Proof Scratch

Classic Update - General Case

Bernardo Amorim

Universidade Federal de Minas Gerais

April 21, 2020

Some definitions

Definition

Let's call the sequence $P(a_i|a_j) = (a_i, a_k, ..., a_{k+l})$ *a path from* a_i *to* a_j*, if:*

- *All elements on the sequence are different.*
- *The first element in the sequence is* a_i*.*
- a_j *doesn't belong to the sequence.*
- *If* a_n *is the* n^{th} *element in the sequence and it has a successor* a_{n+1}*,* $In(a_n, a_{n+1}) > 0$*.*
- *If* a_n *is the last element in the sequence* $In(a_n, a_j) > 0$*.*

Definition

Let's call N_i *the neighbourhood of the agent* a_i*, which is defined as:*

$$\{a_j \ \in A \mid \exists \ P(a_j|a_i) \ \}$$

An useful lemma

Lemma

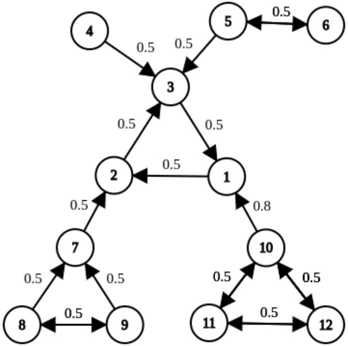
$$\exists\ P(a_j|a_i) \iff N_j \subseteq N_i$$

Proof.

Let's suppose that $\exists P(a_j|a_i)$.
Now suppose $a_k \in N_j$. By definition this means that $\exists P(a_k|a_j)$. But since $\exists P(a_j|a_i)$, there will also exist $P(a_k|a_i)$. Thus, by definition of neighbourhood $a_k \in N_i$. Since a_k was arbitrary, $N_j \subseteq N_i$. Proving the other side of the implication is trivial given the definition of neighbourhood. □

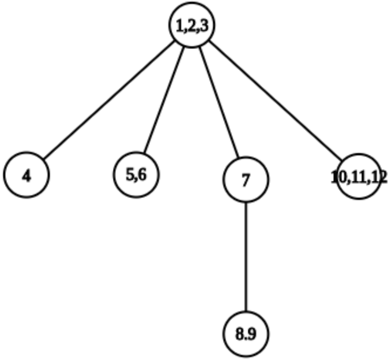
Partial order

Note that the neighbourhoods are basically sets of agents, thus they are a partial order. Keeping that in mind, we will look at the minimal elements of the neighbourhoods, but first let's look at an influence graph.



Partial order representation

Drawing a partial order representation of the neighbourhoods of the previous graph we end up with:



Lemma

Looking at the minimal elements of the graph above we should note that the neighbourhoods of all of them are weakly connected, which leads us to our lemma:

Lemma

$$\nexists a_j \ (N_j \subseteq N_i) \implies N_i \textit{ is weakly connected.}$$

Proof.

Suppose $a_j, a_k \in N_i$. Thus $\exists P(a_k|a_i)$ and $\exists P(a_j|a_i)$.
Since N_i is minimal, there must exist $P(a_i|a_j)$ and $P(a_i|a_k)$. This is true because, according to Lemma 1, $N_j \subseteq N_i$ and $N_k \subseteq N_i$. But if $\nexists P(a_i|a_j)$ or $\nexists P(a_i|a_k)$ then either $N_j \subset N_i$ or $N_k \subset N_i$. Which contradicts our hypothesis. Since $\exists P(a_k|a_i) \wedge \exists P(a_i|a_j)$, $\exists P(a_k|a_j)$. And the same goes the other way around, since both a_j and a_k where arbitrary, it is true for all, which means there is a path between any two agents in the neighbourhood. Does it is weakly connected. $\square$

Next steps

Given that the minimal neighbourhoods are weakly connected and that they don't receive external influence (I didn't prove this property but it kind of follows from 1). It will converge according to the other proof (if it is correct, of course). This might give us a good start on how to continue the proof. But the most useful part of this would be, to me, the ability to assume that the minimal elements have a constant belief, since they converge. This would be really useful but I don't know if I can assume it.

Personal thoughts

After running some experiments, I believe that the general case might not be as random as I first though. Apparently who rules the beliefs in the limits are the minimal elements, because they converge and aren't affected by the others. Also, the simulation I made in the graph showed above actually resulted in the fact that changing the beliefs of agents who aren't from minimal neighbourhoods do not change their belief in the limit.
What I believe now is that the elements who are not minimal converge to the weighted average of all paths from elements in minimal neighbourhoods multiplied by its beliefs.