
FAMILY NAME:

FIRST NAME:

ID No:

STATS 210 FC

Term Test

Date: Thursday 14th April, 2016

Answer **ALL THREE QUESTIONS**. Marks are shown for each question.

1. (a) Clara and Jake are looking for their mum's car key. It could be under the table, behind the bookshelf, or under the dishwasher, each with equal probability. Clara will only search under the table; if the key is there she has a $\frac{3}{5}$ chance of finding it. Jake will only search behind the bookcase; if the key is there he has a $\frac{1}{5}$ chance of finding it.

(i) Find the probability that the car key is found.

(3)

(ii) Suppose the car key has been found. Find the probability that it is found by Jake.

(3)

- (iii) Suppose the car key has not been found. Find the probability that it is under the dishwasher. (3)

- (b) Out of all students graduating with a first class honours degree in statistics from a particular university, 94% passed the mid-semester test in the *Statistical Theory* course. Jim concludes that since he has just passed the mid-semester test, he has an excellent chance of getting a first class honours degree.

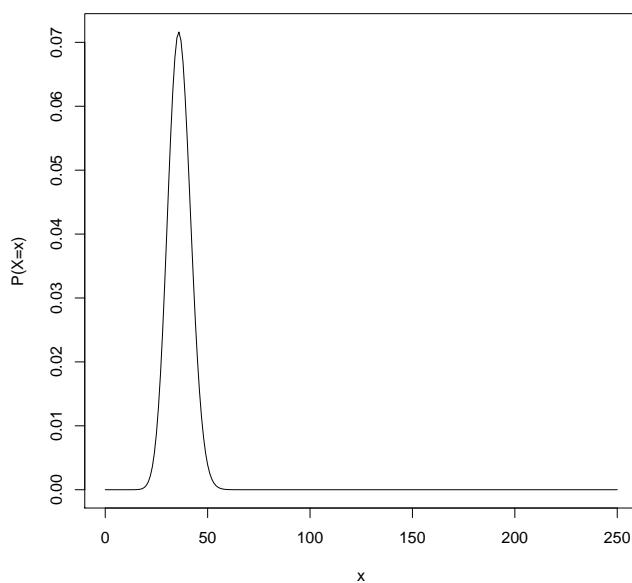
- (i) Explain the main flaw in this argument. (3)

- (ii) Suppose that 80% of students pass the test and 10% of students get a first class honours degree. Find the probability that a randomly selected student who passed the test gets a first class honours degree. (4)

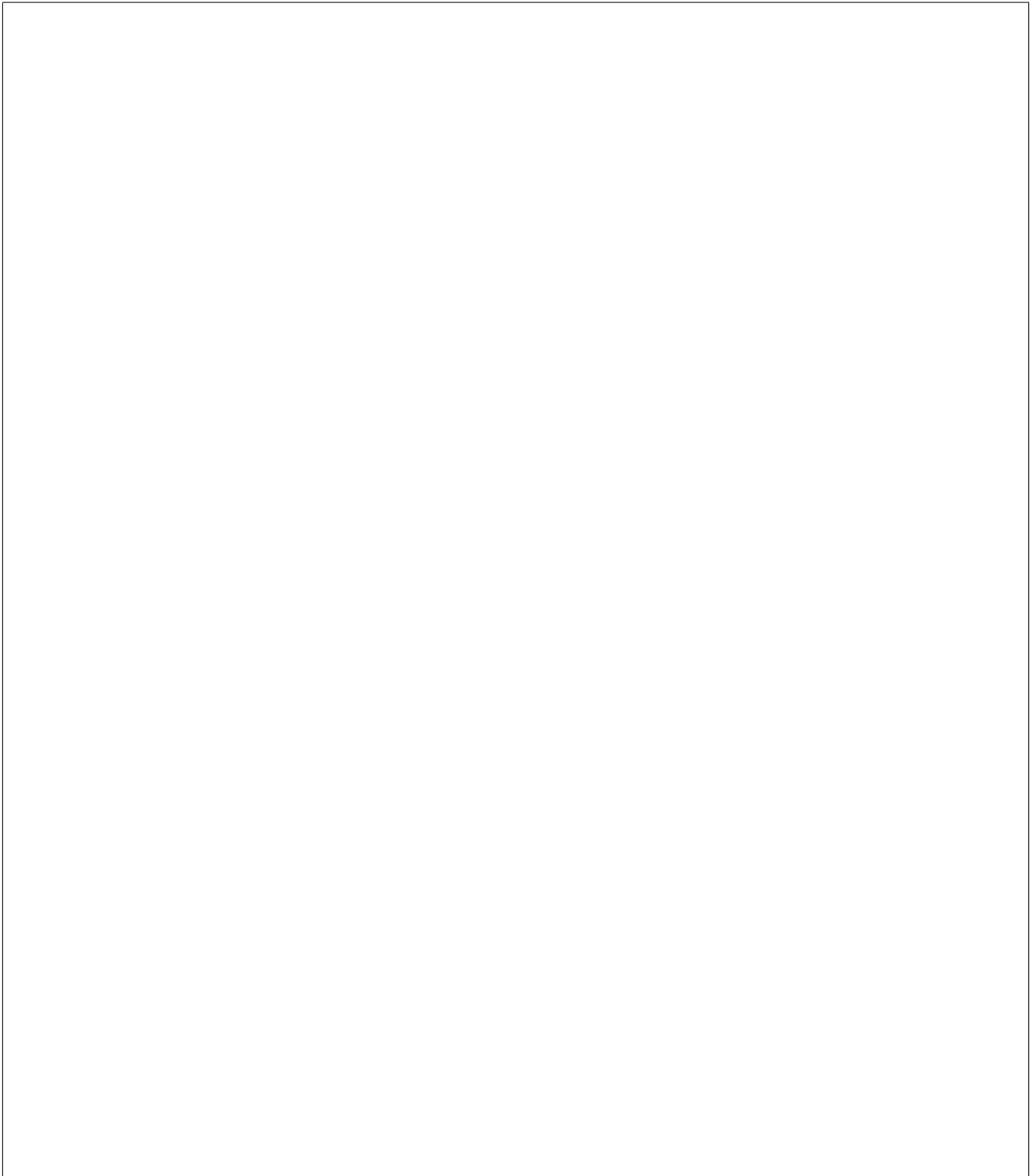
2. The **Fortune 500 list** is a list of the 500 richest businesses in the US. In 2005, a study was done of the chief executive officers (CEOs) of 250 businesses on this list, all of whom were male. It was discovered that 58% of the CEOs were over 6 feet tall (183cm). In the US population, 14.5% of adult men are over 6 feet tall. We wish to test whether these figures provide evidence that there are more tall men among US company CEOs than we would expect by chance.

