



















W2

XO



































University | School of  
of Glasgow | Engineering

10

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10

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Analoge  $\rightarrow$  A/D  $\rightarrow$  Digital processing  $\rightarrow$  D/A  $\rightarrow$  Analoge

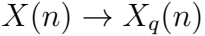
$$\underbrace{X_a(nT)}_{\text{analogue signal}} \equiv \underbrace{x(n)}_{\text{discrete data}}$$

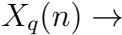














$$X(n) = X_o(n) + X_e(n)$$

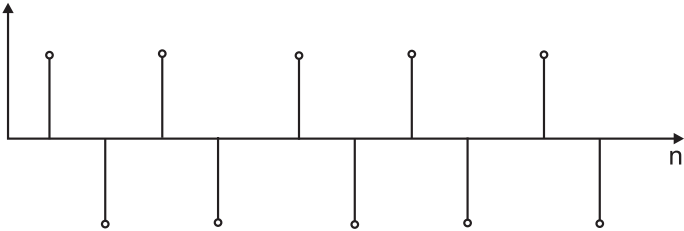
*F* *s*

=

$\frac{1}{T_s}$

$$X_a(t) = X_a(nI) = X_a\left(\frac{n}{F_s}\right)$$

$X(n)$











FOR A GOOD DAY



Handwritten text: *Handwritten*

Normalised frequency:  $f = \frac{F}{F_s}$

*scipy* = 2  $\times$  2

1500 = 1





100% Approved



05



$$x_0(x) = \cos(2\pi x) = 1$$











A pixelated, black and white graphic of the text "E.O. 15". The letters are blocky and have a high-contrast, dithered appearance. The "E" is on the left, followed by a period, then "O", another period, "1", and finally "5" on the right. The overall style is reminiscent of early digital art or a low-resolution scan.



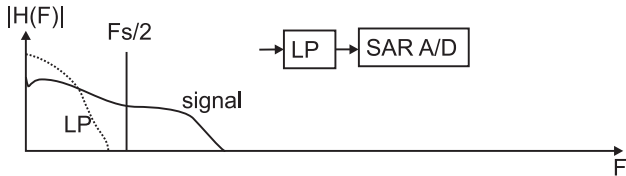
*B*

*<*

$\frac{1}{2}$

*F*<sub>8</sub>

A



B



123456789





$$x_a(t) = \sum_{i=1}^n A_i \cos(2\pi F_i t + \Theta_i)$$



$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}$$

3

5

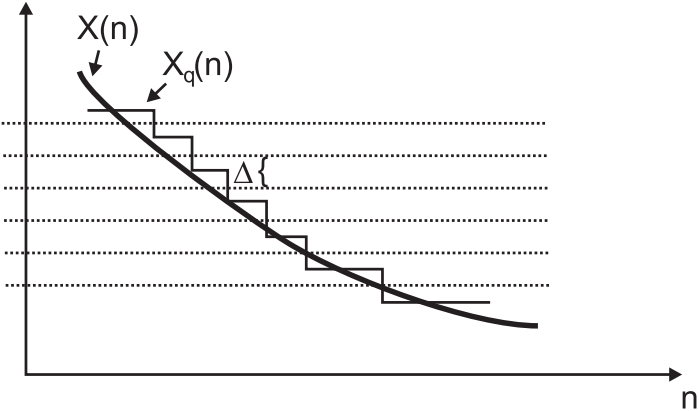
4

7

2

x

$$x_a(t) = \sum_{h=-a}^a x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{h}{F_s}\right)$$



$$\Delta = \text{quantisation step} = \frac{x_{\max} - x_{\min}}{L - 1}$$

1992

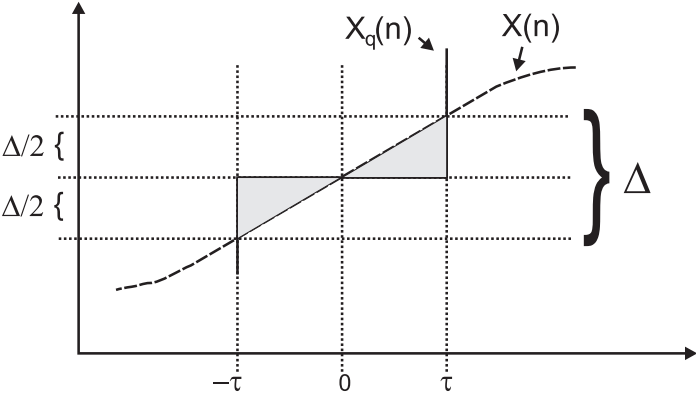
1992



10000



www.english-grammar.com



$$\begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}
 \leq
 e(n)
 \leq
 \begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}$$



$$P_q = \frac{1}{T} \int_0^T e_q^2(t) dt$$







$$\frac{1}{\tau} \int_0^{\tau} \left( \frac{\Delta}{2\tau} \right)^2 t^2 dt$$

$$\frac{\Delta^2}{4\tau^3}$$

$$\int_0^{\tau}$$

$$t^2 dt$$

$$P_q = \frac{\Delta^2}{12\tau^3} = \frac{\Delta^2}{12}$$

$$P_x = \frac{1}{T_p} \int_0^{T_p} (A \cos \Omega t)^2 dt = \frac{A^2}{2}$$

ENVR

$$\frac{P_x}{P_q} = \frac{A^2}{2} \cdot \frac{12}{\Delta^2}$$

6A2



Δ2







$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_1 t}$$















$$c_k = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi k F_1 t} dt$$

$$c_k = c_k^* x_k \quad \text{and} \quad x(t) \text{ is real}$$









$$\cos z = \frac{1}{2} (e^{2i} + e^{-2i})$$





$$c_0 + \sum_{k=1}^{\infty} |c_k| e^{j\theta_k} e^{j2\pi k F_1 t} + \sum_{k=1}^{\infty} |c_k| e^{-j\theta_k} e^{-j2\pi k F_1 t}$$

