Cameras

Robotics

The human eye

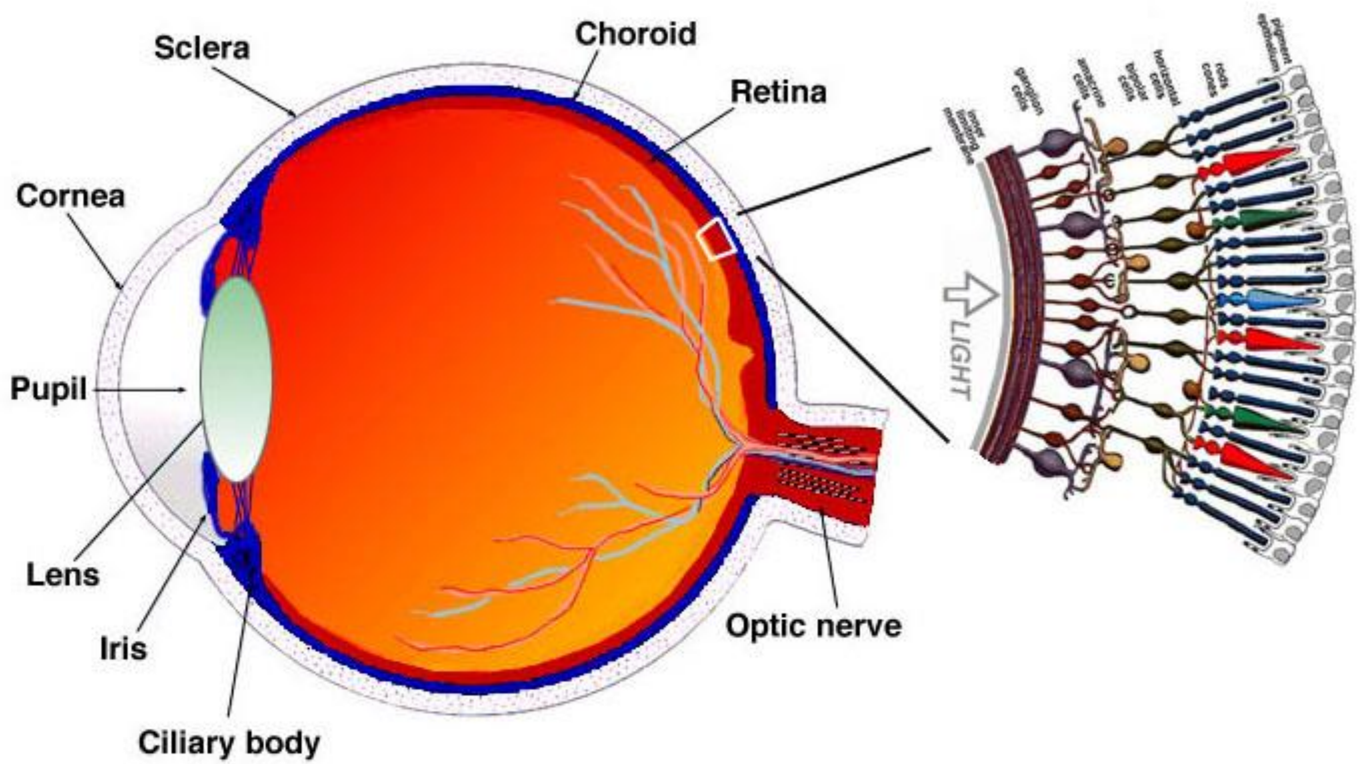
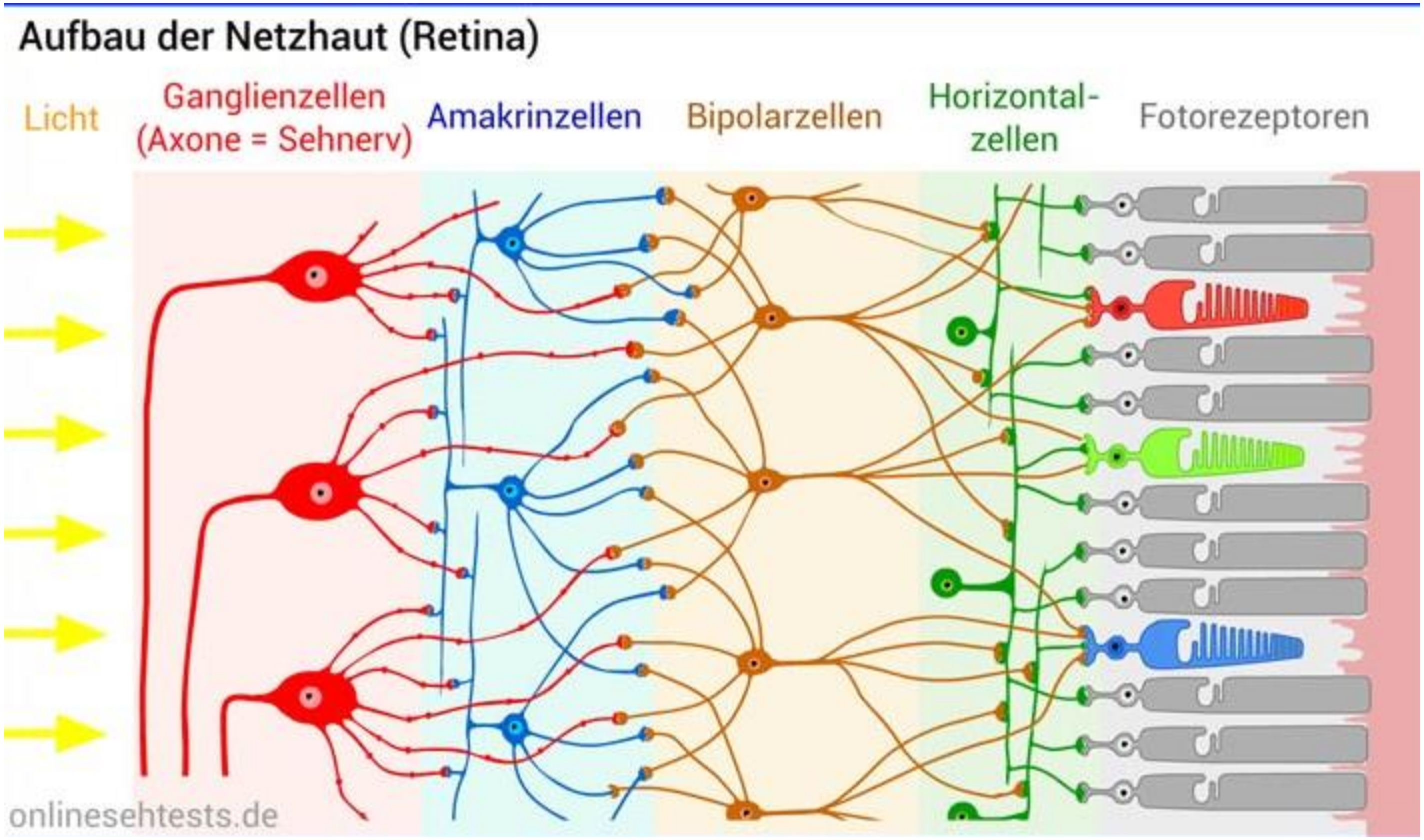


Fig. 1.1. A drawing of a section through the human eye with a schematic enlargement of the retina.

