

Magnetic Diagnostics for Plasmas

bernardo.carvalho@tecnico.ulisboa.pt

APPLAuSE Advanced Program in Plasma Science and
Engineering

Outline

- 1 Magnetic Diagnostics
 - General Principles
 - Global Inductive Magnetic Sensors
 - Plasma Integral quantities
 - Local Inductive Magnetic Probes
 - Plasma Shape
- 2 Integration of signals from inductive sensors
 - Analog Integration
 - Digital Integration
- 3 Non-Integrated signals
 - Non-Integrated signals
 - MHD Instabilities Diagnostics
- 4 Non Inductive Sensors
- 5 Burning plasma experiments

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Outline

1 Magnetic Diagnostics

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

2 Integration of signals from inductive sensors

- Analog Integration
- Digital Integration

3 Non-Integrated signals

- Non-Integrated signals
- MHD Instabilities Diagnostics

4 Non Inductive Sensors

5 Burning plasma experiments

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Portuguese Discoveries, XV - XVI Centuries



Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles
Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

The Portuguese Heroes



Henry the Navigator

1394-1460

King John II

1495

1455-

Vasco da Gama

1460-1524

Pedro Alvares Cabral

1467-1520

Ferdinand Magellan

1480 1521

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

The Instruments



The Caravel



The Astrolabe

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

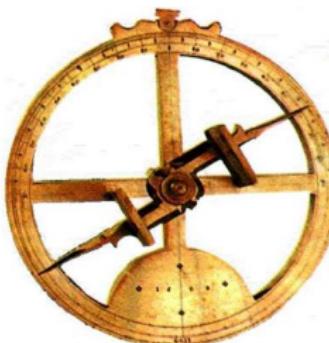
A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

The Instruments



The Caravel



The Astrolabe



The Compass

Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles

Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Magnetic Diagnostics: General Principles

Magnetic measurements provide some of the most fundamental and **essential** information about a fusion plasma:

- I_{plasma} , Internal Inductance ℓ_i , Position and Speed of current centroid, Boundary Shape, Thermal Energy, Currents in the magnet coils, and the strength of the magnetic fields confining the plasma.
- Information about the internal characteristics of the plasma and about asymmetries caused by large-scale MHD instabilities.
- Halo Currents in machine structures.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Magnetic Diagnostics: General Principles

Magnetic measurements provide some of the most fundamental and **essential** information about a fusion plasma:

- I_{plasma} , Internal Inductance ℓ_i , Position and Speed of current centroid, Boundary Shape, Thermal Energy, Currents in the magnet coils, and the strength of the magnetic fields confining the plasma.
- Information about the internal characteristics of the plasma and about asymmetries caused by large-scale MHD instabilities.
- Halo Currents in machine structures.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Magnetic Diagnostics: General Principles

Magnetic measurements provide some of the most fundamental and **essential** information about a fusion plasma:

- I_{plasma} , Internal Inductance ℓ_i , Position and Speed of current centroid, Boundary Shape, Thermal Energy, Currents in the magnet coils, and the strength of the magnetic fields confining the plasma.
- Information about the internal characteristics of the plasma and about asymmetries caused by large-scale MHD instabilities.
- Halo Currents in machine structures.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Magnetic Diagnostics: General Principles

Magnetic measurements provide some of the most fundamental and **essential** information about a fusion plasma:

- I_{plasma} , Internal Inductance ℓ_i , Position and Speed of current centroid, Boundary Shape, Thermal Energy, Currents in the magnet coils, and the strength of the magnetic fields confining the plasma.
- Information about the internal characteristics of the plasma and about asymmetries caused by large-scale MHD instabilities.
- Halo Currents in machine structures.

General Principles II

- Essential for **Equilibrium** Reconstructions
 - Post-discharge full equilibrium codes
 - Real-Time Plasma Shape and Position Control
- Magnetic diagnostics are external, passive and **ROBUST!**
 - The measurements remain valid and useful over the full range of plasma density and temperature as well as during large transient events (disruptions).

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

General Principles II

- Essential for **Equilibrium** Reconstructions
 - Post-discharge full equilibrium codes
 - Real-Time Plasma Shape and Position Control
- Magnetic diagnostics are external, passive and **ROBUST!**
 - The measurements remain valid and useful over the full range of plasma density and temperature as well as during large transient events (disruptions).

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

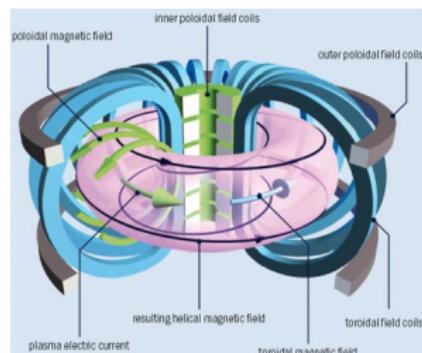
Axisymmetric Configuration of Fusion Devices

Magnetic field

- In a cylindrical coordinate system, (R, Z, ϕ) , \vec{B} can be expressed in terms of two scalar functions, F , and Ψ :

$$\vec{B} = (F \hat{\phi} + \nabla \Psi \times \hat{\phi})/R$$

- \vec{B} field can be separated in
 - Toroidal Field: $\vec{B}_\phi = \frac{F}{R} \hat{\phi}$
 - Poloidal Field: $\vec{B}_p = \frac{\nabla \Psi}{R}$



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

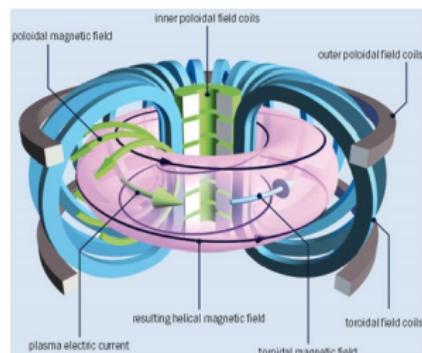
Axisymmetric Configuration of Fusion Devices

Magnetic field

- In a cylindrical coordinate system, (R, Z, ϕ) , \vec{B} can be expressed in terms of two scalar functions, F , and Ψ :

$$\vec{B} = (F \hat{\phi} + \nabla \Psi \times \hat{\phi})/R$$

- \vec{B} field can be separated in
 - Toroidal Field: $\vec{B}_\phi = \frac{F}{R} \hat{\phi}$
 - Poloidal Field: $\vec{B}_\theta = \frac{\nabla \Psi}{R}$



Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles

Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

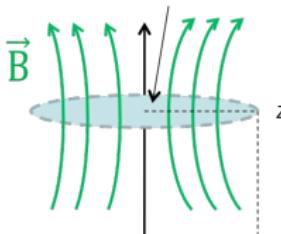
Axisymmetric Configuration

Poloidal Fluxes

Magnetic Flux

Mag. Poloidal flux (PF),
 $\Psi(R, Z)$ over one major circle, at (R, Z) .

$$\begin{aligned}\psi(R, Z) &= \frac{\Psi(R, Z)}{2\pi}, \text{ flux per radian} \\ \Psi(R, Z) &= \iint d\vec{B} \cdot d\vec{S}\end{aligned}$$



Current Flux

Poloidal current function, F , crossing the major circle, at (R, Z) .

$$\begin{aligned}F(R, Z) &= \mu I_{pol} \Psi(R, Z) / 2\pi = R B_\phi \\ I_{pol}(R, Z) &= \iint d\vec{j}_{pol} \cdot d\vec{S}\end{aligned}$$

Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles

Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

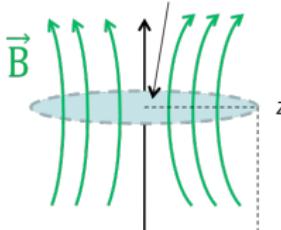
Axisymmetric Configuration

Poloidal Fluxes

Magnetic Flux

Mag. Poloidal flux (PF),
 $\Psi(R, Z)$ over one major circle, at (R, Z) .

