



Eidgenössische Technische Hochschule Zürich  
Swiss Federal Institute of Technology Zurich

Lecture with Computer Exercises:  
Modelling and Simulating Social Systems with MATLAB

**The impact of pedestrians on roundabouts entry**  
...

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Zurich  
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The impact of pedestrians on  
randabout's entry

Verfasst von (in Druckschrift):

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## Contents

## 1 Abstract

We are interested in measuring the pedestrians impact on the entry flow. Indeed, they influence the traffic on roundabout in a direct way, since the cars intending to merge into the circulation flow have to leave the priority to pedestrians on the crosswalk; and in an indirect manner since pedestrians crossing the streets at exits of the roundabout keep vehicles of the circulating flow from making their way out of the roundabout. We look at how the crosswalks location at the roundabout entrance play a role in the capacity entry taking into account different influencing factors and traffic's situations over a normal working day. We use a microscopic approach, where gap acceptance behavior is used to reproduce conflict with pedestrians. The estimated entry capacity results and limitations approach are interpreted.

## 2 Individual contributions

Several researches have been led lately in particular by japanese and germans researchers such that Wui, Kang Nan or Siegloch. Our project is inspired by their advanced works, where we've been trying to understand and summarize them. We derived a simplified theoretical model with simplified formulas and different parameters adapted to our framework and our objectives, which still lead to the coherent results found in the original papers. Indeed, while the storage space between the yield line and the crosswalk was considered as a fix constant we made it a variable and measure its impact on the entry capacity. We also made a MATLAB code reproducing a changing traffic and pedestrian flow over a typical working day.

## 3 Introduction and Motivations

In this project we consider fix small type roundabout 4 single lanes with crosswalks at each entrances which are closed to shopping center area or other attractive places. Those types are pretty common in Switzerland in small and middle towns. We noticed that the crosswalks are located most of the time right at the entrance, which in our opinions had a bad impact on the traffic flow and hence on the entry capacity of the roundabout. The reason for that is always claimed to be the pedestrian's safety. Indeed, vehicles willing to merge into the circulating flow on the roundabout have to slow down and then putting the crosswalk almost at the yield line make the pedestrians crossing the street where the vehicles are at theirs slowest speed and then optimizing safety by limiting fatal crash accident. It seems to us that putting the crosswalks further down the road would improve the traffic flow but decrease the pedestrian's safety since cars are still pretty fast and that, in practice, they tend to

cross randomly the road at the roundabout entrance instead of walking down the road to use the crosswalk. In consequence, there is clearly a trade off between the entry capacity of roundabouts and the pedestrian's safety and as a consequence we can't consider setting up the crosswalks too far from the roundabouts of Switzerland in the case described above to optimize the traffic. In other terms, the question we want to give an answer to is the following :

Is a reasonably small increase of the storage space between the yield line and the crosswalk has a significant positive impact on the entry capacity ?

## 4 Description of the Model

**Gap acceptance theory**( Kang, N., H. Nakamura, and M. Asano. An Empirical Analysis on Critical Gap and Follow-Up Time at Roundabout Considering Geometry Effect)

Our model is based on the gap acceptance theory, which is nothing but the expression of the priority rules applying to unsignalized intersections and in particular to roundabouts (which is a particular case of intersection). It is based on the concept of minor flows trying to merge into or cross the major flows. The minor flow is yielding the priority to minor flow. The considered roundabout is exactly of this type : the minor flow is represented by the cars willing to enter the roundabout, giving way first at pedestrians crossing the street who are hence a major flow. The minor flow intends afterwards to merge into the circulating flow which also have the priority and hence defined another major flow. Using gap acceptance theory we can simplify the model of our roundabout : we can consider that the minor flow is facing two intersections and want to cross the first one in order to merge into the second one. From this point of view, there is no difference between the pedestrian flow and the circulation flow.