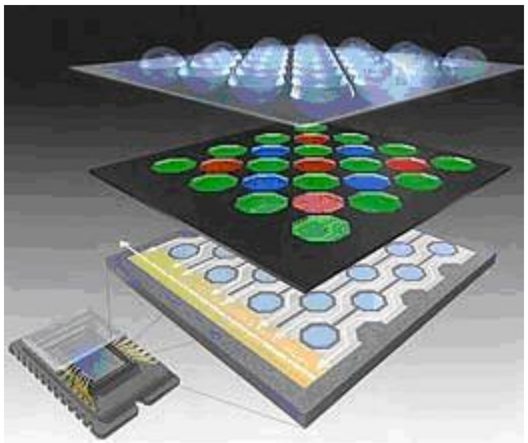
https://youtu.be/qrKZBh8BL_U

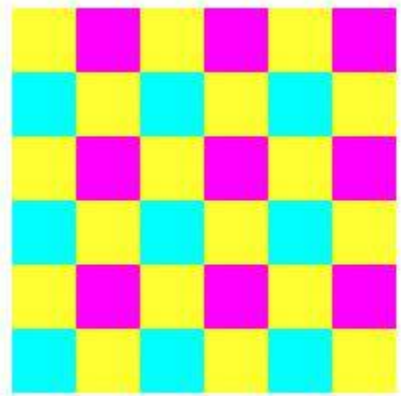
Retina



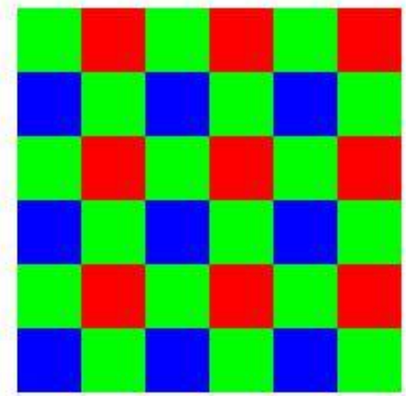
https://youtu.be/GOlGsDS1ZBk

Bayer Pattern

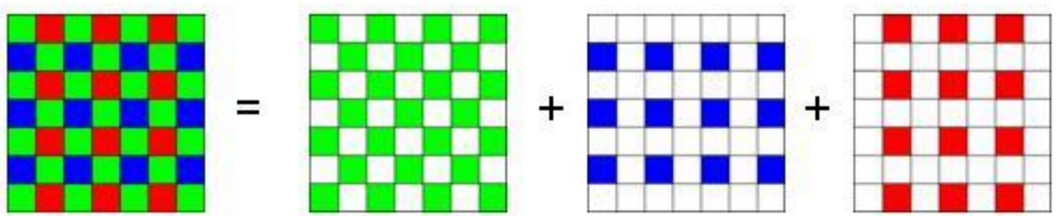


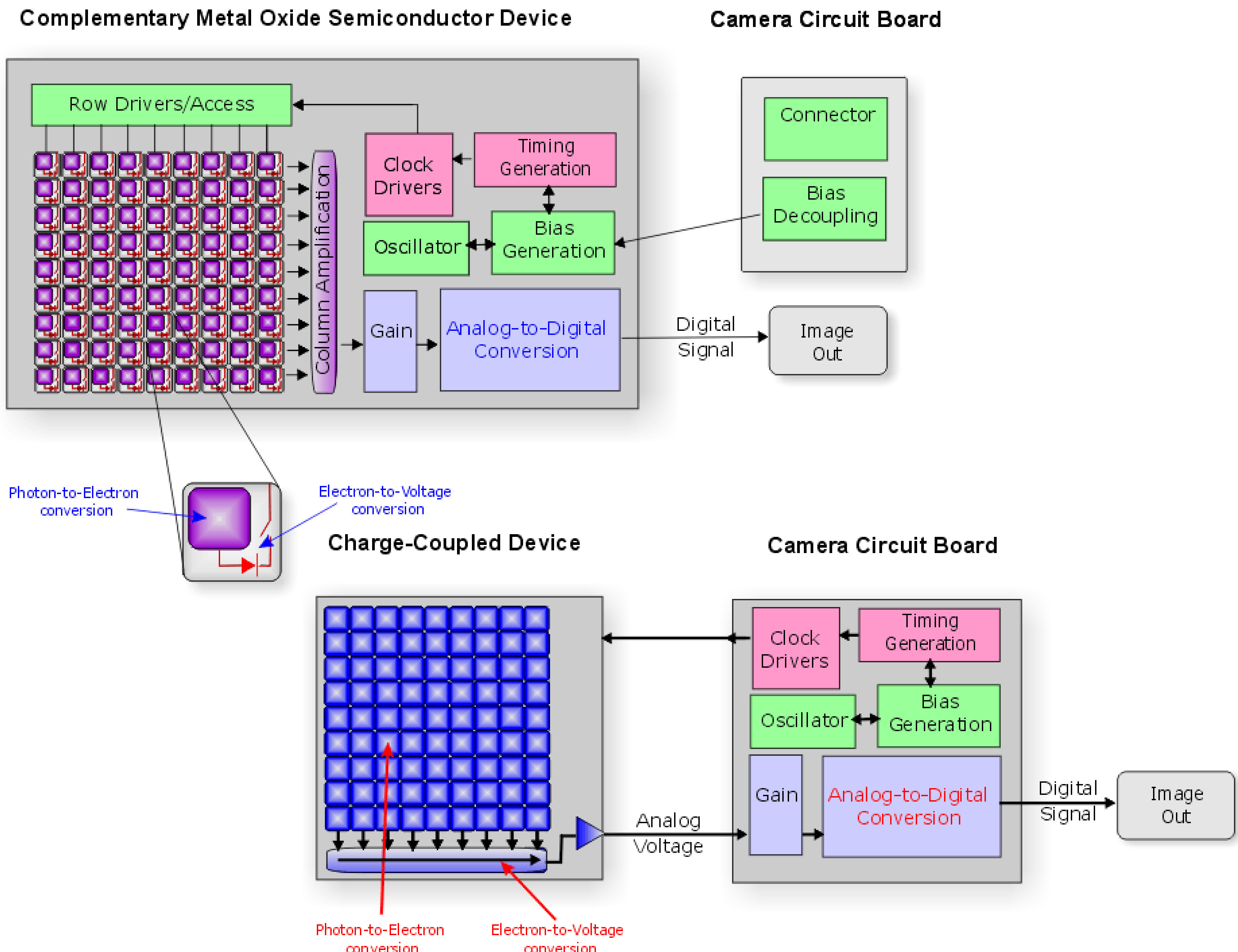


CMY Bayer Pattern
(New DCS 620x)



