

















W2

XO































University | School of
of Glasgow | Engineering

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10

10

10

10

10

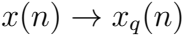


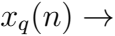
$$\underbrace{x_a(nT)}_{\text{analogue signal}} \equiv \underbrace{x(n)}_{\text{discrete data}}$$















$$x(n) = x(n)I, \quad x(n) = x(n) + x(n)$$

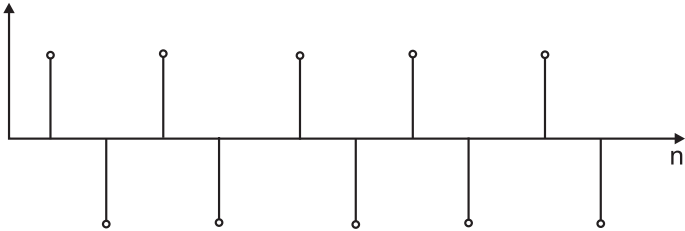
F *s*

=

$\frac{1}{T_s}$

$$x_a(t) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$x(n)$









WORLDWIDE
= ACOPIA



www.A2PI.com

Normalised frequency: $f = \frac{F}{F_s}$

scipy = 2 \times 2

1500 = 1



www.100days.org



05



$$x_0(x) = \cos(2\pi x) = 1$$









A pixelated, black and white graphic of the text "E.O. 15". The letters are blocky and have a high-contrast, dithered appearance. The "E" is on the left, followed by a period, then "O", another period, "1", and finally "5" on the right. The overall style is reminiscent of early digital art or a low-resolution scan.



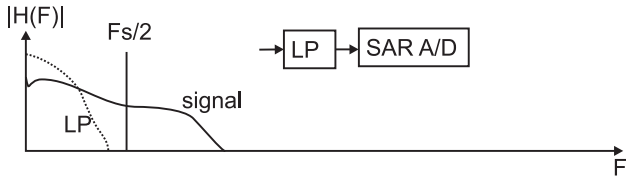
B

<

$\frac{1}{2}$

*F*₈

A



B



2024-2025



$$x_a(t) = \sum_{i=1}^n A_i \cos(2\pi F_i t + \Theta_i)$$



$$g(t) = \frac{\sin 2\pi Bt}{2\pi Bt}$$

3

5

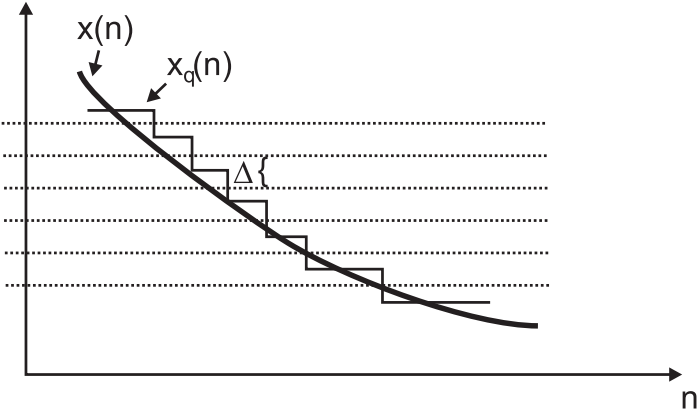
4

7

2

$$x_a(t) = \sum_{h=-a}^a x_a\left(\frac{n}{F_s}\right) g\left(t - \frac{h}{F_s}\right)$$





$$\Delta = \text{quantisation step} = \frac{x_{\max} - x_{\min}}{L - 1}$$

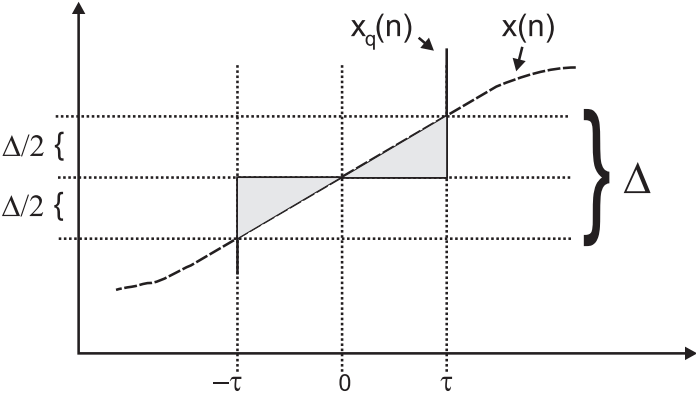
1992

1992

10000



Wiederholung



$$\begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}
 \leq e(n) \leq
 \begin{array}{c}
 \triangle \\
 \hline
 2
 \end{array}$$



$$P_q = \frac{1}{T} \int_0^T e_q^2(t) dt$$





$$\frac{1}{\tau} \int_0^{\tau} \left(\frac{\Delta}{2\tau} \right)^2 t^2 dt$$

$$\frac{\Delta^2}{4\tau^3}$$

$$\int_0^{\tau}$$

$$t^2 dt$$

$$P_q = \frac{\Delta^2}{12\tau^3} = \frac{\Delta^2}{12}$$



$$P_x = \frac{1}{T_p} \int_0^{T_p} (A \cos \Omega t)^2 dt = \frac{A^2}{2}$$

ENVR

$$\frac{P_x}{P_q} = \frac{A^2}{2} \cdot \frac{12}{\Delta^2}$$

6A2



Δ2





$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_1 t}$$













$$c_k = \frac{1}{T_p} \int_{T_p} x(t) e^{-j2\pi k F_1 t} dt$$

$$c_k = c_k^* x_k \quad \text{and} \quad x(t) \text{ is real}$$





CPA



CPA





$$\cos z = \frac{1}{2} (e^{zi} + e^{-zi})$$



$$c_0 + \sum_{k=1}^{\infty} |c_k| e^{j\theta_k} e^{j2\pi k F_1 t} + \sum_{k=1}^{\infty} |c_k| e^{-j\theta_k} e^{-j2\pi k F_1 t}$$

$$c_0 + 2 \sum_{k=1}^{\infty} |c_k| \cos(2\pi k F_1 t + \theta_k)$$









$$X(F) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi Ft} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(F) e^{j2\pi Ft} dF$$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$



