

















W2

XO































University | School of
of Glasgow | Engineering

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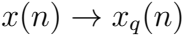


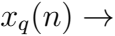
$$\underbrace{x_a(nT)}_{\text{analogue signal}} \equiv \underbrace{x(n)}_{\text{discrete data}}$$















$$x(n) = x(n)I, \quad x(n) = x(n) + x(n)$$

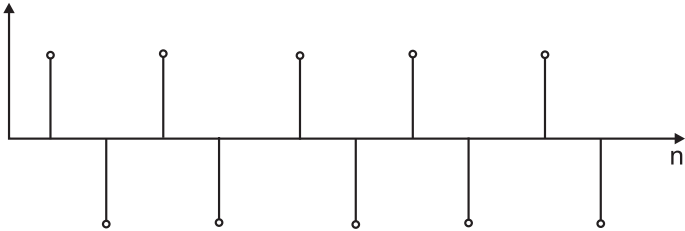
F_S

$=$

$\frac{1}{T_S}$

$$x_a(t) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$x(n)$









WORLDWIDE
= ACOPIA



www.A2PI.com

Normalised frequency: $f = \frac{F}{F_s}$

scipy = 2 π

1500 = 1



www.100days.org



05



$$x_0(x) = \cos(2\pi x) = 1$$









A pixelated, black and white graphic of the text "E.O. 15". The letters are blocky and have a high-contrast, dithered appearance. The "E" is on the left, followed by a period, then "O", another period, and finally "15" on the right. The overall style is reminiscent of early digital art or a low-resolution scan.



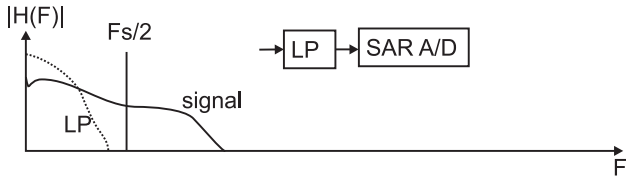
B

<

$\frac{1}{2}$

*F*₈

A



B



10

11

12

13



$$x_a(t) = \sum_{i=1}^n A_i \cos(2\pi F_i t + \Theta_i)$$



$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}$$

3

5

4

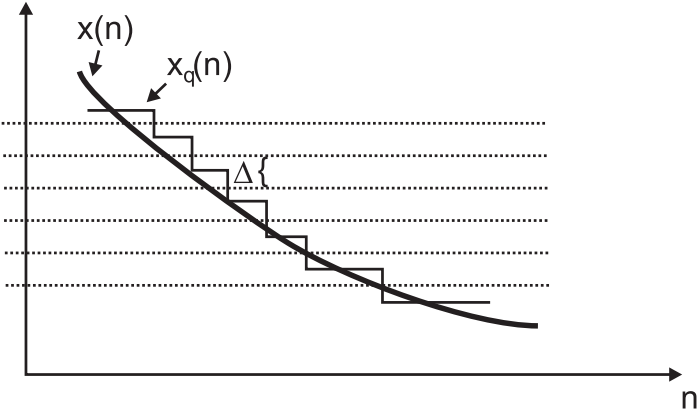
7

2

x

$$x_a(t) = \sum_{h=-a}^a x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{h}{F_s}\right)$$





$$\Delta = \text{quantisation step} = \frac{x_{\max} - x_{\min}}{L - 1}$$

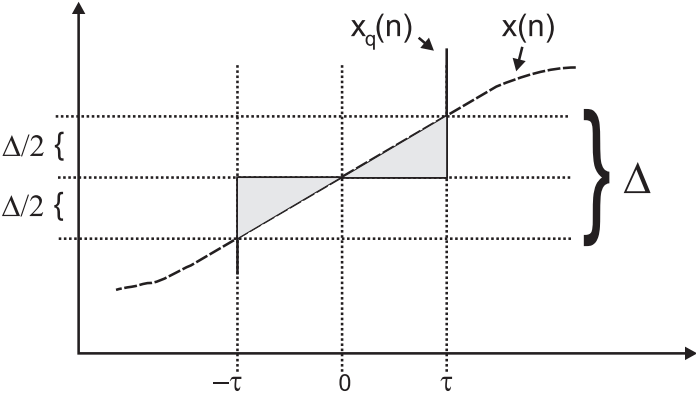
1992

1992

10000



www.oxfordjournals.org



$$\begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}
 \leq e(n) \leq
 \begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}$$



$$P_q = \frac{1}{T} \int_0^T e_q^2(t) dt$$





$$\frac{1}{\tau} \int_0^{\tau} \left(\frac{\Delta}{2\tau} \right)^2 t^2 dt$$

$$\frac{\Delta^2}{4\tau^3}$$

$$\int_0^{\tau}$$

$$t^2 dt$$

$$P_q = \frac{\Delta^2}{12\tau^3} = \frac{\Delta^2}{12}$$

$$P_x = \frac{1}{T_p} \int_0^{T_p} (A \cos \Omega t)^2 dt = \frac{A^2}{2}$$

ENVR

$$\frac{P_x}{P_q} = \frac{A^2}{2} \cdot \frac{12}{\Delta^2}$$

6A2



Δ2





$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_1 t}$$













$$c_k = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi k F_1 t} dt$$

$c_x = c_x^* x$ is read





CPA



CPA





$$\cos z = \frac{1}{2} (e^{zi} + e^{-zi})$$



$$c_0 + \sum_{k=1}^{\infty} |c_k| e^{j\theta_k} e^{j2\pi k F_1 t} + \sum_{k=1}^{\infty} |c_k| e^{-j\theta_k} e^{-j2\pi k F_1 t}$$

