Search and Motion Planning

Prof. Dr. Daniel Göhring

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State Space Search[1]

- States Z (Vertices)
- Operators $Op \subseteq Z \times Z$ (Edges)
- Graph $[Z, Op]$
- Initial states $z_{initial} \in Z$
- Target states $Z_{final} \subseteq Z$
- Cost functions $c : Z \times Z \to R^+$,
  Costs of path $w = z_0 z_1 \cdots z_n \in Z^*$:
  $c(z_0 z_1 \cdots z_n) := \sum_{i=1,\cdots,n} c(z_{i-1}, z_i)$
- Estimator function $\sigma : Z \to R$ (Heuristic) for remaining from z to a target state.

[1]Slides with courtesy of Prof. Dr. Hans-Dieter Burkhard [2]

State Space Search

- Tasks:
- Can a goal state $z \in Z_{final}$ be reached from initial state $z_{initial} \in Z$?
- Find a way from initial state $z_{initial} \in Z$ to a target state $z \in Z_{final}$
- Find an optimal path from initial state $z_{initial} \in Z$ to a target state $z \in Z_{final}$

Complexity (Number of States / Vertices)

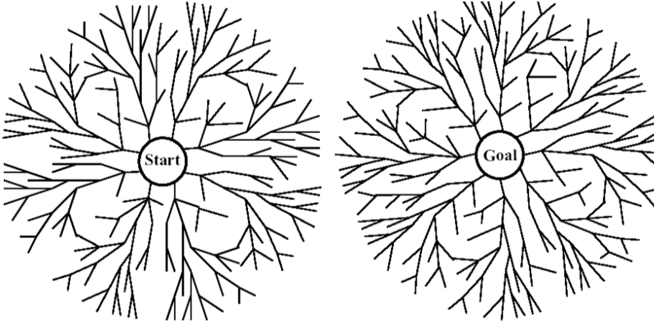
- 8 Puzzle: 9! states
  $9!/2 = 181.440$ reachable
- 15 Puzzle: : 16! states
  $16!/2$ reachable
- Rubik's cube: $12 \cdot 4.3 \cdot 10^{19}$ states
  $1/12$ reachable: $4.3 \cdot 10^{19}$
- Towers of Hanoi: 3^n States for n discs
  solvable in $(2^n) - 1$ moves
- Checkers: approx. 10^{40} games of average length
- Chess: approx. 10^{120} games of average length
- Go: 3^{361} states

Expansion Strategies

- Directions
  - Forward, start with $z_{initial}$ (forward chaining, data driven, bottom up)
  - Reverse z_{final} (backward chaining, goal driven, top down)
  - Bi-directional
- Expansion
  - Depth first
  - Breadth first
- Additional information
  - blind search ("uninformed")
  - heuristic search with σ ("informed")

Bi-Directional Breadth-Search

- Parallel search from start and goal until meeting



- Search depth from both sides only half

Expansion

Data structures:

- OPEN List: A vertex is "open" , if it was constructed but not expanded (neighboring vertices not calculated)
- CLOSED List: A vertex is "closed" , if it was fully expanded (all neighboring vertices are known)
- Further information: Predecessor / successor of vertices for reconstruction of found paths

Heuristic Search for best Way: A*

Costs to reach z' from z:

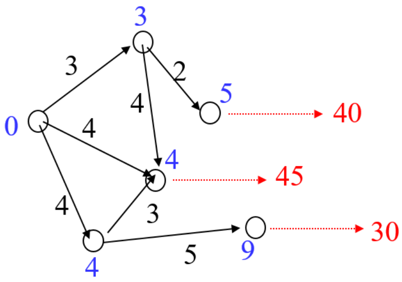
- If z' is reachable from z:
  $g(z, z') := \min\{c(s) | s \text{ path from } z \text{ to } z'\}$
- Else: $g(z, z') := \infty$

Tentative cost calculation during expansion: $G' = [V', E']$ is a (known) partial graph of G

$g'(z, z', G') := \min\{c(s) | s \text{ path in } G' \text{ from } z \text{ to } z'\}$

