Quantum Integer Programming

47-779

Quantum Annealing

Outline

Basic of quantum physics for computing

Superconducting Qubits

Adiabatic Quantum Computing

Quantum Annealing à la D-Wave

Colab: Solving IP via Quantum Annealing

New Prospects in Quantum Annealing

Amazon Braket for Quantum Annealing

Resources

- **Videos**: Login to https://riacs.usra.edu/quantum/login

Introduction to Quantum Computing, Quantum Annealing, NISQ Gate-Model Algorithms

- **Subscribe** to NISQ-QC Newsletter and browse recent work on annealing and optimization: https://riacs.usra.edu/quantum/nisqc-nl

- **D-Wave Leap Tutorials**

Important to understand language and problems
https://www.scottaaronson.com/blog/

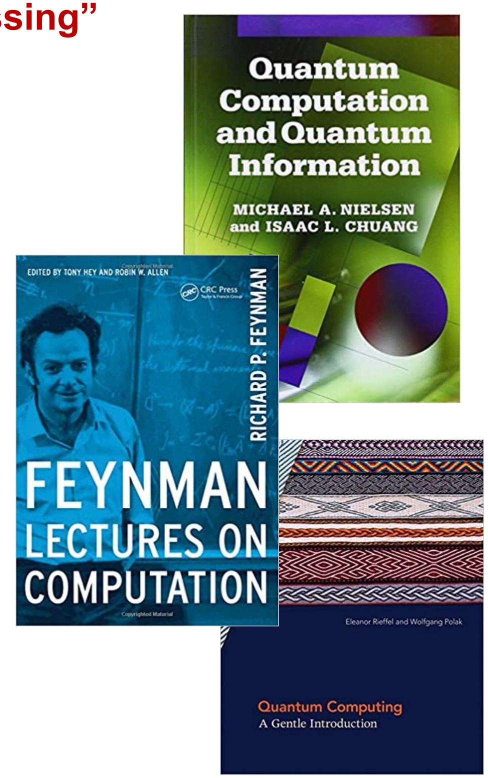
Crash Course in QM

Quantum Mechanics is the physics theory that describes and predicts the outcome of experiments with systems that are sufficiently small, cold and isolated

Quantum Computing uses quantum Mechanics as "information processing"
EXPERIMENT → COMPUTATION

Four concepts that are required to USE (not understand!) QM for QC:

- **QUANTUM STATE** (≡QUBITS, QUBIT REGISTERS, WAVEFUNCTION)
- **QUANTUM COHERENT OPERATIONS** (≡SHRÖDINGER or UNITARY EVOLUTION, GATES)
- **QUANTUM INCOHERENT OPERATIONS** (≡DISSIPATION, DEPHASING, DECOHERENCE, SEMI-CLASSICALITY, DENSITY MATRIX, NOISE)
- **MEASUREMENT** (≡COLLAPSE, PROJECTION, PROBABILITY AMPLITUDE, BORN RULE)



The Quantum State (QUBITs)

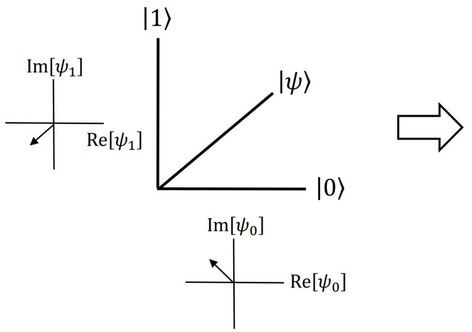
A state is a representation of a physical system through a collection of variables which fully describes its physics within the theory (e.g. in thermodynamics is P,V,T, …)

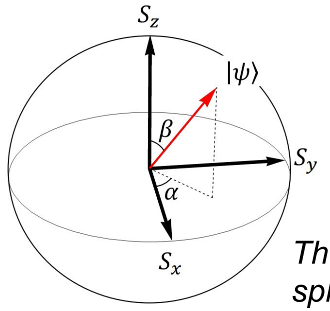
A QUBIT is the simplest quantum state you can think of: a two-level system

Bra-Ket notation for generic quantum states $|\psi\rangle$

The two states define the COMPUTATIONAL BASIS $|0\rangle$ and $|1\rangle$.
Think of them as the X and Y axis of a fixed length arrow.
The quantum state is fully specified by its components on the axes which are complex numbers

Notation for qubit states $|\psi\rangle_{\text{qubit}} = \psi_0|0\rangle + \psi_1|1\rangle$





The Bloch sphere

Insight: QM/QC is mostly linear algebra with complex numbers

The "Exponentiality" of QM

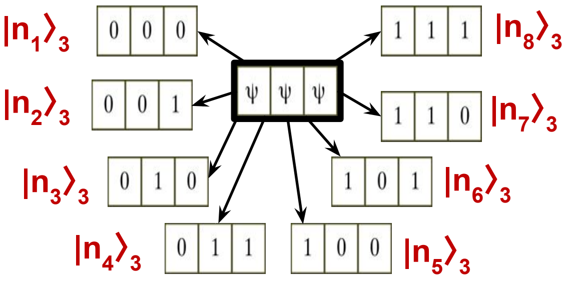
One qubit is fully described by its wavefunction, i.e. 2 complex numbers
N qubits are fully described by their global wavefunction,
a vector with 2^N complex numbers
(probability amplitudes)

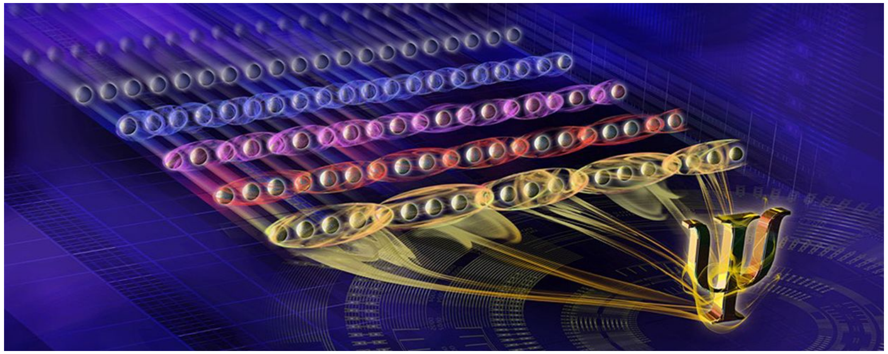
$$|\psi\rangle_{qubit} = \psi_0|0\rangle + \psi_1|1\rangle$$

$$|\psi\rangle_{2qubits} = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$$
$$\neq (\psi_0|0\rangle + \psi_1|1\rangle) \otimes (\phi_0|0\rangle + \phi_1|1\rangle)$$

