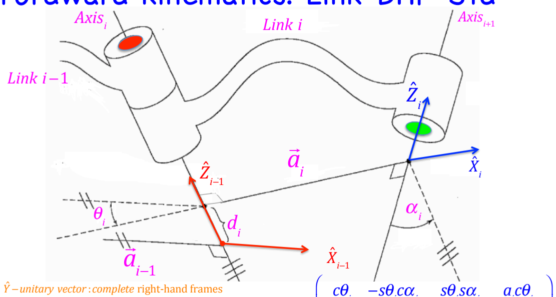




Puma 560 at 4 different poses



Fordward kinematics: Link-DHP-Std

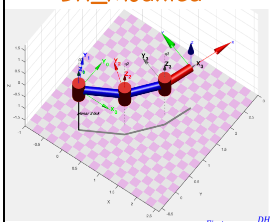


$\hat{Y}$ –unitary vector : complete right-hand frames

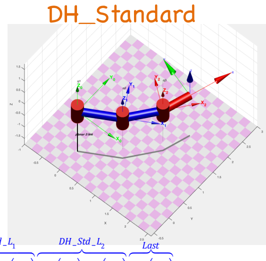
$$^{i-1}_{i}T(\theta_i,d_i,a_i,\alpha_i)=R_Z(\theta_i)D_Z(d_i)D_X(a_i)R_X(\alpha_i)=\begin{pmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & \alpha_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Comparig DH-Std vs DH-Mod

DH_Modified



DH_Standard

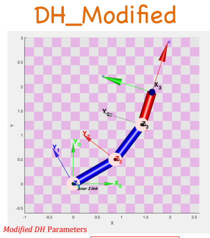


$$DH-Modified;\ ^{EE}_{0}T=\overbrace{R_Z(\theta_1)}^{First}\overbrace{D_X(L_1)R_Z(\theta_2)}^{DH\_Mod\_L_1}\overbrace{D_X(L_2)R_Z(\theta_3)}^{DH\_Std\_L_2}\overbrace{D_X(L_3)}^{Last}$$

$$DH-Standard;\ ^{EE}_{0}T=\underbrace{R_Z(\theta_1)D_X(L_1)}_{DH\_Std\_L_1}\underbrace{R_Z(\theta_2)D_X(L_2)}_{DH\_Std\_L_2}\underbrace{R_Z(\theta_3)D_X(L_3)}_{DH\_Std\_L_3}$$

Comparig DH-Std vs DH-Mod

DH_Modified

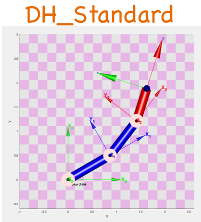


Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3
EE	0	L_3	0	0

$$^{i-1}_{i}T(\alpha_{i-1},a_{i-1},\theta_i,d_i)=\begin{pmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1}d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1}d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

DH_Standard



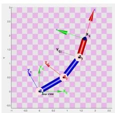
Standard DH Parameters

i	α_i	a_i	d_i	θ_i
1	0	L_1	0	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3

$$^{i-1}_{i}T(\theta_i,d_i,a_i,\alpha_i)=\begin{pmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & \alpha_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

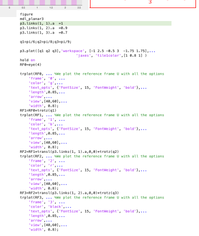
RTB: DH-Std vs DH-Mod

DH_Modified

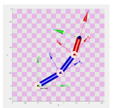


Modified DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3
EE	0	L_3	0	0



DH_Standard



Standard DH Parameters

i	α_i	a_i	d_i	θ_i
1	0	L_1	0	θ_1
2	0	L_2	0	θ_2
3	0	L_3	0	θ_3