University of Colorado at Boulder

ASEN 5044 - Statistical State Estimation for Dynamical Systems

Fall 2024

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Final Project
(Cooperative Air-Ground Robot Localization)

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Part I: Deterministic System Analysis

Part 1.

We are given the Equation Of Motion (EOM) for the Unmanned Ground Vehicle (UGV). The EOM is,

$$\begin{aligned}\dot{\xi}_g &= v_g \cos\theta_g + \tilde{w}_{x,g} \\ \dot{\eta}_g &= v_g \sin\theta_g + \tilde{w}_{y,g} \\ \dot{\theta}_g &= \frac{v_g}{L}\tan\phi_g + \tilde{w}_{\omega,g}\end{aligned}$$

and for the Unmanned Aerial Vehicle (UAV) we have the following EOM,

$$\begin{aligned}\dot{\xi}_a &= v_a \cos\theta_a + \tilde{w}_{x,a} \\ \dot{\eta}_a &= v_a \sin\theta_a + \tilde{w}_{y,a} \\ \dot{\theta}_a &= \frac{v_a}{L}\tan\phi_a + \tilde{w}_{\omega,a}\end{aligned}$$

where $\tilde{w}_a = [\tilde{w}_{x,a}, \tilde{w}_{y,a}, \tilde{w}_{\omega,a}]^T$ and $\tilde{w}_g = [\tilde{w}_{x,g}, \tilde{w}_{y,g}, \tilde{w}_{\omega,g}]^T$ are the process noise for the UAV And UGV respectively. We are also given the following sensing model,

$$y(t) = \begin{bmatrix} \arctan\left(\frac{\eta_a-\eta_g}{\xi_a-\xi_g}\right) - \theta_g \\ \sqrt{(\eta_a-\eta_g)^2 + (\xi_g - \xi_g t)^2} \\ \arctan\left(\frac{\eta_g-\eta_a}{\xi_g-\xi_a}\right) - \theta_a \\ \xi_a \\ \eta_a \end{bmatrix} + \tilde{\mathbf{v}}(t)$$

where $\tilde{\mathbf{v}}(t) \in \mathbb{R}^5$ is the sensor error vector. Finally, we are given the combined states, control inputs, and disturbance inputs as,

$$\begin{aligned}\mathbf{x}(t) &= [\xi_g \quad \eta_g \quad \theta_g \quad \xi_a \quad \eta_a \quad \theta_a]^T, \\ \mathbf{u}(t) &= [\mathbf{u}_g \;\; \mathbf{u}_a]^T, \\ \tilde{\mathbf{w}}(t) &= [\tilde{\mathbf{w}}_\mathbf{g} \;\; \tilde{\mathbf{w}}_\mathbf{a}]^T\end{aligned}$$

The state is $x = [\xi_g \quad \eta_g \quad \theta_g \quad \xi_a \quad \eta_a \quad \theta_a]^T = [x_1 \quad x_2 \; x_3 \quad x_4 \quad x_5 \quad x_6]^T$ and our inputs $u = [\mathbf{u}_g \;\; \mathbf{u}_a]^T = [v_g \;\; \phi_g \;\; v_a \;\; \phi_a]^T = [u_1 \;\; u_2 \;\; u_3 \;\; u_4]^T$. We then have the following,

$$\dot{x} = \begin{bmatrix} \dot{\xi}_g \\ \dot{\eta}_g \\ \dot{\theta}_g \\ \dot{\xi}_a \\ \dot{\eta}_a \\ \dot{\theta}_a \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix}$$

Part II: Stochastic Nonlinear Filtering