STATS 210 **Tutorial 6**

This tutorial is a Revision Tutorial. It is based on a past mid-semester test. The past test included one extra question, asking for a hypothesis test.

1. The police have collected the following statistics about a set of car accidents.

   80% of the accidents were caused by speeding drivers.

   Out of the accidents caused by speeding drivers, the driver had a Full Licence in 87.5% of cases, and a Restricted Licence in 7.5% of cases.

   5% of all accidents were caused by drivers with no licence at all. 15% of accidents were caused by drivers with Full licences who were not speeding.

   The sample space is $\Omega = \{\text{all accidents}\}$. Define the following events:

   $F = \{\text{driver had Full licence}\}$ $S = \{\text{driver was speeding}\}$

   $R = \{\text{driver had Restricted licence}\}$ $\overline{S} = \{\text{driver was not speeding}\}$

   $N = \{\text{driver had No licence}\}$

   You may assume that events F, R, and N form a partition of the sample space Ω.

   (a) Show that $\mathbb{P}(N \mid S) = 0.05$. (2)

   (b) Find $\mathbb{P}(N \cap S)$. (2)

   (c) Are events N and S independent? Show your working. (2)

   (d) What is the probability that an accident was caused by a driver who was either speeding or had no licence? (2)

   (e) Find $\mathbb{P}(F)$. (2)

   (f) Find $\mathbb{P}(R)$. (2)

   (g) Among the drivers in the data above, what is the probability that a Restricted licence driver was speeding? (2)

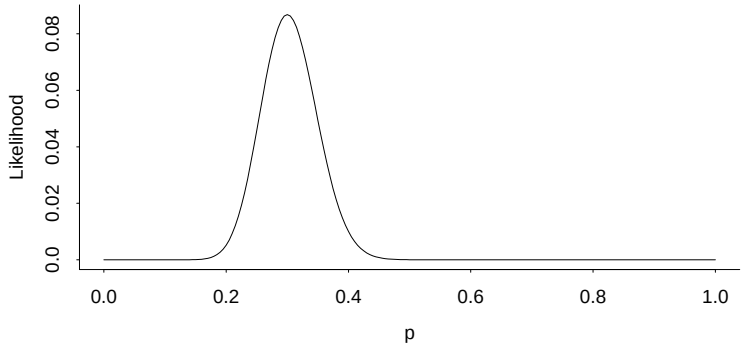
[continued...]

2. Professor Brainpop has 100 students who enter the Really Hard Statistics Exam. He believes that each student will pass the exam with probability p, independently of all other students. When the results arrive, 30 of Brainpop's students have passed the exam. Professor Brainpop wishes to estimate the value of p.

   (a) Write down the likelihood function for this problem, $L(p\,;x)$, substituting the correct value of x. State the range of values of p for which the likelihood is defined. (2)

   (b) Find $\frac{dL}{dp}$, and give three possible solutions to the equation $\frac{dL}{dp} = 0$. (6)

   (c) The likelihood function is plotted here. By referring to the graph and using your answer from (b), find the maximum likelihood estimate of p. (2)



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3. In Question 2, Professor Brainpop knows that the national pass-rate for the Really Hard Statistics Exam is 40%. He is worried that his own students might have a lower pass-rate, so he conducts the following hypothesis test:

$$X \sim \text{Binomial}(100, p)\,; \qquad H_0 \;:\; p = 0.4\,; \qquad H_1 \;:\; p \neq 0.4.$$

With his observation $x = 30$, Prof B's p-value for the test is 0.050.

Dr Draintop has 1000 students, of whom 300 passed the exam. Dr D conducts a similar test to Prof B:

$$X \sim \text{Binomial}(1000, p)\,; \qquad H_0 \;:\; p = 0.4\,; \qquad H_1 \;:\; p \neq 0.4.$$

   (a) Will Dr D's p-value be higher than 0.050, or lower? (2)

   (b) Which teacher out of Prof B and Dr D should be more worried about their students' pass rate being lower than the national 40%? Briefly explain why. (2)

Total: 28