New Polarization Measure Ideas

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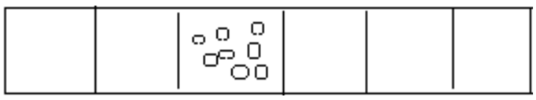
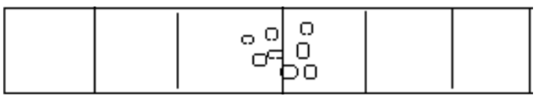
NOTE: How is the weight of y_i defined on the original paper? It says things about the range it operates on, but not about its value.

Problems of the current measure:

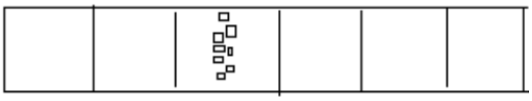
- Small changes in belief make so that an agent is placed in other group, causing discontinuities on polarization. Examples:
  - Polarization = 0:

  - Polarization $\neq 0$:

- It ignores belief differences inside the same bucket which causes constant polarization, although beliefs are getting closer! This is specially problematic with big buckets. I also think that there is an easy fix for part of this problem, which will be explained later. Examples:
  - Polarization = x:

  - Polarization = x:

- It uses "thresholds" do decide which group does an agent belongs to, which makes things harder to prove and also contributes to the discontinuity problem explained above. My opinion now, having read the Esteban-Ray paper is that this might be the hardest problem to solve (assuming it can be solved). The problem is that polarization has as much to do with inner group homogeneity as it has to outer group heterogeneity, and that is where the problem lies: our measure must account, directly or indirectly, for the size of each group. Does, my view now is that the hypothetical new polarization measure still must count the number of agents in each group, but it may be possible to “smooth” things out.

- It only allows us to express opinions on one dimension, which is a limiting factor for measuring more complex situation, when we consider beliefs about many subjects.

To solve this problems I have ideas on some changes in the Esteban Ray measure. So, I am going to use some simulations (in one dimension, two dimensions will be addressed later) as a benchmark so we can compare them.

Ideas:

- Use the weighted averages of the beliefs to represent each group. Apparently (I am not 100% sure of this) the original measure uses the distance from the left side of the bin interval to calculate polarization, but this hides the fact that, inside the same bin, elements might be closer to one side than to another. It also makes polarization constant if no agent change groups, which doesn't make any sense intuitively. So my idea is to calculate the distance using the average position of the elements in each bin.

  Simulation 1 -
  Configurations:

  ```
  pol_sim = InfGraph(num_agts=100)

  pol_sim.build_bel(bel_type="uniform")
  pol_sim.build_graph(graph_type="faintly-connected")

  pol_sim.simulate(time=100, save_pol=True,
  save_bel=True, K=1000, a=1.6)
  ```

  In this simulation we can see that, specially when we have larger bins, polarization tents to stay constant in the old measure, although the beliefs are getting closer.

  - Belief evolution: 1
  - 11 bins: 4
  - 21 bins: 7
  - 201 bins: 10

  Simulation 2 -
  Configurations:

  ```
  pol_sim = InfGraph(num_agts=100)
  ```

```
pol_sim.build_bel(bel_type="trypartite")
pol_sim.build_graph(graph_type="faintly-connected")

pol_sim.simulate(time=20, save_pol=True,
save_bel=True, K=1000, a=1.6)
```

In this simulations we can see the same effect pointed above, specially when we look at the graphic with 3 bins. Apparently this new measurement makes so that larger groups can be used without loosing information between the group transitions.

- Belief evolution: 11
- 3 bins: 14
- 11 bins: 17
- 21 bins: 20
- 201 bins: 23

Simulation 3 -
Configurations:

```
pol_sim = InfGraph(num_agts=2)
pol_sim.build_bel(bel_type="uniform")
pol_sim.change_edge(0,1,0.1)
pol_sim.change_edge(1,0,0.1)

pol_sim.simulate(time=30, save_pol=True,
save_bel=True,k=4,K=1000, a=1.6)
```

I built this case to show the disadvantages of the original measurement. Polarization clearly diminishes steadily, since there are only two agents which are converging, but, in the original measurement, it basically stays constant, and then diminishes abruptly when they change from bins 0 and 3 to bins 1 and 2. While polarization in the new measurement decays as expected.