[if it is equal then it is not <u>entangled</u>/separable]

$$|\psi\rangle_{Nqubits} = \sum_{n=1}^{2^N} \psi_n|n\rangle_n \text{ (binary representation)}$$





Source IEEE

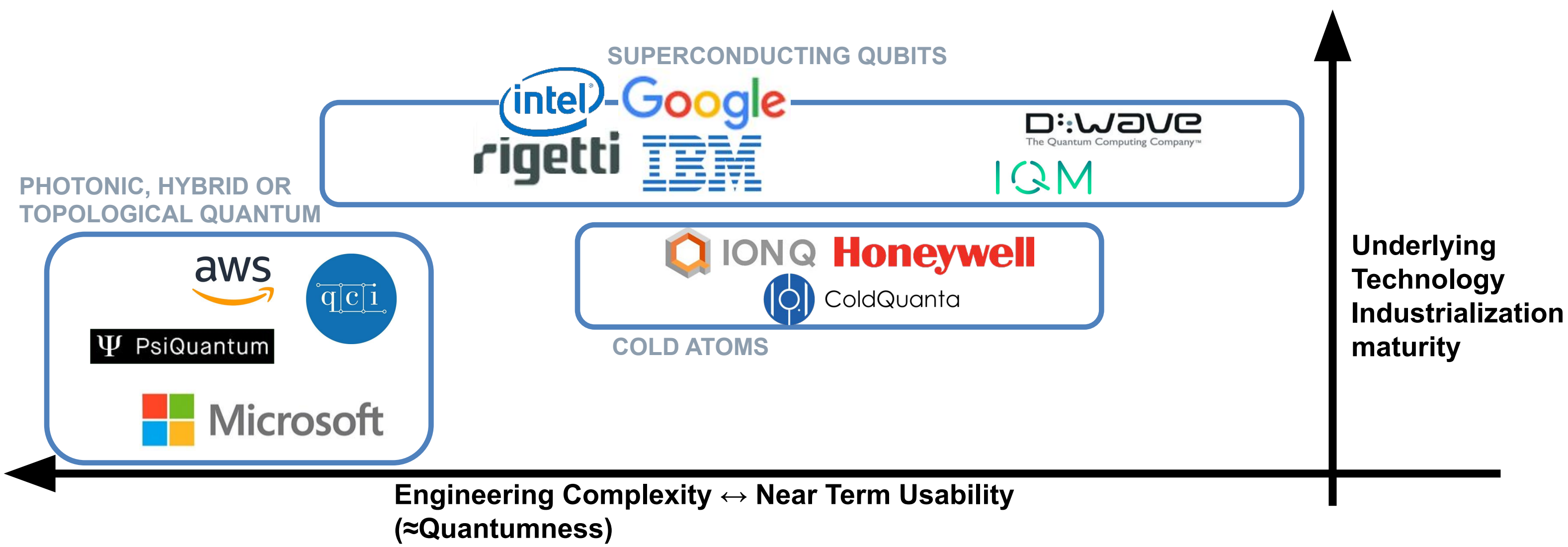
$2^{265} \approx$ estimated number of atoms in the Universe.

There is no way we can store the general wavefunction of even a processor with more than 60 qubits on earth, let alone process it!

Qubits Tech Landscape 10/2020

Qubits have been fabricated with:

- **Single atoms (ions or neutral) trapped and manipulated by lasers**
- **Single photons or photon wavepackets in interferometers or in cavities (modes)**
- **Single electrons trapped in silicon heterostructures (spin qubits)**
- **Magnetic/electric moments of molecules or impurities in materials (diamond, NMR..)**
- **Superconducting circuits**



Disclaimer: *only commercially launched/tech disclosed ... Many more!*

Superconducting Qubits

A Quantum Engineer's Guide to Superconducting Qubits
P. Krantz[1,2,†], M. Kjaergaard[1], F. Yan[1], T.P. Orlando[1], S. Gustavsson[1], and W. D. Oliver[1,3,‡]

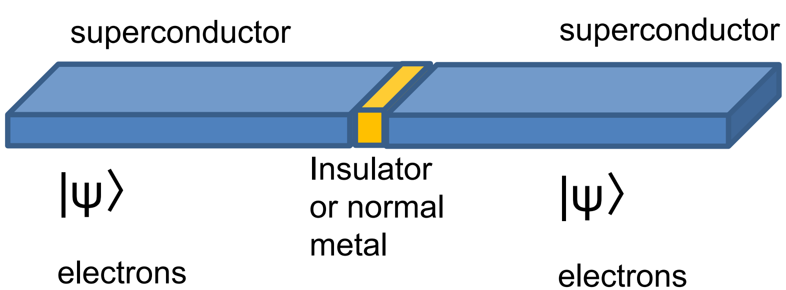
https://arxiv.org/pdf/1904.06560.pdf

Tutorial: Gate-based superconducting quantum computing
Sangil Kwon,[1,a)] Akiyoshi Tomonaga,[1,2] Gopika Lakshmi Bhai,[1,2] Simon J. Devitt,[3] and Jaw-Shen Tsai[1,2]

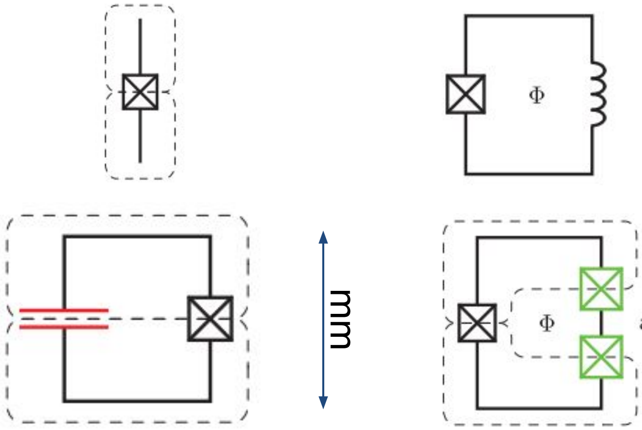
https://arxiv.org/pdf/2009.08021.pdf

Some metals at low temperature becomes superconductors.
Superconductors = electrons becomes correlated/entangled and are described with a single wavefunction «they behave as one», which leads to zero resistance.

