# **Coherent Ising Machines:** non-von Neumann computing using networks of optical parametric oscillators


Peter McMahon

Cornell University


**CMU 47-779 *Quantum Integer Programming***

**6 October 2020**

http://mcmahon.aep.cornell.edu

# Credits

Alireza Marandi, Zhe Wang, Kenta Takata, Robert L. Byer, Yoshihisa Yamamoto. "Network of time-multiplexed optical parametric oscillators as a coherent Ising machine." *Nature Photonics* **8**, 937-942 (2014).

Peter L. McMahon*, Alireza Marandi*, Yoshitaka Haribara, Ryan Hamerly, Carsten Langrock, Shuhei Tamate, Takahiro Inagaki, Hiroki Takesue, Shoko Utsunomiya, Kazuyuki Aihara, Robert L. Byer, M. M. Fejer, Hideo Mabuchi, Yoshihisa Yamamoto. "A fully programmable 100-spin coherent Ising machine with all-to-all connections." *Science* **354**, No. 6312, 614 - 617 (2016).

Ryan Hamerly*, Takahiro Inagaki*, Peter L. McMahon* *et al*. "Experimental investigation of performance differences between coherent Ising machines and a quantum annealer." *Science Advances* **5**, 5, eaau0823 (2019).

Funding: ImPACT Program, Cabinet Office of Japan

*The topic of this talk*:

A physics-based computing machine that provides a novel and scalable approach to solving difficult optimization problems.

# Overview

- **The Ising Problem**
- Ising Machines
- Foundations of All-Optical OPO Ising Machines
- Measurement-Feedback OPO Ising Machines
- Conclusions

# The Ising Problem
## (Combinatorial Optimization Version)

**Problem Statement:** Given couplings between a set of spins, find the configuration that minimizes the energy function:

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This is an NP-hard problem*, and is difficult to solve in practice for moderate-size *N.*

→ Approximate (heuristic) solvers take hours when *N*=10,000.

* Relation to MAX-CUT will be shown later.

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$$\vec{\sigma} \triangleq (\sigma_1, \sigma_2, \ldots, \sigma_N)$$

$$\sigma_k \in \{-1, +1\}$$

$$J \in \mathbb{R}^{N \times N}$$

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- 1D Ising model with nearest-neighbor connections
- 2D Ising model with nearest-neighbor connections
- Generalized Ising model with arbitrary connections

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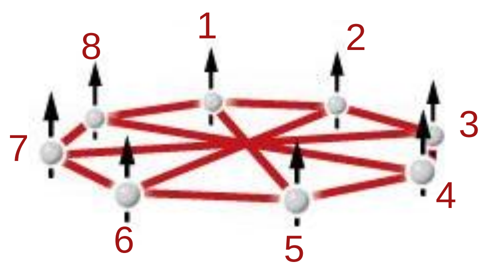


$$J = -\begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

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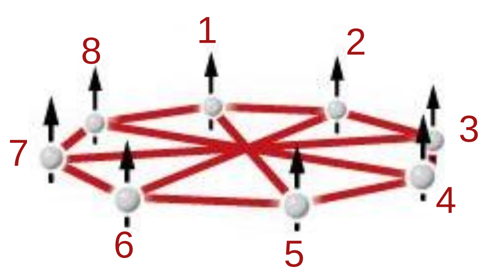


$$J = - \begin{array}{c} \\ \\ \end{array} \begin{array}{cc} \begin{array}{cccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} & \\ \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{array} \end{array}$$

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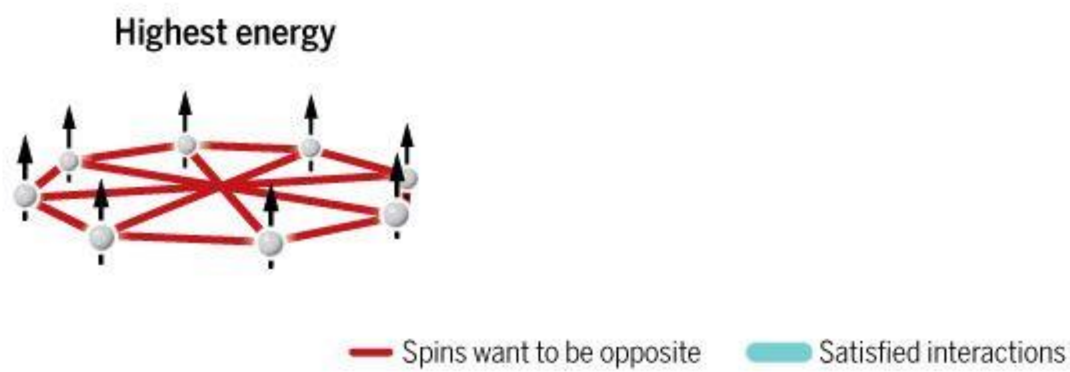

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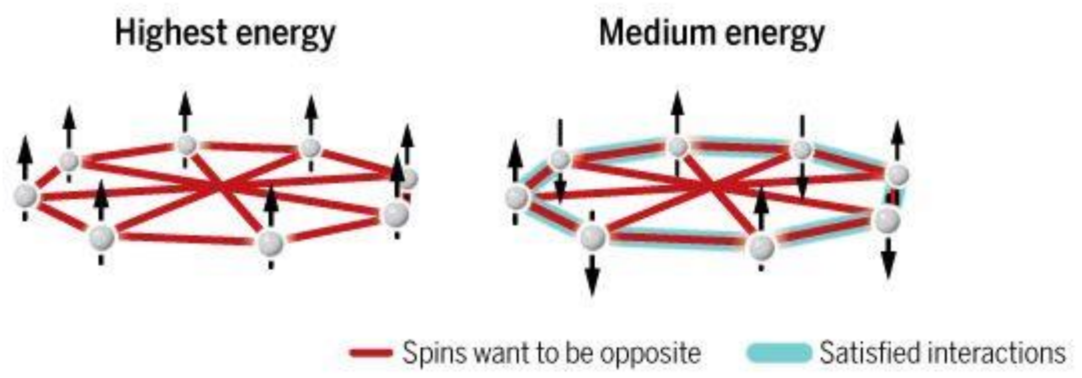

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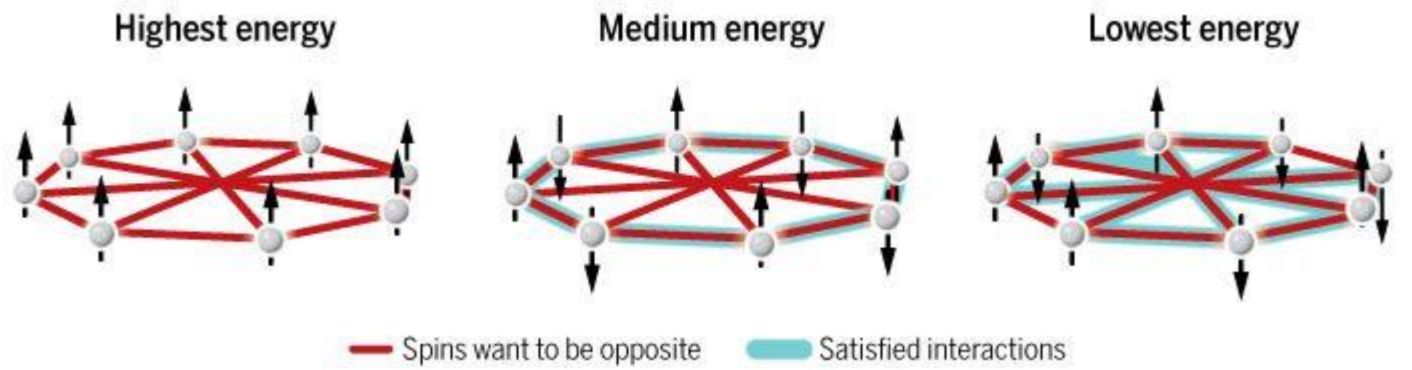


Figure credit: *Science* doi:10.1126/science.354.6310.269

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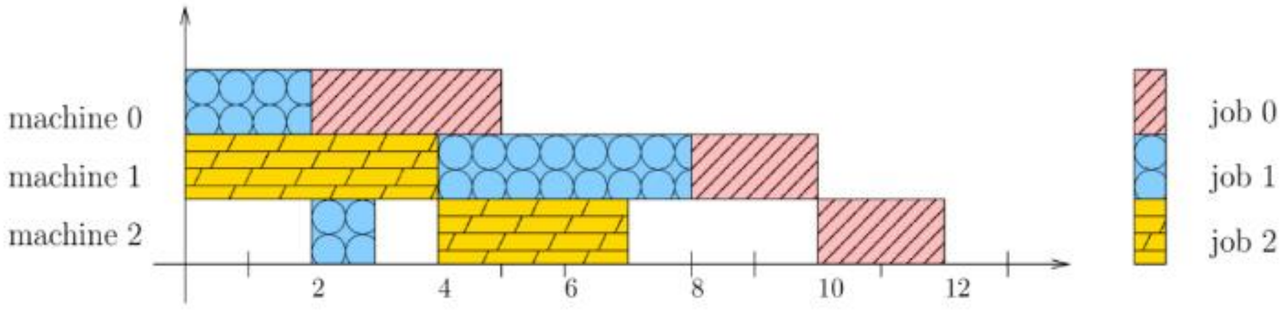


Image credit: developer.google.com

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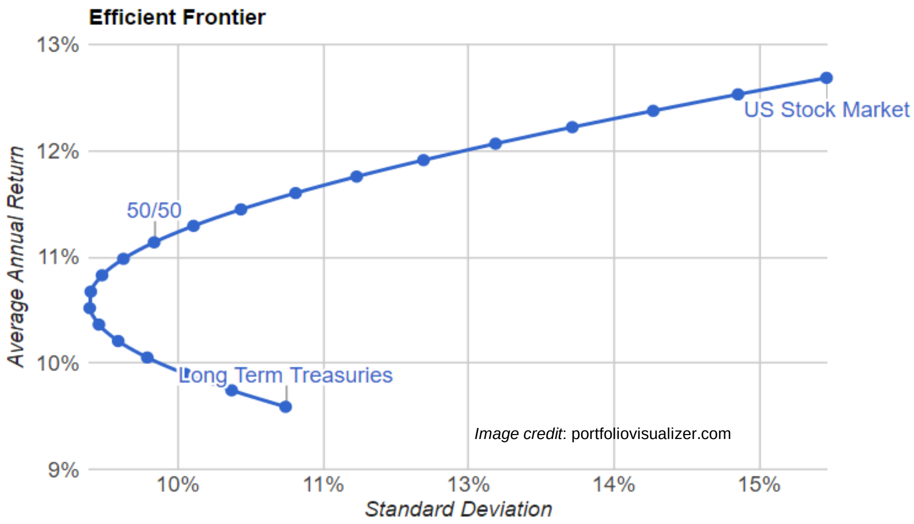

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  - Protein folding

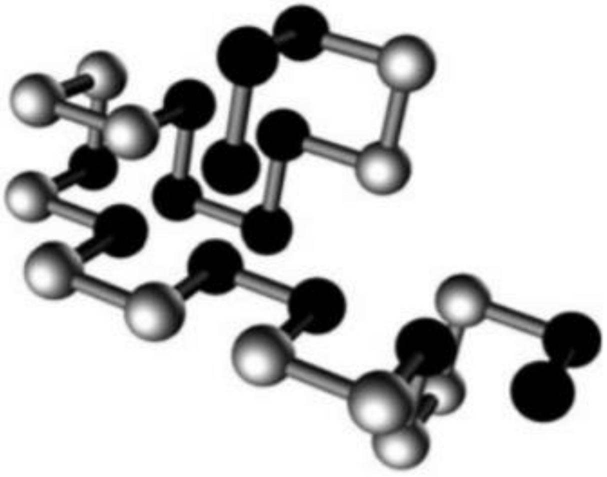

*Image credit*: J. Comp. Bio. 21, 11, pp. 823 (2014)

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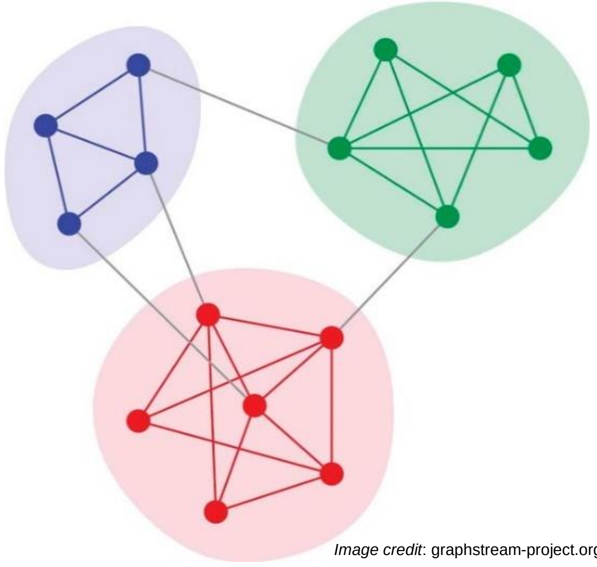

*Image credit*: graphstream-project.org

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- Many interesting discrete optimization problems can be framed as Ising problems:
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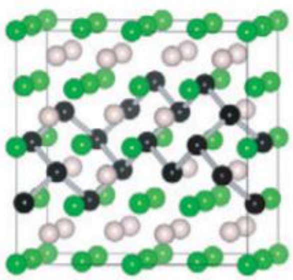
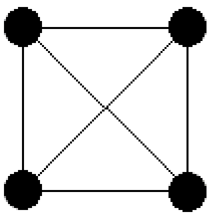
  - Protein folding
  - Graph problems
  - Materials design



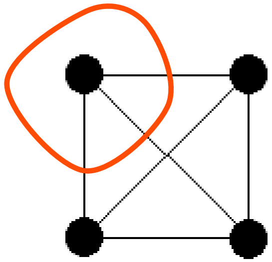
Yuge, et al. *Phys. Rev. B* **77**, 094121 (2008)

# MAX-CUT



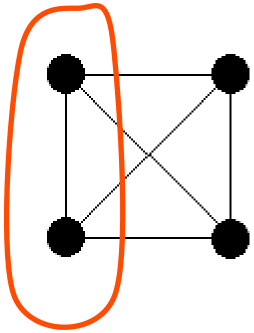
# MAX-CUT

Cut size = 3

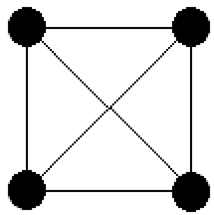


# MAX-CUT

Cut size = 4



# Ising Formulation of MAX-CUT



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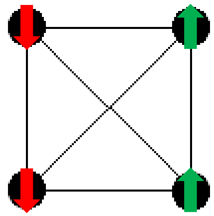

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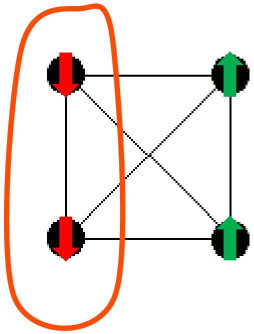

$$H(\vec{\sigma}) = -\sum_{1 \leq i < j \leq N} J_{ij}\sigma_i\sigma_j \qquad J = -\begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

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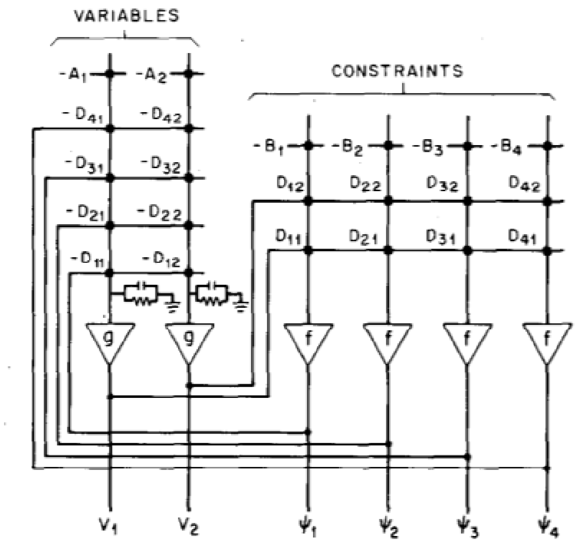


**Neural Networks**
*Biol. Cybern.* 52, 141-152 (1985)

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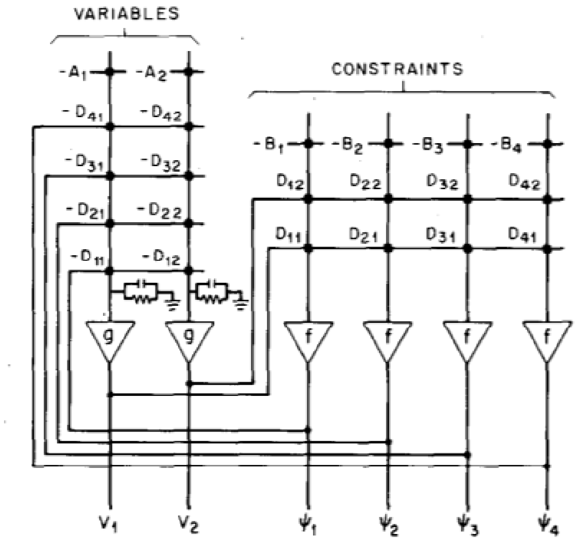


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*Nature* **473**, 194-198 (2011)

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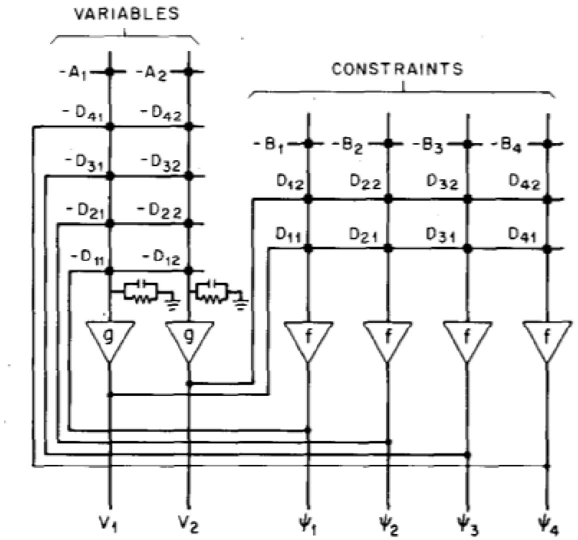


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**Quantum Annealing**
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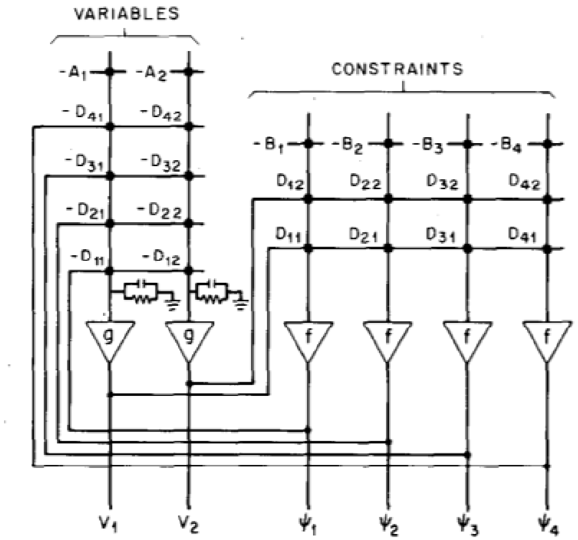
Uses *quantum* dynamics to solve a ***classical*** spin Hamiltonian!

# Ising Machines

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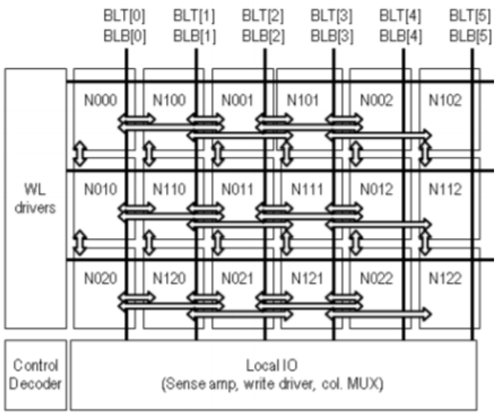


**CMOS Annealers**
*ISSCC* 24.3 (2015)

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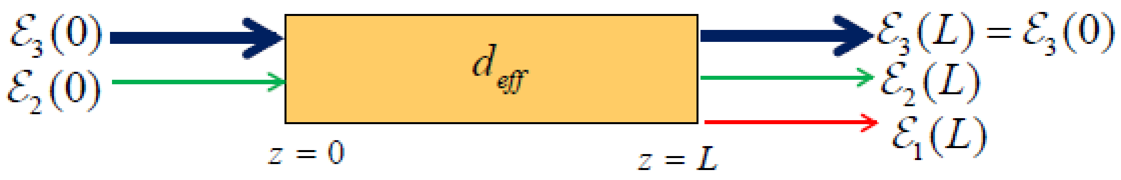
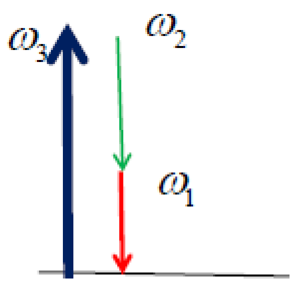
# Optical Parametric Oscillators

**Optical Parametric Amplifier**




Diagrams from: EE346 Lecture Notes, Stanford University (M. Fejer)

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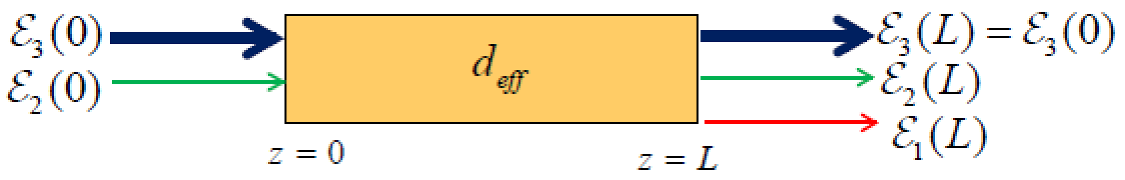
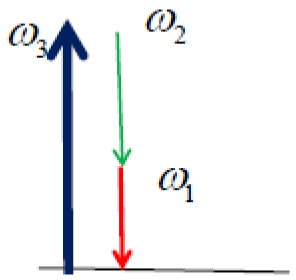
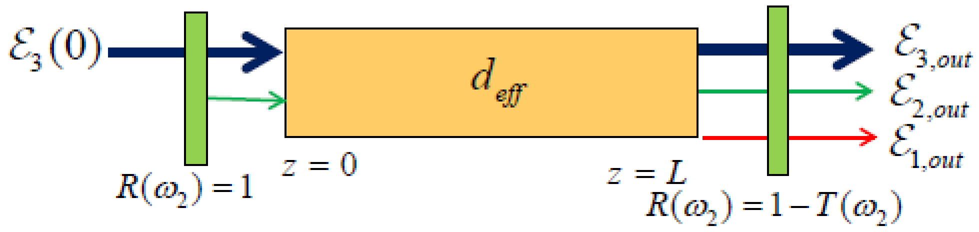



**OPO: Optical Parametric Amplifier in a Cavity**

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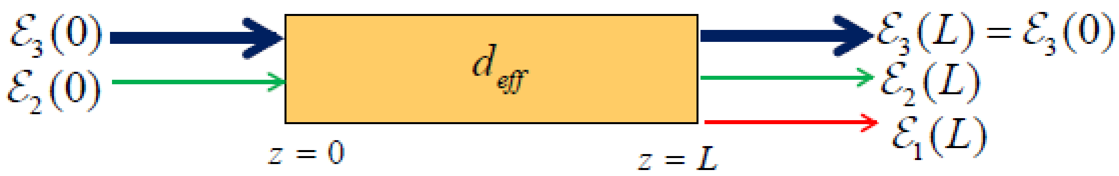


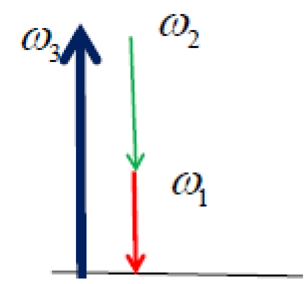


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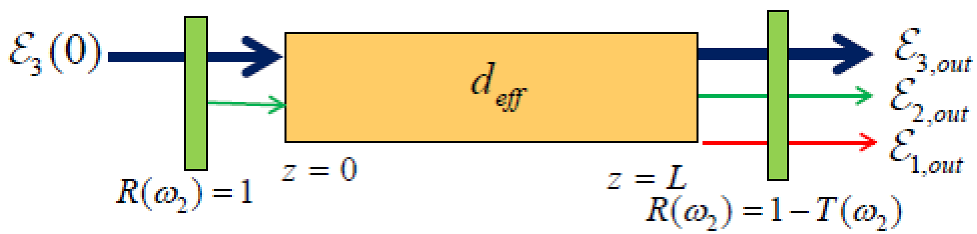


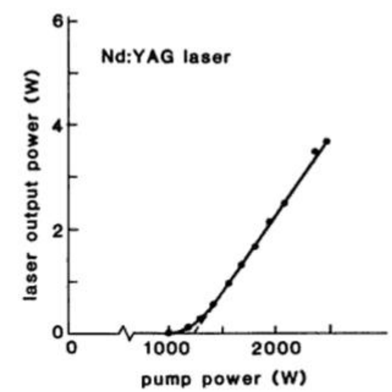


*Diagram credit:*
A. E. Siegman, "Lasers" (1986)

An OPO has a **threshold**, just like a laser does.

