Directed Technical Change and Technology Diffusion

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Abstract

This paper starts by documenting how important technological transitions across history show that declining technologies experience significant patenting activity and innovation for a prolonged period. Although the innovation flow into outdated technologies is smaller than in the rising ones, and contracts over time, the innovation magnitudes are similar and comparable for a prolonged period. To what extent is such significant innovation effort directed to old technologies quantitatively important for economic growth? And is it efficient? Are we innovating too much in old technologies? To answer these questions, I develop a growth model of directed technical change featuring, endogenously, a rising and a declining technology. The former is intrinsically better, and, as it is continuously perfected by cumulative innovation, it becomes cheaper and gradually replaces the latter. Importantly, the competitive equilibrium is inefficient: the share of scientists targeting the old technology at each point in time is above the optimum, i.e., too much innovation is done in old technologies. Although the social value of an innovation is higher than the market value for both technologies, as the spillovers of current innovation on future innovation (building on giants' shoulders) are not internalized by patent holders – this discrepancy is higher for the new technology. A simple calibration for the steel-making industry in the period 1890-1935 estimates that the old technology during the time, the Bessemer process, was responsible for approximately 15 percent of the total gain in productivity of the industry.

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1 Introduction

The adoption of new technologies - and the corresponding decline of old ones - is not instantaneous. Different technology vintages coexist for a long time. Several potential causes are discussed in the literature: the switching to the most modern vintage may involve expensive fixed costs (David, 1990; Mansfield, 1961); or it may have to wait further perfection and streamlining to reduce the new technology's price (Helpman and Trajtenberg, 1994, 1996); or the technical advantages of the modern vintage may be offset by a large stock of human capital and experience already accumulated in the older (Atkeson and Kehoe, 2007; Chari and Hopenhayn, 1991); or the knowledge regarding the use of new techniques may have to diffuse or be constructed slowly through a learning by doing process (Juhász et al., 2021; Jovanovic and Lach, 1989). Variations in expected profitability and fixed costs related to adoption, market size, institutions and trade, all have been used to successfully account for differences in the rate at which new vintages replace older ones (Griliches, 1957; Mansfield, 1961, 1963; Comin and Hobijn, 2004, 2010).

While this extensive literature has contributed to our understanding about the persistence in adoption of old technologies (and in the accumulation of physical and human capital associated to them), less is known about the innovation dynamics they experience throughout their decline. Economic historians have for long highlighted how outdated vintages, while gradually disappearing, keep improving for decades[1]. The economic growth literature has yet to systematically document this pattern, understand the allocation of innovation resources between rising and declining technologies and its consequences for productivity and welfare.

For a set of important technological transitions across history, I show that declining technologies experience significant patenting activity and innovation for a prolonged period. To what extent is such significant innovation effort directed to old technologies quantitatively important for economic growth? And is it efficient? Are we innovating too much in old technologies? To answer these questions, I develop an endogenous growth model of directed technical change in which forward looking innovators choose each period between a rising and a declining technology. The model reproduces the patterns documented in the data and delivers a key finding: in the competitive equilibrium, too much innovation is done in the old technology relative to the rising one. The share of

[1] See, for example, Mokyr (1990), Graham (1956), Harley (1972, 1973).

research effort allocated to the declining technology is above the optimal. Moreover, I show how the aggregate growth rate can be decomposed into the contributions of each vintage - a decomposition that can be applied to assess the quantitative importance of old technologies for economic growth, as I illustrate with a simple calibration exercise.

The first contribution of this paper, therefore, is to provide evidence on how innovation in old technologies is persistent and quantitatively significant. I do this for a set of technology transitions in industries that played a key role in the process of economic growth, including steel manufacturing, shipbuilding, engines and motors, and communications. For each (among a total of 12) technology studied, I construct measures of innovation effort directed to it across time. This is done by linking US issued patents and technologies if two conditions are simultaneously satisfied: the patent text contains keywords related to a technology and it is classified into a class associated to such technology. Since the patent classification code does not have a one-to-one map with the techniques we are considering, I have to combine patent classes with keywords in the process of assigning patents to technologies. If a patent is matched to more than one technique, I assign it to the newest one in order to be conservative in assessing the flow of innovation directed to old technologies.

In addition to innovation measures, I leverage data on adoption and diffusion paths associated to each technology. These are the variables usually analysed in the literature. They present the transition - within a sector or in the aggregate economy - between different technologies (for example, between the steam engine and the electric motor when it concerns the supply of power, or between the Open Hearth and the Basic Oxygen methods in the steelmaking industry), defining, therefore, which is the rising or new technology (the one being increasingly adopted) and which one is the declining or outdated one (the technique being substituted away, the one whose market share is shrinking over time). Such adoption measures come from different sources in the literature, including the CHAT dataset (Comin and Hobijn, 2009) and the Grübler (1998)'s dataset.

The analysis of this group of important technology transitions leads to a set of results. As expected, innovation in declining and outdated technologies also follows a downward trend and gradually fade away as the adoption and use of the respective technology does so. However, patenting activity directed to outdated technologies declines only slowly and is significant during a prolonged period, being equivalent, in magnitude, to an important share of the total patent flow directed to their substitutes, the rising new technologies. For many of the technologies analysed, even after decades of steady decline in their adoption shares, the flow of patents remains in levels close to their heydays, and, more importantly, the gap between them and the rising technologies in terms of patent flows, although increasing, does not diverge: the old techniques remain being a considerable part of overall innovation. The allocation of research efforts to declining techniques - at least when it concerns the group of key technologies here studied - is non-trivial, and cannot be disregarded.

Motivated by these results, I develop an endogenous growth model that can reproduce the empirical findings and through the lens of which important positive and normative questions concerning research allocation between rising and declining technologies can be answered. In the model, technologies represent the services delivered by a continuum of specific intermediate varieties, which can be though of representing, as in Helpman and Trajtenberg (1994, 1996), the complementary goods and services that implement a technology and are compatible with it. For simplicity, I assume the existence of two vintages: an old and a new technique, which are perfect substitutes in the production of final consumption goods in this economy (think again on the examples of the steam engine vs electric motor, or in the different methods to produce a given tonnage of steel). Although the new technology is more efficient on average, each final good has its own specificities and the comparative advantages between the old and new technologies vary across them. For example, the Open Hearth method could produce higher quality steel much less prone to fracture under pressure when compared to the Bessemer method (Mokyr, 1990). For some applications, as the construction of building and bridges, this difference was of high importance, while for others, like the construction of rails, it was meaningless[2]. Variations in the relative advantage of the new technology in comparison to the old imply that some goods will adopt the new technology soon, when it still expensive, while others will wait until it has been perfected and its price decreased.

Such perfection of technologies depends on the innovation decisions of a fixed mass of scientists. At each period a scientist choose which technology to target in its innovation efforts. As it is standard in the endogenous growth literature following Romer (1990), Aghion and Howitt (1992) and Jones (1995), past innovation has spillovers into current innovation: scientists “build on giants’ shoulders”, and the costs of innovation in a technology are lower the higher the stock of knowledge accumulated in it. Innovation costs also vary across scientists as a function of their individual comparative advantages

[2] United States Census Office (1890, 1900)

towards each technology. In equilibrium, as result of the positive adoption shares of both technologies, of the positive knowledge stock built within each technology, and of scientists comparative advantages, there are profit opportunities for innovation within both technologies and research is divided among them. As the rising technology becomes more adopted, however, more and more scientists are gradually attracted to it given its larger market size and better future perspectives.

