

## 1 Single Objective

### 1.1 Representation

The chosen representation was a simple list of the order by which each customer is visited. By keeping track of the capacity of the truck, it is possible to know at which point in the trip it is necessary to go back to the warehouse.

By not including zeros in the list, every individual is a valid path representation, and simply by making sure every mutation and crossover keeps the list as a permutation of distinct numbers from 1 to  $N$  is enough to generate valid individuals.

For this reason the crossover chosen was the Partially-mapped crossover (PMX), as it is designed to keep the offsprings as permutations. For mutation, two were experimented with: switching two elements of the individual and inverting some segment of the individual. The second one provided the best results, and was the one used.

Each simulation had an initial population of 40 individuals. Every generation the 3 fittest individuals were kept, and the 37 offsprings were generated by crossovers and mutations of the previous generation.

### 1.2 Results

The results obtained for running the evolutionary algorithm for 30 runs are summarized in table 1.

**Table 1:** Results obtained without heuristic.

#Customers	WHCentral_OrdFile		WHCentral_Ord50		WHCorner_OrdFile		WHCorner_Ord50	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
10	306.00	0.00	306.00	0.00	342.00	0.00	342.00	0.00
30	563.93	36.52	555.53	27.37	644.47	28.65	626.57	32.17
50	873.60	62.28	845.80	46.95	1065.00	47.88	1009.33	45.91

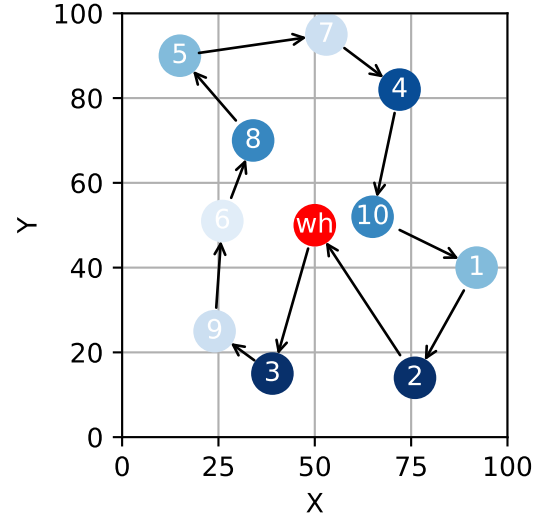
The algorithm was also run with the inclusion of a route obtained through the described heuristic on the initial population. The respective results can be seen on table 2.

All means and standard deviations decreased with this tactic. By looking at figures 3a, 3b and 3c, it is evident that the inclusion of the route generated with the heuristic allows the algorithm to start with a much better candidate solution, guaranteeing that better solutions are found.

## 2 Multiple Objective

### 2.1 Representation

The chosen representation was also a simple list of the order by which each customer is visited, exactly as the single objective. The difference lies on the evaluation function, that keeps track of when it is necessary to go to



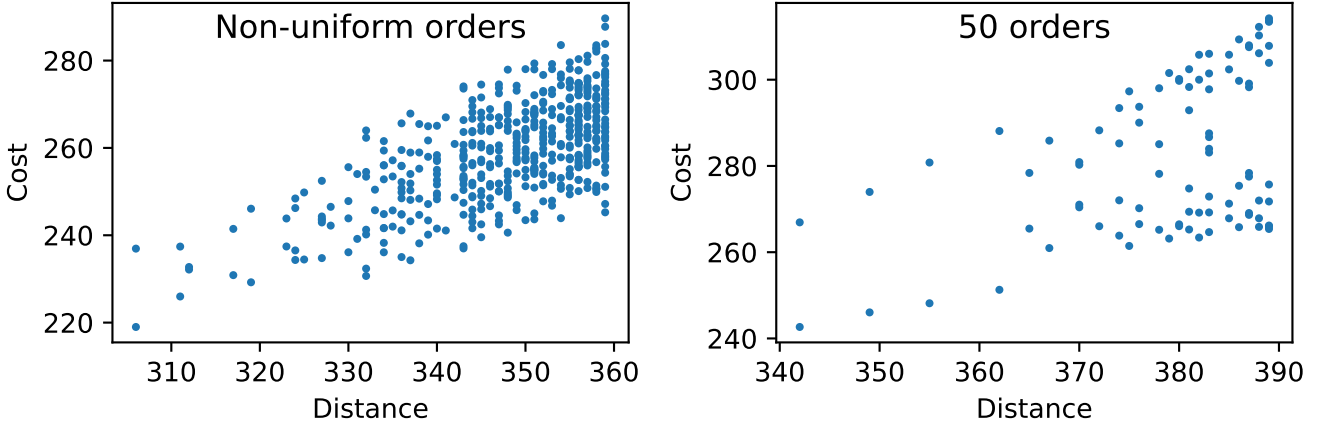
**Figure 1:** Route corresponding to the sequence [3, 9, 6, 8, 5, 7, 4, 10, 1, 2]. Darker tones of blue represent higher amounts of orders.