Figure 1: Gap acceptance

As said before, gap is defined as the headway between two subjects. These gaps are measured in time. the minimum headways are  $\tau_p$  for pedestrians and  $\tau_c$  for the circulating flow, expressed in second. Subjects in minor flow judge the gaps in major flow to decide crossing or merging into major flow. Namely if the gap is bigger than  $t_{C_c}$  for circulating flow and  $t_{C_p}$  for pedestrians (those are called the critical time) then the minor flow takes the decision to merge into the traffic (respectively cross the crosswalk) while rejected gap is defined as the gap in major flows which smaller than these values. This gap judging behaviour is the gap acceptance behaviour. We define  $t_f$  to be the follow up time that is the time for a the second car queuing in the minor flow to follow the vehicle in front in merging into the traffic (resp crossing the crosswalk). For example, if the circulating flow provide a gap  $t$  with the length  $t_{C_c} \leq t \leq t_{C_c} + t_f$  enables the departure of one vehicle, the gap  $t$  with the length  $t_{C_c} + t_f \leq t \leq t_{C_c} + 2 * t_f$  enables the departure of two vehicles, and so on. We define also  $t_{0p} := t_{C_p} - \frac{t_f}{2}$  and  $t_{0c} := t_{C_c} - \frac{t_f}{2}$  to be the intercept gap and determine the minimum physical distance between two given elements of the major flows to insert one car.

We recall that we aim to estimate the roundabout's entry capacity in terms of several factors, and in particular in terms of the storage space left between the yield line and the crosswalk. We state that the entry capacity of the roundabout (  $C_c$ ) is determined by how many vehicles can enter in one accepted gap of circulating flow and how the accepted gaps are provided by circulating vehicles which is related to headway distribution of circulating vehicles. Namely, if  $E(t)$  is defined as the maximum number of vehicles can enter in one available gap at gap size of  $t$  sec,  $h(t)$  is defined as the density function of headway distribution and  $q_c$  the circulating flow in front of entry S ( vehicle/s) we have the following formula.

$$C_c = q_c \int_{-\infty}^{+\infty} E(t)h(t)dt$$

However, in our case, we are interested in standard single-lane roundabout with four legs (North, South, East, West), which includes crosswalks. Therefore, this formula has to be changed considering roundabout features such that the crossing

pedestrians, behaviour of vehicles in the circulating road and so on.

**Overview of the model**( Kang, N., H. Nakamura, and M. Asano. An Empirical Analysis on Critical Gap and Follow-Up Time at Roundabout Considering Geometry Effect)

We here suggest a model to estimate the entry capacity  $C_s$  considering two cases of circulating flow in front of Entry S. Circulating flow is divided in two cases : either it is flowing or queuing due to congested traffic. We are explaining the two cases : Illustration of cases (a) and (b) is shown in Figure 4.1(a) and (b), respectively.

CASE (a)

Under the condition of flowing circulating vehicles, entry capacity  $C_a$  is determined by pedestrians across Entry S, circulating flow passing in front of Entry S and the distance between crosswalk and yield line.

CASE (b)

On the other hand, the vehicles exiting to Exit N, E or W which are blocked by pedestrians across at these exits may lead to a queue in circulating roadway. When the queuing vehicles reaches up to front of Entry S, vehicles at Entry S are prevented from entering roundabout due to these queuing vehicles. Thus, entry capacity  $C_b$  is equal to zero under this condition since vehicles cannot enter roundabout at all.

$P_f$  is defined as the probability of circulating vehicles flowing in front of Entry S in case (a), and  $P_q$  is defined as the probability of queuing vehicles reaching up to front of Entry S in case (b). Accordingly, entry capacity  $C_s$  considering cases (a) and (b) is estimated by :

$$C_s = P_f * C_a + P_q * C_b$$

Since the situations of circulating flow in case (a) and (b) are independent,  $P_f + P_q = 1$ . In addition,  $C_b$  is equal to zero as described in case (b). Thus, we have:

$$C_s = (1 - P_q) * C_a$$

It amounts now to estimate  $C_a$  and  $P_q$ .

**Estimation of  $C_a$**



What follow next is due to the work of Nan Kang, N., H. Nakamura, and M. Asano. An Empirical Analysis on Critical Gap and Follow-Up Time at Roundabout Considering Geometry Effect.

We first estimate  $C_a$ . Entering procedure is divided into two separate parts due to the storage space, first crossing pedestrian flow and then merging into circulating flow. Thus, entry capacity  $C_a$  is determined by circulating flow passing in front Entry S and how many vehicles waiting at the yield line. Moreover, the number of vehicles that can wait at the yield line is determined by pedestrian flow and the maximum number of vehicles which can be stored between crosswalk and yield. Namely,  $n_a$  is defined as the maximum number of vehicles which can be stored between crosswalk and yield line and  $n$  is defined as the number of vehicles which are queuing in the storage space.  $P_n$  is defined as the probability of number of having  $n$  vehicles queuing in the storage space. Sum of the probabilities  $P_n$  for all of the possible number of queuing vehicles  $n$  must be equal to 1. Dependent on the value of  $n_a$ , the entering procedure can be classified into two states that we will sum up afterwards