$$c_0 + 2 \sum_{k=1}^{\infty} |c_k| \cos(2\pi k F_1 t + \theta_k)$$









$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$



$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$



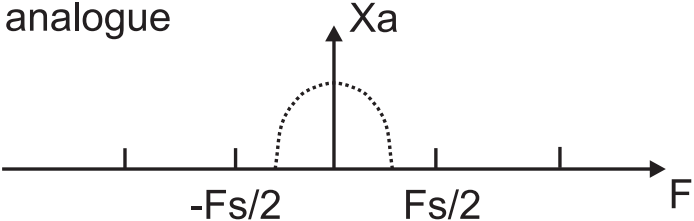
$$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi}$$

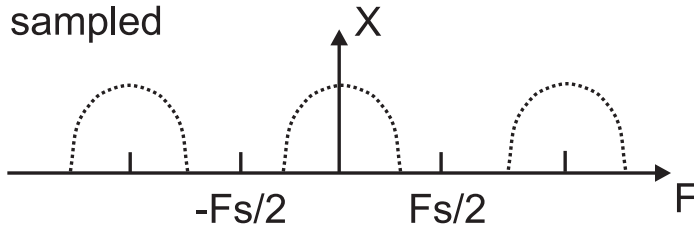
$$\int_{-0.5}^{0.5}$$

$$X(f)e^{j2\pi fn}dt$$

analogue



sampled



W E R

THE UNIVERSITY OF CHICAGO



$$\underbrace{\int_{-0.5}^{0.5} X(f) e^{j2\pi f n} df}_{\text{sampled}} = \int_{-\infty}^{+\infty} \underbrace{X_a(F)}_{\text{cont}} e^{j2\pi n F / F_s} dF$$





$$\frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X\left(\frac{F}{F_s}\right) e^{j2\pi n F/F_s} dF = \int_{-\infty}^{+\infty} X_a(F) e^{j2\pi n F/F_s} dF$$

$$\int_{-\infty}^{+\infty}$$

$$X_a(F)e^{j2\pi n\frac{F}{F_s}}dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s+kF_s}^{+\frac{1}{2}F_s+kF_s} X_a(F) e^{j2\pi n \frac{F}{F_s}} dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} X_a(F - kF_s) \underbrace{e^{j2\pi n \frac{F}{F_s}}}_{kF_s \text{ omit.}} dF$$

$$\underbrace{\int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} \sum_{k=-\infty}^{\infty} X_a(F - kF_s) e^{j2\pi n \frac{F}{F_s}} dF}_{=X(F) \text{ of Eq. 43}}$$



W E R E

$$F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

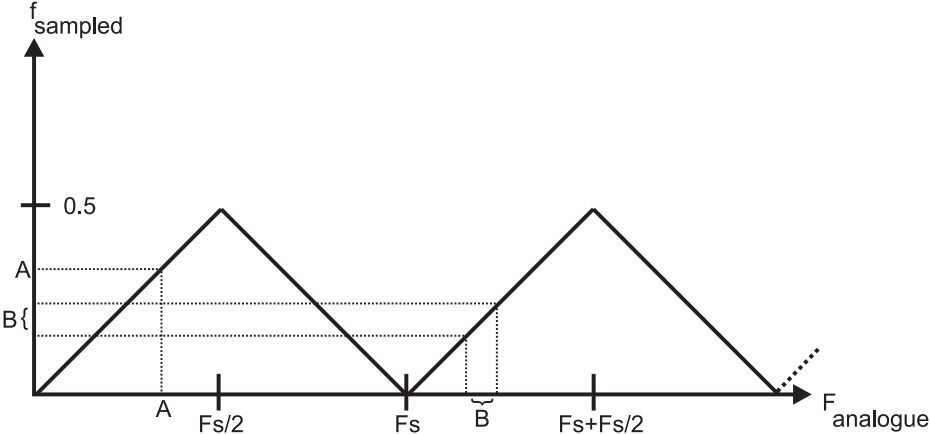


$$F_s \sum_{k=-\infty}^{\infty} X_a[(f-k)F_s]$$

192

12 12

$\frac{1}{2} + 34$   $\frac{1}{2} + 34$





4

and along

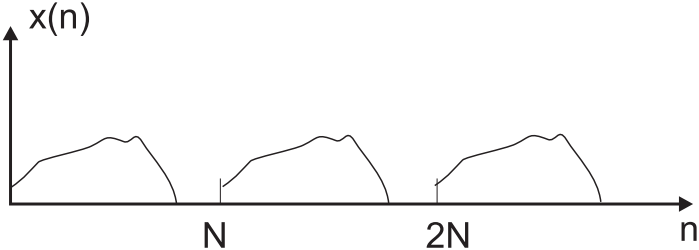
sampled

FOR THE 1929



$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \quad k = 0, 1, 2, \dots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad n = 0, 1, 2, \dots, N-1$$



$$X\left(\frac{2\pi}{N}k\right)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\frac{2\pi}{N}kn}\quad k=0,\ldots,N-1$$