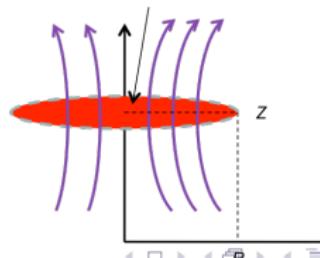
$$\begin{aligned}\psi(R, Z) &= \frac{\Psi(R, Z)}{2\pi}, \text{ flux per radian} \\ \Psi(R, Z) &= \iint d\vec{B} \cdot d\vec{S}\end{aligned}$$



Current Flux

Poloidal current function, F , crossing the major circle, at (R, Z) .

$$\begin{aligned}F(R, Z) &= \mu I_{pol} \Psi(R, Z) / 2\pi = R B_\phi \\ I_{pol}(R, Z) &= \iint d\vec{j}_{pol} \cdot d\vec{S}\end{aligned}$$



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

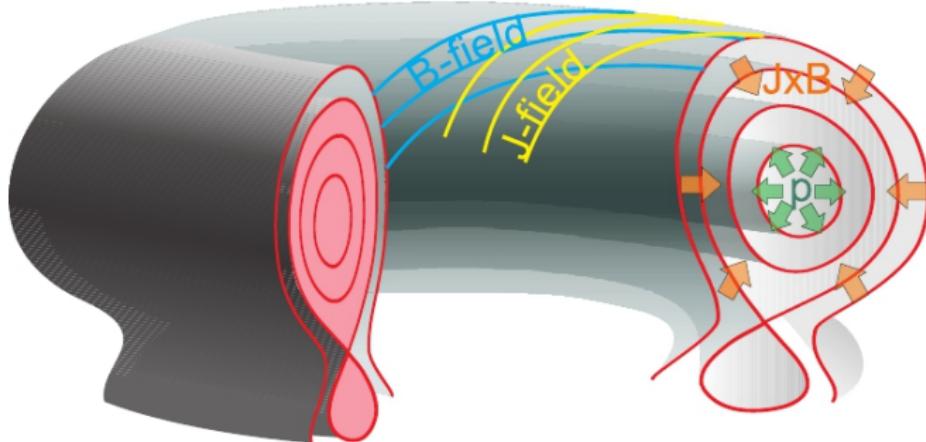
General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Axisymmetric Configuration

Flux Surfaces

Both Fluxes, (Ψ , F) are constant on the Flux Surfaces:



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

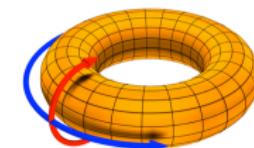
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Toroidal Magnetic Field

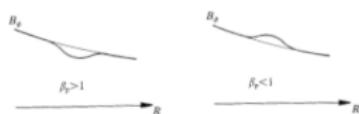
Ampere's Law

$$\nabla \times \vec{B} = \vec{J}_{free}$$
$$\nabla F \times \hat{\phi} = \mu_0 R J_{pol} \text{ (poloidal comp.)}$$

- In vacuum: $\nabla F = 0$, so $B_\phi(R)$ varies only with $1/R$
 - Result: No information from the plasma taken from external local B_ϕ measurements



$$B_\phi = \frac{F}{R}$$



Diamagnetic Paramagnetic

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

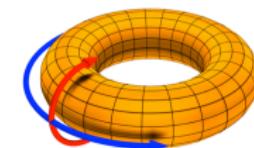
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Toroidal Magnetic Field

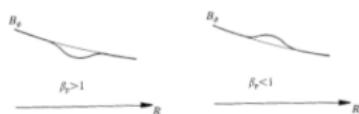
Ampere's Law

$$\nabla \times \vec{B} = \vec{J}_{free}$$
$$\nabla F \times \hat{\phi} = \mu_0 R J_{pol} \text{ (poloidal comp.)}$$

- In vacuum: $\nabla F = 0$, so $B_\phi(R)$ varies only with $1/R$
 - Result: No information from the plasma taken from external local B_ϕ measurements



$$B_\phi = \frac{F}{R}$$



Diamagnetic Paramagnetic

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

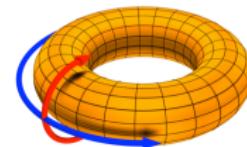
General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Toroidal Magnetic Field

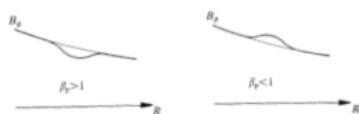
Ampere's Law

$$\nabla \times \vec{B} = \vec{J}_{\text{free}}$$
$$\nabla F \times \hat{\phi} = \mu_0 R J_{\text{pol}} \text{ (poloidal comp.)}$$



- Within Plasma: $B_\phi(R, Z)$ depends on poloidal currents, J_{pol} .
 - Diamagnetic loop can measure the surface integral change.

$$B_\phi = \frac{F}{R}$$



Diamagnetic Paramagnetic

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Poloidal Magnetic Field

Ampere's Law (toroidal comp.)

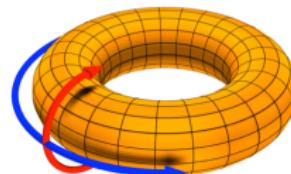
$$\Delta^* \Psi \equiv R \frac{\partial}{\partial R} \left(\frac{1}{R} \frac{\partial \Psi}{\partial R} \right) + \frac{\partial^2 \Psi}{\partial Z^2} = \mu_0 R J_\phi$$

$$\begin{aligned}\Psi(x') &= - \int_{\Omega} G(x, x') J_\phi^{ext}(x) dx \\ &\quad + \oint_{\partial\Omega} \frac{1}{\mu_0 R} \left(\Psi \frac{\partial G}{\partial n} - G \frac{\partial \Psi}{\partial n} \right) dS\end{aligned}$$

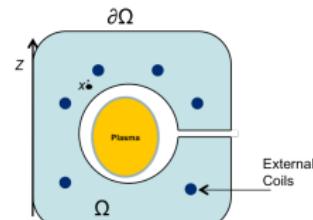
x' = point in Ω

$G(x, x')$ = Green function for Δ^* operator.

$\frac{\partial}{\partial n}$ = normal derivative



$$B_p = \frac{\nabla \Psi}{R}$$



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Equilibrium reconstruction

Basic restriction for the magnetic diagnostics

Green's Theorem

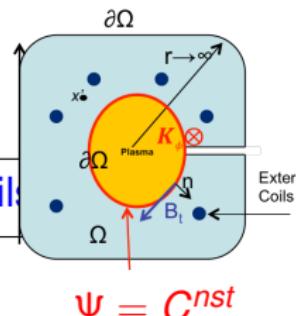
Ω bounded by Plasma and $r \rightarrow \infty$.

$$\Psi(x') =$$

$- \int_{\Omega} G(x, x') J_{\phi}^{ext}(x) dx$, Currents in external coils

$$+ \oint_{\partial\Omega} \frac{1}{\mu_0 R} \Psi \frac{\partial G}{\partial n} dS$$

$$- \oint_{\partial\Omega} \frac{1}{\mu_0 R} G \frac{\partial \Psi}{\partial n} dS$$



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Equilibrium reconstruction

Basic restriction for the magnetic diagnostics

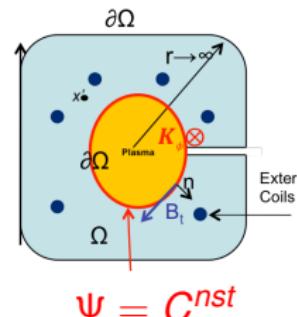
Green's Theorem

Ω bounded by Plasma and $r \rightarrow \infty$.

$$\Psi(x') = - \int_{\Omega} G(x, x') J_{\phi}(x) dx$$

$$+ \oint_{\partial\Omega} \frac{1}{\mu_0 R} \Psi \frac{\partial G}{\partial n} dS, \quad \Psi = C^{nst} \text{ at the boundary}$$

$$- \oint_{\partial\Omega} \frac{1}{\mu_0 R} G \frac{\partial \Psi}{\partial n} dS$$



$$\Psi = C^{nst}$$

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Equilibrium reconstruction

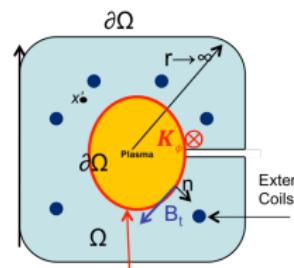
Basic restriction for the magnetic diagnostics

Green's Theorem

Ω bounded by Plasma and $r \rightarrow \infty$.

$$\begin{aligned}\Psi(x') = & \\ & - \int_{\Omega} G(x, x') J_{\phi}(x) dx \\ & + \oint_{\partial\Omega} \frac{1}{\mu_0 R} \Psi \frac{\partial G}{\partial n} dS\end{aligned}$$

$$-\oint_{\partial\Omega} \frac{1}{\mu_0 R} G \frac{\partial \Psi}{\partial n} dS$$



$$\Psi = C^{nst}$$

Term 3 is the only one that depends on internal current, J_{ϕ}^{plasma} .

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Equilibrium reconstruction

Basic restriction for the magnetic diagnostics

BUT solution depends only on B_t distribution on the Plasma boundary. Since many distributions, $J_\phi^{plasma}(r)$, give the SAME B_t .

Reconstruction $\Psi(R, Z)$ in **Vacuum** ☺

External measurements can determine the $\Psi(R, Z)$ anywhere in Ω and B_t on $\partial\Omega$. ☺

Reconstruction $\Psi(R, Z)$ inside **Plasma** ☹

External measurements alone CANNOT distinguish different internal current, $J_\phi(r)^{plasma}$ and $\Psi^{plasma}(R, Z)$ INSIDE the plasma! ☹

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Equilibrium reconstruction

Basic restriction for the magnetic diagnostics

BUT solution depends only on B_t distribution on the Plasma boundary. Since many distributions, $J_\phi^{plasma}(r)$, give the SAME B_t .