State 1,  $1 \leq n \leq n_a$

The illustration of this state is shown in Figure 4.2(a). Under this condition,  $n$  vehicles queue in the space and the maximum number of vehicles which can enter roundabout is dependant on the circulating flow  $q_c$ . Hence, this maximum number of entering vehicles can be described by a function of  $f(q_c)$  that we will define explicitly later on. Thus, entry capacity for  $1 \leq n \leq n_a$   $C_{a1}$  is given by the following equation :

$$C_{a1} = \sum_{n=1}^{n_a} P_n * f(q_c) = (1 - P_0) * f(q_c)$$

State 2,  $n = 0$

$n=0$  represents the situation of no vehicle queuing in storage space. Under this situation, entry vehicle should cross pedestrian flow and circulating flow simultaneously without stopping in the storage space. Thus, entry capacity of state 2  $C_{a2}$  is calculated by a function of circulating flow  $q_c$  and pedestrian flow  $q_p$ ,  $g(q_p, q_c)$  with the probability of  $P_0$  as follow :

$$C_{a2} = P_0 * g(q_p, q_c)$$

Therefore, entry capacity  $C_a$  is given by :

$$C_a = (1 - P_0) * f(q_c) + P_0 * g(q_p, q_c)$$

We determine now  $P_0$  : The follow-up times crossing pedestrian flow and merging into circulating flow are assumed to be identical. According to this, A continuous period, e.g. 1 hour, can be divided into  $m$  intervals of duration  $t_f$ , as  $1h=3600sec=m * t_f$ . Therefore, the situation of the storage space will change at the end of each  $t_f$ . at each end time of  $t_f$  the situation of queuing vehicle in the storage space has  $na + 1$  possibilities and are independents and have for probability  $\frac{1}{na+1}$ . Considering this subdivision of time in terms of  $t_f$  a simple calculations ( which we choose to skip ) leads to  $P_0 = \frac{1}{na+1}$  and hence

$$C_a = \frac{n_a}{na + 1} * f(q_c) + \frac{1}{na + 1} * g(q_p, q_c)$$

It amounts now determine  $f$  and  $g$  which actually represent respectively the maximum entry flow considering only circulating flow and the maximum entry flow considering pedestrian and circulating flow simultaneously without stopping in storage space.

ESTIMATION OF  $f$  (Suzuki, K., and H. Nakamura. TrafAnalyzerThe Integrated Video Image Processing System for Traf Flow Analysis)

The above definition of  $f$  coincide avec  $C_c$  defined of acceptance gap theory. According to Nan Kang We can use  $E(t) = \frac{t-t_{0c}}{t_f}$  if  $t_{0c} \leq t$  and 0 otherwise and  $h(t) = \exp(-q_c \frac{t-\tau_c}{1-\tau_c})$  in the equation to obtain :

$$f(q_c) = \frac{3600}{t_f} (1 - \tau_c \frac{q_c}{3600}) \exp(\frac{-q_c}{3600} (t_{0c} - \tau_c))$$

ESTIMATION OF  $g$  (Wu, N. A Universal Procedure for Capacity Determination at Unsignal-ized (Priority-Controlled) Intersections.)

for the calculation of  $g$ , one can divide the major flow in 4 distinguished periods which when one sums them up allow us to recover the traffic. More specifically, let us decompose the traffic in 3 different stages :

Stage I:

In this stage, the traffic flow in the major stream is divided into 2 states which excludes each other : queuing and queuing-free. In the state of queuing, the vehicles in the major stream stay at the stop line. Departure from the minor stream is not possible in the state of queuing (including discharge queuing for the real-life traffic conditions). In the state of queuing-free all vehicles in the major stream are in motion. Departure from the minor stream is possible in the state of Queuing-free

possible but dependent on the traffic intensity and bunching situation within the major stream. Denoting the probability for the state of queuing by  $p_s$  the probability for the state of queuing-free is then  $p_{0s} = 1 - p_s$ .