RGB Bayer Pattern
(DCS 620)

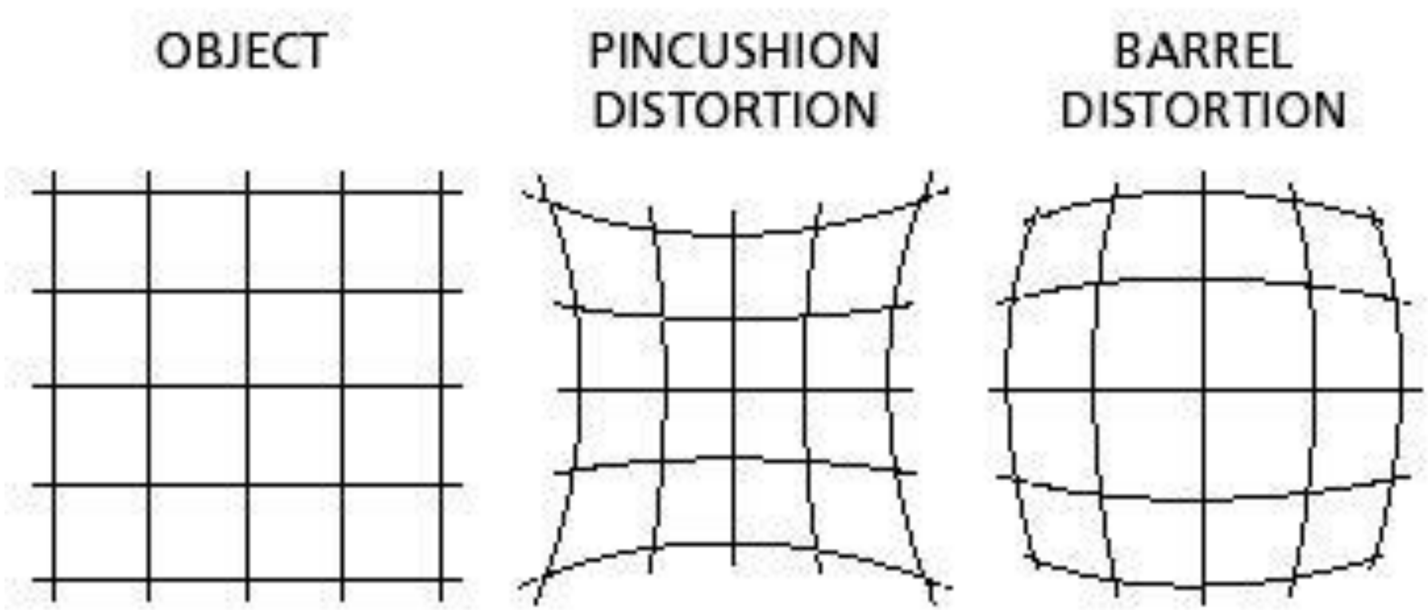



Complementary Metal Oxide Semiconductor Device
Camera Circuit Board
Row Drivers/Access
Clock Drivers
Timing Generation
Oscillator
Bias Generation
Column Amplification
Gain
Analog-to-Digital Conversion
Connector
Bias Decoupling
Digital Signal
Image Out
Photon-to-Electron conversion
Electron-to-Voltage conversion
Charge-Coupled Device
Camera Circuit Board
Clock Drivers
Timing Generation
Oscillator
Bias Generation
Gain
Analog-to-Digital Conversion
Analog Voltage
Digital Signal
Image Out
Photon-to-Electron conversion
Electron-to-Voltage conversion

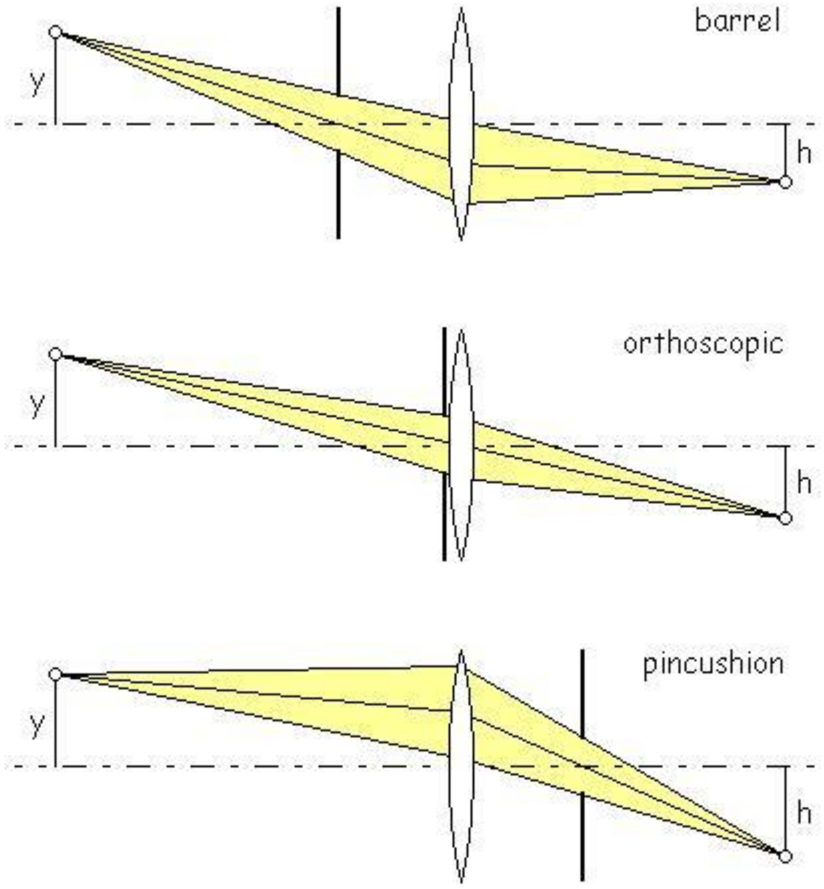
Comparison

CCD	CMOS
Expensive	Cheap
Few suppliers in the world	Many suppliers
Region access	Random access to pixels
Better Signal-to-noise ratio	More stationary noise
More fill area	Less fill area
High sensitivity	Less sensitive
Large sensors available	smaller sensors
High power (can be 1000 times more)	low power
16 bits per pixel	8 bits per pixel
complex interface	simple interface
no Regions of Interest	ROI
complex read electronics	A/D conversion possible
	camera on a chip
homogeneous sensitivity	heterogeneous sensitivity
fill factor 100%	fill factor ¡ 100%, but microlens