Reconstruction $\Psi(R, Z)$ in **Vacuum** ☺

External measurements can determine the $\Psi(R, Z)$ anywhere in Ω and B_t on $\partial\Omega$. ☺

Reconstruction $\Psi(R, Z)$ inside **Plasma** ☹

External measurements alone CANNOT distinguish different internal current, $J_\phi(r)^{plasma}$ and $\Psi^{plasma}(R, Z)$ INSIDE the plasma! ☹

Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

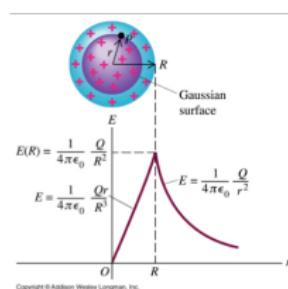
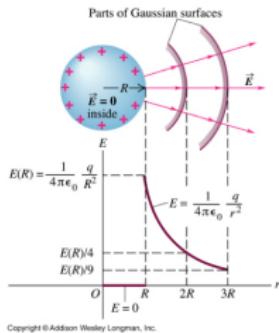
A Bit of History

General Principles

Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Electrostatics Equivalent Parallel Case

The Electric Field outside the Sphere is the same for a given charge surface density or other (infinite) volume distributions.



Reconstruction inside a sphere

External measurements CANNOT distinguish different Volume charge distributions.

Plasma Equilibrium in Fusion Plasmas

Grad-Shafranov Equation

- Combining equation for Ψ with magnetic force balance $\nabla p = \vec{J} \times \vec{B}$ gives G-S equation inside the plasma:

$$\Delta^* \Psi = -\mu_0 R J_\phi = -\mu_0 R^2 \frac{dp}{d\Psi} - \frac{1}{2} \frac{dF^2}{d\Psi}$$

- G-S gives additional constraint on \vec{B} within the plasma but also introduces another unknown scalar functions: the **pressure** and **current**
- Need to make some assumptions on $p(\Psi)$ and $F(\Psi)$ to calculate full plasma equilibrium solution.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

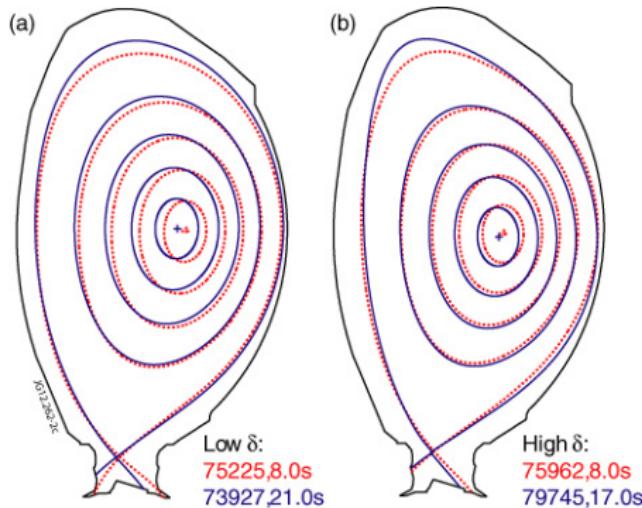
A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Plasma Equilibrium

Example: JET Reconstruction



Magnetic configuration of the (a) low- and (b) high-triangularity plasmas for the hybrid (red) and baseline H-mode (blue) plasmas.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles

Global Inductive Magnetic Sensors

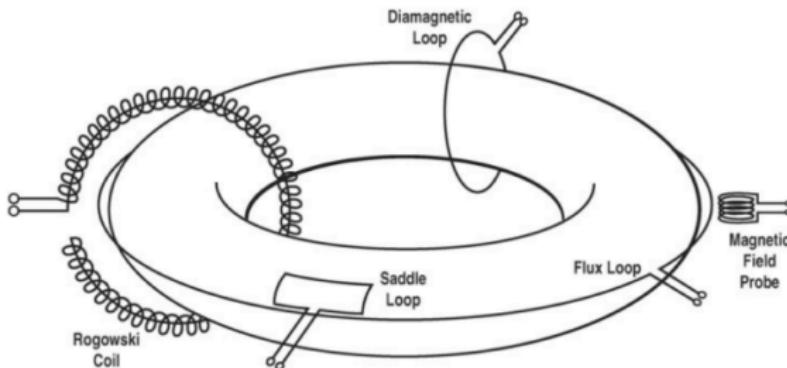
- Plasma Integral quantities

Local Inductive Magnetic Probes

- Plasma Shape

Magnetic Inductive Sensors

Basic Types



Schematic figure of a toroidal plasma, showing the basic types of inductive sensors.

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

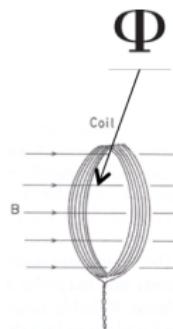
A Bit of History

General Principles

- Global Inductive Magnetic Sensors**
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Magnetic Inductive Sensors

Signal is a time derivative



Mag. Flux on the Sensor Loops

$$\begin{aligned}V_{sensor}(t) &= \oint \vec{E} \cdot d\vec{s} = -\frac{\partial \Phi(t)}{\partial t} = NAB, \text{ Faraday Law} \\ \Phi(t) &= \int_{t_0}^t V(t') dt'\end{aligned}$$

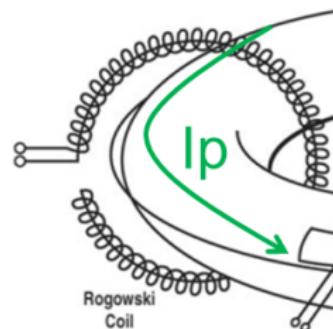
Magnetic Inductive Sensors

Rogowski Coil

- Measures **total electric current** flowing through the enclosed surface,
 - e.g. plasma, plasma + vessel, external coils, passive conductors, Halo Currents, etc.
- If $|\Delta B|/B \ll n$ (n is turns / m), total flux is:

$$\Phi = n \oint_{\ell} \int_A \vec{B} \cdot d\vec{l} dA = nA\mu_0 I_p$$
- Signal is proportional to current time derivative:

$$V(t) = \dot{\Phi} = nA\mu_0 \dot{I}_p$$

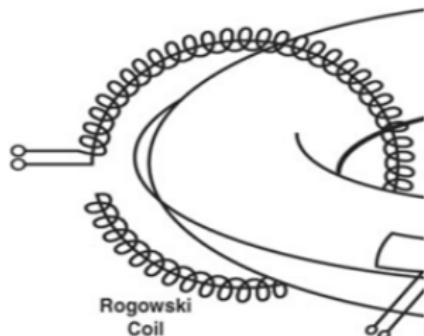


Conducting path from one end must return along the axis to the other end

Magnetic Inductive Sensors

Sinus-cosinus Coil

- A variation of Rogowski Coil but winding density, $n(\theta)$, varies with $\sin(\theta)$ or $\cos(\theta)$
- Used to measure Plasma Displacements 



Magnetic Diagnostics

Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

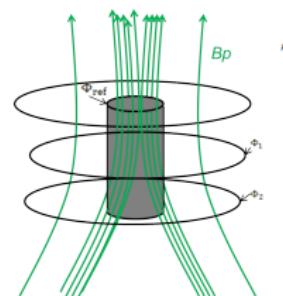
General Principles

Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Magnetic Inductive Sensors

Poloidal Flux Loops

- Measures $\Psi(R, Z)$ on a given (R, Z)
- On Iron Core Tokamaks with ohmic heating, most of the poloidal flux is on the core itself:
 $B = \mu_r \mu_0 H, \mu_r(\text{iron}) \approx 4000$
- To improve the sensitivity, a smaller loop is chosen as reference and subtracted from others: $\Psi_i = \Psi(R, Z) - \Psi_{ref}$



Loop Voltage

Voltage signal from a flux loop is the local one-turn V_{loop} , which drives I_p

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

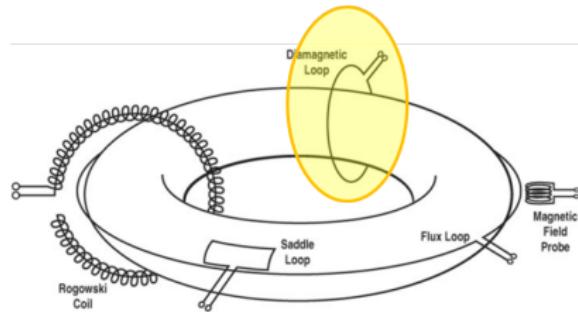
General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Magnetic Inductive Sensors

Diamagnetic Loop

- Measure the toroidal magnetic flux for the purpose of estimating the thermal energy of the plasma $\langle p \rangle \propto W$.
- Normally located in a poloidal plane to minimize coupling to the B_{pol} .
- At low beta, the change in the total toroidal flux is small. $\beta = 2\mu_0 \langle p \rangle / B^2 \ll 1$



A reference signal coupled to $B_{\phi, vacuum}$ is usually subtracted:
 $\Delta\Phi_{Diag} = \Phi_{total} - \Phi_{vacuum}$

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Basic Magnetic Inductive Sensors

Integral quantities

- Total **Plasma Current**, I_ϕ (Rogow.)
- **Ohmic Power** (Rogow+ V. Loop)

$$P \equiv \int_{Vol} \vec{E} \cdot \vec{j} d^3x = V_\phi I_\phi - \frac{\partial}{\partial t} \left(\frac{1}{2} L I_\phi^2 \right)$$

- **Poloidal beta**, β_p , (Rogow+ Diag. Lop)

$\beta_p \equiv 2\mu_0 \langle p \rangle / B_{p,a}^2 (\ll 1)$, $B_{p,a} = \mu_0 I_\phi / \Gamma$, Γ is length of a poloidal plasma contour.

