The Alethe Proof Format

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Abstract

Specifying more rules for alethe in the Cutting Planes reasoning

Contents

1 Introduction

In this article we describe how the cutting planes reasoning can be expressed in the alethe proof format, in order to make use of checkers (carcara).

2 Pseudo Boolean Inequality in Alethe

The Pseudo Boolean format consists of:

$$\sum_i a_i l_i \geq A \tag{1}$$

where,

$$\begin{gathered} A, a_i \in \mathbb{N} \\ l_i \in \{x_i, \overline{x}_i\}, \qquad x_i + \overline{x}_i = 1 \end{gathered} \tag{2}$$

In order to express it in alethe, we define an expression:

`(>= (+ <TERMS> 0) <A>)`

where `<TERMS>` is a list of either:

1. `(* <a_i> <l_i>)` in a plain literal $a_i x_i$.
2. `(* (- 1 <a_i>) <l_i>)` in a negated literal $a_i \overline{x}_i$

and `<A>` is the natural constant A.

3 Cutting Planes Rules in Alethe

3.1 Rule 1: cp_addition

We can apply the rules, using the 'step' syntax:

```
(assume c1 (>= (+ (* 2 x1) 0) 1))
(assume c2 (>= (+ (* 1 (- 1 x1)) 0) 1))
(step t1 (cl (>= (+ (* 1 x1) 0) 1))
    :rule cp\_addition :premises (c1 c2)
)
```

3.2 Rule 2: cp_multiplication

```
(assume c1 (>= (+ (* 1 x1) 0) 1))
(step t1 (cl (>= (+ (* 2 x1) 0) 2))
    :rule cp_multiplication :premises (c1) :args (2)
)
```

3.3 Rule 3: cp_division

```
(assume c1 (>= (+ (* 2 x1) 0) 2))
(step t1 (cl (>= (+ (* 1 x1) 0) 1))
    :rule cp_division :premises (c1) :args (2)
)
```

3.4 Rule 4: cp_saturation

```
(assume c1 (>= (+ (* 2 x1) 0) 1))
(step t1 (cl (>= (+ (* 1 x1) 0) 1))
    :rule cp_saturation :premises (c1)
)
```

References