If two superconductors are separated by a thin barrier, their wavefunction communicates and creates a tunneling current with non-linear properties **(Josephson Effect; Josephson Junctions – Phys. Lett. 1. 251 - 1962)**



LEADING QUBITS DESIGN



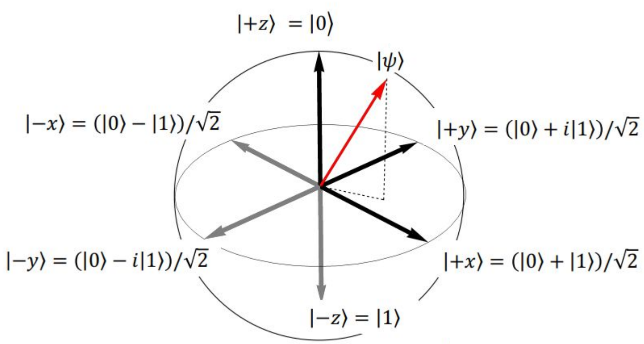
Transmons (e.g. Google, Intel, IBM, Rigetti)

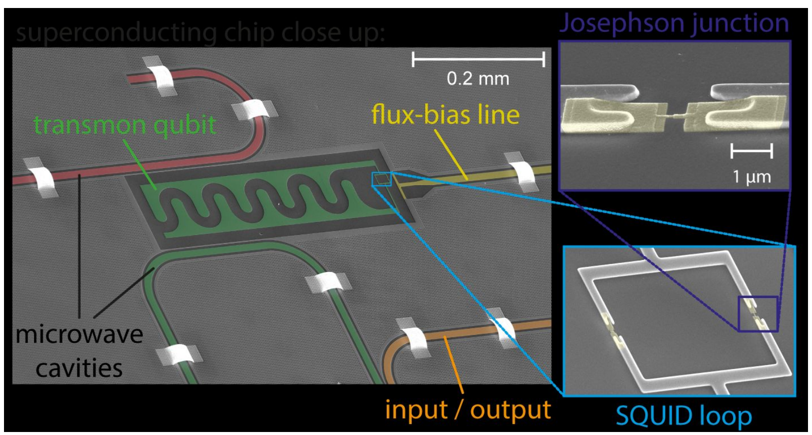
Flux Qubits (e.g. D-Wave)

$$|\psi\rangle_{qubit} = \psi_0|0\rangle + \psi_1|1\rangle$$

Coherent Operations (Gates)

Single Qubit Control





In quantum physics the way a system change state is through the Schrödinger equation.

$$|\psi^I(\Delta t)\rangle = T \exp\left(-\frac{i}{\hbar}\int_0^{\Delta t} \widehat{H}_c^I(t)\, dt'\right)|\psi^I(0)\rangle$$

In QC this means we can create a matrix to transform the state

$$\boxed{|\psi_a^I\rangle = \widehat{U}_c\,|\psi_b^I\rangle \qquad \underline{\mathbb{U} \text{ is a unitary matrix, } \mathbb{U}^\dagger\,\mathbb{U} = 1}}$$

Most important single-qubit unitaries: pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Example: single qubit rotation around the X axis
«transverse field»

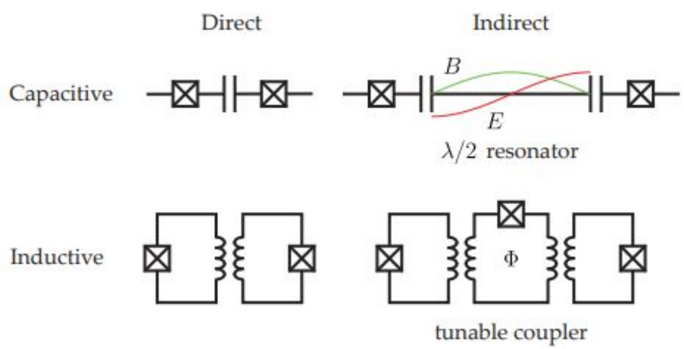
$$R_x(\theta) \equiv e^{-i\frac{\theta}{2}X} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ Exercise $R_x(\pi/2)|0\rangle = ?$

An arbitrary change can be decomposed in maximum 3 axis rotations (Euler)

Two-qubit gates

Many ways to couple multiple superconducting qubits on the chip



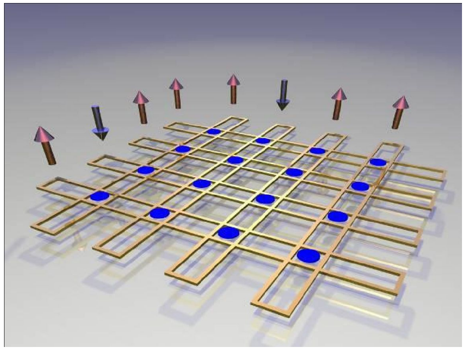
$$|\psi\rangle_{2qubits} = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$$

$$|00\rangle = \begin{bmatrix}1\\0\\0\\0\end{bmatrix},\ |01\rangle = \begin{bmatrix}0\\1\\0\\0\end{bmatrix},\ |10\rangle = \begin{bmatrix}0\\0\\1\\0\end{bmatrix},\ \text{and } |11\rangle = \begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

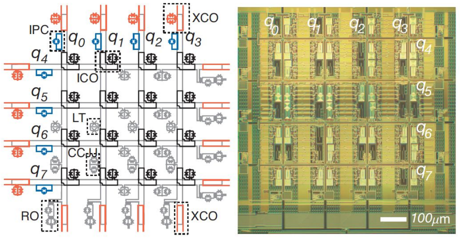
$$X = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix},\quad Y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix},\quad Z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

QUBO to Ising transformation
$|0\rangle \rightarrow s=1$ and $|1\rangle \rightarrow s=-1$
$Z|0\rangle = |0\rangle$ and $Z|1\rangle = -|1\rangle$

Example: D-Wave Flux Qubits



Johnson et al.
Supercond. Sci. Technol. 23 (2010)



Implements the Exp($i\theta Z\otimes Z$) unitary interaction

$$\begin{pmatrix} e^{i\theta} & 0 & 0 & 0\\ 0 & e^{-i\theta} & 0 & 0\\ 0 & 0 & e^{-i\theta} & 0\\ 0 & 0 & 0 & e^{i\theta}\end{pmatrix}$$

