1.(a) Let X be the number of Little-voters out of 1000 respondents.

$$X \sim \text{Binomial}(1000, \text{p}_L)$$

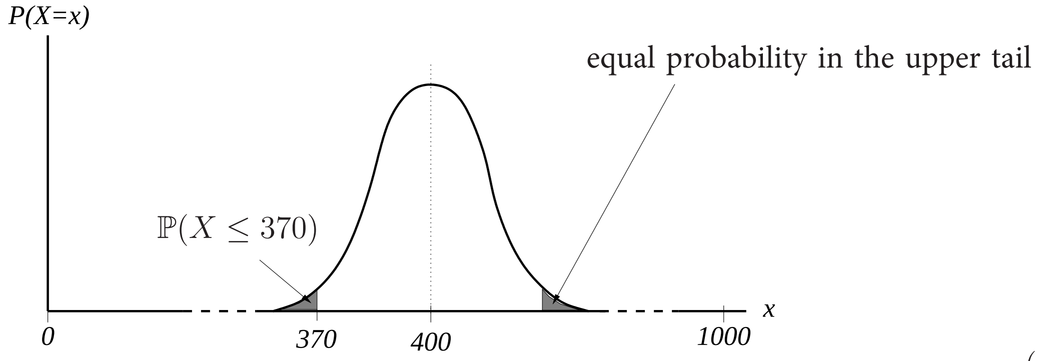
$$H_0: \text{p}_L = 0.4.$$

$$H_1: \text{p}_L \neq 0.4. \qquad \text{(two-sided test)}$$

(3)

(b, c) Under H_0, $X \sim$ Binomial(1000, 0.4).

The probability function of X peaks at about $1000 \times 0.4 = 400$.



(5)

(d) For the p-value, $P(X \leq 370) = \text{F}_X(370)$. Thus
the R command for the p-value is: `2 * pbinom(370, 1000, 0.4)` (2)

(e) Interpretation: about 5.6% of the time when the true value of p is 0.4, the polling of 1000 people will give an answer as extreme as $x = 370$. This p-value is small, but not extremely small. Therefore:
(i) We have some evidence against the null hypothesis that $p = 0.4$.
(ii) We do not have proof against H_0. (We never get definite proof by this method, but this p-value of 0.056 does not come anywhere close to proof.)
(iii) The observed polling is compatible with H_0. A result as extreme as 370/1000 can be expected 5.6% of the time under H_0, so it is certainly compatible with H_0. It is *quite unusual* under H_0, but still *compatible* with H_0. (3)

(f) From the results, we see that both Labour *and* National's results could be compatible with them having equal support at 40%. Clearly they could also be compatible if Labour were a little ahead of National (e.g. 40% for Labour, 39% for National). (2)

Note that this refers to just one poll: if repeated polls continue to put National ahead of Labour, we have good reason to believe that National really is ahead of Labour in the population.

2.(a) Let X be the number of Winsome-voters out of the 1000 respondents.

$X \sim \text{Binomial}(1000, p)$ where p is unknown.

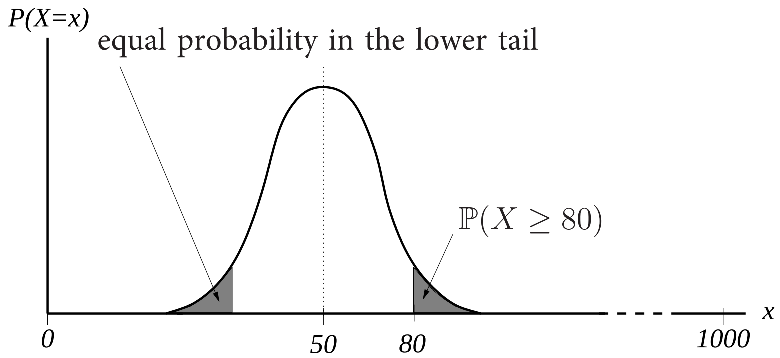
$H_0: \quad p = 0.05.$

$H_1: \quad p \neq 0.05.$ (two-sided test)

(3)

(b, c) Under H_0, $X \sim \text{Binomial}(1000, 0.05)$.

The probability function of X peaks at about $1000 \times 0.05 = 50$.



(5)

(d) For the p-value,

$$\begin{aligned}\mathbb{P}(X \geq 80) &= 1 - \mathbb{P}(X < 80) \\ &= 1 - \mathbb{P}(X \leq 79) \\ &= 1 - F_X(79).\end{aligned}$$

Thus the R command for the p-value is: `2 * (1 - pbinom(79, 1000, 0.05))` (3)

(e) Interpretation: when the true value of p is 0.05, it is extremely unlikely for a polling of 1000 people to give an answer as extreme as $x = 80$. Therefore we have extremely strong evidence against the null hypothesis that $p = 0.05$.

The observed polling is not compatible with the hypothesis that the true level of support is 5%. (2)

(f) The conclusions are very different for the two cases. The difference is that the hypothesized value of p is very small (0.05) in the second case and much larger (0.4) in the first case.

(2)

Explanation: When $p = 0.05$, it is extremely unlikely that a sample proportion of 0.08 will be observed. When $p = 0.4$, on the other hand, it is only slightly unlikely to observe a sample proportion of 0.37. In each case, the shift is 0.03, but the conclusions are different. The result is due to the variance of the Binomial distribution for different values of p, which we will study later in the course.

Total: 30