UNIVERSITY OF COLORADO AT BOULDER

ASEN 5044 - STATISTICAL STATE ESTIMATION FOR DYNAMICAL SYSTEMS

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Final Project (Cooperative Air-Ground Robot Localization)

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Part I: Deterministic System Analysis

Part 1.

We are given the Equation Of Motion (EOM) for the Unmanned Ground Vehicle (UGV). The EOM is,

$$\begin{aligned} \dot{\xi}_g &= v_g \cos\theta_g + \tilde{w}_{x,g} \\ \dot{\eta}_g &= v_g \sin\theta_g + \tilde{w}_{y,g} \\ \dot{\theta}_g &= \frac{v_g}{L}\tan\phi_g + \tilde{w}_{\omega,g} \end{aligned}$$

and for the Unmanned Aerial Vehicle (UAV) we have the following EOM,

$$\begin{aligned} \dot{\xi}_a &= v_a \cos\theta_a + \tilde{w}_{x,a} \\ \dot{\eta}_a &= v_a \sin\theta_a + \tilde{w}_{y,a} \\ \dot{\theta}_a &= \frac{v_a}{L}\tan\phi_a + \tilde{w}_{\omega,a} \end{aligned}$$

where $\tilde{w}_a = [\tilde{w}_{x,a}, \tilde{w}_{y,a}, \tilde{w}_{\omega,a}]^T$ and $\tilde{w}_g = [\tilde{w}_{x,g}, \tilde{w}_{y,g}, \tilde{w}_{\omega,g}]^T$ are the process noise for the UAV And UGV respectively. We are also given the following sensing model,

$$y(t) = \begin{bmatrix} \arctan\left(\frac{\eta_a-\eta_g}{\xi_a-\xi_g}\right) - \theta_g \\ \sqrt{(\xi_g-\xi_a)^2 + (\eta_g-\eta_a)^2} \\ \arctan\left(\frac{\eta_g-\eta_a}{\xi_g-\xi_a}\right) - \theta_a \\ \xi_a \\ \eta_a \end{bmatrix} + \tilde{\mathbf{v}}(t)$$

where $\tilde{\mathbf{v}}(t) \in \mathbb{R}^5$ is the sensor error vector. Finally, we are given the combined states, control inputs, and disturbance inputs as,

$$\begin{aligned} \mathbf{x}(t) &= [\xi_g \quad \eta_g \quad \theta_g \quad \xi_a \quad \eta_a \quad \theta_a]^T, \\ \mathbf{u}(t) &= [\mathbf{u}_g \ \ \mathbf{u}_a]^T, \\ \tilde{\mathbf{w}}(t) &= [\tilde{\mathbf{w}}_\mathbf{g} \ \ \tilde{\mathbf{w}}_\mathbf{a}]^T \end{aligned}$$

The state is $x = [\xi_g \ \eta_g \ \theta_g \ \xi_a \ \eta_a \ \theta_a]^T = [x_1 \ x_2 \, x_3 \ x_4 \ x_5 \ x_6]^T$ and our inputs $u = [\mathbf{u}_g \ \mathbf{u}_a]^T = [v_g \ \ \phi_g \ \ v_a \ \ \phi_a]^T = [u_1 \ \ u_2 \ \ u_3 \ \ u_4]^T$. We then have the following after substituting in our state and input variables,

$$\dot{x} = \begin{bmatrix} \dot{\xi}_g \\ \dot{\eta}_g \\ \dot{\theta}_g \\ \dot{\xi}_a \\ \dot{\eta}_a \\ \dot{\theta}_a \end{bmatrix} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} \mathcal{F}_1(x,u) \\ \mathcal{F}_2(x,u) \\ \mathcal{F}_3(x,u) \\ \mathcal{F}_4(x,u) \\ \mathcal{F}_5(x,u) \\ \mathcal{F}_6(x,u) \end{bmatrix} = \begin{bmatrix} u_1 \cos x_3 \\ u_1 \sin x_3 \\ \frac{u_1}{L} \tan u_2 \\ u_3 \cos x_6 \\ u_3 \sin x_6 \\ \frac{u_3}{L} \tan u_4 \end{bmatrix}$$

$$y = \begin{bmatrix} \arctan\left(\frac{x_5 - x_2}{x_4 - x_1}\right) - x_3 \\ \sqrt{(x_1 - x_4)^2 + (x_2 - x_5)^2} \\ \arctan\left(\frac{x_2 - x_5}{x_1 - x_4}\right) - x_6 \\ x_4 \\ x_5 \end{bmatrix}$$