Diagonal, does not change the state of qubits in computational basis

Ising $Z\otimes Z|s_1 s_2\rangle = s_1 s_2\,|s_1 s_2\rangle$

Measurement

For the purpose of quantum computing, the measurement operation is well defined in the processor, and its effect is taken as a postulate (*the Born rule*):

$$|\psi\rangle_{\text{qubit}} = \psi_0|0\rangle + \psi_1|1\rangle$$

Measurement

Probability to measure 0 $= |\psi_0|^2$

Probability to measure 1 $= |\psi_1|^2$

$|\psi_0|^2 + |\psi_1|^2 = 1$

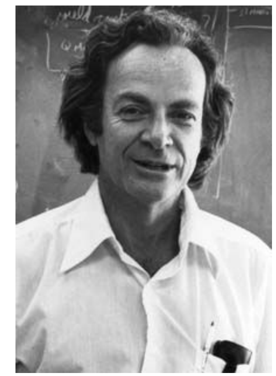
Final state is $|\psi\rangle_{\text{qubit}} = |0\rangle$

Final state is $|\psi\rangle_{\text{qubit}} = |1\rangle$

$|\psi\rangle_{\text{2qubits}} = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$

Probability of measuring 11 is $|\psi_{11}|^2$

From coherent superposition to collapse



If you think you understand quantum mechanics, you don't understand quantum mechanics.

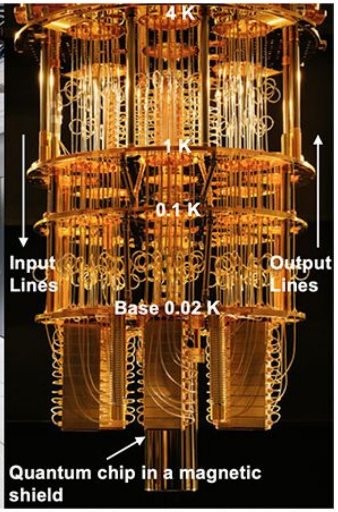
Incoherent Operations (Noise)

Final State after your algorithm if everything was coherent

$|\psi\rangle_{\mathrm{Nqubits}} = \sum_{n=1}^{2^N} \psi_n |\mathrm{solution}(n)\rangle_n$

A random guess algorithm has $|\psi_{n=\mathrm{target}}|^2 \approx 1/\sqrt{2}^N$
A good algorithm has $|\psi_{n=\mathrm{target}}|^2 \gg 1/\sqrt{2}^N$

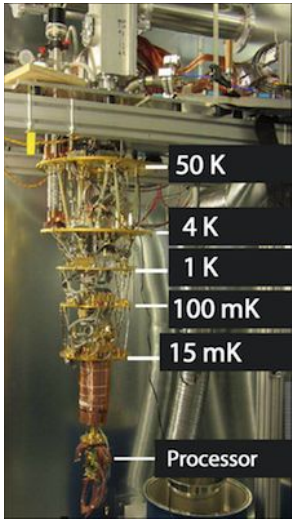
Dilution fridge setup: outside view



Dilution fridge setup: inside view

Source from IBM and D-Wave

BUT....
We are currently using
Noisy-Intermediate-Scale QPUs (NISQ)



$$|\psi^I(\Delta t)\rangle = T\exp\left(-\frac{i}{\hbar}\cancel{\int_0^{\Delta t} \hat{H}_c(t')\,dt'}\right)|\psi^I(0)\rangle$$

The Shrödinger equation applies only approximately and for a limited time

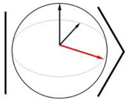
There is interaction with the environment which is always on and it is causing changes to the operations:
→ some measurements happen at random due to the environment and destroy «partially» the wavefunction (decoherence)
→ Some control parameters are imprecise and $\psi_n \approx \psi_n + \delta$
→ Theory of modeling of noise is very phenomenological

A Quantum Optimization Algorithm

(1) Map a QUBO Objective function into Ising form and assign the logical identity of each spin variable to a qubit in the processor.

$x_i = (s_i+1)/2 \rightarrow |x_i\rangle$

(2) Apply single-qubit rotations to every qubit to put the state of the QPU in superposition of all possible solutions of the optimization problem (H gates).



$$|\psi\rangle_{Nqubits} = \frac{1}{\sqrt{2^N}} \sum_{n=1}^{2^N} |solution(n)\rangle_n$$

(3) Apply two level gates and single qubits rotations to change the state, having some smart idea on how to increase the value of $|\psi_{n=target}|^2$ (algorithms are difficult to design because you are doing matrix multiplication with matrices of dimensions $2^N x 2^N$ – nature does it for you you don't need to do it but good luck simulating it)

(4) Measure the state, read the qubits (they are a single bitstring after measurement) and hope to find the target(s).

(5) Repeat the procedure a large number of time and keep the best result.

The Quantum Adiabatic Algorithm (quantum annealing)

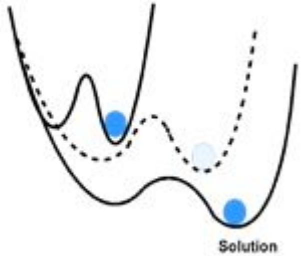
AQC is based on a property of the time-dependent Schrödinger equation – the «adiabatic theorem».

Einstein's "Adiabaten hypothese": "If a system be affected in a reversible adiabatic way, allowed motions are transformed into allowed motions" (Einstein, 1914).

(1) Switch on a quantum interaction in your system
(2) Take the spectrum of possible energies of your quantum system as a function of the degrees of freedom and set the state to a well defined energy (not metastable states) which is ranked n^{th} in order of magnitude (e.g. the second smallest)
(3) Do any Schrödinger evolution (no measurement! no noise!) that changes the energy states «sufficiently slow».
(4) Measure the energy of the state. You will find with 100% probability that the energy is ranked also n^{th}

Albash, Lidar
Rev. Mod. Phys. 90, 015002 (2018)
https://arxiv.org/abs/1611.04471

- Apolloni 1989
- Finnila 1994
- Nishimori 1998
- Brooke 1999
- Fahri 2001



Adiabatic evolution

Adiabatic evolution (e.g. Slow Schrödinger) preserves the energy ranking of your system.

The smallest energy state (ground state) also maps into the ground state at the end.