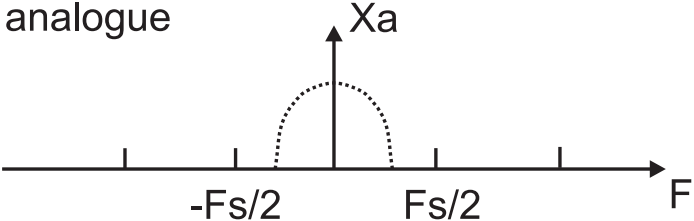
$$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(\omega) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi}$$

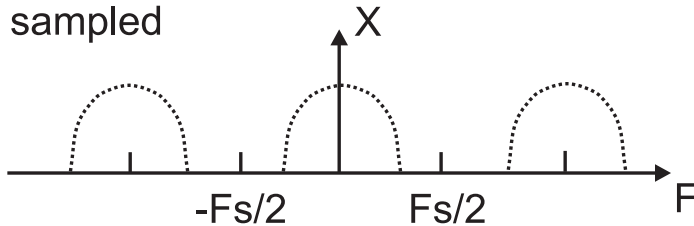
$$\int_{-0.5}^{0.5}$$

$$X(f)e^{j2\pi fn}df$$

analogue



sampled



W E R

THE UNIVERSITY OF CHICAGO

$$\underbrace{\int_{-0.5}^{0.5} X(f) e^{j2\pi f n} df}_{\text{sampled}} = \int_{-\infty}^{+\infty} \underbrace{X_a(F)}_{\text{cont}} e^{j2\pi n F / F_s} dF$$





$$\frac{1}{F_s} \int_{-F_s/2}^{F_s/2} X\left(\frac{F}{F_s}\right) e^{j2\pi n F/F_s} dF = \int_{-\infty}^{+\infty} X_a(F) e^{j2\pi n F/F_s} dF$$

$$\int_{-\infty}^{+\infty}$$

$$X_a(F)e^{j2\pi n\frac{F}{F_s}}dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s+kF_s}^{+\frac{1}{2}F_s+kF_s} X_a(F) e^{j2\pi n \frac{F}{F_s}} dF$$

$$\sum_{k=-\infty}^{\infty} \int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} X_a(F - kF_s) \underbrace{e^{j2\pi n \frac{F}{F_s}}}_{kF_s \text{ omit.}} dF$$

$$\underbrace{\int_{-\frac{1}{2}F_s}^{+\frac{1}{2}F_s} \sum_{k=-\infty}^{\infty} X_a(F - kF_s) e^{j2\pi n \frac{F}{F_s}} dF}_{=X(F) \text{ of Eq. 42}}$$

1992

$$F_s \sum_{k=-\infty}^{\infty} X_a(F - kF_s)$$

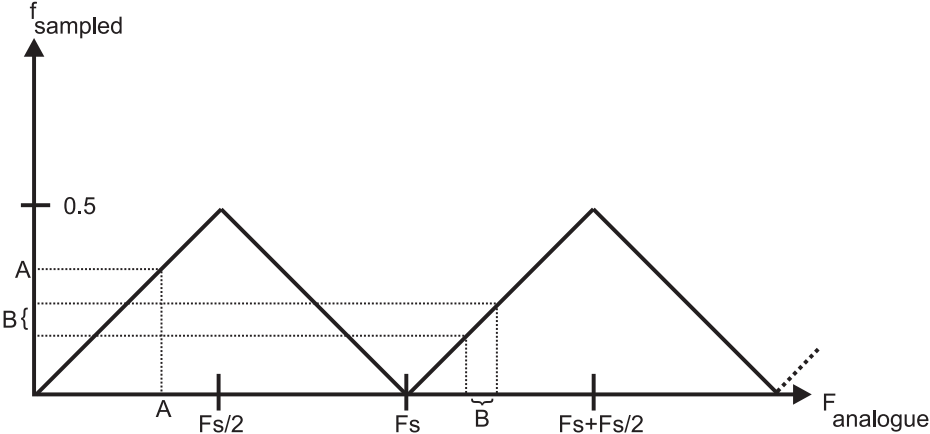


$$F_s \sum_{k=-\infty}^{\infty} X_a[(f-k)F_s]$$

192

12 12

$\frac{1}{2} + 31$ $\frac{1}{2} + 31$





andalogic

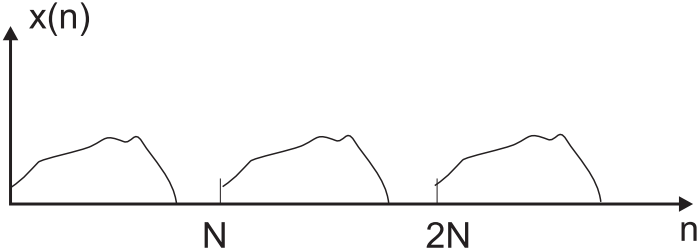
sampled

FOR THE 1929



$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N} \quad k = 0, 1, 2, \dots, N-1$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j2\pi kn/N} \quad n = 0, 1, 2, \dots, N-1$$



$$X\left(\frac{2\pi}{N}k\right)=\sum_{n=-\infty}^{\infty}x(n)e^{-j\frac{2\pi}{N}kn}\quad k=0,\ldots,N-1$$