After characterizing the equilibrium path of the model, I solve for the Pareto optimal allocations. Importantly, the competitive equilibrium is inefficient: the share of scientists targeting the old technology at each point in time is above the optimum. The intuition of this result is as follows. As usual in the endogenous growth literature, the existence of research spillovers not internalized by the market makes the social value of an additional patent higher than its corresponding market value - and this is true here for *both* the new and the old technology. The crucial point in my model, however, is the following: as the old technology is declining, and it will be less adopted and perfected in the future, the spillovers of today's research into tomorrow's research missed by the market are not as large as those missed in the case of the rising technology, exactly because, for the declining one, as it fades away, there is not much innovation left to be done tomorrow. For the new technology, on the other hand, since it is rising and will dominate the economy in the future, the spillovers missed by the market are much more significant, as there is much "innovation to be done tomorrow". As a result, although there is a discrepancy between the social planner and the market in terms of the value given for an innovation in both technologies, such discrepancy is larger for the new technology. Therefore, the planer values the new technology relatively more in comparison to the old than the market does, leading the competitive equilibrium allocation of research to inefficiently overweight the declining technology.

This paper is organized as follows. Section 2 presents the data sources and the construction of the innovation measures; Section 3 presents the empirical results; Section 4 contains the model, including the main theoretical results and the calibration exercise for the steel industry. Section 5 concludes.

2 Data construction: innovation and adoption measures

To investigate the dynamics of innovation in declining and rising techniques, I consider a set of technologies with high relevance in the process of economic growth (see the list in Table 1). I restrict my attention to examples whose declining and ascendant phases are already completed, allowing us an entire view of the process. Although the notion of declining and rising technologies might be *ex post*, as agents in a certain point are not certain about the future success or failure of a technique, the empirical analysis that follows focus on later stages of technology life-cycles. In these later stages, when a declining trend is stable and consolidated (at the same time as more modern technologies robustly expand), uncertainty about each technique's fate is smaller.

When analysing technologies, the goal of this paper is to have measures for the evolution of two variables: innovation intensity and usage intensity. The latter refers to the adoption of a technology in production and consumption - the extent to which it is diffused and used by economic agents. This is the margin usually considered by the literature on technology diffusion, and allow us to tell if a technique is rising (i.e., being diffused and increasingly adopted) or declining (losing relative importance, being less adopted). Table 1 specifies the data sources used to construct such usage intensity series. With respect to the other variable of interest, the innovation intensity directed to each technology, it will be constructed with the use of patent data, as described below.

To study the dynamics of innovation, a natural starting point is patent data[3]. Starting in 1975, the United States Patent and Trademark Office (USPTO) makes patent data available in digital format. Before that, one must rely on Optical Character Recognition (OCR) and other machine learning techniques to process patent images. High quality work has been done (and shared with the community) in terms of processing the universe of patents issued by the USPTO and other patent offices since the XIX century. This is the case of Google patents database, which I will use here. For virtually every patent issued by the United States Patent Office in the period 1850-1990, I collect its patent number; title; publication date; classification code (CPC); description text; assignee and inventors' names.

[3]See Griliches (1990) and Hall et al. (2001) for a discussion on how, despite many limitations, patent data is a rich and insightful source of information related to innovation.

Table 1: List of technologies

	Technologies	Data on adoption/usage
Steel Industry (1880-1990)	Bessemer Process Open Hearth Basic Oxygen Electric Arc	Raw world steel production by technology in million tons. From Grübler (1998).
Shipbuilding (1850-1950)	Sailing ships Non-sailing ships	Tonnage of ships (sailing, motor, steam) in use at midyear in the United States. From CHAT data: Comin and Hobijn (2009).
Engines and motors (1860-1939)	Steam Engine Electric motor	Sources of mechanical power in US manufacture establishments. From Devine (1983).
Communications (1900-1980)	Telegraph Telephone	Number of telegrams sent. From CHAT data: Comin and Hobijn (2009).
Locomotives (1925-1959)	Steam Locomotives Diesel Locomotives	Number of US steam and diesel locomotives operating in major railroads (with more than 5 billion freight ton-miles) in 1925. Data from Mansfield (1963).

To determine whether a patent represents an innovation in a given technology, I combine information from the patent text and from the technology classification code (CPC) assigned to it. The criteria I use is the following: for each technology in Table 1, I first define a list of CPC codes that encompass it, but not necessarily only it. These lists were done based on conversations with engineers and on the technical literature - and can be seen in Table 2. Since the CPC classification code doesn't have a one-to-one map with the technologies we are considering, I can list the codes that exhaust one specific technology - such that a patent related to it must be classified with one of such codes. However, the other direction is not necessarily true: being classified with such a code doesn't guarantee that an invention is related to our specific technology - and that's why I turn to the second stage in the empirical analysis, as described below.

Table 2: CPC patent classes associated (but not restricted) to each technology

Description	Tech. classes
Steel: Bessemer, Open-Hearth, Basic Oxygen, Electric Arc	
Manufacture of iron or steel	C21B - C
Production and refining of metals; Alloys	C22B - C
Casting of metals	B22D
Furnaces; Details or accessories of furnaces	F27B, D
Electric Heating	H05B
Metalloids or compounds; Slag treatment	C01B, C04B
Shipbuilding: Sailing ships, Non-sailing ships	
Ships or other waterborne vessels, related equipment (excluding the subclasses covering water sports)	B63B - J (B63B32 - 34, B63H8)
Engines and motors: steam engine, electric motor	
Machines or Engines in general, Steam Engines	F01
Generation, conversion/distribution of electric power	H02
Communications: Telegraph (uspc)	
Telegraphy; Pulse or digital communications	178, 375
Ship's telegraphs; Railway signals;	246/70-79, 116/21
Optical communications, Facsimile	398/1, 398/100-199, 358/400
Locomotives: steam, diesel	
Steam locomotives or railcars	B61C1
Transmission system locomotives w/ steam engines	B61C9/02-06
Locomotives or railcars with IC engines	B61C5
Transmission system locomotives w/ IC engines	B61C9/08-26

Table 3: List of key words

Technologies	Key words
Bessemer Process	Bessemer steel/process/method/converter/plant Thomas converter/process
Open Hearth	Open Hearth, Siemens Martin OH(SM) furnace/steel/method/process
Basic Oxygen	Basic oxygen, Oxygen(LD) converter, Linz Donawitz BOS, BOF, BOP, OSM, EOF, OBM, AOD, VODC Energy Optmization Furnace, Oxygen bottom maxh
Electric Arc	Electric arc furnace/steel/method Electric steel/furnace steel, EAF
Sailing ships	Sail, Sailing, Mast
Non-sailing ships	Motor, Steam, Engine, Internal combustion
Steam Engine	Steam Engine (and does not contain: internal combustion, combustion engine, wind)
Electric motor	Electric motor/engine
Telegraph	Telegraph, Telegraphy

For each technology in Table 1, one can generate a set of potentially relevant innovations containing all the patents whose CPC classification code is included in Table 2. Then, for each of the patents within this set, I search in its title and text for keywords associated to the technology under consideration - see Table 3 for a list with such words and expressions. The search is made robust to keyword construction concerns (for example, plurals, upper and lower case letters, word spacing, etc). I assign the patent to the technology under consideration if we can indeed find the occurrence of at least one of the relevant keywords in its text. After this procedure is computed for all technologies, if a patent is matched to more than one of them, I assign it to the newest one in order to be conservative in assessing the flow of innovation directed to old technologies. To summarize, a patent is linked to a technology if two conditions are simultaneously satisfied: the patent text contains keywords related to such technology and it is classified into a CPC class associated to such technology.

