STATS 210 **Tutorial 4**

1. John Key and Andrew Little are running for President of the Matchstick Corp. As usual, their election campaign turns into a bitter fight. Their biggest argument is about the statistical properties of the opinion polls. John Key's campaign team contracts Herald Dodgypoll to ask a random sample of 1000 voters whether they plan to vote for Key or not. 410 respondents (41%) say yes. Andrew Little's team employs another statistical company, Callmy Bluffton, to find another 1000 voters and ask whether they plan to vote for Little. 370 respondents (37%) say yes.

   We will perform two hypothesis tests to analyse the results of these polls. Key's test will be used as an example. Little's test is for you to perform in the tutorial.

   In each case, we wish to test the hypothesis that the true level of support for the candidate amongst all voters is 40%. Assume for all the following working that the Matchstick Corp is very very large.

   We use the following formulation:

   (i) For the first poll, X is the number of respondents who say they will vote for Key.
   - $X \sim \text{Binomial}(1000, p_{\text{K}})$ where p_{K} is the unknown true proportion of Key-voters in the Matchstick Corp.
   - *sample answers for the first poll are written in this font at the end of the tutorial.*

   (ii) For the second poll, X is the number of respondents who say they will vote for Little.
   - $X \sim \text{Binomial}(1000, p_{\text{L}})$ where p_{L} is the unknown true proportion of Little-voters in the Matchstick Corp.
   - answers are for you to complete in the tutorial.

(a) Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of X and its parameters. Remember to specify the full distribution of X under the null hypothesis. Use a two-sided test. (3)

(b) Sketch as a curve the shape of the probability function of X under the null hypothesis. Your sketch should have axes labelled x and $\text{P}(X = x)$. Mark on the sketch the upper and lower limits of x, (not to scale), and the value of x where the curve peaks. (3)

(c) Mark the observed value of X on your sketch, so that you can see the tail probabilities required for the p-value. Shade the area under the curve corresponding to the p-value. (2)

(d) Write down the *R* command required to find the p-value for the hypothesis test. (2)

(e) Running the command in part (d) in *R* gives 0.05608583. Interpret this result in terms of strength of evidence against the null hypothesis. You should say, (i) whether we have any evidence against H_0, (ii) whether we have proof against H_0, and (iii) whether or not it is true that the observed polling for Little is compatible with H_0 — in other words, could it happen that the support for Little really is 40% and we nonetheless see an observed polling of 370/1000? (3)

Compare your conclusion from part (e) for Little with the sample answer for Key. Although the example above is fictitious, it is based on real political polls: in NZ we frequently see the two main candidates or parties polling close to 40% and within about 4 points of each other, and the polls are usually taken over 1000 people.

(f) If an opinion poll has National polling at 41% and Labour polling at 37%, does this mean that National is definitely ahead of Labour? Or could the same result emerge if National and Labour had equal support in the population, or even if Labour were ahead of National? (2)

Note: doing it properly

The analysis above does give us insight into the results of opinion polls. However, for a correct analysis of opinion polls, we would usually use slightly different methods, because it is usually a single sample of people who are asked to choose between candidates, not two separate polls as in the Little and Key example. Additionally, in the case of two separate polls, we would usually use a two-sample test to test whether the means could be equal, rather than using two separate hypothesis tests to test whether each observation is individually compatible with a single hypothesized value.

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2. Winsome Peters is also running for President of the Matchstick Corp. His current polling is 80 out of 1000 respondents (8%). If he doesn't get at least 5% of the vote, he will lose his deposit. Winsome wishes to test whether it is possible that he really only has 5% support, given that 80 out of 1000 respondents said that they would vote for him.