$$c_0 + 2 \sum_{k=1}^{\infty} |c_k| \cos(2\pi k F_1 t + \theta_k)$$









$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$



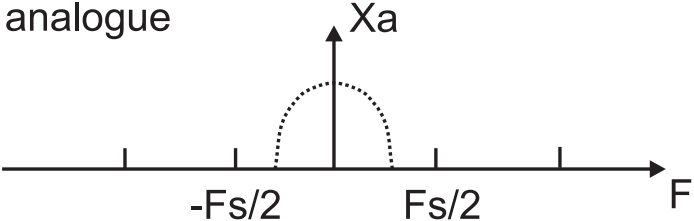
$$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi}$$

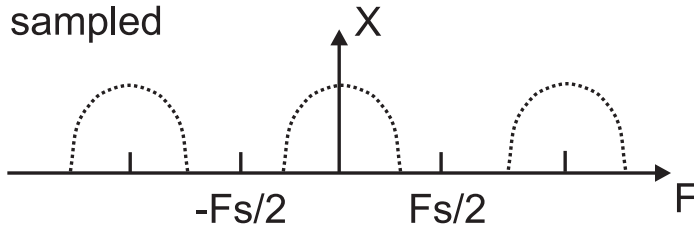
$$\int_{-0.5}^{0.5}$$

$$X(f)e^{j2\pi fn}dt$$

analogue



sampled



W E R

THE UNIVERSITY OF CHICAGO

$$\underbrace{\int_{-0.5}^{0.5} X(f) e^{j2\pi f n} df}_{\text{sampled}} = \int_{-\infty}^{+\infty} \underbrace{X_a(F)}_{\text{cont}} e^{j2\pi n F / F_s} dF$$





$$\frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X\left(\frac{F}{F_s}\right) e^{j2\pi n F/F_s} dF = \int_{-\infty}^{+\infty} X_a(F) e^{j2\pi n F/F_s} dF$$

$$\int_{-\infty}^{+\infty}$$

$$X_a(F)e^{j2\pi n\frac{F}{F_s}}dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s+kF_s}^{+\frac{1}{2}F_s+kF_s} X_a(F) e^{j2\pi n \frac{F}{F_s}} dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} X_a(F - kF_s) \underbrace{e^{j2\pi n \frac{F}{F_s}}}_{kF_s \text{ omit.}} dF$$

$$\underbrace{\int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} \sum_{k=-\infty}^{\infty} X_a(F - kF_s) e^{j2\pi n \frac{F}{F_s}} dF}_{=X(F) \text{ of Eq. 42}}$$

W E R E

$$F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

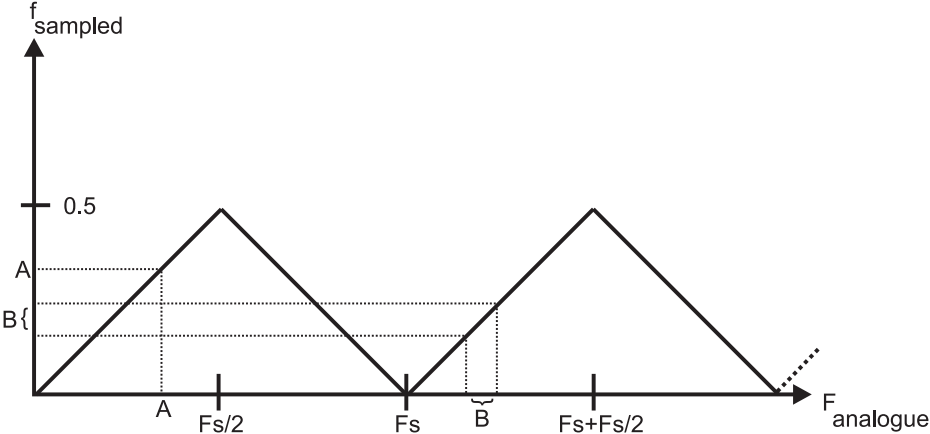


$$F_s \sum_{k=-\infty}^{\infty} X_a[(f-k)F_s]$$

192

1234567890

$\frac{1}{2} + 31$ $\frac{1}{2} + 31$



4

and along

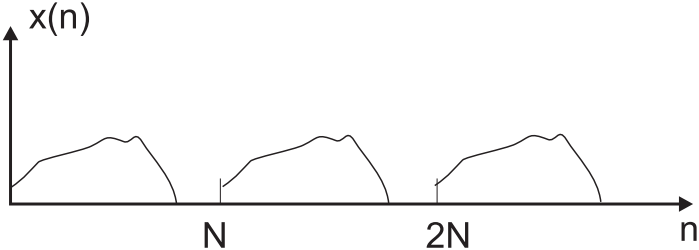
sampled

FOR THE 1929



$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \quad k = 0, 1, 2, \dots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad n = 0, 1, 2, \dots, N-1$$



$$X\left(\frac{2\pi}{N}k\right)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\frac{2\pi}{N}kn}\quad k=0,\ldots,N-1$$

