THE UNIVERSITY OF AUCKLAND

STATS 210

SEMESTER 1, 2018
Campus: City

STATISTICS

Term Test, May 4, 2018

(Time allowed: ONE hour)

LAST NAME: _____

GIVEN NAME: _____

ID No: _____

INSTRUCTIONS

* **Answer all parts of all questions.**
* **Write your name and ID No. at the top of your answer sheet.**
* **Total marks 40. Marks are shown for each question.**

1 Consider an experiment in which the sample space Ω contains four outcomes $\{s_1, s_2, s_3, s_4\}$, and suppose that the probability of each outcome is 1/4. Let the three events A, B, C be defined as follows:

$$\begin{aligned} A &= \{s_1, s_2\} \\ B &= \{s_1, s_3\} \\ C &= \{s_1, s_4\} \end{aligned}$$

(a) Calculate $\mathbb{P}(A)$, $\mathbb{P}(B)$, $\mathbb{P}(C)$, $\mathbb{P}(A \cap B)$ and $\mathbb{P}(A \cap B \cap C)$. Show your working.

[4 marks]

(b) Use the results obtained in part (a) in order to show that A is statistically independent of B.

[1 mark]

(c) Decide if the events A, B, C are mutually independent (or not). Show your working.

[Hint: Use the results obtained in part (a)]

[2 marks]

2 The probability that any child in a certain family will have blue eyes is 1/2, and this feature is inherited independently by different children in the family. Under the hypothesis that there are five children in the family, let X be number of children in the family who have blue eyes.

(a) State the distribution of X, with parameters. Explain your answer.

[2 marks]

(b) Calculate $\mathbb{P}(X \geq 1)$. Show your working.

[2 marks]

(c) Prove that $\mathbb{P}(X \geq 3) = 1/2$. Show your working.

[2 marks]

(d) If it is known that at least one of the five children in the family has blue eyes, what is the probability that at least three of the children have blue eyes? Show all your calculations.

[Hint: Use the results obtained in parts (b)-(c)]

[2 marks]

3 The screens used for a certain type of cell phone are manufactured by three companies, A, B, and C. The proportions of screens supplied by A, B, and C are 0.5, 0.3, and 0.2, respectively, and their screens are defective with probabilities 0.01, 0.02, and 0.03, respectively.

Given that the screen on such a phone is defective, what is the probability that company A manufactured it?

[4 marks]

4 Suppose that the proportion p of defective items in a large population of items is unknown. Suppose also that a random sample of 20 items is drawn from the population, and 8 of them are found to be defective.

(a) Let X denote the number of defective items in the sample. The null hypothesis is that $p = 0.2$. Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of X and its parameters. Remember to specify the full distribution of X, and use a two-sided alternative hypothesis.

[3 marks]

(b) Part of the cumulative distribution function, $F_X(x) = \mathbb{P}(X \leq x)$, under the null hypothesis is shown below. Use the values in Table 1 in order to find $\mathbb{P}(X = 6)$. Show your working. Give your answer to 4 decimal places.

x	$\cdots$	2	3	4	5	6	7	8	$\cdots$
$F_X(x)$	$\cdots$	0.2061	0.4114	0.6296	0.8042	0.9133	0.9679	0.9900	$\cdots$

Table 1: Some values of the cumulative distribution function, $F_X(x)$, under the null hypothesis. The total number of items drawn from the population is 20.

[2 marks]

(c) Sketch as a curve the probability function of X under the null hypothesis. Your sketch should have axes labelled x and $\mathbb{P}(X = x)$, and the value of x where the curve peaks. Also mark the observed value of x so that you can see the tail probabilities required for the p-value, and shade under the curve the area represented by the p-value. Your sketch does *not* need to be an accurate plot of the probabilities above.

[3 marks]

(d) Find the p-value for the hypothesis test using the information provided in Table 1. Interpret the result in terms of the strength of evidence against the null hypothesis.

[4 marks]

5 Consider the experiment described in Question 4 (**8 defective items out of 20 items**). We wish to estimate the probability p that an item is defective.

(a) Write down the likelihood function, $L(p; x)$, substituting the correct value of x. State the range of values of p for which the likelihood function is defined.

[2 marks]

(b) Find $\frac{dL}{dp}$, and give all possible solutions to the equation $\frac{dL}{dp} = 0$.

[4 marks]

(c) The likelihood function is plotted in Figure 1. By referring to the graph and using your answer for part (b), find the maximum likelihood estimate of p and state what this maximum likelihood value represents.

[3 marks]

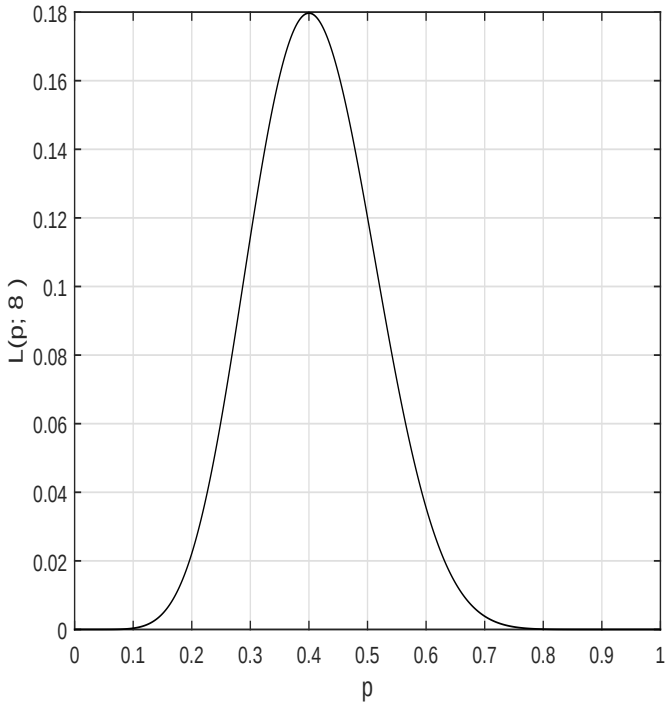

Figure 1: Likelihood function $L(p; 8)$ for the case when the total number of items is 20.