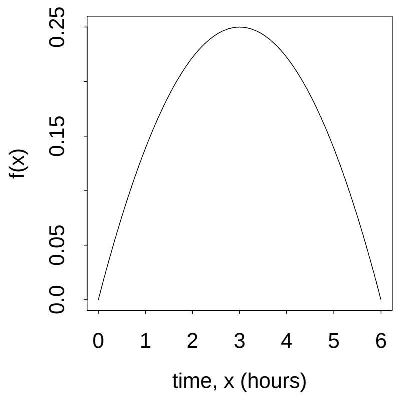
1. How long do you spend on your Stats assignment? Tom thinks his time, X, follows the following **continuous** distribution:



$$f_X(x) = \begin{cases} k(6x - x^2) & \text{for } 0 \le x \le 6, \\ 0 & \text{otherwise.} \end{cases}$$

Here, X is the time taken in hours, $f_X(x)$ is the probability **density** function (PDF) of X, and k is a constant to be determined.

(a) Show that $k = \frac{1}{36}$. [Hint: we need $\int_0^6 f_X(x)\,dx = 1$.] (3)

(b) By integrating $f_X(x)$, find the probability that Tom spends between 3 and 4 hours on his assignment. (3)

(c) Find the cumulative distribution function, $F_X(x)$. Remember to use a **dummy variable** for your integration. Write your answer in the following form:

$$F_X(x) = \begin{cases} \dots & \text{for } x < 0, \\ \dots & \text{for } 0 \le x \le 6, \\ \dots & \text{for } x > 6. \end{cases}$$

(3)

(d) Check your answer for (c) by checking that your formula for $F_X(x)$ when $0 \le x \le 6$ gives $F_X(6) = 1$.
(According to the p.d.f. specified, Tom *must* take less than 6 hours for his assignment.) (1)

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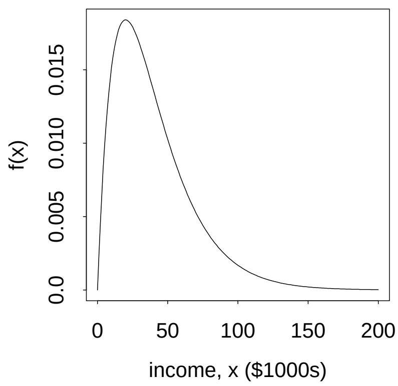
2. The probability density function opposite shows an approximate model for household income in NZ in 1995. The cumulative distribution function for the model is given by:



$$F_X(x) = \begin{cases} 1 - \left(1 + \frac{x}{20}\right) e^{-x/20} & \text{for } 0 < x < \infty, \\ 0 & \text{for } x \le 0. \end{cases}$$

Here, X represents household income for a randomly selected household in thousands of dollars.

(a) Using the cumulative distribution function $F_X(x)$, find the probability that a household had income less than $30,000 in 1995. (2)

(b) Find the probability that a household had income greater than $100,000 in 1995. (2)

(c) Find $\mathbb{P}(30 < X < 50)$. (2)

(d) Show that the p.d.f. of X is $f_X(x) = \dfrac{x}{400} e^{-x/20}$ for $0 < x < \infty$. This is the curve that was plotted in the graph above. [Hint: use the product rule for differentiation.] (4)

Total: 20