- Circular Plasma $\beta_p = 1 - \frac{8\pi B_{\phi,vacuum}}{(\mu_0 I_\phi)^2} \Delta\Phi_{Diag}$
- Non-Circular Plasma

$$\beta_p \approx 1 - \frac{1 + \kappa^2}{2\kappa} \frac{8\pi B_{\phi,vacuum}}{(\mu_0 I_\phi)^2} \Delta\Phi_{Diag}$$

κ is the vertical elongation of plasma

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

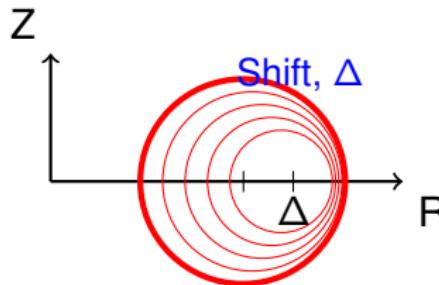
Basic Magnetic Inductive Sensors

Integral quantities II

- Total **Shafranov shift**, Δ . (From Rogow. + Vertical Field Current)

At Large Aspect Ratio approximation:

$$\Delta' = \frac{r}{R} \Lambda, \quad \Lambda = \beta_{pol} + \ell_i/2$$



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

Basic Magnetic Inductive Sensors

Integral quantities III

- **Plasma conductivity** , σ (Rogow+ V. Loop)

$$\hat{\sigma} = \frac{2\pi R}{\pi a^2} \frac{I_\phi^2}{P}, (\text{ if } \frac{\partial}{\partial t} = 0)$$

- **Electron Temperature** , T_e

$$\sigma = 1.9 \times 10^4 \left(\frac{T_e^{3/2}}{Z_\sigma \ln \Lambda} \right)$$

- Z_σ , resistance anomaly determined by ion charge
- $\ln \Lambda$, Coulomb logarithm:

$$\ln \Lambda \approx 31 - \ln(n_e^{1/2}/T_e)$$

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

Global Inductive Magnetic Sensors

Plasma Integral quantities

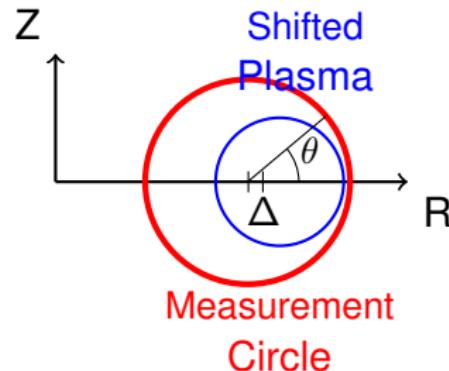
Local Inductive Magnetic Probes

Plasma Shape

Magnetic Inductive Sensors

Plasma Displacement

- Cylindrical Approximation
 $R \gg a$. Plasma Displaced by $\Delta \ll a$



- Measured poloidal field is:

$$\begin{aligned}B_\theta(\theta) &= \frac{\mu_0 I}{2\pi a} \frac{1}{[\sin^2 \theta + (\cos \theta - \Delta/a)^2]^{1/2}} \\&= \frac{\mu_0 I}{2\pi a} \left(1 + \frac{\Delta}{a} \cos \theta\right)\end{aligned}$$

- Displacement can be extracted from the Sinus-cosinus Coils:

Magnetic Diagnostics

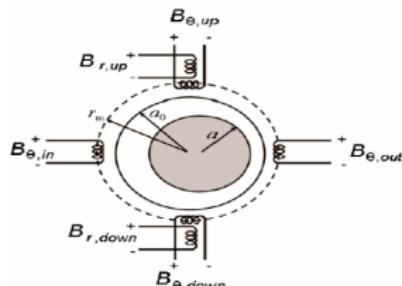
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles
Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Magnetic Probes

- Probes measure components of the local magnetic field strength.
- Usually solenoidal, with dimensions small compared to the gradient scale length of the magnetic field.



$$\Phi_{probe} = N A B_{||}$$

Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

- Global Inductive Magnetic Sensors
- Plasma Integral quantities

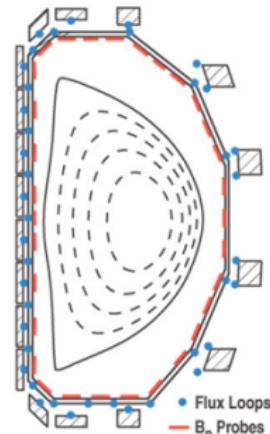
Local Inductive Magnetic Probes

Plasma Shape

Magnetic Local Probes

Shielding

- Probes should be located on the plasma-facing side of the vacuum vessel wall.
- Should be oriented to measure the field tangential to the wall; otherwise, eddy currents in the wall will attenuate the high-frequency part of the signal.



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

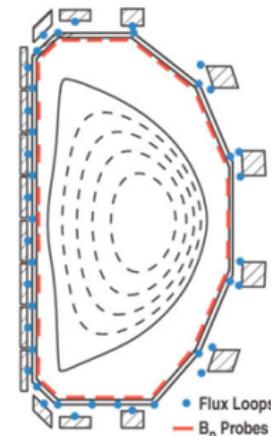
Magnetic Local Probes

Shielding

- Shielding of the tangential field by the conductive wall.

$$\frac{B(\text{inside})}{B_0} = \frac{1+2i\omega\tau_w}{1+i\omega\tau_w}$$
$$\frac{B(\text{outside})}{B_0} = \frac{1}{1+i\omega\tau_w}$$

τ_w is the characteristic time for the magnetic flux to diffuse through the wall



Magnetic Diagnostics

- Integration of signals from inductive sensors
- Non-Integrated signals
- Non Inductive Sensors
- Burning plasma experiments

A Bit of History

General Principles

Global Inductive Magnetic Sensors

Plasma Integral quantities

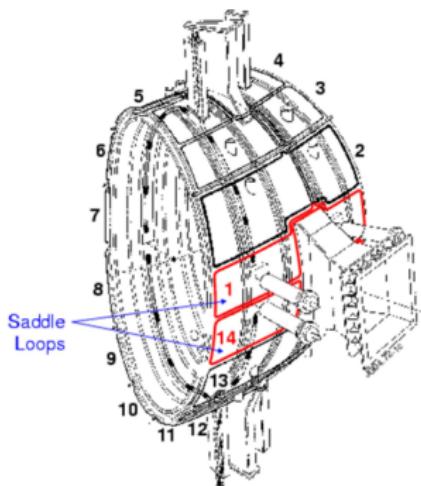
Local Inductive Magnetic Probes

Plasma Shape

Saddle Loops

Can be viewed as

- A large-scale magnetic probe for the magnetic field normal to the surface. $\Phi(\text{saddle}) = N A \langle B_{\perp} \rangle$
- or as probes measuring Flux Difference: .
 $\Phi(\text{saddle}) = N \Delta_{\phi} \Delta_{\psi}$



Magnetic Diagnostics

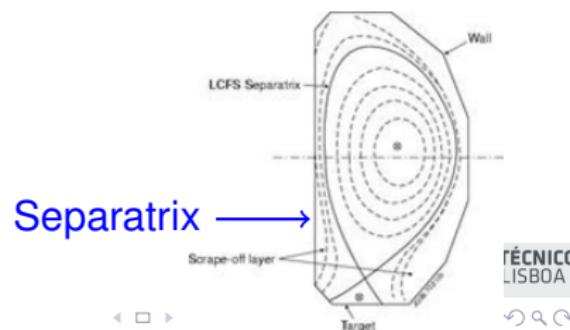
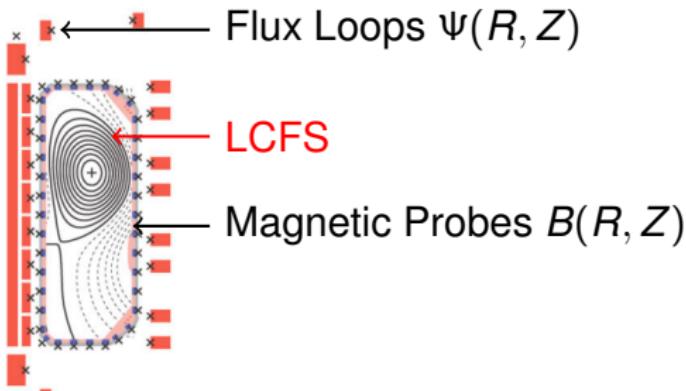
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles
Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Determination of Plasma Shape

