$$\vec{x}_1, \ldots, \vec{x}_N$$

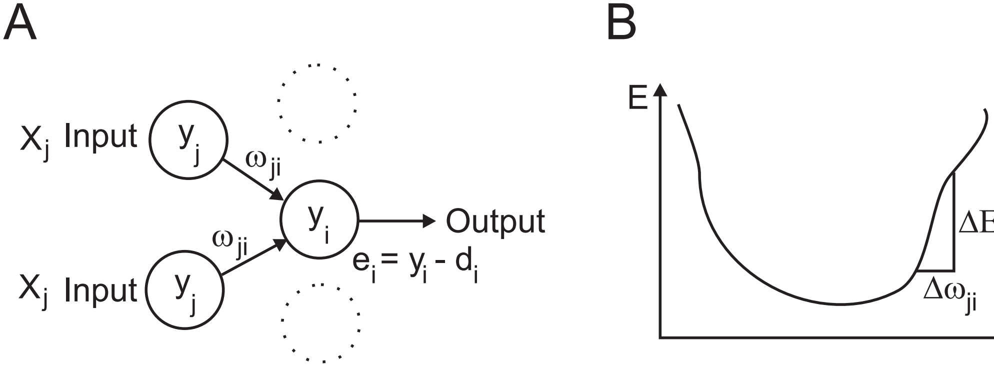
$$\vec{y} = f(\vec{x})$$

$\vec{y}$

$\vec{x}_1, \vec{y}_1$

$\vec{x}_2, \vec{y}_2; \ldots$

f

A
B
X_j Input
y_j
ω_{ji}
y_i
Output
ω_{ji}
$e_i = y_i - d_i$
X_j Input
y_j
E
ΔE
$\Delta\omega_{ji}$

$$y(n)$$

i

n

$$x_j(n) = y_j(n)$$

$$\omega_{ji}$$

$$y_i(n) = \sum_j y_j(n) w_{ji}$$

$$e_i(n) = y_i(n) - d_i(n)$$

$d_i(n)$

$y_i(n)$

$$E = \frac{1}{2}e^2$$

$$\Delta\omega_{ji}$$

=

$$-\mu \frac{\partial E}{\partial \omega_{ji}}$$

ω_{ji}

←

$$\omega_{ji} + \Delta\omega_{ji}$$

E

$$-\mu \frac{1}{2} \frac{\partial (d_i(n) - y_i(n))^2}{\partial \omega_{ji}}$$

$$-\mu \frac{1}{2} \frac{\partial \left(d_i(n) - \sum_j y_j(n) w_{ji} \right)^2}{\partial \omega_{ji}}$$

$$\mu \underbrace{\left(d_i(n) - \sum_j y_j(n) w_{ji} \right)}_{-e_i(n)} \cdot y_j(n)$$

$$-\mu \underbrace{\frac{\partial E}{\partial y_i}}_{-e_i(n)} \underbrace{\frac{\partial y_i}{\partial \omega_{ji}}}_{y_j(n)}$$

$$\mu \cdot e_i(n) \cdot y_j(n)$$

$$\mu << 1$$

$$y_j(n)$$

$$e_i(n)$$

y_j

e_i

y_i

d_i

e_j

e_k

ω_{kj}

$$\frac{\partial E}{\partial \omega_{kj}} = \underbrace{\frac{\partial E}{\partial y_j}}_{\text{trick!}} \frac{\partial y_j}{\partial \omega_{kj}}$$

$$\frac{\partial E}{\partial y_j}$$

$$\frac{\partial E}{\partial y_j} = \sum_i \underbrace{\frac{\partial E}{\partial y_i}}_{-e_i} \cdot \underbrace{\frac{\partial y_i}{\partial y_j}}_{\omega_{ji}}$$

$$\frac{\partial E}{\partial \omega_{kj}} = -\left(\sum_i e_i \omega_{ji}\right) \frac{\partial y_j}{\partial \omega_{kj}}$$

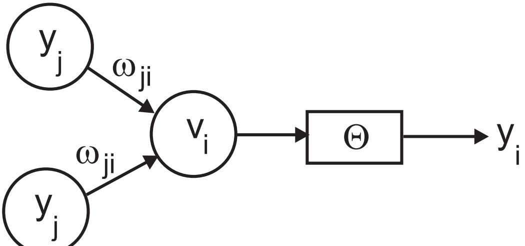
$$y_j = \sum_k y_k(n) w_{kj}$$

$$\frac{\partial E}{\partial \omega_{kj}}$$

$$- \left(\sum_i e_i \omega_{ji} \right) \underbrace{\frac{\partial \left(\sum_k y_k(n) w_{kj} \right)}{\partial \omega_{kj}}}_{y_k}$$

$$- \left(\sum_i e_i \omega_{ji} \right) y_k$$

$$\Delta\omega_{kj} = \mu \cdot y_k \cdot \underbrace{\sum_i e_i \omega_{ji}}_{e_j}$$



y_j
ω_{ji}
v_i
Θ
y_i
ω_{ji}
y_j

Θ

$$y_i(n) = \Theta\left(\underbrace{\sum_j y_j(n) w_{ji}}_{v_i}\right)$$

$$\frac{\partial E}{\partial \omega_{ji}} = \frac{\partial E}{\partial y_i} \underbrace{\frac{\partial y_i}{\partial v_i}}_{\Theta'} \frac{\partial v_i}{\partial \omega_{ji}}$$

Θ'

$$\Delta\omega_{ji} = \mu \cdot \Theta'(y_i) \cdot y_j \cdot e_i$$

$$\Delta\omega_{kj} = \mu \cdot \Theta'(y_j) \cdot y_k \cdot \underbrace{\sum_i e_i \omega_{ji}}_{e_j}$$

$$\Theta(v) = \begin{cases} 0, & \text{if } v < 0 \\ v, & \text{otherwise} \end{cases}$$

$$\Theta(v) = \tanh(v)$$

$$\Theta(v) = \frac{1}{1+e^{-v}}$$