DBA3713 Analytics for Risk Management

Group Project - Option B

Group 2

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Part 1 - Data Exploration and Analysis

In the first part, we first calculated the excess return of the 43 industries.

	Agric	Food	Soda	Beer	Smoke	Toys	Fun	Books	Hshld	Clths	...
0	7.36	1.82	-1.76	-1.42	4.99	1.58	3.35	-1.34	-1.52	4.66	...
1	13.45	7.36	11.15	7.18	11.67	10.19	7.75	14.14	10.20	8.80	...
2	2.14	7.24	10.29	8.57	10.92	5.05	6.26	4.47	4.97	5.57	...
3	4.72	-1.09	3.92	-3.44	3.12	3.39	7.84	-1.66	-2.18	-3.30	...
4	0.59	8.06	7.46	5.88	9.67	2.67	5.42	10.47	4.12	9.29	...

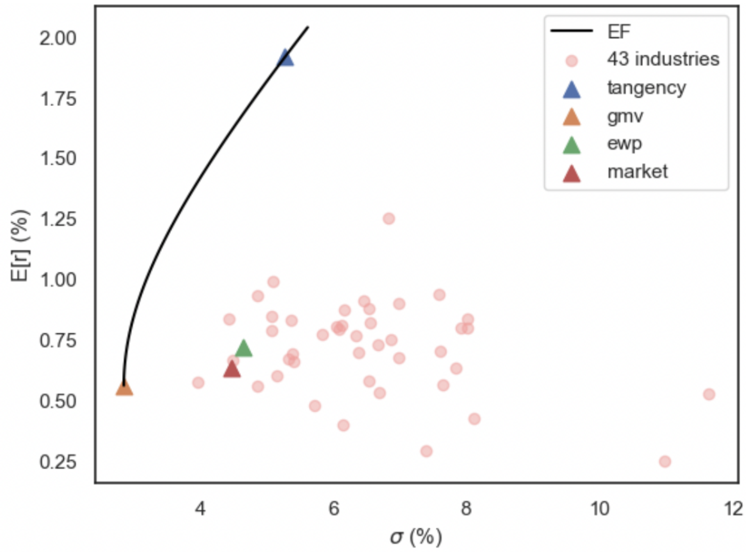
1.2 Basic Portfolio Construction and In-sample Analysis

We then constructed the EWP, GMV and tangency portfolios, and evaluated the in-sample performance using expected return, standard deviation and sharpe ratio.

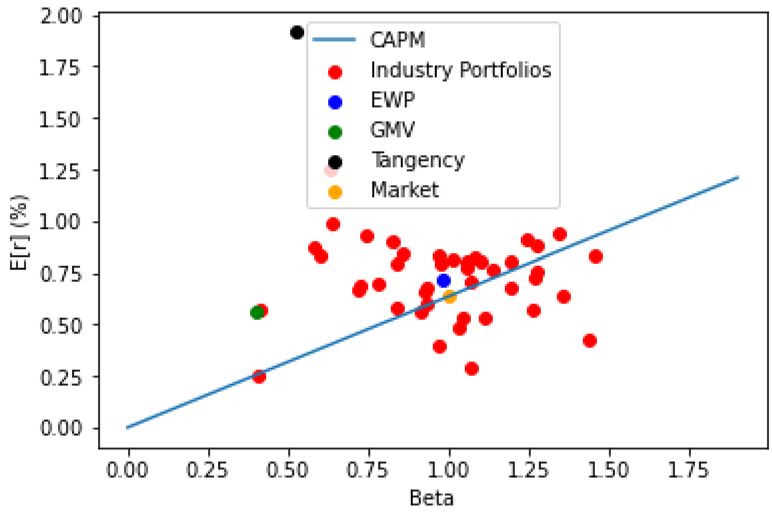
1) In-sample performance for the four portfolios (MKT, EWP, TAN, and GMV)

	EWP	GMV	Tangency	Market
Expected Return	0.718348	0.562142	1.916838	0.636361
Standard Deviation	4.650387	2.860151	5.281517	4.478294
Sharpe ratio	0.154470	0.196543	0.362933	0.142099

2a) The σ vs. E[r] diagram



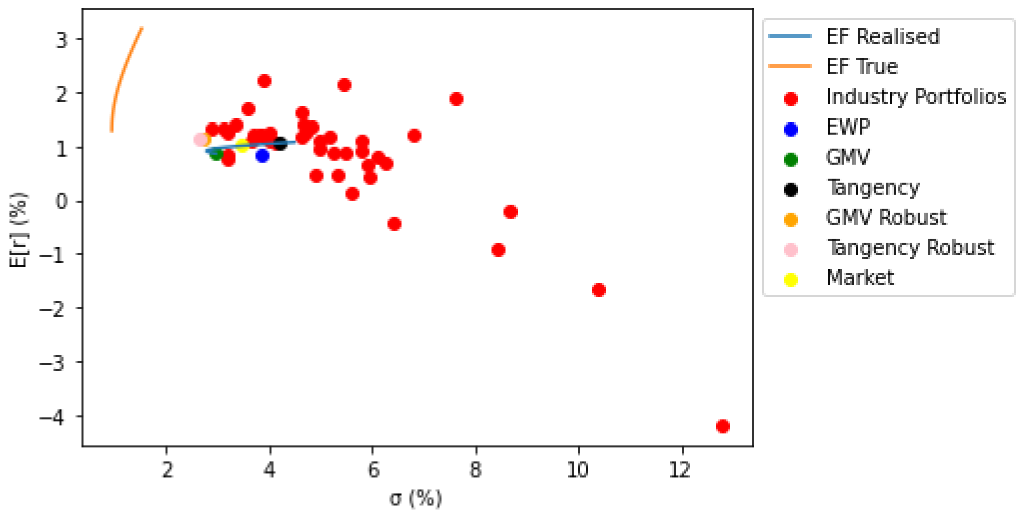
2b) The β vs. E[r] diagram



<u>1.3 Robust Portfolio Construction and Out-of-sample Analysis</u>

For out-of-sample analysis, we constructed the portfolio based on the train set (data from 1986-2010) and evaluated on the test set (data 2011-2015).