We now need to compute the partials with respect to x,

$\frac{\partial \mathcal{F}_1}{\partial x_1} = 0$ $\frac{\partial \mathcal{F}_2}{\partial x_1} = 0$ $\frac{\partial \mathcal{F}_3}{\partial x_1} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_1} = 0$ $\frac{\partial \mathcal{F}_5}{\partial x_1} = 0$ $\frac{\partial \mathcal{F}_6}{\partial x_1} = 0$

$\frac{\partial \mathcal{F}_1}{\partial x_2} = 0$ $\frac{\partial \mathcal{F}_2}{\partial x_2} = 0$ $\frac{\partial \mathcal{F}_3}{\partial x_2} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_2} = 0$ $\frac{\partial \mathcal{F}_5}{\partial x_2} = 0$ $\frac{\partial \mathcal{F}_6}{\partial x_2} = 0$

$\frac{\partial \mathcal{F}_1}{\partial x_3} = -u_1 \sin x_3$ $\frac{\partial \mathcal{F}_2}{\partial x_3} = u_1 \cos x_3$ $\frac{\partial \mathcal{F}_3}{\partial x_3} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_3} = 0$ $\frac{\partial \mathcal{F}_5}{\partial x_3} = 0$ $\frac{\partial \mathcal{F}_6}{\partial x_3} = 0$

$\frac{\partial \mathcal{F}_1}{\partial x_4} = 0$ $\frac{\partial \mathcal{F}_2}{\partial x_4} = 0$ $\frac{\partial \mathcal{F}_3}{\partial x_4} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_4} = 0$ $\frac{\partial \mathcal{F}_5}{\partial x_4} = 0$ $\frac{\partial \mathcal{F}_6}{\partial x_4} = 0$

$\frac{\partial \mathcal{F}_1}{\partial x_5} = 0$ $\frac{\partial \mathcal{F}_2}{\partial x_5} = 0$ $\frac{\partial \mathcal{F}_3}{\partial x_5} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_5} = 0$ $\frac{\partial \mathcal{F}_5}{\partial x_5} = 0$ $\frac{\partial \mathcal{F}_6}{\partial x_5} = 0$

$\frac{\partial \mathcal{F}_1}{\partial x_6} = 0$ $\frac{\partial \mathcal{F}_2}{\partial x_6} = 0$ $\frac{\partial \mathcal{F}_3}{\partial x_6} = 0$ $\frac{\partial \mathcal{F}_4}{\partial x_6} = -u_3 \sin x_6$ $\frac{\partial \mathcal{F}_5}{\partial x_6} = u_3 \cos x_6$ $\frac{\partial \mathcal{F}_6}{\partial x_6} = 0$

and with respect to u,

$\frac{\partial \mathcal{F}_1}{\partial u_1} = \cos x_3$ $\frac{\partial \mathcal{F}_2}{\partial u_1} = \sin x_3$ $\frac{\partial \mathcal{F}_3}{\partial u_1} = \frac{\tan u_2}{L}$ $\frac{\partial \mathcal{F}_4}{\partial u_1} = 0$ $\frac{\partial \mathcal{F}_5}{\partial u_1} = 0$ $\frac{\partial \mathcal{F}_6}{\partial u_1} = 0$

$\frac{\partial \mathcal{F}_1}{\partial u_2} = 0$ $\frac{\partial \mathcal{F}_2}{\partial u_2} = 0$ $\frac{\partial \mathcal{F}_3}{\partial u_2} = \frac{u_1}{L} \sec^2 u_2$ $\frac{\partial \mathcal{F}_4}{\partial u_2} = 0$ $\frac{\partial \mathcal{F}_5}{\partial u_2} = 0$ $\frac{\partial \mathcal{F}_6}{\partial u_2} = 0$

$\frac{\partial \mathcal{F}_1}{\partial u_3} = 0$ $\frac{\partial \mathcal{F}_2}{\partial u_3} = 0$ $\frac{\partial \mathcal{F}_3}{\partial u_3} = 0$ $\frac{\partial \mathcal{F}_4}{\partial u_3} = \cos x_6$ $\frac{\partial \mathcal{F}_5}{\partial u_3} = \sin x_6$ $\frac{\partial \mathcal{F}_6}{\partial u_3} = \frac{\tan u_4}{L}$

$\frac{\partial \mathcal{F}_1}{\partial u_4} = 0$ $\frac{\partial \mathcal{F}_2}{\partial u_4} = 0$ $\frac{\partial \mathcal{F}_3}{\partial u_4} = 0$ $\frac{\partial \mathcal{F}_4}{\partial u_4} = 0$ $\frac{\partial \mathcal{F}_5}{\partial u_4} = 0$ $\frac{\partial \mathcal{F}_6}{\partial u_4} = \frac{u_3}{L} \sec^2 u_4$

Part II: Stochastic Nonlinear Filtering