1.(a) $X \sim \text{Binomial}(7, p)$. (2)

(b) $x = 5$. (1)

(c) $L(p\,;5) = \mathbb{P}(X = 5)$ when $X \sim \text{Binomial}(7, p)$.
Thus

$$\begin{aligned} L(p\,;5) &= \binom{7}{5} p^5 (1-p)^{7-5} \\ &= 21p^5(1-p)^2. \qquad 0 < p < 1 \end{aligned}$$

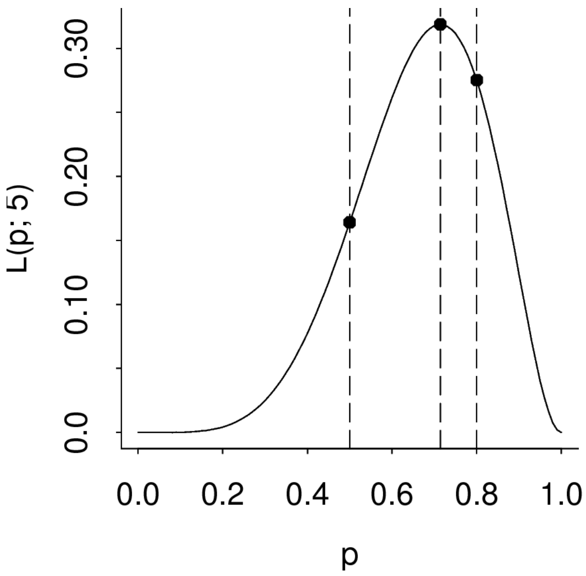
(2)

(d) $L\left(\frac{1}{2}\,;5\right) = 21 \times \left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^2 = 0.164.$ (1)

(e) $L\left(\frac{5}{7}\,;5\right) = 21 \times \left(\frac{5}{7}\right)^5 \times \left(\frac{2}{7}\right)^2 = 0.319.$ (1)

(f) $L\,(0.8\,;5) = 21 \times (0.8)^5 \times (0.2)^2 = 0.275.$ (1)

(g)



(2)

(h)

$$\begin{aligned} \frac{dL}{dp} &= 21 \times \left\{ 5p^4(1-p)^2 + p^5 \times 2(1-p) \times (-1) \right\} \\ &= 21p^4 \left\{ 5(1-p)^2 - 2p(1-p) \right\} \\ &= 21p^4(1-p) \left\{ 5(1-p) - 2p \right\} \\ &= 21p^4(1-p) \left\{ 5 - 7p \right\} \qquad \text{as stated.} \end{aligned}$$

(5)

(i) The maximizing value of p satisfies:

$$\begin{aligned}\frac{dL}{dp} &= 0\\ \Rightarrow\quad 21p^4(1-p)(5-7p) &= 0\\ \Rightarrow\quad p=0,\quad p=1,\quad \text{or}\ \ p&=\frac{5}{7}.\end{aligned}$$

We can see from the sketch that $p \neq 0$ and $p \neq 1$, so the maximizing value of p is

$$p = \frac{5}{7}.$$

(3)

(j) The maximum likelihood estimate, $p = \frac{5}{7}$, is the value of p at which the observation $X = 5$ is more likely than at any other value of p. (2)

2.(a) $L(p\,;4) = \mathbb{P}(X = 4)$ when the parameter takes the value p.
So

$$L(p\,;4) = (1-p)^3 p \qquad \text{for } 0 < p < 1.$$

(2)

(b)

$$\begin{aligned}\frac{dL}{dp} &= -3(1-p)^2 p + (1-p)^3\\ &= (1-p)^2\left\{-3p + (1-p)\right\}\\ &= (1-p)^2(1-4p) \qquad \text{as stated.}\end{aligned}$$

(4)

(c) The maximizing value of p satisfies:

$$\begin{aligned}\frac{dL}{dp} &= 0\\ \Rightarrow\quad (1-p)^2(1-4p) &= 0\\ \Rightarrow\quad p=1,\quad \text{or}\quad p&=\frac{1}{4}.\end{aligned}$$

We can see from the sketch that $p \neq 1$, so the maximizing value of p is

$$p = \frac{1}{4}.$$

(2)

(d) Sammie first succeeds on his 4th jump, so his probability of success could be estimated as 1/4 for each jump. This is the same as the maximum likelihood estimate of 1/4. (2)