Convergence Rates in Constant Cliques Using Classic Update Function

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May 2020

Definition 1.

$$Bel_p^{t+1}(a_i) = Bel_p^t(a_i) + \frac{1}{|A|}\sum_{a_j \in A} In(a_j, a_i)(Bel_p^t(a_j) - Bel_p^t(a_i))$$

Definition 2. We say that a series converges linearly to L if $\exists y \in (0,1)$:

$$\lim_{t\to\infty} \frac{|x_{k+1} - L|}{|x_k - L|} = y$$

Definition 3. We say that a series converges superlinearly (faster than linearly) to L if:

$$\lim_{t\to\infty} \frac{|x_{k+1} - L|}{|x_k - L|} = 0$$

For the sake of simplifying notation, let's call S_t , S. and $Bel_p^t(a_i)$, Bel^t.
As was showed in my previous writing on constant cliques:

$$Bel_p^{t+1}(a_i) = (1-c) \times Bel_p^t(a_i) + \frac{c}{|A|}\ S_t$$

To find the convergence rate of our series, we must remember that it converges to $\frac{S}{|A|}$. That being said, let's go to the calculations:

$$\begin{aligned}
\lim_{t\to\infty} \frac{|Bel^{t+1} - L|}{|Bel^t - L|} &= \lim_{t\to\infty} \frac{|(1-c) \times Bel^t + \frac{c}{|A|}\ S - L|}{|Bel^t - L|} \\
&= \lim_{t\to\infty} \frac{|(1-c) \times Bel^t + \frac{c}{|A|}\ S - \frac{S}{|A|}|}{|Bel^t - \frac{S}{|A|}|} \\
&= \lim_{t\to\infty} \frac{|(1-c) \times Bel^t + \frac{S}{|A|}(c-1)|}{|Bel^t - \frac{S}{|A|}|} \\
&= \lim_{t\to\infty} \frac{|(1-c) \times Bel^t - \frac{S}{|A|}(1-c)|}{|Bel^t - \frac{S}{|A|}|} \\
&= \lim_{t\to\infty} \frac{|(1-c)(Bel^t - \frac{S}{|A|})|}{|Bel^t - \frac{S}{|A|}|}
\end{aligned}$$

Since $c \leq 1$ and $0 \leq 1 - c$:

$$\lim_{t\to\infty} \frac{|Bel^{t+1} - L|}{|Bel^t - L|} = \lim_{t\to\infty} \frac{(1-c)|(Bel^t - \frac{S}{|A|})|}{|Bel^t - \frac{S}{|A|}|} = 1 - c$$

Thus, by the definitions we gave above, if $1 - c = 0$, our series converge superlinearly. This happens when $c = 1$, which makes sense, since this is a special case in which all agent opinions converge right after the first update. On the other hand, if $0 < c < 1$, our series converge linearly. Since the smaller the y, the faster the series converges, the speed is directly proportional to the influence c.