- Belief evolution: 24
- 4 bins: 27

• Fuzzy position. Instead of using the belief (which is sharp) to define the group of the agent. We could say that his opinion has a small "area" of effect, thus an agent could be 60% group 1 and 40% group 2. Conclusion: it does not work. I tried it and it has a giant drawback, if all agents are in the same spot, but this spot is close to the frontier between two bins, polarization doesn't converge.

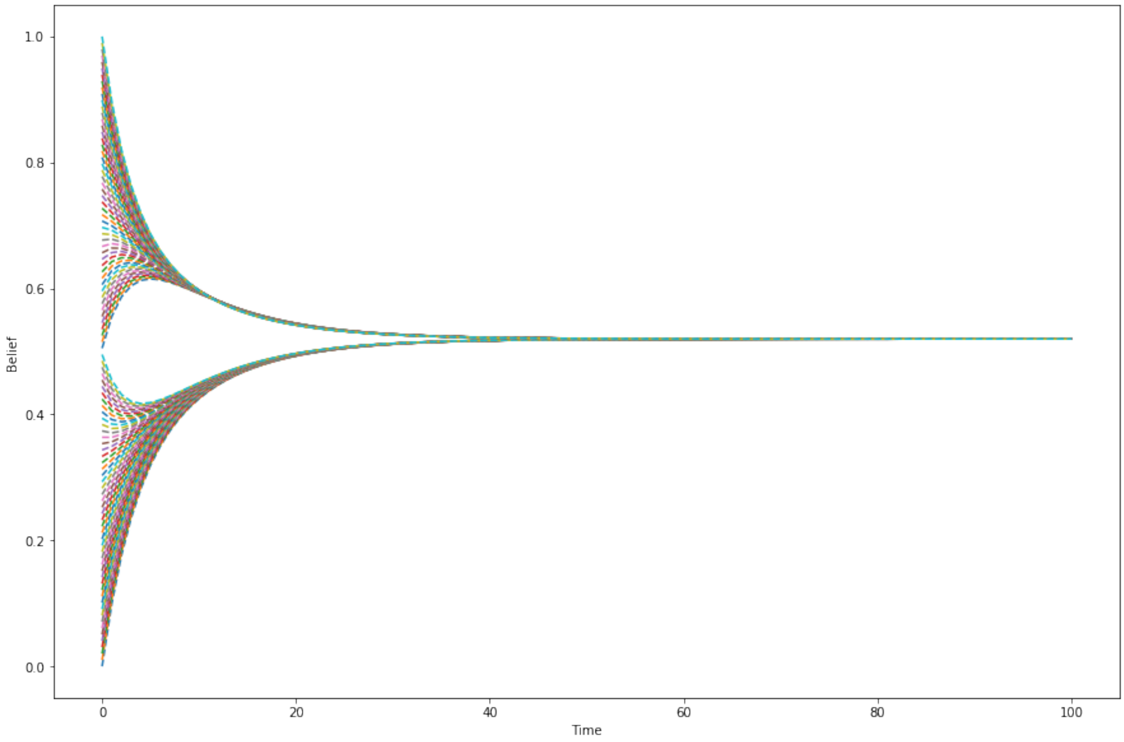
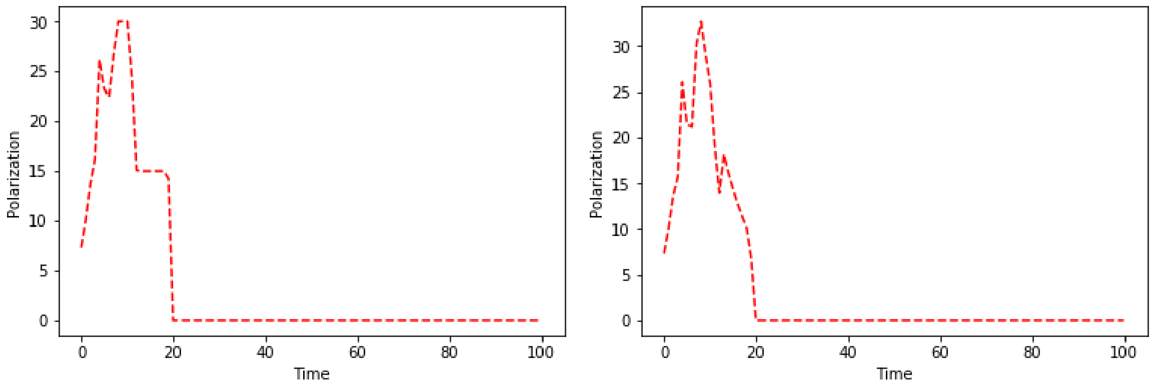


