

# Proof of Belief Convergence

## All Graphs Proof Scratch

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# Influencers definition

## Definition

*Let's call  $I_i$  the influencers of the agent  $a_i$ , which is defined as:*

$$\{a_j \in A \mid \exists P(a_j|a_i) \}$$

# An useful lemma

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## Proof.

Let's suppose that  $\exists P(a_j|a_i)$ .

Now suppose  $a_k \in I_j$ . By definition this means that  $\exists P(a_k|a_j)$ . But since  $\exists P(a_j|a_i)$ , there will also exist  $P(a_k|a_i)$ . Thus, by definition of influencers  $a_k \in I_i$ . Since  $a_k$  was arbitrary,  $I_j \subseteq I_i$ .

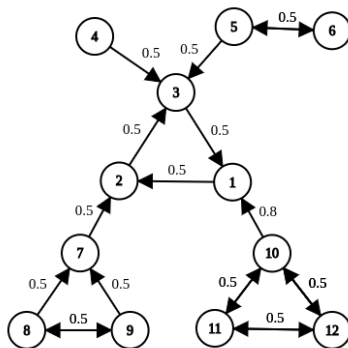
Proving the other side of the implication is trivial given the definition of influencers. □

## Partial order

Note that the influencers are basically sets of agents, thus they are a partial order. Keeping that in mind, we will look at the minimal elements of the influencers, but first let's look at an influence graph.

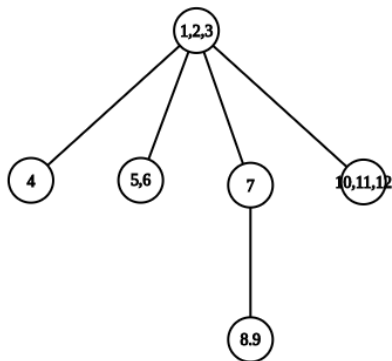
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## Partial order representation

Drawing a partial order representation of the influencers of the previous graph we end up with:



# Lemma

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### Proof.

Suppose  $a_j, a_k \in I_i$ . Thus  $\exists P(a_k|a_i)$  and  $\exists P(a_j|a_i)$ .

Since  $I_i$  is minimal, there must exist  $P(a_i|a_j)$  and  $P(a_i|a_k)$ . This is true because, according to Lemma 1,  $I_j \subseteq I_i$  and  $I_k \subseteq I_i$ . But if  $\nexists P(a_i|a_j)$  or  $\nexists P(a_i|a_k)$  then either  $I_j \subset N_i$  or  $I_k \subset I_i$ .

Which contradicts our hypothesis. Since  $\exists P(a_k|a_i) \wedge \exists P(a_i|a_j)$ ,  $\exists P(a_k|a_j)$ . And the same goes the other way around, since both  $a_j$  and  $a_k$  were arbitrary, it is true for all, which means there is a path between any two agents in the influencers. Does it is weakly connected.  $\square$

## Next steps

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- Given that the minimal influencers are weakly connected and that they don't receive external influence (I didn't prove this property but it kind of follows from 1).
- It will converge according to the other proof (if it is correct, of course). This might give us a good start on how to continue the proof. But the most useful part of this would be, to me, the ability to assume that the minimal elements have a constant belief, since they converge. This would be really useful but I don't know if I can assume it.

## Personal thoughts

- After running some experiments, I believe that the general case might not be as random as I first thought. Apparently who rules the beliefs in the limits are the minimal elements, because they converge and aren't affected by the others.

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- After running some experiments, I believe that the general case might not be as random as I first thought. Apparently who rules the beliefs in the limits are the minimal elements, because they converge and aren't affected by the others.
- Also, the simulation I made in the graph showed above actually resulted in the fact that changing the beliefs of agents who aren't from minimal influencers do not change the beliefs in the limit.
- What I believe now is that the elements who are not minimal converge to the weighted average of all paths from elements in minimal influencers multiplied by its beliefs.