# 47-779. Quantum Integer Programming

Mini-1, Fall 2020

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| **Room:** Zoom Online | **Time:** Tuesday and Thursday 5:20pm-7:10pm |

**Instructors:**



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**Office Hours:** Post your questions in the forum provided for this purpose on Canvas. This course will be conducted online.

**Objectives:** This course is primarily designed for graduate students (and advanced undergraduates) interested in integer programming (with non-linear objective functions) and the potential of near-term quantum and quantum-inspired computing for solving combinatorial optimization problems. By the end of the semester, someone enrolled in this course should be able to:

- Appreciate the current status of quantum computing and its potential use for integer programming
- Access and use quantum computing resources (such as D-Wave Quantum Annealers)
- Set up a given integer program to be solved with quantum computing
- Work in groups collaboratively on a state-of-the-art project involving applications of quantum computing and integer programming

This course is not going to focus on the following topics:

- Quantum Gates and Circuits
- Computational complexity theory
- Quantum Information Theory
- Analysis of speedup using differential geometry, algebraic topology, etc.

**Prerequisite classes and capabilities:** Although this class has no explicit prerequisites we consider a list of recommended topics and skills that the student should feel comfortable with. An undergraduate-level understanding of probability, calculus, statistics, graph theory, algorithms, and linear algebra is assumed. Knowledge of linear and integer programming will be useful for this course. Programming skills are strongly recommended. Basic concepts in physics are recommended but lack of prior knowledge is not an issue as pertinent ones will be covered in the lectures. No particular knowledge in quantum mechanics or algebraic geometry is required.

Students with backgrounds in operations research, industrial engineering, chemical engineering, electrical engineering, physics, computer science, or applied mathematics are strongly encouraged to consider taking this course.

**Tentative Course Outline:**

**Part 1 - Integer programming (classical methods)** ......... 1 week
- Integer Programming basics [1].
- Cutting plane theory and relaxations [1].
- Introduction to Test Sets [2, 3? ? ].
  - Gröbner basis [4, 5? ].
  - Graver basis [6].

**Part 2 - Ising, QUBO** ......................................... 1 week
- Ising model basics [7, 8, 9].
- Simulated Annealing [10, 11].
- Markov-chain Monte Carlo methods [12, 13, 14, 15].
- Benchmarking classical methods [16, 17].
- Formulating combinatorial problems as QUBOs [18].

**Part 3 - GAMA: Graver Augmented Multiseed algorithm** 1 week
- GAMA [19].
  - Applications: Portfolio Optimization [19], Cancer Genomics [20]
  - Quantum Inspired: Quadratic (Semi-)Assignment Problem [21].

**Part 4 - Quantum methods for solving Ising/QUBO** ....... 1 week
- AQC, Quantum Annealing and D-Wave [22, 23, 24, 25, 26, 27, 28].
- QAOA: Quantum Approximate Optimization Algorithm [29, 30, 31].

**Part 5 - Hardware for solving Ising/QUBO** ................. 1 week
- Graphical Processing Units [32, 33, 34].
- Tensor Processing Units [35].
- Complementary metal-oxide-semiconductors (CMOS) [36].
- Digital Annealers [37].
- Oscillator Based Computing [38, 39].
- Coherent Ising Machines [40, 41, 42, 43, 44, 45].

**Part 6 - Other topics and project presentations** ........... 1 week
- Compiling
  - Quantum Annealing [46, 47].
  - Gate-based Noisy Intermediate Scale Quantum (NISQ) devices [48].
- Adiabatic Quantum Computing and Algebraic Geometry
  - Minimizing Polynomial Functions [49].
  - Prime Factorization [50].

**Grading Policy:** Weekly quizzes and scribe (50%), Final Group Project (50%).

- Each lecture will have a short quiz to evaluate the concepts covered in a previous class. The two worst quizzes won't be counted towards the final grade.
- Since this is a new course, we will need your help in compiling the lecture notes by signing up for scribe duties. Each student will complete the scribe duties of one lecture and this will replace one quiz grade. The notes will be written via Overleaf and The sign-up sheet can be found in Canvas.
- The group final project will require the implementation and solution of an integer programming instance. This final project deliverable is a report and a presentation.