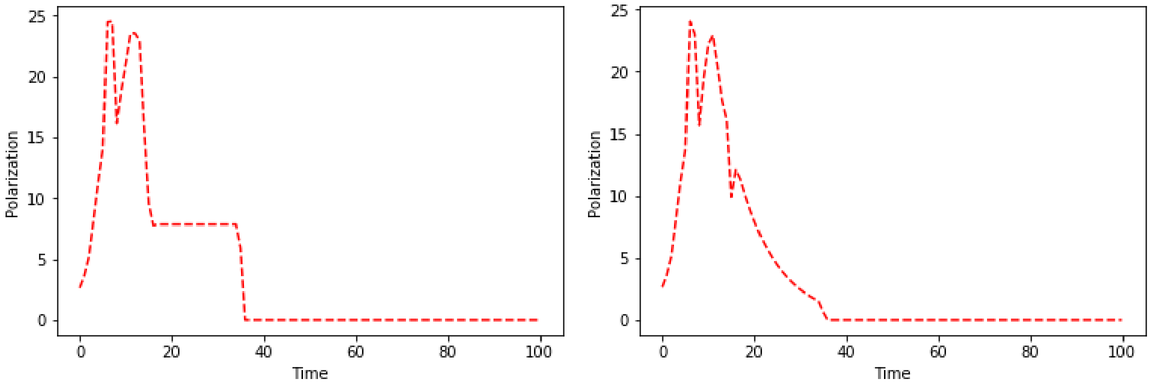
Figure 1: Belief evolution



Original

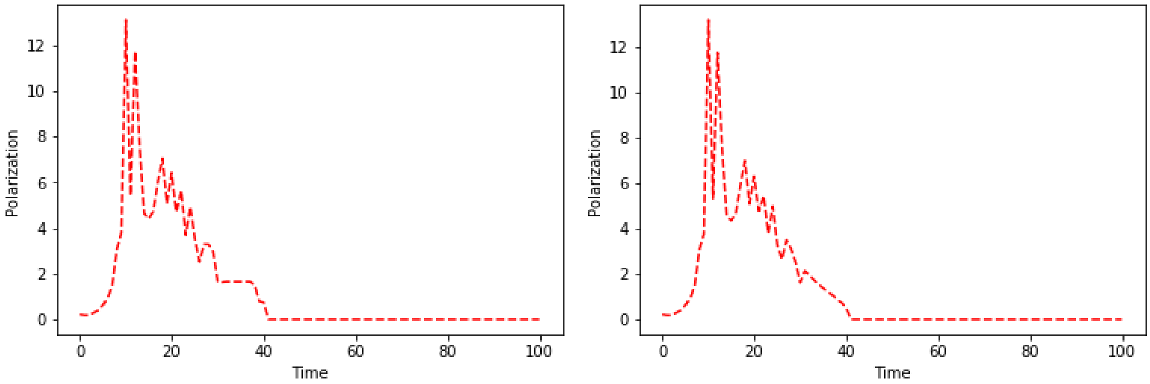
New measure

Figure 4: k = 11



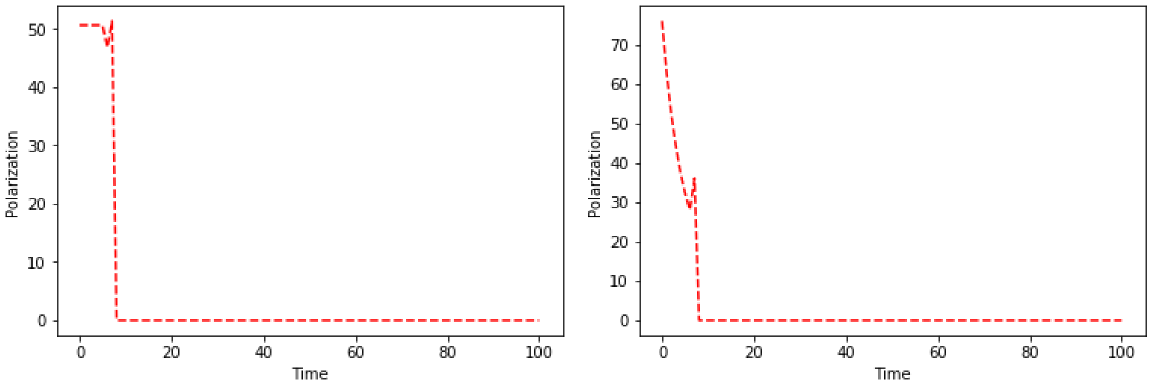