- (a) Let X be the number of CEOs over 6 feet tall from the 250 businesses studied. Formulate the null hypothesis, H_0 , and the alternative hypothesis, H_1 , in terms of the distribution of X and its parameters. Remember to specify the full distribution of X and to use a two-sided alternative hypothesis. (3)

- (b) The probability function of X under the null hypothesis is shown below. Find the approximate value of x where the curve peaks. Also mark the observed value of x so that you can see the tail probabilities required for the p -value, and shade under the curve the area represented by the p -value. (3)



- (c) Estimate the p -value for the hypothesis test using the graph of the probability function above. Interpret the result in terms of the strength of evidence against the null hypothesis. Is the observed data compatible with the null hypothesis? (4)



3. In football, a penalty kick is awarded when a foul that is punishable by a direct free kick is committed within the offending player's own penalty area. Penalties are converted into goals (called a *conversion*) more often than not, even against very talented goal keepers. In the English Premier League 2014-2015 season, a total of 83 penalties were awarded. The conversion was successful on 63 of the 83 opportunities*.

*Note: data available at:

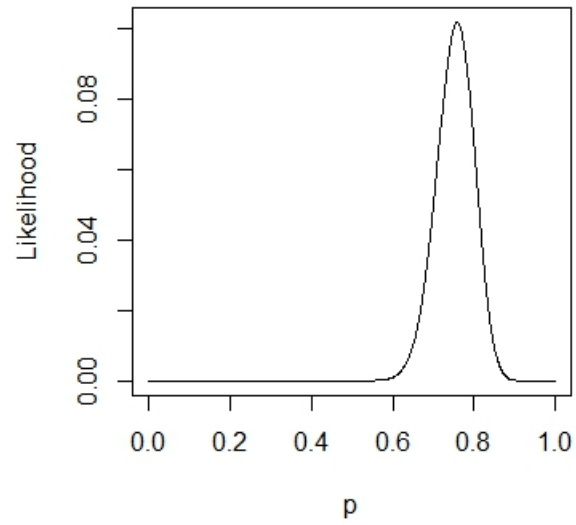
http://www.myfootballfacts.com/Premier_League_Penalty_Statistics.html.

Suppose that every 2014-2015 season English Premier League conversion is successful with probability p , independently of all other conversions. Our data are 63 successful conversions out of 83 attempts. We wish to estimate the probability p that any given conversion is successful.

- (a) Write down the likelihood function, $L(p; x)$, substituting the correct value of x . State the range of values of p for which the likelihood function is defined. (2)

- (b) Find $\frac{dL}{dp}$, and give all possible solutions to the equation $\frac{dL}{dp}=0$. (4)

- (c) The likelihood function is plotted below. By referring to the graph and using your answer for (b), find the maximum likelihood estimate of p and state what this maximum likelihood value represents. (3)



End of Paper

Total: 35