1) The out-of-sample σ vs. E[r] diagram



2) Out-of-sample Performance of MKT, EWP, TAN, TAN-robust, GMV and GMV-robust

	EWP	GMV	Tangency	GMV Robust	Tangency Robust	Market
Expected Return	0.822671	0.863082	1.055857	1.122783	1.141517	1.010833
Standard Deviation	3.858277	2.963800	4.201550	2.708489	2.712153	3.473068
Sharpe ratio	0.213222	0.291208	0.251302	0.414542	0.420890	0.291049

3) Contrast the table for out-of-sample performance with that for in-sample performance. What are your findings and insights? Also, are there any limitations to the current evaluation methodology?

In-sample Performance

	EWP	GMV	Tangency	GMV Robust	Tangency Robust	Market
Expected Return	0.697483	0.467903	2.100185	0.486128	0.539942	0.561467
Standard Deviation	4.792854	2.880749	6.103180	3.050177	3.406652	4.649736
Sharpe ratio	0.145526	0.162424	0.344113	0.159377	0.158496	0.120752

Out-of-sample Performance

	EWP	GMV	Tangency	GMV Robust	Tangency Robust	Market
Expected Return	0.822671	0.863082	1.055857	1.122783	1.141517	1.010833
Standard Deviation	3.858277	2.963800	4.201550	2.708489	2.712153	3.473068
Sharpe ratio	0.213222	0.291208	0.251302	0.414542	0.420890	0.291049

Analysis

In general, the out of sample performance is better (i.e. higher expected return and Sharpe ratio and lower standard deviation). This may suggest that the model was not overfitted, and was able to generalise well on unseen data. Additionally, this might be caused by the fact that during the time period on the test set, the economy is doing quite well compared to during the time period on the train set. This can be seen by the higher return gained by simply investing in Market portfolios or naive EWP strategy.

Comparing the performance between using robust estimators and not, it does seem that the models with robust estimators do have a regularising effect compared to the ones without robust estimators. For example, the naive tangency portfolio strategy has a much better performance (i.e. higher Sharpe ratio) in in-sample performance compared to out-of-sample (even when out-of-sample time period has better economic outlook), indicating a possible overfitting during training. Meanwhile, the tangency portfolio using robust estimators does not perform as well (compared to naive tangency portfolio) during training, but performs much better in test data. The same phenomenon could be seen between GMV and GMV robust portfolios where the GMV robust does have better performance compared to naive GMV.

Our group has identified several limitations in the current evaluation methodology.

Firstly, In a normal train-test split, it is assumed that the train and the test data have the same distribution. However, it is extremely hard to ensure this for time series data such as the one we use. Extreme market events during a period of time can artificially inflate/deflate performance and affect interpretability. This can be mitigated by doing a lot of train-test splits and taking the average performance from many more historical data (e.g. train-test 1: [1950-1970, 1970-1975], train-test 2: [1955-1975, 1975-1980], etc). Although the train and test data will still have different distributions, taking the average performance will promote robustness and interpretability of results.

Furthermore, as time progresses on, we should be able to collect more data for training and come up with a different portfolio using the same methods. However, this evaluation methodology does not allow rebalancing of the portfolio which might make the weights used to be “obsolete” especially towards the end of the testing time period. This is because rapid changes in market conditions suggest that historical data dated longer back may be less applicable in predicting future performance. Additionally, this can make a strategy appear “bad” especially if a market event causes the initial weights to perform badly towards the end. A more realistic and better evaluation methodology will be to allow rebalancing of the portfolio after every fixed time period. Furthermore, the analysis assumes that market conditions are constant over time, the model may not be able to capture the fluctuating market conditions. This suggests the importance of rebalancing the portfolio, as a model that performs well in the past, may not perform as well in the future.

Part 2 - Data Challenge

Overview of Project Approach

The following is a brief summary approach we took to identify the recommended weights for the respective 43 industries. Our group assumes that the investor is a slightly risk-averse investor. In other words, given a group of portfolios with similar expected returns, he/she will choose the one with the lowest standard deviation. Additionally, since we are conducting an out-of-sample analysis, it's important to balance between sample risk and model risk (overfitting and underfitting) so that we can come up with a robust model that is able to generalise well on new and unseen data, which is essential for out-of-sample analysis. Therefore, this suggests a more conservative approach, which may be less likely to be affected by overfitting or fluctuating market conditions, which are crucial aspects of a robust model.

Firstly, we wanted to evaluate the suitable number of data to be evaluated in the train set, to consider more recent data only to minimise the curse of non-stationarity. Next, we will be evaluating the suitable shrinkage parameters for the covariance, beta and expected returns. Building on top of the shrinkage estimators from the first part (constant covariance matrix with shrink constant 0.3 and beta with shrink constant 0.5), we will incrementally find the best estimators with strong structures together with their corresponding shrink constants, starting from estimations for covariance matrix, beta, and finally expected returns.