- 1 Taking measurements of the $\Psi(R, Z)$ and poloidal B_{pol} near the Wall, plus the currents in external Coils allows local Ψ extrapolation.
- 2 Plot the contours of $\Psi(R, Z) = C^{nst}$. Find the Last Closed Flux Surface **LCFS**, or **Separatrix** in Divertor tokamaks



Magnetic Diagnostics

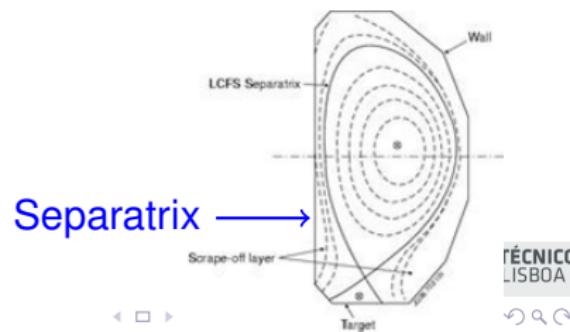
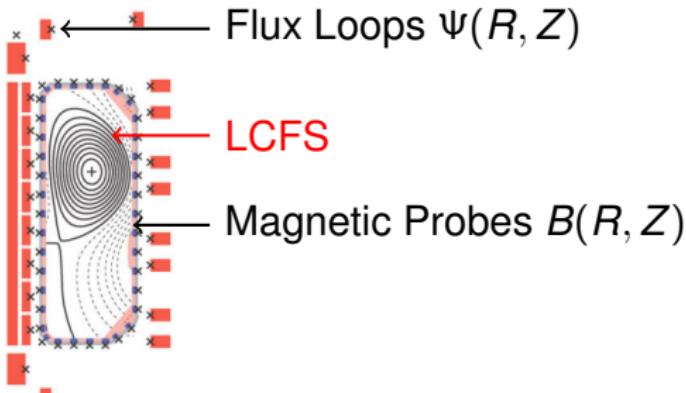
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

A Bit of History

General Principles
Global Inductive Magnetic Sensors
Plasma Integral quantities
Local Inductive Magnetic Probes
Plasma Shape

Determination of Plasma Shape

- 1 Taking measurements of the $\Psi(R, Z)$ and poloidal B_{pol} near the Wall, plus the currents in external Coils allows local Ψ extrapolation.
- 2 Plot the contours of $\Psi(R, Z) = C^{nst}$. Find the Last Closed Flux Surface **LCFS**, or **Separatrix** in Divertor tokamaks



Outline

1 Magnetic Diagnostics

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

2 Integration of signals from inductive sensors

- Analog Integration
- Digital Integration

3 Non-Integrated signals

- Non-Integrated signals
- MHD Instabilities Diagnostics

4 Non Inductive Sensors

5 Burning plasma experiments

Integration of signals from Mag. Sensors

Analog Integration

- To obtain the fluxes and magnetic field values from inductive sensors we must integrate the signal in time:

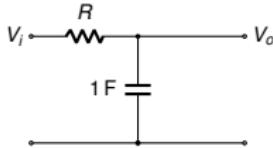
$$V_{out}(t) = -\frac{1}{\tau} \int_0^t V_{in}(t') dt' = \Phi(t)/\tau$$



- Typical loop flux values vary from few $mV.s$ to $V.s$ (e.g. Iron core), so integrator circuits are used with $1\text{ ms} < \tau < 1\text{ s}$.

Integration of signals

Analog Passive Integrator



- Simple Passive Integrator (RC Low Pass Filter, 1st Order, -20dB/dec.), $\tau = RC$

$$V_{out}(\omega) = -\frac{1}{1 + i\omega\tau} V_{in}(\omega) \Rightarrow V_{out}(t) \approx -\frac{1}{\tau} \int_{t_0}^t V_{in}(t') dt'$$

RC Limitation!

The approximation fails for timescales $t \geq RC$.

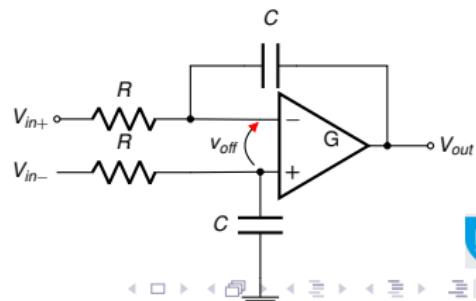
Integration of signals

Analog Active (Op-Amp) Integrator

- Gain is similar to passive integrator, $1/RC$, but timescale is increased from $\sim RC \rightarrow \sim G \cdot RC$ (1 ms to 10 s).

NEW problem: Integrator Drift by OpAmp Input V_{off}

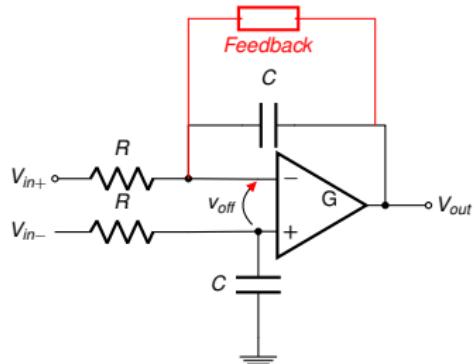
Example: for $RC = 10\text{ ms}$ a $V_{off} = 100\text{ }\mu\text{V}$ integrates to a 0.1 V after 10 s



Integration of signals

Analog Active (Op-Amp) Integrator

- Gain is similar to passive integrator, $1/RC$, but timescale is increased from $\sim RC \rightarrow \sim G \cdot RC$ (1 ms to 10 s).



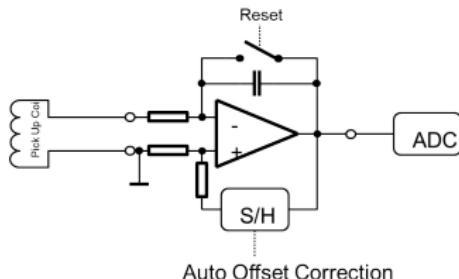
Solution

Basic compensation circuit

Analog Integrators

Advanced Designs

- Automatic drift-compensation:
Measurement of drift before integration, store and compensate offset during integration.
- Drift can be reduced down to several 0.1 mVs @ 1000 s
- But worse values if signal is applied during drift compensation!



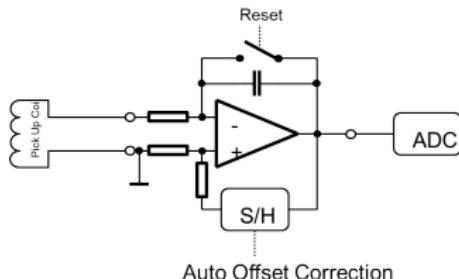
Tore Supra

Drift achieved:
 $< 135 \mu \text{Vs}$ @ 1000 s

Analog Integrators

Advanced Designs

- Automatic drift-compensation:
Measurement of drift before integration, store and compensate offset during integration.
- Drift can be reduced down to several 0.1 mVs@1000s
- But worse values if signal is applied during drift compensation!



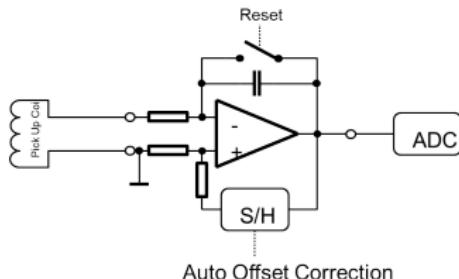
Tore Supra

Drift achieved:
 $< 135 \mu \text{Vs@1000 s}$

Analog Integrators

Advanced Designs

- Automatic drift-compensation:
Measurement of drift before integration, store and compensate offset during integration.
- Drift can be reduced down to several 0.1 mVs@1000s
- But worse values if signal is applied during drift compensation!



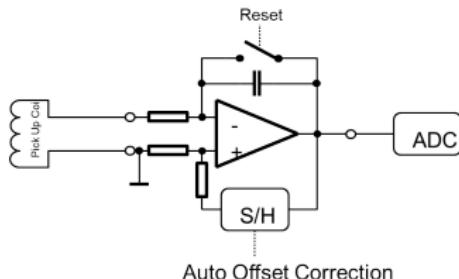
Tore Supra

Drift achieved:
 $< 135 \mu \text{Vs@1000 s}$

Analog Integrators

Advanced Designs

- Automatic drift-compensation:
Measurement of drift before integration, store and compensate offset during integration.
- Drift can be reduced down to several 0.1 mVs @ 1000 s
- But worse values if signal is applied during drift compensation!