**Table 2:** Results obtained with heuristic.

#Customers	WHCentral_OrdFile		WHCentral_Ord50		WHCorner_OrdFile		WHCorner_Ord50	
	Mean	STD	Mean	STD	Mean	STD	Mean	STD
10	306.00	0.00	306.00	0.00	342.00	0.00	342.00	0.00
30	532.97	11.47	533.70	9.97	631.93	14.83	617.70	19.02
50	727.43	26.74	758.07	29.30	930.83	32.94	913.53	30.09

the warehouse (and adds the respective distance to and from the warehouse) and also keeps track of the cost. For normalization purposes, the cost is divided by 1000. When computing the hypervolume, the reference point used was [2000, 2000].

Since the problem is small enough for 10 costumers, an exhaustive search was performed to confirm results. We see on figure 2 that the Pareto front for both cases corresponds to a single point, the same one that minimizes the distance.

**Figure 2:** Exhaustive search for 10 costumers. Points with greater distance and cost values are not shown.

The results obtained are summarized in table 3, and the respective Pareto fronts and hypervolume evolution can be seen on figures 4a, 4b, and 4c.

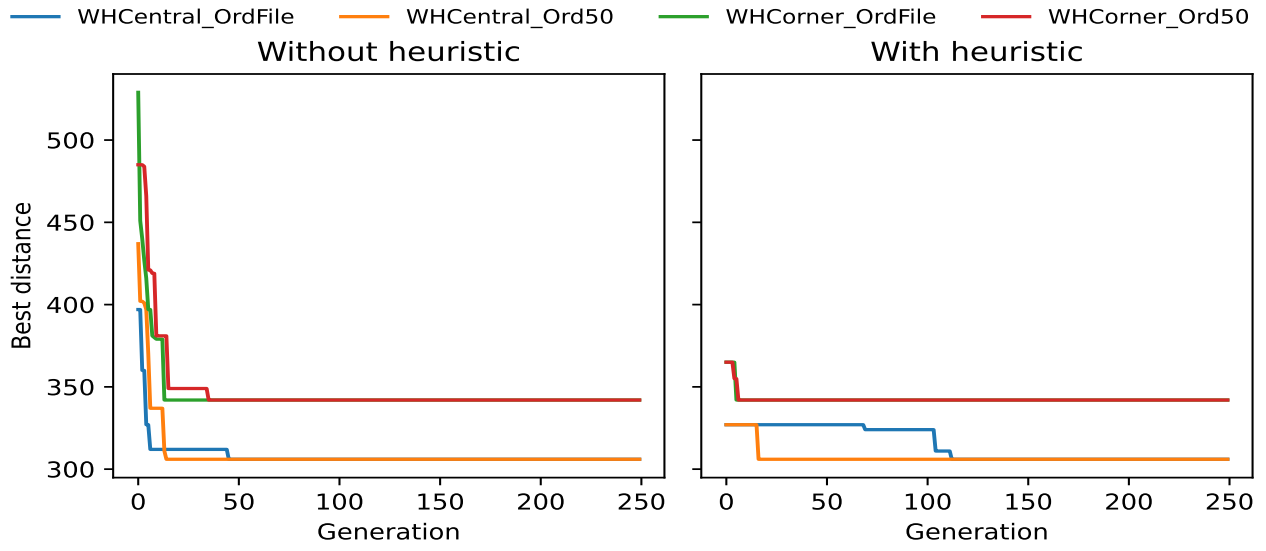
**Table 3:** Results obtained for the multiple objective optimization.