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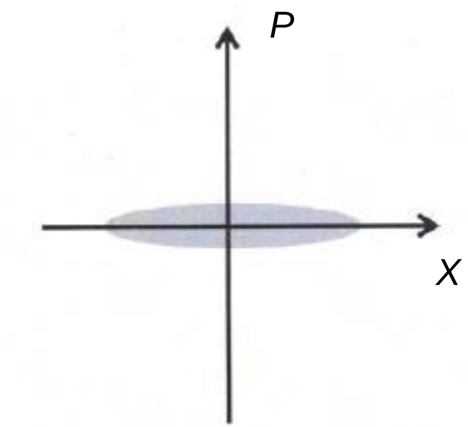
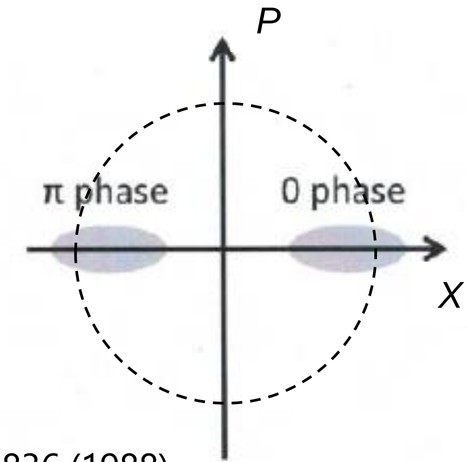
# OPO Phase Properties

**Squeezed vacuum state below threshold**




**Squeezed coherent state above threshold**




M. Wolinski and H. J. Carmichael. *PRL* **60**, 1836 (1988)

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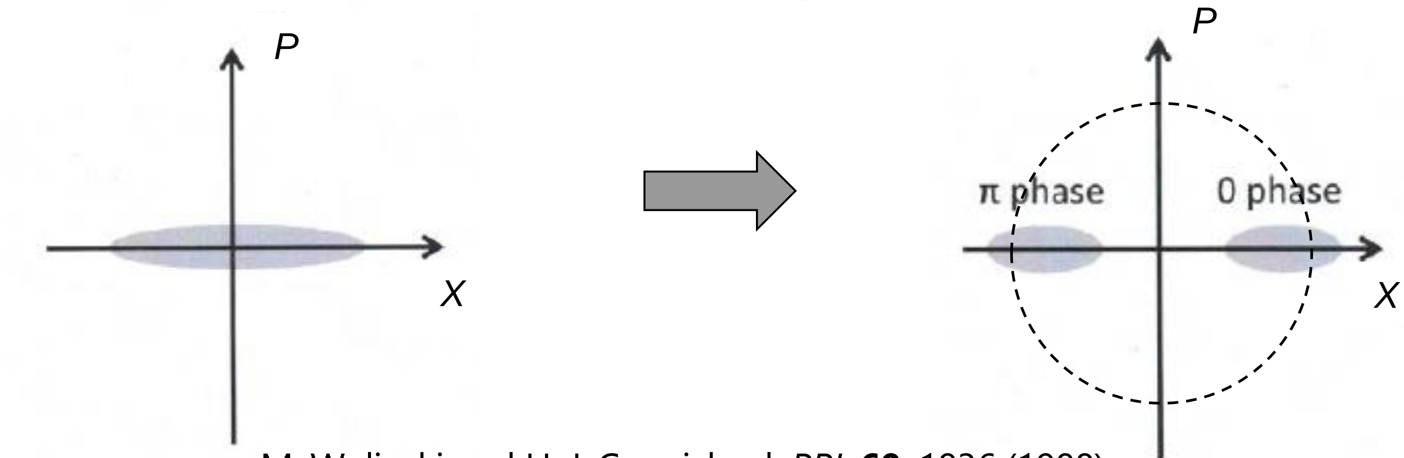


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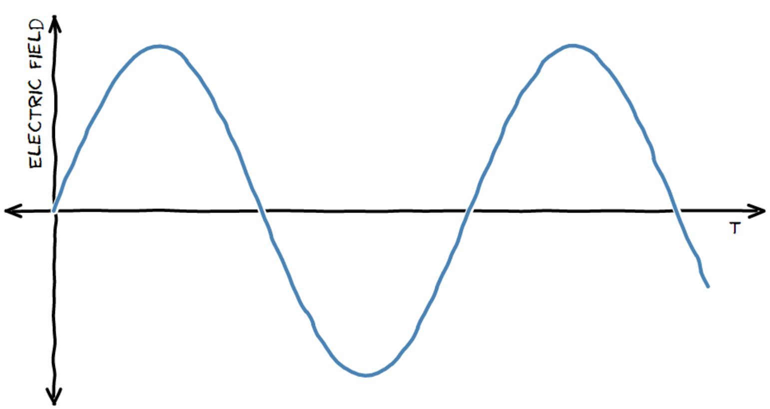

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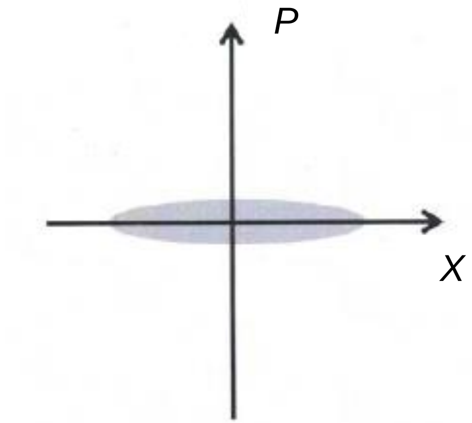


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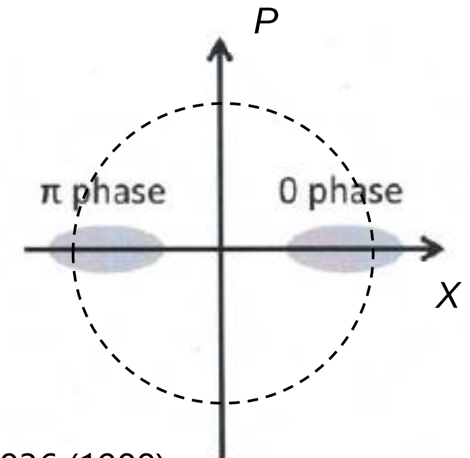


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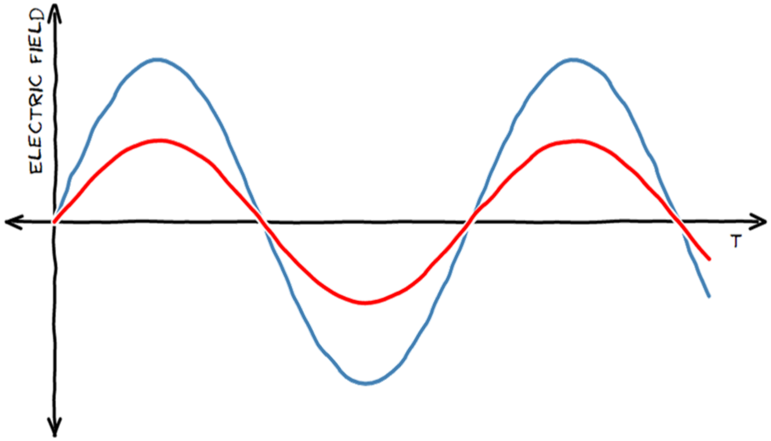

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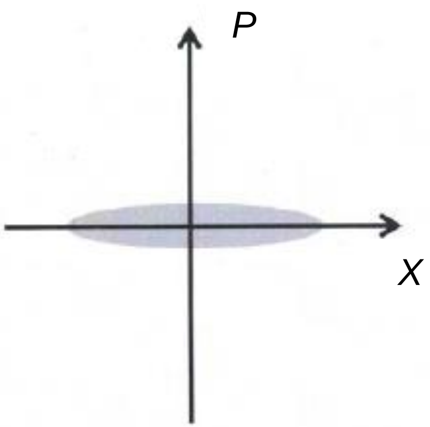


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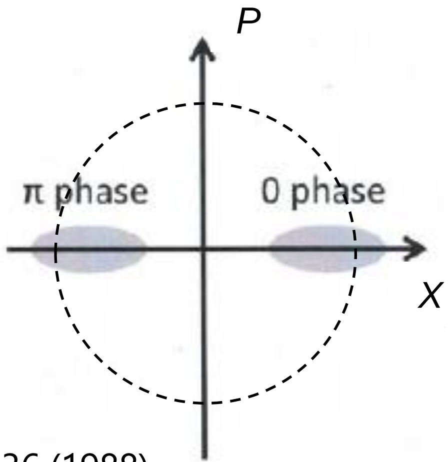


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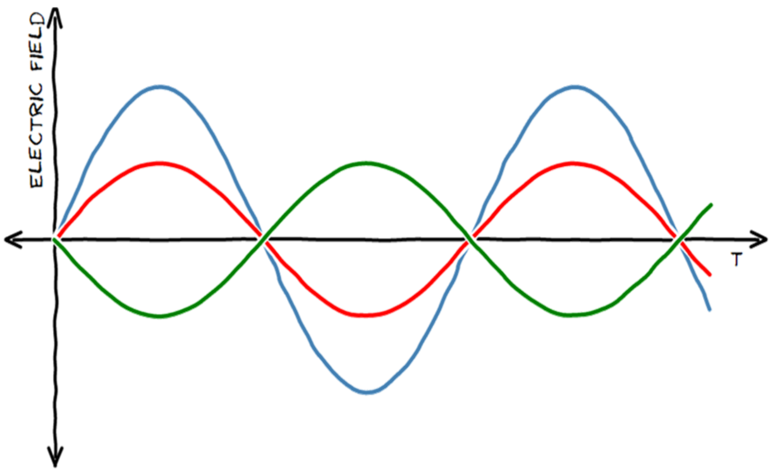

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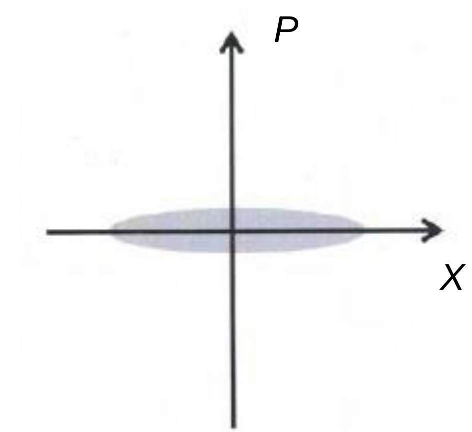


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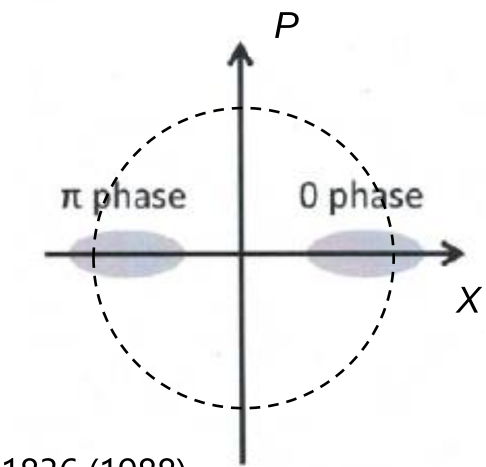


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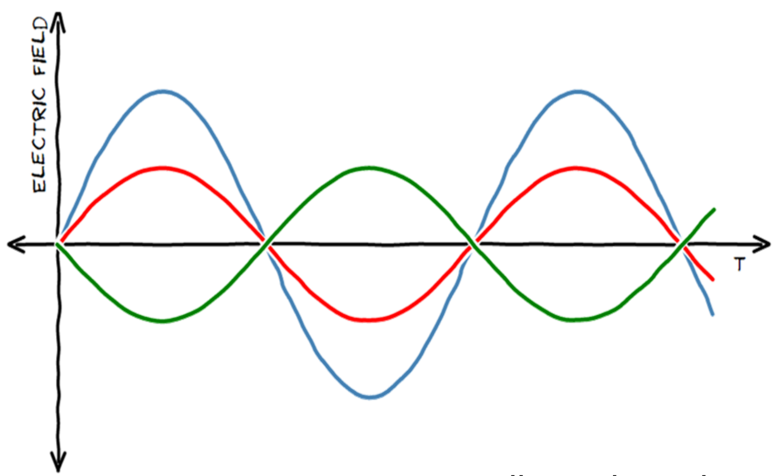
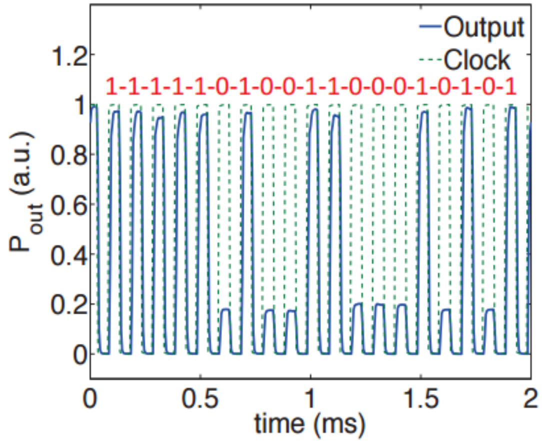
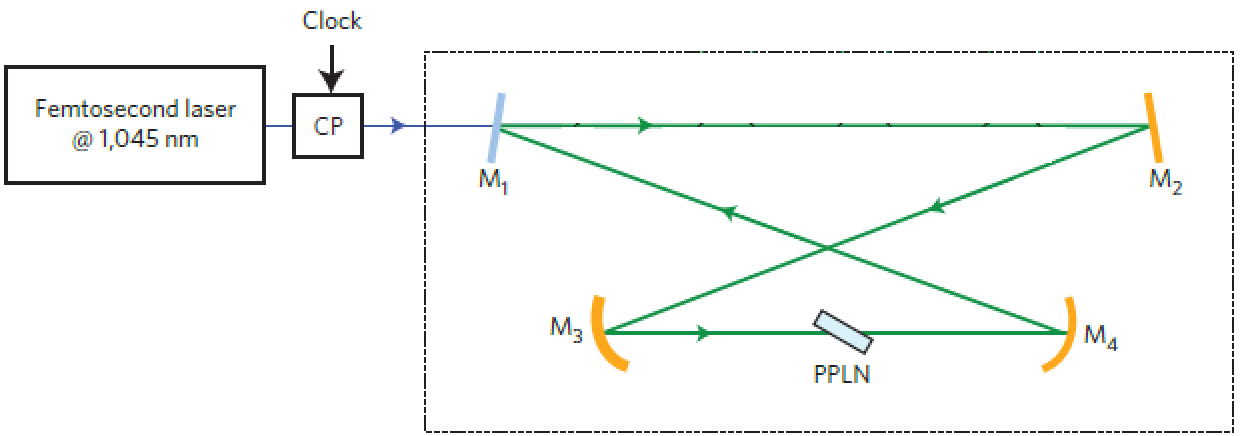



A. Marandi, *et al*. *Optics Express* 20, 17 19322 (2012).

# Time-Multiplexed OPOs




Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

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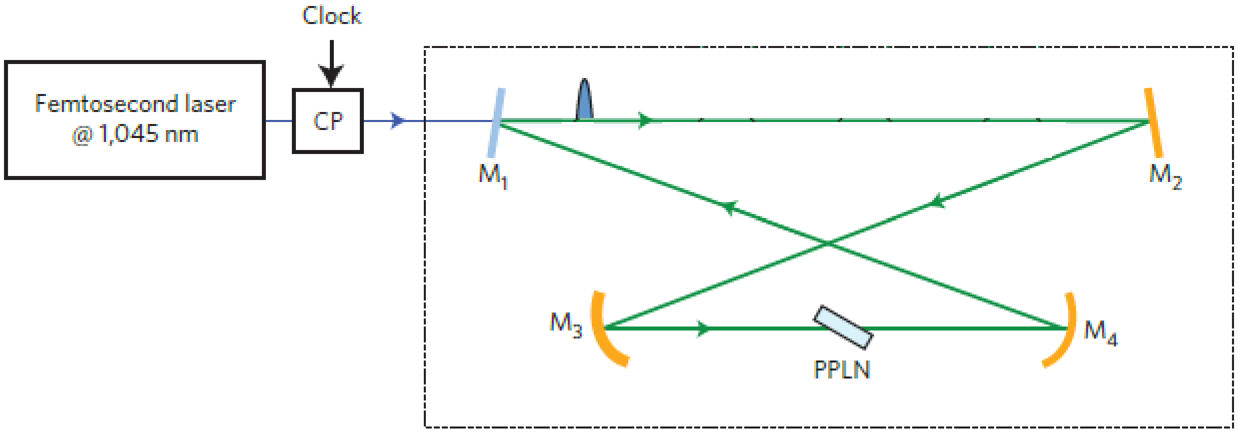
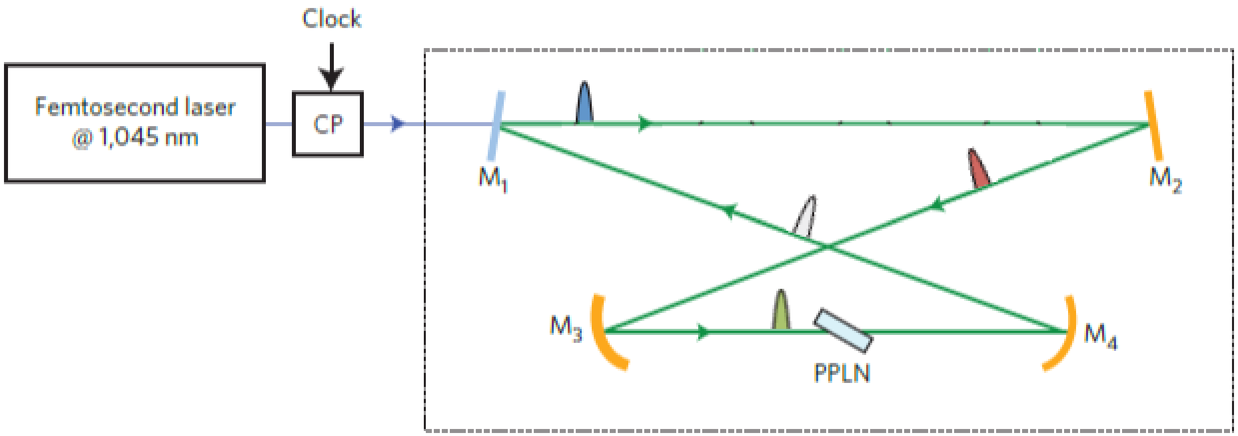



Typical pulsed OPO case:

$$\frac{1}{\textbf{Laser pulse repetition rate}} = \textbf{Cavity roundtrip time}$$

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# Time-Multiplexed OPOs

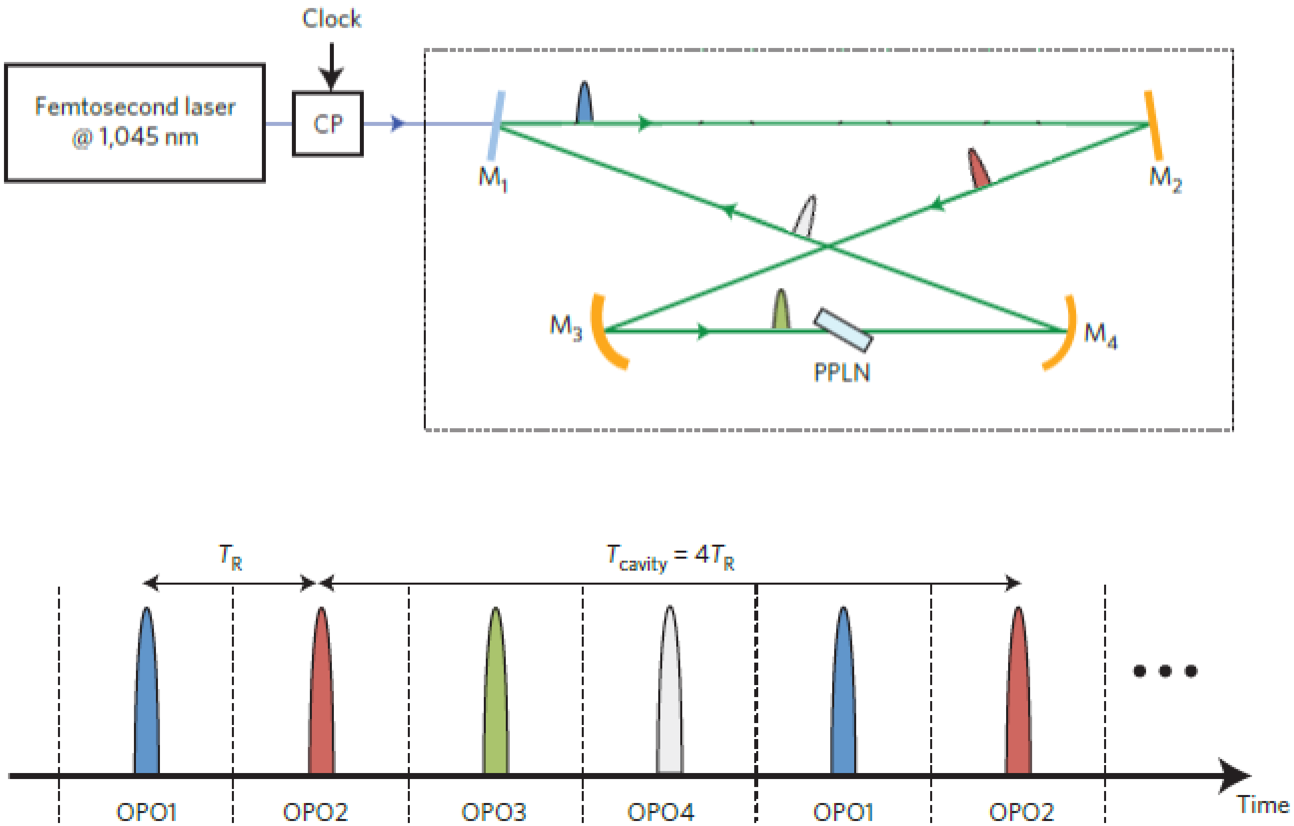



**Time-multiplexed** OPO case:

$$\frac{1}{\text{Laser pulse repetition rate}} = \frac{\text{Cavity roundtrip time}}{\boldsymbol{N_{\text{pulses}}}}$$

