Assignment 2 Hashing Report

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1.
Assuming we are using separate chaining linked list without MTF and not tracking the tail, insertion takes O(n) because we have to traverse through the entire list until the end and point the tail to the newly malloced element. The list is n long in maximum since n elements hashes to one bucket.

The upper bound of the search is O(n) since the element finding could be at the end of the list and to reach the end in linked list, we are required to traverse through all the linked elements.

2.
Insertion still takes O(n/size) for the same reason as number 1, except that the maximum length of the list is now n/size since elements are spread out evenly throughout the buckets.
Upper bound of search is O(n/size) because each bucket has now n/size elements as the input was perfectly spread over all buckets and the element to find is potentially is the last element in the linked list.

3.
Insertion takes O(1) since it is guaranteed no other value can get into the same bucket. If there are repeating elements with same values and they are m of them, it is O(m) for the same reason as before except that the list is at most m long.

Search takes O(1). Two different inputs never hash to same bucket, meaning each input has its own unique bucket. Therefore, the element we search is guaranteed to be either in the first hash if it exists in the table. If it is not, the index must be empty since no other value can take place in it.

4.
Any string that starts with the same character hashes into the same bucket, making the search potentially extremely inefficient since search in linked list takes O(n), n = elements in list, as explained previously.

Example of such string input is:

Algorithms are fun
Alistair Moffat
Alice in the Wonderland
Adafruit
Ani
Andante
Allegro
Andre
Andreas

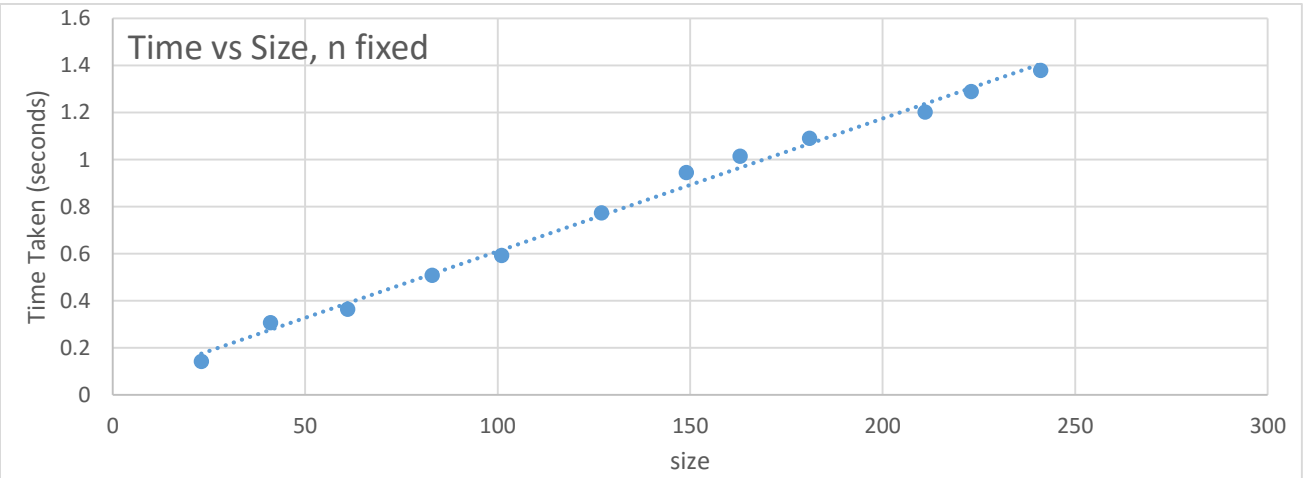
5.
During initial insert, O(1 * MAXSTRLEN) = O(1) time is taken to generate the random numbers, assuming mod operation takes O(1). Each input has maximum length, MAXSTRLEN. Then, each input takes upper bound O(MAXSTRLEN) to calculate the sum then mod, assuming all integer operations take O(1) time. Therefore, first insertion takes O(MAXSTRLEN + MAXSTRLEN * n) = O(n). After that, the insertion takes O(MAXSTRLEN * n) = O(n).