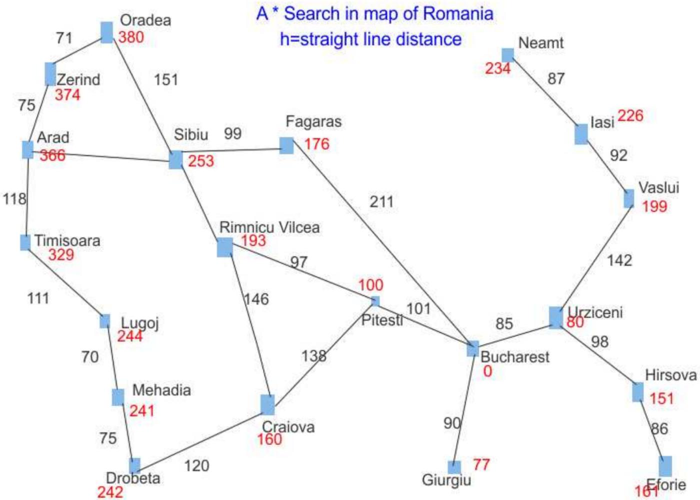
$$\boxed{g'(z, z', G') \geq g(z, z')}$$

Heuristic Search for Best Path



$g'(z_0, z', G')$: so far known costs to reach z' from Start
$\sigma(z')$: estimated costs to reach target state, starting from z'

State Space Search - Example



Algorithm A* (soft form) for Trees

- A*0 (Start) $OPEN := [z_0]$, $CLOSED := []$.
- A*1: (negative exit)
  If $OPEN = []$: EXIT("no").
- A*2: (positive exit)
  If z first vertice in $OPEN$:
  If z is target: EXIT("yes:" z).
- A*3: (expand)
  $OPEN := OPEN - z$. $CLOSED := CLOSED \cup \{z\}$
  $Succ(z) :=$ set of successors of z .
  If $Succ(z) = \{\}$: Goto A*1
- A*4: (Organization of $OPEN$)
  $OPEN := OPEN \cup Succ(z)$ with increasing order of
  $g'(z_0, z', G') + \sigma(z')$
- Goto A*1. Search space as a tree: CLOSED not used.

Algorithm A* (soft form) for Trees and for Cyclic Graphs

- A*0 (Start) $OPEN := [z_0], CLOSED := []$.
- A*1: (negative exit)
  If $OPEN = []$: EXIT("no").
- A*2: (positive exit)
  If z first vertice in $OPEN$:
  If z is target: EXIT("yes:" z).
- A*3: (expand)
  $OPEN := OPEN - z$. $CLOSED := CLOSED \cup \{z\}$
  $Succ(z) :=$ set of successors of z .
  If $Succ(z) = \{\}$: Goto A*1
  $NEW = Succ(z) - \{z'|z' \in Succ(z)$ and $z' \in CLOSED$ and $g'(z_0, z', G') >= g'(z_0, z', G'_{old})\}$

Algorithm A* (soft form) for Trees and for Cyclic Graphs

- A*4: (Organization of *OPEN*)
  $OPEN := OPEN \cup NEW$ with increasing order of
  $g'(z_0, z', G') + \sigma(z')$
- Goto A*1. If search space has cycles: use CLOSED.

Algorithm A* ("soft form")

Definition

$f(z) := \min\{g(z, z_{final}) | z_{final} \in Z_{final}\}$ = real costs from z to target state (vertex)
($f(z_0)$ = cost of the optimal path)
Heuristic function σ is called optimistic (a.k.a. admissible) or underestimating, if $\sigma(z) \leq f(z)$ for all $z \in Z$.

Proposition

Given: Ex. $\delta > 0$ with $c(z, z') > \delta$ for all z,z'. σ is an optimistic / admissible heuristic. Every node has a finite number of successors. One can prove: If solution exists, A* (soft form) finds an optimal path.

Special Cases

$c \equiv 0$:
Search for best path with heuristic σ and no cost function (Hill climbing)
$\sigma \equiv 0$:
Search for best path without heuristic ($s \equiv 0$ ist also optimistic heuristic) - Dijkstra
$c \equiv 1$
($g' \equiv$ Search depth) , $\sigma \equiv 0$: Breadth first search

Influence of Heuristic σ

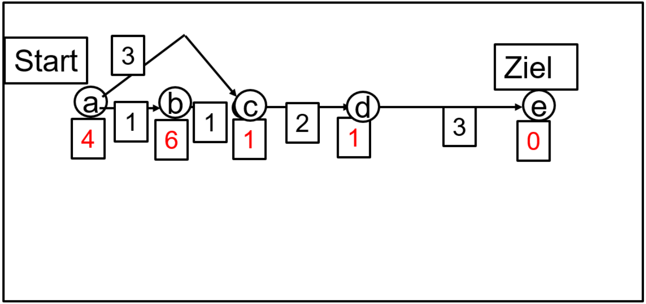
- Trade-off between quality of result and calculation effort
- Criteria for order of extension
- Same order as $g' + \sigma$ is also provided by $a(g' + \sigma) + b$ for arbitrary positive constants a, b .
- optimal order for $\sigma = f$
- σ_2 more efficient than σ_1 if $\sigma_1 \leq \sigma_2 \leq f$ (Hierarchy for heuristic functions)

Algorithm A* (hard form)

- A*0 (Start) $OPEN := [z_0], CLOSED := []$.
- A*1: (negative exit)
  If $OPEN = []$: EXIT("no").
- A*2: (positive exit)
  If z first vertice in $OPEN$:
  If z is target: EXIT("yes:" z).
- A*3: (expand)
  $OPEN := OPEN - z$. $CLOSED := CLOSED \cup \{z\}$
  $Succ(z) :=$ set of successors of z .
  If $Succ(z) = \{\}$: Goto A*1
- A*4: (Organization of $OPEN$)
  $OPEN := OPEN \cup (Succ(z) - CLOSED)$ sorted with increasing order of $g'(z_0, z', G') + \sigma(z')$
- Goto A*1.

