ISING MACHINES

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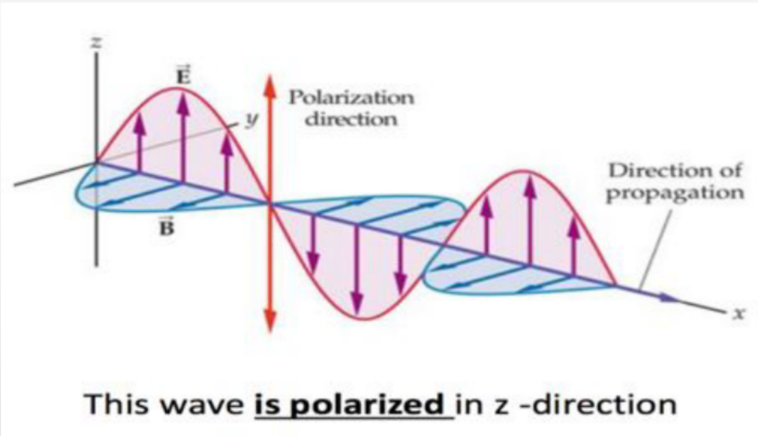
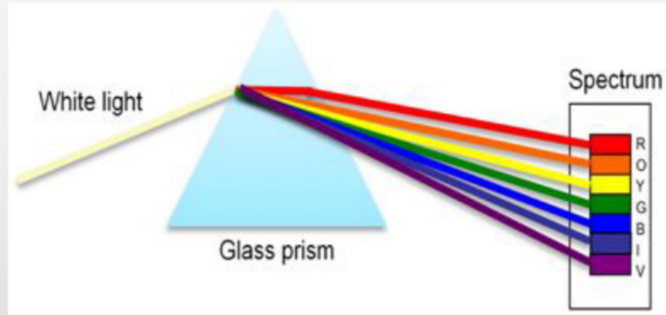
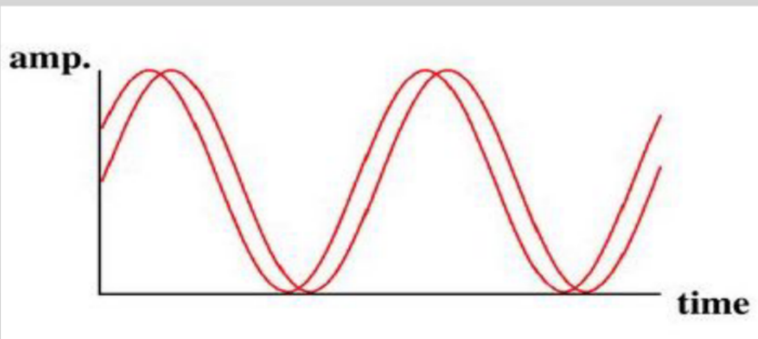
Outline

- Some properties of light
  - Superposition, Entanglement
  - Wave – Particle Duality
  - Classical versus Quantum Experiments
- Photonic Ising Machines
  - Multiplexed in Time – Poor Man's Ising Machine
  - Multiplexed in Space – Spatial Light Modulation
- Chip-based Ising Computing Machines
  - Electronic oscillators
  - Magnetic oscillators

Properties of Light







- Polarization
- Color
  - Wavelength, Frequency
- Phase
- Spatial Modes

How does Quantum Computing work?

- Classical bits versus quantum qubits

A	B
0	0
0	1
1	0
1	1

2 bits of information
State of A and of B

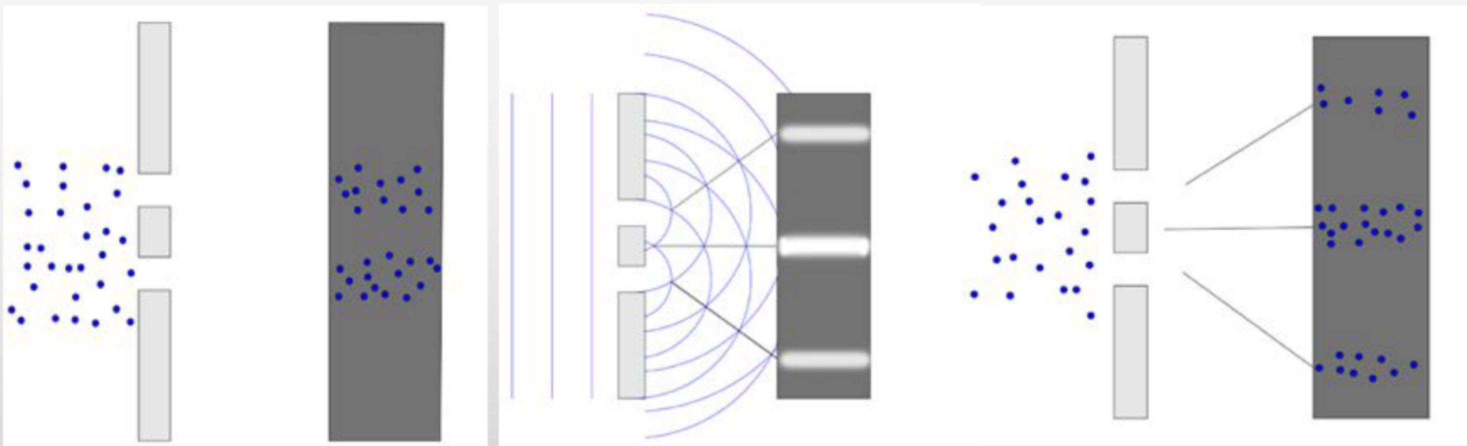
A	B	probability
0	0	α
0	1	β
1	0	γ
1	1	δ

4 bits of information
Superposition state

Waves interfere, we use them for computation

- Extend this to 3 qubits – 8 bits of information
- n-qubits will have 2^n bits of information

How does Photonic Computing work?

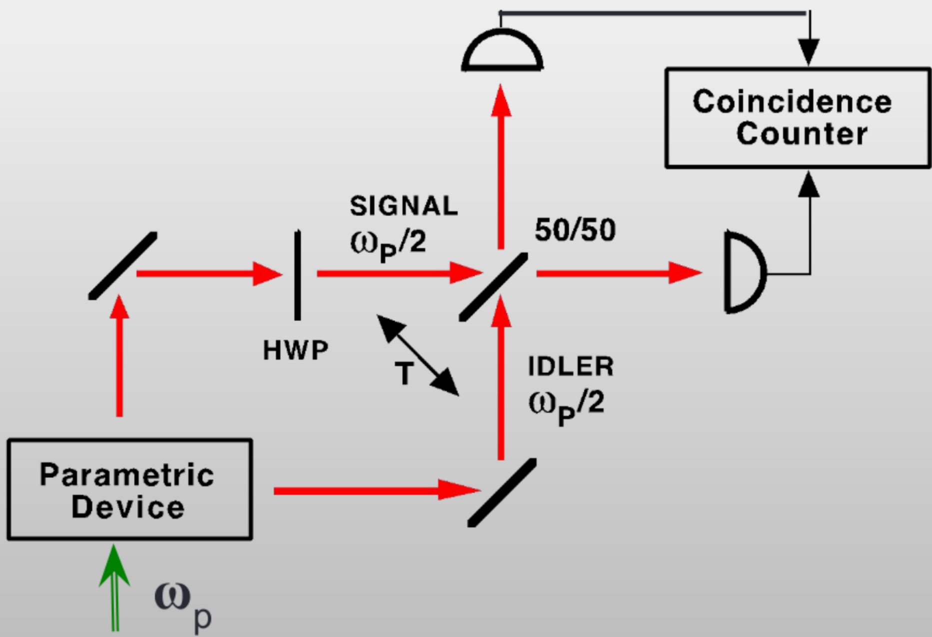


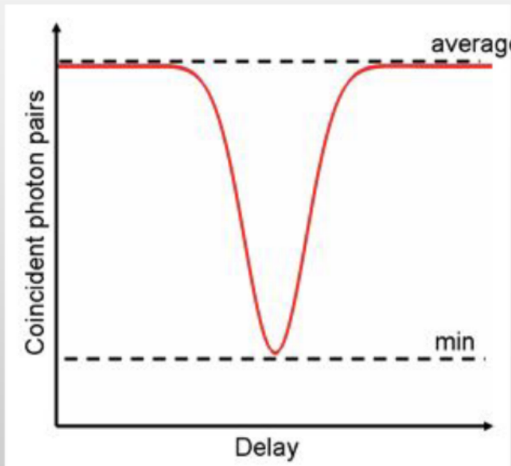
(a) Particles (b) Waves (c) Particles as Waves

- Double slit experiment – wave particle duality
- Analogy – massively parallel computation

Testing a Single Photon Source

- Hong-Ou-Mandel Interferometer

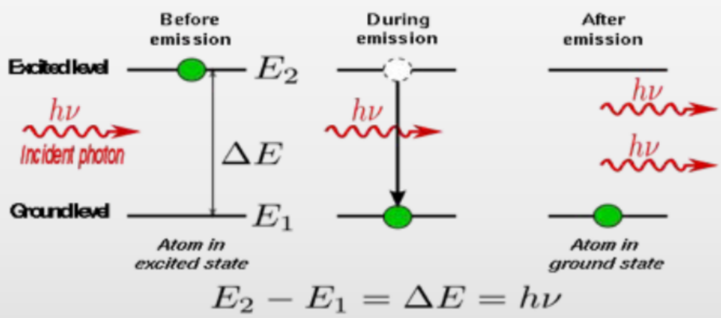


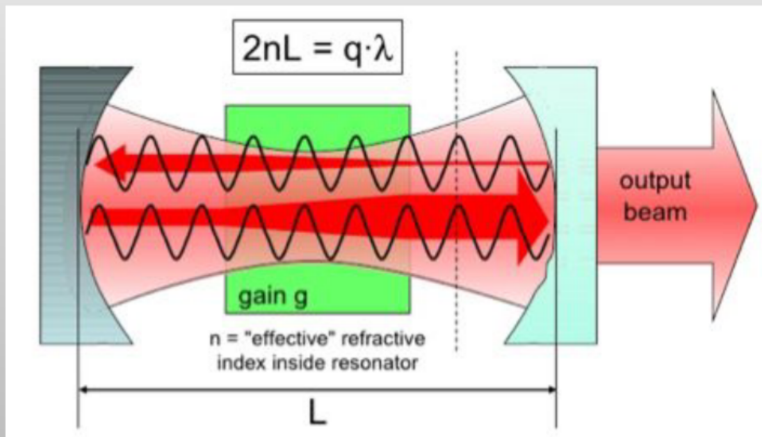


HOM dip as a function of T

Coincidence of counts shows a non-classical dip

How does a laser work?