After we fine-tuned the shrinkage estimators, we compared the performance between the portfolios with and without no-shorting constraint.

Finally, combining all results that we have, which so far have been fine-tuned to minimise standard deviation, we will choose the one with the highest Sharpe ratio as our final model. To come up with the final recommendation, we used this model on the last n data points backdating from 2016, where n is the optimal number of data points found in our experiment.

Analysing Number of Data Points

In our out-of-sample analysis, we took train data as those from 1986 to 2010. However, the curse of non-stationarity states that ancient data might be less relevant for the current decision. Thus, we wanted to explore if taking only more recent data could improve the performance. Therefore, we varied the number of data points we took backdating from 2011.

We used the GMV-robust and TAN-robust introduced in part 1 and evaluated the performance using the appropriate metrics (expected return, standard deviation and sharpe ratio) on the test set. We plotted each metric against the number of data used for training.

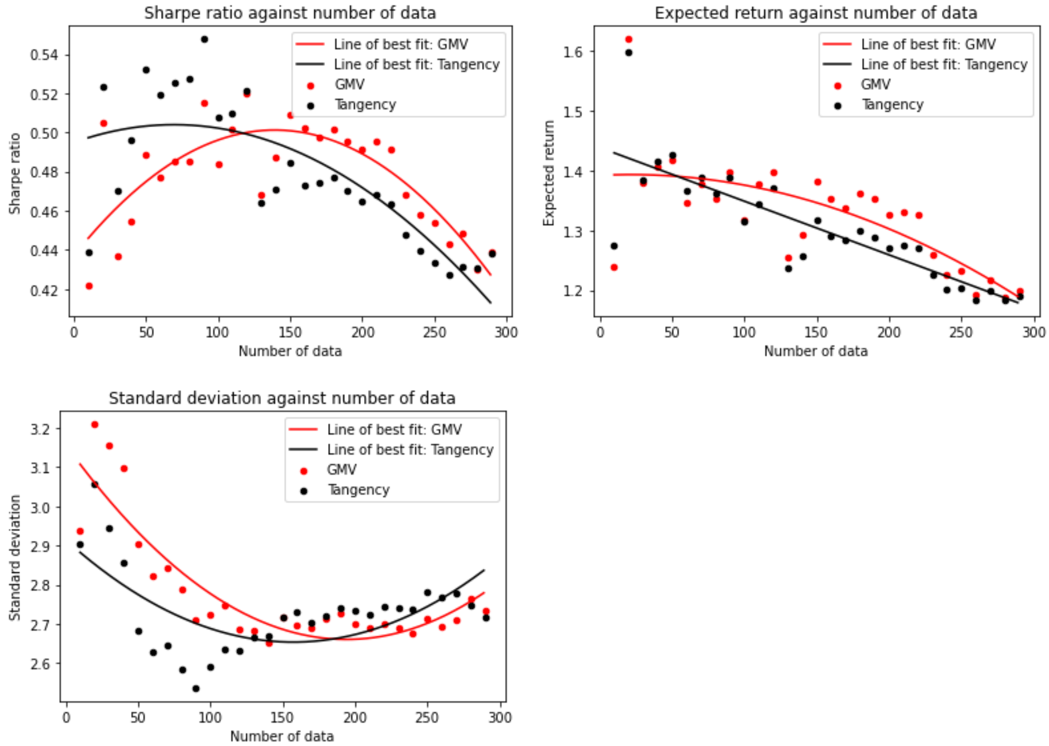


Fig 1: Performance Metrics derived from different number of data points evaluated in train set

Referring to the sharpe ratio plot, we can see that as the number of data points used for training increases, the sharpe ratio increases to a maximum value, before decreasing. When too few data points are used, there is insufficient data to come up with a good estimate of correlation matrix and expected returns. However, when the number of data points increases beyond a certain point, too much old data is irrelevant and hence causes the sharpe ratio to decrease due to the deteriorating quality of the estimate of correlation matrix and expected returns.

To find the optimal number of data points, we took the average of data points corresponding to the lowest standard deviation for the GMV portfolio and the Tangency portfolio as indicated by the line of best fit of degree 2. The number was found to be 176.

From this point onwards, we will only take 176 of the most recent data points (around 14+ years of data) to train our models.

<u>Analysing Various Shrinkage Estimators for Covariance Matrix</u>

In part one, we used a constant correlation matrix with shrinkage constant of 0.3. Higher shrinkage constant indicates a higher level of regularisation when it comes to the covariance matrix estimates. However, it remains to be analysed whether a constant correlation matrix is the best estimate with strong structure and whether the shrinkage constant of 0.3 is the optimal amount of regularisation. We are interested to know how the performance would change if we change the structural assumption and the shrinkage constant.

The performance metrics (expected return, standard deviation and sharpe ratio) was plotted against the shrinkage constant for each of the estimation methods - identity matrix, diagonal matrix and constant correlation matrix. Again, we use GMV and tangency-robust for this section.