Algorithm A* (hard form)

- Problem:
- For optimistic σ the hard form is not always correct



Algorithm A* (hard form)

What kind of heuristic do we need to stay optimal?

Definition

A heuristic σ is called consistent or monotonous, if for all states z', z'' holds: $\sigma(z') \leq g(z', z'') + \sigma(z'')$

Lemma

If σ is consistent, σ is optimistic. The reverse does not necessarily apply

Algorithm A* (hard form)

Proposition

Given: Ex. $\delta > 0$ with $c(z, z') > \delta$ for all z, z' . σ is a consistent heuristic. One can prove: if a solution exists, A^* (hard form) finds an optimal path.

Algorithm A* ("hard form")

- "consistent" functions harder to find then "optimistic"
- It is also possible to use an optimistic heuristic with weaker pruning of search space
- Erase states in OPEN (or Succ(z)) only if new evaluation is worse than earlier evaluation.

Memory Saving Variants of Algorithm A*

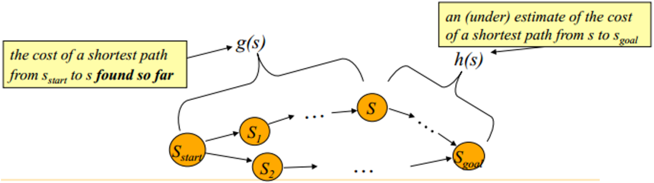
- Iterative Deepening A* (IDA*) in analogy to IDA: Depth first till boundary $g'(z_0, z', G') + s(z')$ is exceeded from earlier iteration
- SMA* (simplified memory-bounded A*)

Anytime (or Weighted) A*

- If the heuristic σ (sometimes denoted as h) is closer to the real costs, less vertices have to be expanded
- But "inflating" the heuristics can lead to a heuristic which is not optimistic (admissible)

Heuristics in Heuristic Search[2]

- Dijkstra's: expands states in the order of $f = g$ values
- A* Search: expands states in the order of $f = g + \sigma$ values
- Weightes A*: expands states in the order of $f = g + \varepsilon\sigma$ values, $\varepsilon > 1$ = bias towards states that are closer to goal



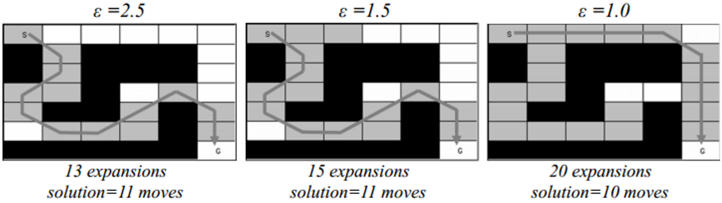
[2]Slides from Maxim Likhachev

Suboptimality

- Heuristic includes factor ε
- Suboptimality is bounded by factor ε
  - The length of the found solution is not longer than Ït times the optimal solution
- Exampe:
  - costs from cell to cell are 1
  - Heuristic is the larger of coordinate difference from current cell to goal cell
  - Start is upper left cell
  - Goal is lower right
  - Obstacles black
  - Free space white
  - Expanded cells gray

Anytime Search based on weighted A*

- Constructing anytime search based on weighted A*:
  - find the best path possible given some amount of time for planning
  - do it by running a series of weighted A* searches with decreasing ε:



Anytime A*

- Problem:
  - Running A* with increasing ε each time from scratch can be very expensive
  - Many states remain the same through various iterations
  - A solution for reusing the search results is described in ARA* (Likhachev [4])
- ARA* : an efficient version of the above that reuses state values within any search iteration

