THE UNIVERSITY OF AUCKLAND

STATS 210

SEMESTER 1, 2017
Campus: City

STATISTICS

Term Test, May 2, 2017

(Time allowed: ONE hour)

LAST NAME: _____

GIVEN NAME: _____

ID No: _____

INSTRUCTIONS

* **Answer all parts of all questions.**
* **Write your name and ID No. at the top of your answer sheet.**
* **Total marks 41. Marks are shown for each question.**

1 A family has three children, each of which is equally likely to be a boy or a girl independently of the others. Define the events:

$$\begin{aligned} A &= \{\text{all the children are of the same sex}\}, \\ B &= \{\text{there is at most one boy}\}, \\ C &= \{\text{the family includes a boy and a girl}\}. \end{aligned}$$

(a) Show that A is statistically independent of B. [Hint: Calculate $\mathbb{P}(A)$, $\mathbb{P}(B)$ and $\mathbb{P}(A \cap B)$].

[4 marks]

(b) Show that B is statistically independent of C.

[3 marks]

2 Assume that we have two boxes:

- Box_1 contains 1 white ball and 999 red balls.
- Box_2 contains 1 red ball and 999 white balls.

We randomly select a box, and then pick a ball at random from the selected box. We examine the ball and we find it is red. Calculate the probability that the ball came from Box_1. Show your working.

[6 marks]

3 I toss a coin 20 times and get 15 heads.

(a) Let X be the number of heads out of 20 tosses. The null hypothesis is that the coin is fair. Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of X and its parameters. Remember to specify the full distribution of X, and use a two-sided alternative hypothesis.

[3 marks]

(b) Part of the cumulative distribution function, $F_X(x) = \mathbb{P}(X \leq x)$, under the null hypothesis is shown below. Use the values in Table 1 in order to find $\mathbb{P}(X = 15)$ and $\mathbb{P}(X = 4)$. Show your working. Give your answers to 4 decimal places.

x	$\cdots$	12	13	14	15	16	17	$\cdots$
$F_X(x)$	$\cdots$	0.8684	0.9423	0.9793	0.9941	0.9987	0.9998	$\cdots$

Table 1: Some values of the cumulative distribution function, $F_X(x)$, under the null hypothesis. The total number of tosses is 20.

[3 marks]

(c) Sketch as a curve the probability function of X under the null hypothesis. Your sketch should have axes labelled x and $\mathbb{P}(X = x)$, and the value of x where the curve peaks. Also mark the observed value of x so that you can see the tail probabilities required for the p-value, and shade under the curve the area represented by the p-value. Your sketch does *not* need to be an accurate plot of the probabilities above.

[3 marks]

(d) Find the p-value for the hypothesis test using the probability function above. Interpret the result in terms of the strength of evidence against the null hypothesis.

[4 marks]

(e) I keep tossing the same coin until the total number of tosses is 40. The number of heads is 30.

Let Y be the number of heads out of 40 tosses. Again, the null hypothesis is that the coin is fair. Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of Y and its parameters. Remember to specify the full distribution of Y, and use a two-sided alternative hypothesis.

Find the p-value for the hypothesis test using the values of the cumulative distribution function given in Table 2. Interpret the result in terms of the strength of evidence against the null hypothesis. Compare this interpretation with the one obtained in part (d). If your interpretations are different, explain why.

y	$\cdots$	25	26	27	28	29	30	$\cdots$
$F_Y(y)$	$\cdots$	0.9597	0.9808	0.9917	0.9968	0.9989	0.9997	$\cdots$

Table 2: Some values of the cumulative distribution function, $F_X(x)$, under the null hypothesis. The total number of tosses is 40.

[4 marks]

4 Consider the experiment described in part (e) of Question 3 (**30 heads out of 40 tosses**). We wish to estimate the probability p of getting heads.

(a) Write down the likelihood function, $L(p; y)$, substituting the correct value of y. State the range of values of p for which the likelihood function is defined.

[2 marks]

(b) Find $\frac{dL}{dp}$, and give all possible solutions to the equation $\frac{dL}{dp} = 0$.

[4 marks]

(c) The likelihood function is plotted in Figure 1. By referring to the graph and using your answer for part (b), find the maximum likelihood estimate of p and state what this maximum likelihood value represents.

[3 marks]

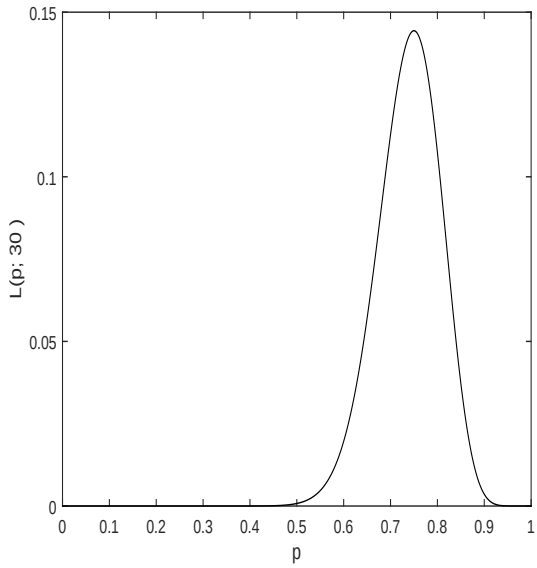

Figure 1: Likelihood function $L(p; 30)$ for the case when the total number of tosses equals 40.

(d) Consider the experiment described at the beginning of Question 3 (**15 heads out of 20 tosses**). We wish to estimate the probability p of getting heads. The likelihood function is plotted in Figure 2. Without making any calculations, find the maximum likelihood estimate of p. Explain your answer.

[2 marks]

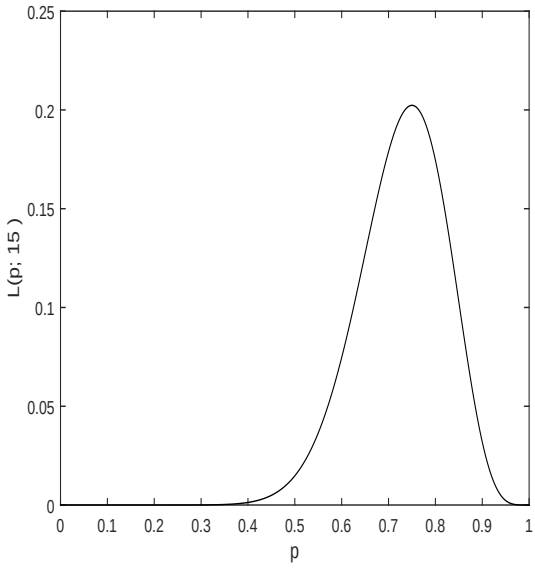

Figure 2: Likelihood function $L(p; 15)$ for the case when the total number of tosses equals 20.