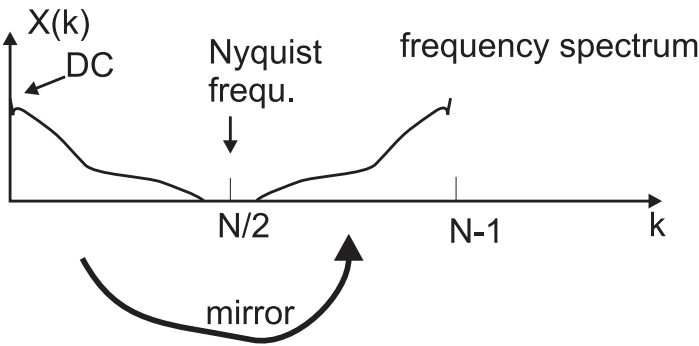
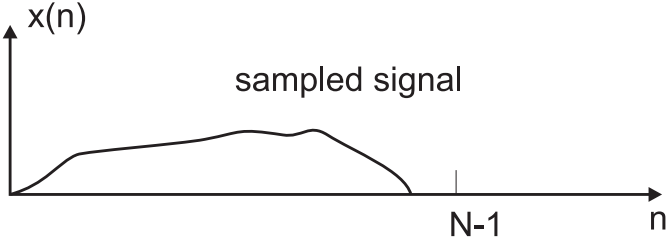
$$X\left(\frac{2\pi}{N}k\right)$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j \frac{2\pi}{N} kn}$$

$$\sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n-lN) e^{-j\frac{2\pi}{N}kn}$$

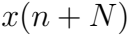
$$\sum_{n=0}^{N-1} \underbrace{\sum_{l=-\infty}^{\infty} x(n - lN)}_{\text{Periodic repetition!}} e^{-j\frac{2\pi}{N}kn}$$

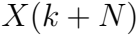
Periodic repetition!



WAVE 20









$$x(n) \text{ is real} \Rightarrow x^*(k) = x(N-k)$$

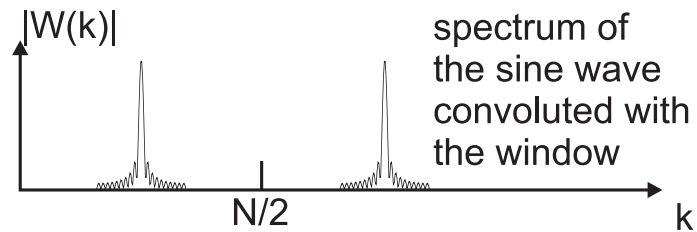
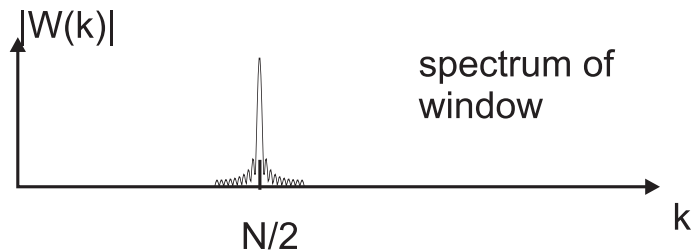
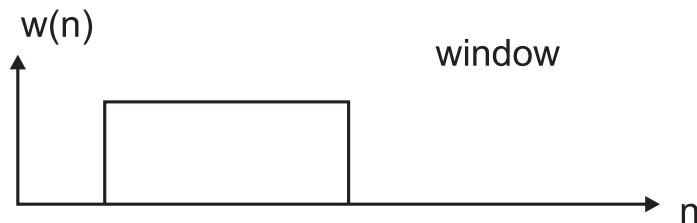
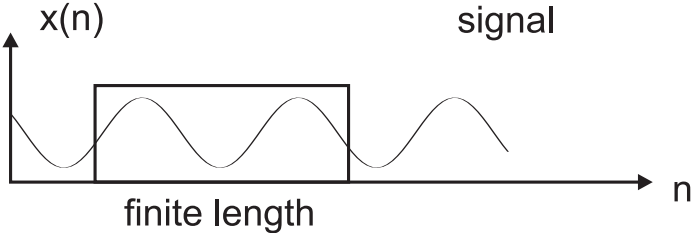




X 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

1992年12月21日

$$x_3(m) = \sum_{n=0}^{N-1} x_1(n)x_2(m-n)$$



Wiederholung









2023-03-23





$$W(\omega) = \frac{\sin(\omega L/2)}{\sin(\omega/2)} e^{-j\omega(L-1)/2}$$

WAVES IN * WAVES



Wavelength

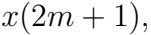
Wew

$$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

WAVELENGTH



2020



$$m = 0, \dots, \frac{N}{2} - 1$$

W2mk
N



W2mk
N2

$$\sum_{m=0}^{N/2-1} x(2m)W_N^{2mk} + \sum_{m=0}^{N/2-1} x(2m+1)W_N^{k(2m+1)}$$

$$\sum_{m=0}^{N/2-1} x(2m) W_{N/2}^{mk} + W_N^k \sum_{m=0}^{N/2-1} x(2m+1) W_{N/2}^{mk}$$