Original New measure

Figure 7: k = 21



Original New measure

Figure 10: k = 201



Original New measure

Figure 14: k = 3

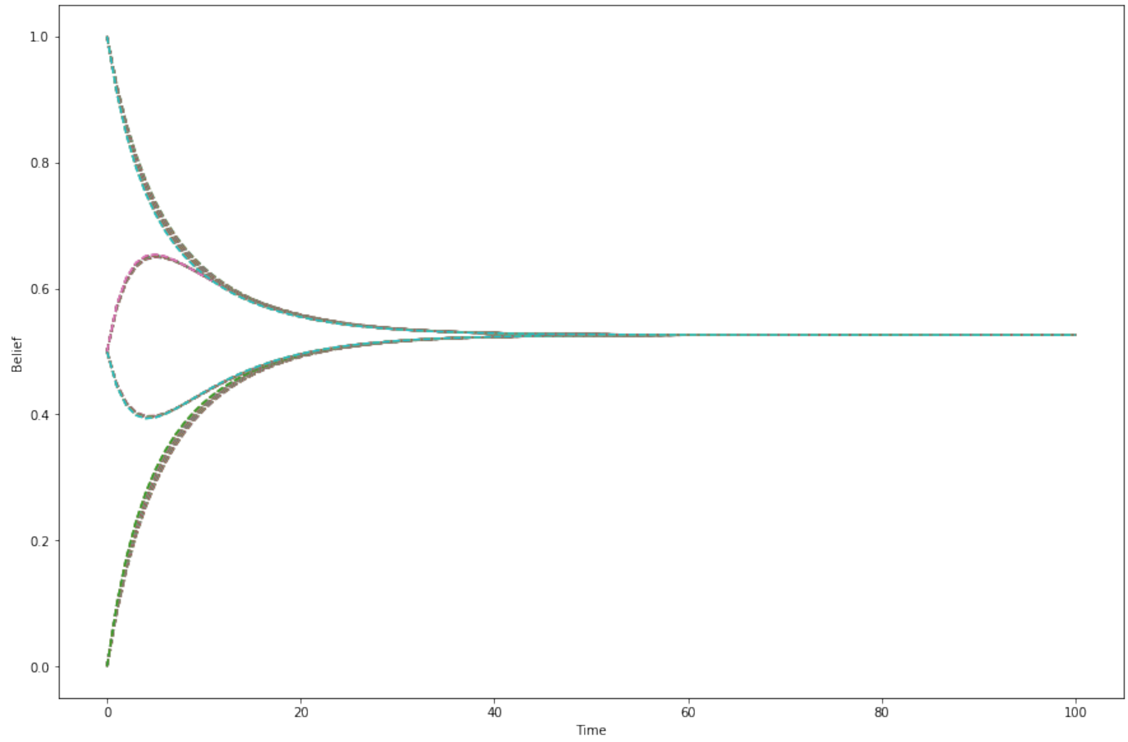
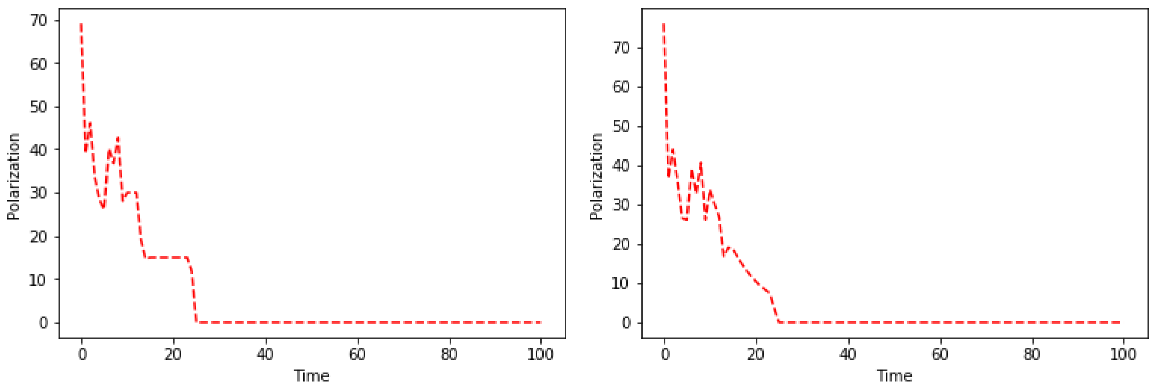


