Trying to Fix Lemma 2

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I didn't have had the time to do this nearly as formal as I would like, but here is my idea for fixing my mistakes in lemma 2.

My argument in lemma 2 actually shows that, if exists a path of length l between two agents a_i and a_k, than the belief of a_k in the time t influences the belief of a_j in the time $t+l$, I just used a special a_k, which I chose as the one that holds the belief min_t at the time t. The problem in my argument is that I stated that it is valid for all beliefs in the time $t+|A|-1$, but I only proved that the inequality holds for a_i in the time $t+|P(a_k|a_i)|$. Since the length of the paths can be smaller than $|A|-1$, the argument is incomplete.

Given that this is the problem with the argument, my plan to fix it is as follows:

Instead of jumping directly into lemma 2, we could use it as a corollary:

$$Bel_p^{t+|P(a_k,a_i)|} \leq max_t + \frac{In_{path-min}(min_t - max_t)}{|A|^{|P(a_k|a_i)|}}$$

Now we can prove lemma 2 using some inductive reasoning. If there is a path of length $|A|-1$ from a_k to a_i, lemma 2 holds.

If this path doesn't exist, then a smaller path exist. Thus a_i was influenced by min_t in some time step before $t+|A|-1$. But the belief in the next step is influenced by the step before. Thus we can carry it throughout time until $t+|A|-1$. My idea to do so is listed above:

$$\begin{aligned} Bel_p^{t+|A|-1}(a_i) &= Bel_p^{t+|A|-2}(a_i) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \\ &= Bel_p^{t+|A|-3}(a_i) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-3}(a_i|a_j) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \end{aligned}$$

And we can repeat this step until $Bel_p^{t+|P(a_k,a_i)|}$ appears in the inequity. When it does, we can use the corollary explained above to substitute it and transform the equation in an inequity (note that this expansion could go way longer, I am using a small case to simplify things):

$$\begin{aligned} &Bel_p^{t+|A|-1}(a_i) \leq \\ &max_t + \frac{In_{path-min}(min_t - max_t)}{|A|^{|P(a_k|a_i)|}} + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-3}(a_i|a_j) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \end{aligned}$$

We should also replace the occurrences of it in the first summation:

Let's call $\alpha = \frac{In_{path-min}(min_t - max_t)}{|A|^{|P(a_k|a_i)|}}$, to save space.

$$\begin{aligned} &Bel_p^{t+|A|-1}(a_i) \\ &\leq max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-3}(a_i|a_j) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \\ &= max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A} In(a_j,a_i)(Bel_p^{t+|A|-3}(a_j) - Bel_p^{t+|A|-3}(a_i)) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \\ &= max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A} In(a_j,a_i)(Bel_p^{t+|A|-3}(a_j) - max_t - \alpha) + \frac{1}{|A|}\sum_{a_j \in A} Bel_p^{t+|A|-2}(a_i|a_j) \end{aligned}$$

We can then take one element out of the summation: Let's call the second summation S_2, because our space is pretty limited.

$$
\begin{aligned}
&Bel_p^{t+|A|-1}(a_i) \\
&= max_t + \alpha + \frac{1}{|A|}\left(\sum_{a_j \in A\setminus\{a_i\}} In(a_j,a_i)(Bel_p^{t+|A|-3}(a_j) - max_t - \alpha) + Bel_p^{t+|A|-3}(a_i|a_i)\right) + S_2 \\
&= max_t + \alpha + \frac{1}{|A|}\left(\sum_{a_j \in A\setminus\{a_i\}} In(a_j,a_i)(Bel_p^{t+|A|-3}(a_j) - max_t - \alpha)\right) + S_2
\end{aligned}
$$

By the definition of max and the fact that it is decreasing:

$$
\begin{aligned}
&Bel_p^{t+|A|-1}(a_i) \\
&\leq max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A\setminus\{a_i\}} In(a_j,a_i)(max_t - max_t - \alpha) + S_2 \\
&= max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A\setminus\{a_i\}} In(a_j,a_i)(-\alpha) + S_2
\end{aligned}
$$

Since the influence can be at most 1:

$$
\begin{aligned}
Bel_p^{t+|A|-1}(a_i) &\leq max_t + \alpha + \frac{1}{|A|}\sum_{a_j \in A\setminus\{a_i\}} (-\alpha) + S_2 \\
&= max_t + \alpha + \frac{(|A|-1)(-\alpha)}{|A|} + S_2 \\
&= max_t + \frac{\alpha}{|A|} + S_2
\end{aligned}
$$

Now we know that $max_t + \frac{\alpha}{|A|}$ is the biggest value of $Bel_p^{t+|A|-2}$ possible, thus we can repeat the argument until we to lemma 2, basically carrying the influence of min_t (which is hidden in α) until the time $t+|A|-1$.

I tried to explain it as well as I could, although I don't think it was very successful.