BEWAAR + WENIG





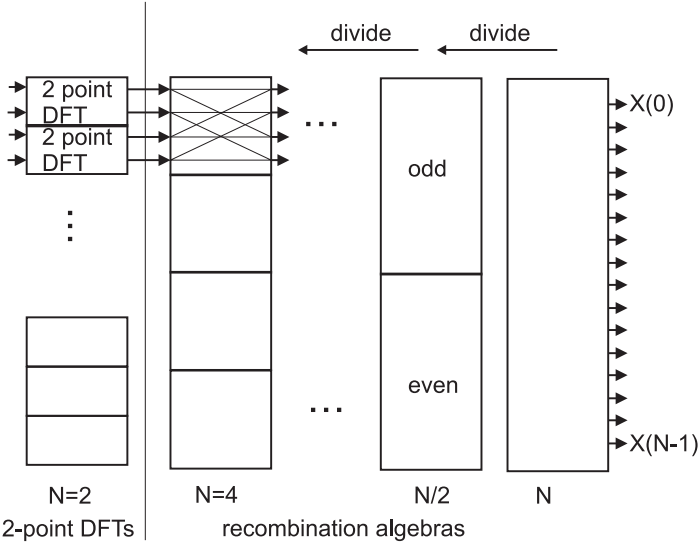


$$2 \cdot \sin \left(\frac{\pi}{2} \right) = \pi$$

BE

BEWE

Beethoven, Beethoven,



$$X_1(k) = X_2(k) + IV_1(k)X_3(k)$$



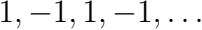


DOG XNO

$$x(0) + \underbrace{W_2^0}_{1} x(1) = x(0) + x(1)$$

visist freqv. x1

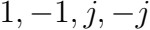
$$x(0) + \underbrace{W_2^{-1}}_{-1} x(1) = x(0) - x(1)$$

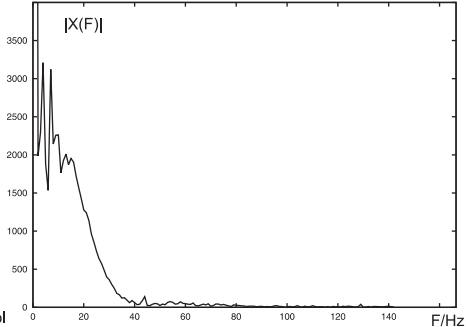
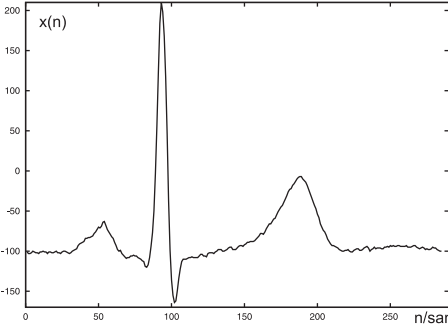


W

1002

W

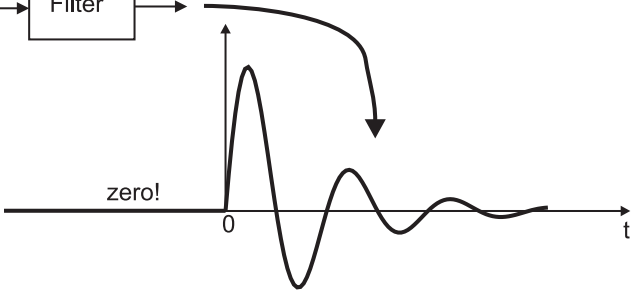


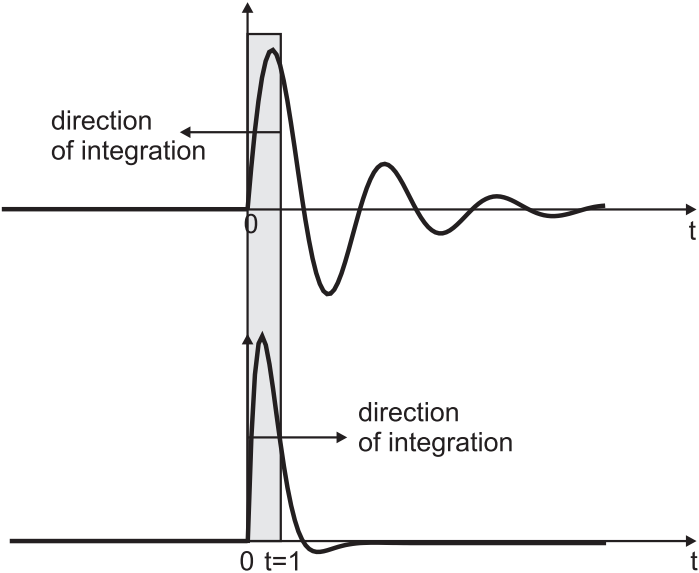




219 X 14

delta
pulse







$$h(t) * x(t) = \int_{-\infty}^{\infty} h(t - \tau) x(\tau) d\tau$$



$$h(n) * x(n) = \sum_{n=-\infty}^{\infty} h(n)x(n-n)$$



500

900





$$y(t) = \int_{-\infty}^{\infty} h(t - \tau) \delta(\tau) d\tau = h(t)$$



$$\mathcal{L}\{h(t)\} = H(s) = \int_0^{\infty} h(t) e^{-st} dt$$

$$\int f(\tau) d\tau \Leftrightarrow \frac{1}{s} F(s)$$

$$\frac{d}{dt} f(t) \Leftrightarrow sF(s)$$

for the first time

1995

$$\int_0^{\infty} \underbrace{h(t-T)}_{\text{causal}} e^{-st} dt$$

$$\int_0^{\infty}$$

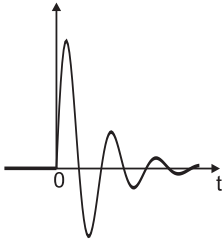
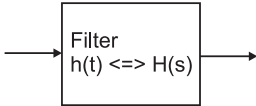
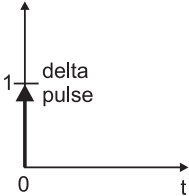
$$h(t)e^{-s(t+T)}dt$$