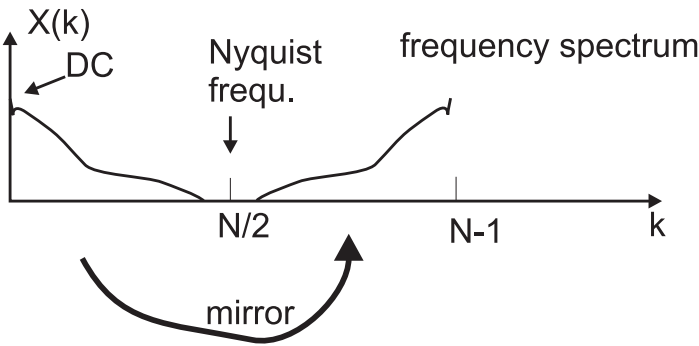
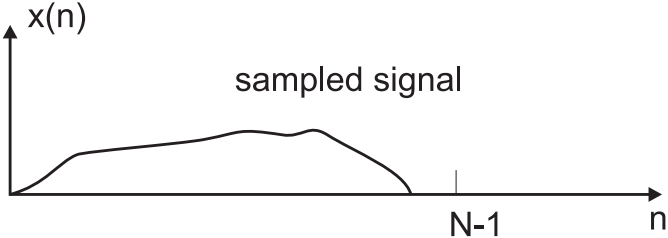
$$X\left(\frac{2\pi}{N}k\right)$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}kn}$$

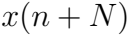
$$\sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n - lN)}_{\text{Periodic repetition!}} e^{-j\frac{2\pi}{N}kn}$$

Periodic repetition!

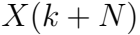


WAVEZ











$$x(n) \text{ is real} \Rightarrow x^*(k) = x(N-k)$$



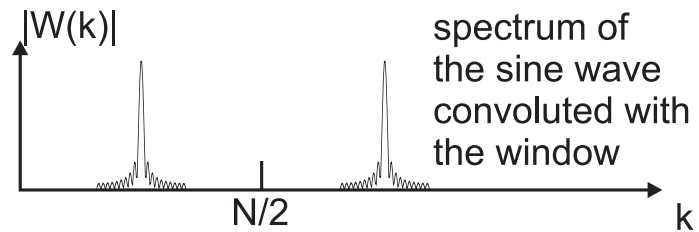
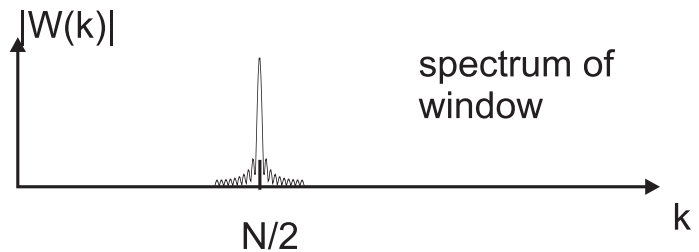
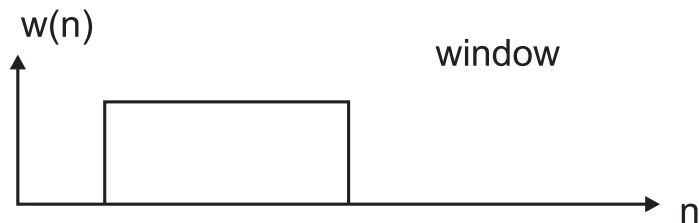
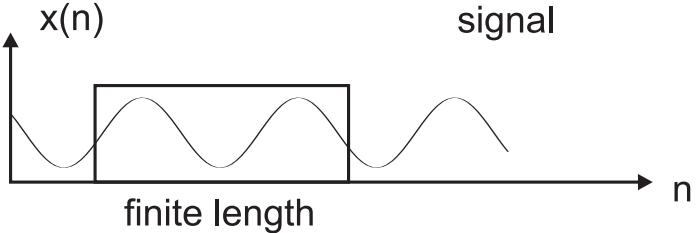




1992年12月21日

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2(m-n)$$





*Wiederholung*









2025-02-27







$$W(\omega) = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

WAVES IN \* WAVES



Wavelength

Wew

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

A pixelated, black and white graphic of the text "WIN-2020". The letters are blocky and have a dithered appearance. The "W" and "N" are larger and more prominent, while the "I" is smaller. The hyphen and the numbers "2020" are also pixelated. The overall style is reminiscent of early digital art or video game titles.





2020

$x^2 + 1$

$$m = 0, \dots, \frac{N}{2} - 1$$

W2mk  
N



Wmk  
N2

$$\sum_{m=0}^{N/2-1} x(2m)W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)W_N^{k(2m+1)}$$

$$\sum_{m=0}^{N/2-1} x(2m) W_{N/2}^{mk} + W_N^k \sum_{m=0}^{N/2-1} x(2m+1) W_{N/2}^{mk}$$