Stimulated Emission is in phase with the incoming photon

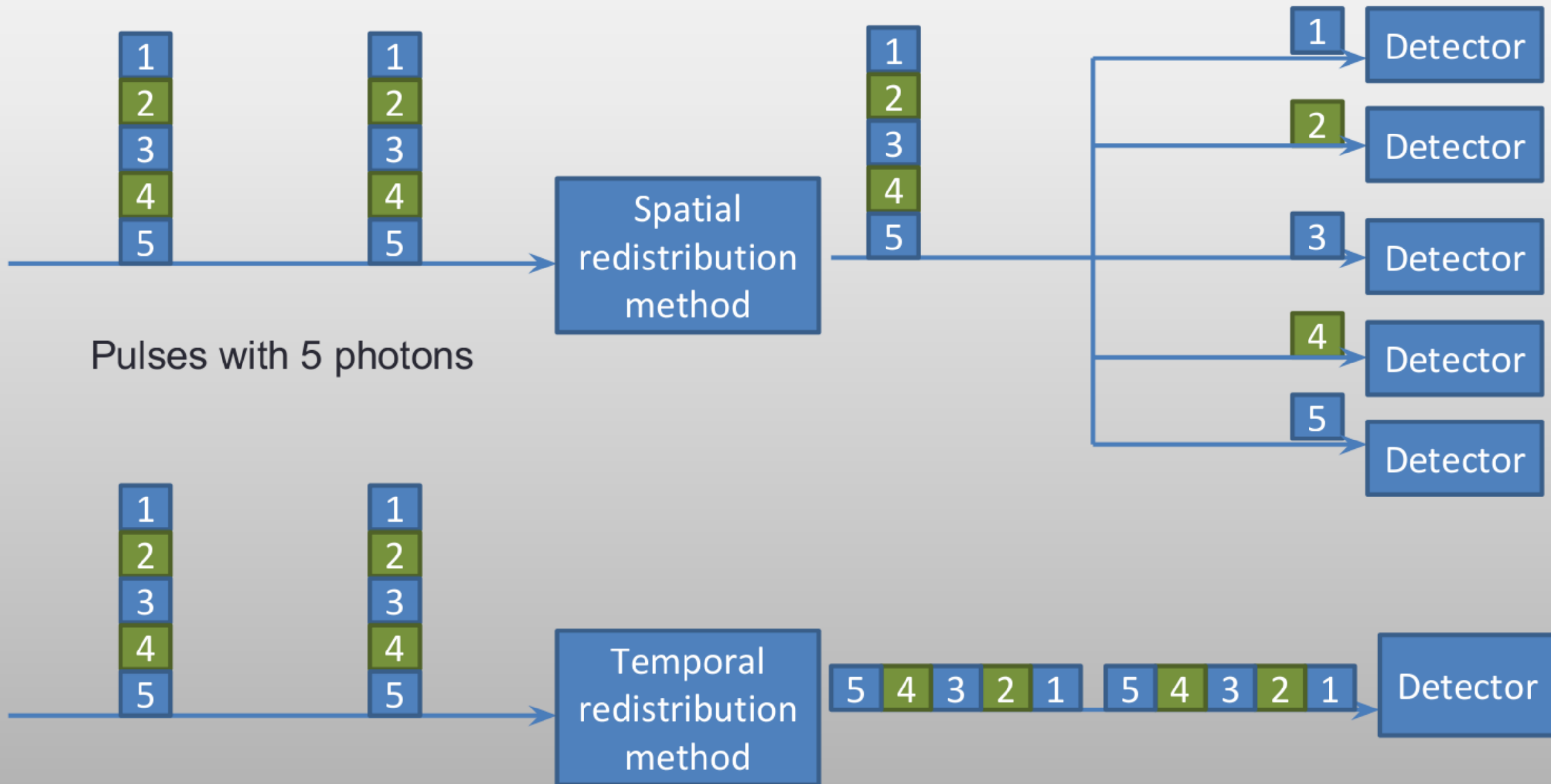
A laser is Coherent. All the photons are locked in phase.

Can measure a coherence time.

- Optical wave (photons) oscillating inside a resonant cavity

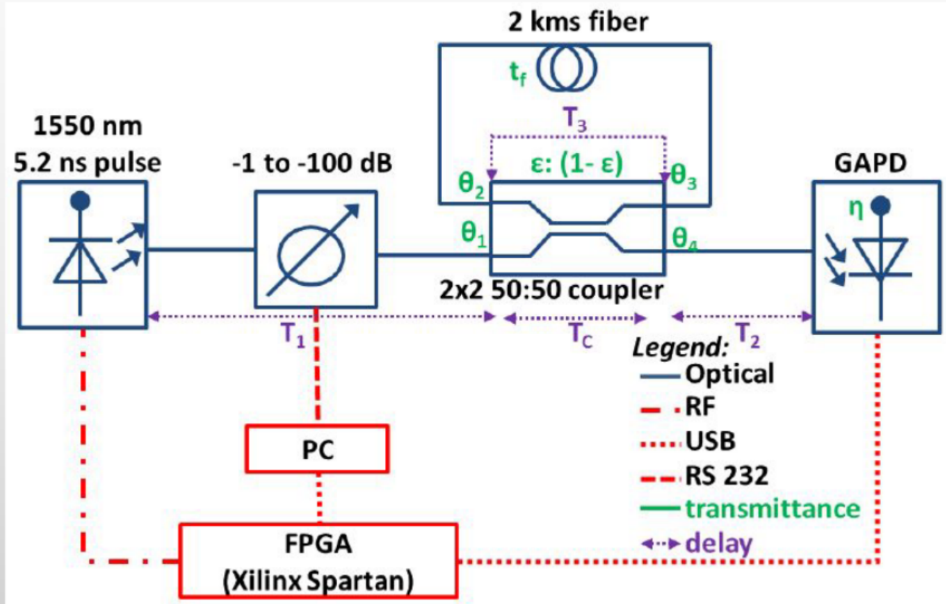
How do we resolve photon numbers?

- Redistribution into different spatial or temporal bins

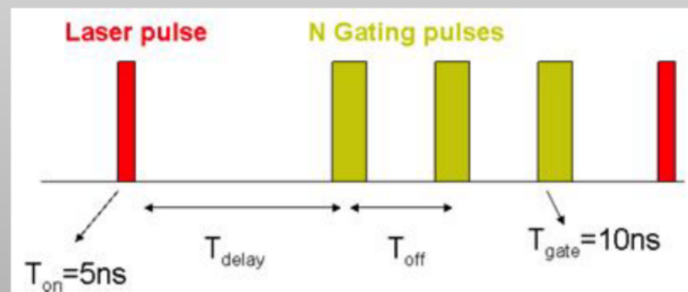


Temporal redistribution

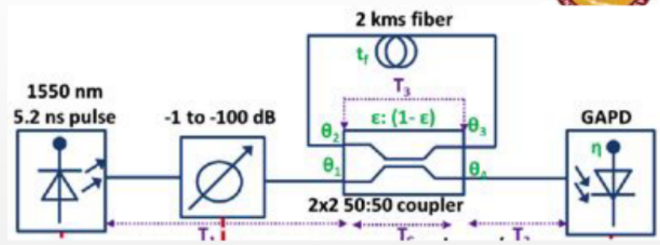
- Average power reduced by half for each circulation