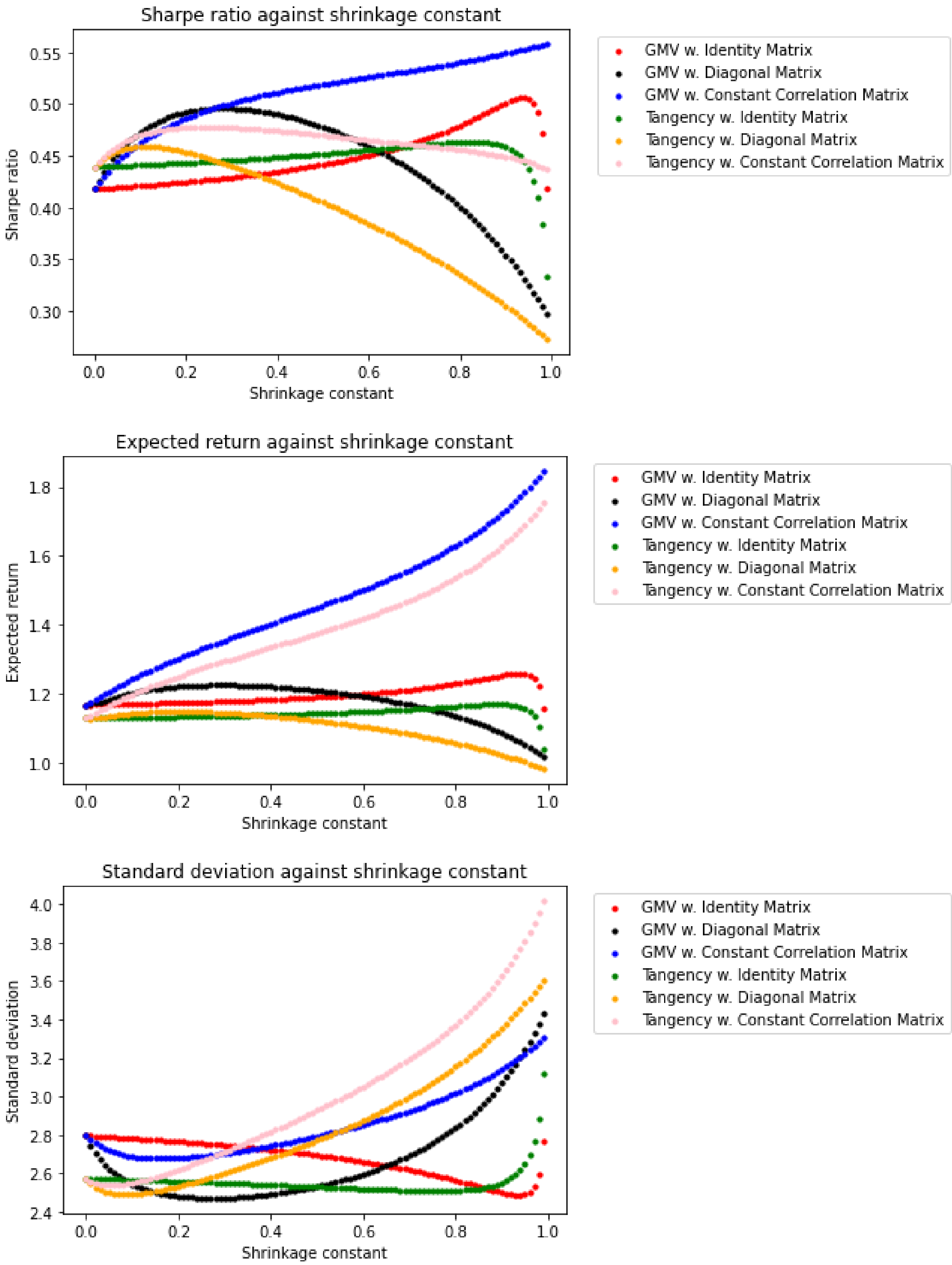


Fig 2: Performance Metrics evaluated based on Shrinkage Constant of Covariance Matrix

The following plot below shows the expected return against standard deviation for the 6 kinds of portfolio above.

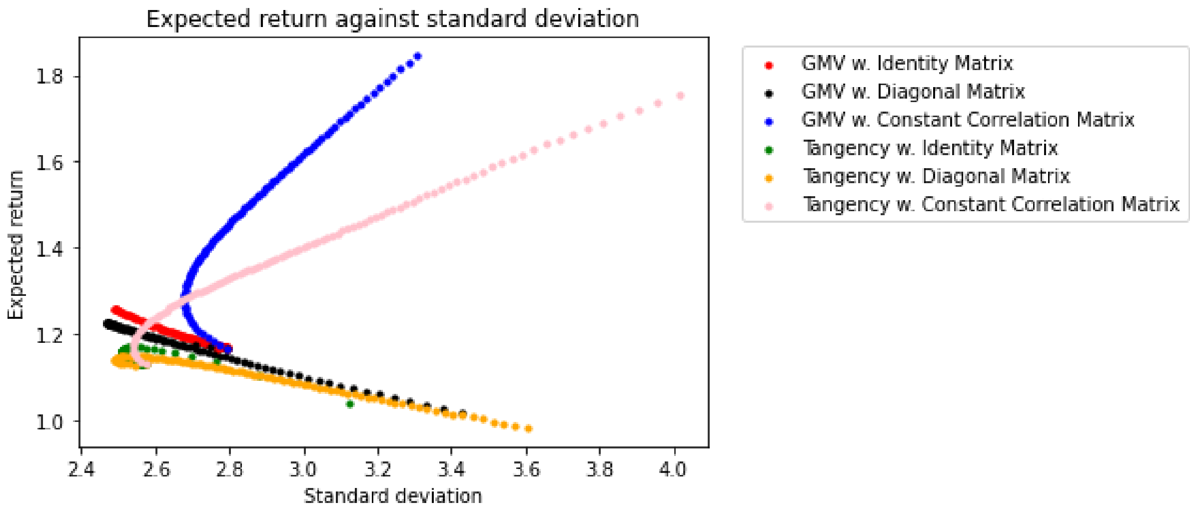


Fig 3: Relationship between Expected Returns and Standard Deviation Derived from shrinkage constant of covariance matrix

For each model + shrinkage estimator pair, we take the one with the lowest standard deviation (i.e. the leftmost point in the plot above). Below is a summary table of the best performance of each model + shrinkage estimator pair.

