STATS 210	**Tutorial 7**

1. Venus is playing tennis. Every time she serves, the serve is successful with probability 0.64. With probability 0.36, the serve is deemed a Fault. Assume all serves are independent of each other.

   (a) Let Y be the number of Faults that Venus makes before she makes a successful serve. State the distribution of Y, with parameters. (2)

   (b) Find $\mathbb{P}(Y = 0)$, $\mathbb{P}(Y = 1)$, and $\mathbb{P}(Y = 2)$. (3)

   (c) If Venus makes two Faults in a row before a successful serve, she loses a point. The probability that Venus does *not* lose a point can be written $\mathbb{P}(Y \ ? \ 2)$, where the '?' should be replaced by one of the symbols $<$, $>$, $=$, $\leq$, or $\geq$. State which symbol should be used instead of the '?'. (2)

   (d) Find the probability that Venus does not lose a point. You should use the answers to both (b) and (c). (2)

---

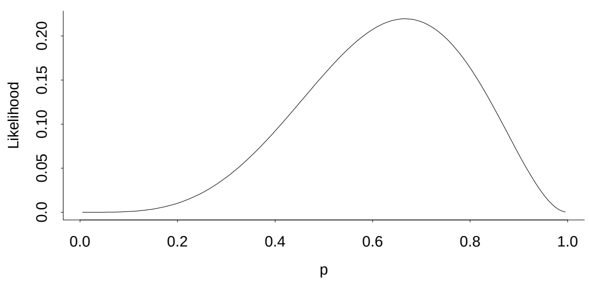
2. In tennis, players aim to win 4 points before they lose 3 points, otherwise they reach Deuce and it becomes harder to win the game. Suppose that Venus has probability p of winning each point, and each point is independent of all other points. Let X be the number of points that Venus *loses* before she wins 4 points.

   (a) State the distribution of X, with parameters. (3)

   (b) For a single game, we make the observation that Venus loses 2 points before she wins 4 points. State the observed value of X. (1)

   (c) We wish to estimate the value of p. Write down the likelihood function for this problem, $L(p\,;\,x)$, substituting the correct value of x from part (b). Remember to state the range of values of p for which the likelihood is defined. (2)

   (d) By differentiating $L(p\,;\,x)$, find the maximum likelihood estimate of p. You should make reference to the graph of the likelihood function, shown below. (4)



---

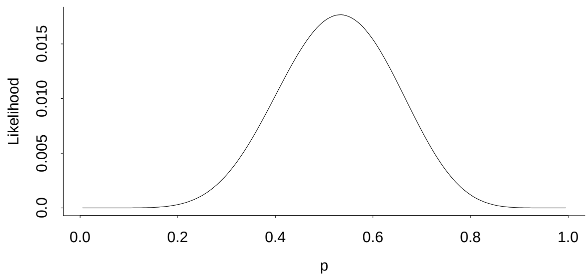
3. Suppose that we make two observations of Venus's X instead of one. For the first observation, Venus loses 2 points before she wins 4 points. For the second observation, Venus loses 5 points before she wins 4 points. Assume that the two observations are independent.

(a) Think of p as being unknown again. In terms of the unknown value of p, write down an expression for $\mathbb{P}(X_1 = 2,\ X_2 = 5)$ when X_1 and X_2 are independent and each have the distribution given in part 2(a).

[Hint: remember that $\mathbb{P}(X_1 = x_1\,, X_2 = x_2) = \mathbb{P}(X_1 = x_1)\mathbb{P}(X_2 = x_2)$ when X_1 and X_2 are independent.] (3)

(b) We wish to estimate p given these *two* observations. Write down the likelihood function, $L(p\,;\,2,\ 5)$. Remember to state the range of values of p for which the likelihood is defined. (2)

(c) By differentiating $L(p\,;\,x)$, show that the maximum likelihood estimate of p is now $\widehat{p} = \frac{8}{15}$. You should make reference to the graph of the likelihood function, shown below. (4)



(d) Explain in one sentence why you would expect $\widehat{p} = \frac{8}{15}$ to be the correct answer.

[Hint: how many points did Venus win in total? How many did she lose in total?] (2)

Total: 30