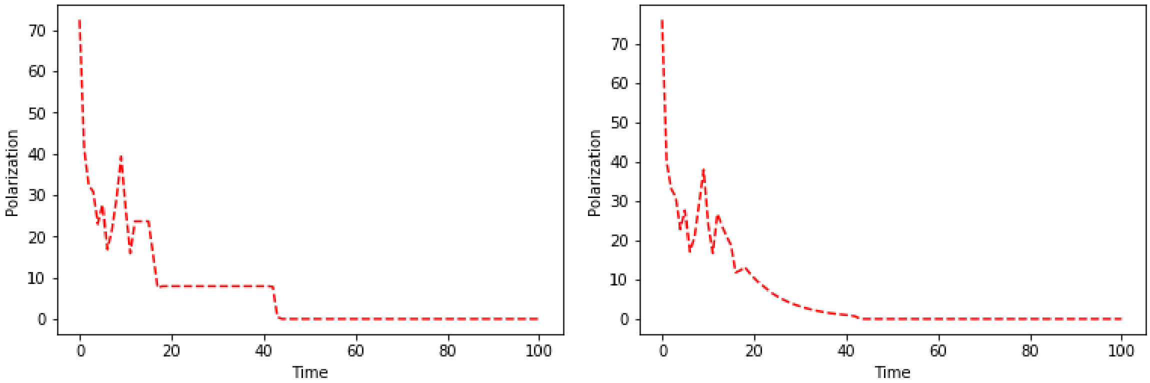
Figure 11: Belief evolution



Original

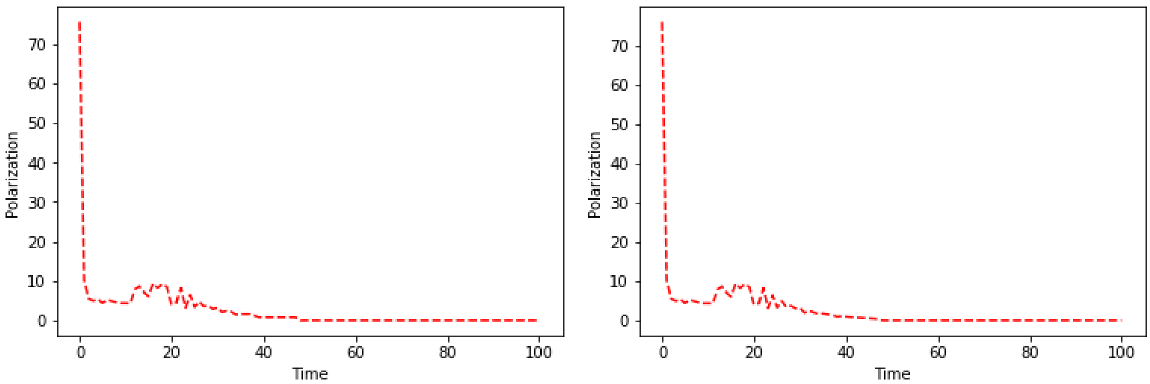
New measure

Figure 17: k = 11



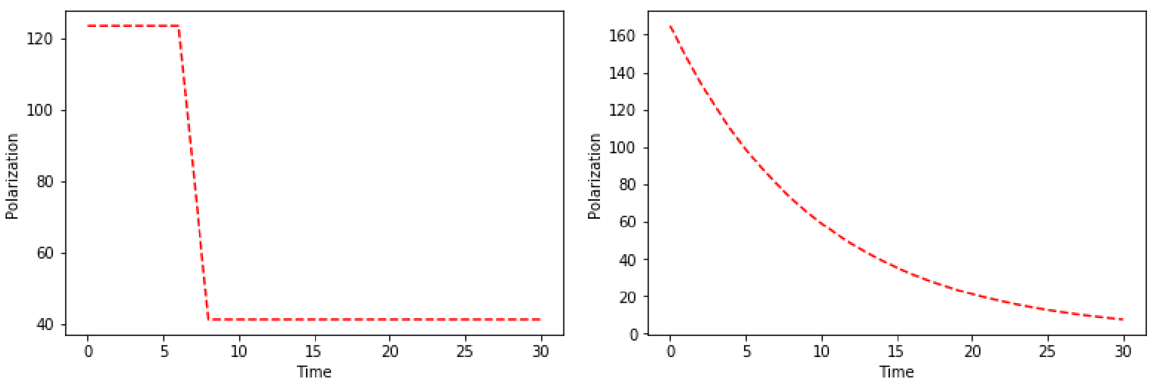