Stage II:

In the stage II, the traffic flow in the queuing free state is also divided into 2 sub-states which excludes each other: bunching and bunching-free under the condition of queuing free. In the state of bunching, the vehicles in the major stream is in motion with the minimum gaps  $\tau_c$ . Departure from the minor stream is not possible in the state of bunching. In the state of bunching-free the gaps between the vehicles are larger than  $\tau_c$  and distributed by chance. Departure from the minor stream is in the state of Bunching-free possible but dependent on the traffic intensity within the major stream in this state. Denoting the probability for the state of Bunching under the condition of Queuing-free by  $p_b = Pr(bunching|queuingfree)$ , the probability for the state of bunching-free the condition of queuing-free is then  $p_{0b} = Pr(bunchingfree|queuingfree) = 1 - p_b$ .

Stage III:

In the stage III, the traffic flow in the bunchingfree state under the condition of queuingfree is divided again into 2 sub-sub-states which excludes each other : single-vehicle and vehicle-free (Free-space) under the condition of (bunching-free — queuing-free). In the state of single-vehicle, vehicles in the major stream are moving independently from each other. In the front of a vehicle, a time period of the length  $t_0$  is closed for the minor stream. The total closing time by the vehicles in the major stream is the sum of the set  $\{t \leq t_{0c}\}$ . Departure from the minor stream is not possible for the state of single-vehicle. In the state of free-space there is no vehicle in the major stream. Departure from the minor stream in the state of free-space is carried out with the saturation capacity  $K = \frac{1}{t_f}$ . Denoting the probability for the state of single-vehicle under the condition of (bunchingfree | queuingfree) by  $p_f = Pr[singlevehicle|(bunchingfree|queuingfree)]$ , the probability for the state of vehicle-free (free-space) under the condition of (bunchingfree | queuingfree) is then  $p_{0f} = Pr[vehiclefree|(bunchingfree|queuingfree)] = 1 - p_f$ .

Thus, the major stream can be divided into four regimes 1) that of state of Free-space (Vehicle- free), 2) that of state of Single-vehicle, 3) that of state of Bunching, and 4) that of state of Queuing. According to the definition of the conditioned probabilities, the probabilities  $p_s$ ,  $p_b$ ,  $p_f$  are completely independent of each other. They are to be determined according to the queuing theory. Let us omit the pedestrians for the

moment and come back to that later. the formula for the determination of  $g$  reads:

$$\begin{aligned} g &= Pr[Vehiclefree|(Bunchingfree|Queuingfree)] \\ &= K * p_{0s} * p_{0b} * p_{0f} \end{aligned}$$

with  $K$  to be the capacity in the free space state.

We use now queuing theory to calculate  $p_{0s}$ ,  $p_{0b}$ ,  $p_{0f}$  and  $K$ .

#### 1) Determination of $p_{0s}$

The probability for the state of Queuing-free in the major stream  $p_s$  can in general (approximately according to the M/G/1 queuing system) be estimated with the degree of saturation,  $x_c = \frac{\exp(t_{Cc}-t_f) * (\exp(t_f * q_c) - 1)}{q_c}$ . The probability for the Queuing-free state in the major stream  $p_{0s}$  then reads  $p_{0s} = 1 - p_s = 1 - x_c$ .

#### 2) Determination of $p_{0b}$

One can assume that bunching formation in the traffic in motion within the major stream is independent of the queuing saturation (Bunching during discharge queuing belongs to Queuing). Accordingly, it is true that  $Pr(Bunchingfree|Queuingfree) = Pr(Bunchingfree)$ . The probability of Bunching  $p_b$  in the major stream is simple the portion of the sum of the minimum gap  $\tau_c$  for all vehicles. Namely,  $p_b = q_c * \tau_c$  and then we have  $p_{0b} = 1 - q_c * \tau_c$ .

#### 3) Determination of $p_{0f}$

It is identical to the probability that the gap in the major stream  $t$  is larger than zero-gap  $t_{0c}$  under the condition that the gap  $t$  is larger than the minimum gap  $\tau_c$ . We choose to skip the calculation to assume that  $p_{0f} = \exp(-q_f(t_{Cc} - \tau_c))$  with  $q_f = \frac{(1-q_c*\tau_c)*q_c}{1-q_c*\tau_c}$ .

#### 4) Determination of $K$

Vehicle-free state the reciprocal of the mean service time of the queuing system. The mean service time in the vehicle free state is equal to the mean move-up time  $t_f$   $K = \frac{1}{t_f}$ .