BEWAARDE  
+ WERKEN  
BESCHERME









$$2 \cdot \sin \left( \frac{\pi}{2} \right) = \frac{\pi}{2}$$

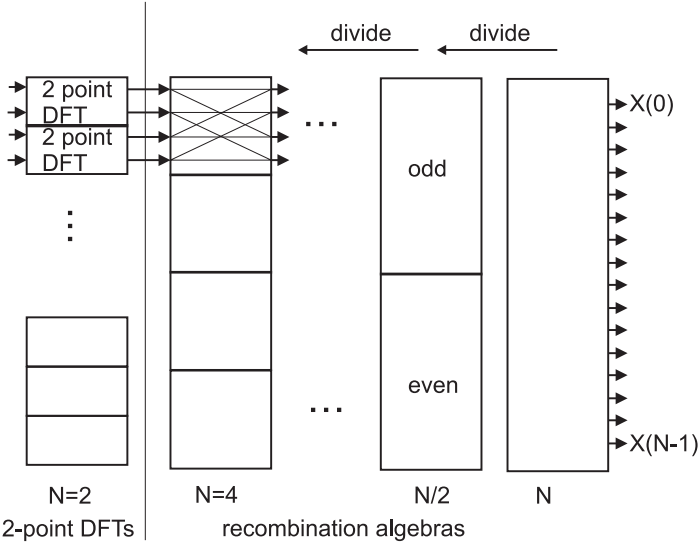
BEAD

BEWE

Beethoven, Beethoven,

100%





$$X_0(k) = X_{\text{re}}(k) + jX_{\text{im}}(k)$$





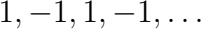
DOG XNO

$$x(0) + \underbrace{W_2^0}_{1} x(1) = x(0) + x(1)$$

visist freqv. x1

$$x(0) + \underbrace{W_2^{-1}}_{-1} x(1) = x(0) - x(1)$$

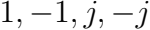


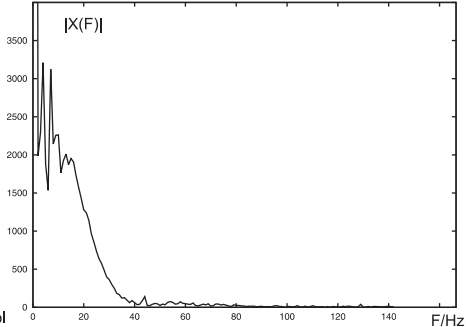
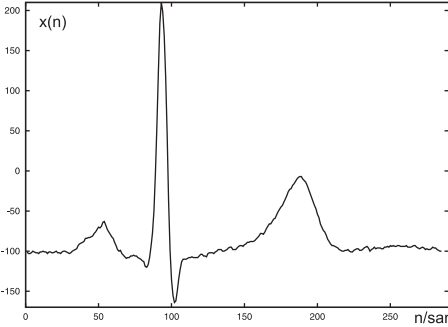


W

1002

W

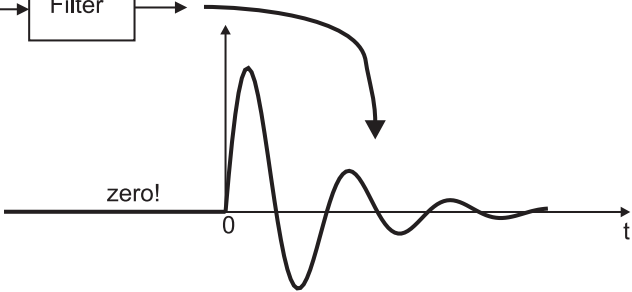


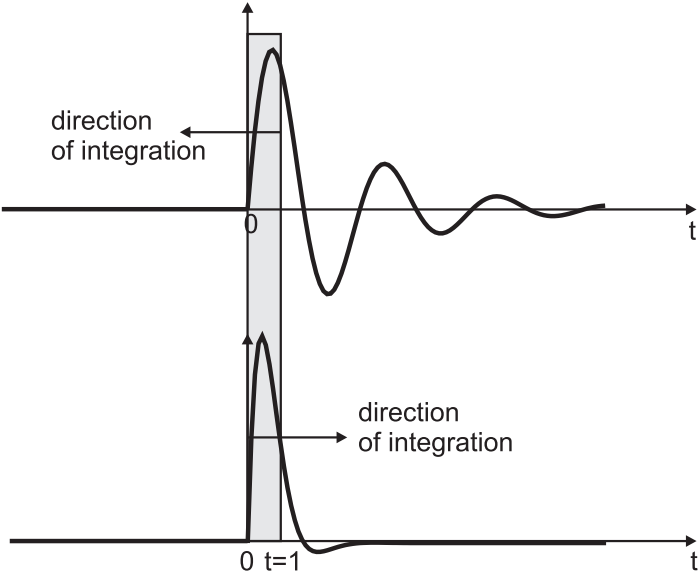




2100X1000

delta  
pulse









$$h(t) * x(t) = \int_{-\infty}^{\infty} h(t - \tau) x(\tau) d\tau$$



$$h(n) * x(n) = \sum_{n=-\infty}^{\infty} h(n)x(n-n)$$



500

900







$$y(t) = \int_{-\infty}^{\infty} h(t-\tau) \delta(\tau) d\tau = h(t)$$



$$\mathcal{L}\{h(t)\} = H(s) = \int_0^{\infty} h(t) e^{-st} dt$$

$$\int f(\tau) d\tau \Leftrightarrow \frac{1}{s} F(s)$$

$$\frac{d}{dt} f(t) \Leftrightarrow sF(s)$$

for the first time

1995



$$\int_0^{\infty} \underbrace{h(t-T)}_{\text{causal}} e^{-st} dt$$

$$\int_0^{\infty}$$

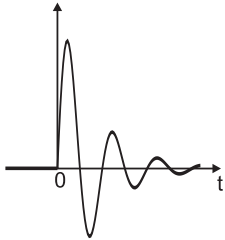
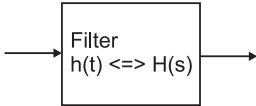
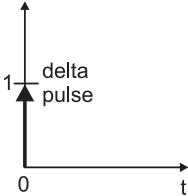
$$h(t)e^{-s(t+T)}dt$$