IDEA: map objective function into energy. Start from easy problem to solve with known solution and modify slowly to difficult. Measure unknown solution

The Quantum Adiabatic Algorithm for Ising Machines

(1) map objective function into energy of a quantum Ising system

$$H_P = \sum_{ij} J_{ij} Z_i \otimes Z_j + \sum_i h_i Z_i$$

$H|s_1 s_2 \ldots s_N\rangle = E_N | s_1 s_2 \ldots s_N\rangle$

$\exp(iH)\, |s_1 s_2 \ldots s_N\rangle = e^{iE_N} |s_1 s_2 \ldots s_N\rangle$

(2) Start from easy problem to solve with known solution

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$H_D = \Gamma \sum_i X_i$$ (transverse field)

$R_x(\pi/2)|0\rangle = |1\rangle$
$R_x(\pi/2)|1\rangle = |0\rangle$

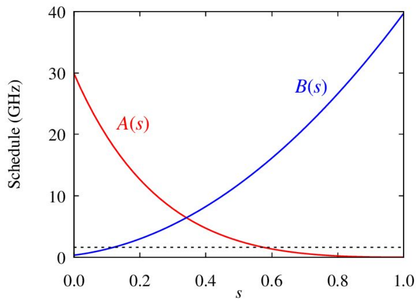
$$|\psi\rangle_{Nqubits} = \frac{1}{\sqrt{2^N}} \sum_{n=1}^{2^N} |solution(n)\rangle$$

If this field is always on and constant the minimum energy state is the **all-superposed state**

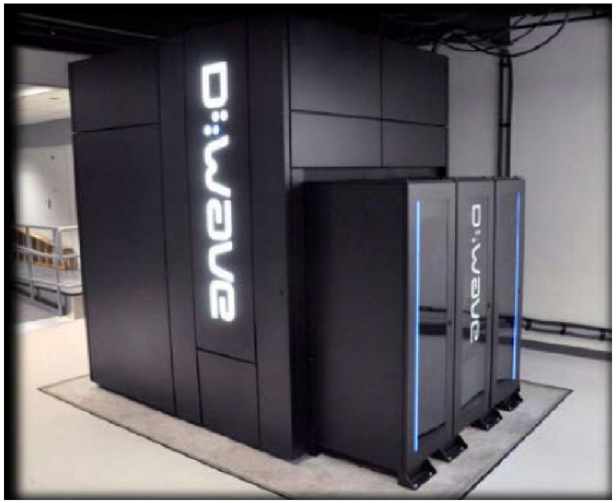
(3) Do any Schrödinger evolution (no measurement! no noise!) that changes the energy states «sufficiently slow».

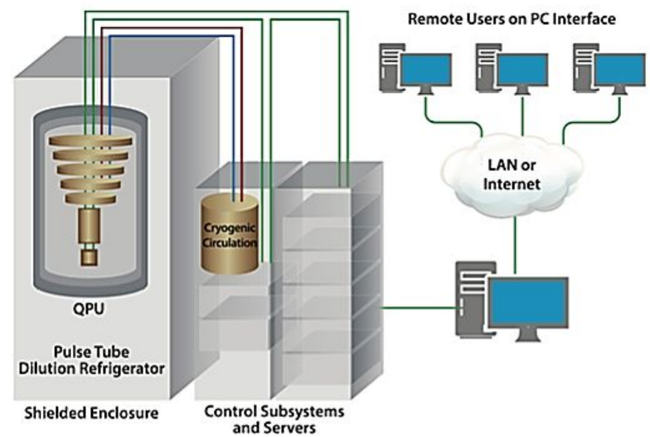
How slow? It depends on the problem, on H_D and on the <u>Annealing Schedule</u>
No way to predict efficiently. Try!

$H = A(t)\, H_D + B(t)\, H_P$



Quantum annealing à la D-Wave

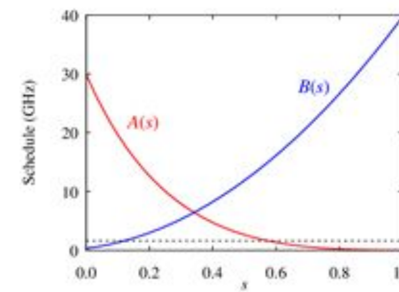


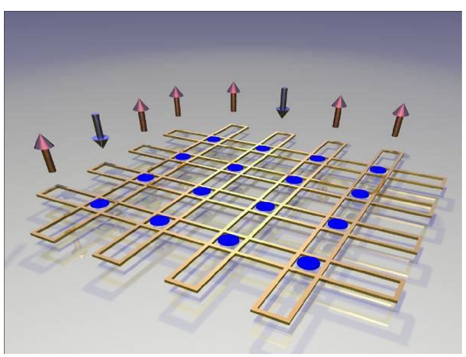


J_{ij} and h_i have maximum value and fluctuating intrinsic control errors:

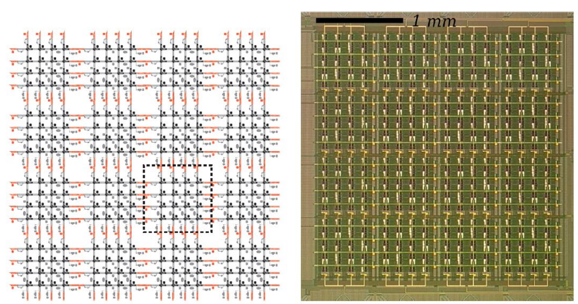
$$J_{ij} + \delta J$$

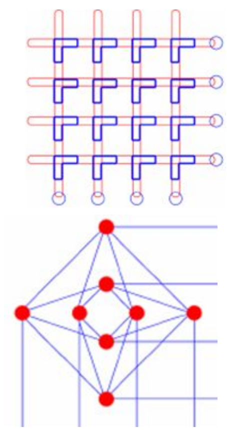
$$h_i + \delta h$$

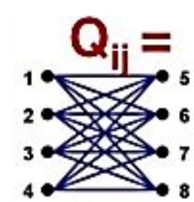




A 128-qubit chip composed of a 4 × 4 array of eight-qubit unit cells.