We recall that so far we've been ignoring the pedestrian's flow. In our model, we consider that the pedestrian's flow can be represented the same way before, i.e like the circulating flow. Hence, the theory above applies and then since the intersections are considered as parallel it amounts us to multiply by the equivalent of  $p_{0b}$ ,  $p_{0s}$  and  $p_{0f}$  for the pedestrian's flow replacing the corresponding parameters.

Overall, we get the following formula for  $g$ .

and hence  $C_a$  is fully determined and depends explicitly on  $n_a$ . This allows us through a simulation to measure the impact of the gap between the yield line and the crosswalk on the entry capacity.

### **Estimation of $P_q$ ( Wu, N. A Universal Procedure for Capacity Determination at Unsignal-ized (Priority-Controlled) Intersections)**

We recall that at downstream exits (referring to N, E or W), exit vehicles have conflict with pedestrians and cross pedestrian flow by available gaps of pedestrians. Thus, a queue of exit vehicles may be generated when there is no available gap in pedestrian flow. Exit vehicles and pedestrians form a queuing system. First, this of course depends highly on the roundabout's size, that in our case is fixed (we suppose that between two exits next to each other have the length to store two vehicle). For any exit X,  $P_X$  is defined as the probability of 2 vehicles queuing in circulating roadway.  $P_q$  is calculated as the maximum value of  $P_X$  over the entries. In our simulation we will make no difference between the different exits, taking higher values than we took for the subject entry  $S$ . According to queue theory,  $P_q$  can be calculated by

$$P_q = (1 - \frac{\lambda}{\mu_p}) * (\frac{\lambda}{\mu_p})^2$$

Here, we have that  $\lambda$  is related to the circulating flow passing in front of Entry S  $q_c$  and to  $\alpha$  the proportion of demand of vehicles exiting from the downstream exits in circulating flow passing in front of Entry S; namely  $\lambda = \frac{\alpha * q_c}{3600}$ . For  $\mu_p$ , since pedestrian flow plays a central role and exit vehicles cross pedestrian flow depending on available gaps of pedestrians, we can use the same decomposition we've been using for  $g$  above (computing  $p_{0s}$ ,  $p_{0b}$  and  $p_{0f}$  with the parameters corresponding to the pedestrian's flow) leading to  $\mu_p = 1 - (1 - x_p) * (1 - q_p * \tau_p)$ .

**Overall we have an explicit estimation for  $C_s$ .** In the next chapter we are going to implement it and find out how big the impact of  $n_a$  on  $C_s$  is.

## **5 Implementation**

We have done two implementation with Matlab, one called distance and one called daytime. They're both build similar, but are giving a different look at the mater.

**The basic structure of daytime and distance:**

As said before both are build similar. Both have a main script, in which the data storing and the plotting is done. To see the linking have a look at the flowchart shown to the left. In the parameters script, all constant parameters are implemented. It makes it easy to adjust some changes in the constant parameters. Both implementations consist of a bunch of functions. Which are called from function or script one step higher and give back a value to the calling scrip or function. To see the purpose of every function have a look at the matlab code, where everything is documented. Because of the simplification of our model, some extra constants had to be added, just that our simulation gives results in the same hight as the simulations we used to abstract our model.

## THE IMPLEMENTATION "DISTANCE"

In this one, the whole calculation of the entry capacity  $c_s$  depends from the circulating flow  $q_c$  and the distance between the crosswalk and the yield line  $n_a$ . With that we produced two different plots. One in figure 2 which shows the affect of a growing  $n_a$  (from one to five cars) with a constant  $q_c = 800$  [cars/hour]. This one shows clearly the affect of the distance  $n_a$ . And an other one in figure 1 where  $q_c$  and  $n_a$  are variable. In both cases the produced date gets stored to the txt-file called *cs1*

## THE IMPLEMENTATION "DAYTIME"

In the implementation of daytime we simulated, with the same basic structure as in distance, the entry capacity  $c_s$  during 24 hours. The sampling rate was an half an hour. During this day we variated  $q_c$ , the circulating flow in front of the subject entrance, and  $q_{cprime}$ , the circulating flow in front of the main exit. We also variated  $q_p$ , the pedestrians crossing the crosswalk at the subject entrance, and  $q_{pprime}$ , the pedestrians crossing the crosswalk at the main entrance. The data for them was set for a usual roundabout in the city. This was done for  $n_a = 1$  and  $n_a = 2$ . The percentage advantage of  $n_a = 2$  compare to  $n_a = 1$  over a day, was computed and

