1. Mr Fixit offers his services as a mender of stuck windows and doors. He says he can fix any stuck window in just 10 to 20 minutes. The time it takes him to fix a stuck window is X minutes, where X is a continuous random variable with probability density function:

$$f_X(x) = \begin{cases} \dfrac{k}{x^2} & \text{for } 10 < x < 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that the constant k is $k = 20$. (2)

(b) Sketch the probability density function, $f_X(x)$, for $-\infty < x < \infty$. (2)

(c) Find the probability that Mr Fixit takes more than 15 minutes to fix a stuck window. (2)

(d) Find the cumulative distribution function of X, $F_X(x) = \mathbb{P}(X \leq x)$. Remember to give the value of $F_X(x)$ for ***all*** $x \in (-\infty, \infty)$. (2)

(e) Using the distribution function, $F_X(x)$, check your answer to part (c) for $\mathbb{P}(X > 15)$, and find also $\mathbb{P}(12 < X < 14)$. (2)

(f) Find the average time taken for Mr Fixit to fix a window, $\mathbb{E}(X)$. (2)

(g) Find $\mathbb{E}(X^2)$, and hence find $\text{Var}(X)$. (2)

(h) Mr Fixit has 10 stuck windows he needs to fix today. Let T be the total time he spends fixing stuck windows today. Say which of the following is the **correct** description of T: description A or description B.

A. $T = 10X$.

B. $T = X_1 + X_2 + \ldots + X_{10}$, where each X_i has the same distribution as X for $i = 1, \ldots, 10$. (2)

(i) Assuming the times taken to fix the 10 windows are independent, find $\mathbb{E}(T)$ and $\text{Var}(T)$. You will need to use your answers to (f), (g), and (h). Specify when you **do** need to use independence, and when you **do not** need to use independence. (2)

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2. **Exam Question, 2008**

A shuttle bus waits at the airport until it is loaded with passengers. Passengers arrive by themselves, according to a Poisson process with rate $\lambda = 20$ passengers per hour. The bus is full when 8 passengers have arrived.

Note: for the exam you will need to have the pdf, cdf, and mean of the Exponential distribution from memory, so you can state them WITHOUT calculation here!

(a) Let T be the **time** the bus driver has to wait for the first passenger to arrive, in hours. State with parameters the distribution of T. (2)

(b) State $\mathbb{E}(T)$, the expected time waited for the first passenger to arrive, in hours. (2)

(c) Find the probability the bus driver has to wait more than 5 **minutes** for the first passenger to arrive. (2)

(d) Find the expected time waited for 8 passengers to arrive. (2)

Total: 26