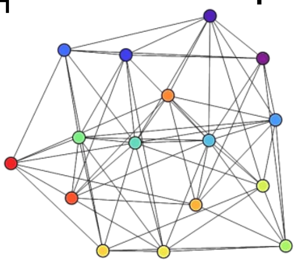




Minor Embedding

Topological Embedding

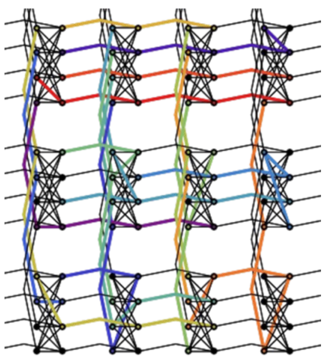
(n_H hardware qubits)



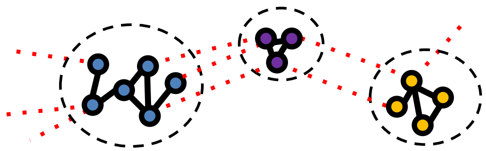
$$\mathcal{E}(i) : \{1, \ldots, n_L\} \to 2^{\{1,\ldots,n_P\}}$$

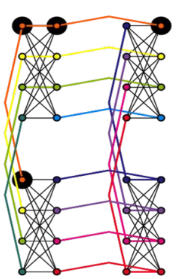
Assign “colors” to connected sets of qubits

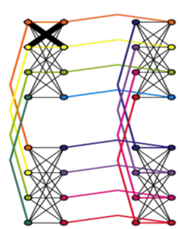
(n_P logical bits)

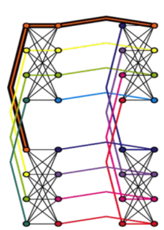


Parameter Setting







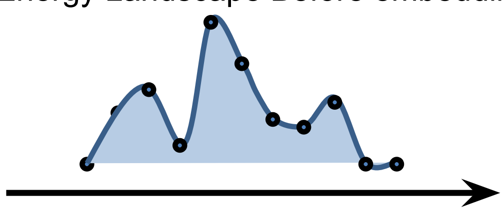


$$\sum_{j \in \mathcal{E}(i)} h'_j = h_i$$

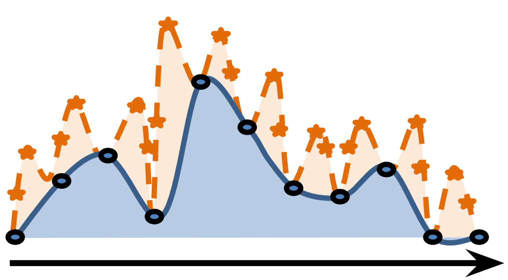
$$\sum_{j_1 \in \mathcal{E}(i_1)} \sum_{j_2 \in \mathcal{E}(i_2)} J'_{j_1 j_2} = J_{i_1 i_2}$$