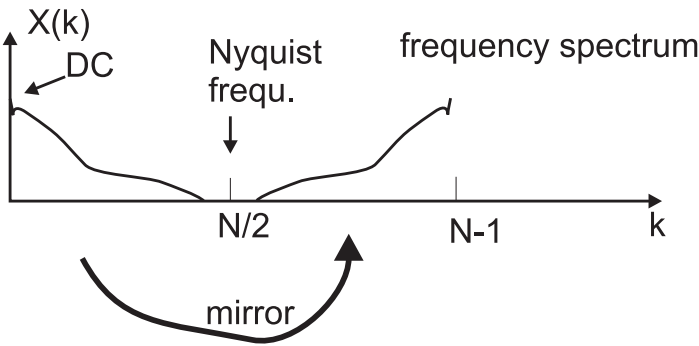
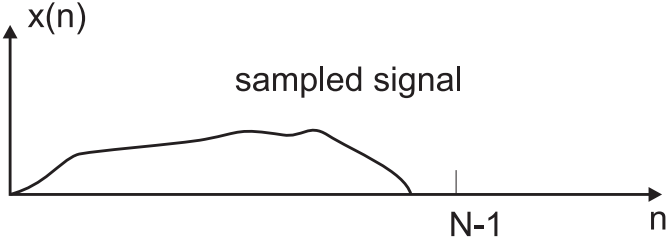
$$X\left(\frac{2\pi}{N}k\right)$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}kn}$$

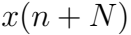
$$\sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n - lN)}_{\text{Periodic repetition!}} e^{-j\frac{2\pi}{N}kn}$$

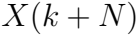
Periodic repetition!



WAVEZ





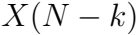




$$x(n) \text{ is real} \Rightarrow x^*(k) = x(N-k)$$

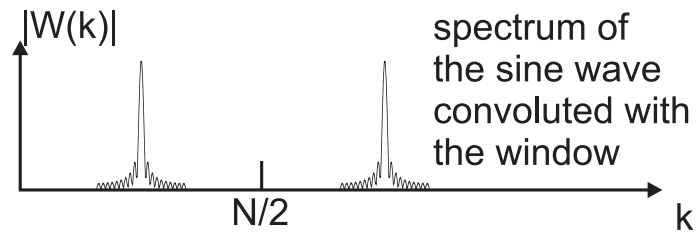
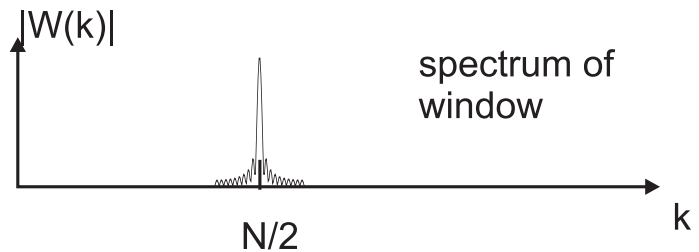
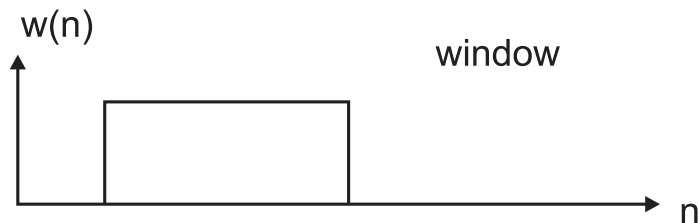
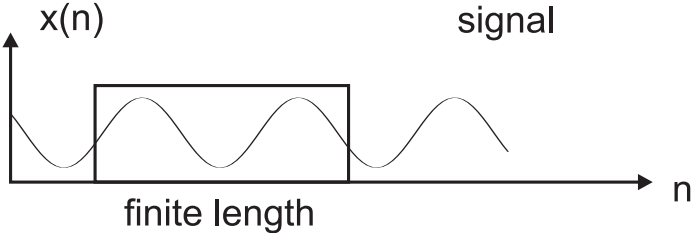


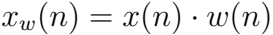




1992年12月21日

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2(m-n)$$













2025-02-27





$$W(\omega) = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

WAVES ARE * WAVES



Wavelength

Wew

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

WAVELENGTH



2020

$x^2 + 1$

$$m = 0, \dots, \frac{N}{2} - 1$$

W2mk
N



W2mk
N2

$$\sum_{m=0}^{N/2-1} x(2m)W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)W_N^{k(2m+1)}$$

$$\sum_{m=0}^{N/2-1} x(2m) W_{N/2}^{mk} + W_N^k \sum_{m=0}^{N/2-1} x(2m+1) W_{N/2}^{mk}$$