#Customers	Min Cost		Min Dist	
	Dist	Cost	Dist	Cost
10	306.0	219.0	306.0	219.0
30	627.0	264.38	528.0	288.93
50	945.0	430.6	864.0	476.77

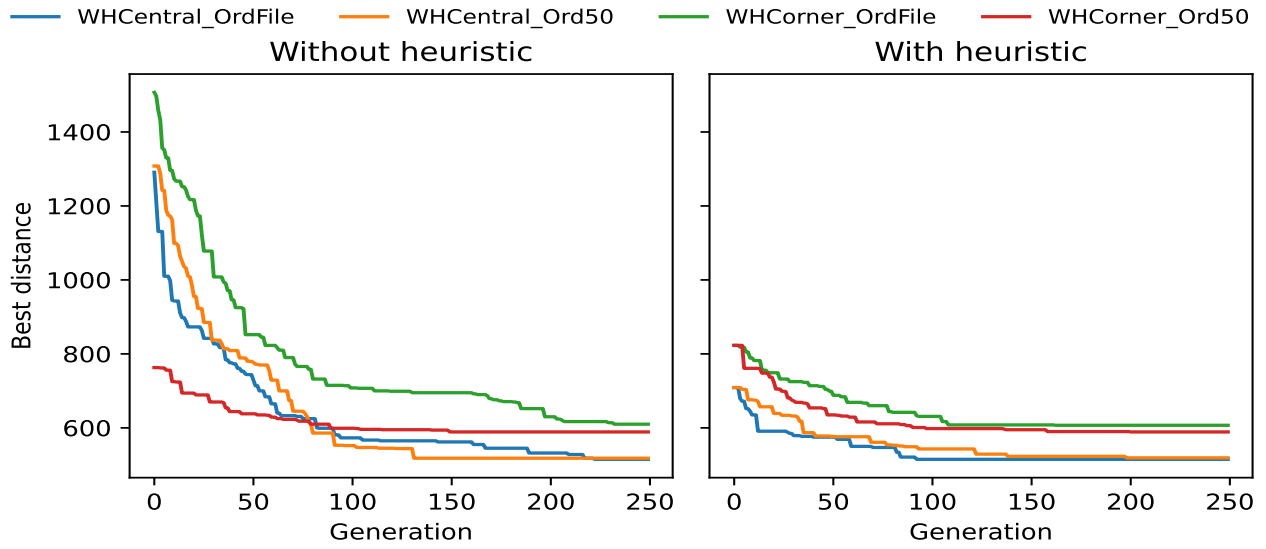
### 3 Concluding Remarks

In general, the results obtained were really good. On the 10 customer problem, the evolutionary algorithm always achieves the best solution (confirmed by exhaustive search). For 50 costumers, both the distance in the single-objective and the hypervolume in the multi-objective problems show an evolving trend, suggesting that with more generations it would be possible to achieve even better results. As expected, using the heuristic as a candidate solution yielded better results, because the evolution starts already with a great candidate solution.

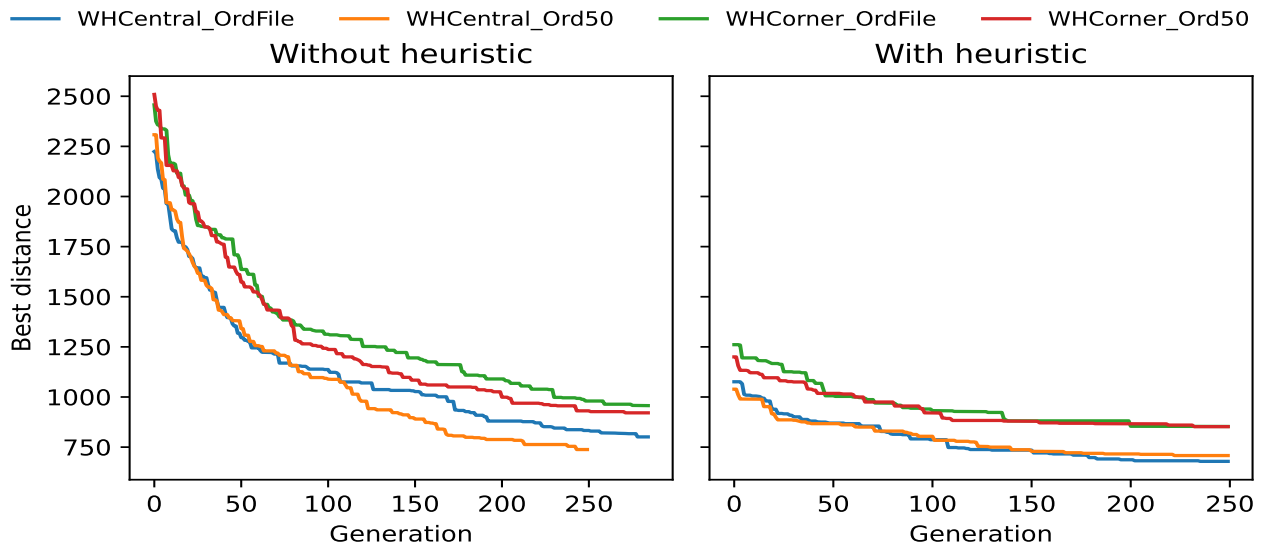
The results of the minimal distance obtained by the multi objective were in line with the results on the single objective problem, despite not being able to achieve the best individuals previously obtained.