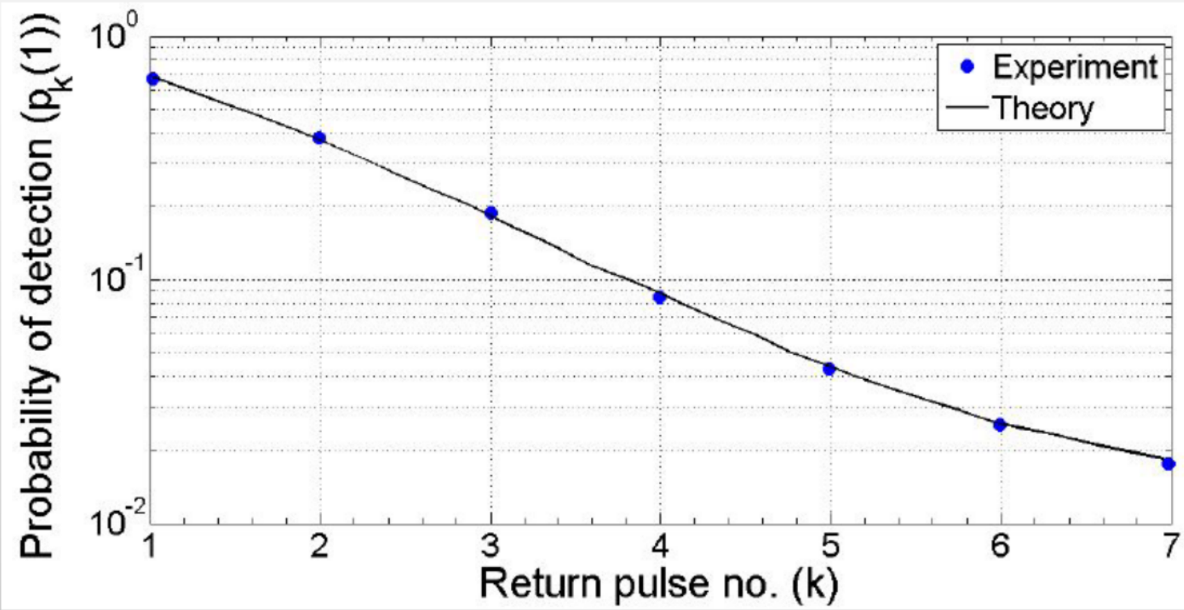


- Synchronized detection using gating at GAPD



Detection probabilities

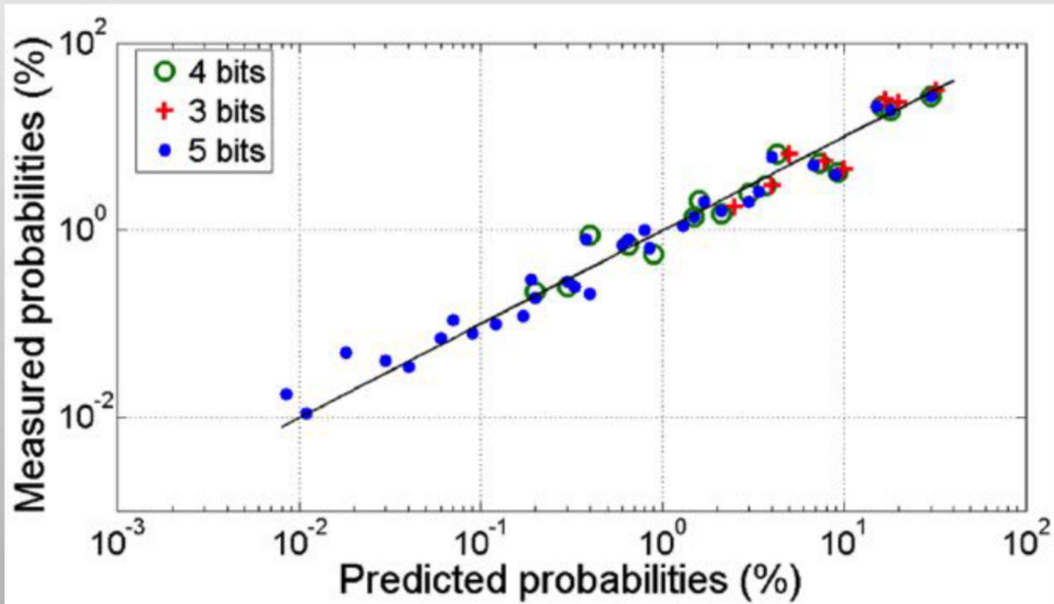




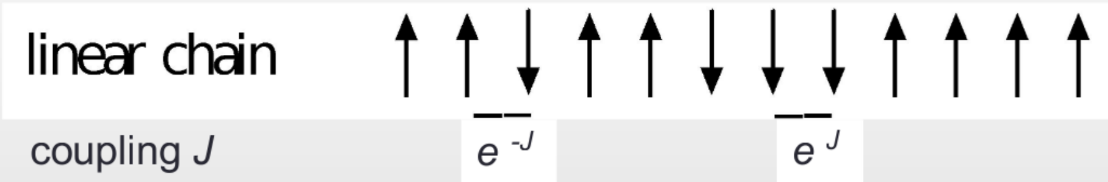
- Decreasing detection probabilities with subsequent redistribution

Predicting detections

- If it is classical, we can predict patterns
  - 10…, 11
  - 100…, 101, 110, 111…
  - 1000, 1001, 1010, 1011 … etc



The Ising Model

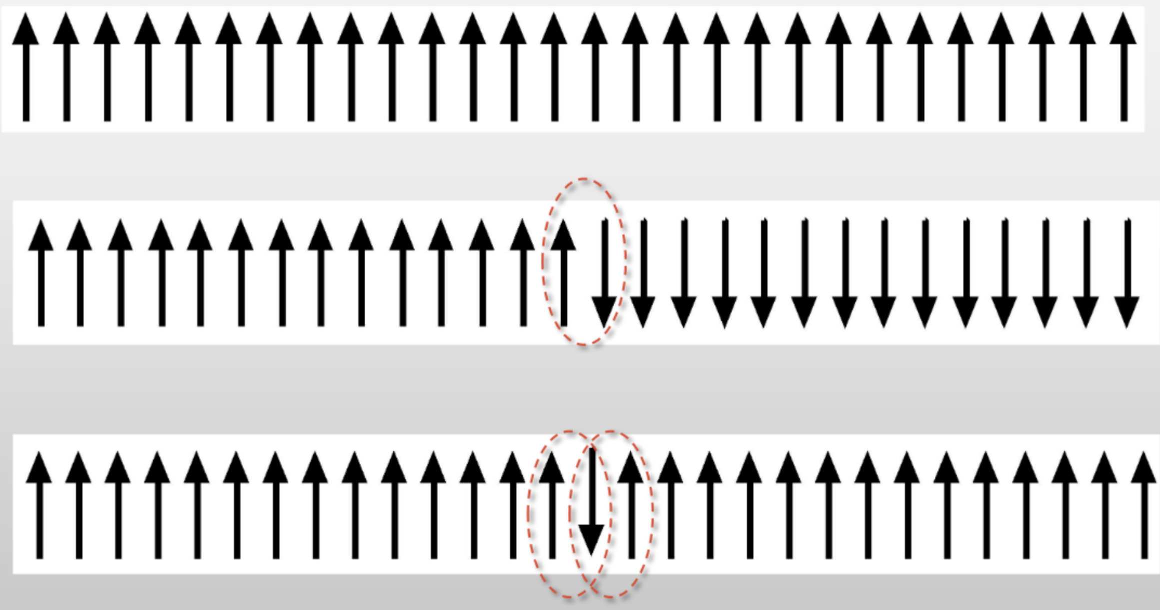


- Popular model in Statistical Physics
- Phase Transition
- Percolation Theory
- Ferromagnetic vs Antiferromagnetic

$$H_{\mathrm{Ising}} = -\frac{1}{2}\sum_{mn}^{N} J_{mn}\sigma_m\sigma_n .$$

$$J = \begin{cases} > 0 & \text{ferromagnetic} \quad \uparrow\uparrow\uparrow\uparrow\uparrow\uparrow \\ < 0 & \text{antiferromagnetic} \quad \downarrow\uparrow\downarrow\uparrow\downarrow\uparrow \end{cases}$$

Bloch Wall



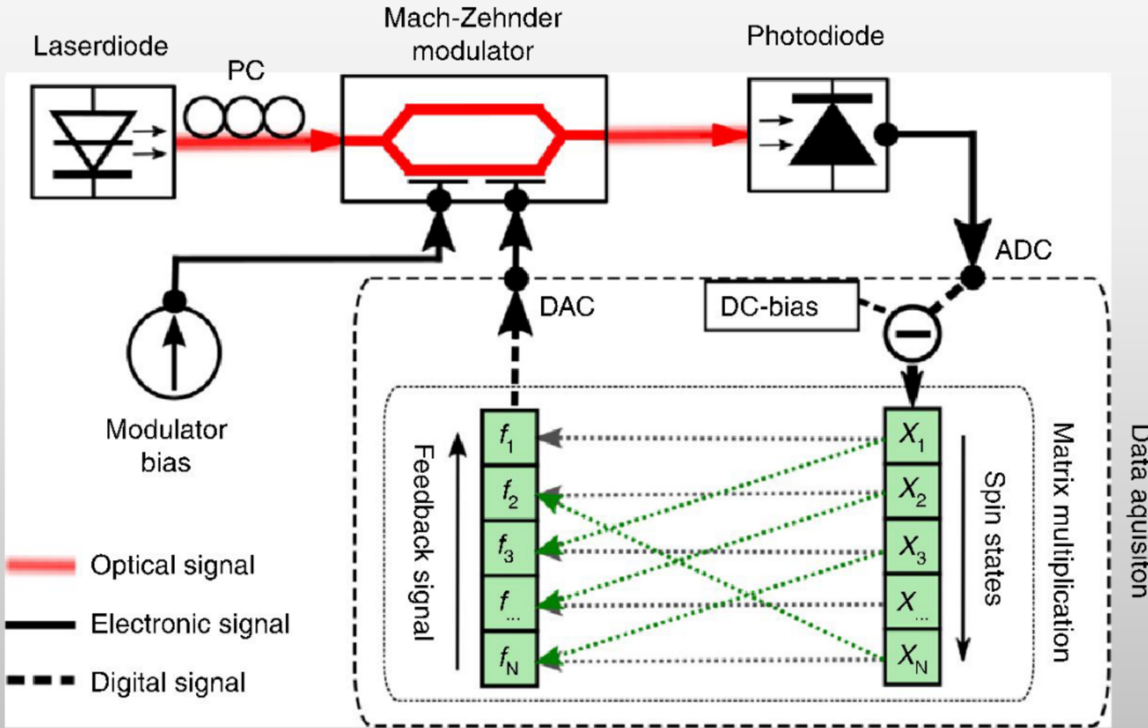
$E = 0$

$E = e^{-J}$

$E = e^{-2J}$

- Spins will orient themselves based on the minimum energy configuration
- Costs less energy to create a Wall than to flip one spin

The Poor Man's Ising Machine



- Mach Zender Modulator with optoelectronic feedback
- Bohm et al, Nature Communications, 10:3538,2019.