Distortion

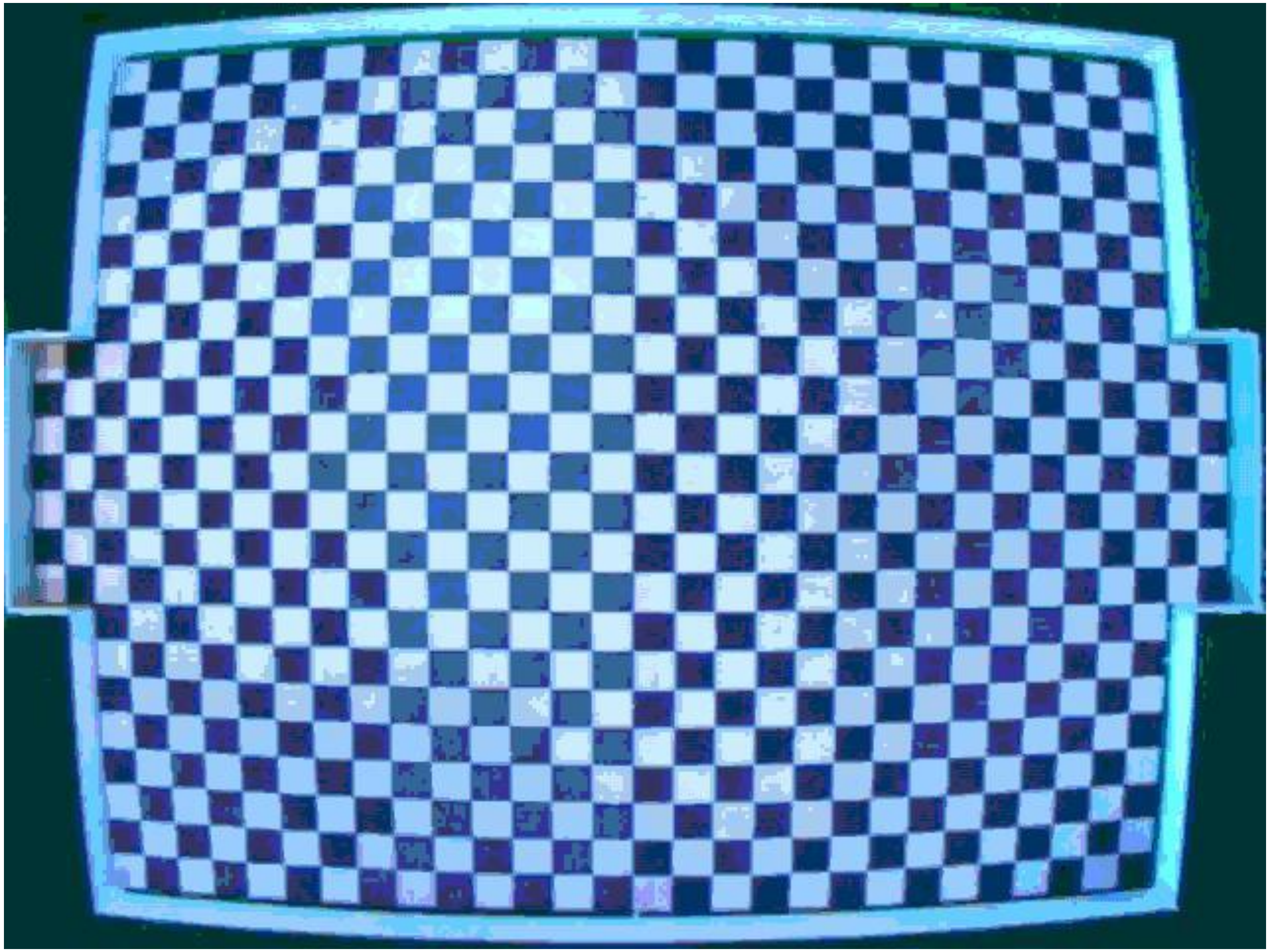


Distortion

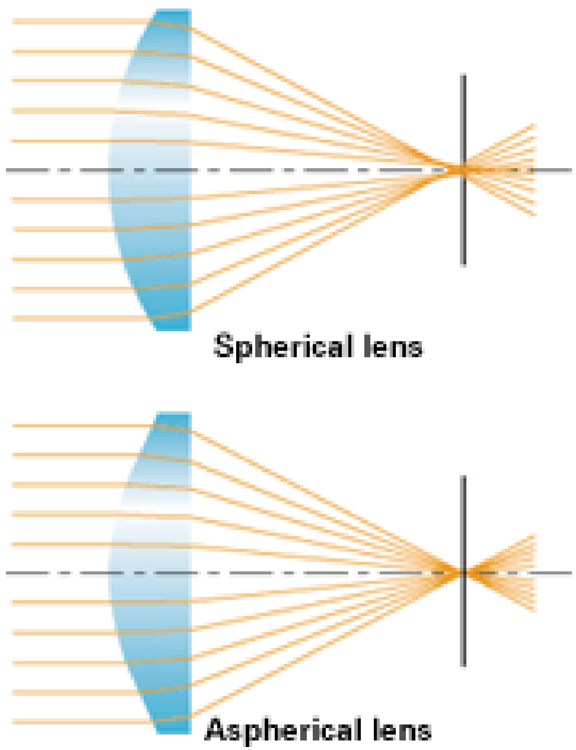


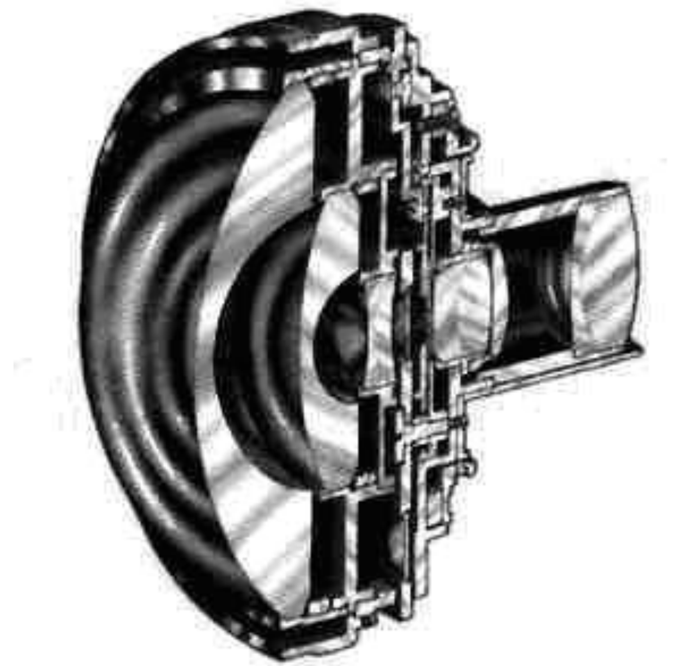
https://youtu.be/dg3kIKphB5E

Barrel distortion