Some final comments about the linking of innovations and technologies are in order. Regarding the shipbuilding industry, the CPC classification code, within the group designated for ships, reserves some subgroups specifically for water sports. As detailed in Table 2, I exclude the patents from such subgroups as the focus of the analysis in on ships as a transportation technology.

3 Evidence of innovation in declining technologies

Figure 1 shows, for each of the technologies listed in Table 1, two adjoint plots: the bottom one containing the evolution of innovation measures, and the top plot displaying the measures of adoption, as described in Section 2.

Figure 1a considers the steam engine and electric motor cases. After playing a prominent role in the industrial revolution and economic expansion of the XIX century, the steam engine was in decline in the beginning of the XX century. The emergence of the internal combustion engine and the electric motor led to a substitution away from it, as can be seen in the percentage of total horsepower in US manufacture supplied by different sources, as represented in the usage share plot in Figure 1a. The steam engine adoption in the manufacture sector went through a stable and persistent decline in the first decades, exemplifying the economy wide perception during this period of the steam engine as an outdated technology.

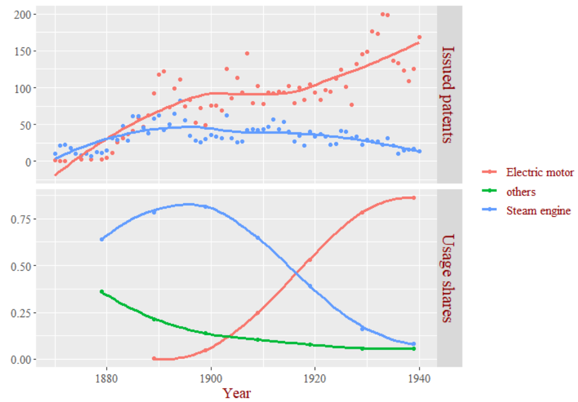
In the upper plot of Figure 1a, we can see how innovation in the steam engine was nonetheless significant and prolonged over this period: in the 1920s and 1930s, for example, when the decline trend in adoption of this technology had been going on for decades, its patenting flow was at levels not very different from the beginning of the century or the past century. The crucial point is: although, as expected, innovation related to the steam engine was facing a declining trend, such decline was slow, with persistent patenting activity comparable in magnitude to the flow of innovation in the rising technology of the period, the electric motor. The electric motor's innovation curve lies above the steam engine's, what was expected, but we should notice how we can plot both in the same scale: innovation in the obsolete technology, the steam engine, reached for a long period more than one third of the innovation in the electric motor.

Figure 1b presents one classic example of technology transition: the case between the Bessemer and the Open Hearth processes in steel production. The breakthrough innovation of Henry Bessemer in 1856 increased steelmaking productivity by a factor of 70 (Schauwinhold and Toncourt, 2011), allowing steel to be produced in large scale for the first time in history. Bessemer's steelmaking processes revolutionized its industry and in few years was responsible for more than 80% of total production in the world (Grübler, 1998). By this time, a new production method also emerged in Europe from the work of two engineers, Carl Siemens and Pierre-Emile Martin: the Open-Hearth (or Siemens-Martin) process. Although slower than the Bessemer, the Open Heath yielded higher quality steel, overcoming some of the weakness of the former - as, for example, the fractures recurrently experienced by the Bessemer steel when submitted to high pressures. As discussed by Mokyr (1990), this allowed the Open Hearth production method to diffuse and, by the beginning of the XX century, prominent entrepreneurs in the industry considered it to be the technology of the future. The plot for usage shares in Figure 1b shows the steady rise of the Open Hearth method in the share of world raw steel production starting in 1880 and continuing throughout the first half of the XX century. Concomitantly, the Bessemer process gradually shrink, facing stable decline and obsolescence in the XX century[4].

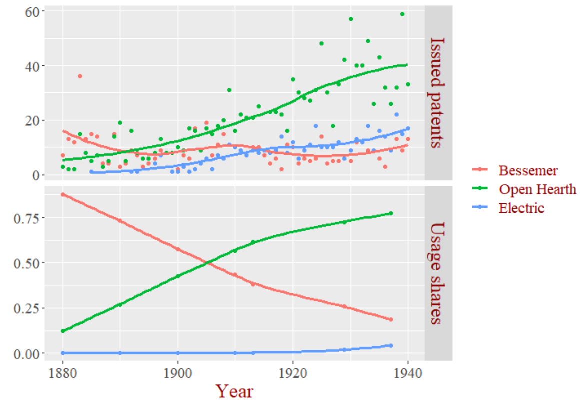
When we turn to the innovation efforts directed to steelmaking technologies, we ob-

[4]See Figure 1c for the evolution of the Bessemer steel production share in the second half of the century.

