

Tom has a complicated social life involving lots of girlfriends, although for some reason he has no other friends. He gets three different types of phone calls: all his calls are either wrong numbers, girlfriends, or somebody trying to sell something to him.

Some calls have bad interference, and this seems to be particularly common when somebody is trying to sell something. Most of Tom's calls are short, but some are quite long.

Define the following events:

W = “call is a wrong number” G = “call is from a girlfriend”

S = “call is from somebody trying to sell something”

B = “call has bad interference” L = “call is long (5 minutes or more)”

- (a) Which **one** of the following is an appropriate sample space, Ω , for events W , G , S , B , and L ?

A. $\Omega = \{ \text{girlfriends} \}$

B. $\Omega = \{ \text{Tom's phone calls} \}$

C. $\Omega = \{ \text{types of phone call} \}$

(2)

- (b) Using the sample space you have named in (a), Tom's phone calls have the following probabilities. A call is a wrong number with probability 0.2. Half of Tom's calls are from girlfriends. All the rest of his calls are from somebody trying to sell something.

The probability of bad interference is 0.3. Long calls comprise 10% of Tom's calls.

Write down all the information given in this paragraph as probability statements. Your answer should consist of 5 statements. [Hint: the first statement is $\mathbb{P}(W) = 0.2$.] (4)

- (c) Say whether or not the following sets form a partition of Ω :

(i) W , G , and S .

(ii) B and L .

(2)

- (d) For each of the following statements, say whether they are **true** or **false**:

(i) $B = S$.

(ii) $W \subseteq \Omega$.

(iii) $W \cup S = \overline{G}$.

(iv) $W \cap G = \emptyset$.

(v) $W \cup G \cup S = \Omega$.

(vi) $B \cup L = \Omega$.

(6)

- (e) Tom gets annoyed by bad interference and by wrong numbers. Overall, 40% of his phone calls suffer from one or both of these problems. What is the probability that a phone call suffers from **both** of these problems?
[Hint: first write down probability statements for the information given, and for the information you are asked to find.] (4)
- (f) What is the probability that a call has bad interference and is **not** a wrong number? (3)
- (g) One day, Tom says in annoyance that he ‘always gets bad interference whenever someone is selling something.’ Given the numbers we have already calculated, is it possible that Tom is correct about this? Show your working.
[Hint: consider the partition theorem for $\mathbb{P}(B)$.] (3)
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Total: 24