	GMV w. Identity Matrix	GMV w. Diagonal Matrix	GMV w. Constant Correlation Matrix	Tangency w. Identity Matrix	Tangency w. Diagonal Matrix	Tangency w. Constant Correlation Matrix
Expected Return	1.259603	1.225680	1.280742	1.160242	1.141047	1.173608
Standard Deviation	2.489615	2.471401	2.678393	2.509630	2.491074	2.539323
Sharpe Ratio	0.505943	0.495945	0.478176	0.462316	0.458054	0.462174

Table 1: Summary of Performance Metrics for portfolios with different Covariance Matrix

Based on Fig 3, we observed that apart from the curve for GMV with Constant Correlation Matrix (CCM), the highest expected returns for the respective portfolios happen to be near the points of lowest standard deviation. Therefore, our group believes that it would be suitable to select the model with the lowest standard deviation as a benchmark.

For the next part (analysing beta shrinkage constant), we need to pick one covariance shrinkage estimator used by Tangency portfolios since beta is only used to estimate CAPM's expected return and thus only affects Tangency portfolio. We will be using the Tangency portfolio with Diagonal Covariance Matrix (DCM) as it has the lowest standard deviation 2.49 (3.sf), even though it has a lower sharpe ratio relative to the Identity Matrix (IDM), to maintain consistency with our premise of a risk-averse investor (i.e. given a group of portfolios with similar expected returns, he/she will choose the one with the lowest standard deviation). Additionally, DCM might have a better balance of model and sample risk (the simplistic assumption of IDM might be subjected to model risk), therefore may be more suitable in achieving our goal in identifying a more robust model.

At the Tangency Portfolio with DCM, we derived the optimal shrinkage constant for the covariance matrix which is 0.08. As our aim is to identify the most robust model, we will be applying this optimal covariance matrix shrinkage constant in the model building for the next step, where we will be evaluating the Beta Shrinkage Constant.

<u>Analysing Beta Shrinkage Constant</u>

Similarly, another shrinkage that was used in part 1 was the shrinkage on beta estimate. WIth the optimal covariance shrinkage constant we have identified in the previous step, we will be evaluating the change in the respective performance metrics for varying values of beta shrinkage constant.

Therefore, using a tangency-robust portfolio which employs a diagonal covariance matrix to adjust for a more robust covariance matrix, we plotted the metrics (expected return, standard deviation and sharpe ratio) against the beta shrinkage.

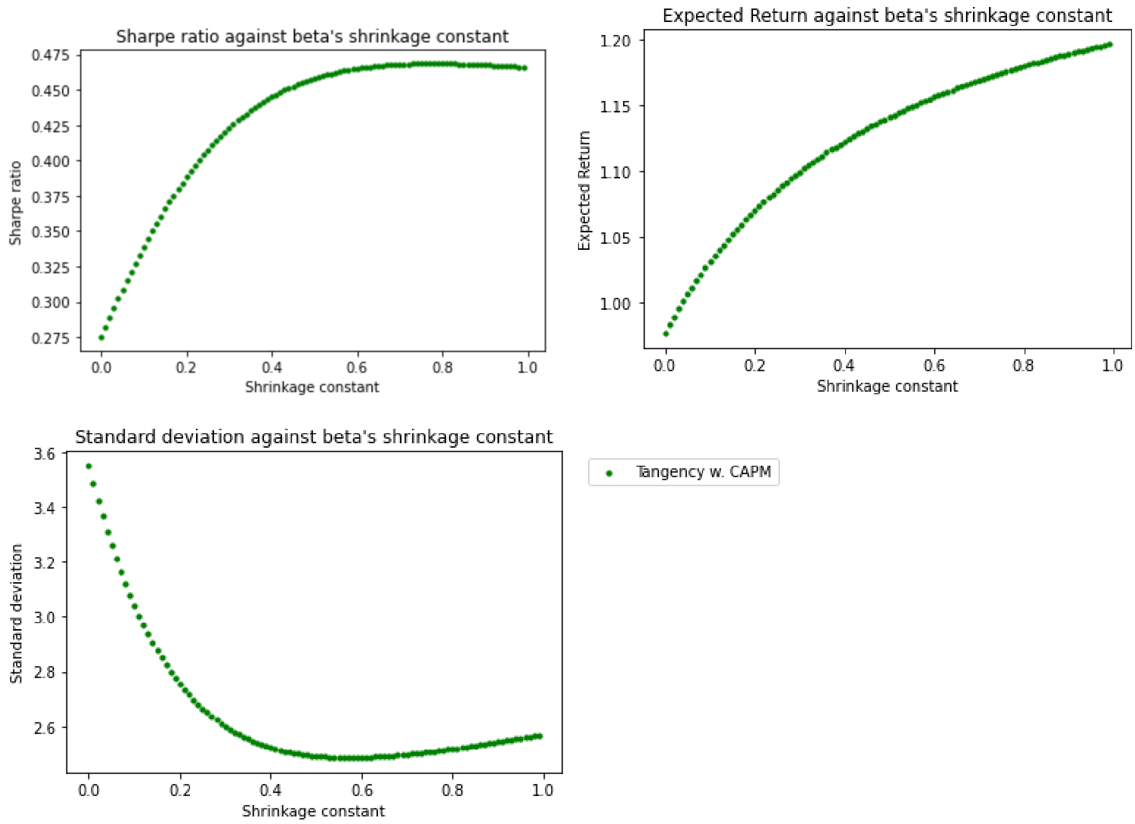


Fig 4: Performance Metrics evaluated based on Beta's Shrinkage Constant

Inline with our approach to choose parameters based on the lowest standard deviation, the beta shrinkage constant that corresponds to the lowest standard deviation is 0.57.

