Proof of Belief Convergence and Other Discoveries

Weakly Connected - Classic Update

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min^t and max^t definition

Definition

Let's call min^t and max^t the minimum and maximum of the beliefs in time t over all agents, respectively. Thus:

$$min^t = \min_{a_i \in A} Bel_p^t(a_i) \text{ and } max^t = \max_{a_i \in A} Bel_p^t(a_i)$$

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- This means that both *min* and *max* are monotonic.
- It can be showed (Corollary 2) that this implies that *min* and *max* converge to L and U, respectively.

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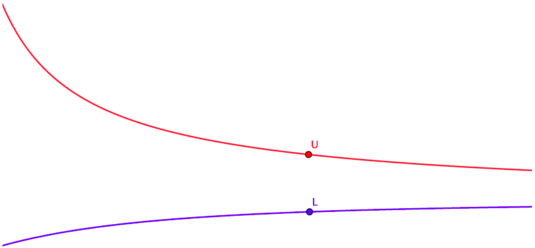

Theorem 1

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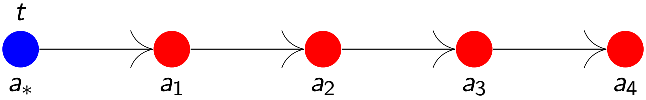
- Using this information we choose an extreme agent a_*^t in time t (the agent who holds the belief min^t) and try to trace the influence it exerts in the rest of the society.
- It can be showed (Theorem 1) that doing so guarantees us that a_*^t influences every a_i by a factor of δ^t.

What the theorem means?

- The theorem states that the influence exerted by a_*^t guarantees that, by the time $t + |P(a_*^t \to a_i)|$ every agent is less extreme than max^t by a factor of at least δ^t.



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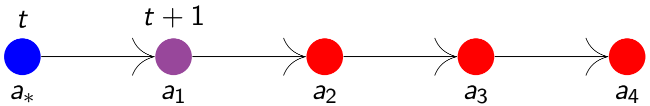

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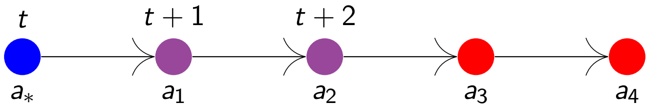

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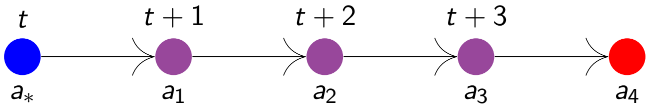

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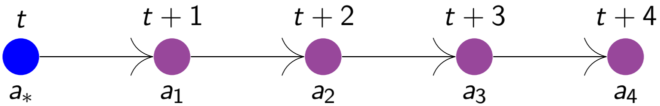

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- To solve this we can use an important piece of information to acquire an idea about the agents belief in the same time step.

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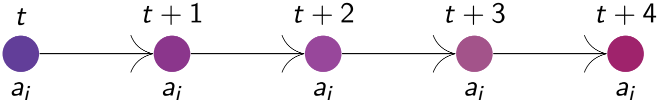

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    - The smallest non negative influence.
    - The size of the society.
    - L and U, the limits of minimum and maximum.

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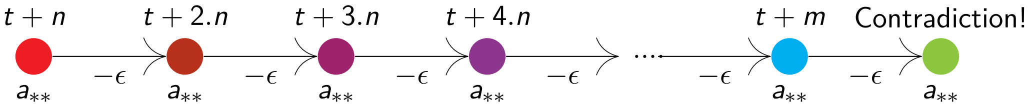

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- Since the limits of maximum and minimum are equal, every belief equal in the limit, as we wanted to prove.

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- In general the proof is the same as the one showed above but with different constant factors.
- There are some corner cases we must address, but we solve it using a similar approach used with a_*^t in the proof above.

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- The idea is, for short, to use the proof showed above in weakly connected subgraphs and thus guaranteeing conversion of each subgraph.

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- Experiments showed that, under the backfire-effect update function, *min* and *max* are not monotonic.
- Since this property is crucial for the proof showed above, I don't think that this is the way to prove convergence in this case.

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- In some graphs, the initial belief of some agents does not affect their own belief in the limit. (found via experiments).
- In the graph "unrelenting influencers" the belief in the limit *seems* to be equal to the average of the beliefs of the influencers weighted by their influence (found via experiments).