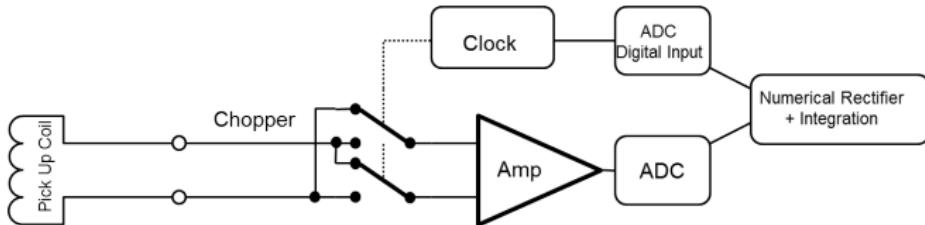


Tore Supra

Drift achieved:
 $< 135 \mu \text{Vs} @ 1000 \text{ s}$

Digital Integrators

Chopper Input



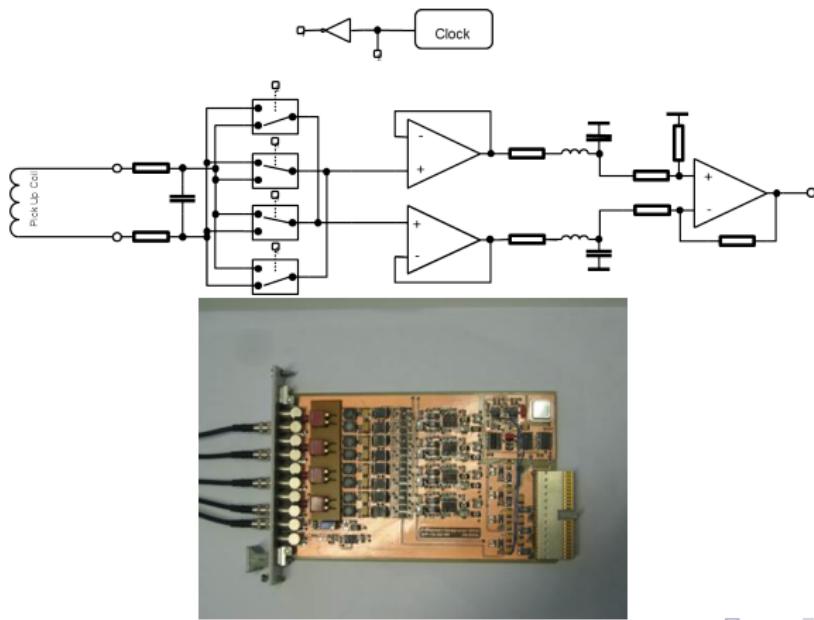
- Dynamic range limited by input stage
- Not affected by input stage semiconductors
- Complex offset correction algorithms feasible

Magnetic Diagnostics
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

Analog Integration
Digital Integration

Digital Integrators

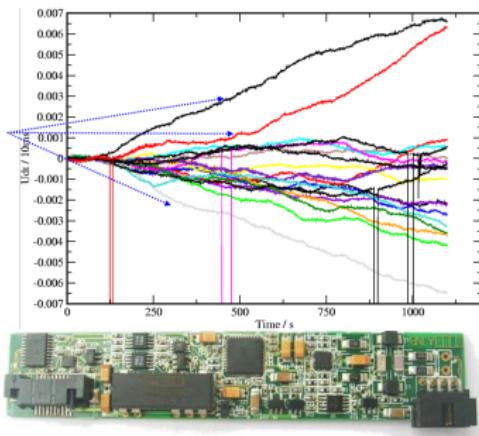
W7-X / IST ADC chopper module



Digital Integrators

IST prototype

- 100s drift compensation
- 1000s run Cold Integrator



Outline

1 Magnetic Diagnostics

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

2 Integration of signals from inductive sensors

- Analog Integration
- Digital Integration

3 Non-Integrated signals

- Non-Integrated signals
- MHD Instabilities Diagnostics

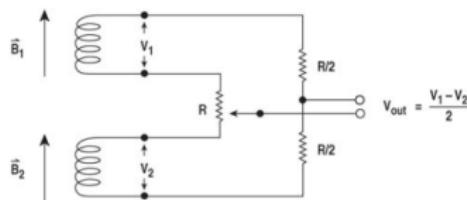
4 Non Inductive Sensors

5 Burning plasma experiments

Non-Integrated Signal

Inductive probes

- Usually as linear combinations of flux Loop and/or field probes (analog adders)
- Direct information on Plasma Speed:
 $V(t) \propto Vel_{R,Z}(t)$
 - Used as controllable variables for active control



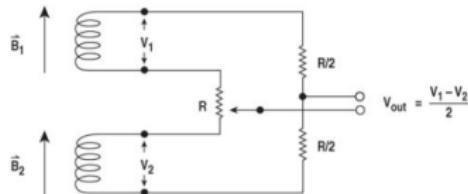
Non-Integrated Signal

Inductive probes

- High Frequency MHD plasma instabilities detection and control

$$V(t) \propto B(t) \sim \omega B(t)$$

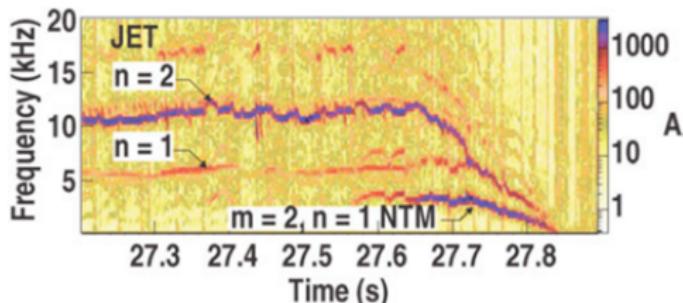
- Mirnov coils, poloidal or toroidal arrays oriented to measure $B_{pol}(t)$



Non-Integrated Signal

MHD Instabilities Diagnostics

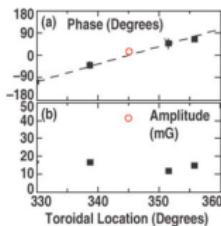
- Analyses Techniques: Spectrogram (Fourier analysis of successive short-time windows).
- Other techniques:
 - Wavelet analysis.
 - Hilbert transform.
 - Singular Value Decomposition (SVD).



Non-Integrated Signal

MHD Instabilities mode number detection

- Usual MHD perturbation: $\delta B(t) \propto \delta B(r) \cos(m\theta + n\phi - \omega t)$
- Mode numbers m, n are determined by the phase shift between equally separated Mirnov coils, by Fourier Analysis.



Identification of the toroidal mode number $n = 7$ of a compressional Alfvén eigenmode in NSTX.

Non-Integrated Signal

Auto and Cross Correlation between two signals

Defining a dot product between 2 functions

$$\langle f, g \rangle = \int_{-\infty}^{\infty} f^*(t)g(t)dt$$

Cross-correlation :

$$\begin{cases} (f * g)(\tau) = \int_{-\infty}^{\infty} f^*(t)f(t + \tau)dt & \text{Continuous,} \\ (f * g)[n] = \sum_{m=-\infty}^{\infty} f^*[n]g[n + m] & \text{Discrete.} \end{cases}$$

Auto-correlation :

$$\begin{cases} (f * f)(\tau) = \int_{-\infty}^{\infty} f^*(t)f(t + \tau)dt & \text{Continuous,} \\ (f * f)[n] = \sum_{m=-\infty}^{\infty} f^*[n]f[n + m] & \text{Discrete.} \end{cases}$$

Non-Integrated Signal

Auto and Cross Correlation between two signals

Cross-correlation of real life finite sampled signals

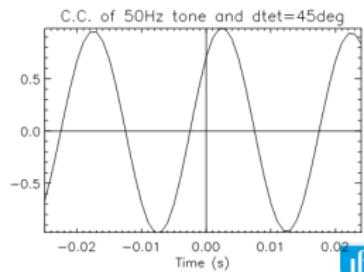
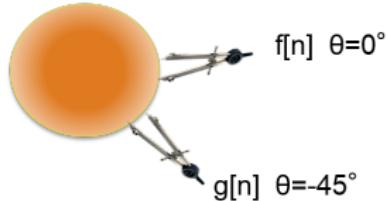
- Real life signals have only limited number of samples
- Cross-correlation easier to interpret when bounded in [-1,1]
- Lag vector (index-n) also finite
- MATLAB/Octave Function $r = xcorr(x, y)$

$$(f * g)[n] = \begin{cases} \frac{\sum_{m=1}^{N-|n|} (f[m+|n|] - \bar{f})(g[m] - \bar{g})}{\sqrt{\sum_{m=1}^N (f[m] - \bar{f})^2} \sqrt{\sum_{m=1}^N (g[m] - \bar{g})^2}} & \text{if } n < 0, \\ \frac{\sum_{m=1}^{N-n} (f[m] - \bar{f})(g[n+m] - \bar{g})}{\sqrt{\sum_{m=1}^N (f[m] - \bar{f})^2} \sqrt{\sum_{m=1}^N (g[m] - \bar{g})^2}} & \text{if } n \geq 0. \end{cases}$$