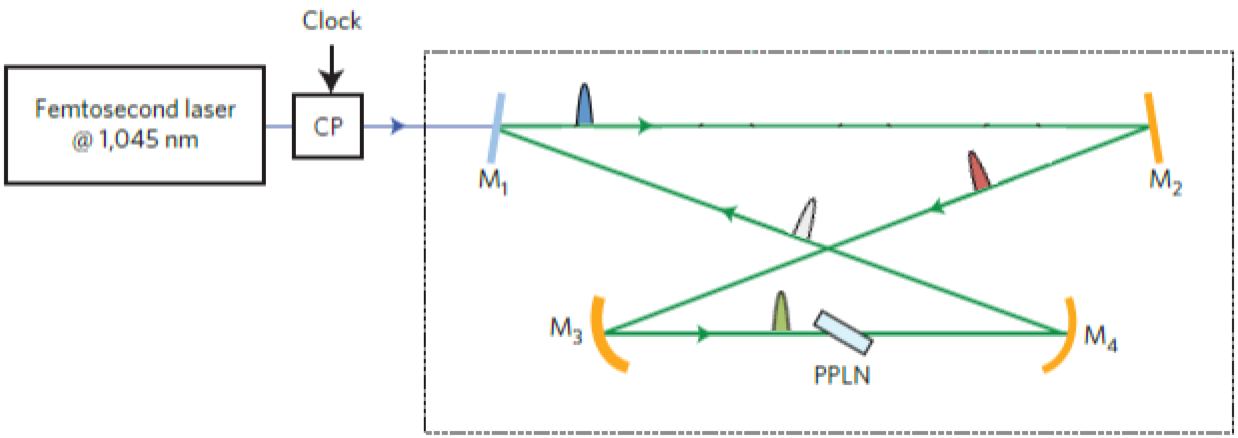
Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# Time-Multiplexed OPOs




Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# From OPOs to Ising Machine



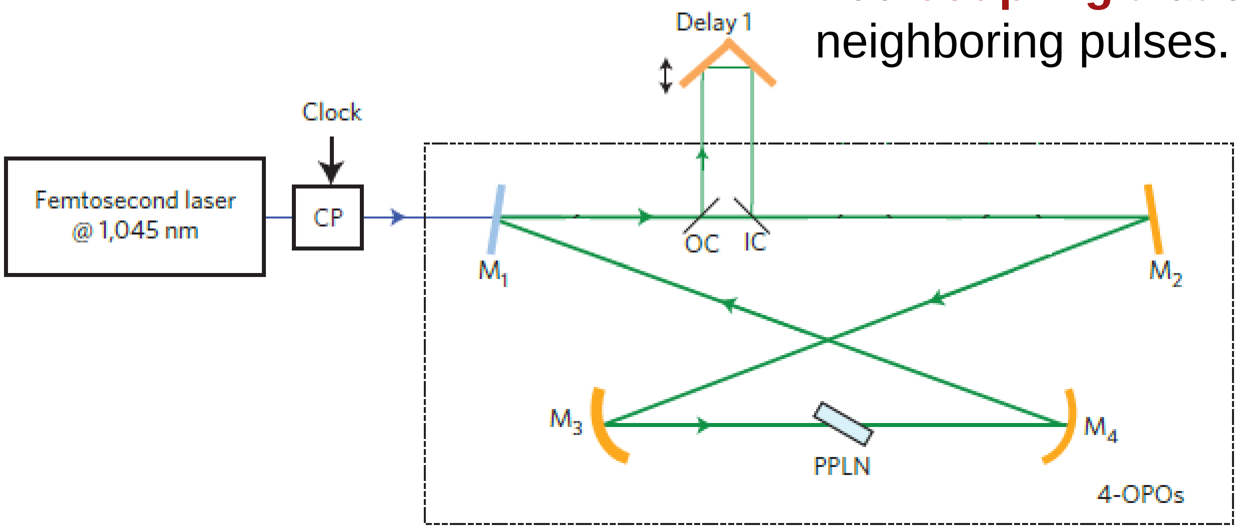

So far we just have ***N*** uncoupled “spins”.
(Recall that each pulse has phase 0 or π).

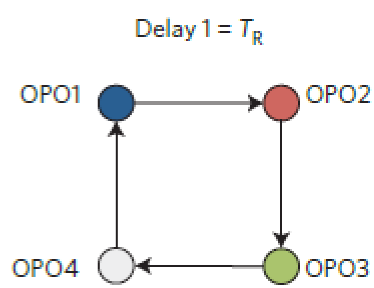
$$H = 0$$

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# From OPOs to Ising Machine

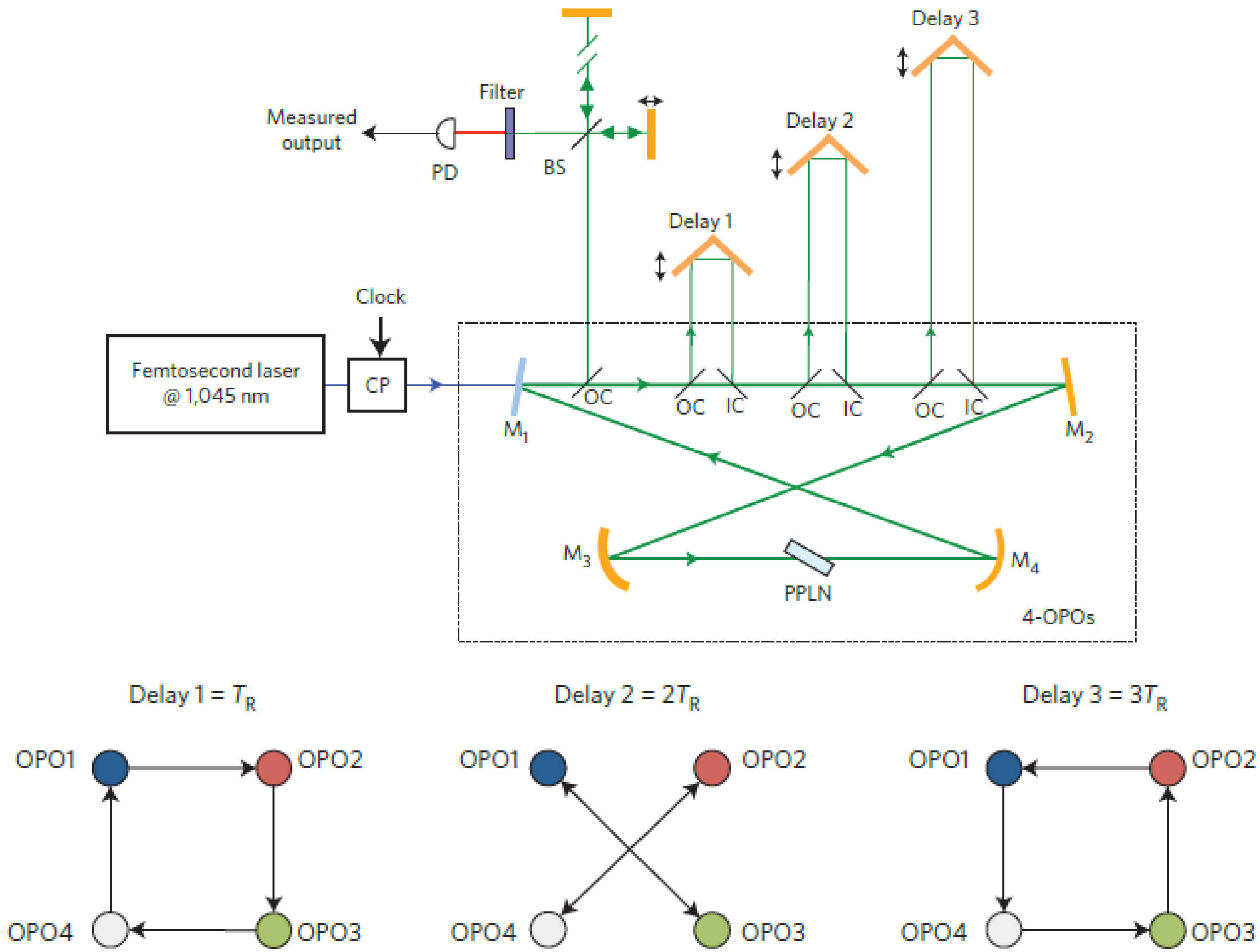
Add **coupling** that connects neighboring pulses.



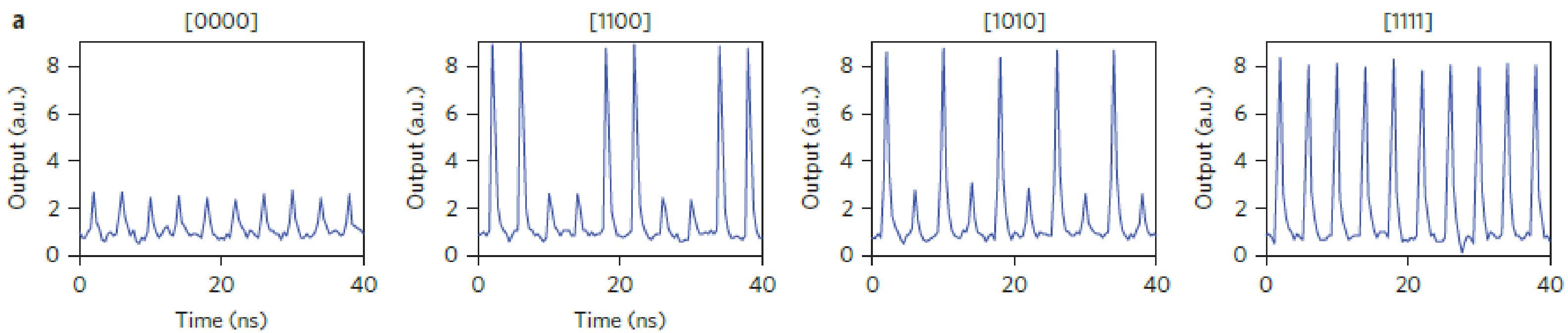

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

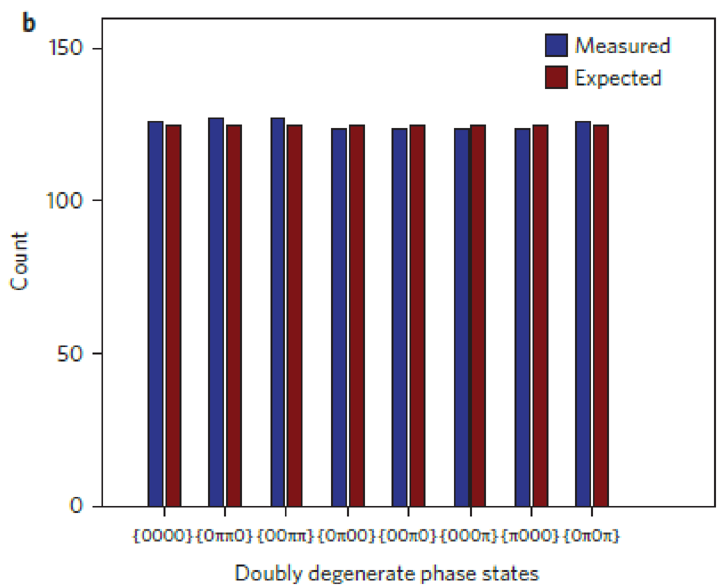
# $N$ = 4 Ising Machine




Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# $N$ = 4 Ising Machine Results



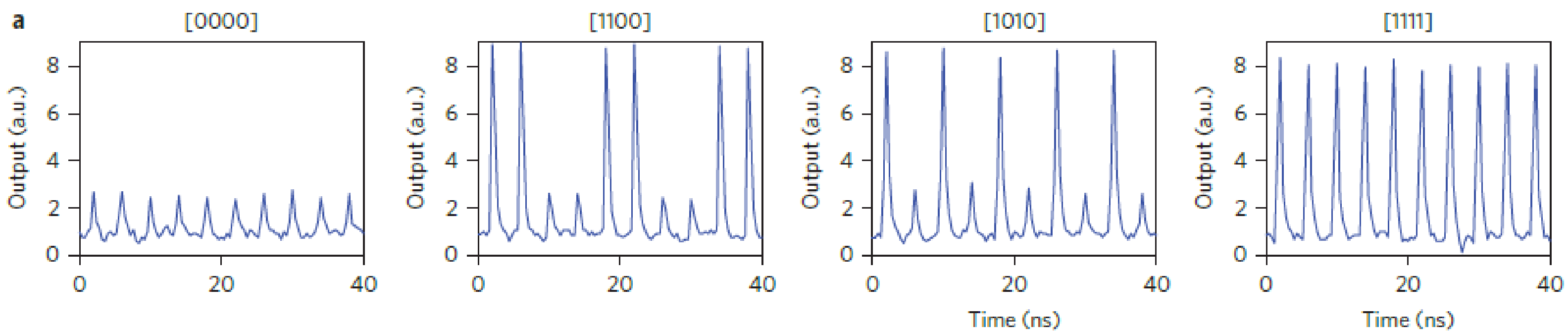




$$H = 0$$

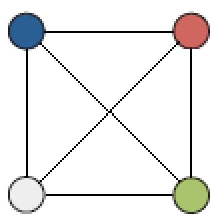
(no coupling)

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# $N$ = 4 Ising Machine Results

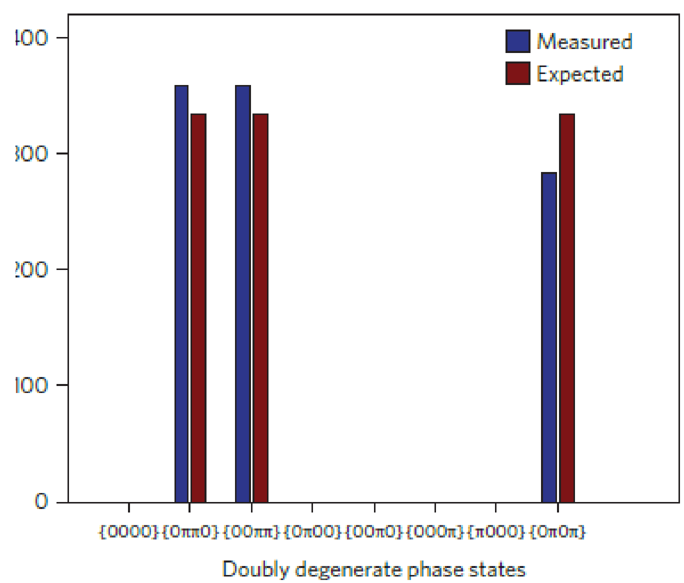



**MAX-CUT**



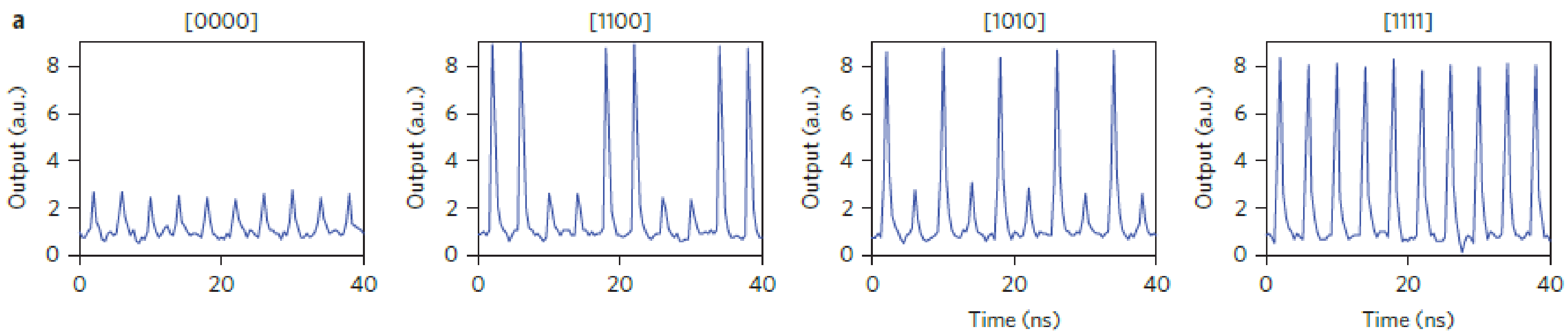
$$H = -\sum_{ij}^{N} J_{ij}\sigma_i\sigma_j$$

**(with coupling)**

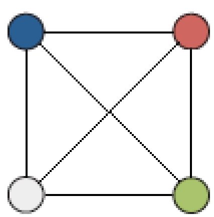



Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# $N$ = 4 Ising Machine Results




**MAX-CUT**



$$H = -\sum_{ij}^{N} J_{ij}\sigma_i\sigma_j$$

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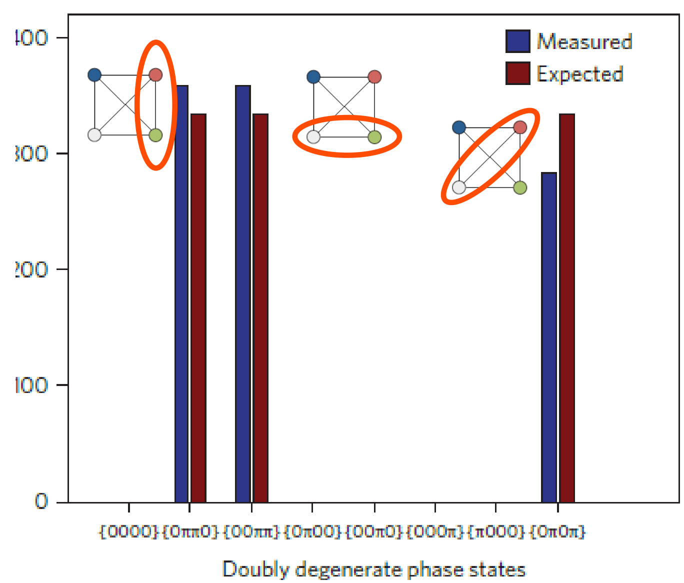
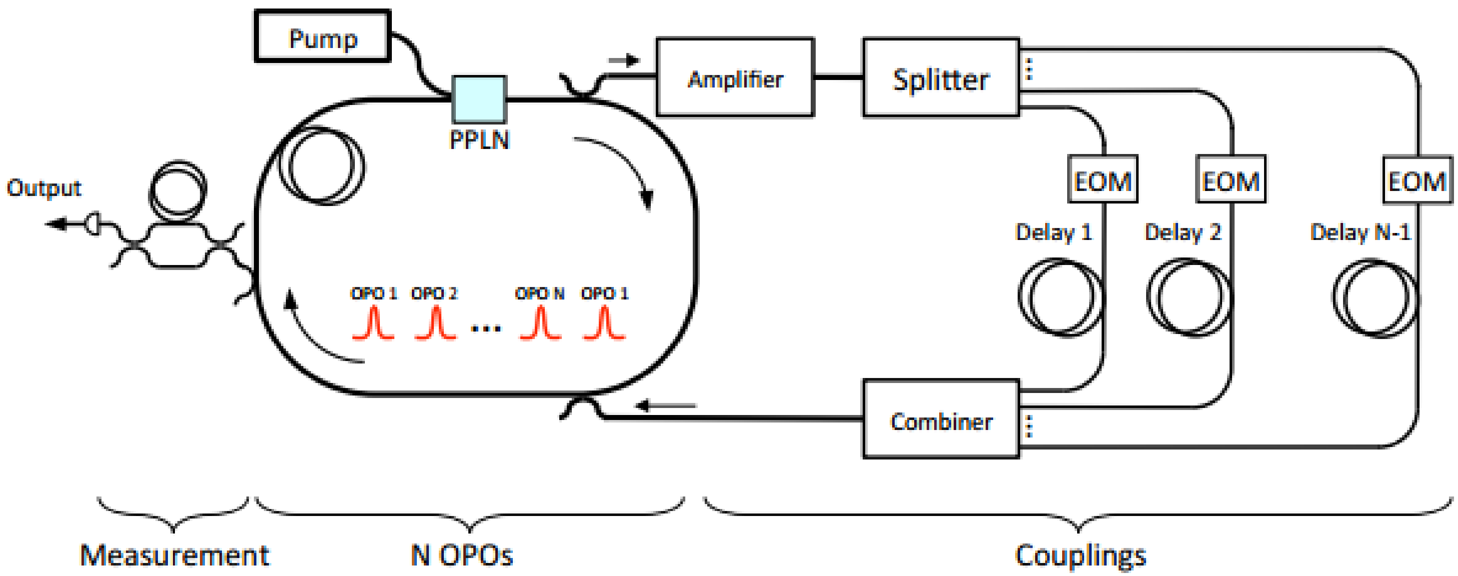



Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

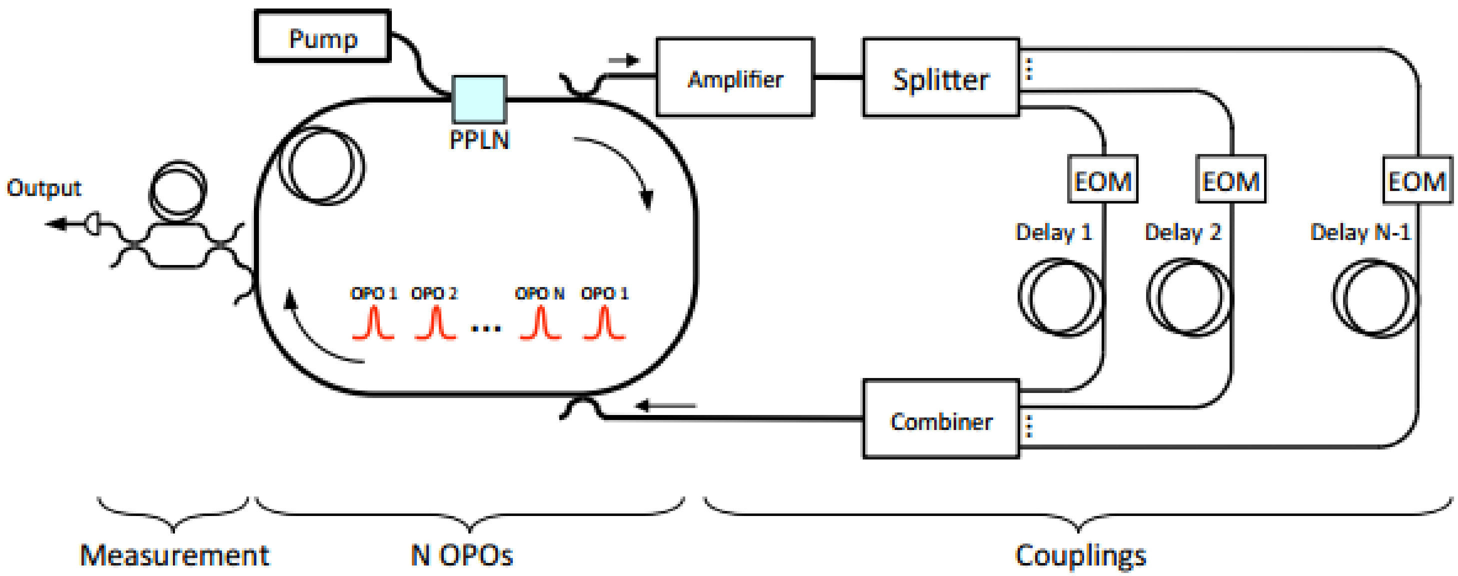
# Achieving Arbitrary Connectivity




**Note:** with temporal control of couplings, coupling strength can change during computation (similar to annealing schedule in AQC)

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014). [Supplementary Information].