The Optical-Electronic-Optical Model

$$x_n[k+1] = \cos^2(f_n[k] - \pi/4 + \zeta_n[k]) - \frac{1}{2}.$$

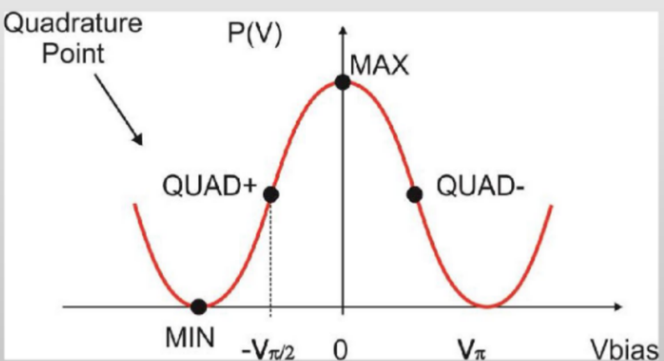
$$f_n[k] = \alpha x_n[k] + \beta \sum_m J_{mn} x_m[k].$$

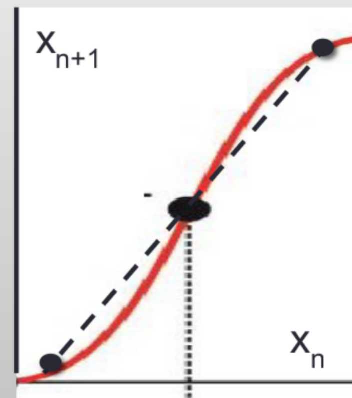
$$\sigma_n = sig_n(x_n[k]$$

- Self bias term α
- Coupling coefficient β
- Weights between spins J
- What does the optics do?
  - Nonlinear function – $\cos^2$

MZM Transfer Function

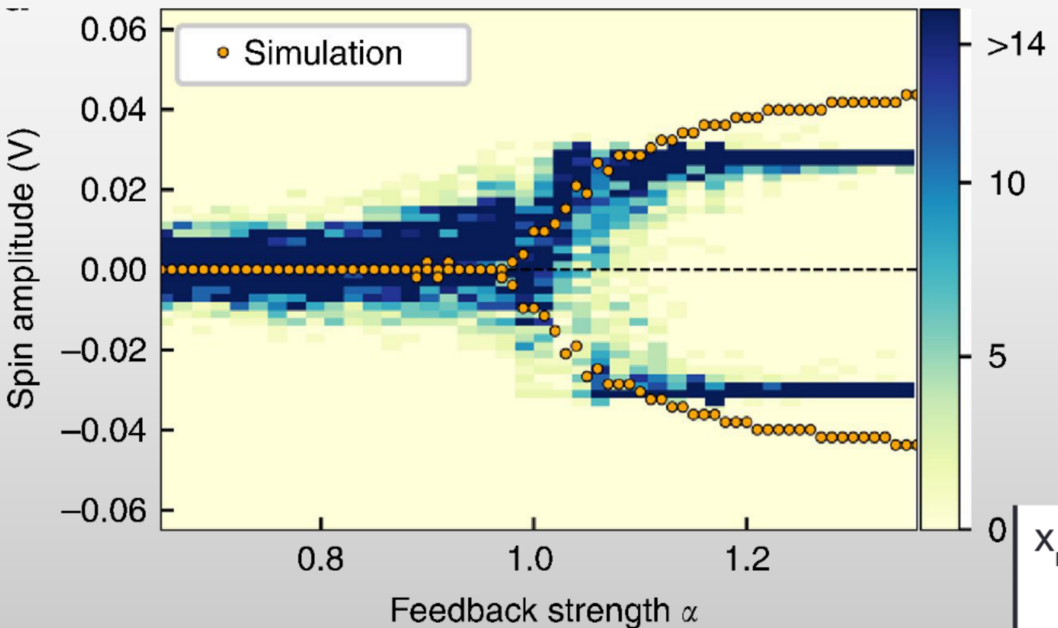
$$I_{out}(t) = T_{mod}\frac{I_{in}}{2}\left[1 + cos\left(\frac{\pi}{V_\pi}V(t) - \phi\right)\right]$$

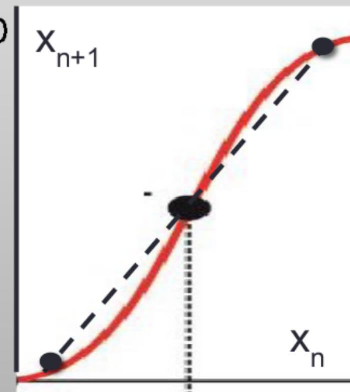




- Find the correct bias point, so we have two solutions - bifurcation

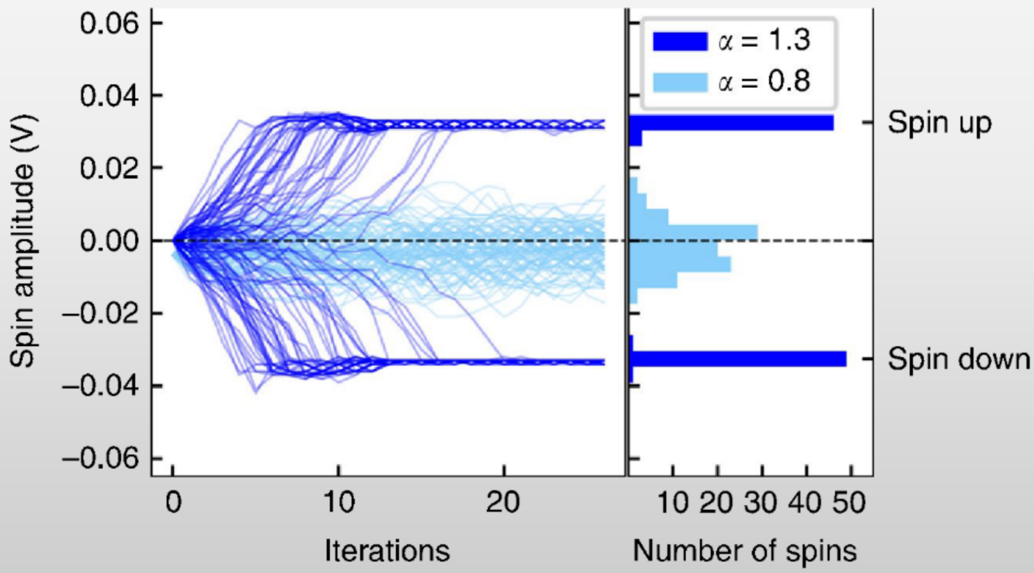
Pitchfork Bifurcation





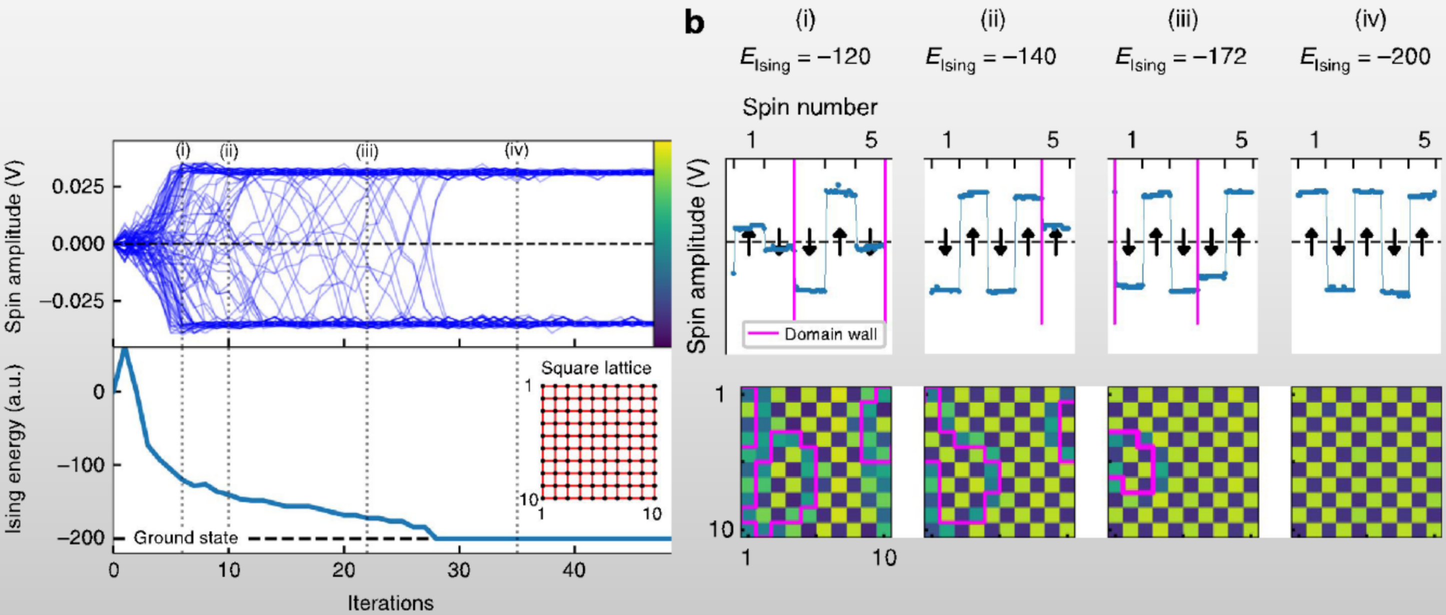
- Nonlinear function has two solutions
- Uncoupled spins