BEWAARDE
+ VERBODEN





WZQ

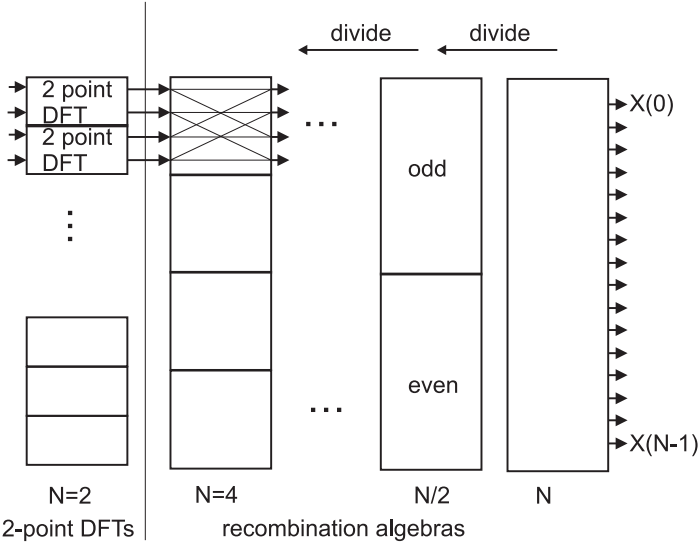
$$2 \cdot \sin \left(\frac{\pi}{2} \right) = \frac{\pi^2}{2}$$

BE

BEWE

Beethoven, Beethoven,

100% 100%



$$X_0(k) = X_{\text{re}}(k) + jX_{\text{im}}(k)$$



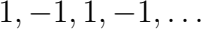


DOG XNO

$$x(0) + \underbrace{W_2^0}_{1} x(1) = x(0) + x(1)$$

visist freqv. x1

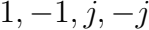
$$x(0) + \underbrace{W_2^{-1}}_{-1} x(1) = x(0) - x(1)$$

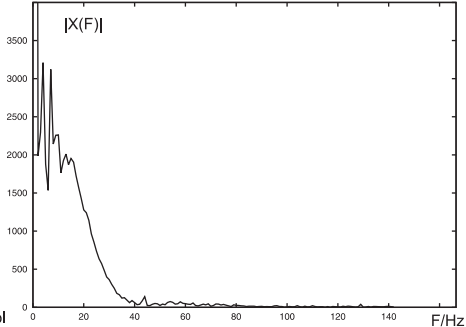
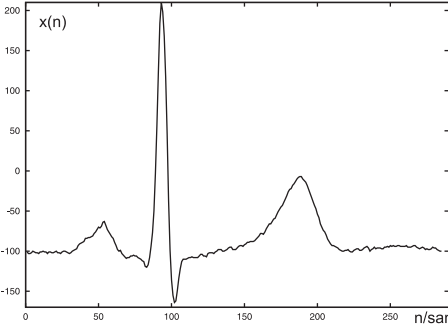