$$\int_0^{\infty} h(t) e^{-st} e^{-sT} dt$$

$$e^{-sT} \underbrace{\int_0^{\infty} h(t) e^{-st} dt}_{H(s)}$$

CHISEL

1845



$$d(t) = h(t) * x(t) = B(s) \cdot X(s)$$



desiderata pro



sp/ise impulse response





How do we

The image displays the word "AIRBORNE" in a highly pixelated, black-and-white font. The letters are thick and blocky, with a jagged, digital appearance. The 'A' is particularly prominent, featuring a wide base and a sharp, pointed top. The 'I' is a simple vertical bar. The 'B' has a rounded, almost circular shape. The 'O' is a simple circle. The 'R' has a curved tail. The 'N' is a simple vertical bar with a horizontal crossbar. The 'E' is a simple horizontal bar with a vertical crossbar. The 'A' is a simple vertical bar with a horizontal crossbar. The 'I' is a simple vertical bar. The 'B' has a rounded, almost circular shape. The 'O' is a simple circle. The 'R' has a curved tail. The 'N' is a simple vertical bar with a horizontal crossbar. The 'E' is a simple horizontal bar with a vertical crossbar. The overall style is reminiscent of early computer graphics or video game titles.

$$\tau_w = \frac{d\phi(w)}{dw}$$





$$x(t) = \sum_{n=0}^{\infty} x(n) \delta(t - nT) \quad \text{Sampled signal}$$

19

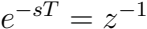
$$\sum_{n=0}^{\infty} x(n) e^{-snT}$$

$$\int_0^{\infty}$$

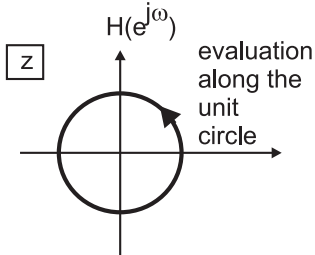
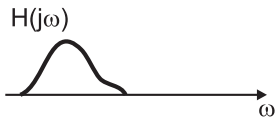
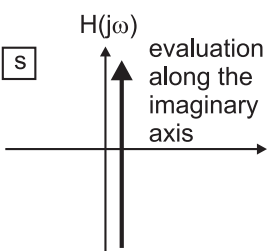
$$x(n)e^{-st}dt$$

$$\sum_{n=0}^{\infty} x(n) \underbrace{\left( e^{-sT} \right)^n}_{z^{-1} = e^{-sT}}$$

$$\sum_{n=0}^{\infty} x(n) (z^{-1})^n \qquad \text{z-transform}$$





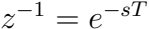


19











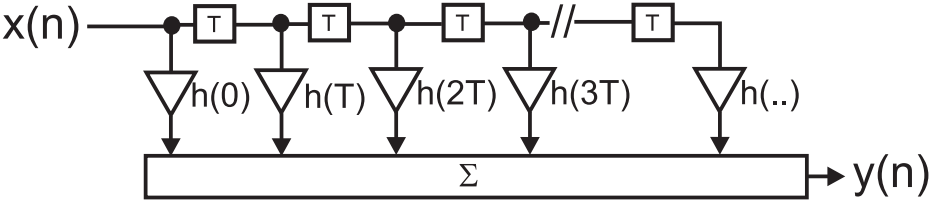




How easy?

HERNANDEZ

airborne



$$h(t) = \sum_{n=0}^{\infty} h(nT) \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} h(nT) \underbrace{\left( e^{-sT} \right)^n}_{z^{-1}}$$







$$H(z)=\sum_{n=0}^{\infty}h(nT)(z^{-1})^n$$



12

$$H(z)X(z) = \underbrace{\sum_{n=0}^{\infty} h(nT)z^{-n}}_{H(z)} X(z)$$

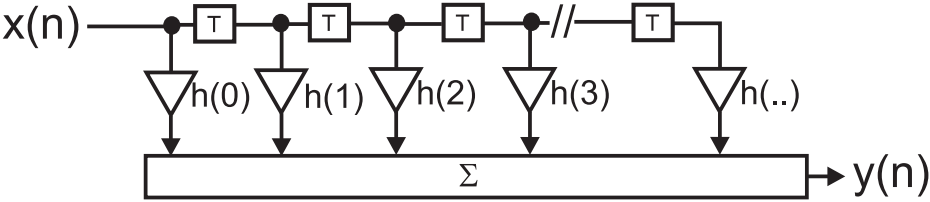
1925





$$H(z)X(z) = \underbrace{\sum_{m=0}^M h(mT)z^{-m}}_{H(z)} X(z)$$







123456789

$$\sum_{m=0}^M h_{\text{analogue}}(mT) z^{-n} X(z)$$

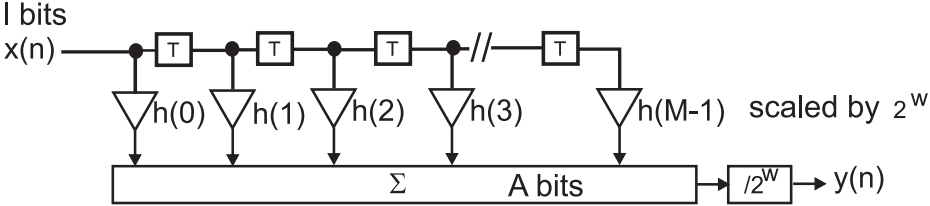
$$\sum_{m=0}^M h_{\text{digital}}(m) z^{-n} X(z)$$

$$\sum_{m=0}^M h(m) z^{-m} X(z)$$



$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m)$$









2W

,

W

=

15





1002

11

4-1-1992









$$\tau_w = \frac{d\phi(w)}{dw}$$



$$A(w) = B(w) - w + w$$

EW

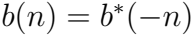




$$h(n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(e^{i\omega}) e^{i\omega n} d\omega$$