Pitchfork Bifurcation



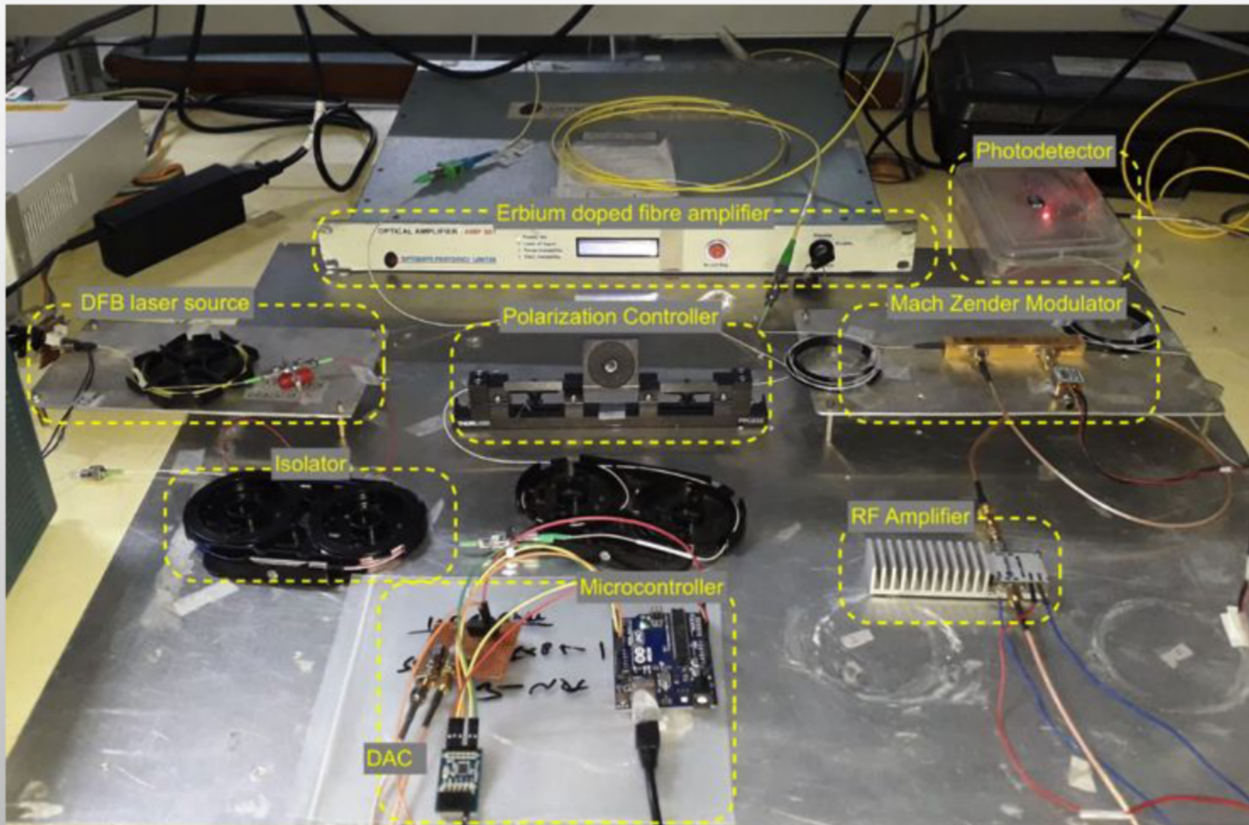
- With 100 spins
- Tune α to observe the bifurcation

Square Lattice



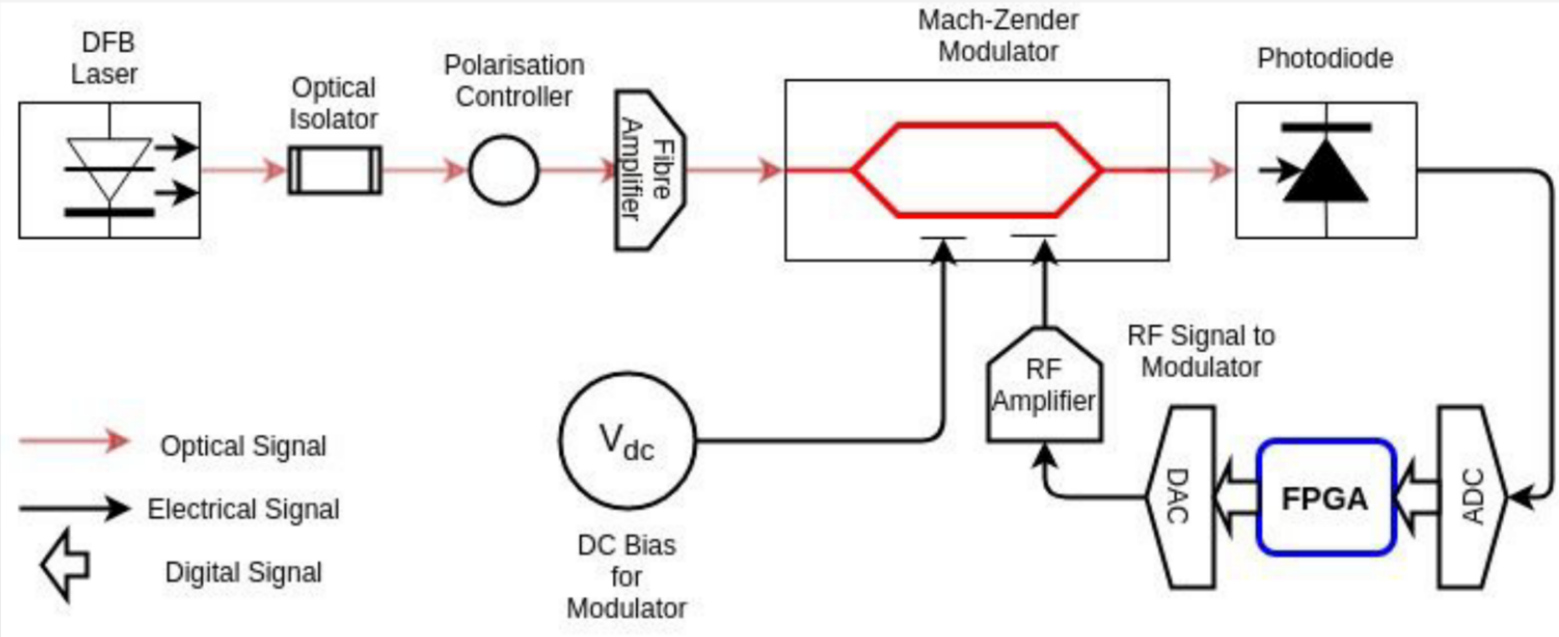
- 10 x 10 lattice, $\alpha = 0.25$, $\beta = 0.29$
- Can observe domain walls where spins are aligned up
- Lowest energy for the checkerboard pattern

The Poor Man's Ising Machine



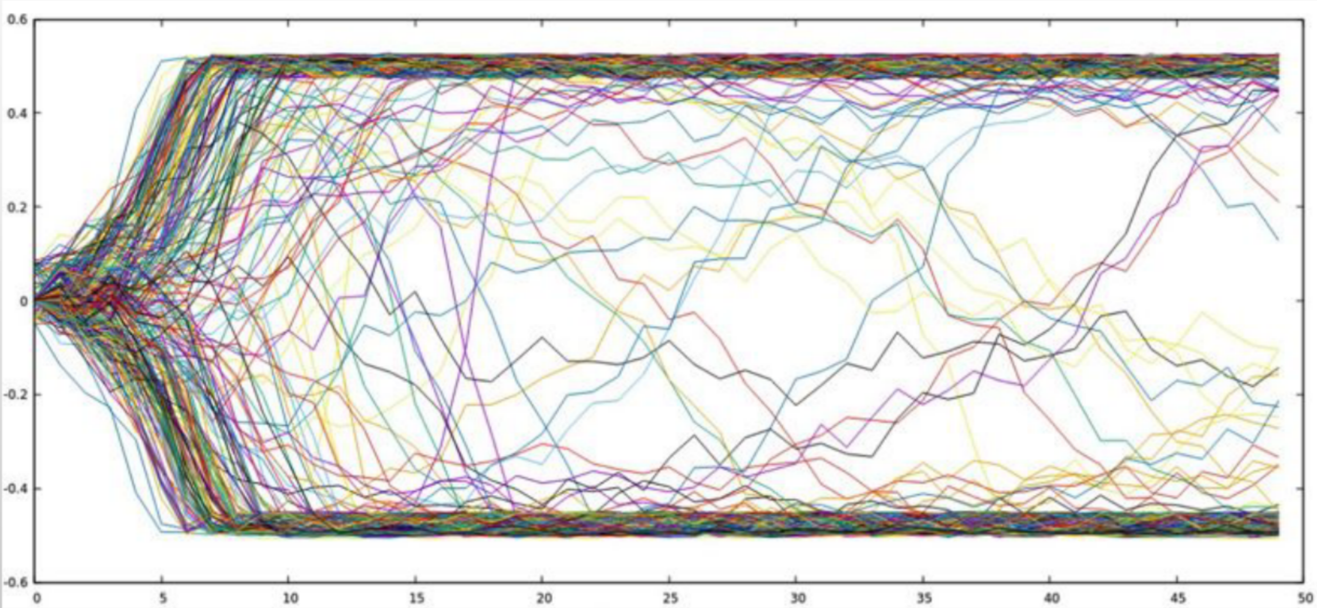
- Not quite Poor….about $10,000

The Poor Man’s Ising Machine



- It is actually a little more complicated
- Needed an optical isolator, a polarization controller, and an optical amplifier