Non-Integrated Signal

Cross Correlation as wavenumber m, n estimator

$$\begin{aligned} f[n] &= \cos(w_1 t[n] - k\theta_1) \\ g[n] &= \cos(w_1 t[n] - k\theta_2) \\ w_1 t[n + lag_{max}] - k\theta_2 &= w_1 t[n] - k\theta_1 \\ \sim w_1(t[n + lag_{max}] - t[n]) &= k(\theta_2 - \theta_1) \\ k &= \frac{w_1 \cdot t[lag_{max}]}{\theta_2 - \theta_1} \end{aligned}$$

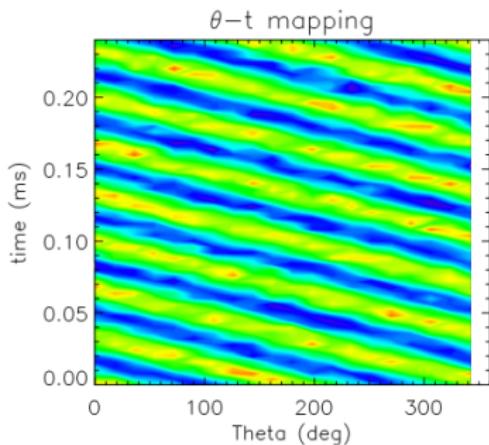


Non-Integrated Signal

$\theta - t$ space visual analysis

Example

- Magnetic coils : $i = 1 \dots 12$ eq. spaced in poloidal plane
- $s_i(t) = \cos(\pi f t + m\theta_i) + \text{noise}(\mu = 0, \sigma = 0.2)$

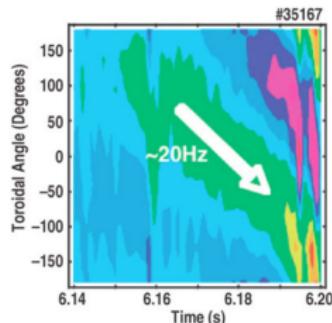


- Wave front propagates clockwise
- Two periods for fixed t ($m=2$)
- Period $\approx 0.03ms$
 $f 30kHz$

Non-Integrated Signal

MHD Nonrotating Modes

- Nonrotating modes are most commonly detected with toroidal arrays of **saddle coils**
- At low frequencies involved the field perturbation penetrates the vacuum vessel.



Time evolution of an RWM in JT-60U measured with an array of eight saddle coils.

Outline

1 Magnetic Diagnostics

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

2 Integration of signals from inductive sensors

- Analog Integration
- Digital Integration

3 Non-Integrated signals

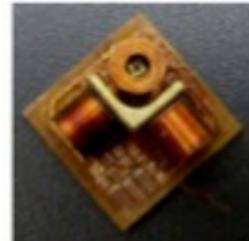
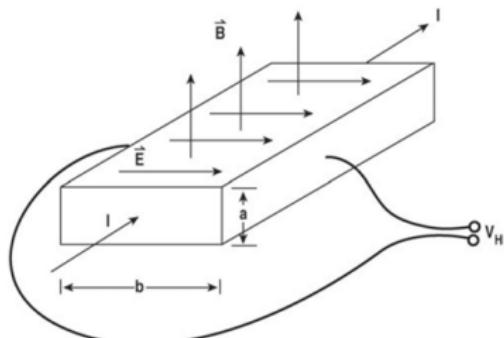
- Non-Integrated signals
- MHD Instabilities Diagnostics

4 Non Inductive Sensors

5 Burning plasma experiments

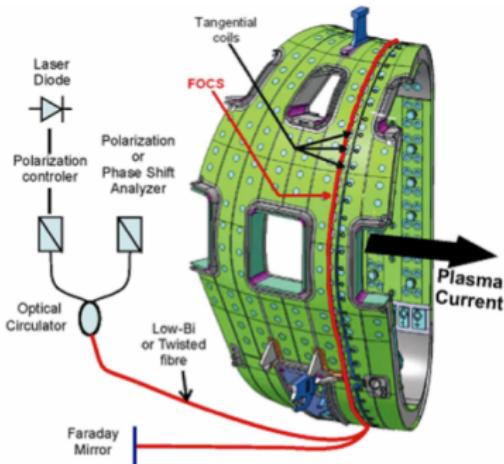
Non Inductive Sensors

- Signal has direct Value:
 $v(t) \propto B(t)$.
- Hall Probes are relatively simple and inexpensive.
 $V_H = \frac{1}{qn} \frac{I_H B}{a}$,
 $\frac{1}{qn}$ is **Hall coefficient**, is a property of each material.
- low frequency, low sensivity, very sensible to radiation!



Non Inductive Sensors II

- Resistive Shunts:
 - Measuring halo currents flowing between the plasma and plasma-facing components
- Faraday rotation current measurements:

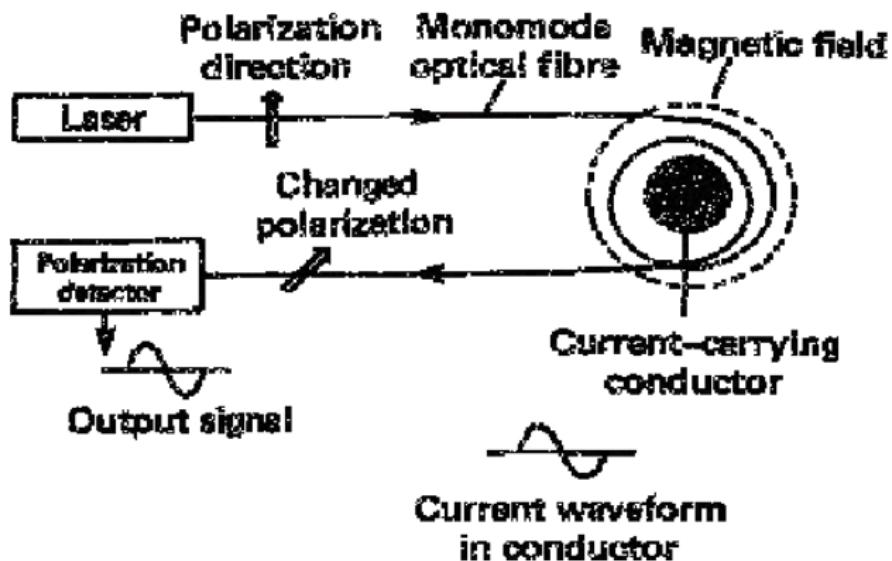


Schematic overview of a fiber-optic Faraday rotation measurement

device.

Non Inductive Sensors III

Faraday effect

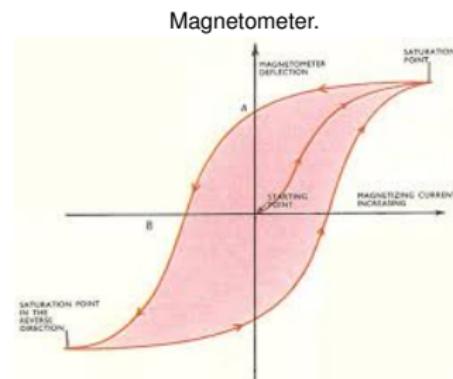
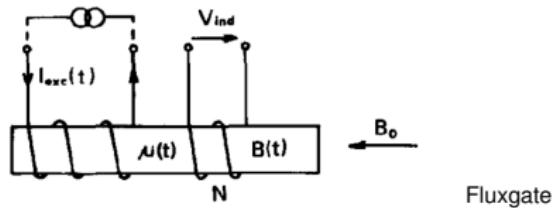


Non Inductive Sensors IV

Fluxgate Magnetometer

Basic sensor configuration.

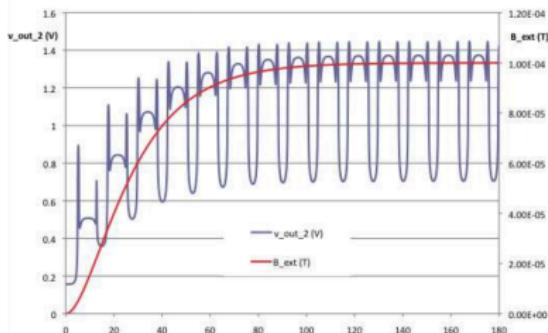
- The core is excited by an AC current I_{exc} .
- The signal induced in the sense coil V_{ind} at the second harmonic of I_{exc} proportional to the external field B_o .