Analysing Different Structural Assumptions for Expected Returns

Similar to the optimal covariance matrix shrinkage constant we have derived in the first step, we will be applying the optimal beta shrinkage constant we have identified in the previous step to train the model to explore the shrinkage parameters used to estimate the expected returns. Therefore, we will use the tangency portfolio with diagonal matrix as the covariance matrix estimate with strong structure, together with the optimal shrinkage parameters that we have identified previously.

We will be exploring the estimated expected returns based on 3 approaches of strong structures:

1. assume that all assets have equal returns
2. assume that all assets have equal Sharpe ratio
3. assume that CAPM holds true and predict the asset's return based on its beta.

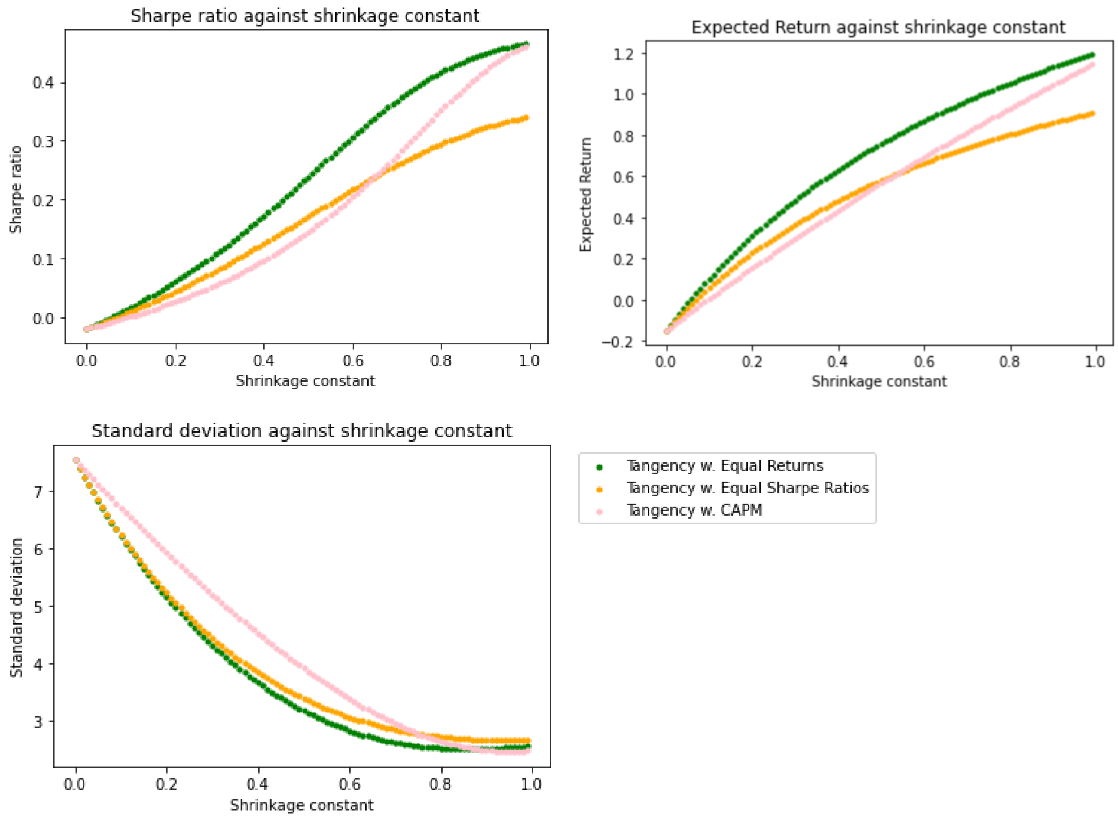


Fig 5: Performance Metrics evaluated based on Shrinkage Constant

Consistent with our aim to ensure a robust model that is able to generalise well on unseen data, for each model + shrinkage pair, we will take the one with the lowest standard deviation. Below shows the summary table for the performance of the models.

	Tangency w. Equal returns	Tangency w. Equal Sharpes	Tangency w. CAPM
Expected Return	1.098623	0.886189	1.098135
Standard Deviation	2.510298	2.664325	2.467376
Sharpe Ratio	0.437647	0.332613	0.445062

Table 2: Summary of Performance Metrics for Tangency portfolio using various approaches of strong approaches to estimate expected returns

As can be seen from Fig 5, the shrinkages that lead to the lowest standard deviation tend to be very close to 1 (i.e. much greater weight to the estimation with strong structure). For the estimation with equal returns assumption, the best shrinkage constant is 0.86, for the estimation with equal Sharpe ratio assumption, the best shrinkage constant is 0.95, while for the estimation with CAPM formula, the best shrinkage constant is 0.95. Based on our criteria of choosing results with the lowest standard deviation, the estimator with equal returns assumption performs the best with standard deviation of 2.51.