$$J'_{j_1, j_2} < |h_i| - \sum_{k=1}^{n} |J_{ik}|$$

Energy Landscape Before embedding

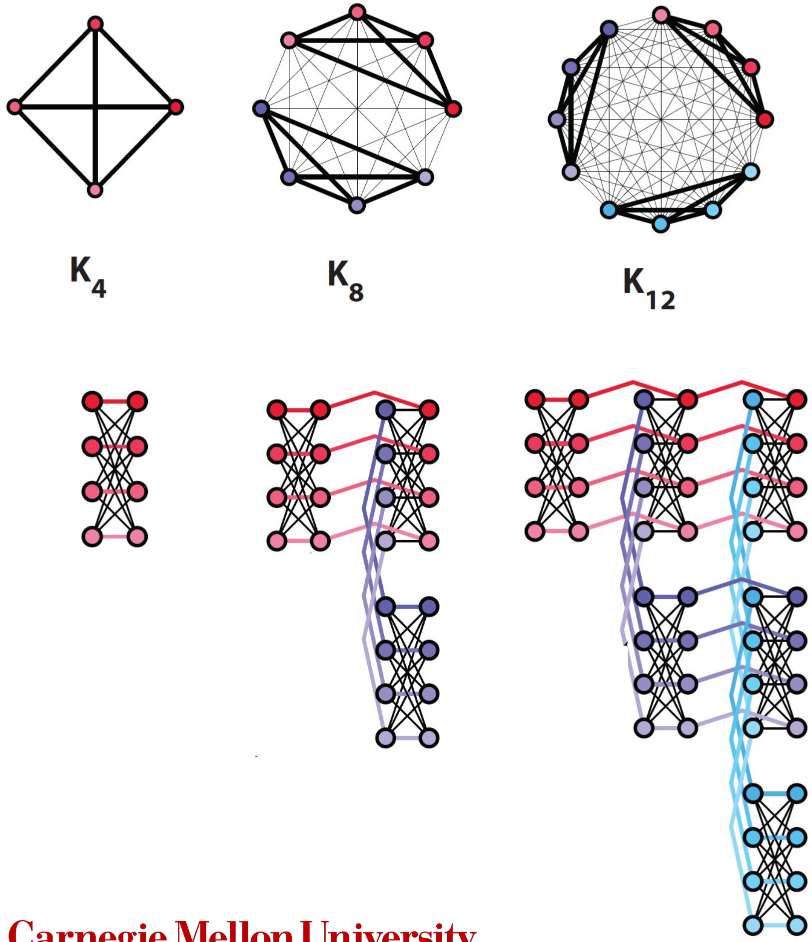


Energy Landscape After embedding

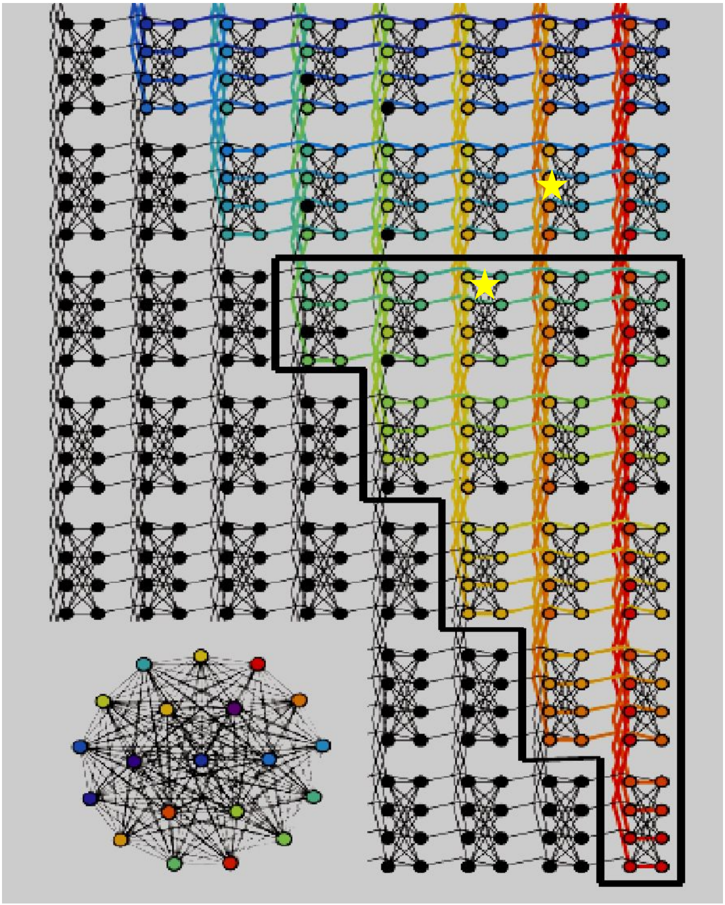


Minor Embedding of a fully connected graph

Systematic Rule for Embedding



Quadratic overhead

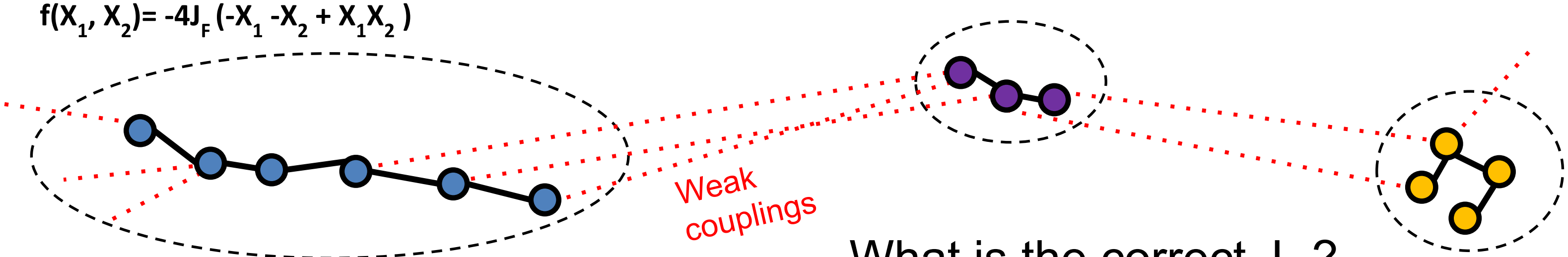


Unembedding

Ferromagnetic Coupling

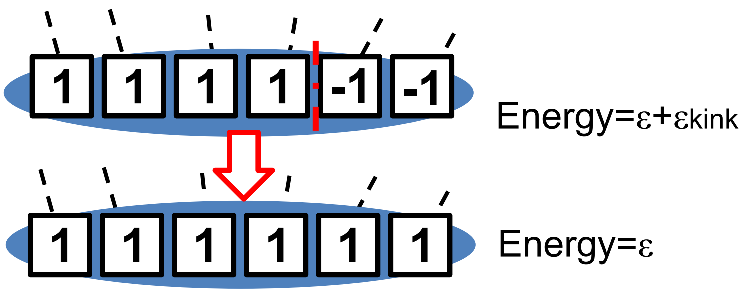
$f(S_1, S_2) = -J_F s_1 s_2$

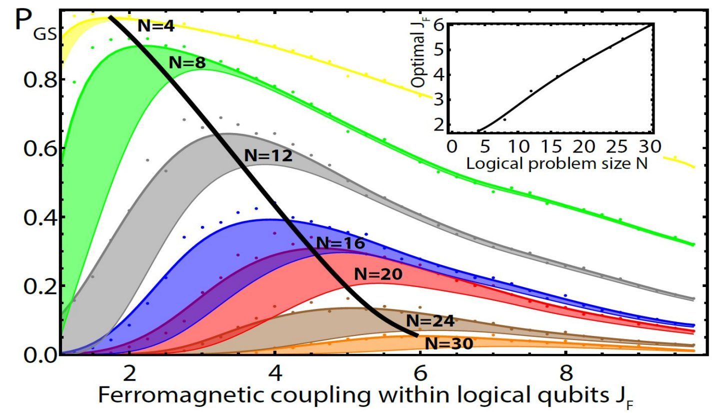
$f(X_1, X_2) = -4J_F(-X_1 - X_2 + X_1X_2)$



What is the correct J_F ?

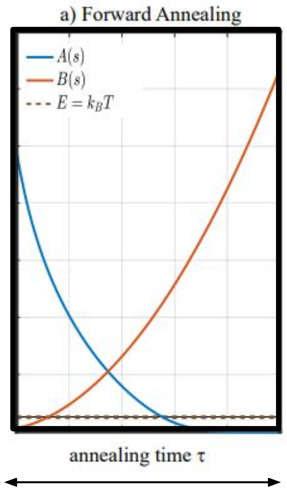
Majority Voting



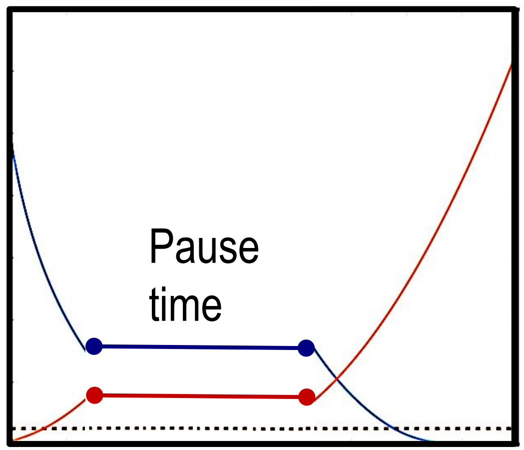


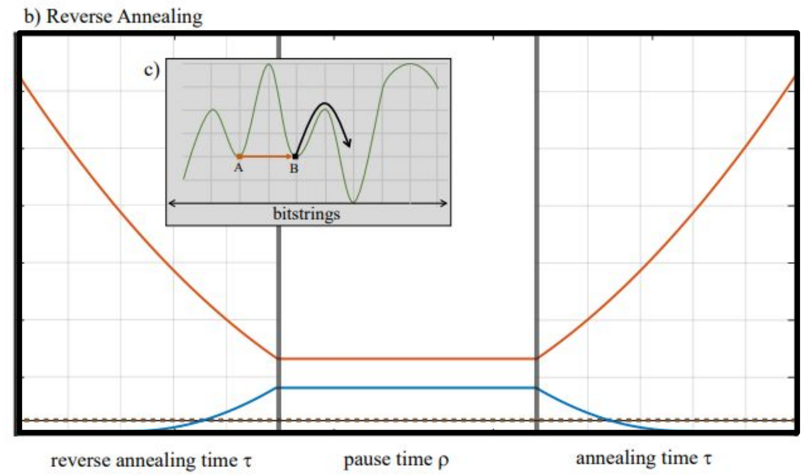
Not too large, not too small. Trial and error.
(See Venturelli et al. https://journals.aps.org/prx/abstract/10.1103/PhysRevX.5.031040)