**Project description:** The final project accounts for 50% of the total grade. This project will be completed in groups of 2-4 students and will reflect the understanding of the students of the material covered in the lecture. The components of this project are the following

- Identify a problem that can be posed as an Integer Program. Discuss the importance of this problem.
- Solve instances of the identifies problem using classical tools. Identify which are the sources of complexity while solving this problem.
- Model the problem as a Quadratic Unconstrained Binary Optimization (QUBO). Verify that the reformulation of the problem is valid, in the sense that it represents the original problem.
- Solve the resulting QUBO using non-conventional methods, e.g. Quantum Annealing, QAOA, simulated annealing in GPUs/TPUs, etc. Compare at least two different methods.
- Write a report outlining the different approaches used and highlighting the knowledge obtained while developing the project.
- Hold a final presentation in front of the class reporting the findings of the project.

**Important Dates:**

No Final Exam. Presentations in weeks 6-7.

**Course Policy:**

- Auditing students are encouraged to participate actively in the lectures.
- Regular attendance is essential and expected.
- Please sign up using Canvas.

**Highlights:** The specific skills that students will gain that will make them "quantum ready" for industry or further academic research in this course are:

1. Classical
   (a) Given a practical problem (from supply chain or physics or anything else), formulate it as a non-linear integer program. We will provide a few practical problems, but we encourage you to suggest one that you are already working on or are interested in.
   (b) Solve such formulations via classical solvers.
2. Quantum
   (a) Reformulate the problem to be quantum ready by making it in the form of a QUBO.
   (b) Solve the QUBO brute force through D-Wave or IBM (via QAOA).
3. Hybrid Quantum-Classical
   (a) Reformulate the problem again in the form suitable for GAMA.
   (b) Solve GAMA compatible formulation via D-Wave and/or via QAOA.

Regarding USRA

1. Access to D-Wave systems might be available via written proposals to the University Space Research Association (USRA). See `https://riacs.usra.edu/quantum/rfp` for terms and conditions. The course will discuss proposal preparation.
2. Students of this course are encouraged to apply to the Feynman Academy Internship program `https://riacs.usra.edu/quantum/qacademy` that sponsors research projects at NASA Ames Research Center.

**Academic Honesty:** Lack of knowledge of the academic honesty policy is not a reasonable explanation for a violation. Any form of plagiarism can earn you a failing grade for the entire course. For more information you can refer to CMUs policies on academic integrity. When in doubt, add a citation.

**Casual References:** There is no single text book for the course. This is a short list of various interesting and useful books that will be mentioned during the course. You need to consult them occasionally.

- Georges Irfah, *The Universal History of Computing*, John Wiley & Sons, 2001.
- A. Das and B.K. Chakrabarti (Eds.). *Quantum Annealing and Related Optimization Methods*, Springer-Verlag, 2005.
- Eleanor G. Rieffel and Wolfgang H. Polak, *Quantum Computing: A Gentle Introduction*, MIT Press, 2011.
- Richard J. Lipton and Kenneth W. Regan, *Quantum Algorithms via Linear Algebra. A Primer*, MIT Press, 2014.

**Student Resources:** If you have a disability and require accommodations, please contact Catherine Getchell, Director of Disability Resources, 412-268-6121, getchell@cmu.edu. If you have an accommodations letter from the Disability Resources office, we encourage you to discuss your accommodations and needs with us as early in the semester as possible. We will work with you to ensure that accommodations are provided as appropriate.

As a student, you may experience a range of challenges that can interfere with learning, such as strained relationships, increased anxiety, substance use, feeling down, difficulty concentrating and/or lack of motivation. These mental health concerns or stressful events may diminish your academic performance and/or reduce your ability to participate in daily activities. CMU services are available, and treatment does work. You can learn more about confidential mental health services available on campus at: `http://www.cmu.edu/counseling/`. Support is always available (24/7) from Counseling and Psychological Services: 412-268-2922.

## References


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