Estimating the Fisher Matrix for SNe Ia surveys

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A qualitative approach to the problem of comparing two different simulations can be to compare how many objects pass the standard `SNANA` quality cuts and how the signal-to-noise ratio, SNR, is distributed within the survey, together with the recovery of the SALT2 α and β parameters. However, a Fisher matrix approach can be used, as described below.

For this, we first define the likelihood $\mathcal{L}$ of the parameters $\boldsymbol{\theta}$:

$$-2\log\mathcal{L} = \sum_{i=1}^{N}\left(\frac{y_i - \bar{y}_i(\boldsymbol{\theta})}{\sigma_i}\right)^2. \tag{1}$$

This is simply the χ^2 for all objects in a given data-set. Now, a Fisher matrix can be defined for the likelihood(**?**, chapter 11):

$$\mathcal{F}_{\alpha\beta} \equiv -\left\langle \frac{\partial^2(\log\mathcal{L})}{\partial\theta_\alpha\partial\theta_\beta}\right\rangle. \tag{2}$$

This is roughly the inverse of the covariance matrix of the estimated parameters, and represents the total information that the dataset can provide. On this section, a semi-analytical model for the specific case of supernovae datasets will be presented.

We first note that

$$\frac{\partial^2\log(\mathcal{L})}{\partial\theta_\alpha\partial\theta_\beta} = \sum_i\left(\partial_\alpha\partial_\beta\bar{y}_i\frac{(y_i-\bar{y}_i)}{\sigma_i^2} - \frac{\partial_\alpha\bar{y}_i\partial_\beta\bar{y}_i}{\sigma_i^2}\right), \tag{3}$$

where $\partial_i \equiv \frac{\partial}{\partial\theta_i}$.

For cosmological information, we take $y = \mu$, that is, our measurements are the distance moduli of each object. Now, we define the distribution $N(z)$, the number of supernovae observed in a given redshift bin. This allows the sum over all observations to be approximated by a integration over the average values of μ and σ_μ over each bin, that is

$$\sum_i \to \int \mathrm{d}z\frac{\mathrm{d}N}{\mathrm{d}z},$$

leading us to:

$$\begin{aligned}\frac{\partial^2\log(\mathcal{L})}{\partial\theta_\alpha\partial\theta_\beta} = \int\mathrm{d}z\,\frac{\mathrm{d}N}{\mathrm{d}z}\,\Bigg(&\partial_\alpha\partial_\beta\bar{\mu}(z,\boldsymbol{\theta})\frac{\langle\mu\rangle_{z\text{ bin}} - \bar{\mu}(z,\boldsymbol{\theta})}{\langle\sigma^2\rangle_{z\text{ bin}}}\\ &- \frac{\partial_\alpha\bar{\mu}(z,\boldsymbol{\theta})\partial_\beta\bar{\mu}(z,\boldsymbol{\theta})}{\langle\sigma^2\rangle_{z\text{ bin}}}\Bigg).\end{aligned} \tag{4}$$

We can approximate this equation by noting that the $\langle\mu\rangle_{z\text{ bin}} - \bar{\mu}(z,\boldsymbol{\theta})$ term is, on average, zero or close to zero. This leaves us with

$$\frac{\partial^2\log(\mathcal{L})}{\partial\theta_\alpha\partial\theta_\beta} \approx -\int\mathrm{d}z\,\partial_\alpha\bar{\mu}(z,\boldsymbol{\theta})\partial_\beta\bar{\mu}(z,\boldsymbol{\theta})\frac{\frac{\mathrm{d}N}{\mathrm{d}z}}{\langle\sigma^2\rangle_{z\text{ bin}}}. \tag{5}$$

Finally, we note here that we have a product of two terms: one that is only model dependent and one that is survey dependent. In order to maximize the survey information, we have to maximize the second term. This term can be separated in bins of quality k, measured by the signal-to-noise ratio:

$$\frac{\mathrm{d}N}{\mathrm{d}z}\langle\sigma^2\rangle^{-1} = \sum_k \frac{\mathrm{d}N_k}{\mathrm{d}z}\frac{1}{\langle\sigma_k^2\rangle}. \tag{6}$$

This serves as a measure of the overall quality of the survey.