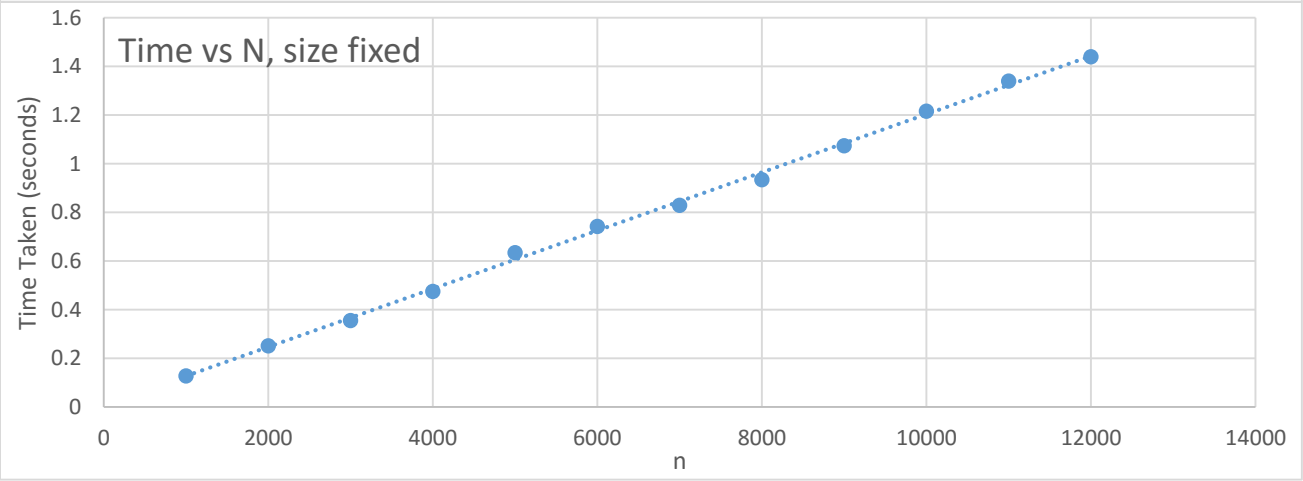
Search takes O(1) because calculating the sum takes O(1) time as explained previously, assuming no collision has taken place during hashing.

6.
The function first initialises the universal hash function with the provided seed, making sure it has random constants we expect it to. This is important since collide_dumb uses rand().

Then, it randomly assign random size within range, then assign random values within range to each characters, then check if it hashes to 0. If it does, print the value and start again with new size. If it doesn't, for the same size, we randomly re-assign values and try again. If we tried until certain number of times and still could not find a string that hashes to 0, we change the size and try again.

Theoretically, the function can run for indefinite amount of time due to its randomness. However, in real life, it shows O(n + size) behaviour according to the data collected (with MAXLEN 256).





7.
The algorithm works with the following steps:

1. Set random size. Set index of the character to work on (must have constant != 0). We will do the rest of the steps assuming the index is 0.

2. Fill in the string with random characters, excluding the index to change (ie. 0).

3. Calculate constant[i] * char[i] from index 1 to the end (since we are working with index 0), then add them up.

4. Calculate the value (let as A) required to make (sum + A) % size = 0 by A = size – (sum % size).

5. Now we know that we should satisfy constant[0] * char[0] = A mod size.
   constant[0] * char[0] = A + size * d for any integer d.
   constant[0] * char[0] + size * d = A

   Then, solve Extended Euclid for constant[0] and size.
   This gives X and Y such that constant[0] * X + size * Y = 1 because size is prime and gcd(size, any integer) gives 1.

