(a) Yes, events R, O, and G do form a partition of the sample space.

- There is no overlap between the events, because a Zoglin cannot be two different colours at once.
- The events collectively cover all possible outcomes in Ω, because

$$\mathbb{P}(R) + \mathbb{P}(O) + \mathbb{P}(G) = 1.$$

(2)

(b) No, events T and L do not form a partition of the sample space. This is clear because of the red Zoglins, 50% satisfy T, and 20% satisfy L. Therefore there must be at least 30% of red Zoglins that satisfy neither T nor L. So T and L do not cover all possible outcomes.

Additionally, we are not told that there is no overlap between T and L: it might be possible for a Zoglin to have two heads *and* a long nose. (2)

(c) Information given:

$$\begin{aligned} \mathbb{P}(T \mid R) &= 0.50 \qquad & \mathbb{P}(L \mid R) &= 0.20\,; \\ \mathbb{P}(T \cap O) &= 0.02 \qquad & \mathbb{P}(L \cap O) &= 0.06\,; \\ \mathbb{P}(T \mid G) &= 0.60 \qquad & \mathbb{P}(L \mid G) &= 0.05. \end{aligned}$$

(5)

(d) We only need to make a calculation for $\mathbb{P}(T \mid O)$, as the other two probabilities are given directly.

$$\mathbb{P}(T \mid O) = \frac{\mathbb{P}(T \cap O)}{\mathbb{P}(O)} = \frac{0.02}{0.2} = 0.1.$$

Thus:

$$\mathbb{P}(T \mid R) = 0.5\,; \qquad \mathbb{P}(T \mid O) = 0.1\,; \qquad \mathbb{P}(T \mid G) = 0.6.$$

(2)

(e) Using the Partition Theorem, because R, O, and G form a partition of Ω:

$$\begin{aligned} \mathbb{P}(T) &= \mathbb{P}(T \mid R)\mathbb{P}(R) + \mathbb{P}(T \mid O)\mathbb{P}(O) + \mathbb{P}(T \mid G)\mathbb{P}(G) \\ &= 0.5 \times 0.4 + 0.1 \times 0.2 + 0.6 \times 0.4 \\ &= 0.46. \end{aligned}$$

(2)

(f) $\mathbb{P}(T \cup R)$ is the probability that a Zoglin has two heads, ***or*** is red, ***or*** both.

$$\begin{aligned}\mathbb{P}(T \cup R) &= \mathbb{P}(T) + \mathbb{P}(R) - \mathbb{P}(T \cap R) \\ &= 0.46 + 0.4 - \mathbb{P}(T \mid R)\mathbb{P}(R) \\ &= 0.46 + 0.4 - 0.5 \times 0.4 \\ &= 0.66.\end{aligned}$$

(2)

(g) No, it is not possible to calculate $\mathbb{P}(T \cap L \mid R)$, because we are not given any information about the overlap between T and L, for any Zoglins. (2)

(h) Let

$$F = \text{"Zoglin is friendly"} \qquad C = \text{"Zoglin is crafty"} \qquad V = \text{"Zoglin is vicious"}$$

Information given:

$$\begin{array}{lll} \mathbb{P}(F \mid R) = 1 & \mathbb{P}(C \mid R) = 0 & \mathbb{P}(V \mid R) = 0\,; \\ \mathbb{P}(F \mid O) = 0.5 & \mathbb{P}(C \mid O) = 0.4 & \mathbb{P}(V \mid O) = 0.1\,; \\ \mathbb{P}(F \mid G) = 0.2 & \mathbb{P}(C \mid G) = 0.5 & \mathbb{P}(V \mid G) = 0.3. \end{array}$$

(8)

(i) Yes, events F, C, and V do form a partition of the sample space.

- There is no overlap between them, because they are mutually exclusive (given).
- They collectively cover all possible outcomes. This is because all Zoglins are red, orange, or green. In the red category, the probabilities of F, C, and V sum to 1. In the orange category they sum to 1, and in the green category, they sum to 1. Thus every Zoglin has to be one of F, C, or V.

(2)

(j) Use the Partition Theorem with the partition R, O, and G:

$$\begin{aligned}\mathbb{P}(F) &= \mathbb{P}(F \mid R)\mathbb{P}(R) + \mathbb{P}(F \mid O)\mathbb{P}(O) + \mathbb{P}(F \mid G)\mathbb{P}(G) \\ &= 1 \times 0.4 + 0.5 \times 0.2 + 0.2 \times 0.4 \\ &= 0.58.\end{aligned}$$

(3)