(a) Define an appropriate random variable X, as in Question 1. Formulate the null hypothesis and alternative hypothesis, in terms of the distribution of X and its parameters. Remember to specify the full distribution of X under the null hypothesis. Use a two-sided test. (3)

(b) Sketch as a curve the shape of the probability function of X under the null hypothesis. Your sketch should have axes labelled x and $P(X = x)$. Mark on the sketch the upper and lower limits of x, (not to scale), and the value of x where the curve peaks. (3)

(c) Mark the observed value of X on your sketch, so that you can see the tail probabilities required for the p-value. Shade the area under the curve corresponding to the p-value. (2)

(d) Write down the *R* command required to find the p-value for the hypothesis test. (3)

(e) Running the command in part (d) in *R* gives 6.97729e-05. Interpret this result in terms of strength of evidence against the null hypothesis. Is the observed polling for Winsome compatible with the hypothesis that the true level of support for Winsome is 5%? (2)

(f) Compare your result for question 2(e) with your result for question 1(e). In each case, we were testing a hypothesis that the true p was a shift of 3 percentage points away from the observed polling: testing 5% support given 8% polling in Q2; and testing 40% support given 37% polling in Q1. Are the conclusions the same? What is the primary di erence between the two situations? (2)

Total: 30

Sample answers for the Key poll in Question 1

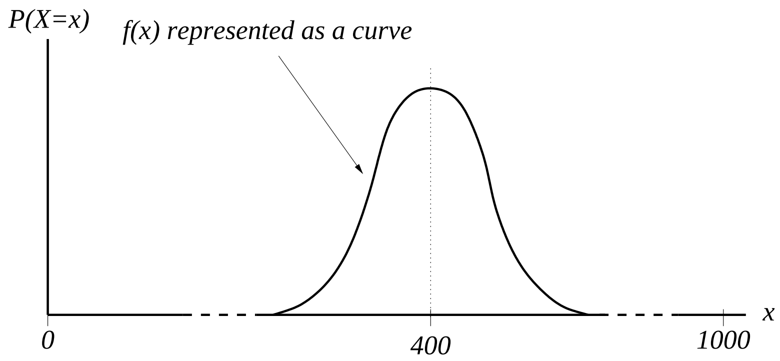
(a) *Let X be the number of Key-voters out of 1000 respondents.*

$$
\begin{aligned}
X &\sim \textit{Binomial}(1000,\ p_K) \\
H_0 &: p_K = 0.4. \\
H_1 &: p_K \neq 0.4. \qquad \textit{(two-sided test)}
\end{aligned}
$$

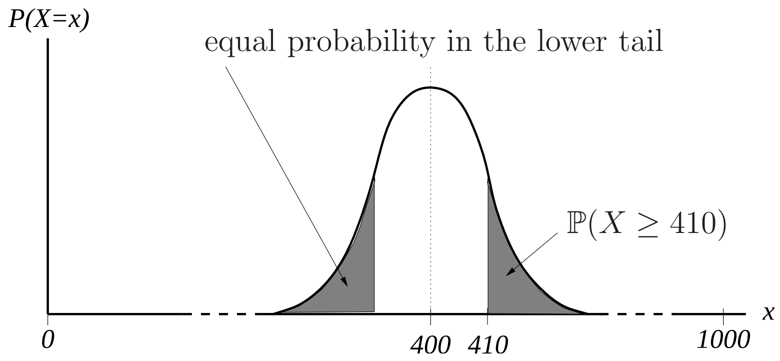
(b) *Under H_0, $X \sim$ Binomial$(1000, 0.4)$.*

The probability function of X peaks at about $1000 \times 0.4 = 400$.

Sketch:



(c)



(d) *For the p-value,*

$$
\begin{aligned}
\mathbb{P}(X \geq 410) &= 1 - \mathbb{P}(X < 410) \\
&= 1 - \mathbb{P}(X \leq 409) \\
&= 1 - F_X(409).
\end{aligned}
$$

Thus the R command for the p-value is: `2 * (1 - pbinom(409, 1000, 0.4))`

(e) *For the Key test, running the command in (d) in R gives the result* `0.5388626`.

Interpretation: over 53% of the time when the true value of p is 0.4, the polling of 1000 people will give an answer as extreme as $x = 410$. This p-value is large and therefore we have no evidence against the null hypothesis that $p = 0.4$. The observed polling is compatible with the possibility that the true support for Key is 40%.

(Note that this does not imply that p really is 0.4; it just tells us that the observation is **compatible** *with this possibility.)*