Convergence Rates in Cliques Using Classic Update Function

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March 2020

Definition 1

$$Bel_p^{t+1}(a_i) = (1-c) \times Bel_p^t(a_i) + \frac{c}{|A|}\ S_t \tag{1}$$

As was showed in my previous writing on cliques.

Definition 2 *We say that a series converges linearly to L if* $\exists y \in (0,1)$*:*

$$lim_{k\to\infty}\frac{|x_{k+1}-L|}{|x_k-L|} = y \tag{2}$$

Definition 3 *We say that a series converges superlinearly (faster than linearly) to L if:*

$$lim_{k\to\infty}\frac{|x_{k+1}-L|}{|x_k-L|} = 0 \tag{3}$$

For the sake of simplifying notation, let's call S_t , S. and $Bel_p^t(a_i)$, Bel^t.

To find the convergence rate of our series, we must remember that it converges to $\frac{S}{|A|}$. That being said, let's go to the calculations:

$$\begin{aligned}
lim_{t\to\infty}\frac{|Bel^{t+1}-L|}{|Bel^t-L|} &= lim_{t\to\infty}\frac{|(1-c)\times Bel^t + \frac{c}{|A|}\ S - L|}{|Bel^t - L|}\\
&= lim_{t\to\infty}\frac{|(1-c)\times Bel^t + \frac{c}{|A|}\ S - \frac{S}{|A|}|}{|Bel^t - \frac{S}{|A|}|}\\
&= lim_{t\to\infty}\frac{|(1-c)\times Bel^t + \frac{S}{|A|}(c-1)|}{|Bel^t - \frac{S}{|A|}|}\\
&= lim_{t\to\infty}\frac{|(1-c)\times Bel^t - \frac{S}{|A|}(1-c)|}{|Bel^t - \frac{S}{|A|}|}\\
&= lim_{t\to\infty}\frac{|(1-c)(Bel^t - \frac{S}{|A|})|}{|Bel^t - \frac{S}{|A|}|}
\end{aligned} \tag{4}$$

Since $c \leq 1$ and $0 \leq 1-c$:

$$\begin{aligned} lim_{t\to\infty}\frac{|Bel^{t+1}-L|}{|Bel^{t}-L|} &= lim_{t\to\infty}\frac{(1-c)|(Bel^{t}-\frac{S}{|A|})|}{|Bel^{t}-\frac{S}{|A|}|} \\ &= 1-c \end{aligned} \tag{5}$$

Thus, by the definitions we gave above, if $1-c=0$, our series converge superlinearly. This happens when $c=1$, which makes sense, since this is a special case in which all agent opinions converge right after the first update. On the other hand, if $0<c<1$, our series converge linearly. Since the smaller the y, the faster the series converges, the speed is directly proportional to the influence c, which also makes sense intuitively.