

Proof of Belief Convergence

All Graphs Proof Scratch

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Definition

Let's call I_i the influencers of the agent a_i , which is defined as:

$$\{a_j \in A \mid \exists P(a_j|a_i) \}$$

An useful lemma

Lemma

$$\exists P(a_j|a_i) \iff I_j \subseteq I_i$$

Proof.

Let's suppose that $\exists P(a_j|a_i)$.

Now suppose $a_k \in I_j$. By definition this means that $\exists P(a_k|a_j)$. But since $\exists P(a_j|a_i)$, there will also exist $P(a_k|a_i)$. Thus, by definition of influencers $a_k \in I_i$. Since a_k was arbitrary, $I_j \subseteq I_i$.

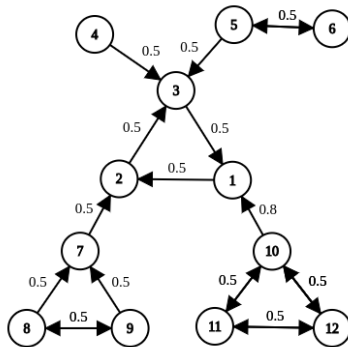
Proving the other side of the implication is trivial given the definition of influencers. \square

Partial order

Note that the influencers are basically sets of agents, thus they are a partial order. Keeping that in mind, we will look at the minimal elements of the influencers, but first let's look at an influence graph.

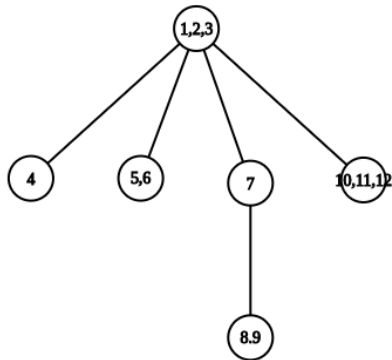
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Partial order representation

Drawing a partial order representation of the influencers of the previous graph we end up with:



Lemma

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Proof.

Suppose $a_j, a_k \in I_i$. Thus $\exists P(a_k|a_i)$ and $\exists P(a_j|a_i)$.

Since I_i is minimal, there must exist $P(a_i|a_j)$ and $P(a_i|a_k)$. This is true because, according to Lemma 1, $I_j \subseteq I_i$ and $I_k \subseteq I_i$. But if $\nexists P(a_i|a_j)$ or $\nexists P(a_i|a_k)$ then either $I_j \subset N_i$ or $I_k \subset I_i$.

Which contradicts our hypothesis. Since $\exists P(a_k|a_i) \wedge \exists P(a_i|a_j)$, $\exists P(a_k|a_j)$. And the same goes the other way around, since both a_j and a_k where arbitrary, it is true for all, which means there is a path between any two agents in the influencers. Does it is weakly connected. \square

Next steps

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- Given that the minimal influencers are weakly connected and that they don't receive external influence (I didn't prove this property but it kind of follows from 1).
- It will converge according to the other proof (if it is correct, of course). This might give us a good start on how to continue the proof. But the most useful part of this would be, to me, the ability to assume that the minimal elements have a constant belief, since they converge. This would be really useful but I don't know if I can assume it.

- After running some experiments, I believe that the general case might not be as random as I first thought. Apparently who rules the beliefs in the limits are the minimal elements, because they converge and aren't affected by the others.

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- Also, the simulation I made in the graph showed above actually resulted in the fact that changing the beliefs of agents who aren't from minimal influencers do not change the beliefs in the limit.
- What I believe now is that the elements who are not minimal converge to the weighted average of all paths from elements in minimal influencers multiplied by its beliefs.