# Achieving Arbitrary Connectivity



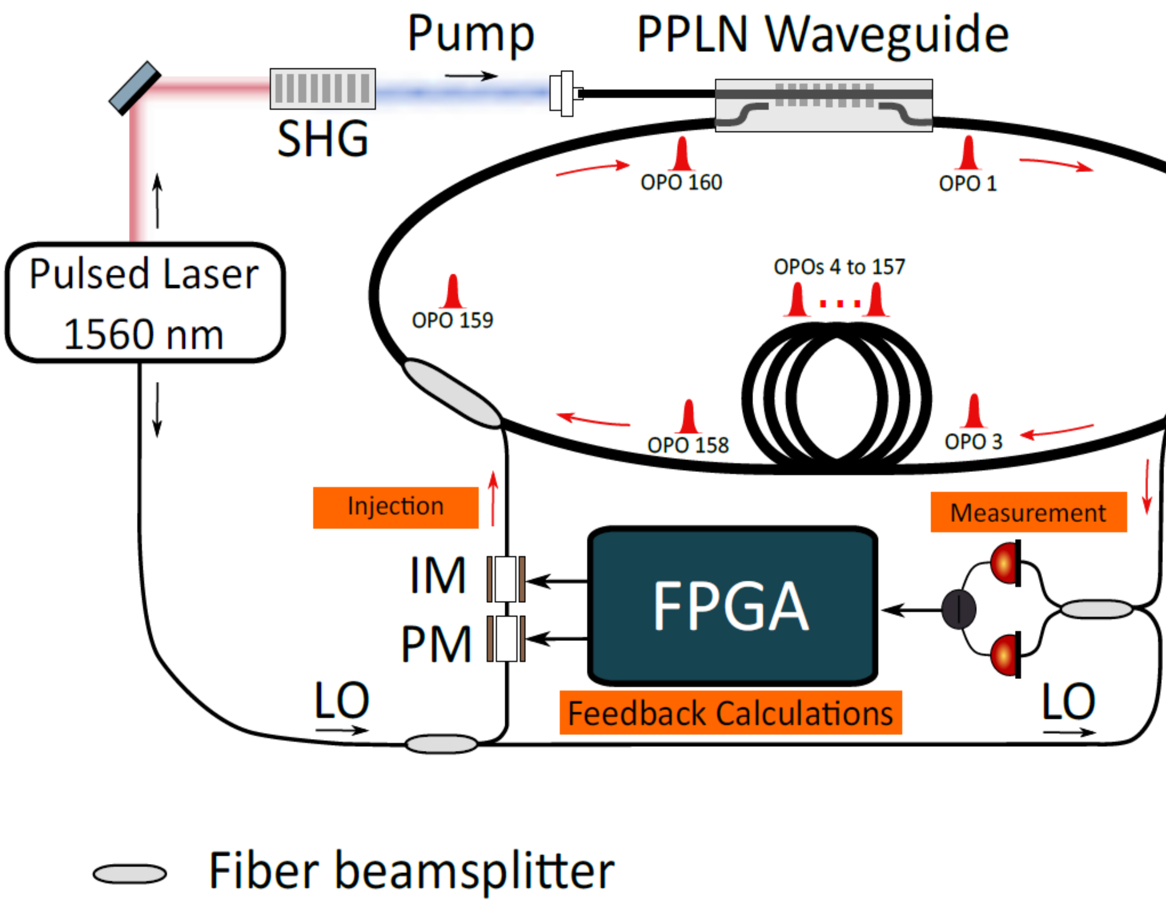

Want ***N* > 128**, preferably $N$ >> 1000.
→ Cost, loss, and phase stabilization issues!

Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014). [Supplementary Information].

# Overview

- The Ising Problem
- Ising Machines
- Foundations of All-Optical OPO Ising Machines
- **Measurement-Feedback OPO Ising Machines**
- Conclusions

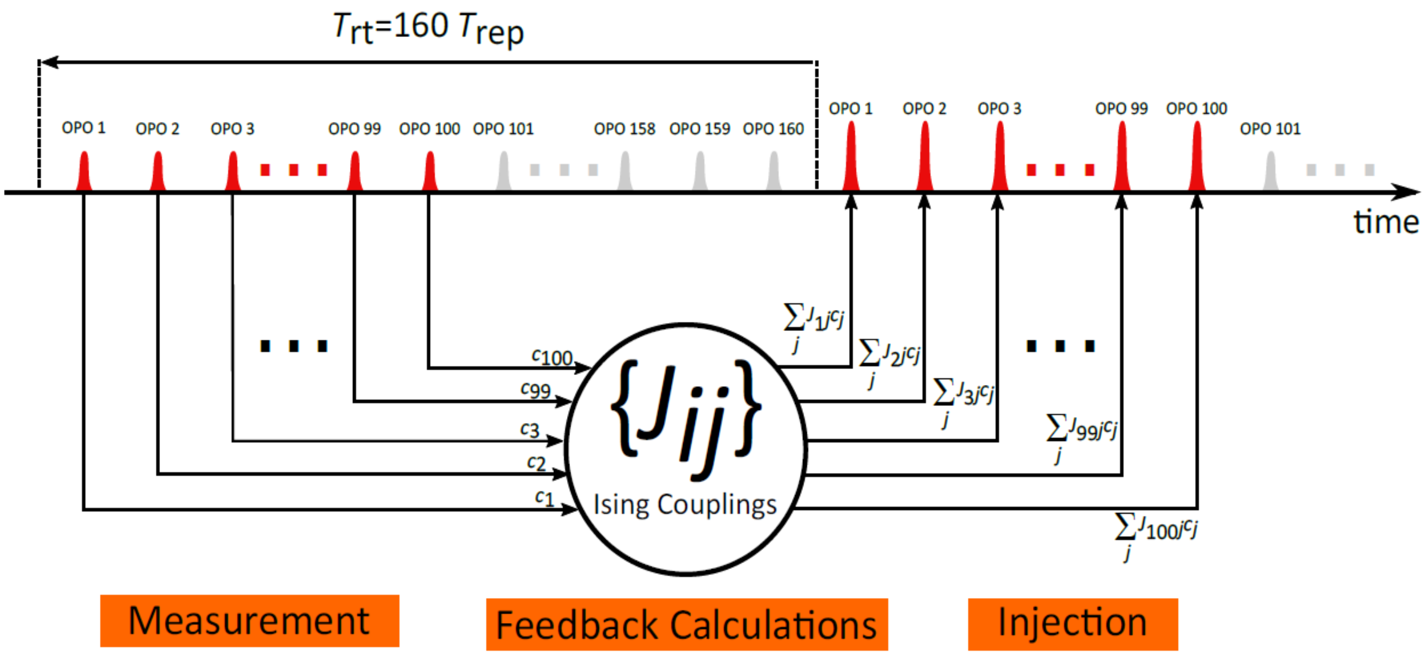
# Measurement-Feedback OPO Ising Machine



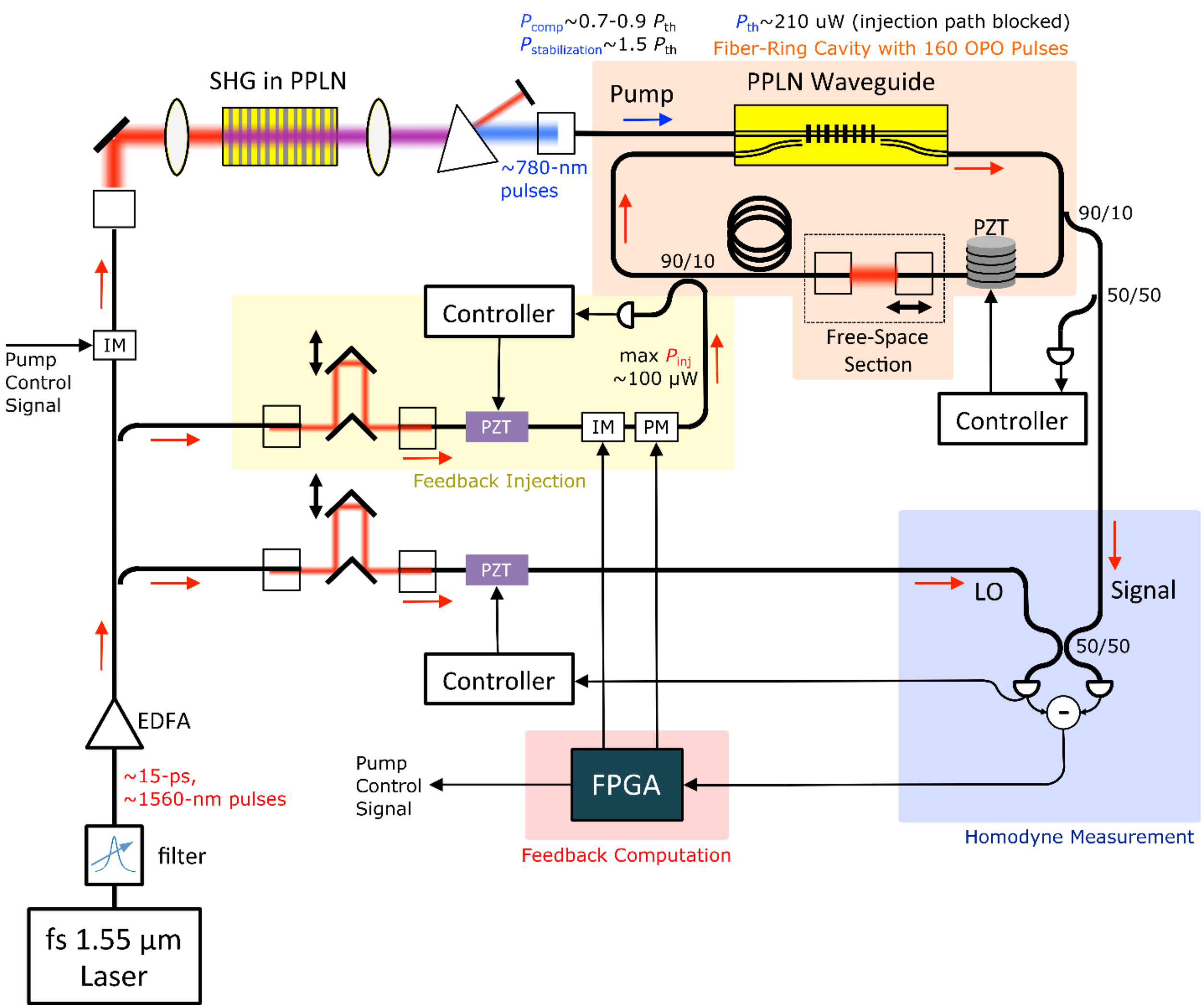

FPGA computes feedback for $i$th pulse: $\sum_{j=1}^{N=100} J_{ij} c_j$

i.e., **one *N*-dim vector-vector dot product per pulse**

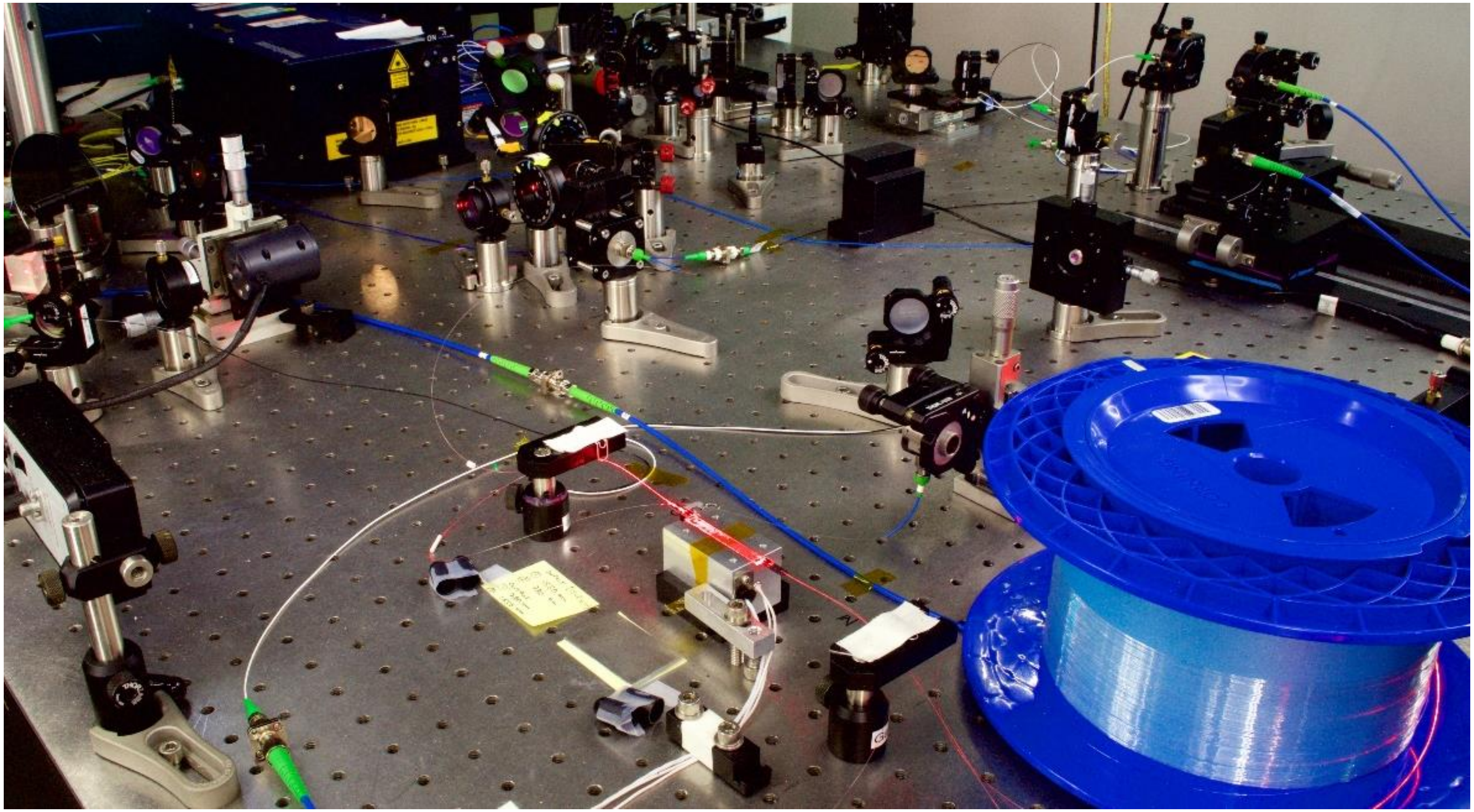
# Feedback Calculations



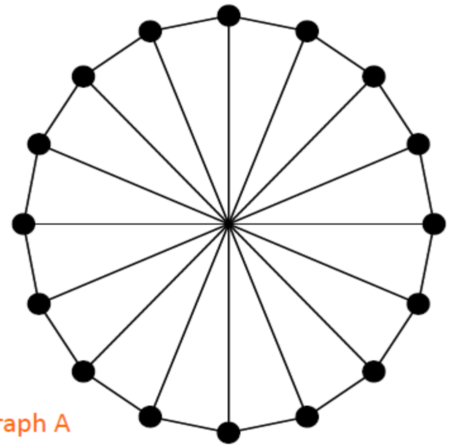
# Experimental Realization



# Experimental Realization

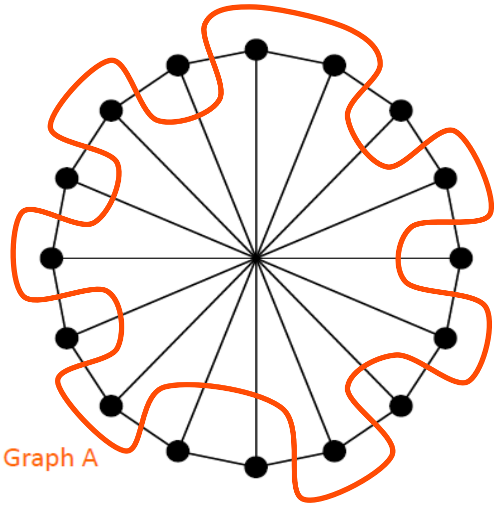


# MAX-CUT on small graphs



Graph A

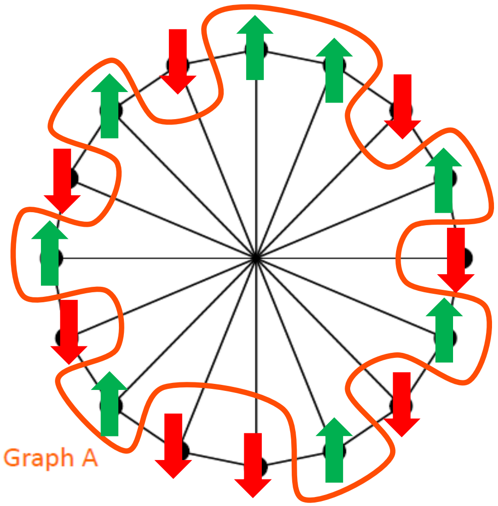
# MAX-CUT on small graphs



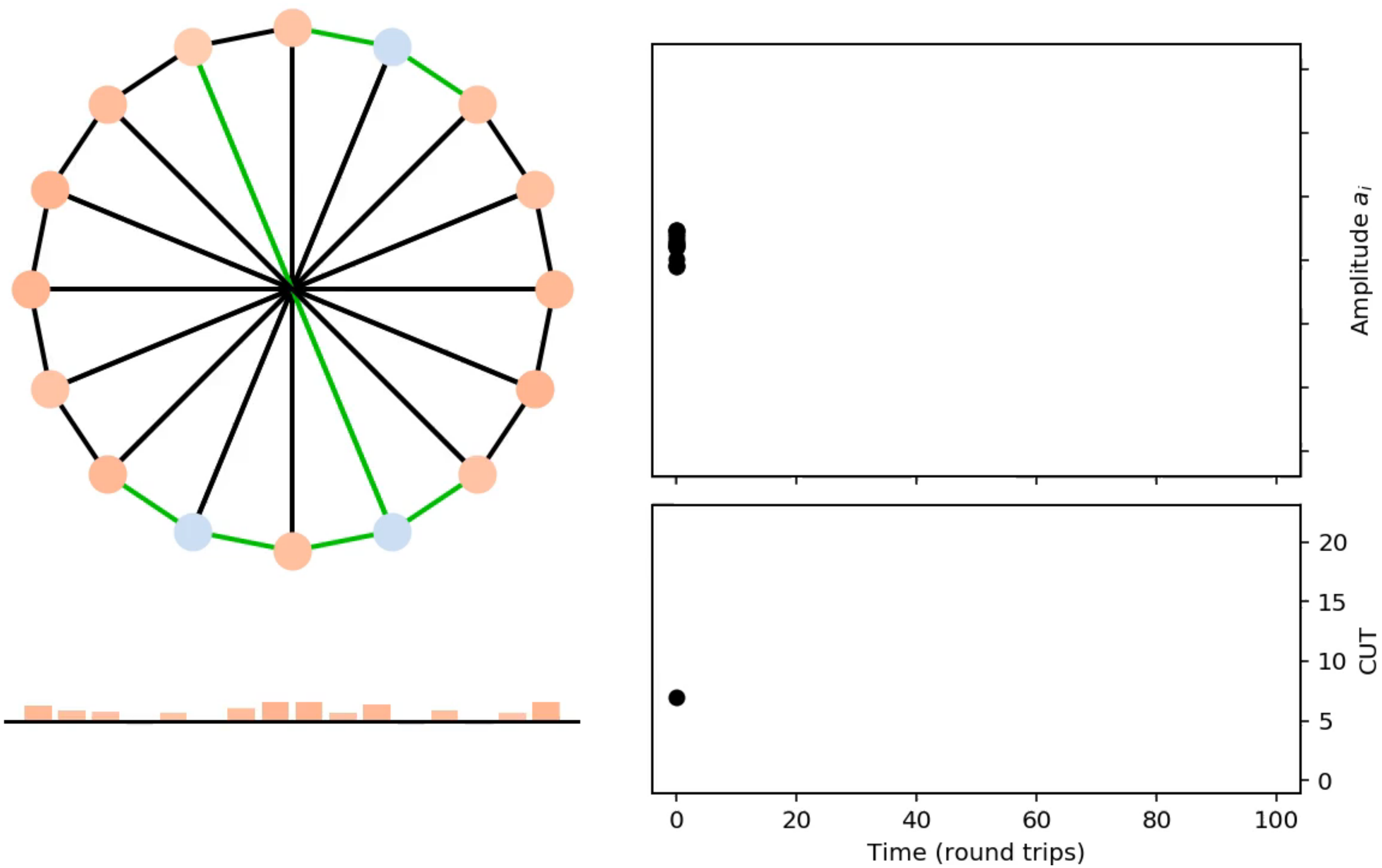

**22** edges are crossed

→ Size of “maximum cut” is 22

# MAX-CUT on small graphs



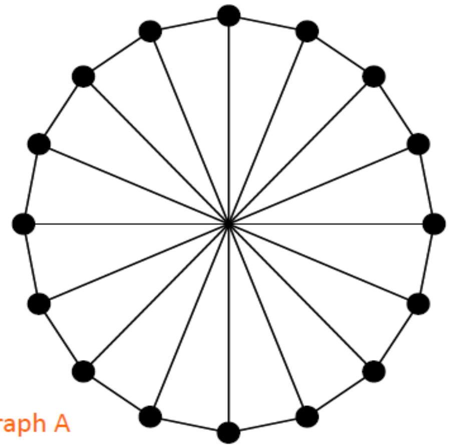
# MAX-CUT on small graphs



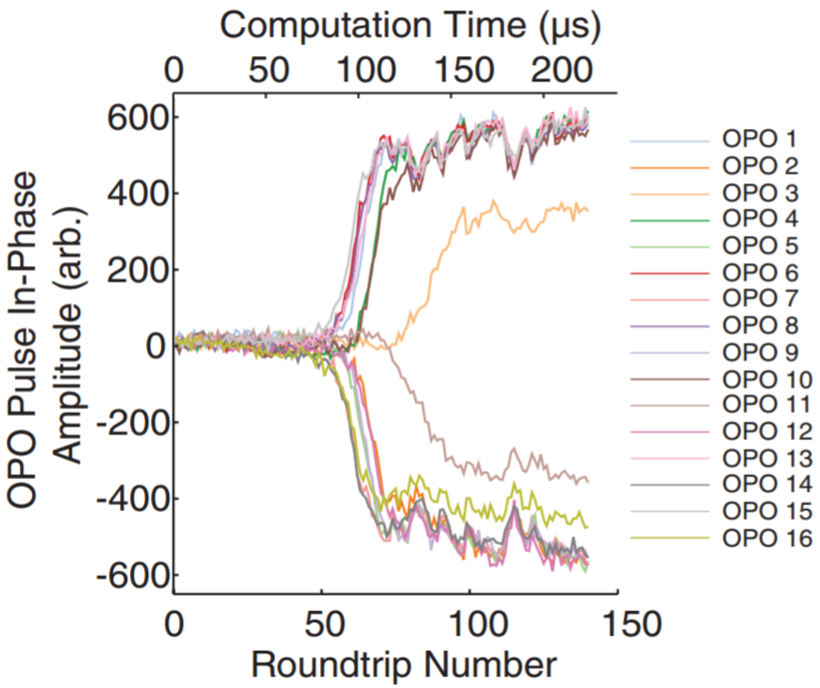



Video prepared from data in P.L. McMahon*, A. Marandi*, et al. *Science* **354**, No. 6312, 614 - 617 (2016) by Ryan Hamerly.