$$\int_0^{\infty} h(t) e^{-st} e^{-sT} dt$$

$$e^{-sT} \underbrace{\int_0^{\infty} h(t) e^{-st} dt}_{H(s)}$$

CHISEL

1845



$$d(t) = h(t) * x(t) = B(s) \cdot X(s)$$

desiderata pro



sp/ise impulse response





How do we

[illegible]

$$\tau_w = - \frac{d\phi(w)}{dw}$$



$$x(t) = \sum_{n=0}^{\infty} x(n) \delta(t - nT) \quad \text{Sampled signal}$$



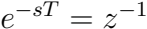
$$\sum_{n=0}^{\infty} x(n) e^{-snT}$$

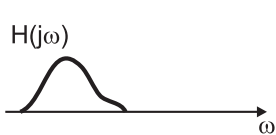
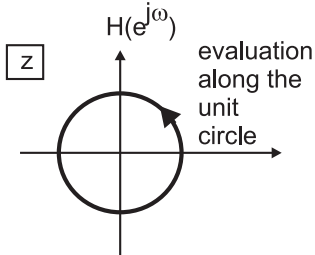
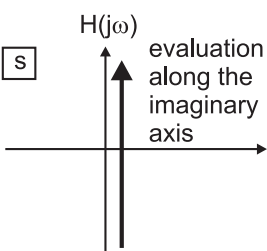
$$\int_0^{\infty}$$

$$x(n)e^{-st}dt$$

$$\sum_{n=0}^{\infty} x(n) \underbrace{\left(e^{-sT} \right)^n}_{z^{-1} = e^{-sT}}$$

$$\sum_{n=0}^{\infty} x(n) (z^{-1})^n \qquad \text{z-transform}$$



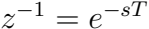


19









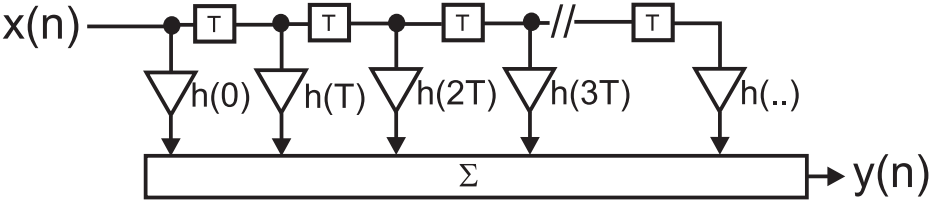




How easy?

REVIEW

airborne



$$h(t) = \sum_{n=0}^{\infty} h(nT) \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} h(nT) \underbrace{\left(e^{-sT} \right)^n}_{z^{-1}}$$





$$H(z) = \sum_{n=0}^{\infty} h(nT)(z^{-1})^n$$



12

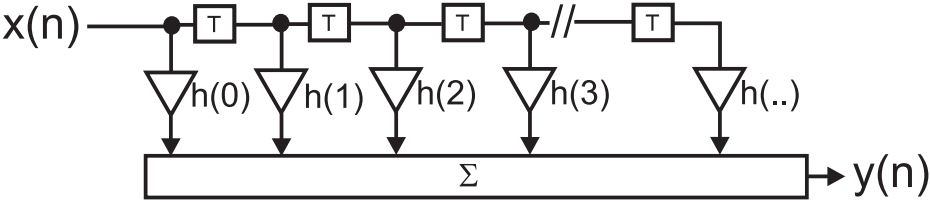
$$H(z)X(z) = \underbrace{\sum_{n=0}^{\infty} h(nT)z^{-n}}_{H(z)} X(z)$$

1925





$$H(z)X(z) = \underbrace{\sum_{m=0}^M h(mT)z^{-m}}_{H(z)} X(z)$$





123456789

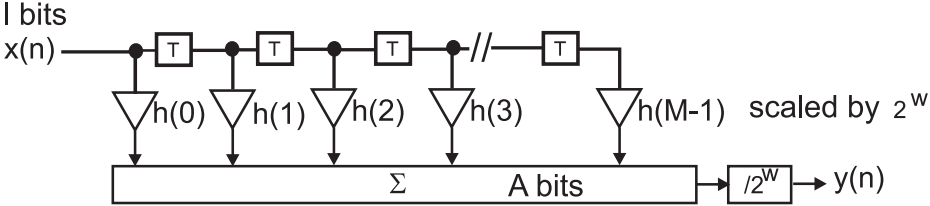
$$\sum_{m=0}^M h_{\text{analogue}}(mT) z^{-n} X(z)$$

$$\sum_{m=0}^M h_{\text{digital}}(m) z^{-n} X(z)$$

$$\sum_{m=0}^M h(m) z^{-m} X(z)$$



$$y(n) = \sum_{m=0}^{M-1} h(m)x(n-m)$$







2W

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W

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15





1002

111

4-1-1992







$$\tau_w = - \frac{d\phi(w)}{dw}$$



$$A(w) = B(w) - w + w$$

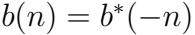
EW



$$h(n) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(e^{i\omega}) e^{i\omega n} d\omega$$