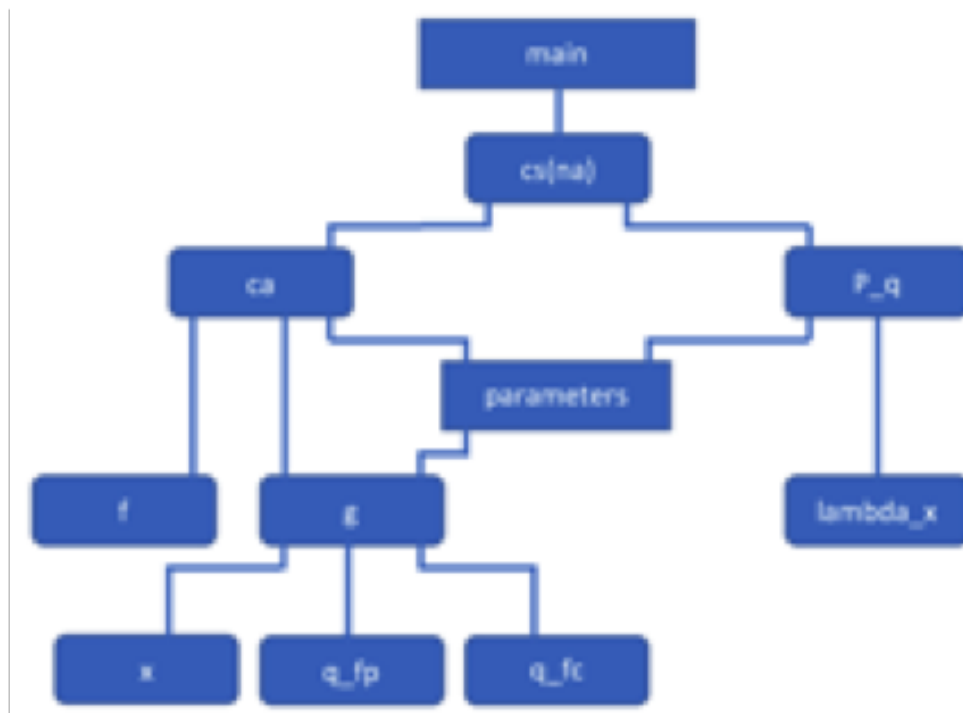


Figure 2:  $cs$  in terms of  $n_a$  and  $q_c$

displayed to the command window. Also we plotted the data to figure 3, which shows  $c_s(n_a)$ . All the produced data got stored in a txt-file called Daytimecs.

parameter	value	unit
tau_p	2	sec
tau_c	2.2	sec
t_fp,t_fc	3.2	sec
t_Cc	4.5	sec
t_Cp	6.2	sec
t_Oc	$t_{Cc} - (0.5 * t_{fc})$	sec
t_Op	$t_{Cp} - (0.5 * t_{fp})$	sec
q_p	$(1/3600) * 100$	cars/sec
q_pprime	$(1/3600) * 50$	cars/sec
q_m	$(1/3600) * 50$	cars/sec
q_mprime	$(1/3600) * 50$	cars/sec
q_c	800/not constant	cars
q_cprime= $(1/3600) * 400$	$(1/3600) * 400$	cars/sec
alpha_x	0.2	
n_b	2	cars
n_a	not constant	cars

Figure 3:  $cs$  in terms of  $n_a$  and  $q_c$

parameter	value	unit
$\tau_p$	2	sec
$\tau_c$	2.2	sec
$t_{fp}, t_{fc}$	3.2	sec
$t_{Cc}$	4.5	sec
$t_{Cp}$	6.2	sec
$t_{Oc}$	$t_{Cc} - (0.5 \cdot t_{fc})$	sec
$t_{Op}$	$t_{Cp} - (0.5 \cdot t_{fp})$	sec
$q_p$	not constant	cars/sec
$q_{pprime}$	not constant	cars/sec
$q_m$	$(1/3600) \cdot 50$	cars/sec
$q_{mprime}$	$(1/3600) \cdot 50$	cars/sec
$q_c$	not constant	cars/sec
$q_{cprime}$	not constant	cars/sec
$\alpha_x$	0.2	cars
$n_b$	2	cars
$n_a$	2	cars
$t$	not constant	hours

Figure 4:  $cs$  in terms of  $n_a$  and  $q_c$

## 6 Simulation Results and Discussion

Comparison of our work to Esimation of Roundabout Entry Capacity That Considers Conflict with Pedestrians written by Dr. Eng. Nan Kang and Dr. Eng. Hideki Nakamura

With the first implementation we wanted have a look at the impact on the entry capacity of a increasing space between the crosswalk and the yield line.