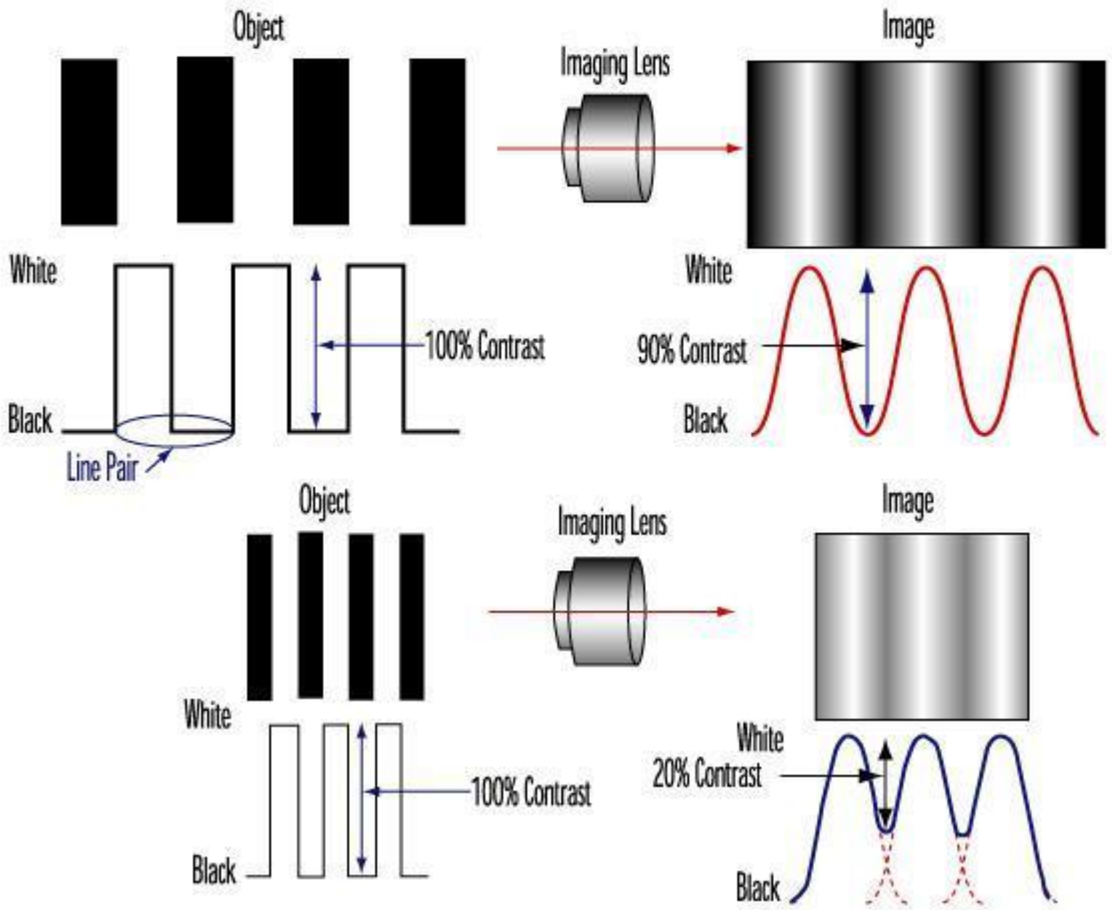
Lenses



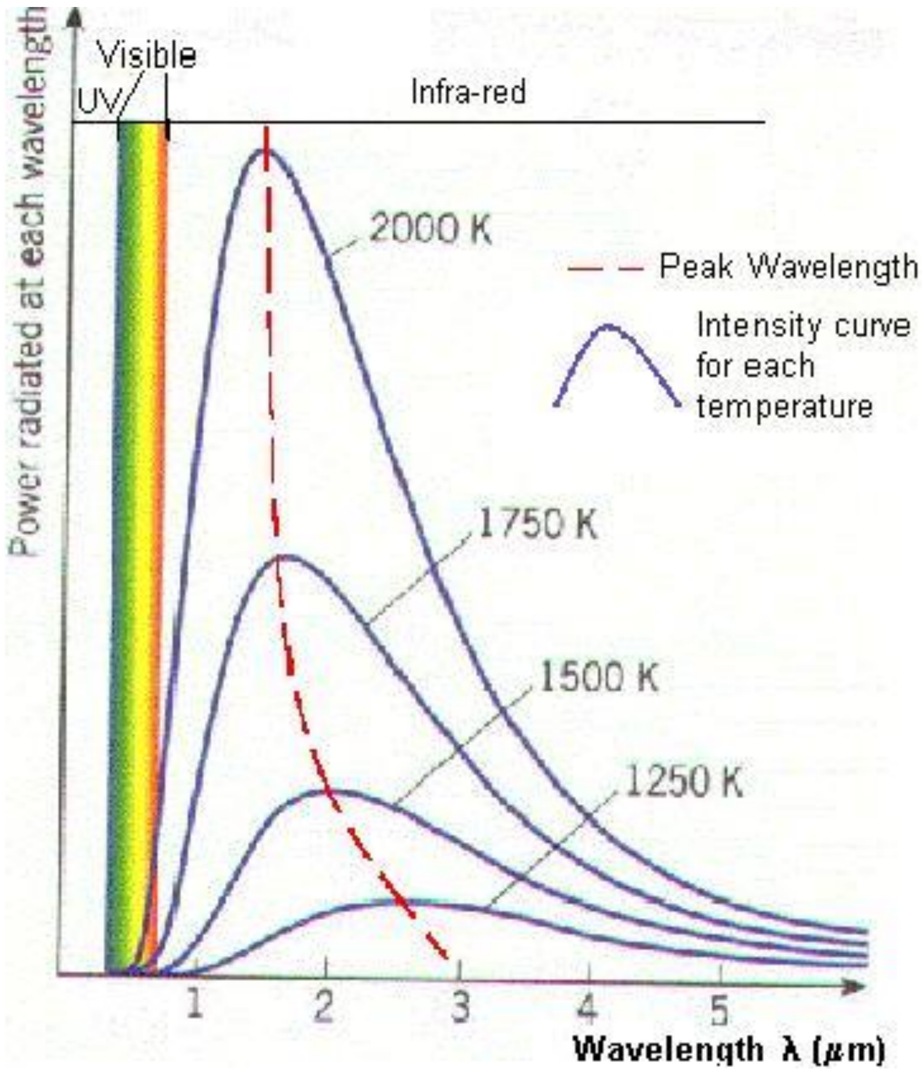


https://youtu.be/B7qrgrrHry0

Loss of resolution



Black body radiation



Color constancy

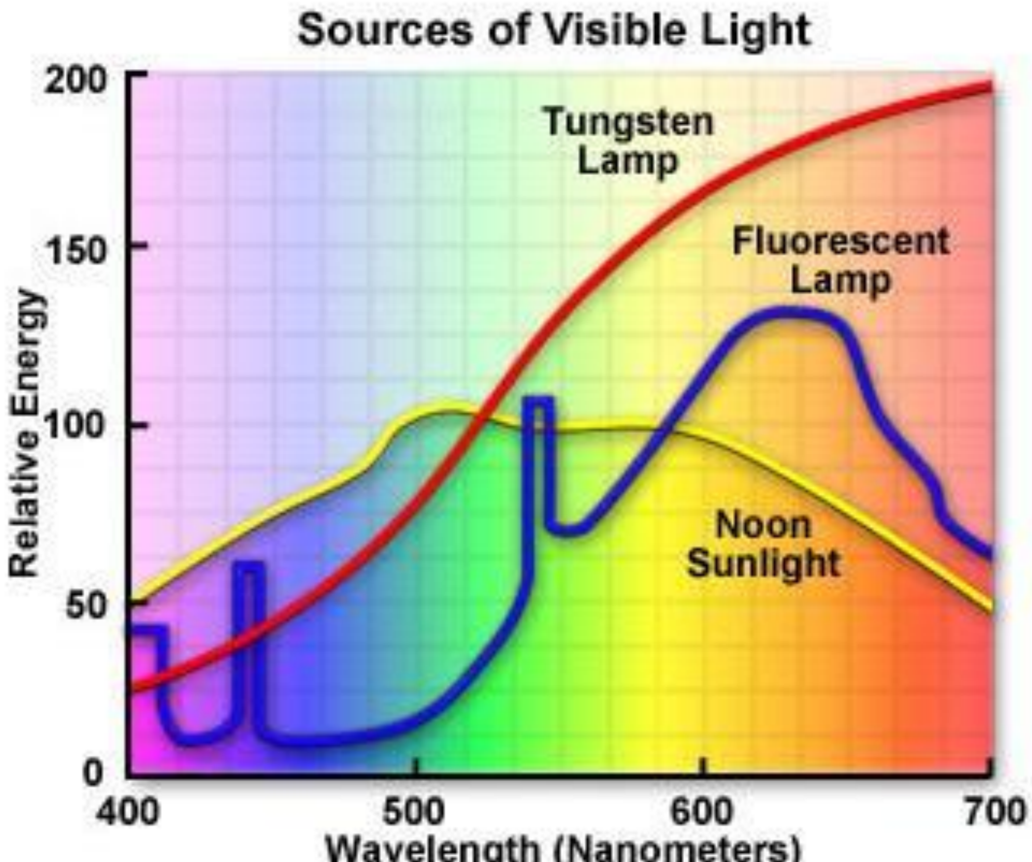


