1.(a) Yes: $X \sim \text{Binomial}(n = 500,\ p = 0.14)$.

(b) Yes: $X \sim \text{Binomial}(n = 8,\ p = 0.7)$.

(c) No: not a fixed number of trials (attempts until the first success).

(d) Yes: $X \sim \text{Binomial}(n = 12,\ p = 0.02)$.

(e) No: not a fixed number of trials (coffees) before she runs out of sugar. (10)

2.(a) $X \sim \text{Binomial}(n = 10,\ p = 0.2)$. (2)

(b)

$$p_2 = \mathbb{P}(X = 2) = \binom{10}{2}(0.2)^2(0.8)^8 = 0.30 \text{ to 2 d.p.}$$

$$p_3 = \mathbb{P}(X = 3) = \binom{10}{3}(0.2)^3(0.8)^7 = 0.20 \text{ to 2 d.p.}$$ (2)

(c) $c_0 = \mathbb{P}(X \leq 0) = \mathbb{P}(X = 0) = 0.11$.
$c_4 = \mathbb{P}(X \leq 4) = \mathbb{P}(X \leq 3) + \mathbb{P}(X = 4) = 0.88 + 0.09 = 0.97$.
$c_5 = \mathbb{P}(X \leq 5) = \mathbb{P}(X \leq 4) + \mathbb{P}(X = 5) = 0.97 + 0.02 = 0.99$. (3)

(d) Looking for $\mathbb{P}(X \geq 6)$.
This is $\mathbb{P}(X \geq 6) = 1 - \mathbb{P}(X \leq 5) = 1 - F_X(5)$.
So:

$$\mathbb{P}(\text{get at least 6 out of 10 by guessing}) = 1 - F_X(5) = 1 - c_5 = 1 - 0.99 = 0.01.$$ (3)

(e) A sum of independent Binomial random variables, sharing the same value of p, is also Binomial. So $T \sim \text{Binomial}(40, 0.2)$. (2)

(f)

$$\mathbb{P}(T = 10) = \binom{40}{10}(0.2)^{10}(0.8)^{30} = 0.107 \text{ to 3 d.p.}$$ (2)

(g) We need:

$$\mathbb{P}(T \geq 20) = 1 - \mathbb{P}(T \leq 19) = 1 - F_T(19).$$

Now

$$\begin{aligned} F_T(19) &= F_T(18) + \mathbb{P}(T = 19) \\ &= 0.9999148 + \binom{40}{19}(0.2)^{19}(0.8)^{40-19} \\ &= 0.9999148 + 0.0000635 \\ &= 0.9999783. \end{aligned}$$

So

$$\mathbb{P}(T \geq 20) = 1 - \mathbb{P}(T \leq 19) = 1 - 0.9999783 = 0.0000217.$$

The probability that Tom passes the Cecil component just by guessing is about 2 in 100 thousand. No, I wouldn't recommend this strategy. Tom would be better off doing a bit of study. (4)