Introduction to Signal Processing

Definition of a Signal

- A signal is a function of an independent variable such as time, distance, position, or temperature. Some examples of biomedical signals are Electrocardiogram (ECG), electroencephalogram (EEG) and magnetoencephalogram (recording magnetic fields, MEG)
- A signal is said to be continuous when its domain is the set of real numbers, and discrete otherwise
- Discrete signals are presented as sequences of numbers called samples
- An analog signal is a real-valued continuous signal
- A digital signal is discrete in time and value

Definition of Signal Processing

- Signal processing usually refers to: **Signal generation**, **Modifying signals**, **Extracting information from signals**.
- Signal processing benefits from improvements in the areas of electrical engineering, applied mathematics, statistics, mathematical information technology.

Definition of Frequency

- ”Frequency is the number of occurrences of a repeating event per unit time”
- The unit of frequency is hertz (symbol Hz, 1 Hz = 1 / s)

Sampling, Sampling Rate and theorem

- Sampling is the process of converting a continuous signal to a discrete one.
- Sampling rate, usually denoted by **fs**, is the number of samples per second collected from a continuous signal
- Sampling rate is given in the unit of **hertz**

"If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/2B seconds apart" –**Shannon**.

For example, consider the human hearing sense. The human hearing range is about from 20 Hz to 20 kHz, so the sampling frequency of audio signals must be at least 40 kHz to include all audible frequencies (audio compact discs use 44.1 kHz)

Signal Processing Domains

- Signals are usually studied in time-domain (with respect to time), frequency-domain (with respect to frequency), time and frequency domains simultaneously, using some time-frequency representation (**TFR**)
- **Fourier transforms** can be used to transform signals from time-domain to frequency-domain, and vice versa
- Time-frequency representations can be computed using short-time Fourier transform (**STFT**) or wavelets

Discrete Fourier Transform (DFT)

- The frequency-domain representation of a digital time-domain signal x[t] can be calculated using **the discrete Fourier transform** (referred as DFT or the analysis equation)

Inverse DFT

- The time-domain representation of a frequency-domain signal X[k] can be calculated using the inverse discrete Fourier transform (referred as IDFT)

Fast Fourier Transform (FFT)

- Fast Fourier transform (FFT) is ”the most important algorithm of our lifetime”. It is needed at least for: Signal processing (convolution, digital filters), Fast multiplication of large integers, Solving partial differential equations, Magnetic resonance imaging (MRI)
- The computational complexity of the discrete Fourier transform is **O(N^2)** where N is the signal length
- FFT produces the exact same result in **O(N log N)** operations

Magnitude and phase Spectrums

- DFT is complex valued → carries information about magnitude and phase
- The magnitude spectrum of a frequency-domain signal X is given by its absolute value **|X|**
- The phase spectrum of a frequency domain-signal X is given by its argument **arg(X)**

Convolution

- Discrete convolution of signals f and g is defined as: $(f * g)[n] = \mathrm{Sum}(f[n] \cdot g[n - m])$ or $\mathrm{Sum}(f[n - m] \cdot g[m])$ where m goes from – infinity to + infinity
- The resulting new signal is usually viewed as a modified or filtered version of one of the original signals

Convolution Theorem

- Convolution has an important property known as convolution theorem. Given two signals f and g, the convolution theorem states that $\mathbf{F\{f * g\} = F\{f\} \cdot F\{g\}}$ where F is used to denote DFT.
- By applying IDFT, denoted by F ^−1 , on the both hand sides of the equation, one gets $\mathbf{f * g = F^{-1}\{F\{f\} \cdot F\{g\}\}}$, so it is possible to calculate convolution efficiently by using Fourier transforms.

Digital Filters

- Low-pass filters: Pass low frequencies, attenuate high frequencies
- High-pass filters: Pass high frequencies, attenuate low frequencies
- Band-pass filters: Pass frequencies within a specified range, attenuate frequencies outside that range
- Band-stop filters: Attenuate frequencies within a specified range, pass frequencies outside that range
- If a filter is linear and time-invariant (LTI), it is completely characterized and best described by its frequency response

Frequency Response

- If a filter is linear and time-invariant, its frequency response is given by either

$F\{y\}/F\{x\}$, where **x is the input signal** and **y is the output signal**

Or by taking the Fourier transform of the impulse response of the filter, which is the filter's response to Kronecker's delta

$\delta(n) = \{$

$1, n = 0$

$0, n <> 0$

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