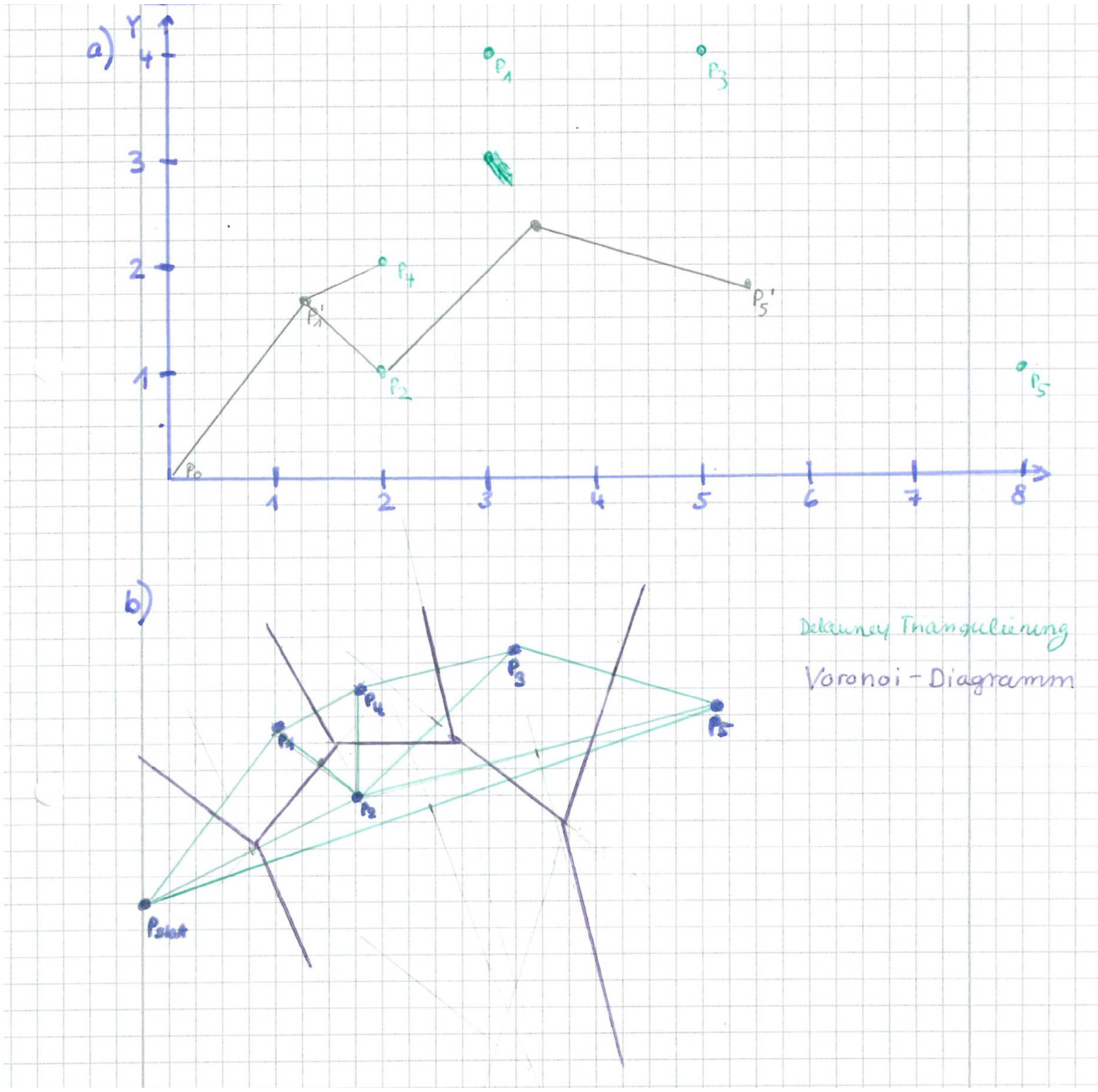
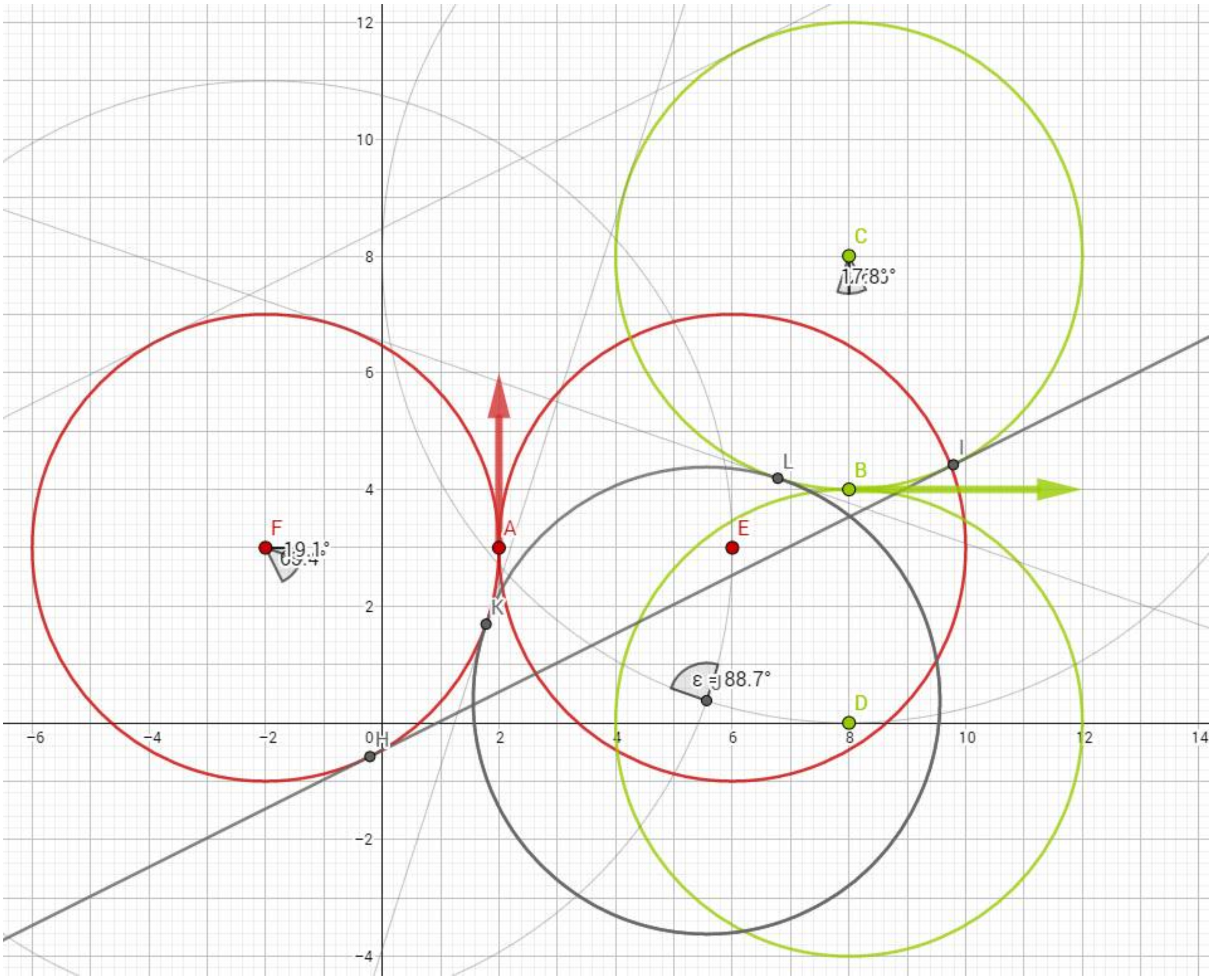
Task 1