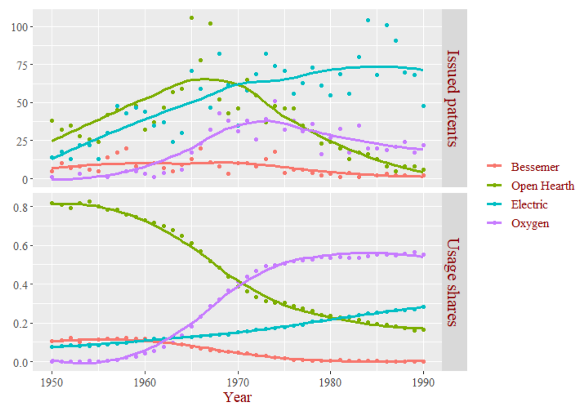
Figure 1: Innovation and adoption paths



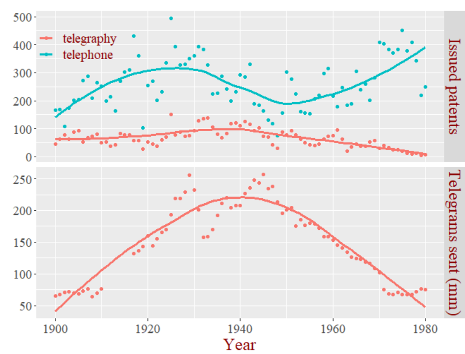
(a) Steam engine and electric motor



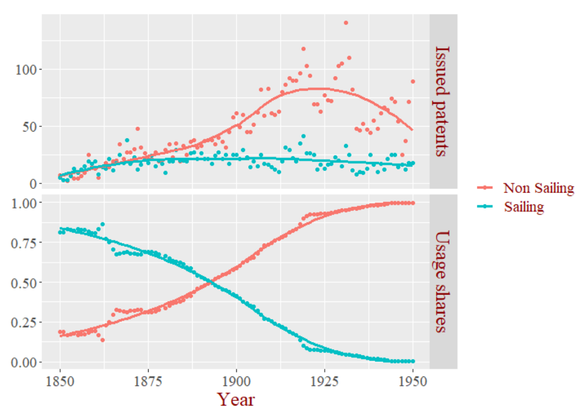
(b) Steelmaking



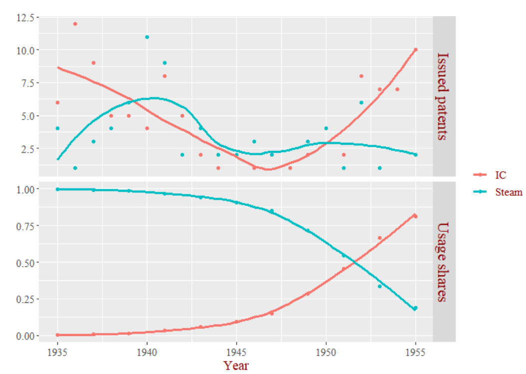
(c) Steelmaking (after 1950)



(d) The telegraph



(e) Shipbuilding



(f) Locomotives

Notes: Panels (a)-(f) contain the evolution of adoption (bottom plot) and innovation measures (upper plot) for different technologies. Innovation measures in the upper plots represent the patent flow associated to each technology - and were constructed based on each patents' text and classification code, as described in Section 2. The bottom plot in Panel (a) shows the percentage of total horsepower in US manufacture provided by different sources (Devine, 1983). The bottom plots in Panels (b) and (c) show the share of different steelmaking technologies on total raw steel production (Grübler, 1998). The bottom plot in Panel (d) shows the total number of telegrams sent each year in millions (Comin and Hobijn, 2009). The bottom plot in Panel (e) shows the percentage of total merchant ship fleet (tonnage) by ship type in use at midyear, in the United States Comin and Hobijn (2009). The bottom plot in Panel (f) shows the shares of steam and diesel locomotives operating in major US railroads (with more than 5 billion freight ton-miles) in 1925 (Mansfield, 1963).

serve, as expected, a smaller patent flow related to the Bessemer process in comparison to the Open Hearth during our period of analysis. Importantly, however, as the Bessemer's adoption steadily declined during the first decades of the XX century, its related patent activity remained stable, barely declining over time. During the 1930s and 1940s, after decades of a stable decline trend in adoption, Bessemer patents remained in a level close to its heydays. It it is known that the total number of patents issued per year has been growing across time - which raises the question of whether the stability in Bessemer's patents are in fact an indication of significant innovation. The innovation related to the Open Hearth process, the rising technology during the period, is useful to establish a reference. As we can see in the upper plot of Figure 1b, the two technologies series can be plotted in the same scale during all the period without a major divergence, indicating that it is not a pure time effect driving the significance of Bessemer's innovation.

In the second half of the XX century, the steel industry witnessed another important technological transition, which can be seen in Figure 1c. After decades of expansion, the Open Heath method entered into its declining phase in the 1960s after the revolutionary basic oxygen method, which had been recently invented, started to be commercialized. The use of concentrated oxygen top-blowed into a converter to carbon reduce the pig iron and transform it into steel created a process that was simultaneously cheap, fast and able to produce high quality steel. Approximately 10 years after its introduction, the Oxygen method had already overcame the Open Hearth in total production of raw steel world wide. Moreover, another method, the electric arc furnace, also rise in the period, contributing to the Open Hearth decline. Invented in the 1920s, it was in the second half of the century that the electric arc process gained prominence - as it was proved efficient to small scale steel production and expanded into this market niche.

The innovation path in the steel industry in the second half of the century presents similar patterns to the ones discussed for the first half of the century and the steam engine and electric motor cases. As the Open Hearth steadily decline and lose production share, innovation associated to it also did, and was overcame by the two rising steelmaking process of the period. However, this decline in Open Hearth innovation was smooth, patent activity persisted for a long a time and maintained levels plotted in the same scale and order of magnitude as those of the Oxygen blowing and Electric processes.

Figure 1d also shows similar patterns for the telegraph case. During the second half of the XX century, not only in relative but also in absolute terms it faced decline and obsolescence - as can be seen by the number of telegrams sent shrinking steadily. Patenting activity declined during the period, but very smoothly, remaining active across the decades. Moreover, as a comparison, notice how the innovation flow associated to telephone technologies, which were rising, is again comparable to the one related to the declining technology.

A secular transition between sailing ships and steam or motor ships took place between 1850-1950 in the United States - and can be seen in the bottom plot of Figure 1e. A consistent decline in the relative importance of sailing ships over decades was accompanied by gradual improvements in engines and motors and in the production of metallic hulls capable of supporting heavy motors and their fuel stock. The century of decline and obsolescence of the sailing ship was, nonetheless, characterized by a significant and persistent innovation effort directed to it. In the upper plot of Figure 1e, similarly to the patterns highlighted for the previous technologies, we see how patenting activity barely declined over the period. Innovation into sailing ships remained an important fraction of that into the rising technologies, the steam and motor ships, during all period. This is consistent with findings from economic historians who highlighted the important improvements in efficiency and productivity undergone by the sailing ship during its declining phase (Graham, 1956; Harley, 1972, 1973).

In Figure 1f, it is possible to observe the substitution away from steam locomotives in the years 1940-1960, with the diesel ones rapidly spreading in major American railroads. Despite the small number of patents associated to locomotive technologies, we observe, not surprisingly, how the flow into the diesel type becomes higher during these decades. Mansfield (1963) points out that, between 1946 and 1955, most railway firms decide to dieselize completely, which explains the fast transition between technologies observed in this case. Consistent with that, innovation in steam locomotives also disappear by the end of the 1950s. Notice, however, how in the beginning of the 1950s and in the 1940s, amidst the decline of the steam locomotive, innovation in such technology was still present - and present in comparable magnitudes to the diesel type.

To summarize, the analysis of a group of important technologies in the process of economic growth provides evidence that (i) innovation in declining and outdated technologies, as one would expect, follows a declining trend, and is less intensive than innovation in the rising technologies, (ii) however, such patenting activity directed to outdated technologies declines only slowly and is significant during a prolonged period, (iii) being equivalent, in magnitude, to an important share of the total patent flow directed to their substitutes, the rising new technologies. What should be highlighted, therefore, is that the allocation of research efforts to declining techniques - at least when it concerns this group of key technologies we analysed - is non-trivial, and cannot be disregarded. Is this efficient? Are we innovating too much in old technologies? To what extent is such significant innovation effort directed to old technologies quantitatively important for economic growth?