W

1002

W

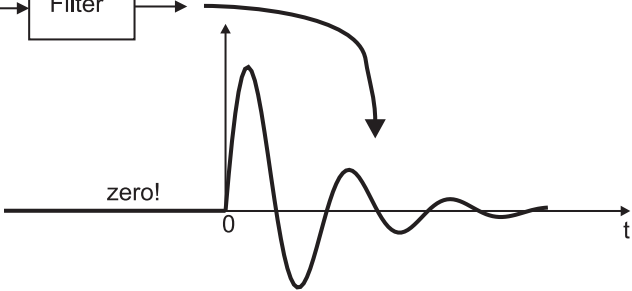


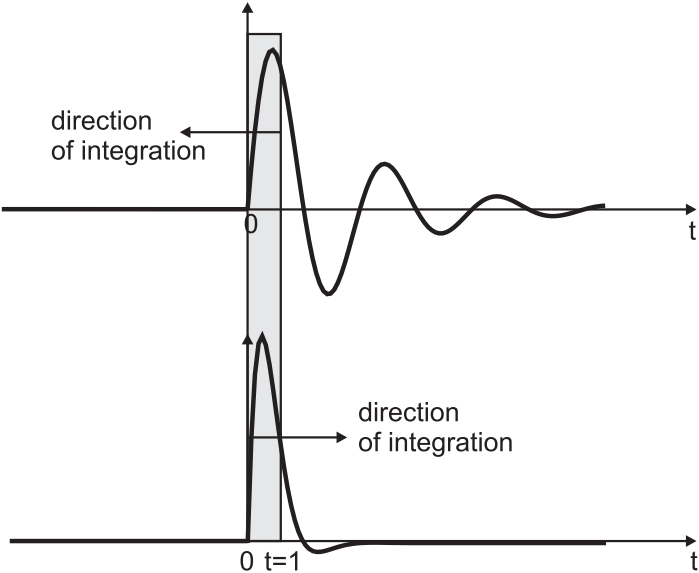




2100X400

delta
pulse







$$h(t) * x(t) = \int_{-\infty}^{\infty} h(t - \tau) x(\tau) d\tau$$



$$h(n) * x(n) = \sum_{n=-\infty}^{\infty} h(n)x(n-n)$$



500

900





$$y(t) = \int_{-\infty}^{\infty} h(t-\tau) \delta(\tau) d\tau = h(t)$$



$$\mathbb{L}\mathbb{T}(h(t)) = H(s) = \int_0^{\infty} h(t) e^{-st} dt$$

$$\int f(\tau) d\tau \Leftrightarrow \frac{1}{s} F(s)$$

$$\frac{d}{dt} f(t) \Leftrightarrow sF(s)$$

for the first time

1995

$$\int_0^{\infty} \underbrace{h(t-T)}_{\text{causal}} e^{-st} dt$$

$$\int_0^{\infty}$$

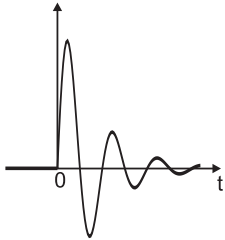
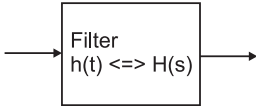
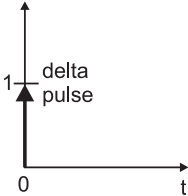
$$h(t)e^{-s(t+T)}dt$$

$$\int_0^{\infty} h(t) e^{-st} e^{-sT} dt$$

$$e^{-sT} \underbrace{\int_0^{\infty} h(t) e^{-st} dt}_{H(s)}$$

CHISEL

1845



$$d(t) = h(t) * x(t) = B(s) \cdot X(s)$$

desiderata pro



sp/ise impulse response





How do we

0 = 25

$$\tau_w = \frac{d\phi(w)}{dw}$$



$$x(t) = \sum_{n=0}^{\infty} x(n) \delta(t - nT) \quad \text{Sampled signal}$$



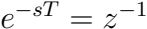
$$\sum_{n=0}^{\infty} x(n) e^{-snT}$$

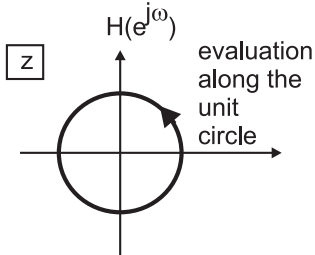
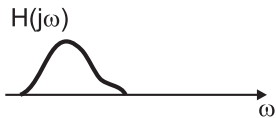
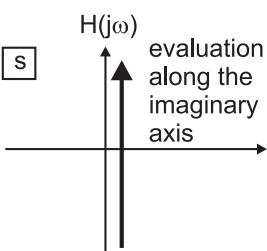
$$\int_0^{\infty}$$

$$x(n)e^{-st}dt$$

$$\sum_{n=0}^{\infty} x(n) \underbrace{\left(e^{-sT} \right)^n}_{z^{-1} = e^{-sT}}$$

$$\sum_{n=0}^{\infty} x(n) (z^{-1})^n \qquad \text{z-transform}$$



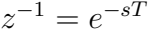


19









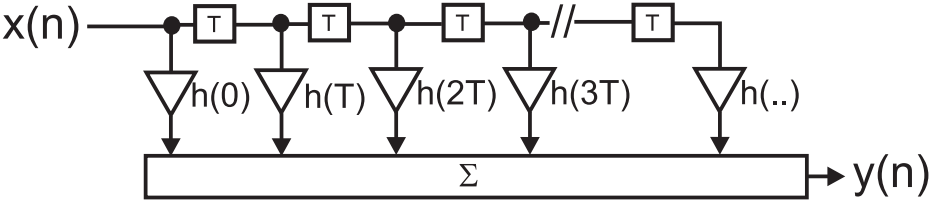




How easy?

HERBARIUM

airborne



$$h(t) = \sum_{n=0}^{\infty} h(nT) \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} h(nT) \underbrace{\left(e^{-sT} \right)^n}_{z^{-1}}$$





$$H(z) = \sum_{n=0}^{\infty} h(nT)(z^{-1})^n$$



12

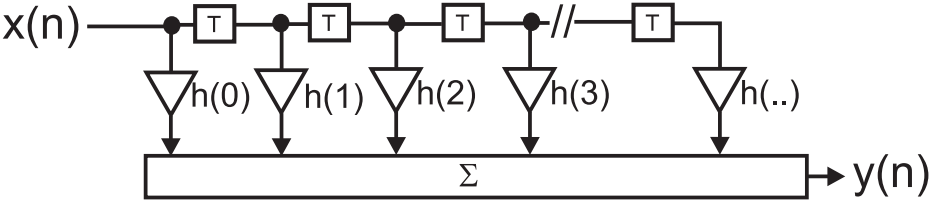
$$H(z)X(z) = \underbrace{\sum_{n=0}^{\infty} h(nT)z^{-n}}_{H(z)} X(z)$$