Task 2

We used Geogebra to construct the relevant Points and circles:



The possible paths can be described as follows:

LSL: point A to H (on left red circle) + Segment H, I + point I to B (on upper green circle)

RSL: not possible, because changing directions requires the tangent to go though the mid-point of segment (E, D). Since the circles with r = 4 around E and D overlap, this tangent does not exist.

LRL: point A to H (on left red circle) + K to L (on grey circle) + point L to B (on upper green circle)

To get the lengths of those paths we need to calculate the position of all those points and the enclosed angles:

1. Kreismittelpunkte nach Lage in der Skizze mit Radius

$$M_{LA} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 4 \\ 0 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \qquad M_{LB} = \begin{pmatrix} 8 \\ 4 \end{pmatrix} + \begin{pmatrix} 0 \\ 4 \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \end{pmatrix}$$

2. Schnittpunkte der Tangenten mit Kreisen.

Länge der Tangente: Abst. der Mittelp. der Kreise:

$$\vec{b} = \begin{pmatrix} 8 \\ 8 \end{pmatrix} - \begin{pmatrix} -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$$

$$|\vec{b}| = \sqrt{10^2 + 5^2} = \sqrt{125} \approx 11{,}2$$

$$\left.\begin{aligned} \vec{H} &= M_{LA} - r(\vec{e}_z \times \vec{e}_b) \\ \vec{I} &= \vec{H} + \vec{b} \end{aligned}\right\} \text{Schnittpunkte}$$

$\vec{e}_z$ und $\vec{e}_b$ seien dabei die Einheitsvektoren, die in die Richtung der virtuellen z Achse und $\vec{b}$ zeigen.

Ab hier werden die Werte hässlich:

$\vec{e}_b = \begin{pmatrix} 5/\sqrt{125} \\ 10/\sqrt{125} \end{pmatrix}$ und damit wollen wir nicht weiterrechnen.

Es folgt daher eine Bestimmung aus der Konstruktion (siehe Skizze).

LSL:

Since we know that the angle a between B, C and I is the same as the one on the red circle between point (-2, -1), F and H, the path length becomes:

$$a = \arcsin(5/(125)^{0.5}) = 0.46$$
$$L1 = (3/2\,\pi + 0.46)\,4 = 24.63$$
$$S2 = (5^2+10^2)^{0.5} = 11.18$$
$$L2 = (2\,\pi - 0.46)\,4 = 23.29$$
$$L1 + S2 + L2 = (3.5\,\pi)\,4 + ||H-I||_2 = (3.5\,\pi)4 + (125)^{0.5} = 59.1$$

LRL:

Since the center of the grey circle J is at some 'messy' place, calculations of the desired angles become very messy. We thus state the length of the path like this:

$$L1 = \mathrm{ang}(K, F, A)\,4$$
$$R2 = \mathrm{ang}(K, J, L)\,4$$
$$L3 = \mathrm{ang}(L, C, B)\,4$$
$$L1 + R2 + L3 = 4*(2\,\pi - \mathrm{ang}(K,F,A) + \mathrm{ang}(K, J, L) + \mathrm{ang}(L, C, B))$$

Both lengths could be easily determined using some straight-forward functions in any programming language.