4 Theory

I now present an endogenous growth model that can shed light on the questions raised by the empirical analysis: do we innovate too much in old technologies? Are old technologies important for economic growth? After presenting the environment and the equilibrium proprieties, the first question is answered by solving for the Pareto optimal allocations. I also discuss how the growth rate of the economy can be decomposed into the contribution of old and new technologies - and present a simple calibration exercise for the steel industry to exemplify that.

Final goods Total output is produced with a continuum of final goods indexed by $i \in [0, 1]$:

$$C = Y = \exp\left(\int_0^1 \ln y(i) di\right)$$

The technology for the production of each final good is:

$$y(i) = l(i)^{\alpha(i)} x(i)^{1-\alpha(i)}$$

Here, $l(i)$ represents labor while $x(i)$ represents the services of either one of the currently available technologies $\{A, B\}$:

$$x(i) = \lambda_A(i) x_A(i) + \lambda_B(i) x_B(i)$$

Notice that the efficiency of each technology is heterogeneous across the continuum of goods. The parameters $\lambda_A(i)$ and $\lambda_B(i)$ capture the relative efficiency of technologies A and B - and are *specific to each good* i. They represent the comparative advantages patterns. Although one technology may be on average more efficient than the other, the relative efficiency of A and B varies depending on the specificities of each final good $y(i)$. As an illustration consider the case of steelmaking technologies. The Bessemer process yielded steel that could relatively easily fracture under pressure - and such risk represented a cost in its use. The Open Hearth method, on the other hand, produced higher quality steel - much less brittle and prone to fractures when compared to the Bessemer (Mokyr, 1990). The extent to which this difference between the methods was important depended on the specific good or service using steel as an input. For some applications, as the construction of building and bridges, this difference was of high importance, while for others, like the construction of rails, it was meaningless[5]. Variations in the relative advantage of the new technology in comparison to the old imply that some goods will adopt the new technology soon, when it still expensive, while others will wait until it has been perfected and its price decreased.

I assume that each good i draws, for each technology $\tau \in \{A, B\}$, a value for $\lambda_\tau(i)$ from an Exponential distribution: $\lambda_\tau \sim \text{Exp}(\beta_\tau^{-1})$. Draws are independent across i and τ. A higher β_τ indicates a more efficient technology, on average, across the cross section of goods. Instead of the Exponential, the Fréchet and other extreme value distributions commonly used in the discrete choice literature could be used, with no important changes to the results derived.

Technologies Technologies represent the services delivered by a continuum of intermediate varieties of measure N_τ:

$$x_\tau = \left[\int_0^{N_\tau} \chi_\tau(n)^{\frac{\sigma-1}{\sigma}} dn \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

As in Helpman and Trajtenberg (1994, 1996), such varieties $n \in [0, N_\tau]$ are the complementary goods and services that implement a technology τ and are compatible with it. A steam engine consists of numerous pieces, performing different and complementary functions. A firm wanting to use it, beyond acquiring the engine with all such pieces, also needs to rely on a set of specific engineering and maintenance services. Technologies are

[5] United States Census Office (1890, 1900)

perfected over time by the increase in the availability of such components (increase in the measure N_τ), allowing a more efficient division of tasks between them. It would be also possible to focus on quality improvements (instead of horizontal expansion).

Each intermediate variety $\chi_\tau(n) \in [0, N_\tau]$ is produced by a monopolist - and requires one unit of labor to be produced. Its price is $p_\tau(n) = \mu w$, where w is the wage and and μ is the optimal mark-up $\mu = \frac{\sigma}{\sigma-1}$.

Final goods are competitively produced. The minimum cost to produce a good $i \in [0, 1]$ equals its price and is given by:

$$p(i) = \mu w \div \max\left\{\lambda_A(i)N_A^{\frac{1}{\sigma-1}}, \lambda_B(i)N_B^{\frac{1}{\sigma-1}}\right\}^{1-\alpha(i)} \tau(i) \tag{2}$$

where $\tau(i)$ is a good specific constant.

Adoption shares Adoption decisions by a final good i producer depends, thus, on the development stage of each technology, $\{N_\tau\}$ - and also on their good i specific intrinsic efficiency, $\{\lambda_\tau(i)\}_\tau$. Cost minimization and the properties of the Exponential distribution imply that the share of final goods adopting a technology τ is:

$$s_\tau = \frac{\beta_\tau N_\tau^{\frac{1}{\sigma-1}}}{\sum_{k\in\{A,B\}} \beta_k N_k^{\frac{1}{\sigma-1}}} \tag{3}$$

Labor market clearing is imposed by requiring that the total demand to produce final and intermediate goods add up to the total supply, exogenously given by L. I also normalize the final good price to unit. With that, the individual technology choices of each final good i can be aggregated:

Proposition 1. *In equilibrium, aggregate output is given by*

$$Y = L\left(\beta_A N_A^{\frac{1}{\sigma-1}} + \beta_B N_B^{\frac{1}{\sigma-1}}\right)^{1-\bar{\alpha}} \kappa \tag{4}$$

where κ is a constant and $\bar{\alpha} = \int_0^1 \alpha(i)di$.

In growth rates, this implies that:

$$g_Y = \frac{1-\bar{\alpha}}{\sigma-1}\left[s_A g_{N_A} + s_B g_{N_B}\right] \tag{5}$$

where g_x denotes $\dot{x}/x$.

Equation (5) shows that aggregate growth can be decomposed into the contribution from the different technologies. The more a technology is improving and perfecting (i.e., the higher g_{N_τ}) the larger its contribution. But the extent to which it is diffused and being adopted in production of final goods (s_τ) is also key: the larger the usage share, the more the improves in a technology are translated into economic growth. Notice how aggregate growth is a weighted average of the growth in each technology, where the weights are given by the usage shares.

Innovation There is an unitary mass of scientists. Innovation is directed: each period, a scientist chooses between innovating at rate $\varphi_A N_A^\delta$ if he targets technology A, or at the rate $\varphi_B N_B^\delta$ if he targets B. Here $\delta \leq 1$ measures the amount of research spillovers: the extent to which past research, as reflected in the accumulated knowledge stock, N_τ, makes current research less costlier. Notice that $\delta = 1$ allows the existence of long-run sustained endogenous growth a la Romer (1990), while $\delta < 1$ corresponds the Jones (1995) semi-endogenous case. The parameters φ_A and φ_B capture the comparative advantage pattern of scientists. They independently draw $\varphi_A \sim \text{Exp}(\epsilon_A^{-1})$ and $\varphi_B \sim \text{Exp}(\epsilon_B^{-1})$.

If successful in innovation at technology $\tau \in \{A, B\}$, a scientist is awarded a patent with value v_τ that satisfies:

$$r(t)v_j(t) - \dot{v}_j(t) = \pi_j(t) \tag{6}$$

where the profit flow accrued by the monopolist of a variety related to technology τ equals $\pi_\tau = \frac{1-\bar{\alpha}}{\sigma}\frac{s_\tau Y}{N_\tau}$. Profits are higher when the market size of a technology per firm ($s_\tau Y/N_\tau$) is large.