$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b(n)$$

A pixelated, black and white graphic of the number 30. The digits are rendered in a thick, blocky, and slightly irregular font, characteristic of early digital art or low-resolution computer graphics. The number 3 is on the left, and the number 0 is on the right. The entire image is composed of a grid of black and white pixels, giving it a dithered or aliased appearance.



$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b^*(-n)$$

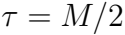


$$h(x + \pi) = e^{2i\phi} h(-x + \pi)$$









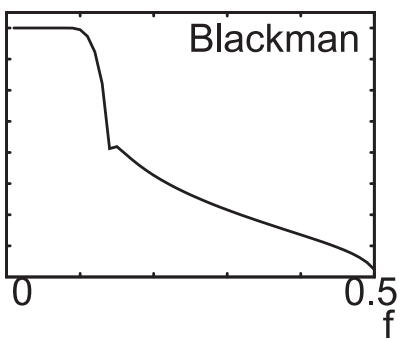
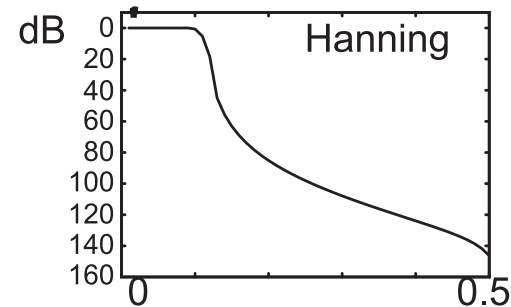
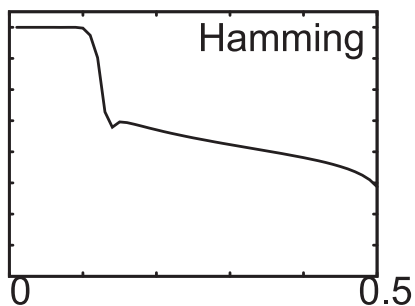
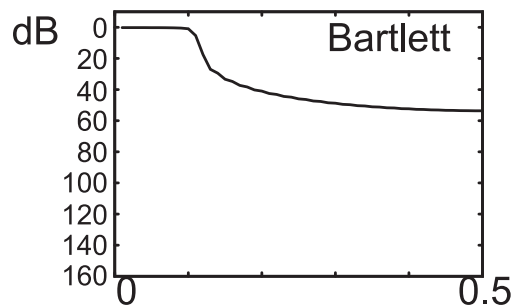
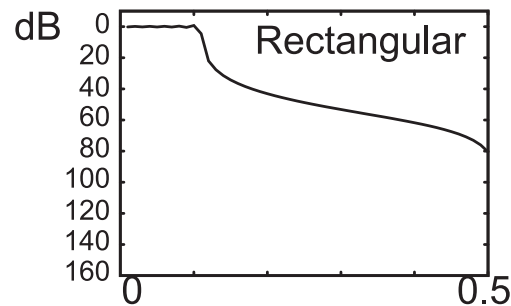




$$h(n+1)/2) = (1)h(n+1)/2)$$



$$|H(f)|$$



12





$$H(z)X(z) = \sum_{n=0}^N \underbrace{h(nT)v(nT)} z^{-n} X(z)$$

$$w(n) = a - (1 - a) \cos\left(\frac{2\pi n}{M}\right)$$





$$v(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{N}\right) + 0.08 \cos\left(\frac{4\pi n}{N}\right)$$









W2E50

00000





$$|H(e^{j\omega})| = \underbrace{B(e^{j\omega})}_{\text{real}}$$

$$|H(e^{j\omega})| = \begin{cases} 1 & \text{for } |\omega| \leq \omega_c \\ 0 & \text{for } \omega_c < |\omega| \leq \pi \end{cases}$$





$$\frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \int_{-\omega_c}^{+\omega_c} e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \left[ \frac{1}{jn} e^{j\omega n} \right]_{-\omega_c}^{+\omega_c}$$

$$\frac{1}{2\pi jn} \left( e^{j\omega_c n} - e^{-j\omega_c n} \right)$$



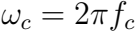
$$\frac{1}{2j} (e^{2j} - e^{-2j})$$



$$\frac{1}{2}(e^{zj} + e^{-zj})$$



$$h(n) = \begin{cases} \frac{1}{\pi n} \sin \omega_c n & \text{for } n \neq 0 \\ \frac{\omega_c}{\pi} & \text{for } n = 0 \end{cases}$$



$$h(n) = \begin{cases} \frac{\omega_c}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 - \frac{\omega_c}{\pi} & \text{for } n = 0 \\ -\frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

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$$h(n) = \begin{cases} \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_2 n) - \sin(\omega_1 n)) & \text{for } n \neq 0 \end{cases}$$