Annealing Schedule Parameters



Time for annealing (if AQC controls performance)





First pause time

reversal time

These are all parameters that influence performance.

Additional parameters:
gauge, anneal offset, J_F

Only for elegant problems they can be derived ab-initio. In the real world you have some physics guidance for best guess then you use a heuristics to find them

Annealing Schedule Parameters

Power of Pausing: Advancing Understanding of Thermalization in Experimental Quantum Annealers

Jeffrey Marshall,[1,2,3] Davide Venturelli,[1,4] Itay Hen,[3,5] and Eleanor G. Rieffel[4]

Why and when is pausing beneficial in quantum annealing?

Huo Chen[1,2] and Daniel A. Lidar[1,2,3,4]

Ferromagnetically shifting the power of pausing

Zoe Gonzalez Izquierdo,[1,2,3] Shon Grabbe,[2] Stuart Hadfield,[2,3] Jeffrey Marshall,[2,3] Zhihui Wang,[2,3] and Eleanor Rieffel[2]

Comparing relaxation mechanisms in quantum and classical transverse-field annealing

Tameem Albash[1,2] and Jeffrey Marshall[3,4,*]

Search range in experimental quantum annealing

Nicholas Chancellor and Viv Kendon
Department of Physics; Joint Quantum Centre (JQC) Durham-Newcastle
Durham University, South Road, Durham, DH1 3LE, UK
(Dated: August 26, 2020)

Reverse Quantum Annealing Approach to Portfolio Optimization Problems

Davide Venturelli[1] Alexei Kondratyev[2]

[1] *USRA Research Institute for Advanced Computer Science, Mountain View, CA 94035, USA*
[2] *Data Analytics Group, Financial Markets, Standard Chartered Bank, London EC2V 5DD, UK*

Towards Hybrid Classical-Quantum Computation Structures in Wirelessly-Networked Systems

Minsung Kim[1,2,3], Davide Venturelli[2,3], and Kyle Jamieson[1]
[1] *Princeton University*
[2] *USRA Research Institute for Advanced Computer Science*
[3] *NASA Ames Research Center, Quantum Artificial Intelligence Laboratory*

Relevant literature to understand performance on pause parameters.

Solving IP via Quantum Annealing

Let’s solve our classical problem using Quantum Annealing

https://colab.research.google.com/github/bernalde/QuIP/blob/master/notebooks/Notebook%207%20-%20DWave.ipynb

New Prospects in Quantum Annealing

THEORY for coherent QA

Prospects for Quantum Enhancement with Diabatic Quantum Annealing

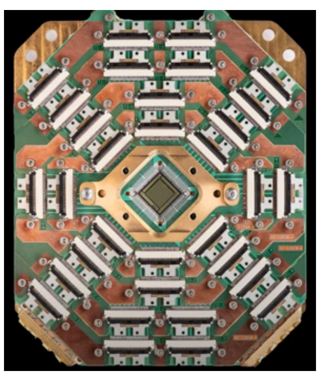
E.J. Crosson
Center for Quantum Information and Control (CQuIC), Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131, USA

D.A. Lidar
Departments of Electrical & Computer Engineering, Chemistry, Physics & Astronomy
Center for Quantum Information Science & Technology (CQIST)
University of Southern California, Los Angeles, California 90089, USA

https://arxiv.org/pdf/2008.09913.pdf

	Forward Adiabatic	Forward Diabatic	Reverse Adiabatic	Reverse Diabatic
Coherent or $\mathcal{C}$-coherent (unitary, closed system dynamics in a low-energy subspace $\mathcal{C}$); enabled by quantum error suppression	State remains close to instantaneous ground state. Provable oracular speedups, but otherwise generally efficiently simulable by QMC. **Questionable promise.**	Enables universality. No known efficient classical simulation methods. Numerical examples of speedup over adiabatic-coherent case, including in a quantum walks framework. **Promising.**	Enables a superpolynomial quantum-classical separation in an oracular setting. **Promising.**	Enables an exponential quantum speedup in an oracular setting. **Promising.**
Weakly-Decoherent (open system in the weak-coupling limit sense: decoherence in the energy eigenbasis)	State remains close to instantaneous Gibbs state. No known examples of speedups. Also generally efficiently simulable by QMC. **Questionable promise.**	Correlates well with QMC and SVMC in experimental studies. **Questionable promise.**	Examples known of exponential improvement over forward adiabatic, but remains generally efficiently simulable by QMC. **Questionable promise.**	Simulating full open system dynamics is classically intractable. Enables an exponential quantum speedup in an oracular setting at sufficiently low temperatures. **Promising.**

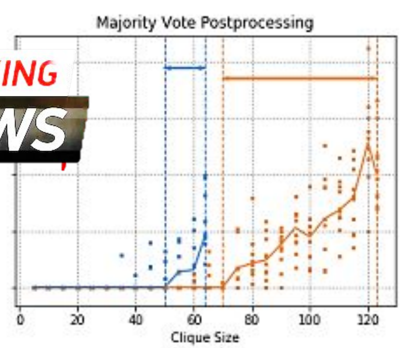
D-WAVE News



https://youtu.be/jMY2Pnq4pgM

https://youtu.be/7Y8y5a54v-E

https://www.dwavesys.com/sites/default/files/14-1049A-A_The_D-Wave_Advantage_System_An_Overview_0.pdf



Tuned Chain Strength						
n :	10	20	30	40	50	60
2000Q	4	6	7.3	8.4	12	10
Advantage	4	6	6	8	8	10