$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b(n)$$

A pixelated, black and white graphic of the number 30. The digits are rendered in a thick, blocky, and slightly irregular font, characteristic of early digital art or low-resolution computer graphics. The number 3 is on the left, and the number 0 is on the right. The entire image is composed of a grid of black and white pixels, giving it a dithered or aliased appearance.



$$h(n+\tau)=\frac{1}{2\pi}e^{i\phi}b^*(-n)$$



$$n(x + \pi) = e^{2i\phi} n^* (-x + \pi)$$





1 = 100%

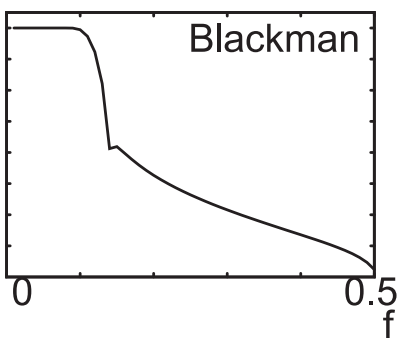
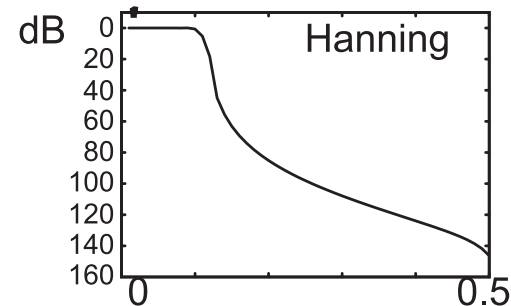
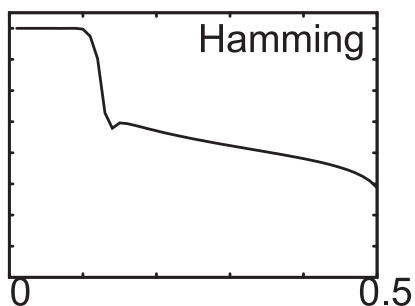
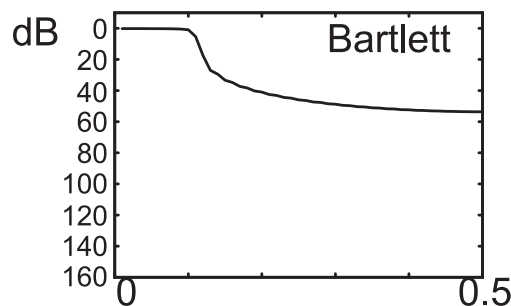
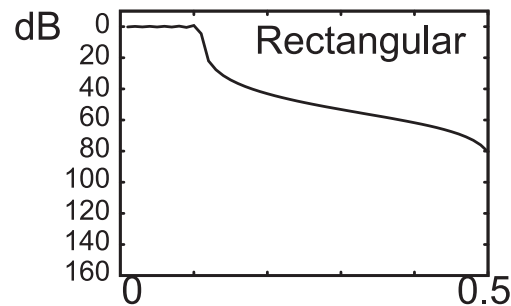




$$h(n+1) \equiv (h(n) + 1) \bmod 2$$



$$|H(f)|$$



12



$$H(z)X(z) = \sum_{n=0}^N \underbrace{h(nT)w(nT)} z^{-n} X(z)$$

$$w(n) = a - (1 - a) \cos\left(\frac{2\pi n}{M}\right)$$





$$v(n) = 0.42 + 0.5 \cos\left(\frac{2\pi n}{N}\right) + 0.08 \cos\left(\frac{4\pi n}{N}\right)$$







W2E50

00000





$$|H(e^{j\omega})| = \underbrace{B(e^{j\omega})}_{\text{real}}$$

$$|H(e^{j\omega})| = \begin{cases} 1 & \text{for } |\omega| \leq \omega_c \\ 0 & \text{for } \omega_c < |\omega| \leq \pi \end{cases}$$



$$\frac{1}{2\pi} \int_{-\pi}^{\pi} H(e^{j\omega}) e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \int_{-\omega_c}^{+\omega_c} e^{j\omega n} d\omega$$

$$\frac{1}{2\pi} \left[\frac{1}{jn} e^{j\omega n} \right]_{-\omega_c}^{+\omega_c}$$

$$\frac{1}{2\pi jn} \left(e^{j\omega_c n} - e^{-j\omega_c n} \right)$$

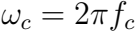


$$\frac{1}{2j} (e^{2j} - e^{-2j})$$



$$\frac{1}{2}(e^{zj} + e^{-zj})$$

$$h(n) = \begin{cases} \frac{1}{\pi n} \sin \omega_c n & \text{for } n \neq 0 \\ \frac{\omega_c}{\pi} & \text{for } n = 0 \end{cases}$$



$$h(n) = \begin{cases} \omega_c & \text{for } n = 0 \\ \frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 - \frac{\omega_c}{\pi} & \text{for } n = 0 \\ -\frac{1}{\pi n} \sin(\omega_c n) & \text{for } n \neq 0 \end{cases}$$