16 x 16 spin lattice

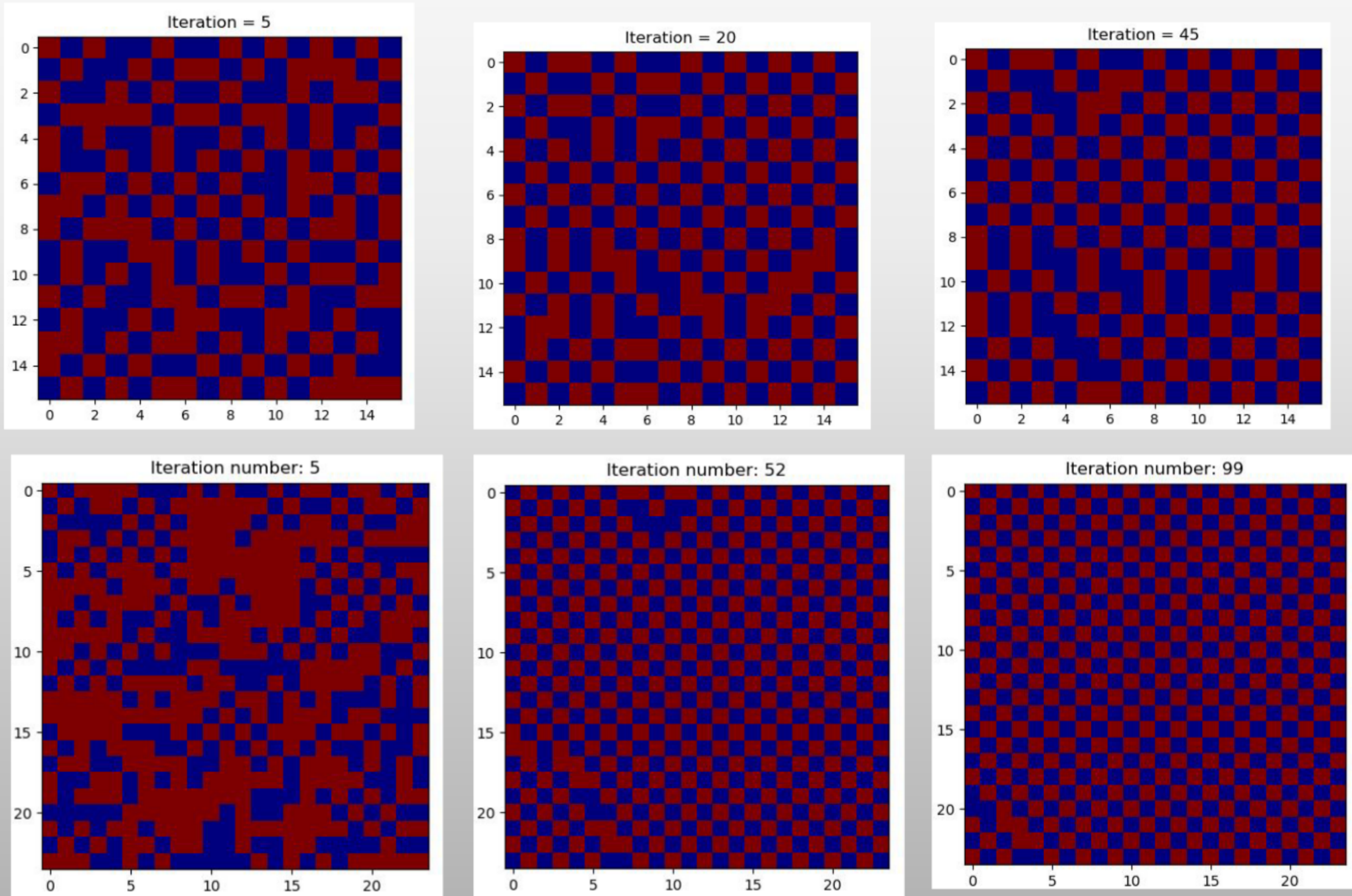


Spin Value

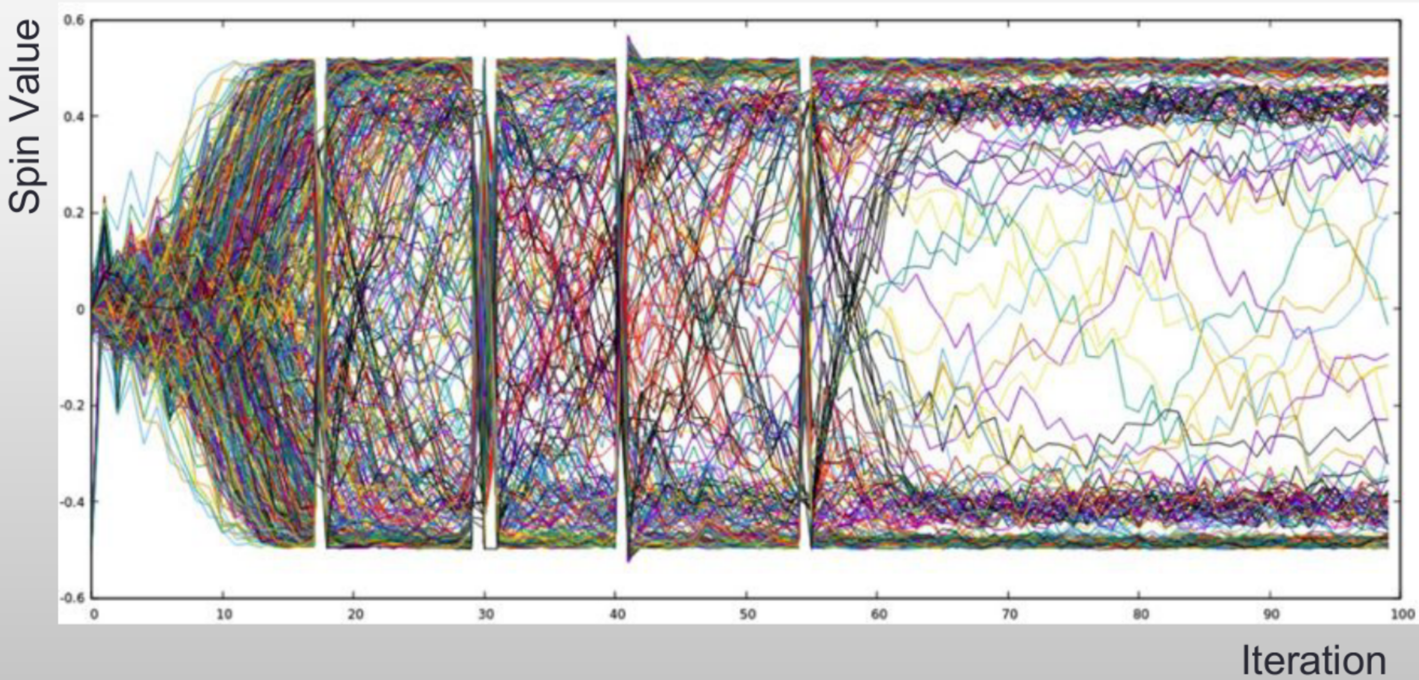
Iteration

- Nearest Neighbours on a square lattice
- Gautham and Parth, IIT Madras

Spin lattice (16x16, 24x24)



24 x 24 spin lattice



- Discontinuities are under investigation
- Gautham and Parth, IIT Madras

How well does it scale?

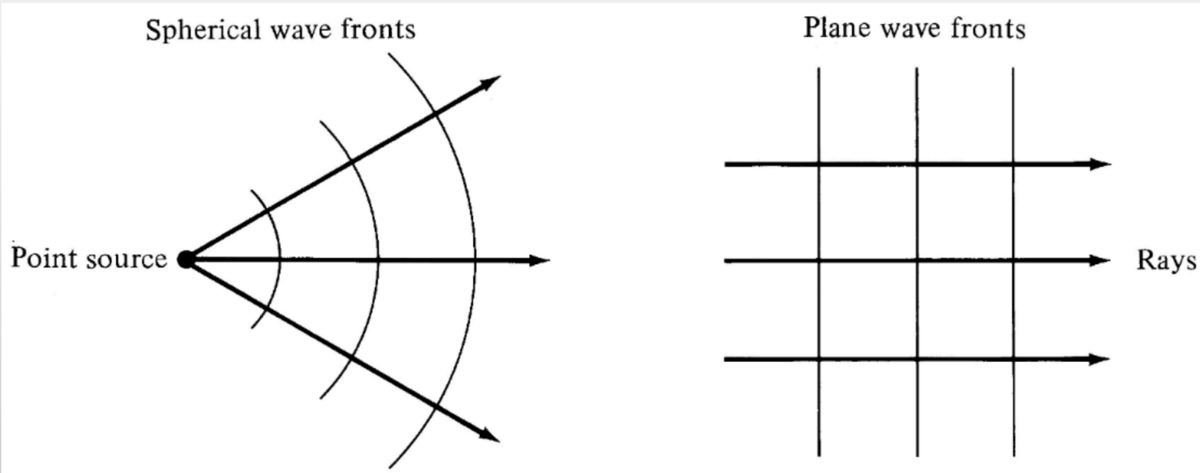
- Photonic Chips, NUS,Singapore

Spatial Light Ising Machine (SLIM)

Large-Scale Photonic Ising Machine by Spatial Light Modulation

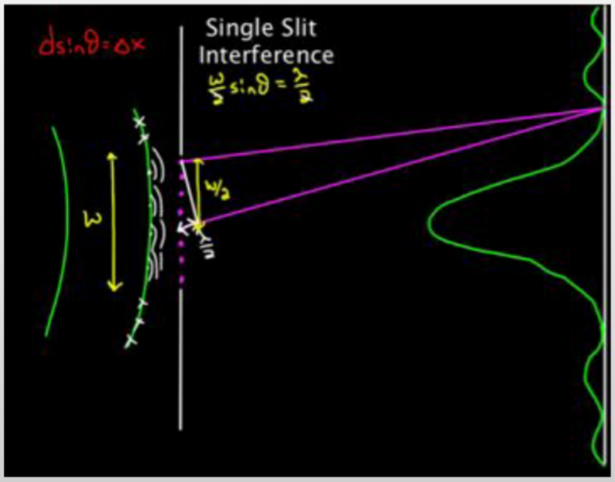
PHYSICAL REVIEW LETTERS **122,** 213902 (2019)

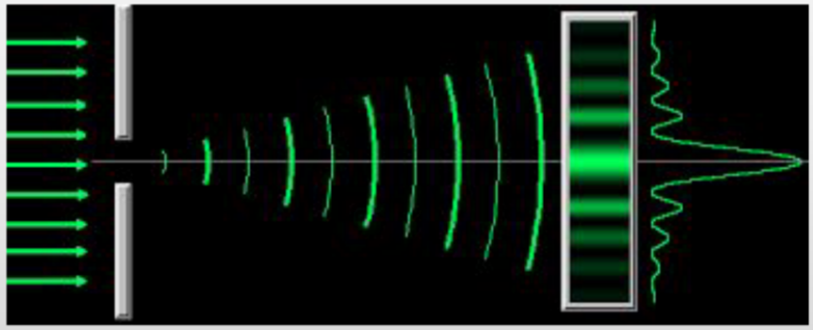
D. Pierangeli,[1,2,*] G. Marcucci,[1,2] and C. Conti[1,2]



- Optical beams have spatial divergence
- Interactions are between different rays - phase