(a) 10 costumers

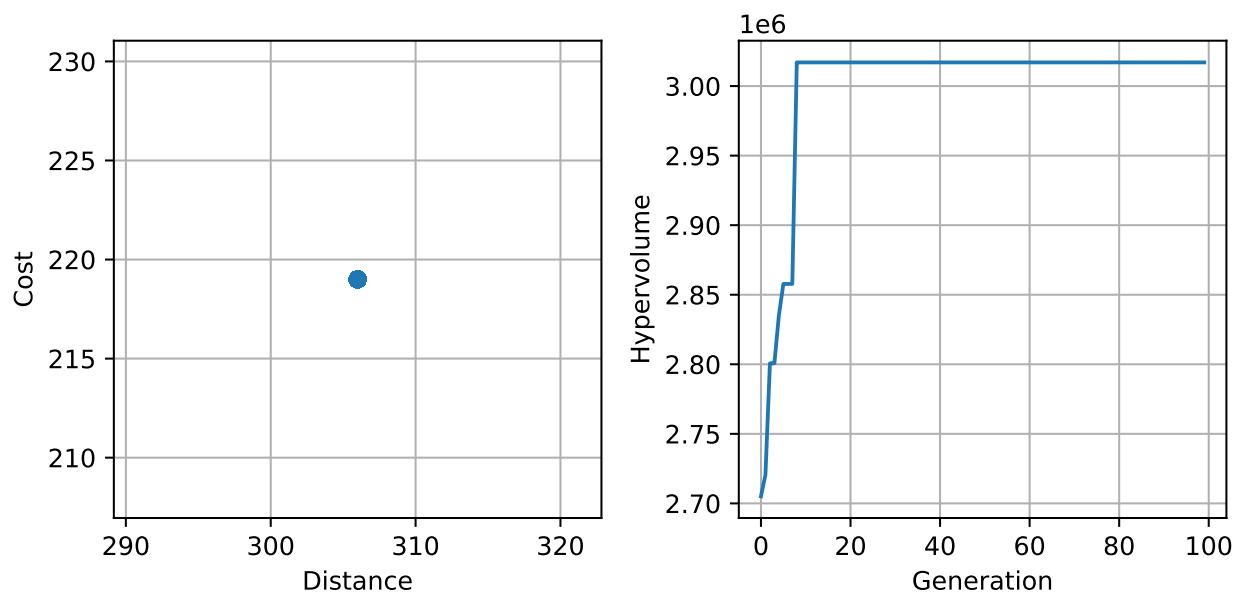


(b) 30 costumers

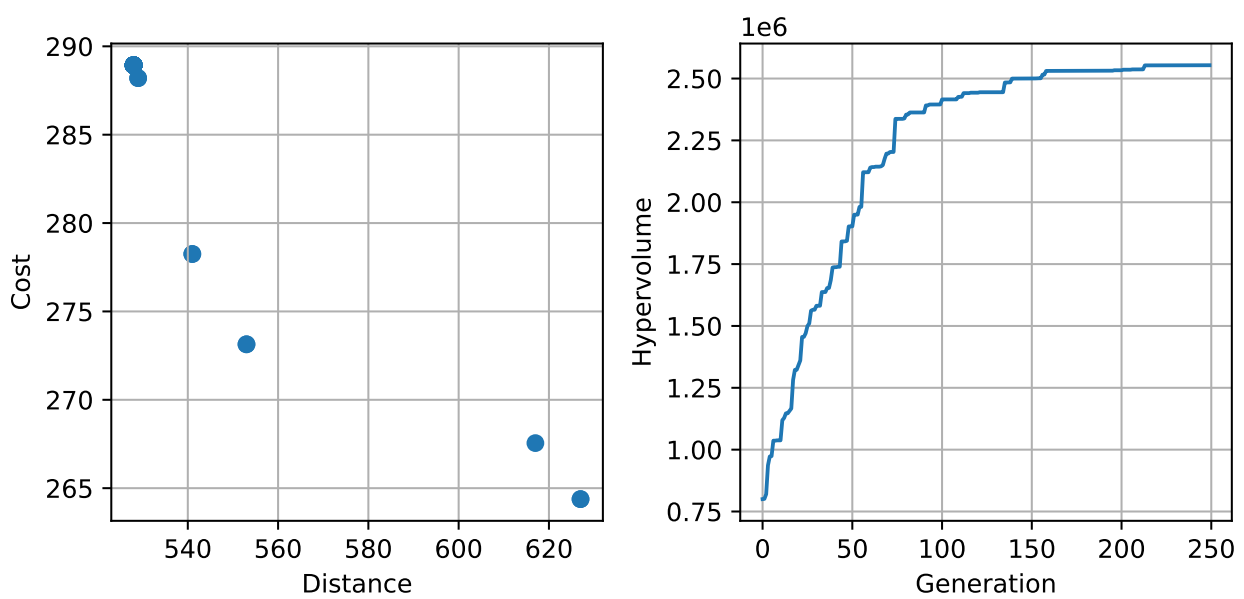


(c) 50 costumers

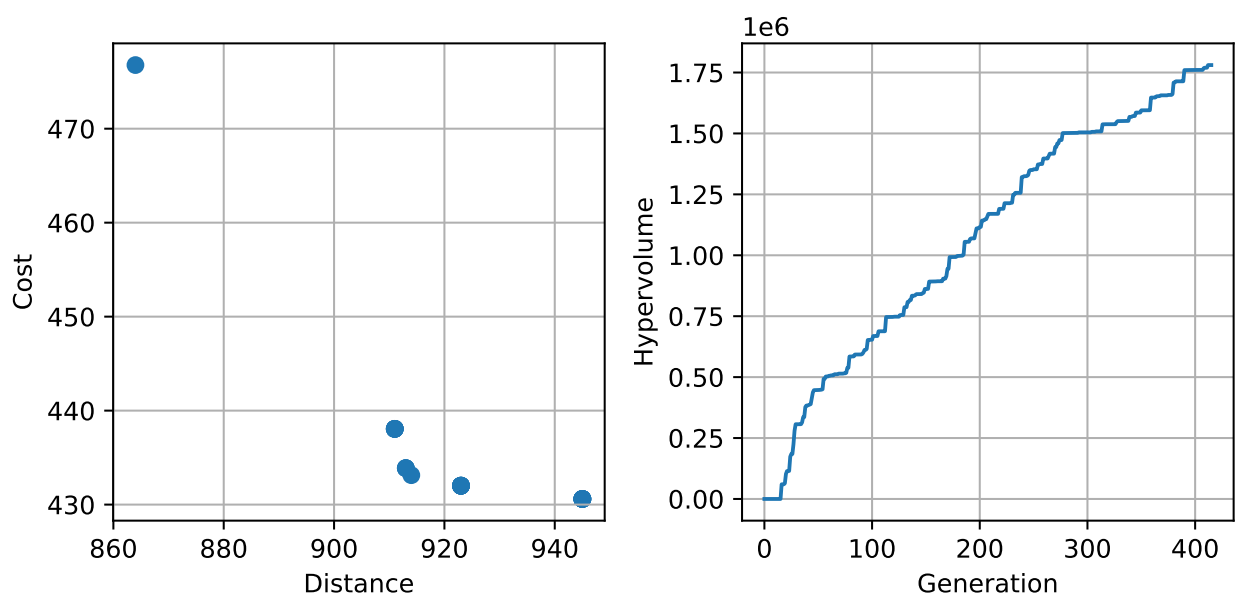
**Figure 3:** Evolutions of the best fitness obtained.



(a) 10 costumers



(b) 30 costumers



(c) 50 costumers

**Figure 4:** Pareto curves and evolution of the hypervolume.