# MAX-CUT on small graphs



Graph A



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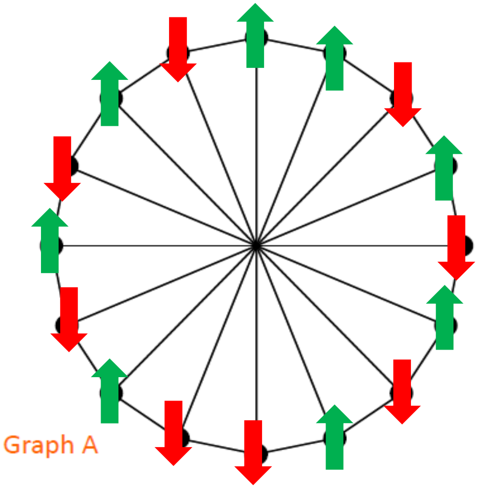


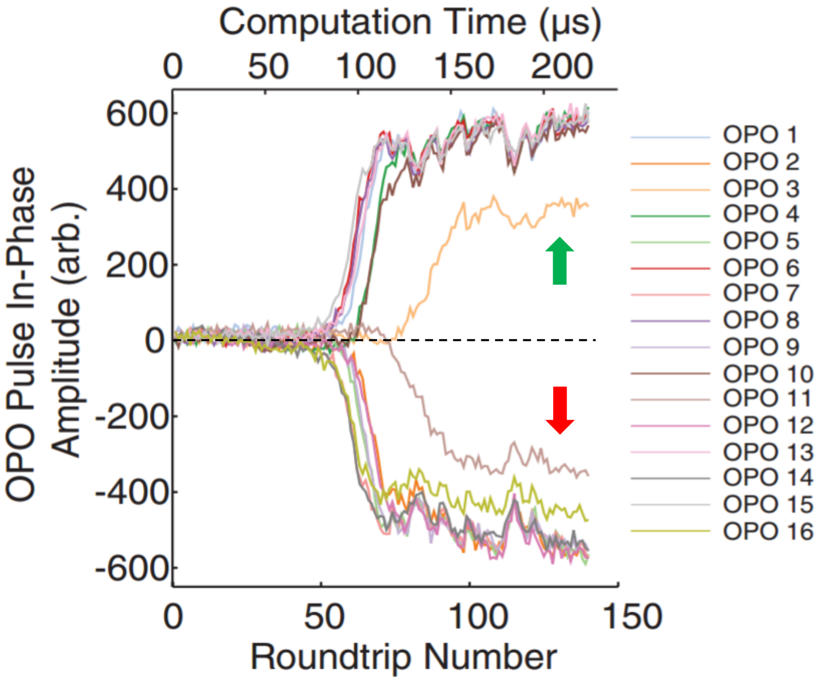

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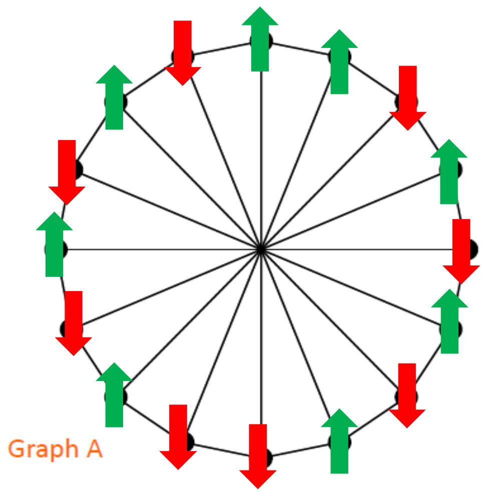


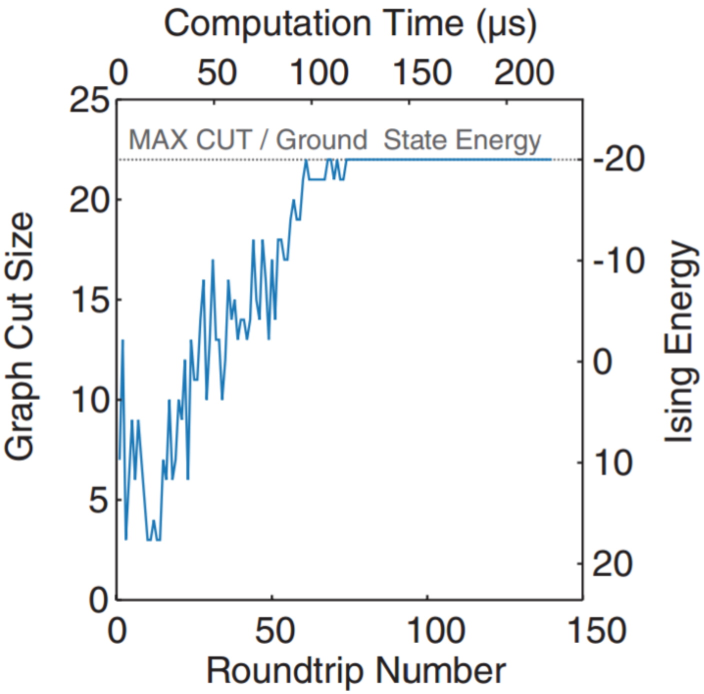

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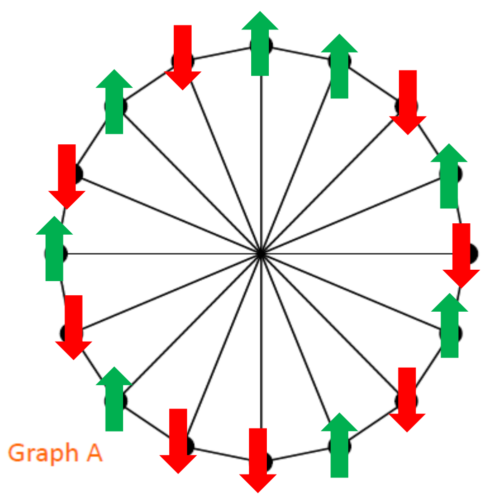


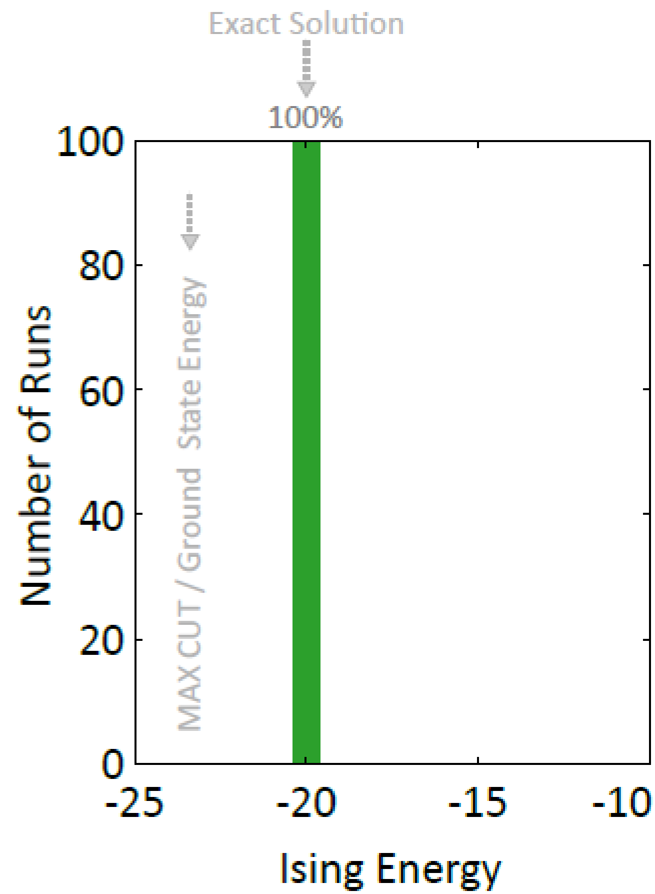

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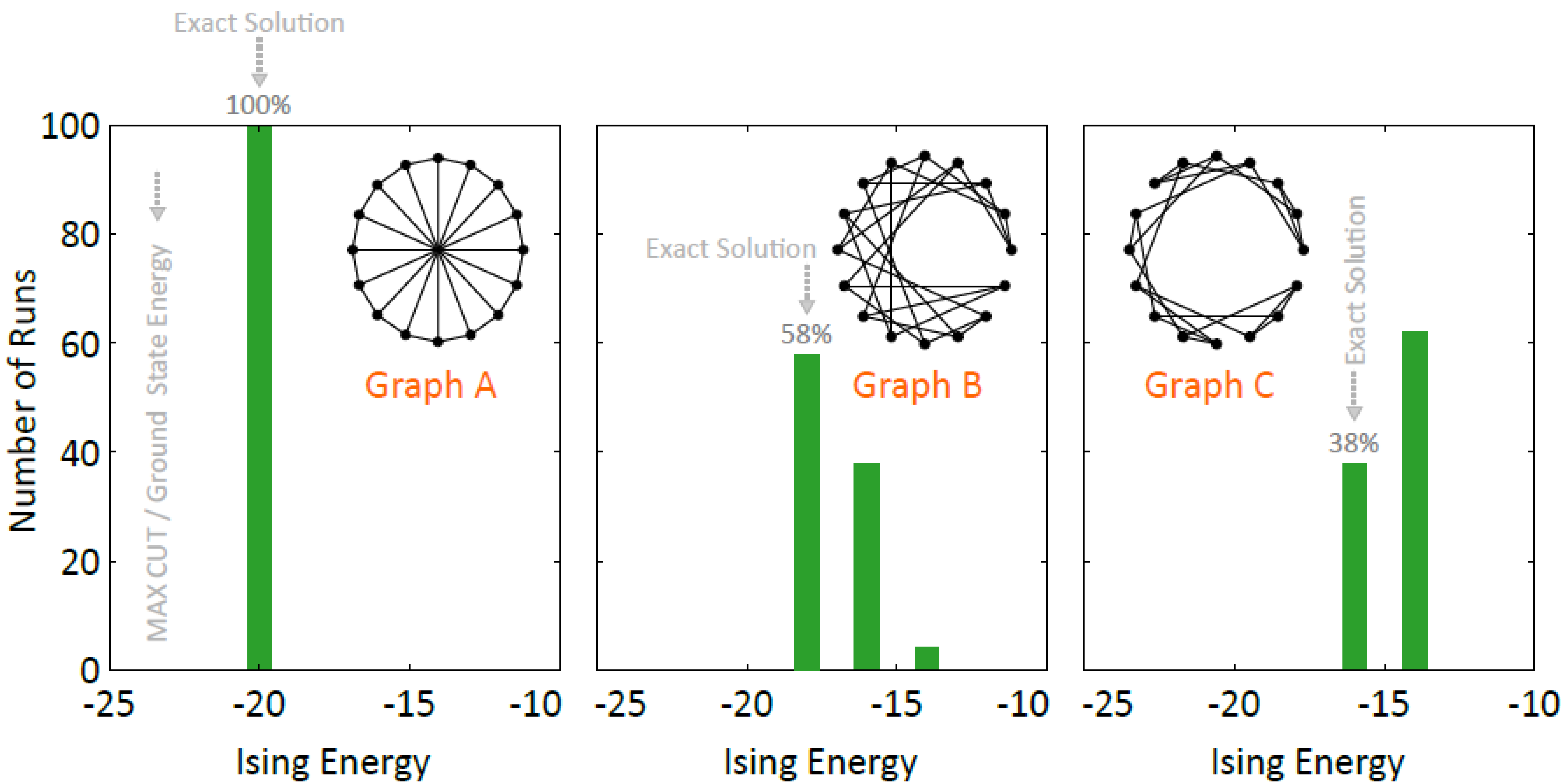
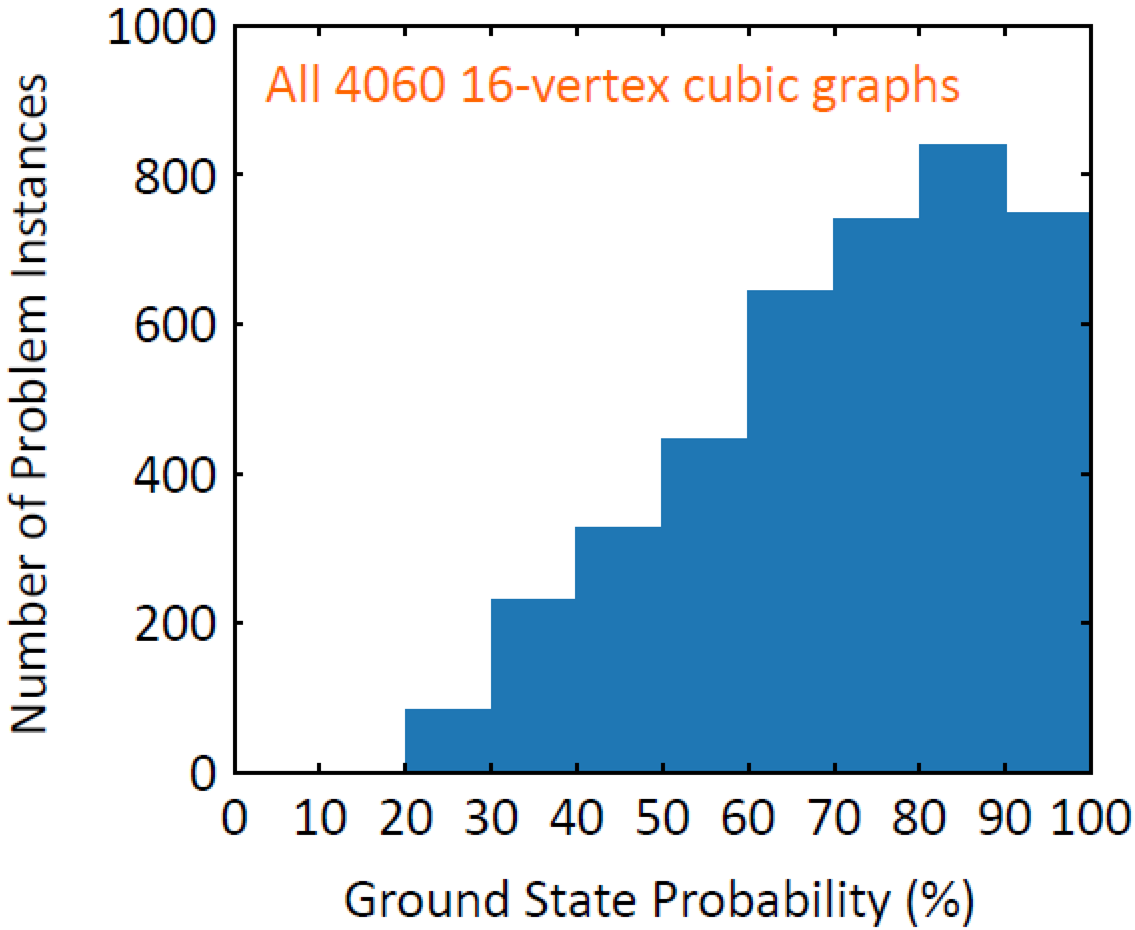
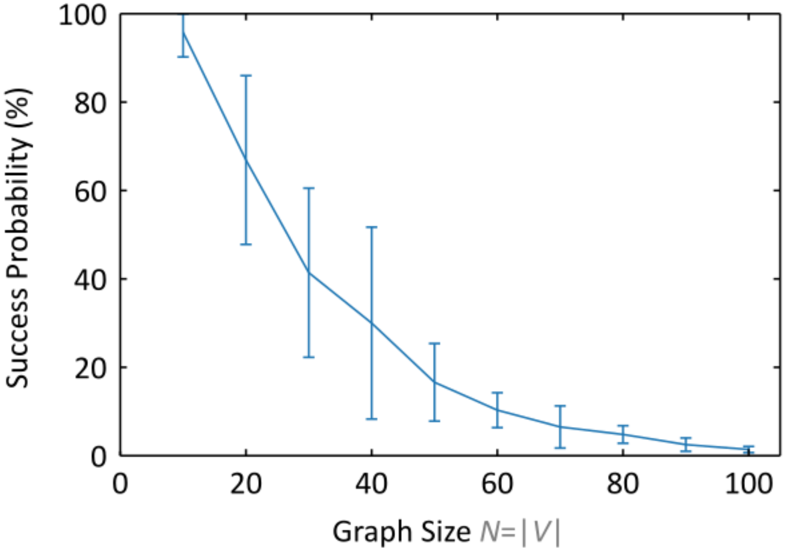


# MAX-CUT on small graphs




**Main point:** we can solve **every** *N*=16 cubic graph instance, so the previous success probabilities we showed were not just lucky data points.

# Scaling of cubic graphs



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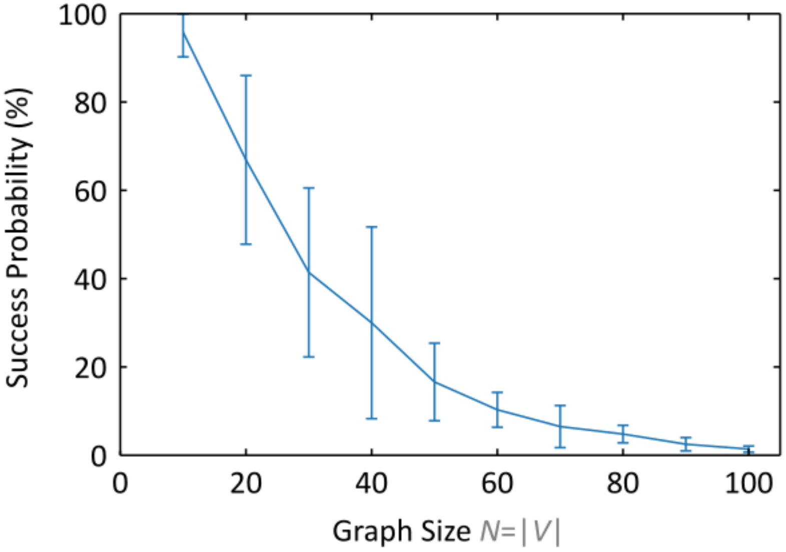
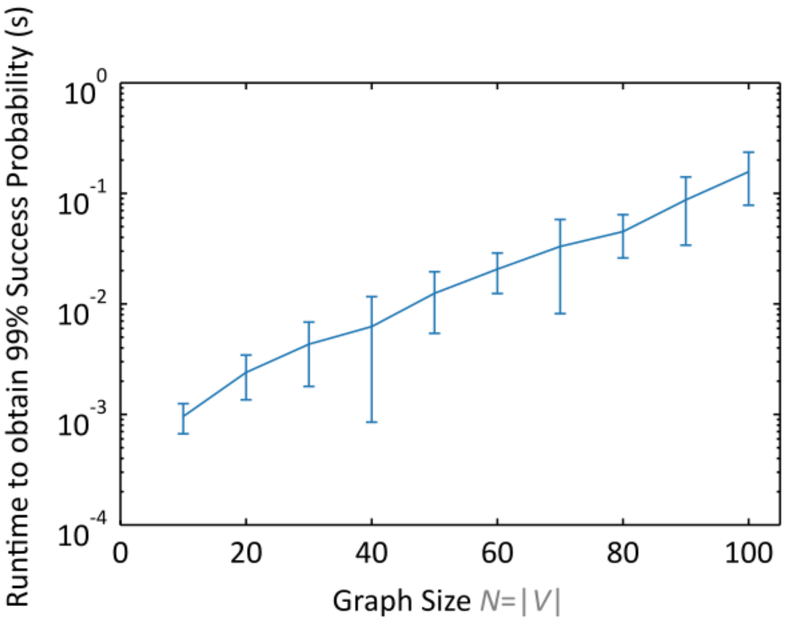
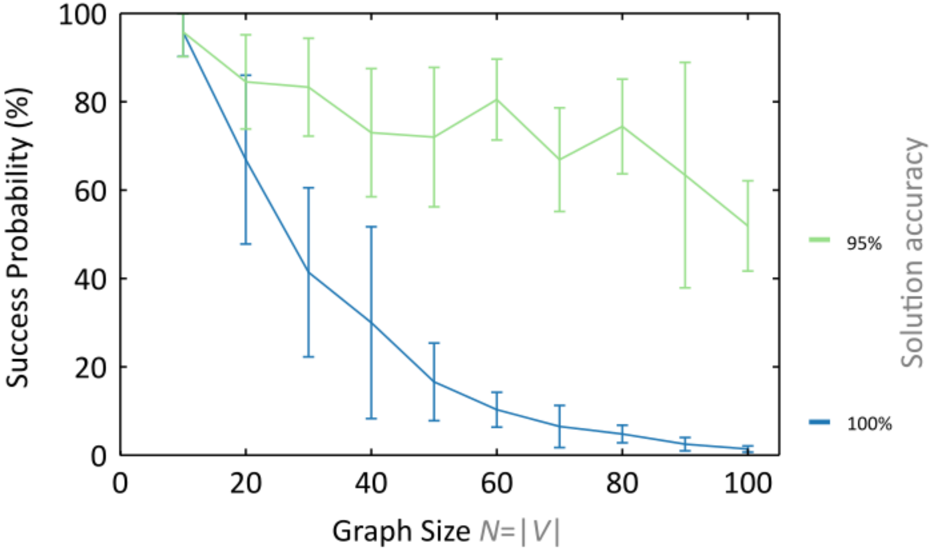
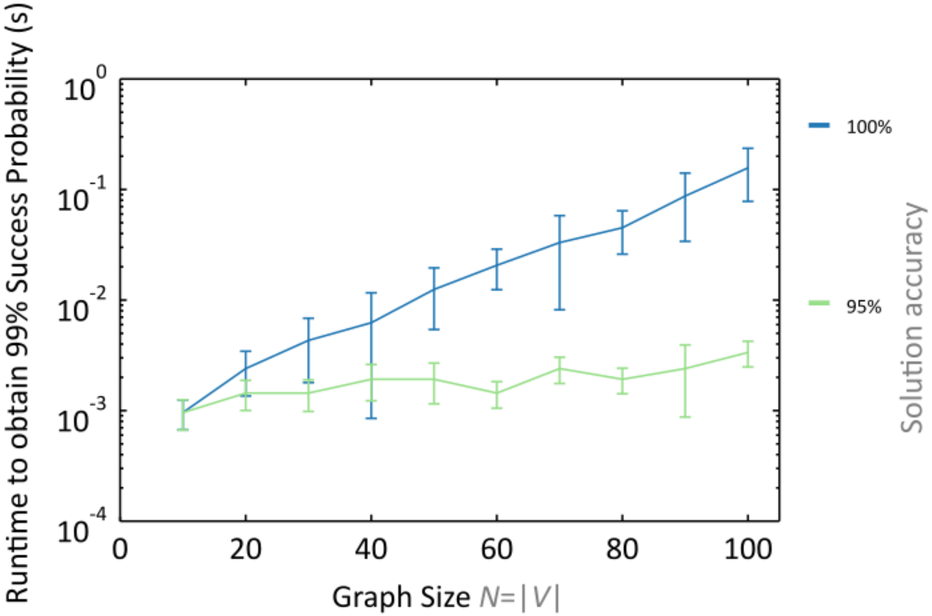