2021-2021

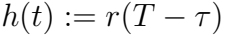
$$h(n) = \begin{cases} \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_2 n) - \sin(\omega_1 n)) & \text{for } n \neq 0 \end{cases}$$

$$h(n) = \begin{cases} 1 - \frac{\omega_2 - \omega_1}{\pi} & \text{for } n = 0 \\ \frac{1}{\pi n} (\sin(\omega_1 n) - \sin(\omega_2 n)) & \text{for } n \neq 0 \end{cases}$$

$$e(t) = \int_0^t \underbrace{s(\tau)}_{\text{signal}} \underbrace{r(\tau)}_{\text{template}} d(\tau)$$

signal template

$$e(t) = \int_0^{\infty} s(\tau) h(t - \tau) d\tau$$








$$\int_0^T s(\tau)r(T-(t-\tau))d\tau$$

$$\int_0^T s(\tau)r(T-t+\tau)d\tau$$

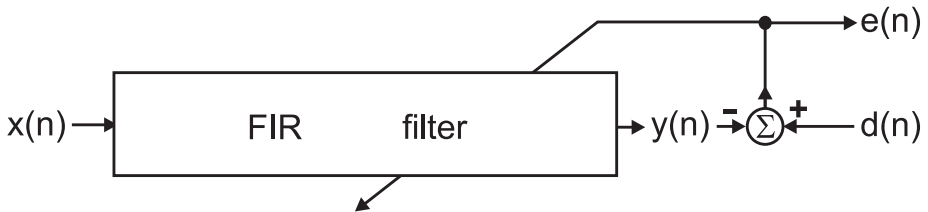
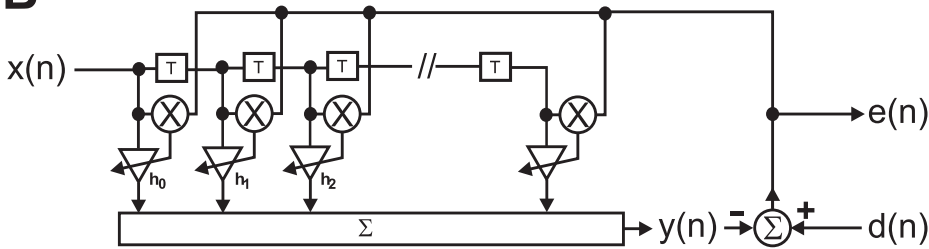


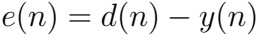
$$E(I) = \int_0^{\infty} S(\tau) r(\tau) d\tau$$

$$h(t) \coloneqq r(T - t)$$


matched

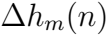
filter!

A**B**



1e32

$$\Delta h_m = -\mu \frac{\partial \left(\frac{1}{2} e(n)^2 \right)}{\partial h_m}$$





$$\begin{aligned}
 & \ln \ln(x+1) = \ln \ln(x) + \frac{1}{x} \\
 & \ln \ln(x) = \ln \ln(x) + \frac{1}{x}
 \end{aligned}$$



es ist ein









$$-\mu \frac{1}{2} \frac{\partial (d(r) - y(r))^2}{\partial h_m}$$

$$-\mu \frac{1}{2} \frac{\partial \left(d(n) - \sum_{m=0}^{M-1} h_m \cdot x(n-m) \right)^2}{\partial h_m}$$

Handwritten text in a cursive script, likely a signature or name, rendered in a pixelated, black and white format. The text is split into two lines by a horizontal separator. The first line contains the characters "Wendell" and the second line contains "Wendell".

www.pearsoned.com

discrete + 50Hz noise

Wishes in

es ist ein

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be

$$H(s) = \frac{1}{s + b}$$

$$h(t) = \sum_{n=0}^{\infty} e^{-bnT} \cdot \delta(t - nT)$$

$$H(s) = \sum_{n=0}^{\infty} e^{-bnT} \underbrace{e^{-nsT}}_{z^{-1}^n}$$

12

$$\sum_{n=0}^{\infty} e^{-bnT} z^{-n}$$

$$\sum_{n=0}^{\infty} \left(e^{-bT} z^{-1} \right)^n$$

1



$$1 - e^{-bT} z^{-1}$$

$$H(s) = \frac{1}{s + b} \iff H(z) = \frac{1}{1 - e^{-bT} z^{-1}}$$

20

—

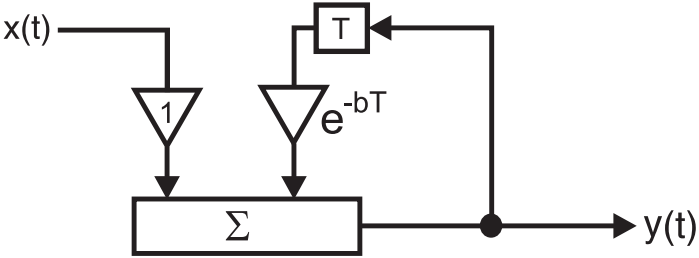
9001

A pixelated, black and white graphic. On the left, the letters 'H' and 'A' are rendered in a blocky, digital font. To the right of these letters is a long, curved line that starts high and ends low, resembling a stylized comma or the number '9'. The entire image has a low-resolution, pixelated aesthetic.

A large, pixelated, black and white graphic of the number 9, resembling a stylized, blocky font. The number is composed of many small squares, giving it a digital or retro aesthetic. It is centered on a white background.

A pixelated black and white illustration. On the right side, there is a large, stylized number '9' composed of many small squares, giving it a digital or retro aesthetic. On the left side, there are two horizontal bars, each made of a series of dark gray squares, resembling a stylized '99' or a digital display. The entire image is set against a plain white background.