No-Shorting Constraints

Lastly, we will be evaluating the impact of imposing a no-shorting constraint on the respective model performance. Allowing short positions might increase the volatility and complexity of the portfolio. This could result in more extreme weights and may increase the need for portfolio rebalancing. Therefore, we want to explore whether imposing the constraint will lead to lower portfolio standard deviation. As our goal is to identify a robust model that is able to generalise well on unseen data, we believe that adopting a model with lower standard deviation would be most appropriate.

For all models featured in Table 1 and Table 2, we will construct the portfolios again, but now with no-shorting constraint. The following table shows the performance of each portfolio with no-shorting constraint.

	GMV, V: Identity	GMV, V: Diagonal	GMV, V: Constant	Tangency, V: Identity, E: Equal returns	Tangency, V: Identity, E: Equal Sharpe	Tangency, V: Identity, E: CAPM	Tangency, V: Diagonal, E: Equal returns	Tangency, V: Diagonal, E: Equal Sharpe	Tangency, V: Diagonal, E: CAPM	Tangency, V: Constant, E: Equal returns	Tangency, V: Constant, E: Equal Sharpe	Tangency, V: Constant, E: CAPM
Expected Return	1.113752	1.120987	1.138533	1.627500	-1.662667	1.702333	1.627500	-1.662667	1.702333	1.408777	-1.662667	1.115025
Standard Deviation	2.669651	2.637509	2.655496	4.611338	10.376357	3.595610	4.611338	10.376357	3.595610	3.417581	10.376357	3.579672
Sharpe Ratio	0.417190	0.425017	0.428746	0.352934	-0.160236	0.473448	0.352934	-0.160236	0.473448	0.412215	-0.160236	0.311488

Table 3: Summary of portfolio performance with no-shorting constraint

By imposing the no-shorting constraint, based on the portfolio performance summary in Table 3, the GMV portfolio with a diagonal matrix has the lowest standard deviation of 2.64 (3.sf). However, in general, standard deviation will increase upon the addition of no-shorting constraint.

Final Recommendation

Here is a summary of our findings thus far:

1. Optimal number of data points to use in our training set: **176**
2. Optimal strong structure for covariance matrix and shrinkage constant: **Tangency Portfolio with diagonal matrix with shrinkage constant 0.08**
3. Optimal Beta Shrinkage constant: **0.57**
4. Optimal strong structure for estimating expected returns and shrinkage constant: **Assume equal expected returns, with shrinkage constant 0.86**

The following figure shows the expected return vs standard deviation plot of selected portfolios from table 1, 2, and 3.

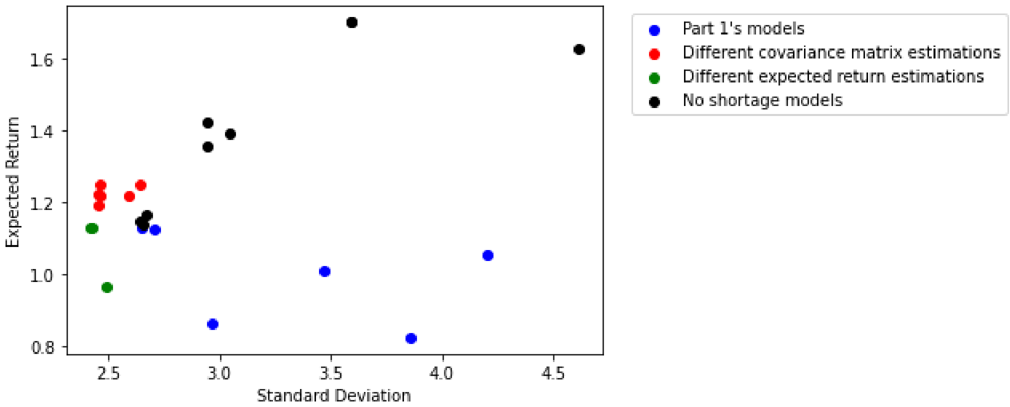


Fig 5: Expected Returns plotted against Standard Deviation for portfolios evaluated

Based on the graph in Fig.5, we observe that the performance portfolios with parameters from the original model are generally dominated by the portfolios with optimal tuned parameters that we derived in the data challenge. This suggests that the portfolio with tuned parameters yield higher returns for a given level of standard deviation. Furthermore, the portfolios from the data challenge are able to get lower standard deviation than any of the portfolios from the original models in part 1.

The following table shows the results summary of all models we tried in the Data Challenge.