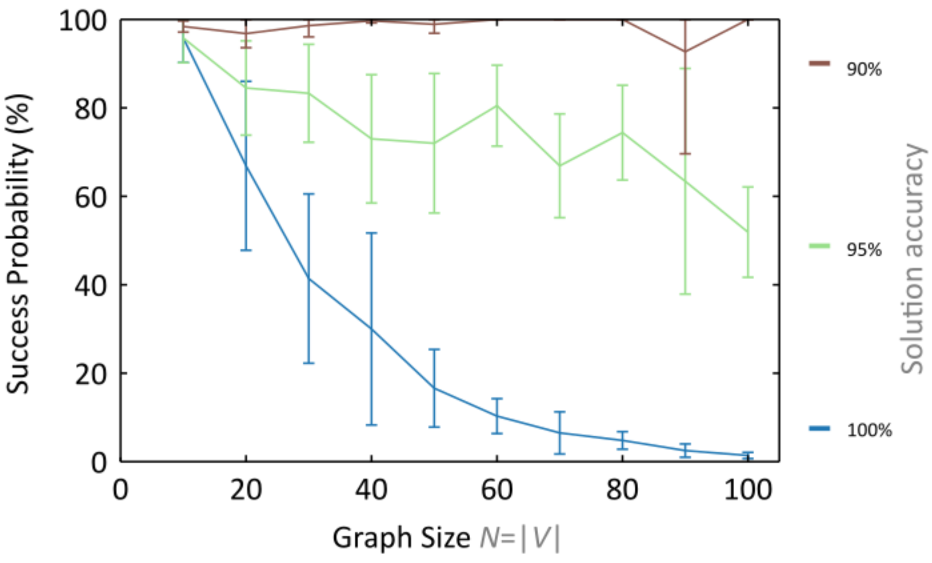



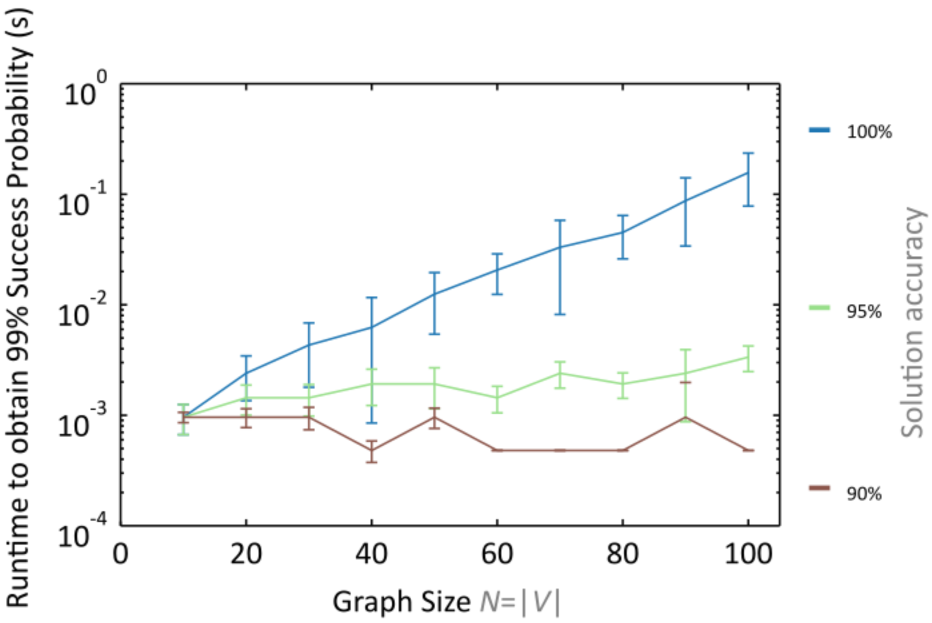
# Scaling of cubic graphs



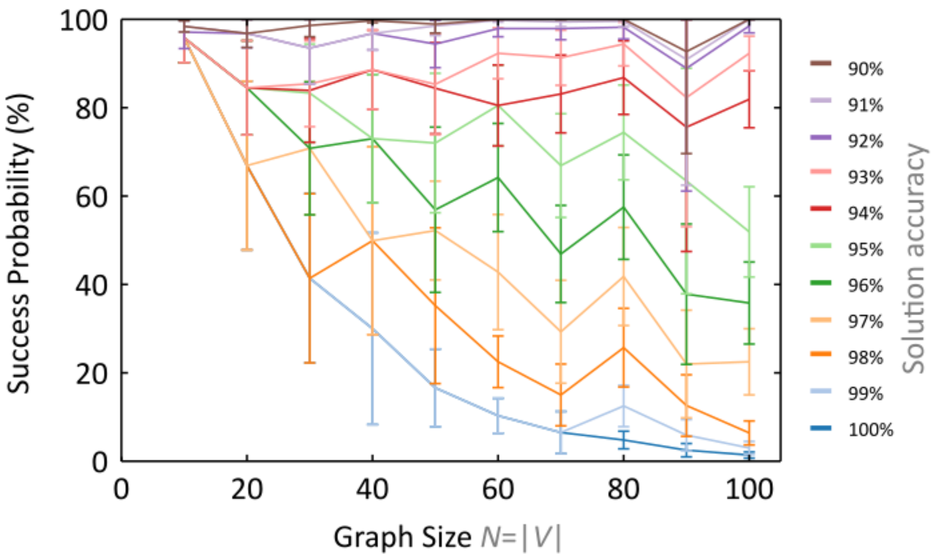



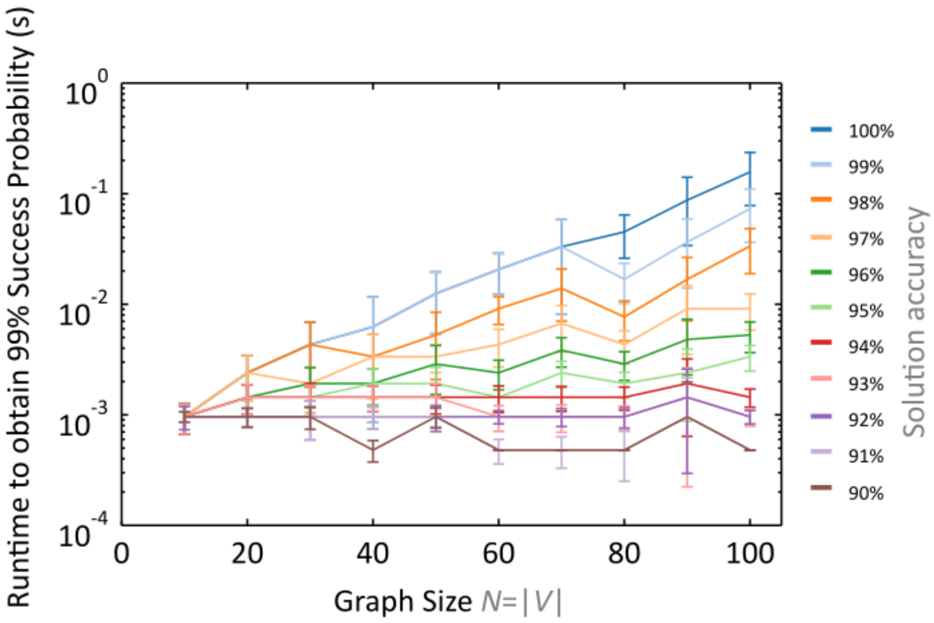
# Scaling of cubic graphs



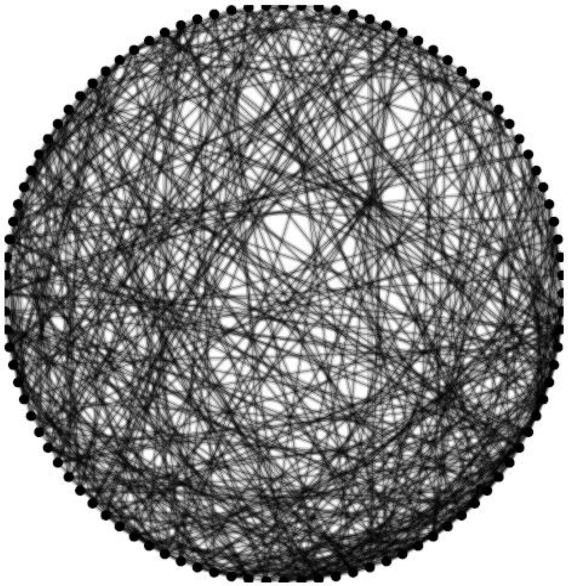
# Scaling of cubic graphs



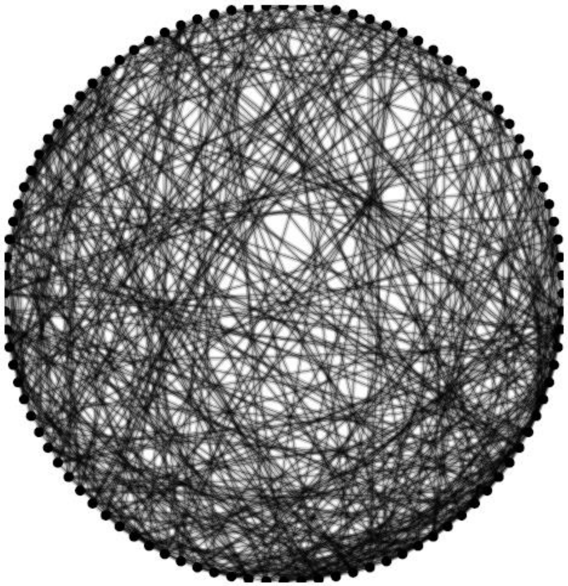
# What about non-cubic graphs?

# Dense(r) Random Graph



100 vertices; 495 edges

# Dense(r) Random Graph



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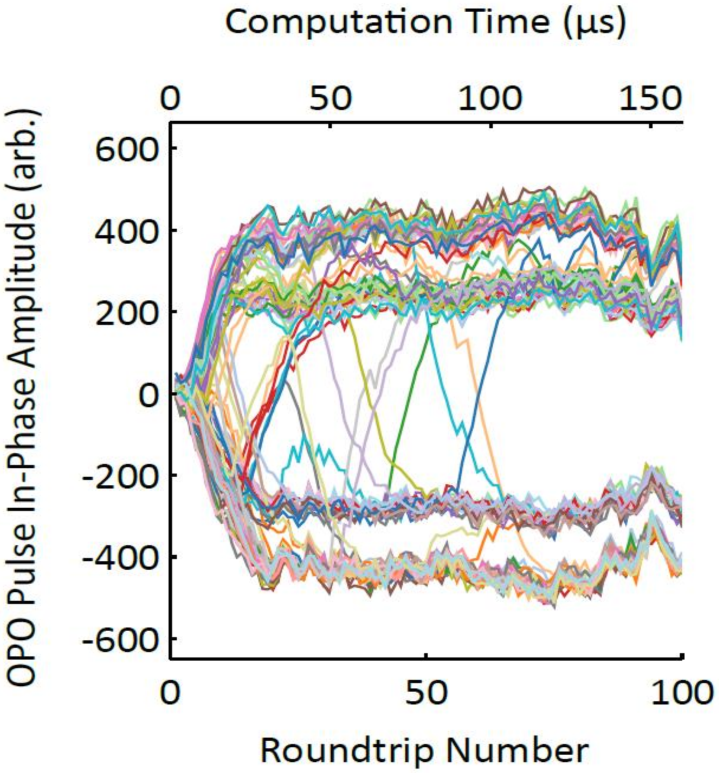
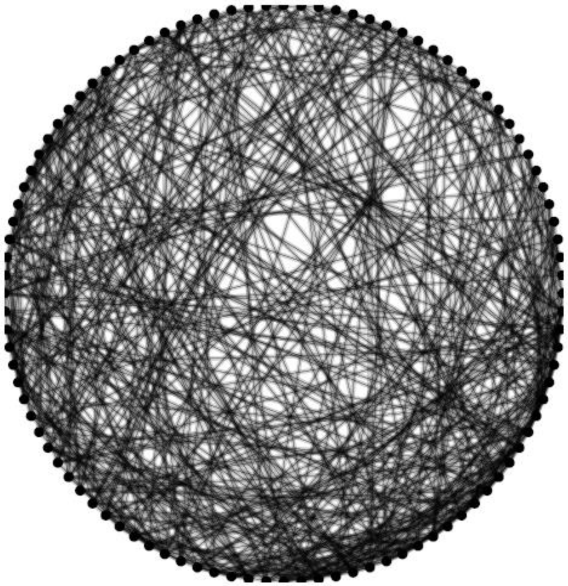


# Dense(r) Random Graph



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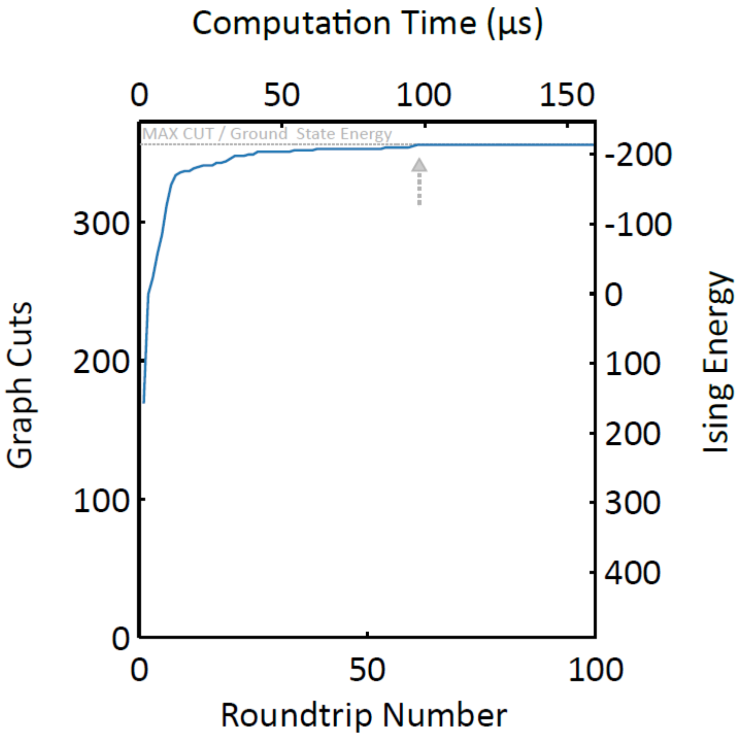
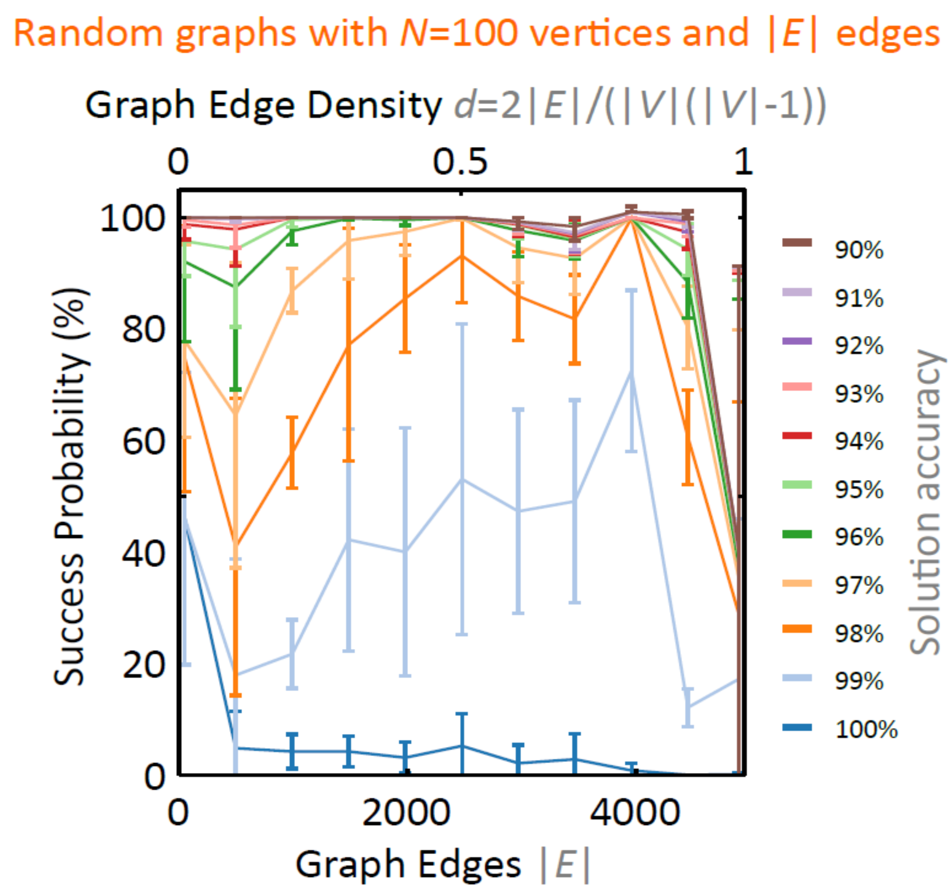


# Sensitivity to Edge Density




**Main point:** system seems to be able to find exact and approximate solutions for a large range of graphs, including both sparse and dense graphs.

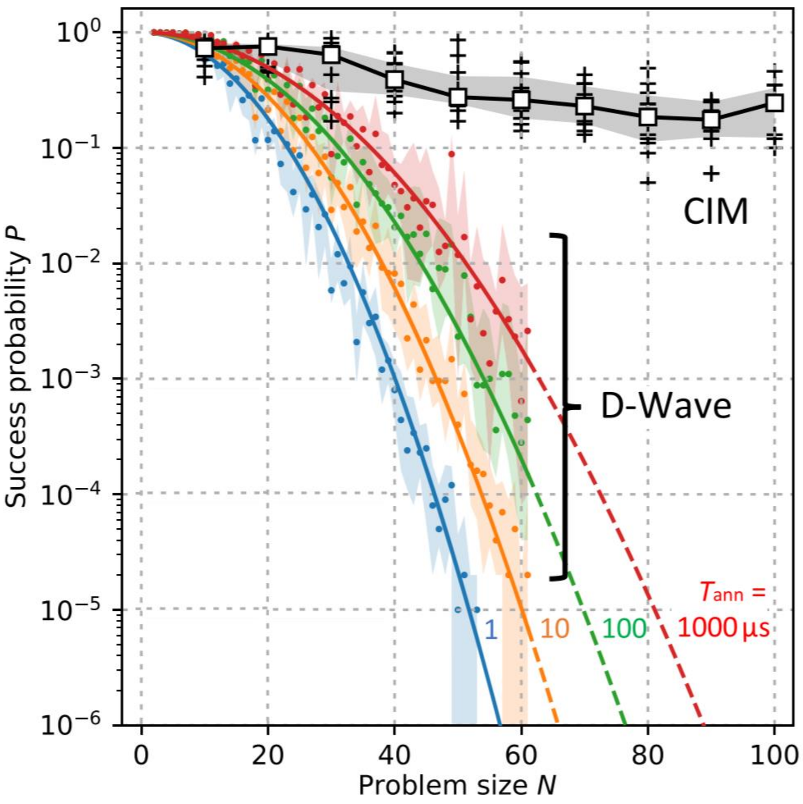
# Comparison to Quantum Annealing

Credit: dwavesys.com

M.W. Johnson, et al. *Nature* **473**, 194-198 (2011)
T.F. Ronnow, et al. *Science* **345**, 6195, 420-424 (2014)
S. Boixo, et al. *Nature Comm.* **7**, 10327 (2016)

# Comparison to Quantum Annealing

*Problem class*: Sherrington-Kirkpatrick spin-glass problems
($J_{ij}$ are -1 or +1 uniformly at random)






(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: Sherrington-Kirkpatrick spin-glass problems
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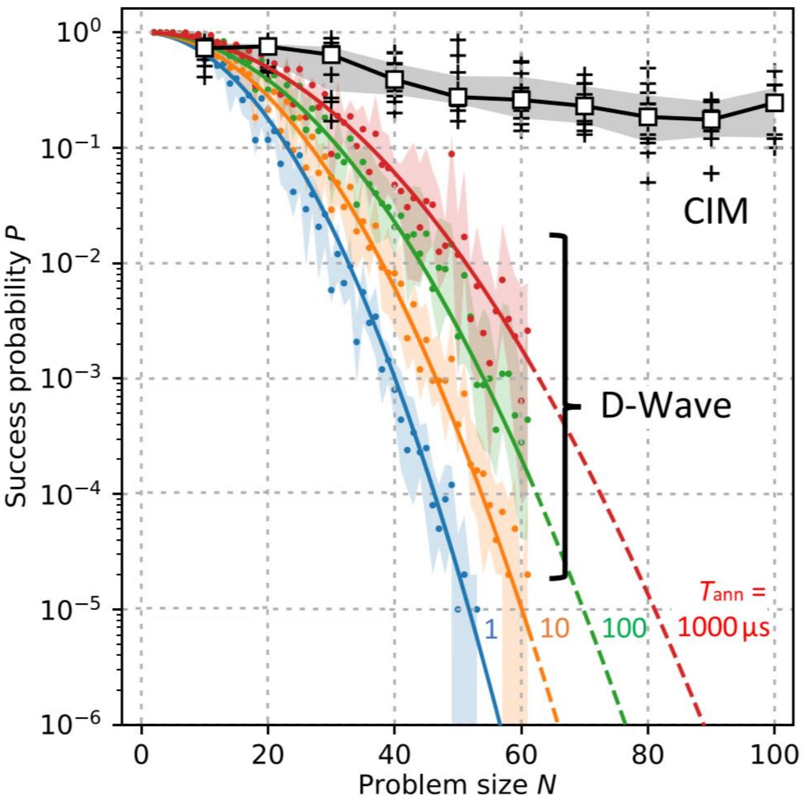



For any given size *N*, which annealing time should we choose?

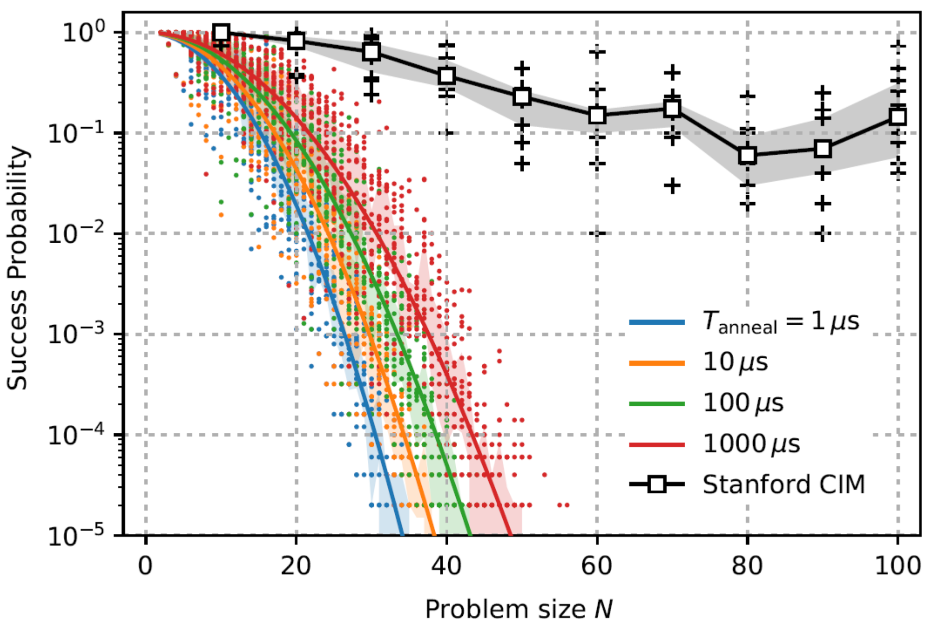
If we want to predict performance for larger problem sizes than we can currently solve, which annealing time should we choose?



