University of Colorado at Boulder

ASEN 5044 - STATISTICAL STATE ESTIMATION FOR DYNAMICAL SYSTEMS

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Professor: Dr. Nisar Ahmed

Final Project (Cooperative Air-Ground Robot Localization)

TEAM MEMBERS:

NICHOLAS MARTINEZ
WHIT WHITTALL
MICHAEL BERNABEI

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Part I: Deterministic System Analysis

Part 1.

We are given the Equation Of Motion (EOM) for the Unmanned Ground Vehicle (UGV). The EOM is,

$$\begin{split} \dot{\xi_g} &= v_g \cos \theta_g + \tilde{w}_{x,g} \\ \dot{\eta_g} &= v_g \sin \theta_g + \tilde{w}_{y,g} \\ \dot{\theta_g} &= \frac{v_g}{L} \tan \phi_g + \tilde{w}_{\omega,g} \end{split}$$

and for the Unmanned Aerial Vehicle (UAV) we have the following EOM,

$$\dot{\xi_a} = v_a \cos \theta_a + \tilde{w}_{x,a}$$

$$\dot{\eta_a} = v_a \sin \theta_a + \tilde{w}_{y,a}$$

$$\dot{\theta_a} = \frac{v_a}{L} \tan \phi_a + \tilde{w}_{\omega,a}$$

where $\tilde{w}_a = [\tilde{w}_{x,a}, \tilde{w}_{y,a}, \tilde{w}_{\omega,a}]^T$ and $\tilde{w}_g = [\tilde{w}_{x,g}, \tilde{w}_{y,g}, \tilde{w}_{\omega,g}]^T$ are the process noise for the UAV And UGV respectively. We are also given the following sensing model,

$$y(t) = \begin{bmatrix} \arctan\left(\frac{\eta_a - \eta_g}{\xi_a - \xi_g}\right) - \theta_g \\ \sqrt{(\eta_a - \eta_g)^2 + (\xi_g - \xi_g t)^2} \\ \arctan\left(\frac{\eta_g - \eta_a}{\xi_g - \xi_a}\right) - \theta_a \\ \xi_a \\ \eta_a \end{bmatrix} + \tilde{\mathbf{v}}(t)$$

where $\tilde{\mathbf{v}}(t) \in \mathbb{R}^5$ is the sensor error vector. Finally, we are given the combined states, control inputs, and disturbance inputs as,

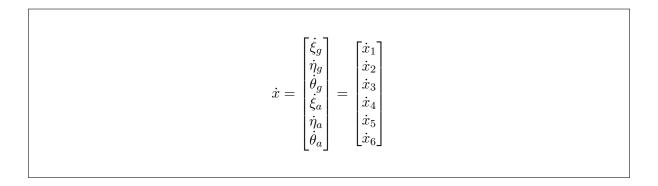
$$\mathbf{x}(t) = \begin{bmatrix} \xi_g & \eta_g & \theta_g & \xi_a & \eta_a & \theta_a \end{bmatrix}^T,$$

$$\mathbf{u}(t) = \begin{bmatrix} \mathbf{u}_g & \mathbf{u}_a \end{bmatrix}^T,$$

$$\tilde{\mathbf{w}}(t) = \begin{bmatrix} \tilde{\mathbf{w}}_g & \tilde{\mathbf{w}}_\mathbf{a} \end{bmatrix}^T$$

The state is $x = \begin{bmatrix} \xi_g & \eta_g & \theta_g & \xi_a & \eta_a & \theta_a \end{bmatrix}^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{bmatrix}^T$ and our inputs $u = \begin{bmatrix} \mathbf{u}_g & \mathbf{u}_a \end{bmatrix}^T = \begin{bmatrix} v_g & \phi_g & v_a & \phi_a \end{bmatrix}^T = \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T$. We then have the following,

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Part II: Stochastic Nonlinear Filtering