STATS 210 **Tutorial 2**

The inhabitants of Planet Zog come in three different colours: red, orange, and green. (This diversity can be very useful: for example, different coloured Zoglins can be employed three at a time as traffic lights.) In the whole population, 40% of Zoglins are red, while 20% are orange and 40% are green.

The sample space can be written as $\Omega = \{\text{Zoglins}\}$. We can formulate events as follows:

$$R = \text{“Zoglin is red”} \qquad O = \text{“Zoglin is orange”} \qquad G = \text{“Zoglin is green”}\,.$$

From the information given, the events have probabilities

$$\mathbb{P}(R) = 0.4 \qquad \mathbb{P}(O) = 0.2 \qquad \mathbb{P}(G) = 0.4.$$

(a) Do the events R, O, and G form a partition of the sample space Ω? Explain why or why not. (2)

Some Zoglins have two heads, and some have long noses. Of the red Zoglins, 50% have two heads, while 20% have long noses. Two-headed orange Zoglins constitute 2% of the population, while 6% of Zoglins are of the long-nosed orange variety. 60% of green Zoglins are two-headed, but only 5% have long noses.

Let T be the event that a Zoglin has two heads, and L be the event that a Zoglin is long-nosed.

(b) Do the events T and L form a partition of the sample space? Explain why or why not. (2)

(c) Write down all the information in the paragraph above as probability statements. Your answer should consist of 6 statements.

[Hint: the first statement is $\mathbb{P}(T \,|\, R) = 0.5$. Be on the lookout for statements that are expressed in a different way.] (5)

(d) Write down the values of $\mathbb{P}(T \,|\, \text{colour})$ for each of the three colours R, O, and G. Some of these were already calculated in part (c), whereas others were not. (2)

(e) Find $\mathbb{P}(T)$. (2)

(f) Find $\mathbb{P}(T \cup R)$, and describe in words what this represents. (2)

(g) From the information given, is it possible to calculate $\mathbb{P}(T \cap L \,|\, R)$ (the probability that a red Zoglin is both two-headed and long-nosed)? Explain why or why not. (2)

Zoglins can be either friendly, or crafty, or vicious. Red Zoglins are always friendly, but orange Zoglins are friendly only with probability 0.5; they are crafty with probability 0.4, and with probability 0.1 they are vicious. Green Zoglins are friendly with probability 0.2, crafty with probability 0.5, and vicious with probability 0.3.

Formulate events F, C, and V for the character types of Zoglins. You may assume that events F, C, and V are mutually exclusive.

(h) Write down probability statements corresponding to the information given above. Your answer should involve 9 statements, each linking one character type, F, C, or V, with one colour, R, O, or G. (8)

(i) Do the events F, C, and V form a partition of the sample space? Explain why or why not. (2)

(j) Find the probability that a Zoglin is friendly. (3)

Total: 30