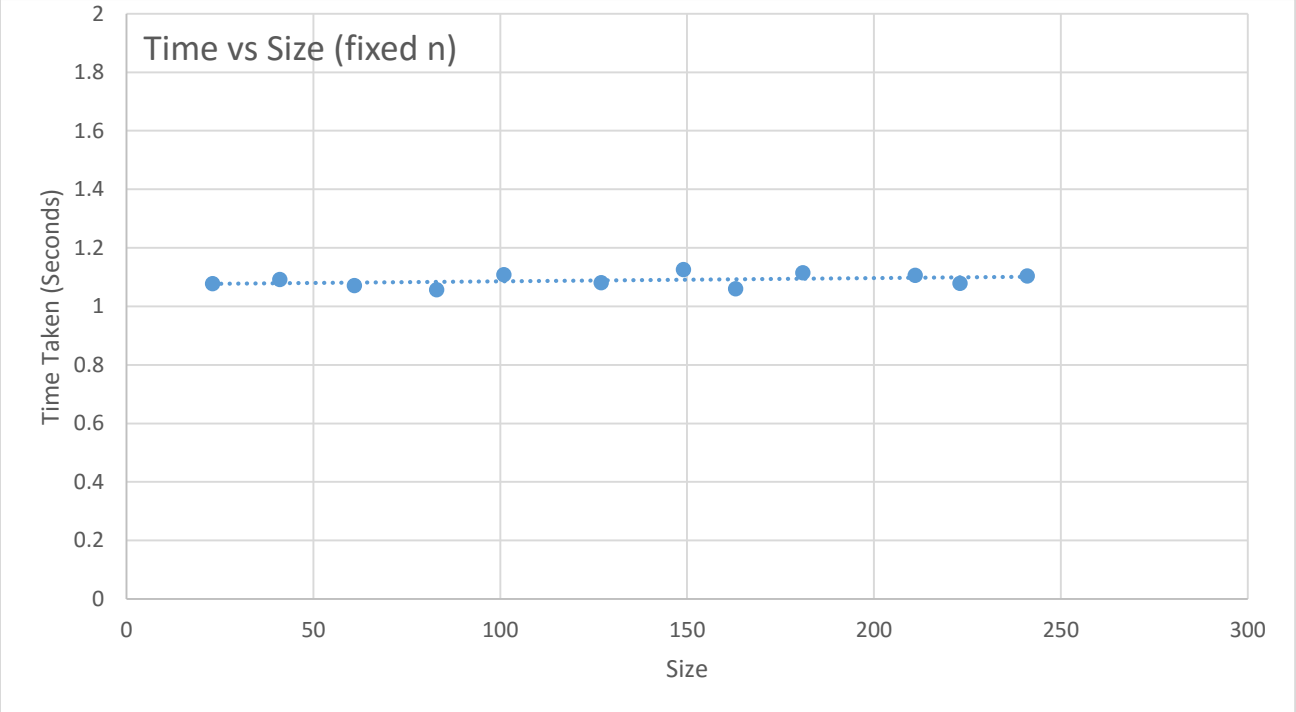
6. Multiply the above equation by A, constant[0] * X * A + size * Y * A = A.
   Now we know that A * X is the character to fill up for index 0.