	Expected Return	Standard Deviation	Sharpe Ratio
Part 1: Simple EWP	0.822671	3.858277	0.213222
Part 1: Simple GMV	0.863082	2.963800	0.291208
Part 1: Simple Tangency	1.055857	4.201550	0.251302
Part 1: Simple GMV Robust	1.122783	2.708489	0.414542
Part 1: Simple Tangency Robust	1.141517	2.712153	0.420890
Part 1: Simple Market	1.010833	3.473068	0.291049
Covariance Matrix Estimation: Tuned GMV w. Identity Matrix	1.259603	2.489615	0.505943
Covariance Matrix Estimation: Tuned GMV w. Diagonal Matrix	1.225680	2.471401	0.495945
Covariance Matrix Estimation: Tuned GMV w. Constant Correlation Matrix	1.280742	2.678393	0.478176
Covariance Matrix Estimation: Tuned Tangency w. Identity Matrix	1.160242	2.509630	0.462316
Covariance Matrix Estimation: Tuned Tangency w. Diagonal Matrix	1.141047	2.491074	0.458054
Covariance Matrix Estimation: Tuned Tangency w. Constant Correlation Matrix	1.173608	2.539323	0.462174
Expected Return Estimation: Tuned Tangency w. Equal returns	1.098623	2.510298	0.437647
Expected Return Estimation: Tuned Tangency w. Equal Sharpes	0.886189	2.664325	0.332613
Expected Return Estimation: Tuned Tangency w. CAPM	1.098135	2.467376	0.445062
No shorting: GMV, V: Identity	1.113752	2.669651	0.417190
No shorting: GMV, V: Diagonal	1.120987	2.637509	0.425017
No shorting: GMV, V: Constant	1.138533	2.655496	0.428746
No shorting: Tangency, V: Identity, E: Equal returns	1.627500	4.611338	0.352934
No shorting: Tangency, V: Identity, E: Equal Sharpe	-1.662667	10.376357	-0.160236
No shorting: Tangency, V: Identity, E: CAPM	1.702333	3.595610	0.473448
No shorting: Tangency, V: Diagonal, E: Equal returns	1.627500	4.611338	0.352934
No shorting: Tangency, V: Diagonal, E: Equal Sharpe	-1.662667	10.376357	-0.160236
No shorting: Tangency, V: Diagonal, E: CAPM	1.702333	3.595610	0.473448
No shorting: Tangency, V: Constant, E: Equal returns	1.408777	3.417581	0.412215
No shorting: Tangency, V: Constant, E: Equal Sharpe	-1.662667	10.376357	-0.160236
No shorting: Tangency, V: Constant, E: CAPM	1.115025	3.579672	0.311488

Table 4: summary of all Model performance conducted in Data challenge

As we have optimised for minimum standard deviation up to this point, for our final decision criteria, we will use Sharpe ratio to also incorporate expected return in our final suggestion. The following bar chart shows the Models with the Top 5 highest Sharpe Ratio from all the models we have evaluated in Table 1, 2, and 3.

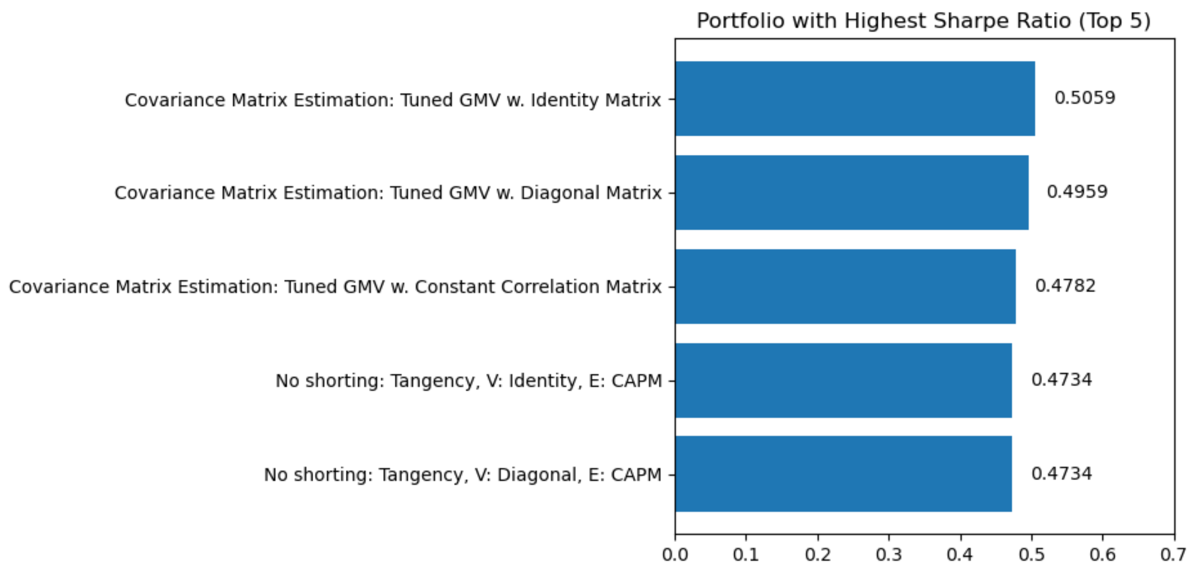


Fig. 6: Portfolio with Highest Sharpe Ratio (Top 5)

We will be choosing the model which yields the highest sharpe ratio, as it suggests higher investment returns relative to the amount of risk taken. Thus, we will be using the GMV w. Identity Matrix model for our final recommendation to determine weights that will be assigned to the 43 industries portfolio.

We then used the GMV w. Identity Matrix model on the last 176 data points backdating from 2016. Our final recommendations are as follows:

**[0.04898899, 0.1259614 , 0.04968953, 0.08767367, 0.00359237,
0.0084413 , -0.08347591, 0.00941781, 0.09293651, 0.04503157,
0.07681686, 0.06477104, 0.071028 , 0.0011516 , -0.01643011,
-0.05974249, -0.01470823, -0.0296616 , -0.07685128, 0.02247653,
-0.03347079, -0.01516133, -0.03370364, 0.00521007, -0.05363443,
0.07507713, 0.06178109, -0.03554517, -0.00527716, 0.08133908,
0.10877446, 0.03751592, 0.02840878, 0.03704924, 0.01908054,
-0.02325337, -0.00515426, 0.04308696, -0.00199605, 0.07892396,
0.03676749, 0.094317 , 0.07275694]**