SEMESTER 1, 2015
Campus: City

STATISTICS

Term Test, April 28, 2015

(Time allowed: ONE hour)

LAST NAME: ____________________

GIVEN NAME: ____________________

ID No: ____________________

INSTRUCTIONS

* **Answer all parts of all questions.**
* **Write your name and ID No. at the top of your answer sheet.**
* **Total marks 32. Marks are shown for each question.**

CONTINUED

1 Assume that the sample space is $\Omega = \{s_1, s_2, s_3, s_4\}$. We have that

$$\begin{aligned}\mathbb{P}(s_1) &= 0,\\ \mathbb{P}(s_2) &= 1/2,\\ \mathbb{P}(s_3) &= 0,\\ \mathbb{P}(s_4) &= 1/2.\end{aligned}$$

We define the events:

$$\begin{aligned}A &= \{s_1, s_2\},\\ B &= \{s_2, s_3\}.\end{aligned}$$

Calculate $\mathbb{P}(A)$, $\mathbb{P}(B)$, $\mathbb{P}(A \cap B)$ and $\mathbb{P}\left(\left(A \cap \overline{B}\right) \cup \left(\overline{A} \cap B\right)\right)$.

[5 marks]

CONTINUED

2 Toss a coin twice and observe the result. The sample space is $\{HH, HT, TH, TT\}$. We assume that

$$\begin{aligned}\mathbb{P}(HH) &= p^2,\\ \mathbb{P}(HT) &= p \times q,\\ \mathbb{P}(TH) &= p \times q,\\ \mathbb{P}(TT) &= q^2,\end{aligned}$$

where $p > 0$, $q > 0$ and $p + q = 1$.

(a) Show that $\mathbb{P}(\Omega) = 1$. [Hint: $(p+q)^2 = p^2 + 2 \times p \times q + q^2$].

[1 mark]

(b) We define the events:

$$\begin{aligned}A &= \{\text{Heads at first toss}\},\\ B &= \{\text{Heads at second toss}\}.\end{aligned}$$

Prove that A and B are statistically independent.

[4 marks]

CONTINUED

3 A box contains three coins:

- Two of them are regular coins. Whenever tossing such a coin, we have

$$\mathbb{P}(H) = \mathbb{P}(T) = 1/2.$$

- The third one is a fake two-headed coin. When tossing it, we have

$$\mathbb{P}(H) = 1.$$

(a) You pick a coin at random from the box and toss it (without checking if it is regular or fake). What is the probability that it lands heads up?

[4 marks]

(b) You pick a coin at random from the box and toss it, and get heads. What is the probability that it is the two-headed coin?

[2 marks]

4 A pair of fair dice is rolled 10 times. The 10 rolls are independent. For the n-th trial, $n \in \{1, 2, \dots, 10\}$, let S_n denote the sum of the two faces and let F_n be the event $\{S_n \neq 7\}$.

(a) Show that $\mathbb{P}(F_n) = 5/6$ for all $n \in \{1, 2, \dots, 10\}$.

[2 marks]

(b) We define the event:

$$A = \{\text{At least one of } S_1, S_2, \dots, S_{10} \text{ is equal to } 7\}.$$

Prove that $\mathbb{P}(A) = 1 - (5/6)^{10}$.
[Hint: It might be easier to start by computing the probability of $\overline{A}$].

[2 marks]

5 According to the results of a recently published census, which was carried out of all staff and PhD students housed in a STATS Department and covered the years 2000-2014, the total offspring number is 36, comprising 25 boys and 11 girls.

(a) Let X be the number of boys out of 36 offspring. The null hypothesis is that the members of the STATS Department are equally likely to have sons and daughters. Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of X and its parameters. Remember to specify the full distribution of X under the null hypothesis, and use a two-sided alternative hypothesis.

[2 marks]

(b) Part of the probability function, $f_X(x) = \mathbb{P}(X = x)$, and part of the cumulative distribution function, $F_X(x) = \mathbb{P}(X \leq x)$, under the null hypothesis are shown below. Find the missing values p_1 and c_1 showing your working. Give your answers to 4 decimal places.

x	$\cdots$	21	22	23	24	25	$\cdots$
$f_X(x)$	$\cdots$	0.0810	0.0552	0.0336	p_1	0.0087	$\cdots$
$F_X(x)$	$\cdots$	0.8785	c_1	0.9674	0.9856	0.9943	$\cdots$

[3 marks]

(c) Sketch as a curve the probability function of X under the null hypothesis. Your sketch should have axes labelled x and $\mathbb{P}(X = x)$, and the value of x where the curve peaks. Also mark the observed value of x so that you can see the tail probabilities required for the p-value, and shade under the curve the area represented by the p-value. Your sketch does *not* need to be an accurate plot of the probabilities above.

[3 marks]

(d) Find the p-value for the hypothesis test. Interpret the result in terms of the strength of evidence against the null hypothesis.

[4 marks]