Ideas for Belief Update Function (changing the fraction)

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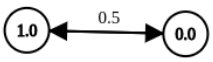
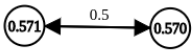
The belief formula multiplies the belief update function by $\frac{1}{|A|}$, which has a big drawback, the same subgraph behaves differently based on the size of the society, even though the subgraph is the same. Example:

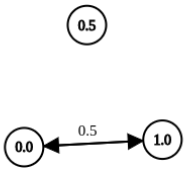
- Before:



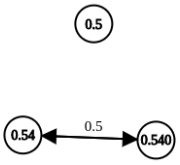
- After:



- Before:



- After:



This an small example, but it shows the presence of more agents, even though not influencing others directly, influences the end result, which is not a desired behaviour of the belief update.

My idea to fix this would be to substitute $|A|$ in the fraction by $|I_i|$, being $I_i = \{a_j | I(a_j, a_i) \geq 0\}$, and this small change would also give us the ability to model puppets pretty easily.

I believe that is clear that doing this guarantees us that a subgraph would behave in the same way independently on the rest of the society (if no external agent influences this subgraph, of course). So I am just going to show that it gives us the ability to model puppets (even puppets of a group). Suppose that a_i is a puppet of a_k, the only agent that has positive influence in it (1):

$$\begin{aligned} B^{t+1}(a_i) &= \frac{1}{I_i} \sum_{a_j \in A} (B^t(a_i) + I(a_j, a_i)(B^t(a_j) - B^t(a_i))) \\ &= \sum_{a_j \in A} (B^t(a_i) + I(a_j, a_i)(B^t(a_j) - B^t(a_i))) \\ &= B^t(a_i) + I(a_k, a_i)(B^t(a_k) - B^t(a_i)) \\ &= B^t(a_i) + B^t(a_k) - B^t(a_i) \\ &= B^t(a_k) \end{aligned}$$

Thus, this makes it easy to model puppets (you can also make an agent be a puppet of a group of agents, which is also not hard to see).

Although it is pretty limited, I think that this is an small change that has positive effects on the model. Another good thing is that, under the assumption that $I(a_i, a_i) > 0$ (i.e. no puppets), the old proofs on belief convergence still hold (probably with different constant factors, but the ideas stay the same).