1925





$$H(z)X(z) = \underbrace{\sum_{m=0}^M h(mT)z^{-m}}_{H(z)} X(z)$$





123456789

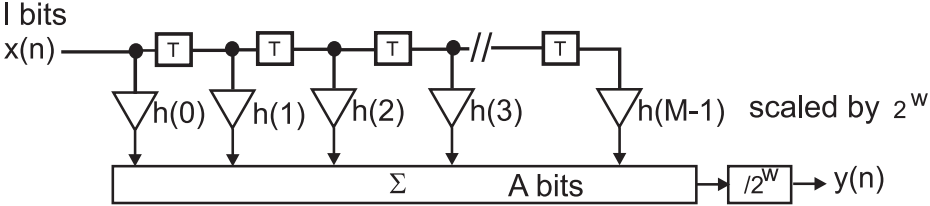
$$\sum_{m=0}^M h_{\text{analogue}}(mT) z^{-n} X(z)$$

$$\sum_{m=0}^M h_{\text{digital}}(m) z^{-n} X(z)$$

$$\sum_{m=0}^M h(m) z^{-m} X(z)$$



$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m)$$







2W

,

W

=

15





1002

111

4-1-1992







$$\tau_w = \frac{d\phi(w)}{dw}$$



$$A(w) = B(w) - w + w$$

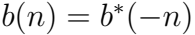
EW



$$h(n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(e^{i\omega}) e^{i\omega n} d\omega$$

$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b(n)$$





$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b^*(-n)$$



$$n(n+1) = e^{2i\phi} n^* (-n+1)$$





1 = 100%

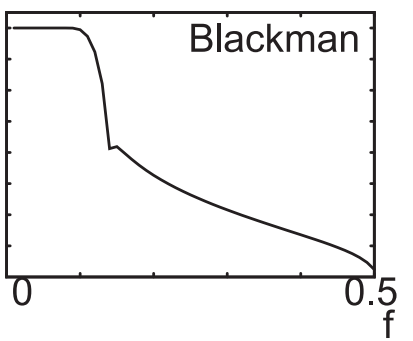
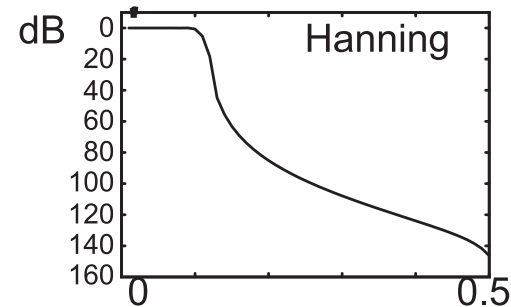
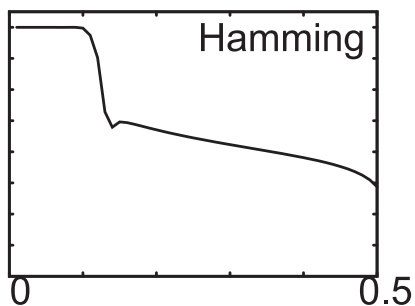
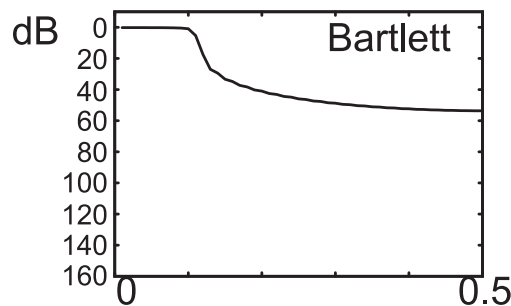
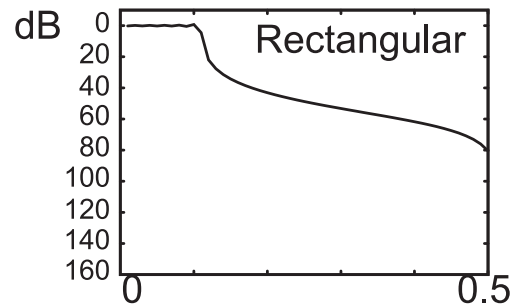




$$h(n+1) \equiv h(n+1)^k h(n+1)$$



$$|H(f)|$$



12



$$H(z)X(z) = \sum_{n=0}^N \underbrace{h(nT)w(nT)} z^{-n} X(z)$$

$$w(n) = a - (1 - a) \cos\left(\frac{2\pi n}{M}\right)$$





$$v(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{N}\right) + 0.08 \cos\left(\frac{4\pi n}{N}\right)$$







W2E50

00000





$$|H(e^{j\omega})| = \underbrace{B(e^{j\omega})}_{\text{real}}$$

$$|H(e^{j\omega})| = \begin{cases} 1 & \text{for } |\omega| \leq \omega_c \\ 0 & \text{for } \omega_c < |\omega| \leq \pi \end{cases}$$



$$\frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \int_{-\omega_c}^{+\omega_c} e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \left[\frac{1}{jn} e^{j\omega n} \right]_{-\omega_c}^{+\omega_c}$$

$$\frac{1}{2\pi jn} \left(e^{j\omega_c n} - e^{-j\omega_c n} \right)$$

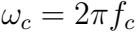


$$\frac{1}{2j} (e^{2j} - e^{-2j})$$



$$\frac{1}{2}(e^{zj} + e^{-zj})$$

$$h(n) = \begin{cases} \frac{1}{\pi n} \sin \omega_c n & \text{for } n \neq 0 \\ \frac{\omega_c}{\pi} & \text{for } n = 0 \end{cases}$$



$$h(n) = \begin{cases} \frac{\omega_c}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 - \frac{\omega_c}{\pi} & \text{for } n = 0 \\ -\frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