Forward looking scientists sort between the two technology fields. In equilibrium, the share of them targeting technology τ at a given point in time is:

$$z_\tau = \frac{\epsilon_\tau v_\tau N_\tau^\delta}{\sum_k \epsilon_k v_k N_k^\delta} \tag{7}$$

Intuitively, the higher the value obtained with a successful innovation directed to technology τ, v_τ, the more scientists are attracted to it. But also the chances to obtain such a successful innovation matter: a higher accumulated knowledge stock, N_τ, weighted by the intensity of spillovers, δ, makes it less costlier to innovate in a given field, attracting more scientists. The same holds for the efficiency parameter ϵ_τ.

Evolution of available varieties Given scientists optimal decision and the innovation technology, with the functional forms assumed, the total measure of varieties for each technology $\tau \in \{A, B\}$ evolves as:

$$\begin{aligned} \dot{N}_\tau &= N_\tau^\delta z_\tau \mathbb{E}\{\varphi_\tau | \text{chose } j\} \\ &= N_j^\delta z_j \epsilon_j (1 + z_{j'}) \end{aligned} \tag{8}$$

The representative household problem[6] delivers the standard Euler equation:

$$r = \rho + g_C \tag{9}$$

where total consumption must equal total output.

Definition 1 (Equilibrium)**.** For a given initial stock of varieties $[N_{A,0}, N_{B,0}]$, an *equilibrium* consists of time paths for aggregate consumption and output $[C_t, Y_t]_t^\infty$, time paths for the stock of machines $[N_{A,t}, N_{B,t}]_t^\infty$, time paths for the allocation of scientists $[z_{A,t}, z_{B,t}]_t^\infty$, time paths for the usage shares $[s_{A,t}, s_{B,t}]_t^\infty$, and time paths for the real interest rate $[r_t]_t^\infty$, such that the system of ODEs given by the HJBs (6) and the laws of motion for varieties (8) is solved - while being consistent with the optimal allocation of scientists (7), the equilibrium usage shares (3) and output (4), and the Euler equation (9).

4.1 Asymptotic steady state

Under a restriction on parameters, the model admits an unique asymptotic steady state in which the usage and research shares converge to constants. Such constants are determined by the intrinsic efficiency of technologies in production and research, i.e., by β_τ and ϵ_τ, and not by the initial conditions. The more efficient a technology is, the more dominant

[6] I am assuming log instantaneous utility over consumption, $log(C_t)$, and discount rate ρ.

it will be in the long run in terms of usage and innovation shares. Interestingly, these long-run shares do not depend on the initial conditions $\{N_A(0), N_B(0)\}$. If technology A is on average more efficient when used by final good producers and researchers (β_A, ϵ_A higher than β_B, ϵ_B), even if it starts from a lower perfection stage $N_A(0) < N_B(0)$, it will in the long run have a higher share in production and research than technology B. This is different from the result reached in Acemoglu et al. (2012) - where the model features path dependence, and the better technology (in their case the green technology) might never take off if it is initially sufficiently behind the worse one (the dirty) in terms of knowledge stock[7].

Proposition 2. *Suppose $(\sigma - 1)(1 - \delta) > \frac{1}{2}$. Then, there **exists** an **unique** asymptotic steady state in which the (i) the growth rate converges to zero, and (ii) and the usage and research shares of each technology converge to constant and positive values.*

In this unique asymptotic steady state, consider $s_A^* = \lim_{t\to\infty} s_A(t)$ and $N^* = \lim_{t\to\infty} N_A(t)/N_B(t)$. Then:

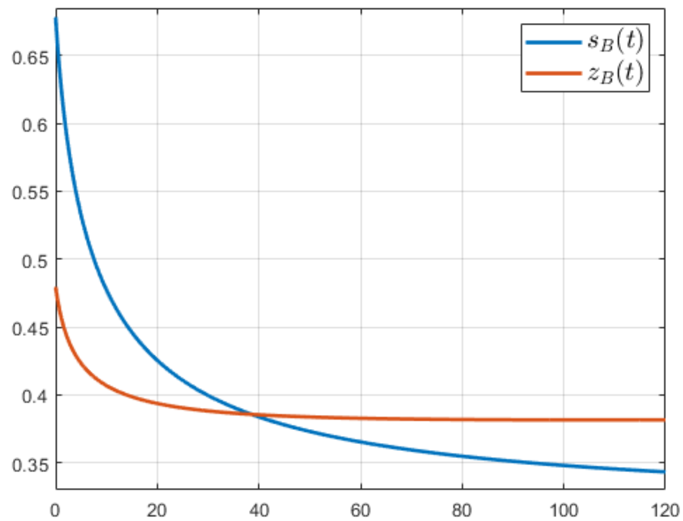
1. *s_A^* and N^* do not depend on the initial conditions $\{N_A(0), N_B(0)\}$.*
2. *s_A^* and N^* are increasing in A's relative research efficiency $\frac{\epsilon_A}{\epsilon_B}$.*
3. *s_A^* and N^* are increasing in A's relative production efficiency $\frac{\beta_A}{\beta_B}$.*

Given an arbitrary initial condition $\{N_{A,0}, N_{B,0}\}$, how does the system converge (if at all) to the asymptotic steady state described in Proposition 2?. It can be show that the convergence indeed always occur, and, moreover, it is monotonic. The technology which starts below its potential (equivalently, the one with relative high β_τ but a low relative initial perfection stage, $N_\tau(0)$) will steadily grow, while the other will constantly shrink.

Proposition 3. *Suppose $(\sigma-1)(1-\delta) > \frac{1}{2}$. Then, for any initial conditions $(N_A(0), N_B(0))$, the system converges to the unique asymptotic steady state. Moreover, the convergence is **monotonic**: if $N(0) = \frac{N_A(0)}{N_B(0)} \le N^*$, then, for every t, $g_{N_A}(t) \ge g_{N_B}(t)$ and $g_{s_A}(t) \ge 0 \ge g_{s_B}(t)$.*

[7] In Acemoglu et al. (2012), scientists consider only the next period profits when choosing in which technology to innovate, while here they are forward-looking with respect to the entire equilibrium path. Also, their model doesn't feature positive innovation in both technologies during the transition path. Innovation happens in only one of them as the economy converges to a BGP, in which either one technology totally dominates the economy, or in which the two technologies coexist.

Figure 2: Numerical exercise: the decay of technology B (and the rise of A)



Notes: In this simulation, technology A is calibrated to be the new and promising one, with $\frac{\beta_A}{\beta_B} = \frac{\epsilon_A}{\epsilon_B} = 1.5$ and $\frac{N_A(0)}{N_B(0)} = 1/100$

Now that we characterized the long-run environment of this economy and also how it converges to it given any initial conditions, we can step back and form some intuition about the evolution of innovation in the outdated technology in comparison to the rising one. Asymptotically, the relative shares in production and innovation converge to:

$$\frac{s_A^*}{s_B^*} = \frac{\beta_A}{\beta_B} (N^*)^{\frac{1}{\sigma-1}}$$
$$\frac{z_A^*}{z_B^*} = \frac{\beta_A}{\beta_B} \frac{\epsilon_A}{\epsilon_B} (N^*)^{\frac{1}{\sigma-1}+\delta-1}$$

where $N^* = \lim_{t\to\infty} N_A(t)/N_B(t)$, as defined in Proposition 2. Using this asymptotic expressions to form intuition and illustrate the mechanisms, notice that as $\frac{N_A(t)}{N_B(t)}$ evolves, $\frac{z_A}{z_B}$ tends to change less than $\frac{s_A}{s_B}$. This is because the research gap, beyond being affected by $\frac{s_A}{s_B}$ directly (i.e., by $N^{\frac{1}{\sigma-1}}$), is also affected by spillovers (N^{δ}) and competition effects (N^{-1}) whose net effect is negative $\delta - 1 \leq 0$. As the economy moves from $N(0)$ to N^*, the relative shares in research and production contract for the outdated technology, and the contraction is more pronounced over time for the usage shares than for the research shares. This illustrates how innovation in the outdated technology is persistent along the equilibrium path.

