(a) **Answer B:** $\Omega = \{$ Tom's phone calls $\}$. Events $W$, $G$, $S$, $B$, and $L$ all describe attributes of individual phone calls, so $\Omega$ must be a set of phone calls. (2)

(b) The first statement is $\mathbb{P}(W) = 0.2$ (given). The other four are:

(i) $\mathbb{P}(G) = 0.5$.

(ii) $\mathbb{P}(S) = 1 - 0.5 - 0.2 = 0.3$.

(iii) $\mathbb{P}(B) = 0.3$.

(iv) $\mathbb{P}(L) = 0.1$.

(4)

(c) Say whether or not the following sets form a partition of $\Omega$:

(i) $W$, $G$, and $S$: **yes**. They don't overlap, and their probabilities sum to 1 overall.

(ii) $B$ and $L$: **no**. Their probabilities don't sum to 1.

(2)

(d) For each of the following statements, say whether they are **true** or **false**:

(i) $B = S$: **false.** Despite $\mathbb{P}(B) = \mathbb{P}(S)$, the **events** $B$ and $S$ are not the same.

(ii) $W \subseteq \Omega$: **true.** All events are subsets of the sample space.

(iii) $W \cup S = \overline{G}$: **true.** This is because events $W$, $S$, and $G$ are a partition of $\Omega$.

(iv) $W \cap G = \emptyset$: **true.**

(v) $W \cup G \cup S = \Omega$: **true.** Events $W$, $S$, and $G$ are a partition of $\Omega$.

(vi) $B \cup L = \Omega$: **false.** Their probabilities do not sum to 1.

(6)

(e) Information given: $\mathbb{P}(B \cup W) = 0.4$. Information wanted: $\mathbb{P}(B \cap W)$.

$$\begin{aligned} \mathbb{P}(B \cup W) = 0.4 &= \mathbb{P}(B) + \mathbb{P}(W) - \mathbb{P}(B \cap W) \\ 0.4 &= 0.3 + 0.2 - \mathbb{P}(B \cap W) \\ \Rightarrow \quad \mathbb{P}(B \cap W) &= 0.1. \end{aligned}$$

(4)

(f) Want to find $\mathbb{P}(B \cap \overline{W})$. By the Partition Theorem:

$$\begin{aligned} \mathbb{P}(B) &= \mathbb{P}(B \cap W) + \mathbb{P}(B \cap \overline{W}) \\ 0.3 &= 0.1 + \mathbb{P}(B \cap \overline{W}) \\ \Rightarrow \quad \mathbb{P}(B \cap \overline{W}) &= 0.2. \end{aligned}$$

The probability that a call has bad interference and is **not** a wrong number is 0.2. (3)

(g) Tom's claim is that $B \cap S = S$, so that all calls involving sellers also have bad interference. (Another way of putting it is that $S \subseteq B$.) This would mean that $\mathbb{P}(B \cap S) = \mathbb{P}(S) = 0.3$. Consider

$$\begin{aligned} \mathbb{P}(B) &= \mathbb{P}(B \cap W) + \mathbb{P}(B \cap G) + \mathbb{P}(B \cap S) \\ 0.3 &= 0.1 + \mathbb{P}(B \cap G) + \mathbb{P}(B \cap S). \end{aligned}$$

Thus, it is **not** possible that $\mathbb{P}(B \cap S) = 0.3$, because this would mean that $\mathbb{P}(B \cap G)$ would have to be negative! Tom's claim can **not** be correct. (3)