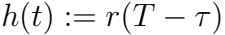
$$h(n) = \begin{cases} 1 - \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_1 n) - \sin(\omega_2 n)) & \text{for } n \neq 0 \end{cases}$$

$$e(t) = \int_0^t \underbrace{s(\tau)}_{\text{signal}} \underbrace{r(\tau)}_{\text{template}} d(\tau)$$

signal template



$$e(t) = \int_0^{\infty} s(\tau) h(t - \tau) d\tau$$








$$\int_0^T s(\tau)r(T-(t-\tau))d\tau$$

$$\int_0^T s(\tau)r(T-t+\tau)d\tau$$



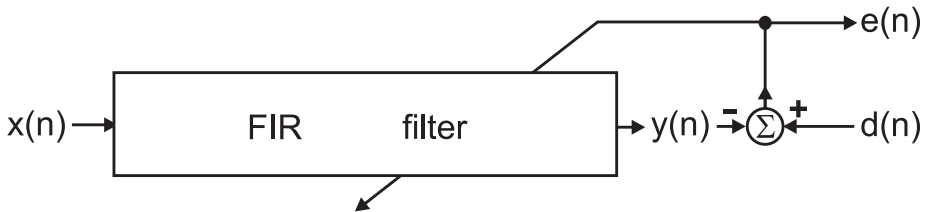
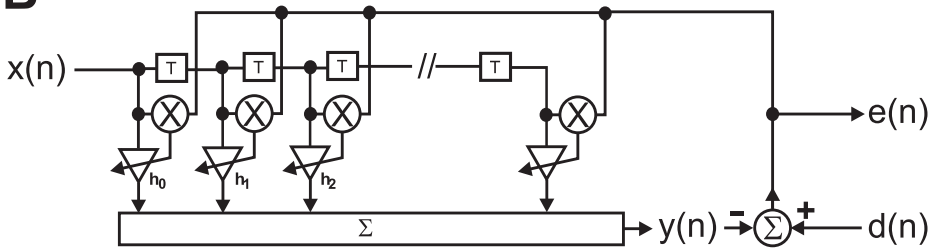
$$E(I) = \int_0^{\infty} S(\tau) r(\tau) d\tau$$

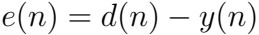


$$h(t) \coloneqq r(T - t)$$


*matched*

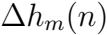
*filter!*

**A****B**



1e32

$$\Delta h_m = -\mu \frac{\partial \left( \frac{1}{2} e(n)^2 \right)}{\partial h_m}$$





$$\ln n(n+1) = \ln n(n) + \ln(n+1)$$





espresso











$$-\mu \frac{1}{2} \frac{\partial (d(r) - y(r))^2}{\partial h_m}$$



$$-\mu \frac{1}{2} \frac{\partial \left( d(n) - \sum_{m=0}^{M-1} h_m \cdot x(n-m) \right)^2}{\partial h_m}$$

Handwritten text in a cursive script, likely a signature or a name, rendered in a pixelated, black and white style. The text is written on a white background and consists of several connected, flowing strokes.

www.pearsoned.com

*discrete + 50Hz noise*

*Wishes in* 50Hz *radio*

es ist ein

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=

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be

$$H(s) = \frac{1}{s + b}$$



$$h(t) = \sum_{n=0}^{\infty} e^{-bnT} \cdot \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} e^{-bnT} \underbrace{e^{-nsT}}_{z^{-1}{}^n}$$

12

$$\sum_{n=0}^{\infty} e^{-bnT} z^{-n}$$

$$\sum_{n=0}^{\infty} \left( e^{-bT} z^{-1} \right)^n$$

1



$$1 - e^{-bT} z^{-1}$$

$$H(s) = \frac{1}{s + b} \iff H(z) = \frac{1}{1 - e^{-bT} z^{-1}}$$

20

—

9001



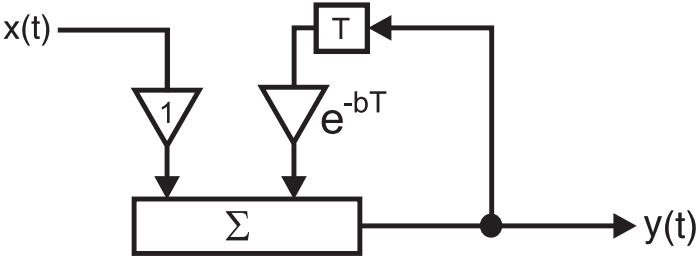


A pixelated, black and white graphic featuring the letters 'H' and 'A' in a bold, blocky font. To the right of these letters is a large, stylized, pixelated question mark. The entire graphic is composed of a grid of black and white pixels, giving it a retro, digital appearance.

A pixelated, black and white graphic of a stylized letter 'S' or '9'. The character is rendered in a thick, irregular, hand-drawn style with a prominent, jagged border. The interior of the character is solid black, while the border is composed of various shades of gray, giving it a stencil-like or distressed appearance. The overall shape is a large, open 'S' or '9' that curves around the center of the image.

A pixelated black and white illustration. On the right side, there is a large, stylized number '9' composed of many small squares, giving it a digital or retro aesthetic. On the left side, there are two horizontal bars, one above the other, also composed of small squares. The top bar is slightly longer than the bottom one. The overall image has a high-contrast, low-resolution appearance.