The numerical exercise presented in Figure 2 exemplifies this intuition. Technology A is calibrated to be the new and promising one, with high potential, but low initial perfection stage. Research in the outdated technology initially jumps below its usage share. But after the first years, the share of research follows a more stable and smoother path than the usage share, being persistent and stable throughout the equilibrium path. This pattern qualitatively matches the data.

4.2 Pareto optimal allocations

We are interested in the optimal allocation of scientists. The planner problem can be divided into two parts: a static and a dynamic one. In the former, for each period t, taking as given the state $(N_A(t), N_B(t))$ vector, the planner needs to optimally allocate production resources to obtain the maximum level of output and consumption. This part is trivial: total output produced by the planner has the same functional form of the expression we derived for the competitive equilibrium - as shown by the comparison of (10) with (4). The only difference is the presence of a larger constant $\tilde{\kappa} > \kappa$, as result of the gains from correcting the monopoly distortion.

The second part of the problem, the dynamic one, is the most import for us. Here, the planner chooses the allocation of scientists, and hence the evolution of the state vector $(N_A(t), N_B(t))$, taking as given the optimal level of consumption obtained at each period given the state. We can write the dynamic problem as:

$$\underset{z_A(t),N_A(t),N_B(t)}{\text{Max}} \int_0^\infty \exp(-\rho t) \ln C(t) dt$$

$$\text{s.t.}$$

$$C(t) = L\left(\beta_A N_A^{\frac{1}{\sigma-1}} + \beta_B N_B^{\frac{1}{\sigma-1}}\right)^{1-\bar{\alpha}} \tilde{\kappa} \tag{10}$$

$$\begin{aligned} \frac{dN_A(t)}{dt} &= \epsilon_A N_A(t)^\delta z_A(t)(2 - z_A(t)) \\ \frac{dN_B(t)}{dt} &= \epsilon_B N_B(t)^\delta (1 - z_A(t)^2) \end{aligned} \tag{11}$$

where we have substituted $z_B(t) = 1 - z_A(t)$ out.

Let $\mu_j(t)$ be the shadow price of an additional technology j patent for the planner. It can be shown that the allocation of scientists to technology j implemented by the planner satisfies:

$$z_j(t) = \frac{\epsilon_j \mu_j(t) N_j(t)^\delta}{\sum_k \epsilon_k \mu_k(t) N_k(t)^\delta} \tag{12}$$

A comparison of (12) with the competitive equilibrium allocation of research presented in (7) shows that the two expressions are equal (for a given level of the state variables N_j) except that, for the planner, μ_j substitutes for v_j. This is intuitive: in the competitive equilibrium, what matters is the *market* value of one additional patent, v_j, while in the Pareto solution we are concerned with the *social* shadow value of the additional invention, μ_j. We have now to understand how μ_j is different from v_j.

To gain intuition, let's expresses the evolution of μ_j in a comparable way to the evolution of v_j - as presented by the HJB function (6) and reproduced below:

$$\dot{\mu}_j(t) = \left(r(t) - \delta g_{N_j}(t)\right) \mu_j(t) - \pi_j(t) \tag{13}$$

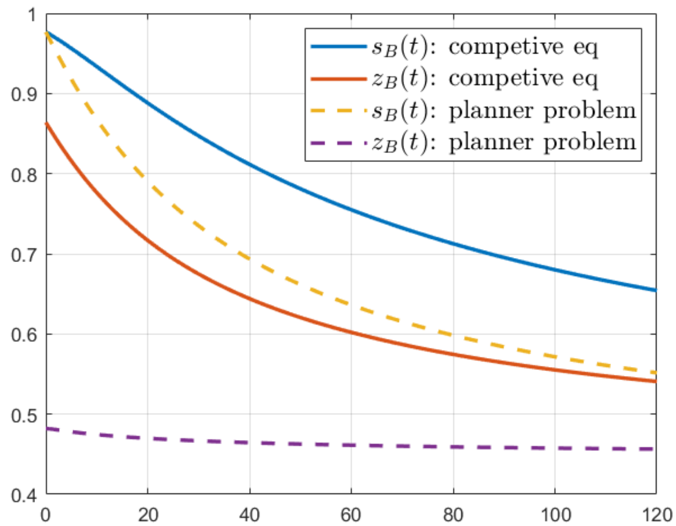
$$\dot{v}_j(t) = r(t) v_j(t) - \pi_j(t) \tag{HJB}$$

where $\pi_j = \frac{1-\bar{\alpha}}{\sigma} \frac{s_j Y}{N_j}$ is the functional form for profits and $r(t) = g_C(t) + \rho$.

As standard in growth models, the planner internalizes the positive spillovers of today's innovation on tomorrow's innovation, and hence values more an additional patent than the market. This can be seen intuitively above by the lower discount factor with which the planner discounts the future compared to the market: $r(t) - \delta g_{N_j}(t) \leq r(t)$. Since the mass of scientists is fixed, the Pareto optimal solution cannot invest more in research. But the allocation of scientists can be different. As we know from (12) and (7), to understand intuitively the difference in such allocation we need to compare the relative importance given to the rising technology in both cases, i.e., μ_A/μ_B and v_A/v_B.

It turns out that the discrepancy between the planner's problem and the market equilibrium is larger for the case of the rising technology than for the declining one. The reason is that, the rising technology is yet to experience a large flow of innovation in the coming periods, meaning that the spillovers we are missing from the under-investment in research today will have a larger bite. For the declining technology, with less and less innovation happening in the future, the effect of more innovation today on the cost of tomorrows research will be less noticed.