2021-2021

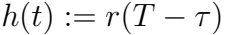
$$h(n) = \begin{cases} \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_2 n) - \sin(\omega_1 n)) & \text{for } n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 - \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_1 n) - \sin(\omega_2 n)) & \text{for } n \neq 0 \end{cases}$$

$$e(t) = \int_0^t \underbrace{s(\tau)}_{\text{signal}} \underbrace{r(\tau)}_{\text{template}} d(\tau)$$

signal template

$$e(t) = \int_0^{\infty} s(\tau) h(t - \tau) d\tau$$







$$\int_0^T s(\tau)r(T-(t-\tau))d\tau$$

$$\int_0^T$$

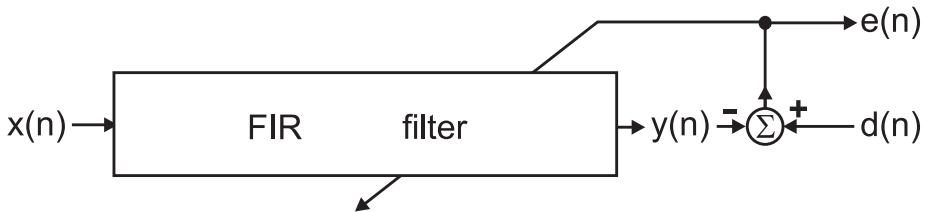
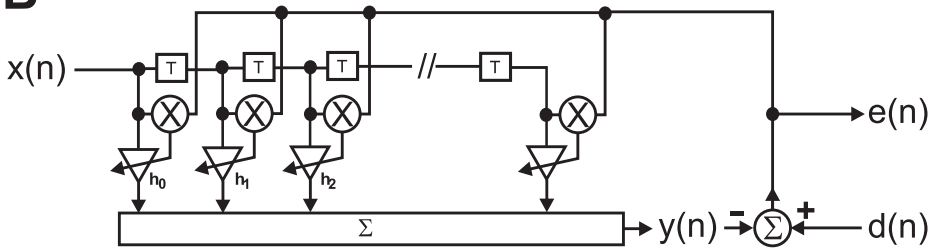
$$s(\tau)r(T-t+\tau)d\tau$$

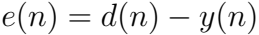


$$E(I) = \int_0^{\infty} S(\tau) r(\tau) d\tau$$

$$\underbrace{h(t) \coloneqq r(T - t)}$$

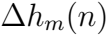
matched filter!

A**B**



1e32

$$\Delta h_m = -\mu \frac{\partial \left(\frac{1}{2} e(n)^2 \right)}{\partial h_m}$$





$$\ln n(n+1) = \ln n(n) + \ln(n+1)$$



es ist ein









$$-\mu \frac{1}{2} \frac{\partial (d(r) - y(r))^2}{\partial h_m}$$

$$-\mu \frac{1}{2} \frac{\partial \left(d(n) - \sum_{m=0}^{M-1} h_m \cdot x(n-m) \right)^2}{\partial h_m}$$

Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, black and white style. The text is split into two lines by a horizontal separator. The first line contains the characters "Wendell" and the second line contains "Wendell".

www.pearsoned.com

disinfectant + 50Hz noise

Wishes in 50Hz *radio*

es ist ein

be a

—

e

—

be

$$H(s) = \frac{1}{s + b}$$

$$h(t) = \sum_{n=0}^{\infty} e^{-bnT} \cdot \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} e^{-bnT} \underbrace{e^{-nsT}}_{z^{-1}^n}$$