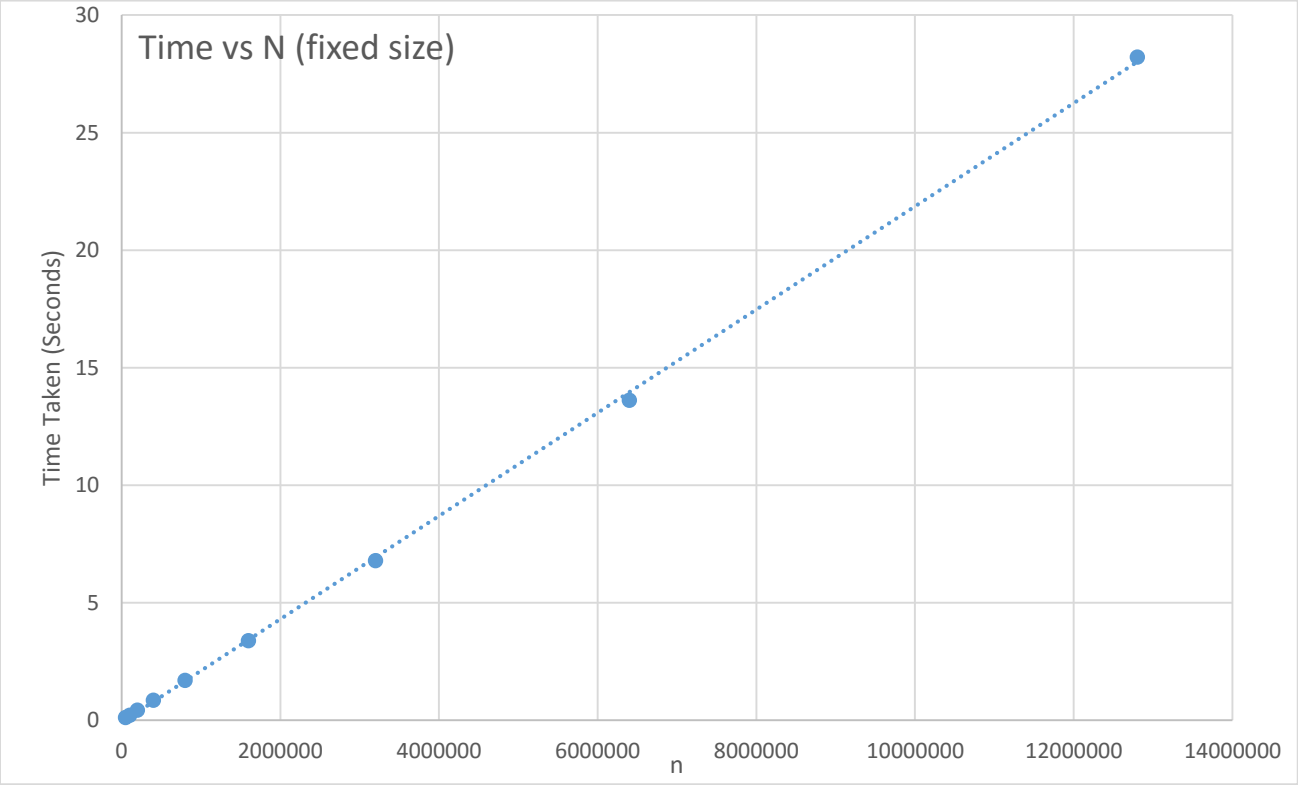
7. We try to make A * X fall into given range of ASCII code for output *prettiness*, by adding appropriate multiple of size. char[0] = A * X + d * size for some integer d. With this value, we are able make universal hash the string into index 0, as we had set in step 4.

   If char[0] = \n or DEL or <=char(32) character, we ditch the solution since we already previously attempted to add minimum multiple of size and try again because those two characters break the string output therefore loads differently when reading. This has almost no impact on the over runtime of the algorithm because, theoretically, this occurs 0.3% of the time but real life results showed less as this had never occurred in MAXLEN=256 and n=12.5 million test with 10 different seeds.

As shown in above steps, the algorithm only uses size for the mod, + and – calculations. Therefore, complexity with size is O(1), assuming mod operation takes O(1) time. This is reflected in the data collected, as the time taken remains constant over fixed values of n with varying size.

True upper bound of the complexity is O(n) a constant time operation is carried out n times. This is also reflected in the data shown in the next page (with MAXLEN 256).


Time vs Size (fixed n)
Time Taken (Seconds)
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
50
100
150
200
250
300
Size


Time vs N (fixed size)
Time Taken (Seconds)
30
25
20
15
10
5
0
0
2000000
4000000
6000000
8000000
10000000
12000000
14000000
n