As 1500 = 1500





W E I R D

$$X(z) = \frac{1}{1 - e^{-bT}z^{-1}}$$

$$\frac{1}{2} \left(\frac{1}{2} + \frac{1}{2} \right)$$



$$2\pi i = 2\pi i - 1 + 1 + 2\pi i$$









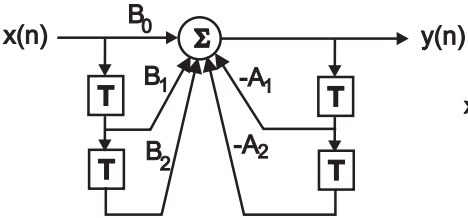


$$H(z) = \frac{\sum_{k=0}^r B_k z^{-k}}{1 + \sum_{l=1}^m A_l z^{-l}}$$

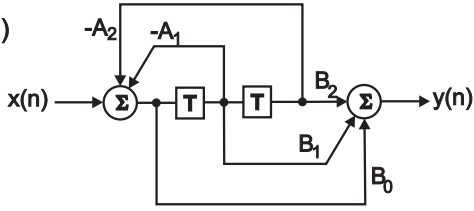


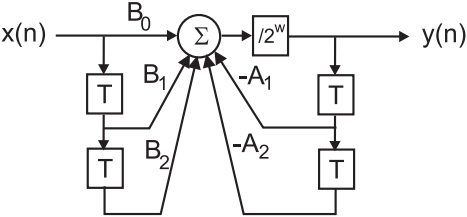


A)



B)



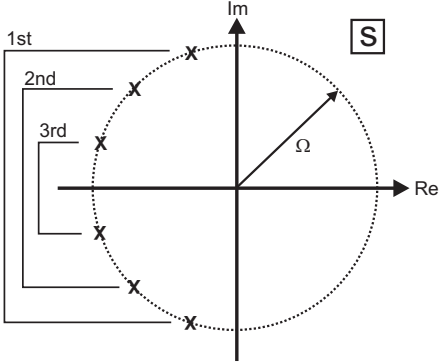






10 + 14 = 24

three 2nd order filters



Handwritten text in a stylized, cursive script, likely a signature or decorative element. The text is rendered in black ink on a white background, featuring thick, dark strokes and a highly decorative, flowing style. The characters are highly stylized and difficult to decipher, but appear to be a mix of letters and flourishes.

Handwritten text in a stylized, cursive script, likely a signature or decorative element. The text is rendered in black ink on a white background, featuring thick, dark strokes and a highly decorative, flowing style. The characters are highly stylized and difficult to decipher, but appear to be a mix of letters and flourishes.

$$|H(\Omega)|^2 = \frac{1}{1 - \epsilon^2 T_N(\Omega/\Omega_p)}$$



$$S = \frac{2}{T} \frac{z - 1}{z + 1}$$



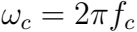


$$j\Omega = \frac{2}{T} \left[\frac{e^{j\omega} - 1}{e^{j\omega} + 1} \right] = \frac{2}{T} j \tan \frac{\omega}{2}$$

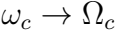


$$\Omega = \frac{2}{T} \tan \frac{\omega}{2}$$

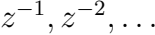
$$\Omega_c = \frac{2}{T} \tan \frac{\omega_c}{2}$$











AB 19 AB 29 AB 39 . . .

$$H(z) = \frac{b}{1 - az^{-1}}$$

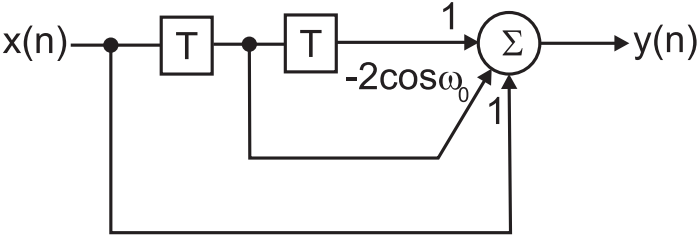


$$|H(e^{j\omega})| = \left| \frac{b}{1 - ae^{-j\omega}} \right|$$

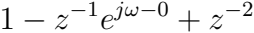
$$\underbrace{y(n)}_{\text{actual estimate}} = a(n) \underbrace{y(n-1)}_{\text{previous estimate}} + b(n) \underbrace{x(n)}_{\text{current data sample}}$$

$$D(x) = E[x] - x$$





1-2021



$$1 - z^{-1} e^{j\omega} + e^{-j\omega} + z^{-2}$$

12cosw + 2

$$H(e^{j\omega}) = \underbrace{(1 - e^{j\omega_0} e^{-j\omega})(1 - e^{-j\omega_0} e^{-j\omega})}_{2 \text{ zeros}}$$

2 zeros



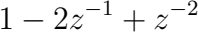


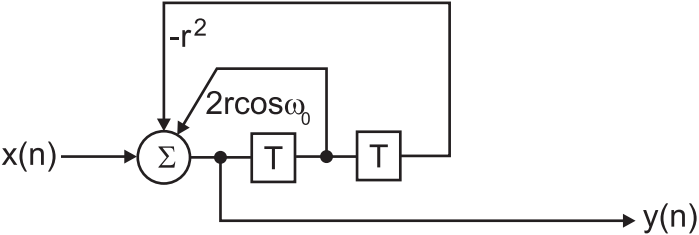






1-2021 1-2021





$$H(z) = \frac{1}{\underbrace{(1 - re^{j\omega_0} z^{-1})(1 - re^{-j\omega_0} z^{-1})}_{2 \text{ poles!}}}$$



$$H(z) = \frac{1}{1 - 2r \cos(\omega_0) z^{-1} + r^2 z^{-2}}$$



$$X(z) \frac{1}{1 - 2r \cos(\omega) z^{-1} + r^2 z^{-2}}$$



$$I_1(z) I_2(z) \cos(\omega z) - I_1(z) I_2(z) - 1$$

$$x(z) + z^{-1}y(z) \cos(\omega_0) - z^{-2}y(z)r^2$$







$$H(z) = \frac{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$$