Figure 2

Calibration

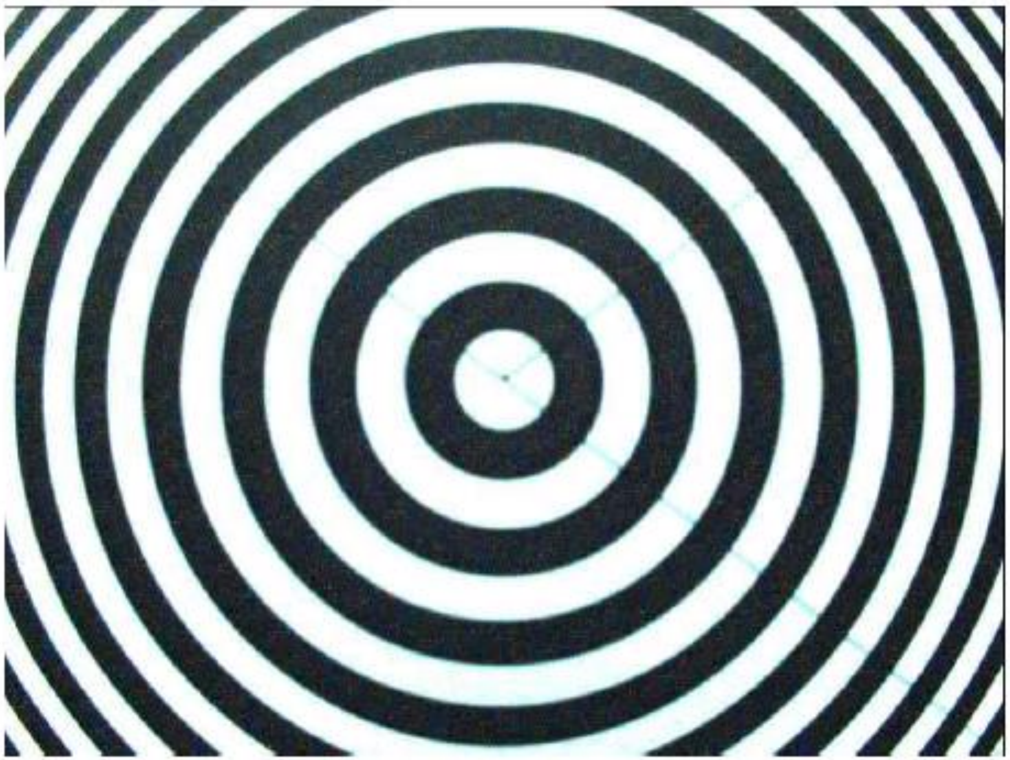


Fig. 2.3. Barrel distortion produced by a 4.2 mm lens. Each ring is of the same width but the camera distorts the pattern.