(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on unweighted graphs with edge density = 50%






(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

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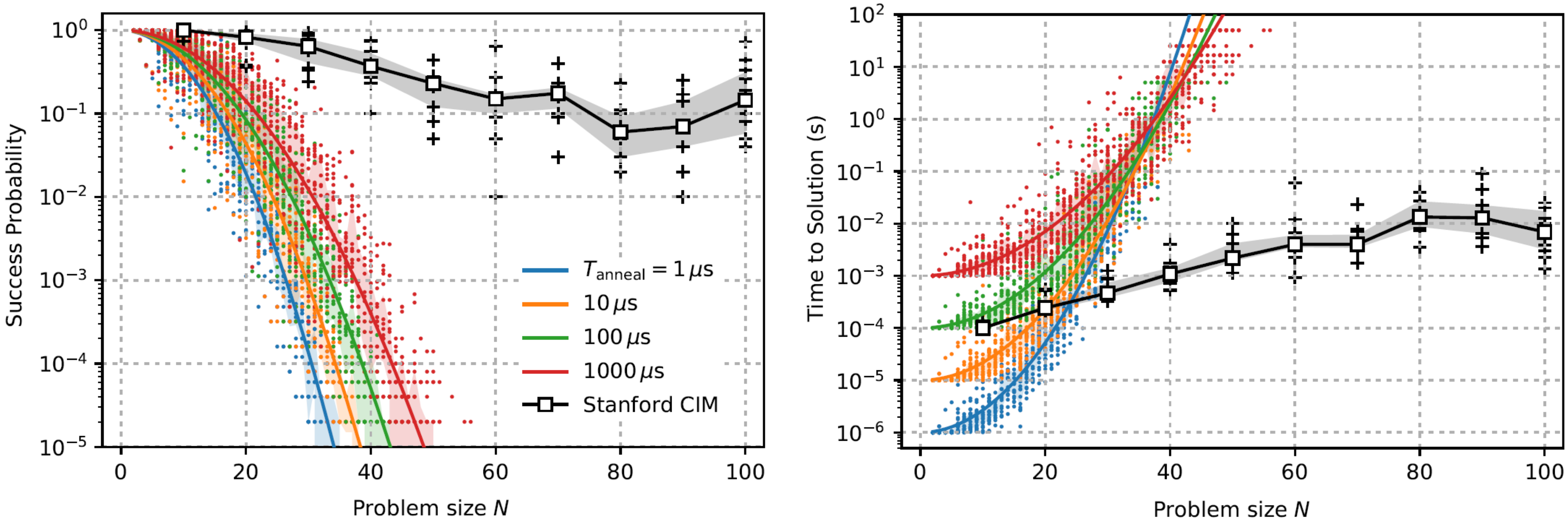
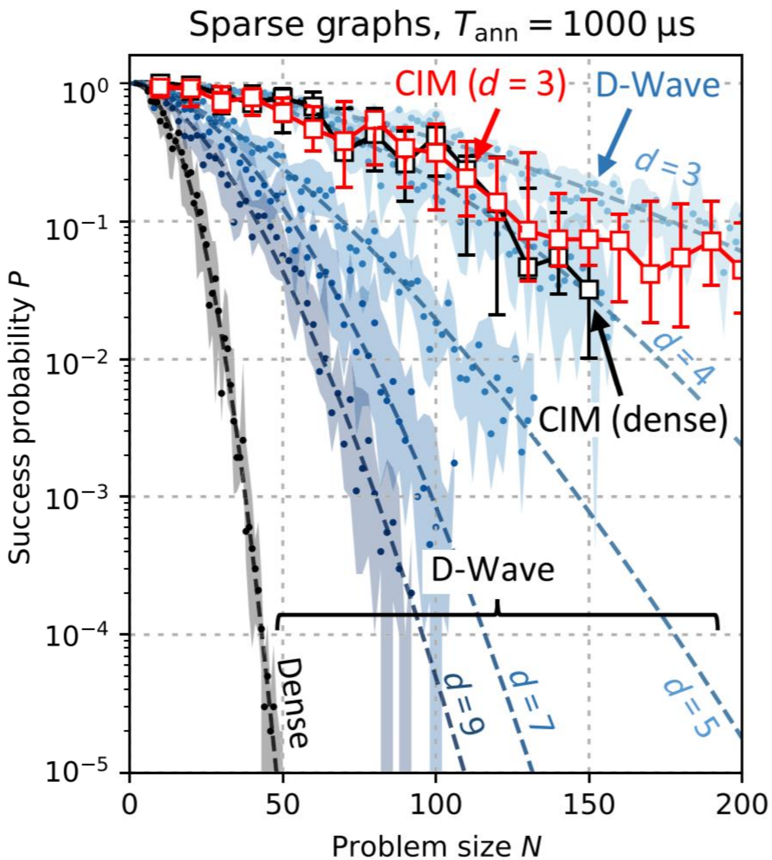





(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on regular graphs with degree *d*

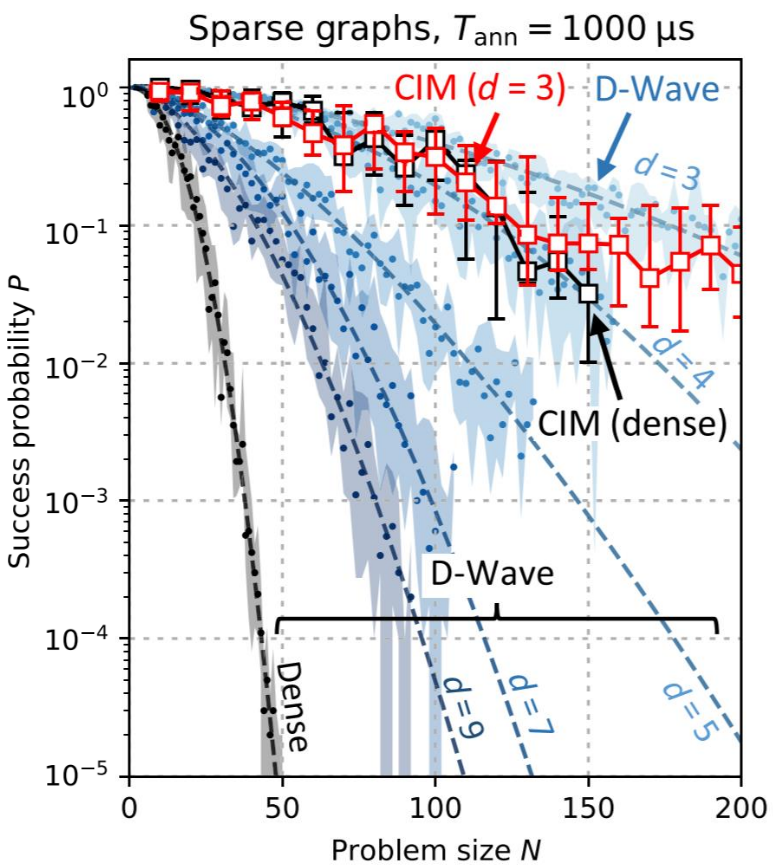





(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on regular graphs with degree *d*




Connectivity makes a big difference!

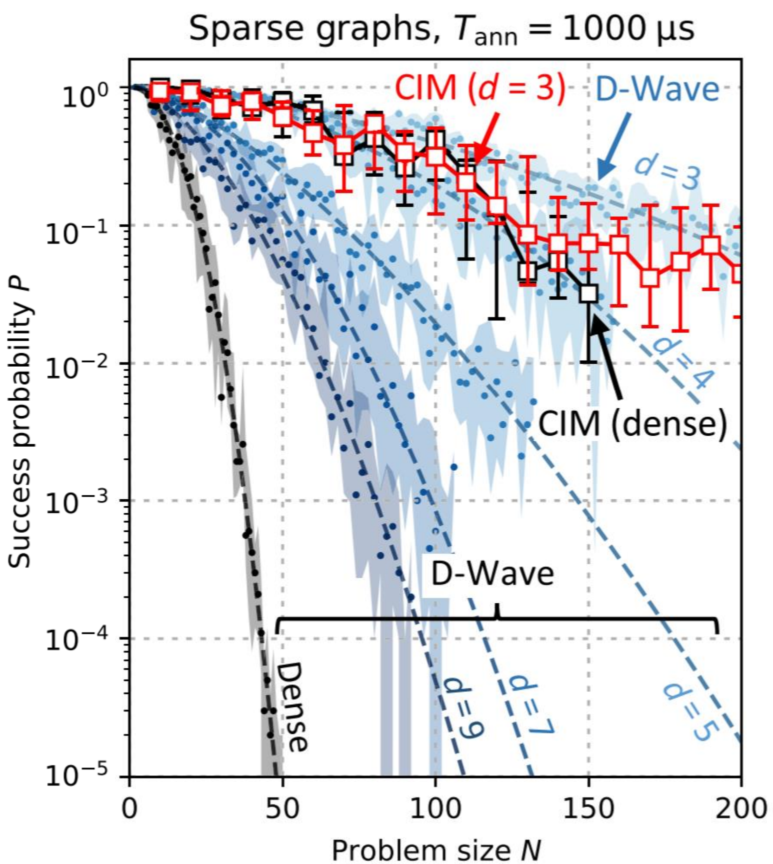
Why?



(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on regular graphs with degree *d*




Connectivity makes a big difference!

Why?

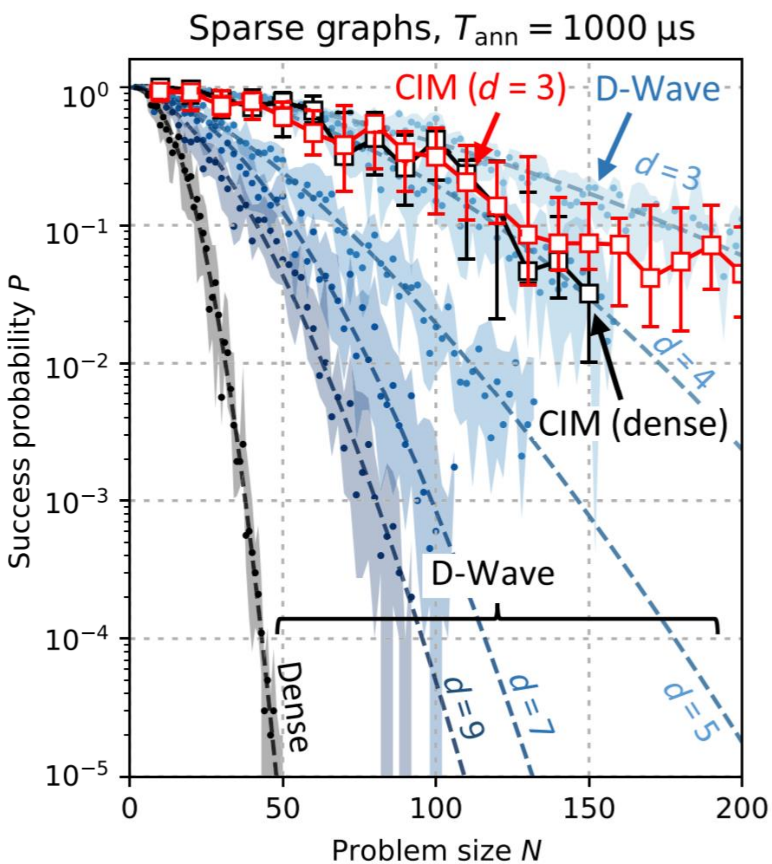
Success probability $P \sim \exp(-\alpha N_{physical})$



(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on regular graphs with degree *d*




Connectivity makes a big difference!

Why?

Success probability $P \sim \exp(-\alpha N_{physical})$

For **dense** problems: $N_{physical} \sim N^2$
For **sparse** problems: $N_{physical} \sim N$

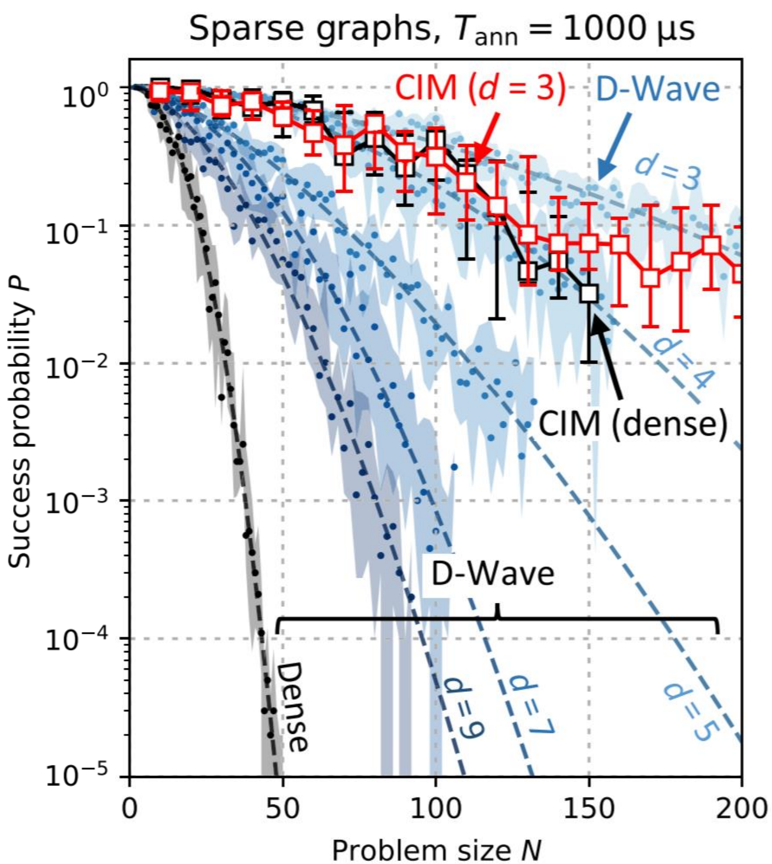
*Note*: α depends on the machine or algorithm



(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

*Problem class*: MAX-CUT on regular graphs with degree *d*




Connectivity makes a big difference!

Why?

Success probability $P \sim \exp(-\alpha N_{physical})$

For **dense** problems: $N_{physical} \sim N^2$
For **sparse** problems: $N_{physical} \sim N$

*Note*: α depends on the machine or algorithm

How can we make quantum annealers with better connectivity?
See: Onodera *et al*. *npj Quantum Information* **6**, 48 (2020).



(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Comparison to Quantum Annealing

| SK | | | | MAX-CUT (dense) | | | | MAX-CUT ($d = 3$) | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | DW2Q | CIM | Factor | $N$ | DW2Q | CIM | Factor | $N$ | DW2Q | CIM | Factor |
| 10 | 6.0 μs | 25 μs | 0.2 | 10 | 6.0 μs | 25 μs | 0.2 | 10 | 1.0 μs | 50 μs | 0.02 |
| 20 | 35 μs | 100 μs | 0.3 | 20 | 0.4 ms | 100 μs | 4 | 20 | 3.0 μs | 100 μs | 0.03 |
| 40 | 6.1 ms | 0.4 ms | 15 | 40 | 6.1 s | 0.4 ms | $10^4$ | 50 | 12 μs | 0.4 ms | 0.03 |
| 60 | 1.4 s | 0.6 ms | 2000 | 55 | $10^4$ s | 1.2 ms | $10^7$ | 100 | 100 μs | 3.3 ms | 0.03 |
| 80* | (400 s) | 1.8 ms | ($10^5$) | 80* | ($10^{11}$ s) | 1.8 ms | ($10^{13}$) | 150 | 2.8 ms | 22 ms | 0.1 |
| 100* | ($10^5$ s) | 3.0 ms | ($10^7$) | 100* | ($10^{19}$ s) | 2.3 ms | ($10^{21}$) | 200 | 11 ms | 51 ms | 0.2 |

Fully-connected graphs — 50%-density graphs — Cubic graphs

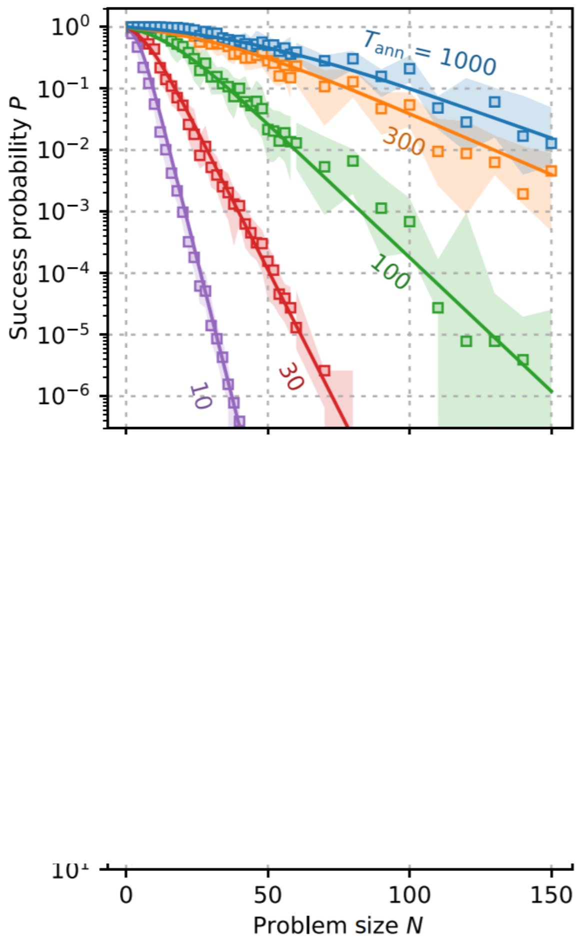
***Note***: CIM runtimes are comparable to those of a laptop running a state-of-the-art solver
→ no speedup from use of optics yet!



(Table from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Choosing Optimal Annealing Times

**CIM** (simulated)






(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

*Problem class*: Density=50% MAX-CUT

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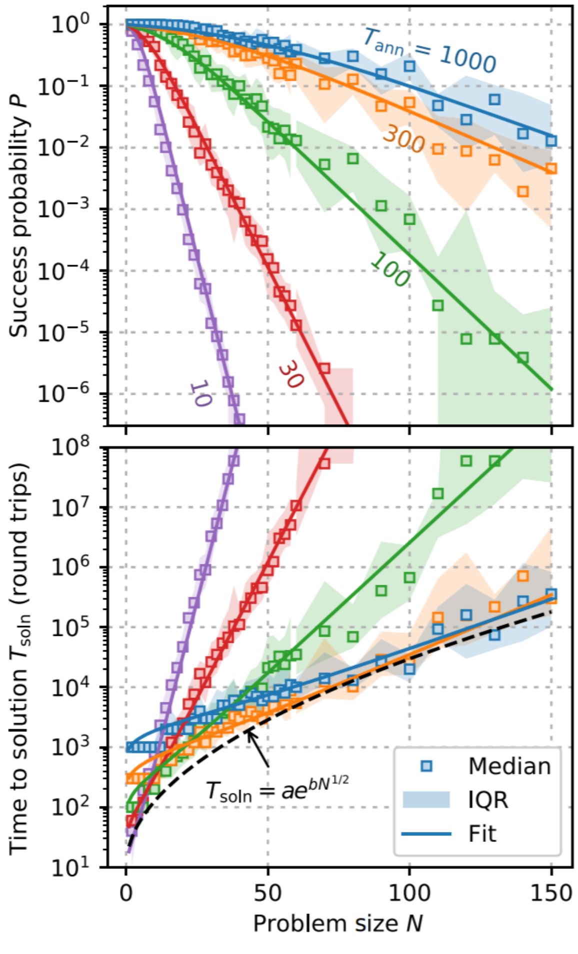




(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

*Problem class*: Density=50% MAX-CUT

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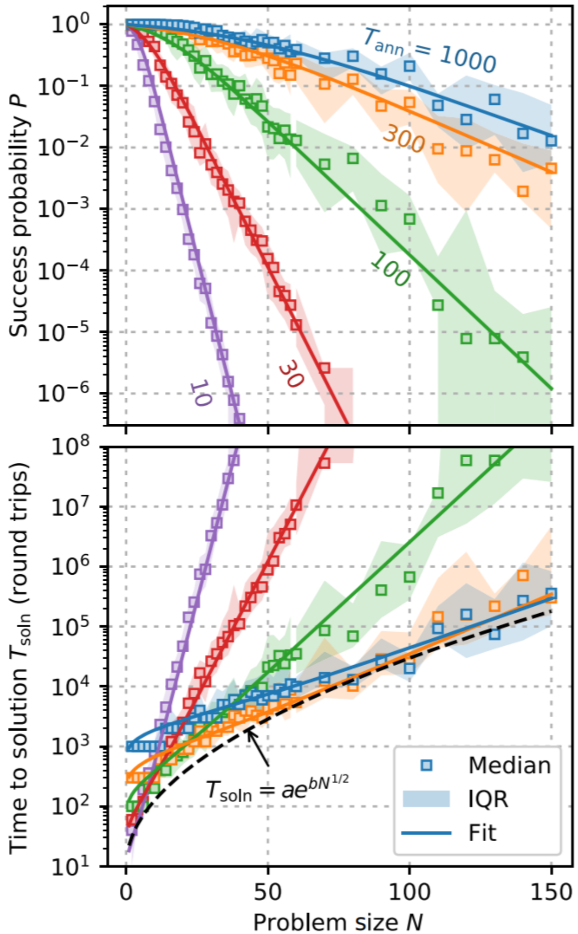
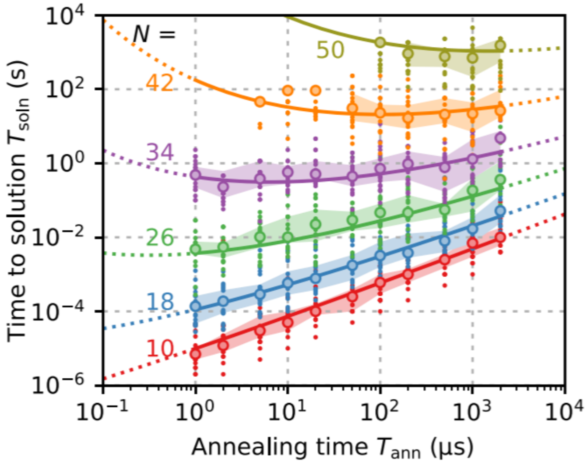
**CIM** (simulated)