Figure 3: Planner's problem vs Competitive Equilibrium

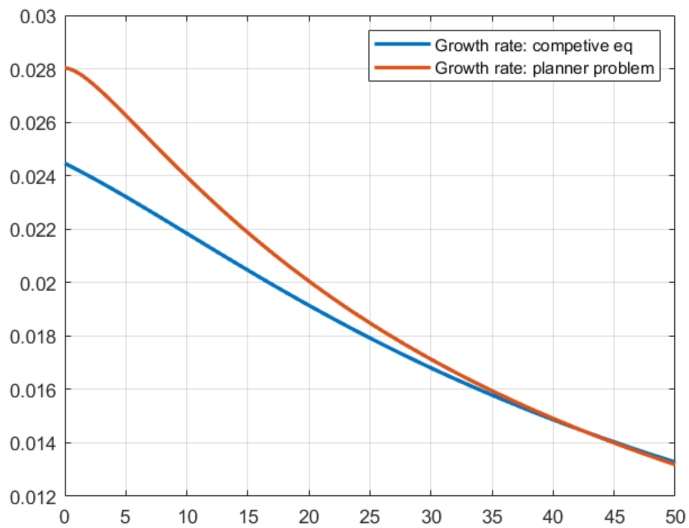


Notes: In this simulation, technology A is calibrated to be the new and promising one, with $\frac{\beta_A}{\beta_B} = \frac{\epsilon_A}{\epsilon_B} = 1.1$ and $\frac{N_A(0)}{N_B(0)} = 10^{-5}$.

In equation (13), we can also see this by observing how the planner discount factor will be lower for the growing technology than for the declining one, i.e., $r(t) - \delta g_{N_A}(t) \leq r(t) - \delta g_{N_B}(t)$. This is a result of the monotonicity property established in Proposition 3, which tells us that, assuming without loss of generality that the parameters are such that A is the rising technology, then, for every t, $g_{N_A}(t) \geq g_{N_B}(t)$.

Therefore, although the planner values more an additional patent in both technologies compared to the market, she values relatively more in the case of the rising technique, and shifts the allocation of researchers to it. The competitive equilibrium allocation of scientists over-weights the declining technology - there is too much innovation on it, as we can see in Figure 3. As a result, the planner allocation can achieve higher growth rates for output and consumption in the beginning of the transition, as seen in Figure 4.

Figure 4: Planner's problem vs Competitive Equilibrium



Notes: In this simulation, technology A is calibrated to be the new and promising one, with $\frac{\beta_A}{\beta_B} = \frac{\epsilon_A}{\epsilon_B} = 1.1$ and $\frac{N_A(0)}{N_B(0)} = 10^{-5}$.

4.3 Simple calibration for the Bessemer vs Open Hearth transition

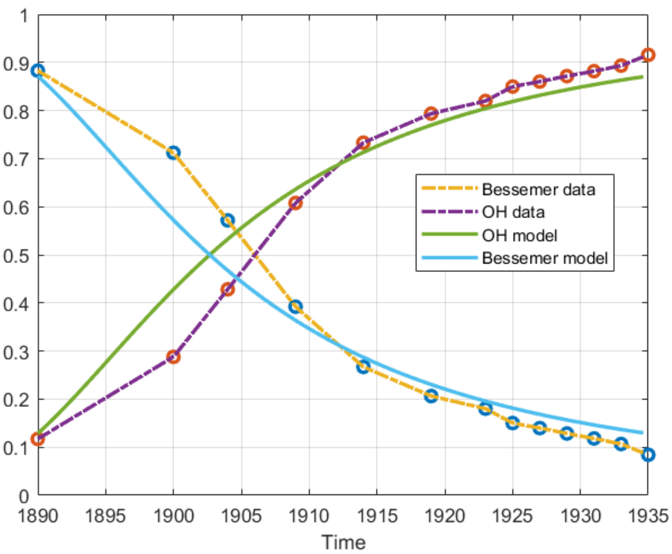
To illustrate how the model can fit the data, I conduct a simple calibration exercise. After externally calibrating σ and ρ with common values in the literature, I chose the other parameters shown in Table 4 so as to minimize the model's distance to four points arbitrarily picked in the evolution of production shares for the Bessemer and Open-Hearth technologies (Figure 5), including the starting year, 1890, the ending year, 1935, and the year in which the two series crossed. According to this exercise, by 1890, the stock of knowledge and varieties associated to the Open Hearth technology was only 2% of the Bessemer's, which was reflected in the small share of the former in the total tonnage of steel produced. However, the model predicts the Open Hearth technology to be intrinsically twice as productive as the Bessemer, which underlies its expansion in the decades to come.

While Figure 5 presents the model fit to the target time series of usage shares, Figure 6 shows the results for the research share directed to the Bessemer technology in the model in comparison with the patenting share associated to this technology - a non targeted variable. Although, as expected, the patenting data at annual frequency contains

Table 4: Calibrated parameters

N_{oh}/N_b	β_{oh}/β_b	ϵ_{oh}/ϵ_b	δ	σ	ρ
0.02	2	2	0.77	4	0.01

Figure 5: Calibration Target - Production shares (Bessemer vs Open Hearth)



noise, notice how the model captures well its HP trend. Even without targeting the path of innovation in the old technology (the Bessemer process) the model does capture it quantitatively well, including the persistence with which it declines.

Finally, we can use the growth rate decomposition derived in Proposition 1 and estimate each technology contribution to the overall growth in the steel industry during the sample period. Results are presented in Table 5. Notice how the model estimates an average annual growth rate of 4.2% throughout the years $1890-1935$, with the Bessemer technology contributing, on average, with 15% of it. This shows how, even during its decline phase, represented by the $1890-1935$ period, the contribution of the Bessemer technology to productivity growth remained significant.

Figure 6: Non Targeted - Patenting share in the Bessemer technology

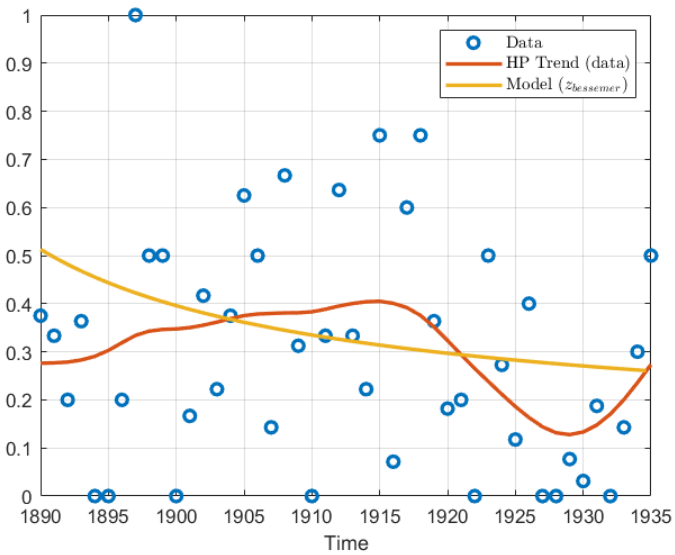


Table 5: Growth decomposition (Steel Industry)

	Average growth	Bessemer	Open - Hearth
1890-1935	0.042	0.006	0.0352

Notes: This growth decomposition uses the expression derived in Proposition 1 for the output growth rate: $g = \frac{1}{\sigma-1}[s_A g_{N_A} + s_B g_{N_B}]$

5 Conclusion

Although the persistence in the use of old technologies has been widely studied, less is known about the innovation dynamics experienced by such technologies along their declining process. This paper argues that this cannot be ignored: quantitatively, important technological transitions across history show that declining technologies experience significant patenting activity and innovation for a prolonged period. Theoretically, an endogenous growth model shows how the competitive equilibrium is inefficient: the share of scientists targeting the old technology at each point in time is above the optimum.

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