1. Let events F ={Full licence}, R ={Restricted licence}, N ={No licence}, S ={Speeding}. Information given:

$$\mathbb{P}(S) = 0.8 \qquad \mathbb{P}(F \mid S) = 0.875 \qquad \mathbb{P}(R \mid S) = 0.075$$

$$\mathbb{P}(N) = 0.05 \qquad \mathbb{P}(F \cap \overline{S}) = 0.15$$

Also, F, R, and N are a partition of Ω.

(a) Because F, R, and N are a partition of Ω,

$$\begin{aligned} \mathbb{P}(F \mid S) + \mathbb{P}(R \mid S) + \mathbb{P}(N \mid S) &= 1 \\ \Rightarrow \quad \mathbb{P}(N \mid S) &= 1 - \mathbb{P}(F \mid S) - \mathbb{P}(R \mid S) \\ &= 1 - 0.875 - 0.075 \\ &= 0.05 \quad \text{as stated.} \end{aligned}$$

(2)

(b) $\mathbb{P}(N \cap S) = \mathbb{P}(N \mid S)\mathbb{P}(S) = 0.05 \times 0.8 = 0.04.$ (2)

(c) We know that $\mathbb{P}(N \cap S) = 0.04$. Now

$$\mathbb{P}(N) \times \mathbb{P}(S) = 0.05 \times 0.8 = 0.04 \quad \text{also.}$$

Thus $\mathbb{P}(N \cap S) = \mathbb{P}(N)\mathbb{P}(S)$, so by definition of independence, N and S <u>are</u> independent. (2)

(d) Looking for $\mathbb{P}(S \cup N)$.

$$\begin{aligned} \mathbb{P}(S \cup N) &= \mathbb{P}(S) + \mathbb{P}(N) - \mathbb{P}(S \cap N) \\ &= 0.8 + 0.05 - 0.04 \\ &= 0.81. \end{aligned}$$

(2)

(e) By the Partition Theorem,

$$\begin{aligned} \mathbb{P}(F) &= \mathbb{P}(F \cap S) + \mathbb{P}(F \cap \overline{S}) \\ &= \mathbb{P}(F \mid S)\mathbb{P}(S) + \mathbb{P}(F \cap \overline{S}) \\ &= 0.875 \times 0.8 + 0.15 \qquad \text{(from information in question)} \\ &= 0.85. \end{aligned}$$

(2)

(f) Because F, R, and N are a partition of Ω,

$$\mathbb{P}(R) = 1 - \mathbb{P}(F) - \mathbb{P}(N) = 1 - 0.85 - 0.05 = 0.1.$$

(2)

(g) Looking for $\mathbb{P}(S \mid R)$. By Bayes' Theorem,

$$\begin{aligned} \mathbb{P}(S \mid R) &= \frac{\mathbb{P}(R \mid S)\mathbb{P}(S)}{\mathbb{P}(R)} \\ &= \frac{0.075 \times 0.8}{0.1} \\ &= 0.6. \end{aligned}$$

(2)

2.(a) X is the number of passes out of 100 students: $X \sim \text{Binomial}(100, p)$ where p is unknown. The observation is $x = 30$ passes.
The likelihood is: $L(p\,; 30) = \mathbb{P}(X = 30)$ when $X \sim \text{Binomial}(100, p)$.
Thus

$$\begin{aligned} L(p\,; 30) &= \binom{100}{30} p^{30}(1-p)^{100-30} \\ &= \binom{100}{30} p^{30}(1-p)^{70} \qquad \text{for } 0 < p < 1. \end{aligned}$$

(2)

(b)

$$\begin{aligned} \frac{dL}{dp} &= \binom{100}{30} \times \left\{30p^{29}(1-p)^{70} + p^{30} \times 70(1-p)^{69} \times (-1)\right\} \\ &= \binom{100}{30} p^{29}(1-p)^{69}\left\{30(1-p) - 70p\right\} \\ &= \binom{100}{30} p^{29}(1-p)^{69}\left\{30 - 100p\right\}. \end{aligned}$$

The likelihood equation is:

$$\begin{aligned} \frac{dL}{dp} &= 0 \\ \Rightarrow \quad \binom{100}{30} p^{29}(1-p)^{69}(30 - 100p) &= 0 \end{aligned}$$

$$\Rightarrow \quad p = 0, \quad p = 1, \quad \text{or} \quad p = \frac{30}{100} = 0.3.$$

The three possible solutions to the likelihood equation are: $p = 0$, $p = 1$, and $p = 0.3$. (6)

(c) We can see from the sketch that $p \neq 0$ and $p \neq 1$ for the maximum, so the maximizing value of p is $p = 0.3$. (2)

3.(a) Dr Draintop's p-value will be **lower** than 0.050. (2)

(b) Dr D should be more worried about her students' pass-rate. Both Dr D and Prof B have the same proportion of passes in their students. However, in her much larger sample, Dr D has *more evidence to detect* that her pass-rate is not the same as the national average of 40%. (2)