Understanding Phase

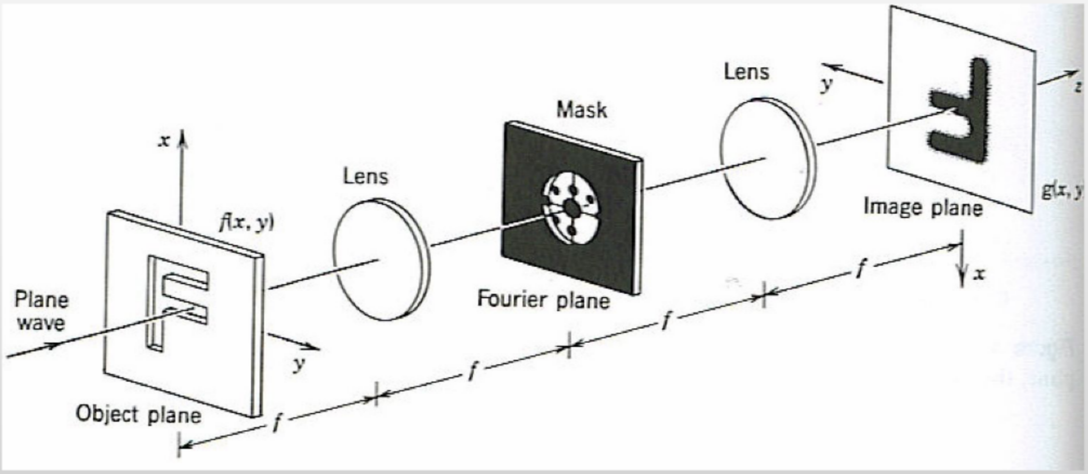




Diffraction from a single slit

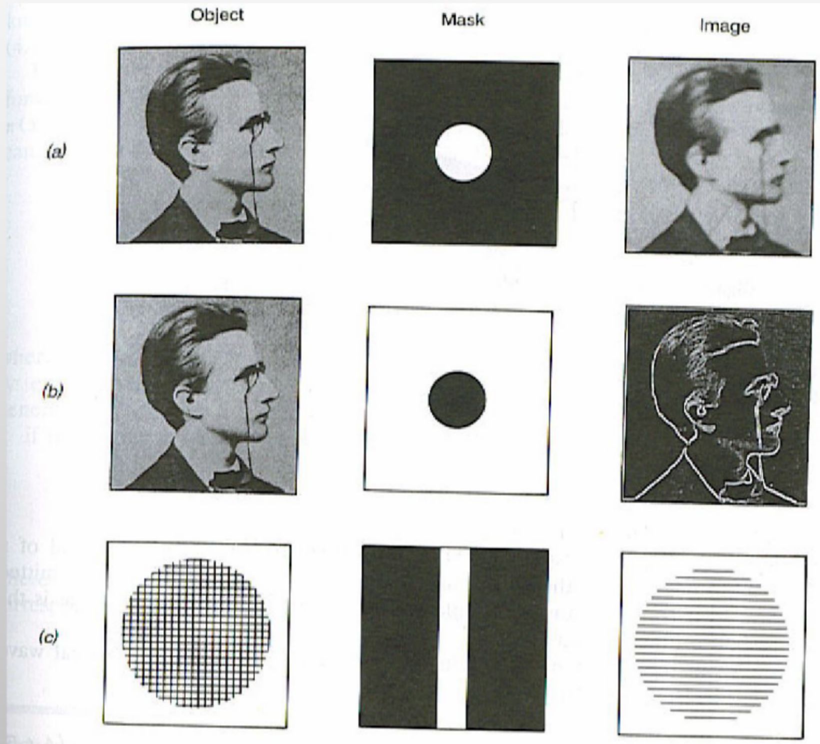
- Different rays of light accumulate a different phase based on their propagation distance from the source
- Think of diffraction in 2-D, you will see rings

Fourier Optics



- The 4-f system
- Every optical ray in 2-D interacts with every other ray
- A lens in 2-D acts as a Fourier Transformer, gives spatial frequency content

Spatial Filters



Blurred

Edge detection

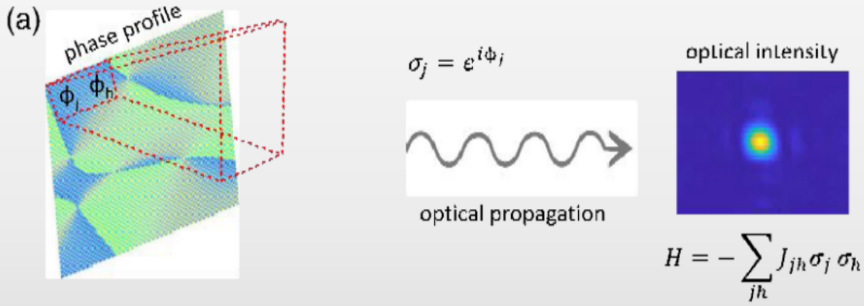
- Easy to design high pass, low pass and vertical pass filters

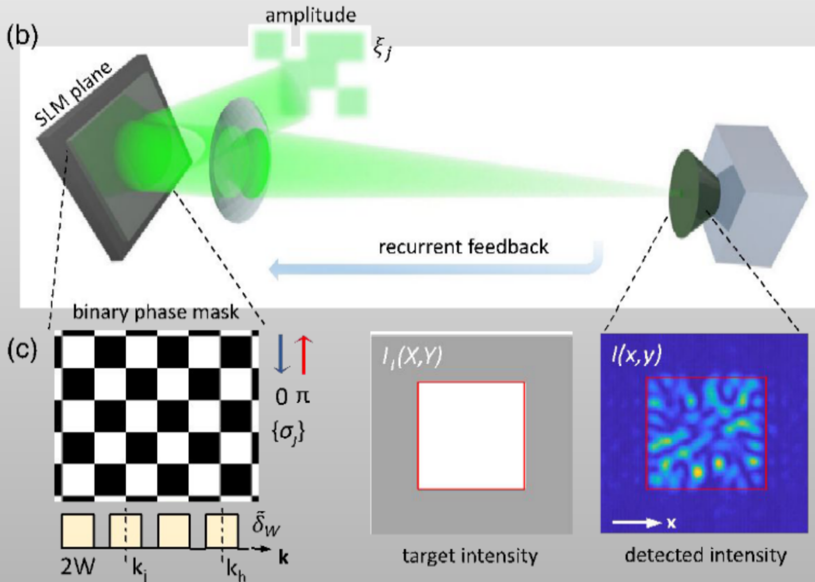
Large-Scale Photonic Ising Machine by Spatial Light Modulation

PHYSICAL REVIEW LETTERS **122,** 213902 (2019)

D. Pierangeli,[1,2,*] G. Marcucci,[1,2] and C. Conti[1,2]

- Amplitude mask
- Phase plane (SLM)
- Camera
- Feedback





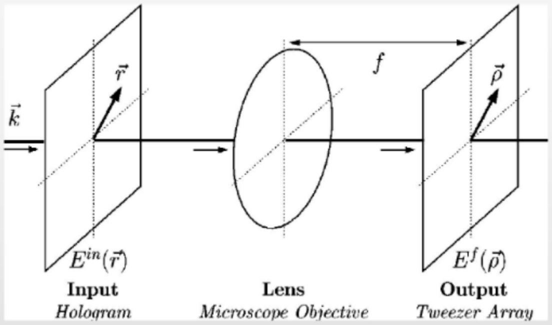
Far field intensity

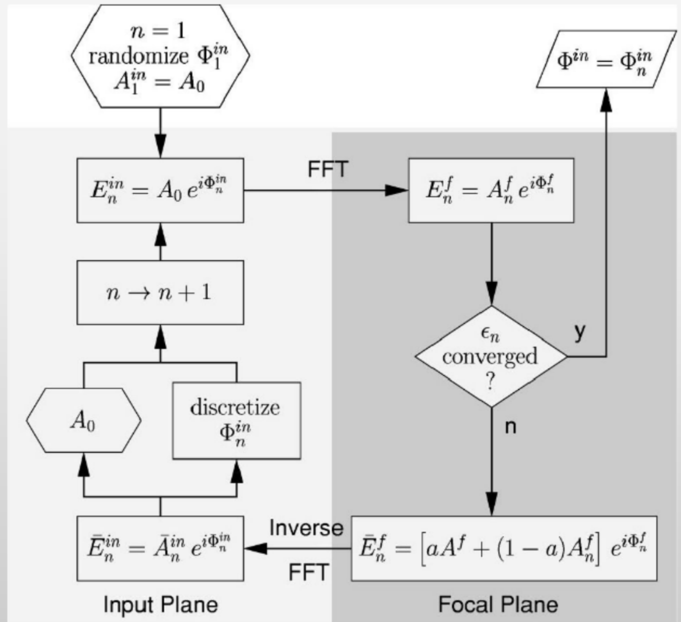
$$I(x) = |E(x)|^2 = \sum_{jh} \xi_j \xi_h \sigma_j \sigma_h \delta_W^2(x) e^{2iW(h-j)x}$$

Fourier transform of a rectangle

$$\delta_W(x) = \sin(Wx)/(Wx)$$

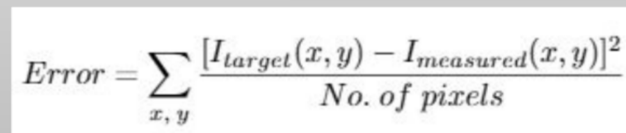
SLIM Algorithm





What are its limitations?