As 1500 = 1500





W E I R D

$$X(z) = \frac{1}{1 - e^{-bT}z^{-1}}$$

$$\frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right)$$





$$2\pi i = 2\pi i - 1 + 1 + 2\pi i$$











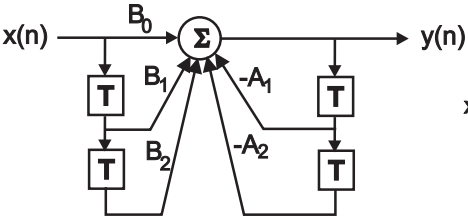
$$H(z) = \frac{\sum_{k=0}^r B_k z^{-k}}{1 + \sum_{l=1}^m A_l z^{-l}}$$



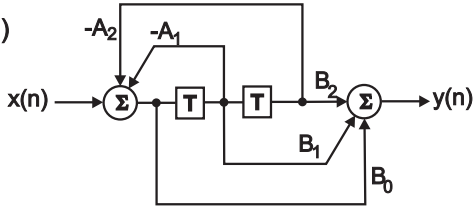


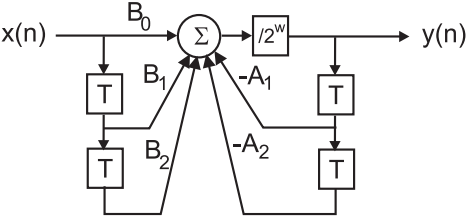


A)



B)



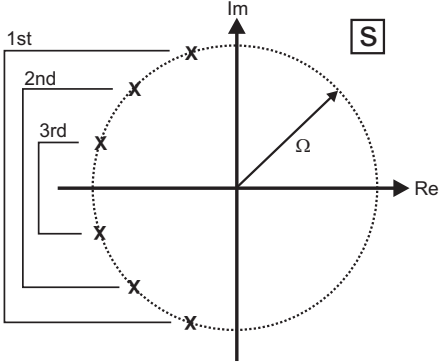






10 + 14 = 24

three 2nd order filters





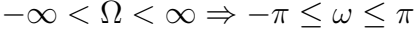
Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, black and white style. The text is split into two parts by a vertical line, with the left part reading "Handwritten" and the right part reading "Text".

Handwritten cursive text: "The" followed by a comma.

Handwritten cursive text: "The" followed by a comma.

$$|H(\Omega)|^2 = \frac{1}{1 - \epsilon^2 T_N(\Omega/\Omega_p)}$$





$$S = \frac{2}{T} \frac{z - 1}{z + 1}$$





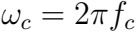


$$j\Omega = \frac{2}{T} \left[ \frac{e^{j\omega} - 1}{e^{j\omega} + 1} \right] = \frac{2}{T} j \tan \frac{\omega}{2}$$

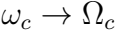


$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

$$\Omega_c = \frac{2}{T} \tan \frac{\omega_c}{2}$$

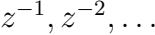




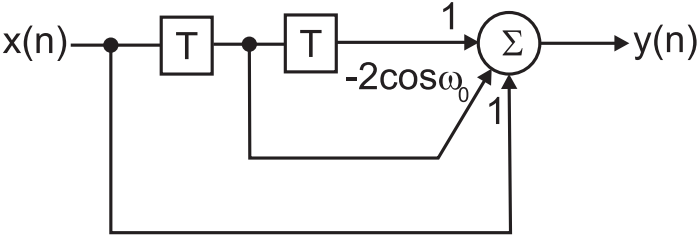








AB 19 AB 29 AB 39 . .



1-2021-2021

1-2010 + 20

$$1 - z^{-1} e^{j\omega} + e^{-j\omega} + z^{-2}$$

12cosw + 2

$$H(e^{j\omega}) = \underbrace{(1 - e^{j\omega_0} e^{-j\omega})(1 - e^{-j\omega_0} e^{-j\omega})}_{2 \text{ zeros}}$$

2 zeros





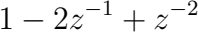


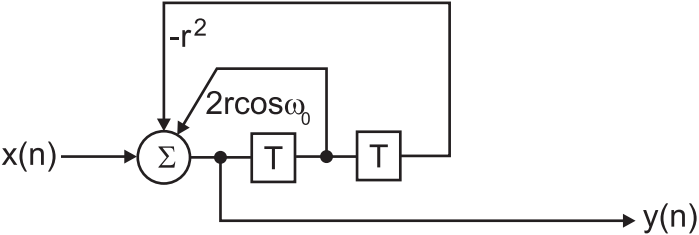






1-2021 1-2021







$$H(z) = \frac{1}{\underbrace{(1 - re^{j\omega_0} z^{-1})(1 - re^{-j\omega_0} z^{-1})}_{2 \text{ poles!}}}$$



$$H(z) = \frac{1}{1 - 2r \cos(\omega_0) z^{-1} + r^2 z^{-2}}$$

1234

$$X(z) \frac{1}{1 - 2r \cos(\omega) z^{-1} + r^2 z^{-2}}$$



$$I_1(z) I_2(z) \cos(\omega z - 1) + I_1(z) I_2(z - 2)$$

$$x(z) + z^{-1}y(z) \cos(\omega_0) - z^{-2}y(z)r^2$$









$$H(z) = \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$$



$$H(z) = \frac{b}{1 - az^{-1}}$$



$$|H(e^{j\omega})| = \left| \frac{b}{1 - ae^{-j\omega}} \right|$$



$$\underbrace{y(n)}_{\text{actual estimate}} = a(n) \underbrace{y(n-1)}_{\text{previous estimate}} + b(n) \underbrace{x(n)}_{\text{current data sample}}$$

$$D(x) = E[x] - x$$