ARA*

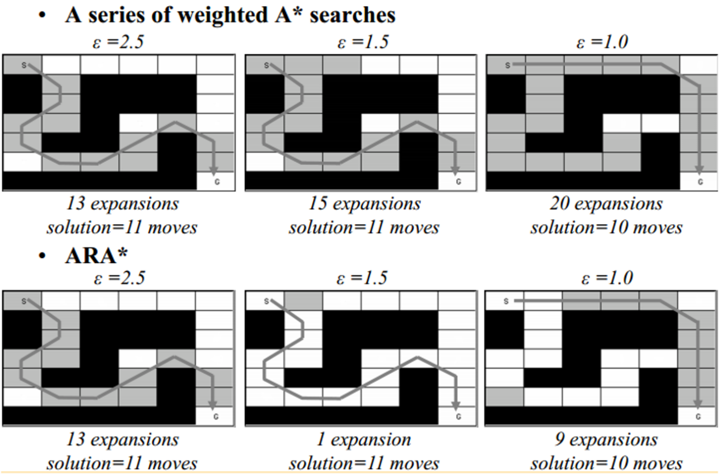
- Efficient series of weighted A* searches with decreasing ε:

ComputePathwithReuse function
while($f(s_{goal})$ > minimum f-value in OPEN)
remove s with the smallest [$g(s) + \varepsilon h(s)$] from OPEN;
insert s into CLOSED;
for every successor s' of s
if g(s') > g(s) + c(s,s')
g(s') = g(s) + c(s,s');
if s' not in CLOSED then insert s' into OPEN;
otherwise insert s' into INCONS

---

set ε to large value;
$g(s_{start}) = 0$; OPEN = {s_{start}};
while $\varepsilon \geq 1$
CLOSED = {}; INCONS = {};
ComputePathwithReuse();
publish current ε suboptimal solution;
decrease ε;
initialize OPEN = OPEN U INCONS;

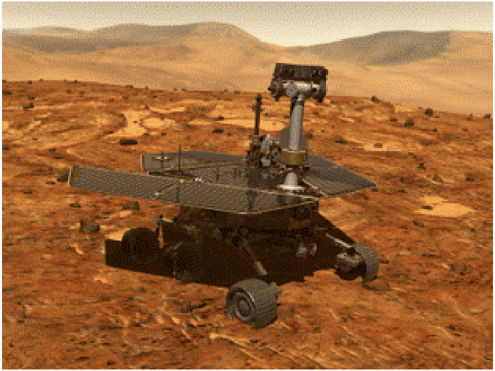
ARA*, slides from Likhachev [4]



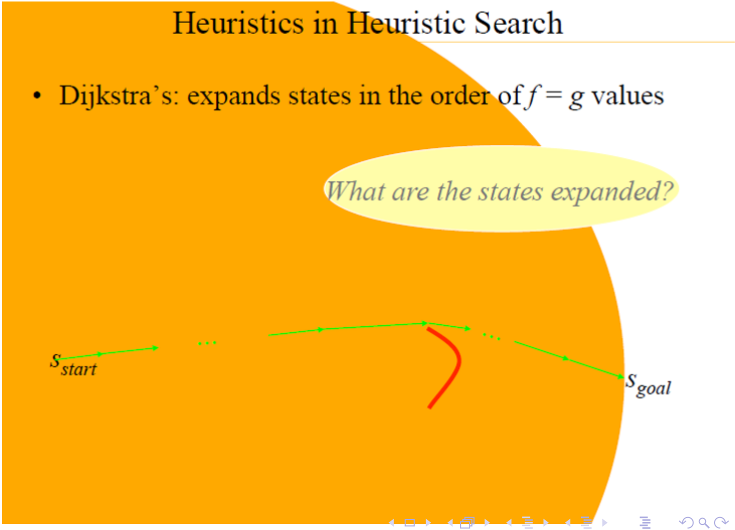
D* and Variations

- D* and its variations have been used on Mars rovers Opportunity and Spirit and by CMU at the DARPA Grand Challenge
- Roughly:
  - D* works as A* but from goal to start
  - Every expanded node "knows" its predecessor
  - When start node (vertex) is the next node, the search is done
  - nodes are marked: NEW (was never in OPEN), OPEN, CLOSE (no longer in OPEN), LOWER, RAISE (cost is higher than last time in OPEN)

D* and Variations



Heuristics in Heuristic Search [4]

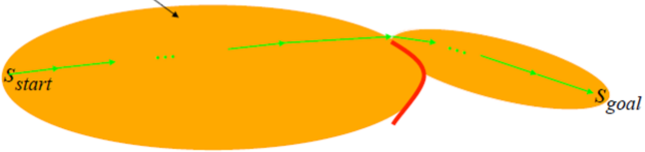


Heuristics in Heuristic Search [4]

Heuristics in Heuristic Search

- A* Search: expands states in the order of $f = g+h$ values

for high-D problems, this results in A being too slow and running out of memory*



Heuristics in Heuristic Search [4]

Heuristics in Heuristic Search

- Weighted A* Search: expands states in the order of $f = g+\varepsilon h$ values, $\varepsilon > 1$
- bias towards states that are closer to goal

Literature

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