https://youtu.be/8R29jGaSdkE

Radial distortion

$$(p_x, p_y) \mapsto (d_r p_x, d_r p_y)$$

$$d_r = a_1 r + a_2 r^2 + a_3 r^3.$$

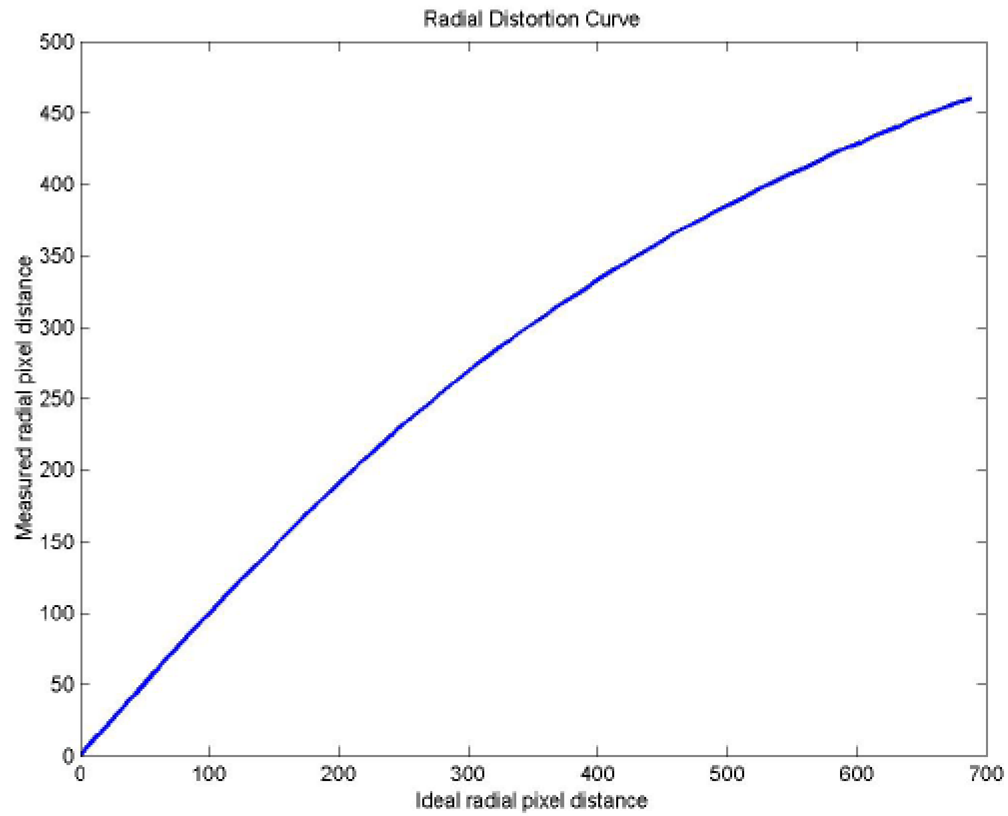
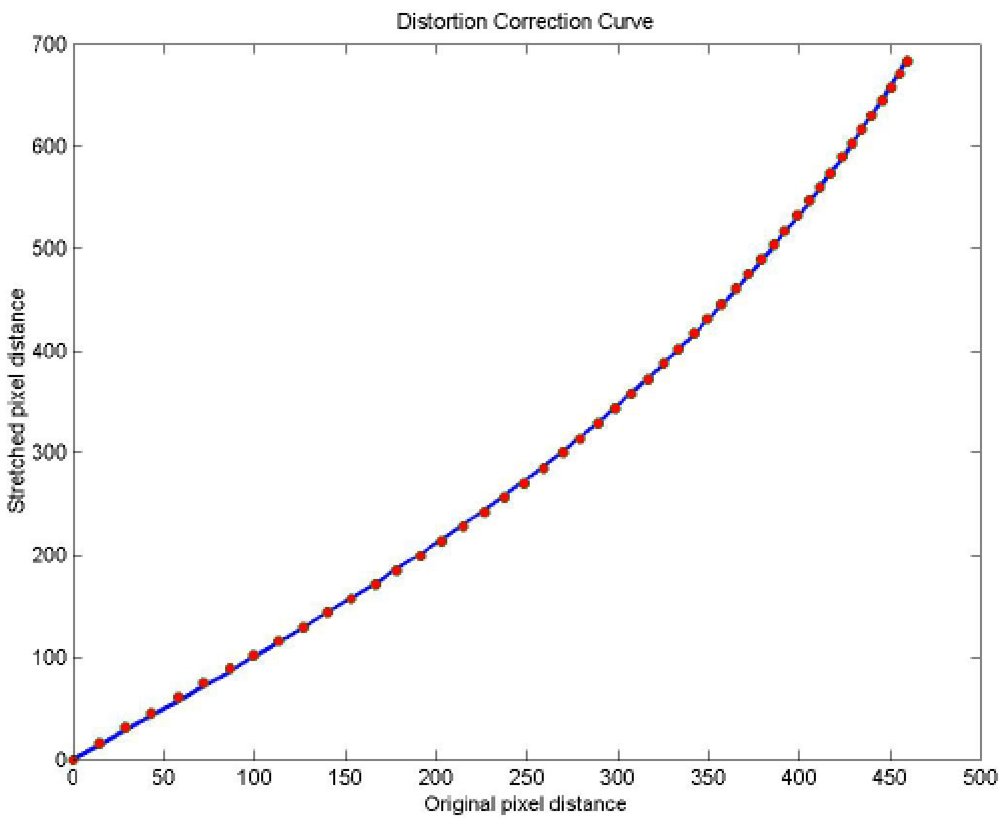


Fig. 2.4. Radial pixel distortion produced by a 4.2 mm lens

$$a_1 = 1.07761103142178$$
$$a_2 = -0.00096240306505$$
$$a_3 = 0.00000401304361$$

Distortion correction



Perspective

Camera coordinate system

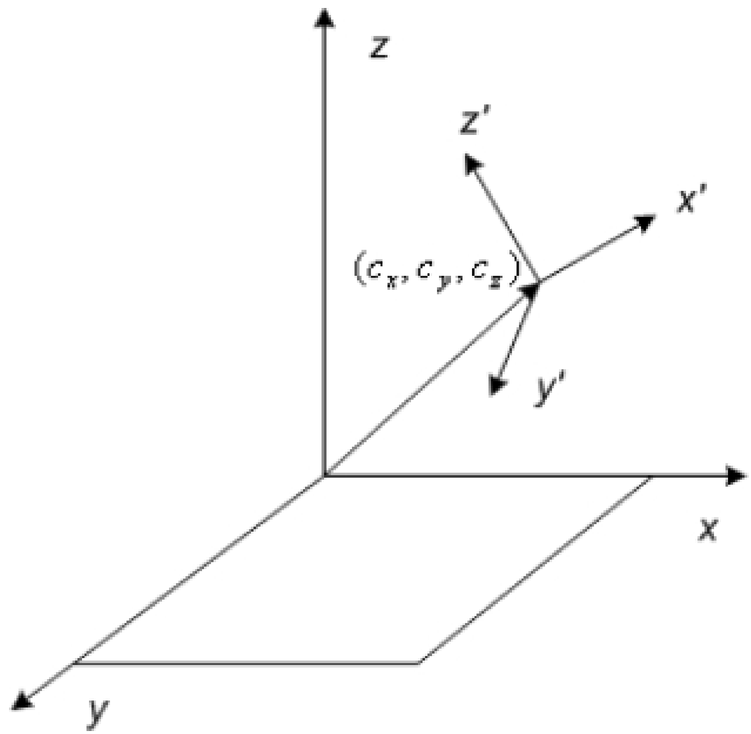


Fig. 2.9. The world's coordinate system and the camera's coordinate system. The camera is positioned at the point (c_x, c_y, c_z) in world coordinates.

Projective Geometry

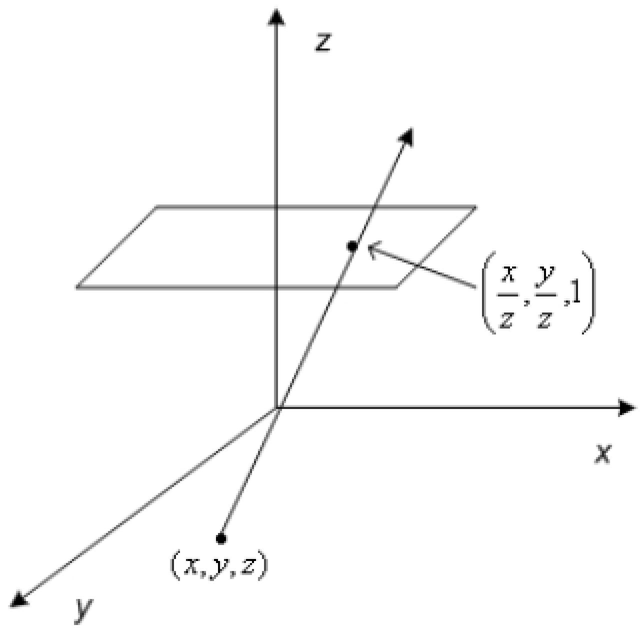


Fig. 2.8. Projective transformation of three dimensional space to the plane $z = 1$

$$f_1((x, y, z)^{\mathrm{t}}) = (x/z, y/z, 1)^{\mathrm{t}}$$

Rotation & Translation

$$(x', y', z')^{\mathrm{t}} = \mathbf{R}((x, y, z)^{\mathrm{t}} - \mathbf{t}) = \mathbf{R}(x, y, z)^{\mathrm{t}} - \mathbf{R}\mathbf{t}$$