Non Inductive Sensors IV

Fluxgate Magnetometer

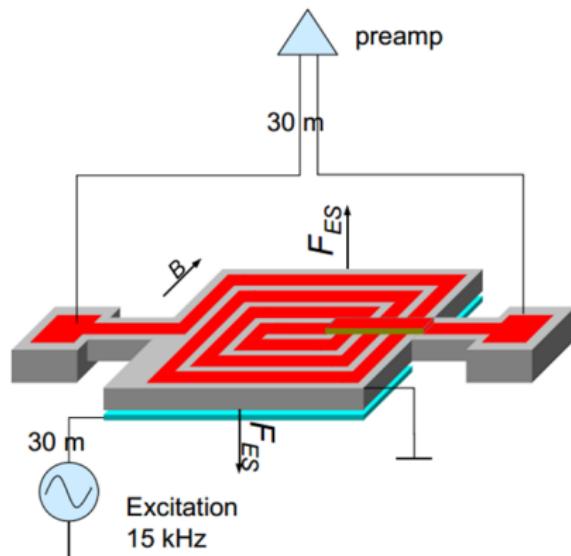
- Capable of measuring low frequency DC/ AC fields in the range $10^{-10} - 10^{-4} T$
- The signal induced in the sense coil V_{ind} at the second harmonic of I_{exc} proportional to the external field B_o .
- Usually double core is adopted



Waveform of sensor signal with variable external magnetic (Red curve).

Non Inductive Sensors V

MEMS magnetic field sensors



Outline

1 Magnetic Diagnostics

- General Principles
- Global Inductive Magnetic Sensors
- Plasma Integral quantities
- Local Inductive Magnetic Probes
- Plasma Shape

2 Integration of signals from inductive sensors

- Analog Integration
- Digital Integration

3 Non-Integrated signals

- Non-Integrated signals
- MHD Instabilities Diagnostics

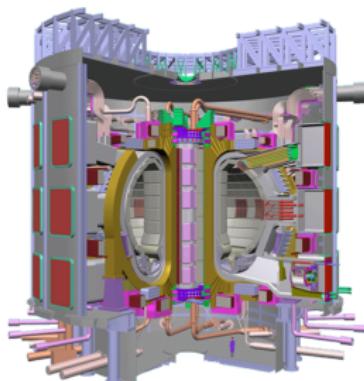
4 Non Inductive Sensors

5 Burning plasma experiments

Burning plasma experiments

ITER

- ITER will be the first burning plasma experiment
- Relative to existing machines (e.g. JET), ITER diagnostic components will be subjected to:
 - High n and γ fluxes.
 - n Heating (now essentially zero).
 - High fluxes of energetic neutral particles.
 - Long pulse lengths.



Magnetic Diagnostics
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

ITER Burning plasma
ITER Diagnostics

ITER diagnostics

Functional requirements

Provide accurate measurements of plasma behavior and performance.

- 3 categories of measurement parameters.

Group	Description	Machine operation?
1	Machine protection and Basic machine control	<i>Advanced</i> operation unable without this
1b	advanced plasma control	Unable without
2	Evaluation and physics studies	Able without

ITER diagnostics

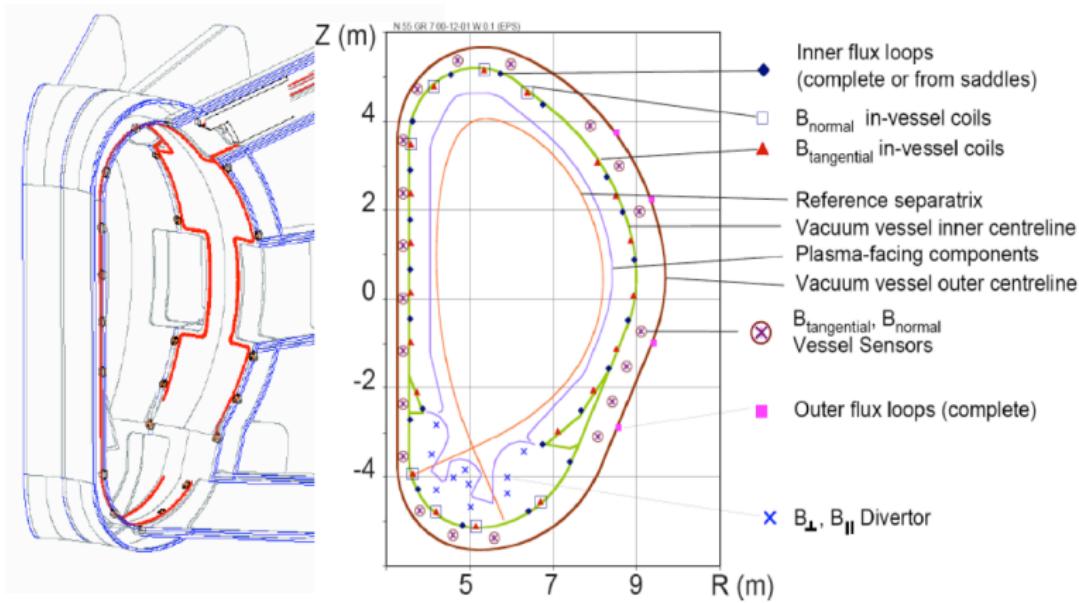
Magnetic Set

Total 1700 sensors, 19 types, > 300 km of cable

Location	n	γ	Radiation/Dose ($\text{cm}^{-2} \text{s}^{-1}$) / MGy	
			Type	Number
In-vessel sensors: Behind blanket modules, fixed on VV inner skin	3. 10^{12} 500	1. 10^{12} 340	Pick-up coils	186
			Rogowski (halo current)	360
			Flux loops	220
	1. 10^{13} 1700	3. 10^{12} 1000	High freq coils	>300
			Pick-up coils	72
			Rogowski (halo current)	48
Ex-vessel sensors Fixed on VV outer skin	1.5 10^{10} 2.5	1. 10^{10} 3.4	Pick-up coils	360
			Steady state sensors	120
			Flux loops	5
			Optic fibre	12
	Inside TFC case (T=4K)	1. 10^{10} 1.7	2. 10^9 0.7	Rogowski (I plasma) ≥ 3

ITER diagnostics

Poloidal Magnetic Set

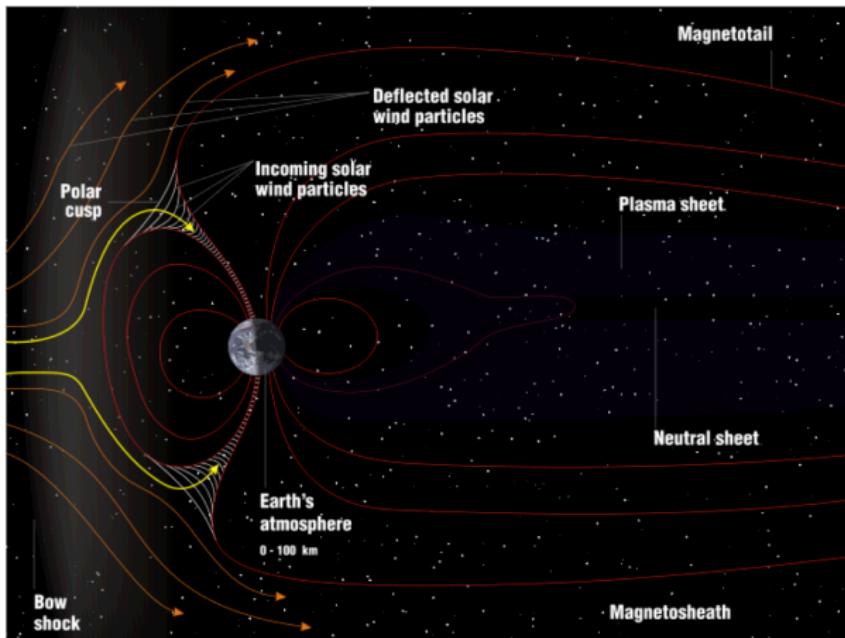


Magnetic Diagnostics
Integration of signals from inductive sensors
Non-Integrated signals
Non Inductive Sensors
Burning plasma experiments

ITER Burning plasma
ITER Diagnostics

Structure of Magnetosphere

Earth Fields



Magnetic Diagnostics

Integration of signals from inductive sensors

Non-Integrated signals

Non Inductive Sensors

Burning plasma experiments

ITER Burning plasma

ITER Diagnostics

Magnetosphere

Plasma Diagnostic



© www.arcticphoto.no



Further Reading I



I. H. Hutchinson

Principles of Plasma Diagnostics.

Cambridge University Press, 2005.



E. J. Strait et al.

Plasma Diagnostics for Magnetic Fusion Research, ch. 2.

Fusion Science and Technology, special Issue , Vol. 53, No.

2, February 2008



A. Woottton

Magnetic Fields and Magnetic Diagnostics for Tokamak

Plasmas, 2008, [web2.ph.utexas.edu/~iheds/](http://web2.ph.utexas.edu/~iheds/magneticfieldsintokamak.pdf)

[magneticfieldsintokamak.pdf](http://web2.ph.utexas.edu/~iheds/magneticfieldsintokamak.pdf)