Original New measure

Figure 20: k = 21



Original New measure

Figure 23: k = 201



Original New measure

Figure 27: k = 201

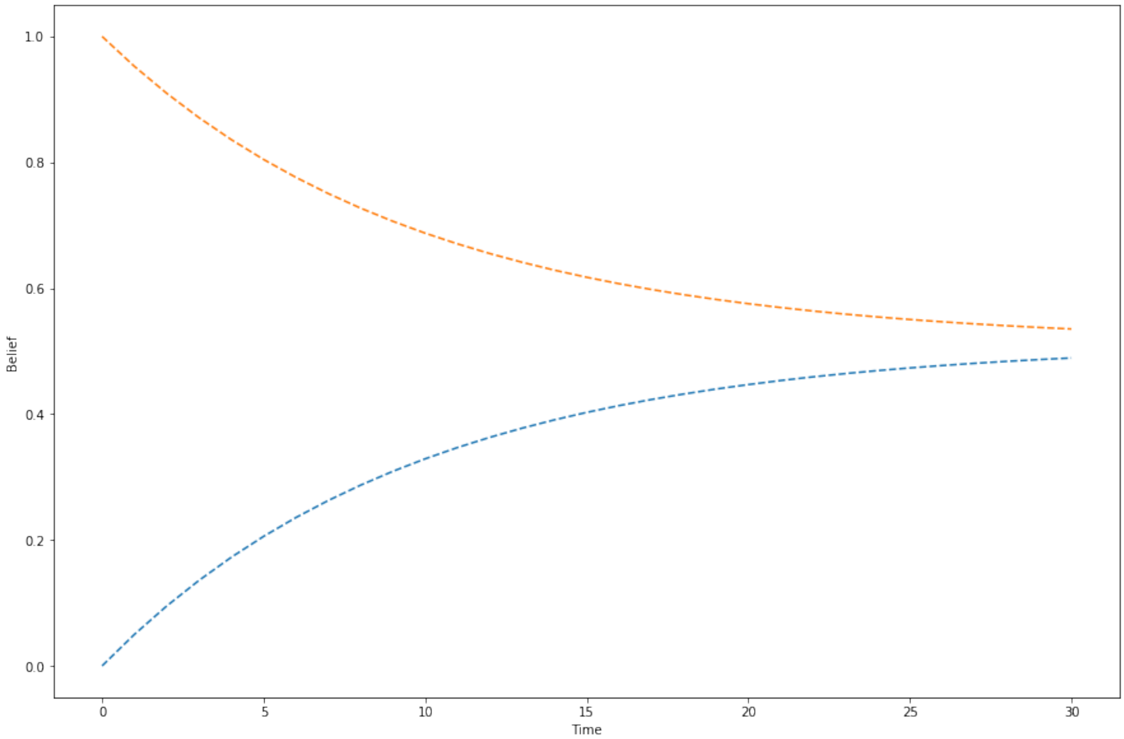


Figure 24: Belief evolution