- We need *Φ(x,y)*
- Write an image to the SLM
- Read 1000 x 1000 pixels off a camera

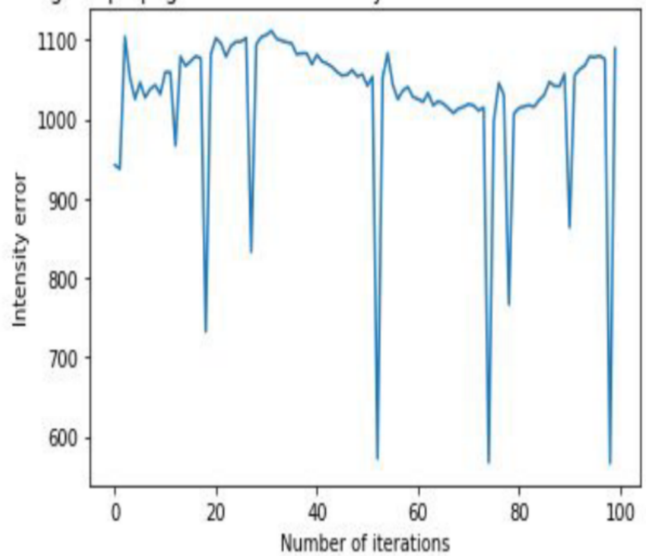


- Computer-generated holographic optical tweezer arrays
- Dufresne et al, Rev. Sci. Instr., **72**, 2001

Convergence Issues

- We lose phase information with a camera
- SLM is slow and resets every so often
- 500 iterations takes 1 hour

Graph showing the propagation of the intensity cost function with the number of iterations

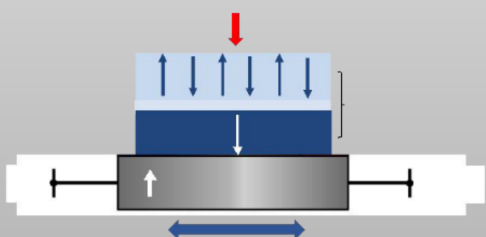


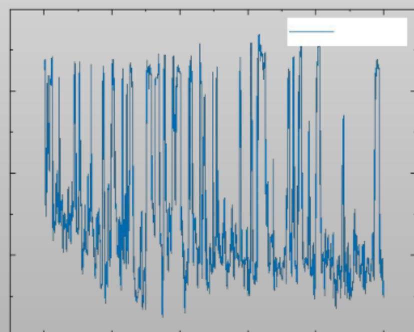
$$Error = \sum_{x,\, y} \frac{[I_{target}(x,y) - I_{measured}(x,y)]^2}{No.\ of\ pixels}$$

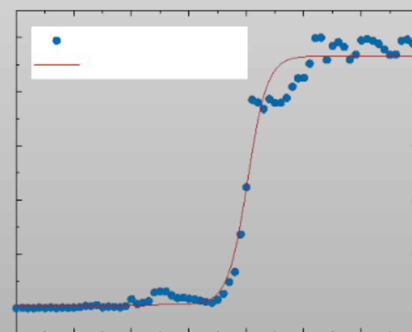
Chip based Ising Computing Machine

- CICM oscillators can be optical, magnetic or electronic

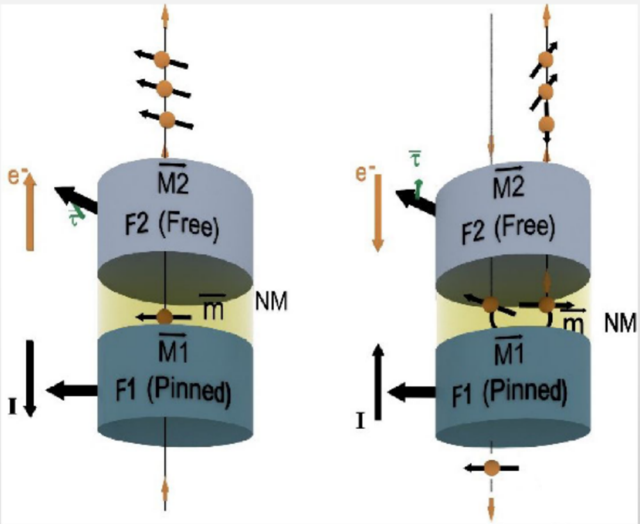
(a) CIM-Spin - ISSCC'20, Kim et al. [47]



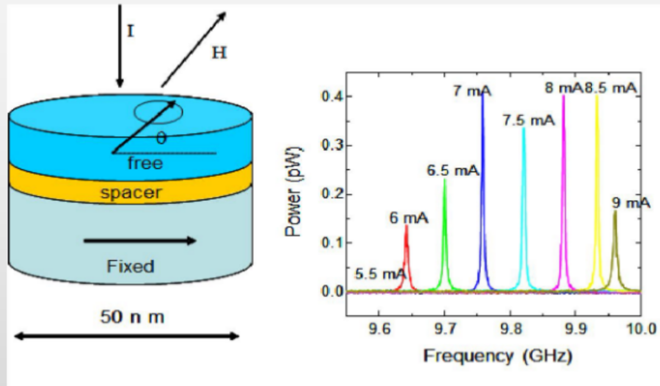




Magnetic Memory, and Oscillators

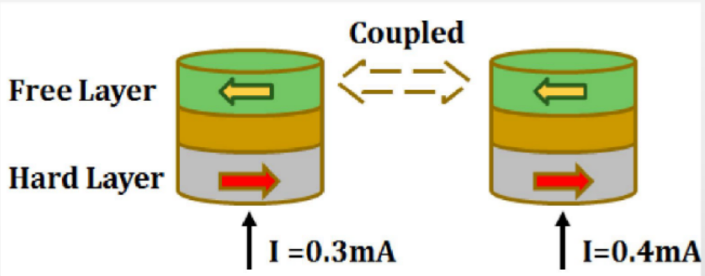


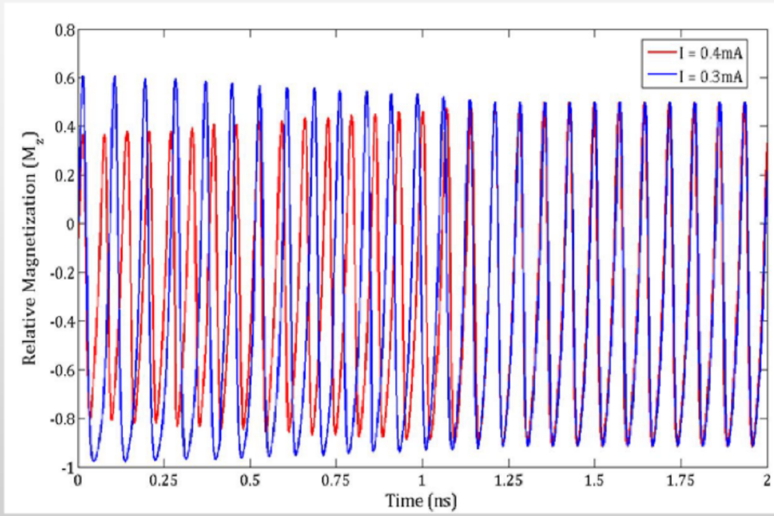
The “free” magnet aligns itself towards a preferred direction determined by the injected current



The “free” magnet oscillates about an effective field (analogous to a top precessing in a gravitational field)

Coupled Nano Oscillators





- Arrays of oscillators can lock together
- How do we change the coupling?

Nano-Patterned Coupled Spin Torque Nano Oscillator (STNO) Arrays – a Potentially Disruptive Multipurpose Nanotechnology

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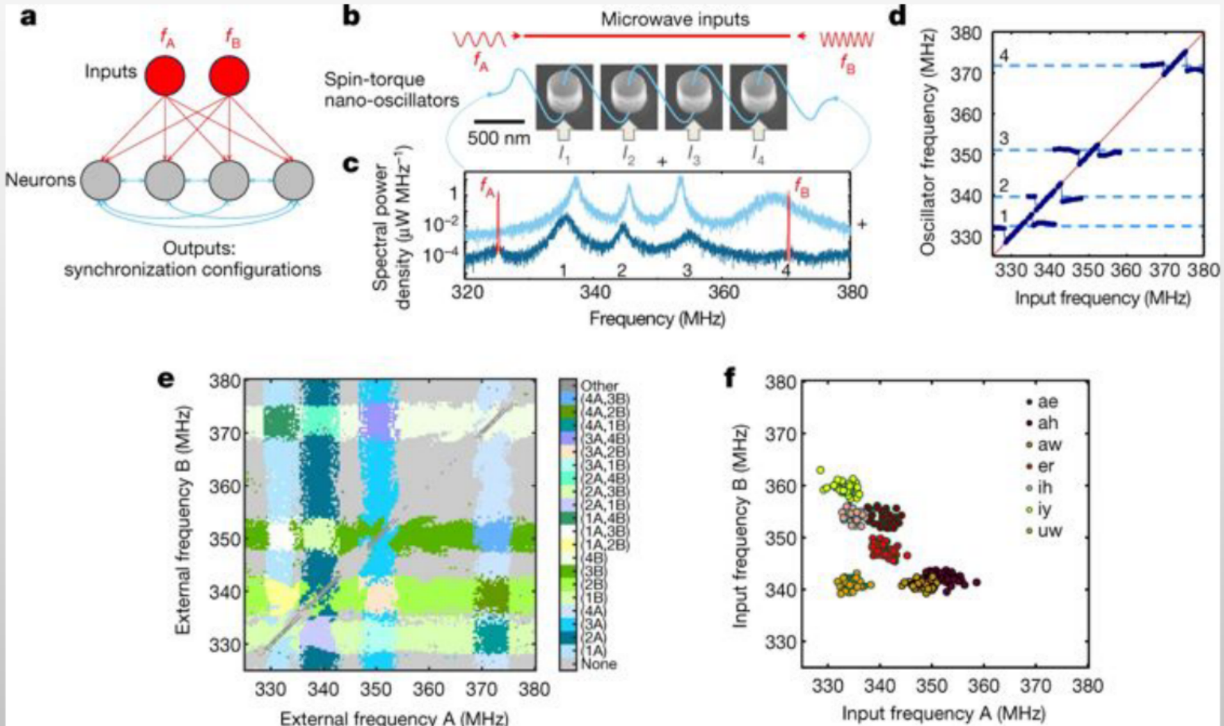
ECE Dept., University of Virginia
Charlottesville, VA, USA

Jiwei Lu, Stuart Wolf
MSE Dept., University of Virginia
Charlottesville, VA, USA

State of the Art



- Can do weighted oscillator networks – vowel recognition
- Romera et al, Nature 563, 2018

Acknowledgements

- Gautham, Parth, Vikram and Vignesh,
- Dr. Nitin Chandrachoodan, IITM

- Dr. Aaron Danner, Prof. Hyunsoo Yang, NUS

- Prof. Sridhar Tayur, CMU

- Centre for Quantum Information, Communication and Computing (quantum.iitm.ac.in)