If all points we are mapping are on the floor, that is, if they have coordinate $z = 0$, then transformation which describes the change of basis reduces to

$$(x', y', z')^{\mathrm{t}} = (\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}')(x, y, 1)^{\mathrm{t}}$$

Composing the projective transformation with the change of coordinates we have

$$(x'', y'', 1)^{\mathrm{t}} = f_1((x', y', z')^{\mathrm{t}}) = f_1((\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}')(x, y, 1)^{\mathrm{t}})$$

We denote the matrix $(\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}')$ by $\mathbf{H}$ and we can write simply

$$(x'', y'', 1)^{\mathrm{t}} = f_1(\mathbf{H}(x, y, 1)^{\mathrm{t}})$$

From pixels to coordinates

$$(x, y, 1)^{\mathrm{t}} = \mathbf{G}(x'', y'', 1)^{\mathrm{t}}$$

where $(x'', y'', 1)$ are the homogeneous coordinates of the image point. Since we know that

$$\mathbf{H}(x, y, 1)^{\mathrm{t}} = (x', y', z')^{\mathrm{t}}$$

and since $x'' = x'/z'$ and $y'' = y'/z'$, it follows that

$$\mathbf{H}(x, y, 1)^{\mathrm{t}} = z'(x'', y'', 1)^{\mathrm{t}}$$

for a certain z'. It is then obvious that $\mathbf{G} = c\mathbf{H}^{-1}$, where c is a proportionality constant.

Computing H

$$\begin{pmatrix} x'' \\ y'' \\ 1 \end{pmatrix} = f_1(\mathbf{H}(x, y, 1)^{\mathrm{t}}) = f_1 \left[\begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \begin{pmatrix} x \\ y \\ 1 \end{pmatrix} \right]$$

Each one of the transformed points yields two linear equations involving the unknown elements of $\mathbf{H}$. The three equations are

$$x' = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{h} \tag{2.1}$$

$$y' = \begin{pmatrix} 0 & 0 & 0 & x & y & 1 & 0 & 0 & 0 \end{pmatrix} \mathbf{h} \tag{2.2}$$

$$z' = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & x & y & 1 \end{pmatrix} \mathbf{h} \tag{2.3}$$

$$x'' = x'/z' \quad \text{and} \quad y'' = y'/z'$$

therefore

$$x''z' = x' \quad \text{and} \quad y''z' = y'$$

From this we deduce that

$$x'' \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & x & y & 1 \end{pmatrix} \mathbf{h} = \begin{pmatrix} x & y & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \mathbf{h}$$
$$y'' \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & x & y & 1 \end{pmatrix} \mathbf{h} = \begin{pmatrix} 0 & 0 & 0 & x & y & 1 & 0 & 0 & 0 \end{pmatrix} \mathbf{h}$$

which we can rewrite as

$$\begin{pmatrix} x & y & 1 & 0 & 0 & 0 & -x''x & -x''y & -x'' \end{pmatrix} \mathbf{h} = 0$$
$$\begin{pmatrix} 0 & 0 & 0 & x & y & 1 & -y''x & -y''y & -y'' \end{pmatrix} \mathbf{h} = 0$$

Four data points

$$\begin{pmatrix} x_0 & y_0 & 1 & 0 & 0 & 0 & -x_0''x_0 & -x_0''y_0 & -x_0'' \\ 0 & 0 & 0 & x_0 & y_0 & 1 & -y_0''x_0 & -y_0''y_0 & -y_0'' \\ x_1 & y_1 & 1 & 0 & 0 & 0 & -x_1''x_1 & -x_1''y_1 & -x_1'' \\ 0 & 0 & 0 & x_1 & y_1 & 1 & -y_1''x_1 & -y_1''y_1 & -y_1'' \\ x_2 & y_2 & 1 & 0 & 0 & 0 & -x_2''x_2 & -x_2''y_2 & -x_2'' \\ 0 & 0 & 0 & x_2 & y_2 & 1 & -y_2''x_2 & -y_2''y_2 & -y_2'' \\ x_3 & y_3 & 1 & 0 & 0 & 0 & -x_3''x_3 & -x_3''y_3 & -x_3'' \\ 0 & 0 & 0 & x_3 & y_3 & 1 & -y_3''x_3 & -y_3''y_3 & -y_3'' \end{pmatrix} \mathbf{h} = 0$$

$$\lambda \mathbf{r}_1 = \mathbf{h}_1$$

and

$$\lambda \mathbf{r}_2 = \mathbf{h}_2$$

Since $|\mathbf{r}_1| = 1$, then $\lambda = |\mathbf{h}_1|/|\mathbf{r}_1| = |\mathbf{h}_1|$ and $\lambda = |\mathbf{h}_2|/|\mathbf{r}_2| = |\mathbf{h}_2|$. We can thus compute the factor λ and eliminate it from the recovered matrix $\mathbf{H}$. We just set

$$\mathbf{H}' = \mathbf{H}/\lambda$$

$$\mathbf{r}_3 = \mathbf{r}_1 \times \mathbf{r}_2$$

Therefore, we can recover from $\mathbf{H}$ the rotation matrix $\mathbf{R}$. We can also recover the translation vector (the position of the camera in field coordinates). Just remember that

$$\mathrm{h}'_3 = -\mathbf{R}\mathrm{t}$$

Therefore the position vector of the camera pin-hole $\mathbf{t}$ is given by

$$\mathbf{t} = -\mathbf{R}^{-1}\mathrm{h}'_3$$

Focal plane

$$K = \begin{pmatrix} \phi & 0 & 0 \\ 0 & \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

then the complete world coordinates to image projective projection is given by

$$\begin{pmatrix} p''_x \\ p''_y \\ 1 \end{pmatrix} = \mathbf{K} f_1(\mathbf{H}(x, y, 1)^{\mathrm{t}})$$

Final mapping

Applying f_1 again we obtain

$$f_1 \left(\mathbf{H}^{-1}\mathbf{K}^{-1} \begin{pmatrix} p''_x \\ p''_y \\ 1 \end{pmatrix} \right) = (x, y, 1)^{\mathrm{t}}$$

The mapping from pixel coordinates to field coordinates is complete.