**D-Wave 2000Q**






(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

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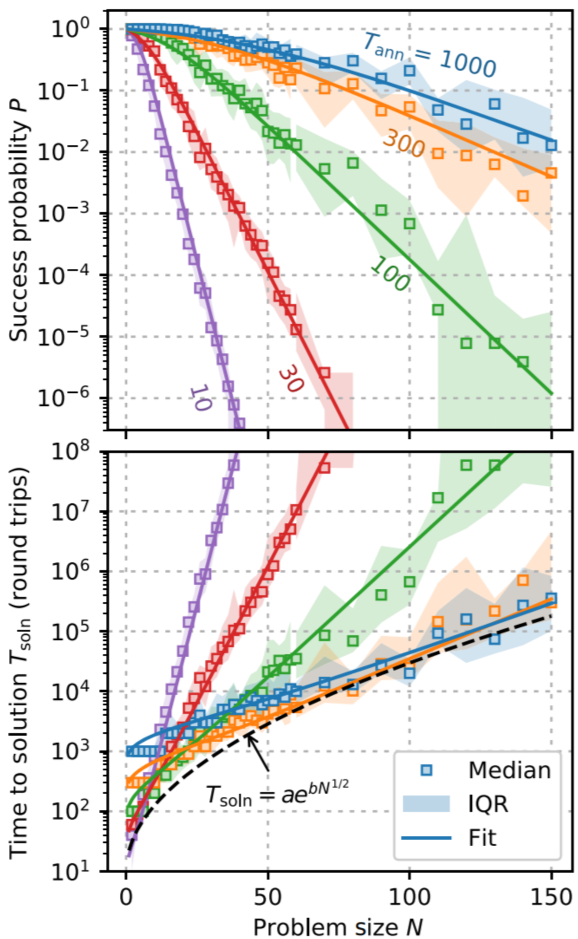
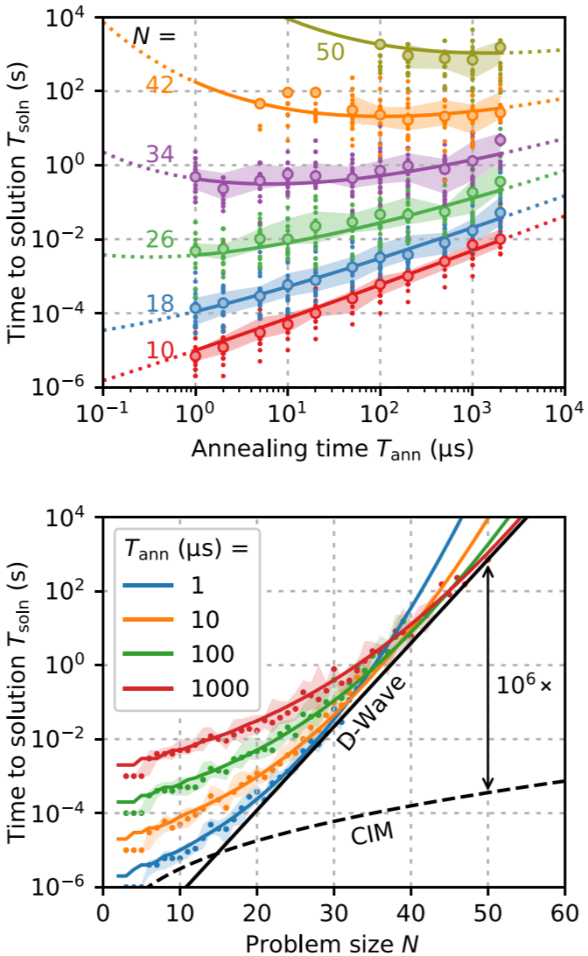
**CIM** (simulated)




**D-Wave 2000Q**

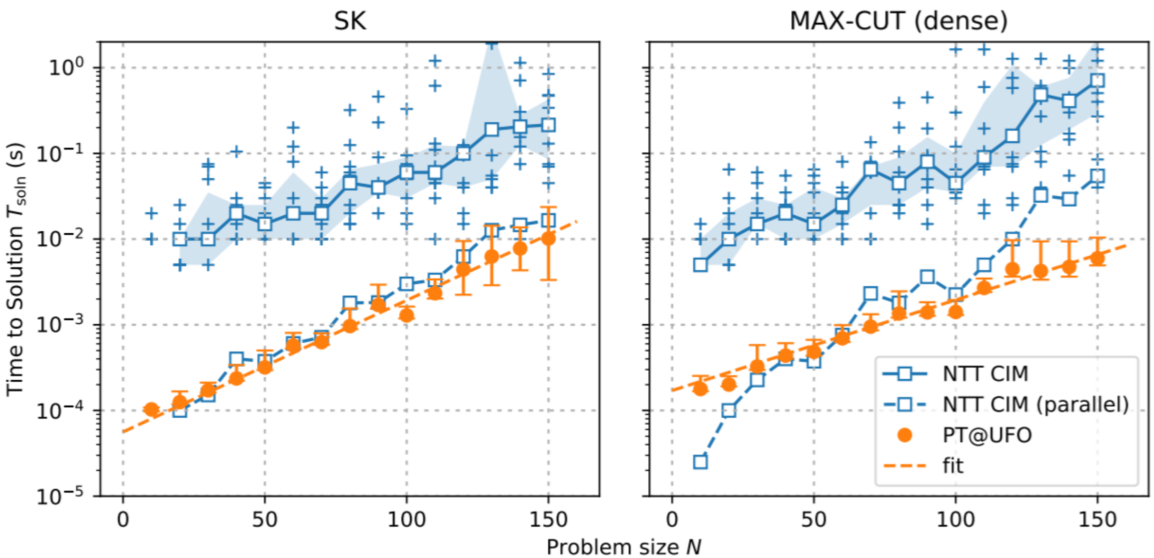





(Figure from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

*Problem class*: Density=50% MAX-CUT

# Comparison to Classical State-of-the-Art






(Table from R. Hamerly*, T. Inagaki*, P.L. McMahon*, et al. arXiv:1805.05217)

# Overview

- The Ising Problem
- Ising Machines
- Foundations of All-Optical OPO Ising Machines
- Measurement-Feedback OPO Ising Machines
- **Conclusions**

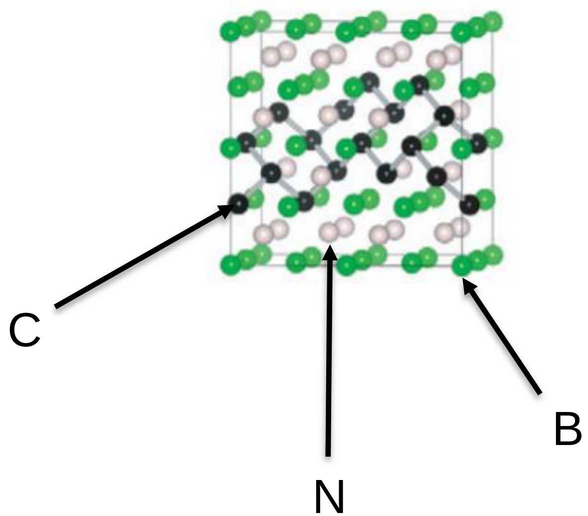
# Summary

- Networks of coupled OPOs provide an alternative platform for physically emulating Classical Ising Spin Hamiltonians.
- $N$ = 100 spin system with 10,000 spin-spin connections (all-to-all) has been implemented.
- System can find exact and approximate solutions for a large range of graphs.
- Time-division multiplexing and measurement-feedback provide the tools to allow scalable all-to-all connectivity in optical systems, and may have some relevance to AQC.

# Backup Slides

# Example Application: Cluster Expansion for Materials

Cubic Carbon Boron Nitride (c-BNC)



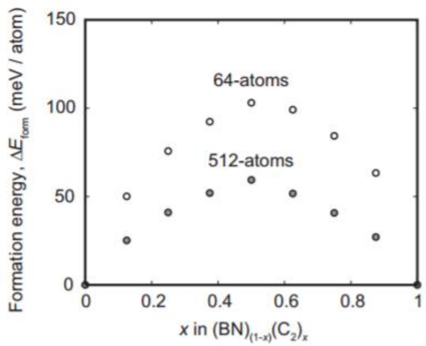




FIG. 4. Ground-state convex hull for $c$-BNC along the composition range of $(BN)_{(1-x)}(C_2)_x$ $(0 \leqslant x \leqslant 1)$. Open and closed circles denote the atomic arrangements in the lowest formation energy within the supercells consisting of 64 and 512 atoms, obtained from the MC simulation with simulated annealing algorithm.

Yuge, et al. *Phys. Rev. B* **77**, 094121 (2008)

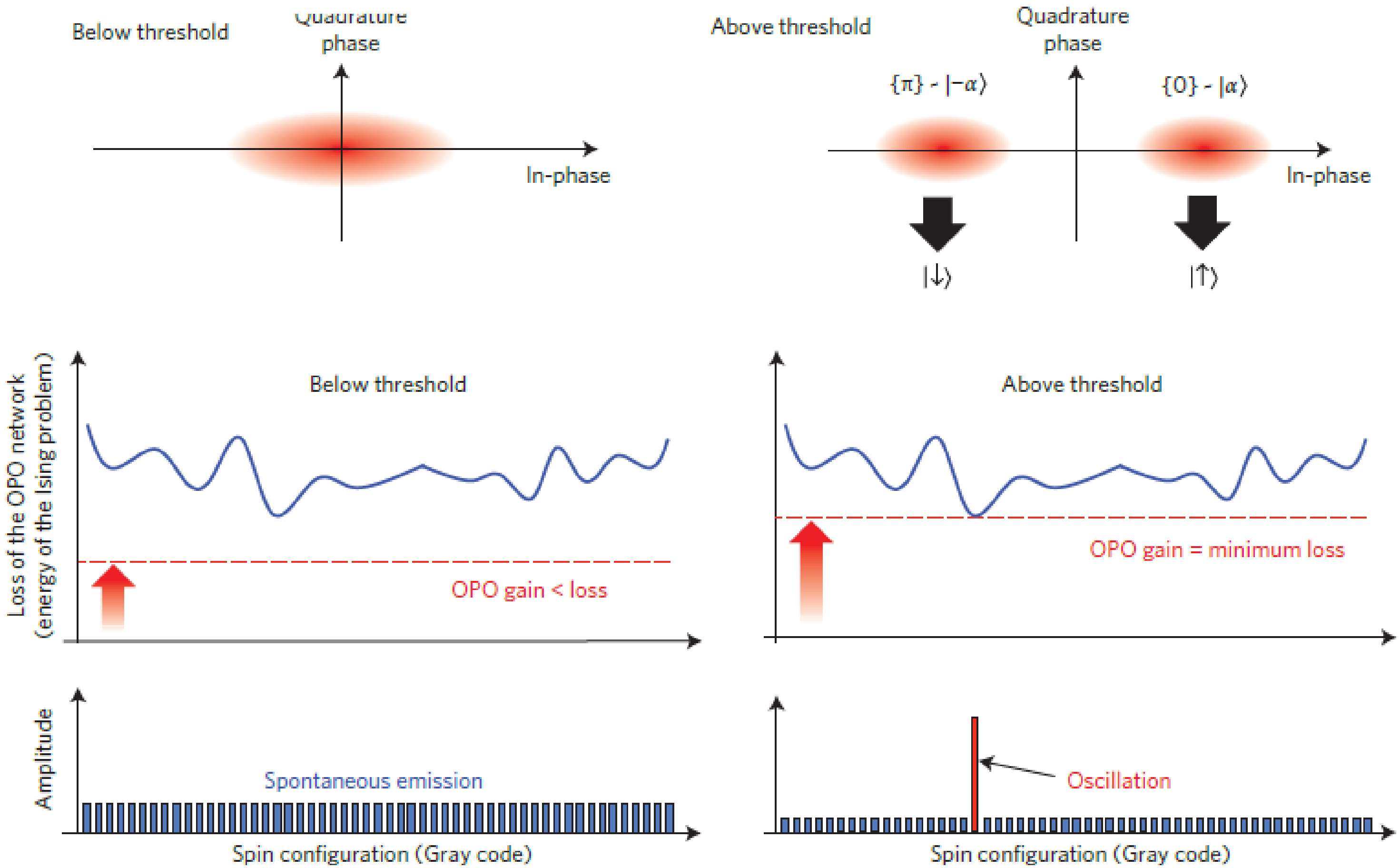
Ising problem with spins: $\sigma_i^{(P)} = \begin{cases} 1 & P \text{ atom at site } i \\ 0 & \text{no } P \text{ atom at site } i \end{cases}$

Collaboration with: Alan Aspuru-Guzik (Harvard) and Libor Veis (Czech Acad. Science)

# OPO Ising Machine Mechanism

$$H = -\sum_{ij}^{N} J_{ij}\sigma_i\sigma_j$$

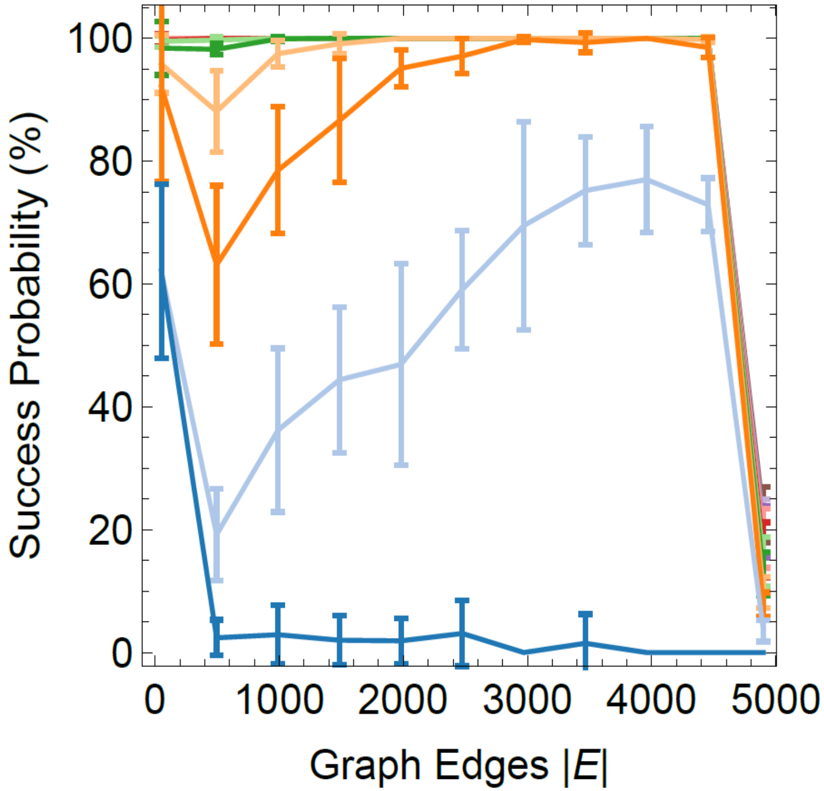
→ design interactions in a network of OPOs to realize this Hamiltonian



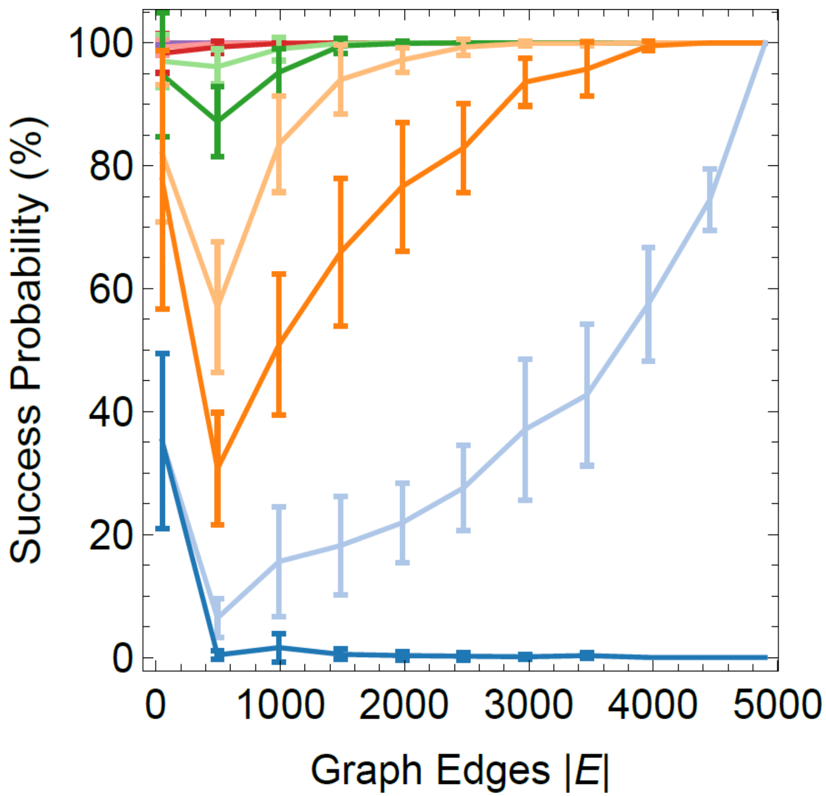



Figures adapted from: A. Marandi, *et al*. *Nature Photonics* 8, 937 (2014).

# Simulations: Density Sensitivity

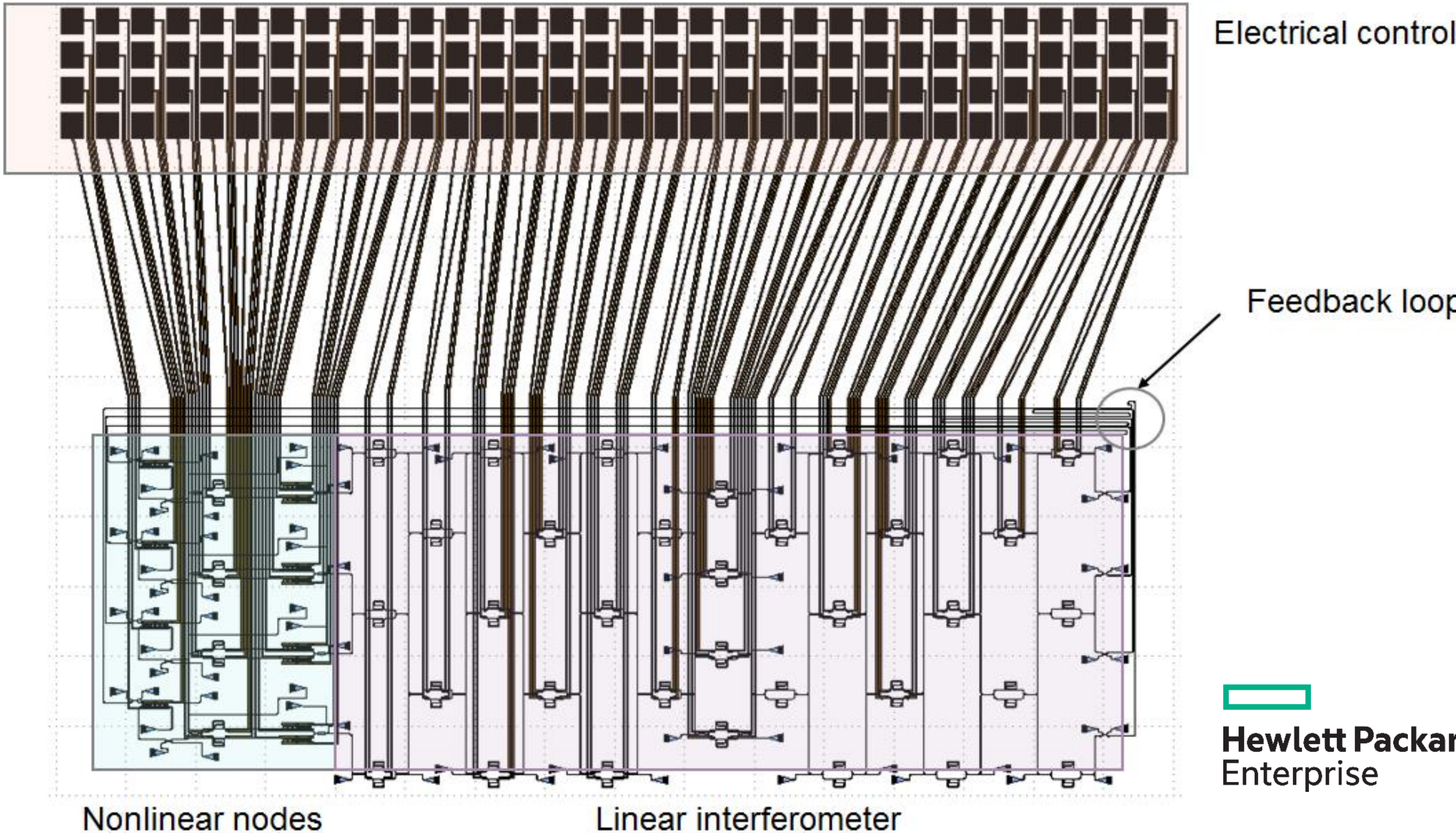



$p_{\mathrm{comp}} = 0.6\ p_{\mathrm{th}}$




$p_{\mathrm{comp}} = 1.3\ p_{\mathrm{th}}$

# Microring Resonator Network

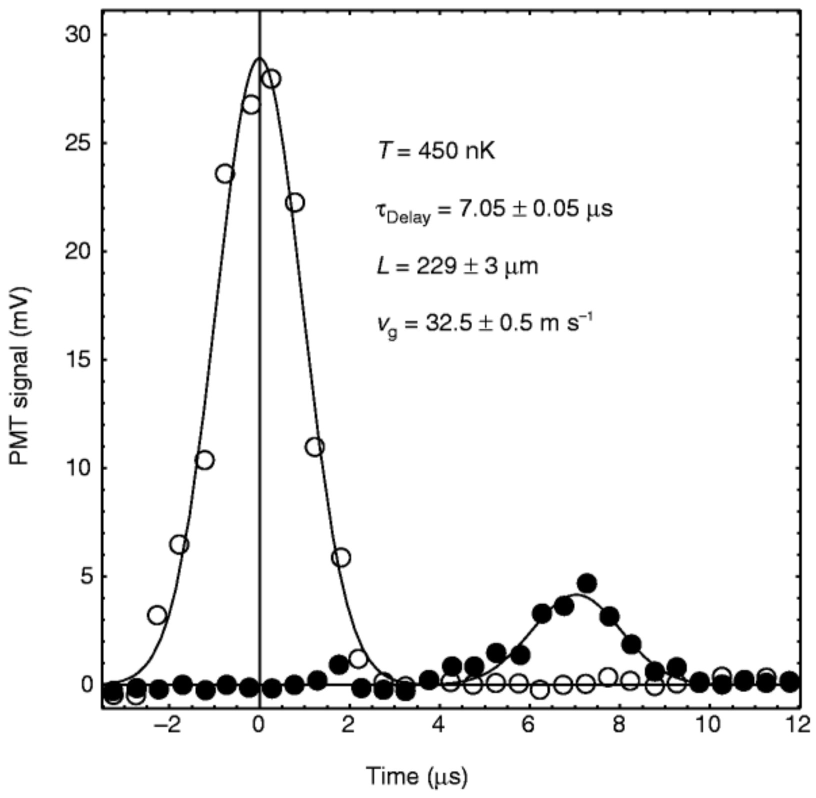



Hewlett Packard Enterprise

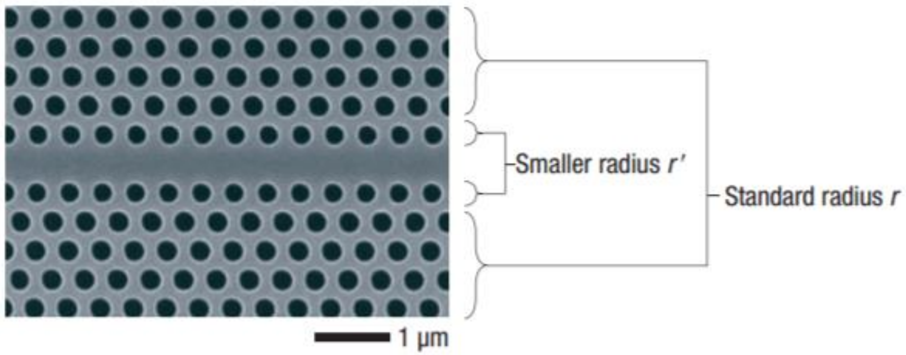


*Slide kindly provided by:* Thomas Van Vaerenbergh (HPE), on behalf of
T. Van Vaerenbergh, J. Pelc, D. Kielpinski, C. Santori, N. Tezak, G. Mendoza, R. Bose and R. Beausoleil

# On-Chip All-Optical Ising Machines using Slow Light




L. Hau, et al. *Nature* **397**, 594-598 (1999)




T. Baba. *Nature Photonics* **2**, 465 (2008)