

sufficed. However, for smaller frequency spacings multiuser detection is required, yielding near-single-user performance in the case  $\Delta f/T = 1/2$ ,  $\Delta\phi = \pi/2$ . (With  $\Delta f/T = 1/2$ ,  $\Delta\phi = 0$ , the composite signal constellation is smaller than  $m^K$  owing to identical signals.)

In Fig. 3, we study the dependency on  $K$  for  $\Delta f/T = 1/2$ ,  $\Delta\phi = \pi/2$  and  $\Delta f/T = 1/3$ ,  $\Delta\phi = 0$ . In the former case, the loss is negligible at BER =  $10^{-3}$  for  $K = 5$  compared to  $K = 1$ . Finally, Fig. 4 shows results for  $K = 2, 3$  and  $5$  in the CDMA (cochannel interference) case  $\Delta f/T = 0$ . It is seen that performance depends strongly on the relative phases between the users. A proper choice, e.g.  $\Delta\phi = \pi/2$  for  $K = 3$ , can give a power/bandwidth efficiency of 2.2 dB/0.8 [Note 1] which is better than for any previously known SCCPM system.

**Conclusions:** Multiuser SCCPM with a high degree of spectral overlap has been investigated using an iterative decoder. Simulation results indicate that for frequency spacings of at least one-third the CPM symbol rate, up to five equal-power users can be detected with a loss of no more than 1.4 dB compared to the single-user case. In addition, with no frequency spacing at all, three users can be detected with a loss of only 0.8 dB, but this is strongly dependent on the relative phases between the users.

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## Shortest path routing algorithm using Hopfield neural network

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A near-optimal routing algorithm employing a modified Hopfield neural network (HNN) is presented. Since it uses every piece of information that is available at the peripheral neurons, in addition to the highly correlated information that is available at the local neuron, faster convergence and better route optimality is achieved than with existing algorithms that employ the HNN. Furthermore, all the results are relatively independent of network topology for almost all source-destination pairs.

Note 1: Power efficiency measured as required  $E_b/N_0$  for BER =  $10^{-3}$ ; bandwidth efficiency measured as double-sided 99% bandwidth normalised to the composite source bit rate.

**Introduction:** Hopfield and Tank [1] initially demonstrated the computational power of the neural network (NN) by applying their models to the travelling salesman problem (TSP). The advantage of the Hopfield neural network (HNN) is the hardware-based rapid computational capability of solving the combinatorial optimisation problem. Subsequently, the shortest path (SP) routing problem was addressed using the HNN under various constraints. In particular, Ali and Kamoun [2] proposed an adaptive algorithm that utilised link cost and topology information. However, the computed paths in this algorithm involved loops, and convergence could be slow. Invoking additional constraint on the flow vector, Park and Choi [3] proposed a modified algorithm. This enhanced the convergence performance, which, however, was heavily dependent on the network topologies. In this Letter, a new approach is proposed to speed up convergence while improving route optimality, and which is relatively insensitive to variations in network topology.

**Proposed approach:** A network topology can be described by the undirected graph  $G = (N, A)$ , where  $N$  is a set of  $N$  nodes (vertices) and  $A$  is a set of links (arcs or edges) [1-3]. The link connection indicator  $\rho_{ij}$  is defined as follows:

$$\rho_{ij} = \begin{cases} 1 & \text{if link from node } i \text{ to node } j \text{ does not exist} \\ 0 & \text{otherwise} \end{cases}$$

There is a cost  $C_{ij}$  associated with each link  $(i, j)$  and they are specified by the cost matrix  $C = [C_{ij}]$ . Source and destination nodes are denoted by  $s$  and  $d$ , respectively. A path inclusion criterion matrix, denoted by  $V = [V_{ij}]$ , represents the output values of neurons and each element is defined as follows:

$$V_{ij} = \begin{cases} 1 & \text{if link from node } i \text{ to node } j \text{ exists in routing path} \\ 0 & \text{otherwise} \end{cases}$$

It is obvious that all the diagonal elements of  $V_{ij}$  must be zero. Using the above definitions, the SP problem can be formulated as a constrained combinatorial optimisation problem:

$$\begin{aligned} &\text{minimise} \quad \sum_{i=1}^N \sum_{j=1, j \neq i}^N C_{ij} \cdot V_{ij} \\ &\text{subject to} \quad \sum_{j=1, j \neq i}^N V_{ij} - \sum_{j=1, j \neq i}^N V_{ji} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \\ &\quad V_{ij} \in \{0, 1\} \end{aligned} \quad (1)$$

The constraint, eqn. 1, ensures that the computed result is indeed a path between a source and a designated destination. To formulate this optimisation problem in terms of the HNN, the computational network requires  $N(N-1)$  neurons since it is organised in an  $(N \times N)$  matrix with all diagonal elements removed. Defining the Lyapunov (energy) function as eqn. 2, its minimisation drives each neuron of the NN into its stable state, thereby providing the SP solution:

$$\begin{aligned} E = & \frac{\mu_1}{2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N C_{ij} \cdot V_{ij} + \frac{\mu_2}{2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N \rho_{ij} \cdot V_{ij} \\ & + \frac{\mu_3}{2} \sum_{i=1}^N \left( \sum_{j=1, j \neq i}^N V_{ij} - \sum_{j=1, j \neq i}^N V_{ji} - \gamma_i \right)^2 + \frac{\mu_4}{2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N V_{ij} \cdot (1 - V_{ij}) \\ & + \frac{\mu_5}{2} \sum_{i=1}^N \sum_{j=1, j \neq i}^N V_{ij} \cdot V_{ji} + \frac{\mu_6}{2} \left[ \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left( \sum_{k=1, k \neq i, j}^N V_{ik} - 1 \right) \cdot V_{ij}^2 \right] \\ & + \frac{\mu_7}{2} \left[ \sum_{i=1}^N \sum_{j=1, j \neq i}^N \left( \sum_{k=1, k \neq i, j}^N V_{kj} - 1 \right) \cdot V_{ij}^2 \right] \end{aligned} \quad (2)$$

where

$$\gamma_i = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{if } i \neq s, d \end{cases}$$

and  $\{\mu_i\}$  are the constants. In eqn. 2, the  $\mu_1$  term minimises the