12

$$\sum_{n=0}^{\infty} e^{-bnT} z^{-n}$$

$$\sum_{n=0}^{\infty} \left(e^{-bT} z^{-1} \right)^n$$

1



$$1 - e^{-bT} z^{-1}$$

$$H(s) = \frac{1}{s+b} \iff H(z) = \frac{1}{1 - e^{-bT}z^{-1}}$$

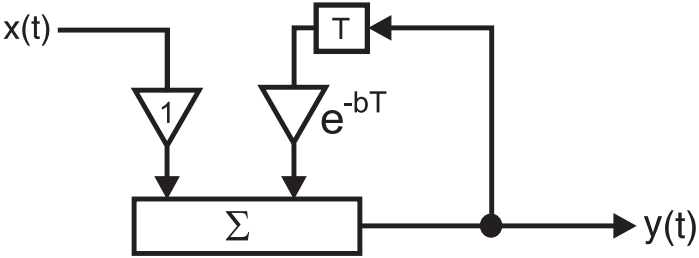
20

—

9001

1995

As 1500 = 1500



1234

W E I R D

$$X(z) = \frac{1}{1 - e^{-bT}z^{-1}}$$

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$



$$2\pi i = 2\pi i - 1 + 1 + 2\pi i$$









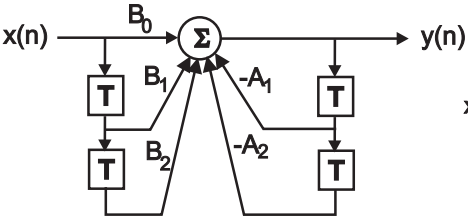


$$H(z) = \frac{\sum_{k=0}^r B_k z^{-k}}{1 + \sum_{l=1}^m A_l z^{-l}}$$

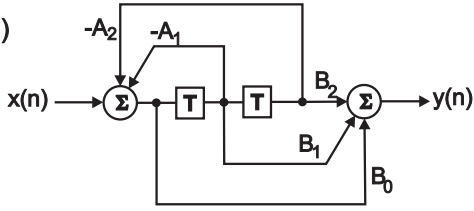


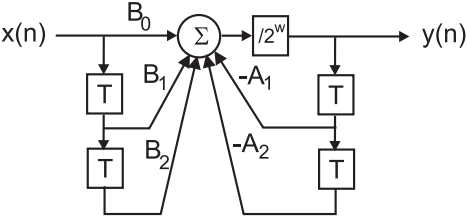


A)



B)



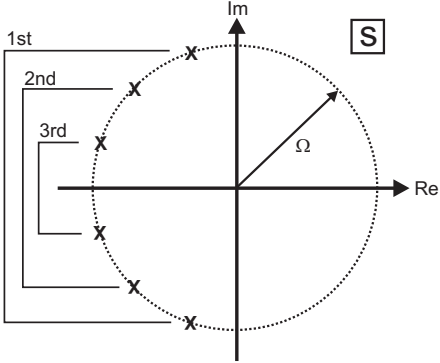






10 + 14 = 24

three 2nd order filters



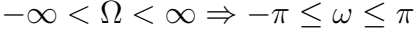
Handwritten text: "HIS" in a stylized, cursive script.

Handwritten cursive text: "The" followed by a comma.

Handwritten cursive text: "The" followed by a comma.

$$|H(\Omega)|^2 = \frac{1}{1 - \epsilon^2 T_N(\Omega/\Omega_p)}$$





$$S = \frac{2}{T} \frac{z - 1}{z + 1}$$



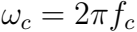


$$j\Omega = \frac{2}{T} \left[\frac{e^{j\omega} - 1}{e^{j\omega} + 1} \right] = \frac{2}{T} j \tan \frac{\omega}{2}$$

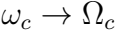


$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

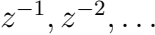
$$\Omega_c = \frac{2}{T} \tan \frac{\omega_c}{2}$$











AB 19 AB 29 AB 39 . . .

$$H(z) = \frac{b}{1 - az^{-1}}$$

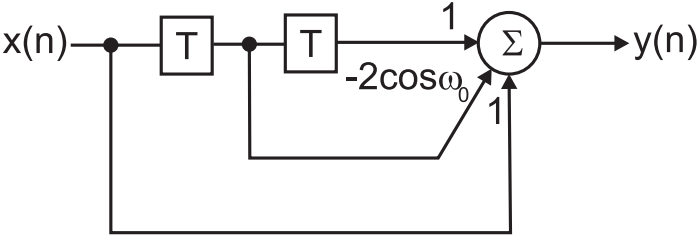


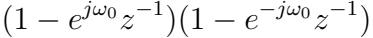
$$|H(e^{j\omega})| = \left| \frac{b}{1 - ae^{-j\omega}} \right|$$

$$\underbrace{y(n)}_{\text{actual estimate}} = a(n) \underbrace{y(n-1)}_{\text{previous estimate}} + b(n) \underbrace{x(n)}_{\text{current data sample}}$$

$$D(x) = E[x] - x$$







1-2010 + 20

$$1 - z^{-1} e^{j\omega} + e^{-j\omega} + z^{-2}$$

12cosw + 2

$$H(e^{j\omega}) = \underbrace{(1 - e^{j\omega_0} e^{-j\omega})(1 - e^{-j\omega_0} e^{-j\omega})}_{2 \text{ zeros}}$$

2 zeros



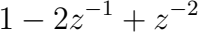


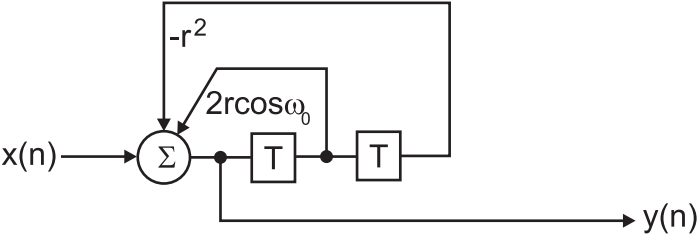






1-2021 1-2021





$$H(z) = \frac{1}{\underbrace{(1 - re^{j\omega_0} z^{-1})(1 - re^{-j\omega_0} z^{-1})}_{2 \text{ poles!}}}$$



$$H(z) = \frac{1}{1 - 2r \cos(\omega_0) z^{-1} + r^2 z^{-2}}$$

1234

$$X(z) \frac{1}{1 - 2r \cos(\omega) z^{-1} + r^2 z^{-2}}$$



$$I_2(r) \cos(\omega z - 1) + I_2(r) z^2 - 2$$

$$X(z) + z^{-1}Y(z) \cos(\omega_0) - z^{-2}Y(z)$$







$$H(z) = \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$$