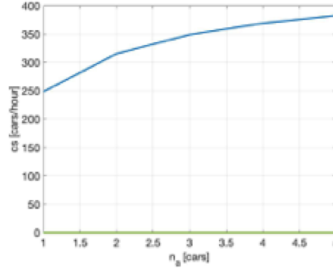


Figure 5:  $cs$  in terms of  $n_a$

The first Graph plotted from distance shows, that there is clearly an effect of a growing  $n_a$  on the entry-capacity  $cs$ . We decided to only consider  $1 \leq n_a \leq 5$ . Because everything else isnt reasonable or in use in switzerland. It shows also, that the effect of a growing  $cs$  gets smaller with a bigger  $n_a$ . The biggest change is from  $n_a = 1$  to 2. At a next point we wanted to see, if the effect also exist, with a growing  $q_c$  and we made a 3D plot.

This Graph shows, that the effect of  $n_a$  on  $c_s$  is not really depending on  $q_c$ . Of course  $c_s$  gets smaller if  $q_c$  gets bigger. But it behaves the same for all the  $n_a$ s. In



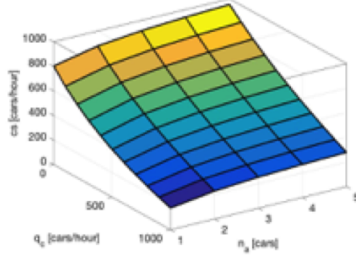


Figure 6:  $cs$  in terms of  $n_a$  and  $q_c$

other words,  $n_a$  has an homogeneous impact on  $c_s$  varying  $q_c$ .

Considering that  $q_{cprime}$ ,  $q_p$  and  $q_{pprime}$  are taken as constant in the first implementation, the results of distance were only a hint in which direction the research should go. Indeed, according to the previous results, the most significant difference is when  $n_a$  varies from 1 to 2. As a consequence, in the second implementation, we set  $n_a = 2$  and compared it with  $n_a = 1$ , so we could see the advantage of having a slightly bigger gap, which isn't very usual in Switzerland. This gap is still relatively small and it would still be reasonable for pedestrians to use it. Since we know that  $n_a = 3, 4, 5$  don't provide a lot of change in terms of results, there wouldn't be necessary to negatively impact the convenience of pedestrians by putting the crosswalk further down the road.

As seen in the plot above collected from the implementation daytime, the smallest entry-capacity, is at 7.30 am, it is near to zero, and there is not even a difference between  $n_a = 2$  and  $n_a = 1$ . The same applies at 6.30 pm. There also is not a big change between 0.00 am and 7.30 am, the reason in this case is that we are not having a lot of pedestrians, and that the amount of traffic is small. As seen in the main function of daytime (see appendix) the biggest density of pedestrians is between 7 am in the morning and 8.30 pm. In this time the effect from the various  $n_a$ s, is also the biggest. This matches with common sense, because if you have a high pedestrian rate the probability that a pedestrian blocks a car gets bigger, therefore  $c_s$  gets smaller. Also if you have a bigger space between the crosswalk and the yield line, the before mentioned probability gets smaller and  $c_s$  gets bigger. That explains the difference between the blue and the red line. Calculated over a day, we have with  $n_a$  equal to 2 a 11% higher entry-capacity than with  $n_a = 1$ . Although the data for this day was set by us, it clearly shows an advantage of a bigger  $n_a$ .

## 7 Summary and Outlook

Our simulation shows clearly that it would be a progress to build a roundabouts with a space for to cars between the crosswalk and the yield line. Clearly you would have to do further researches. One next step would be to set the values of our simulation with collected data from a roundabout in Switzerland and compare the results. Which also points at the biggest problem of our research, we didnt consider actual data. Also one would have to do some research, in aspect of the safety of the pedestrians with  $n_a = 2$  and compare it with the case  $n_a = 1$ . To Conclude one could say, that our model gives a hint that a little bigger  $n_a$  could make a non-neglected improvement on the roundabout capacity. In another hand, a couple of roundabouts features were not considered in this model, which could also provide different results and lead to different conclusions.

## 8 References

Wu, N. A Universal Procedure for Capacity Determination at Unsignal-ized (Priority-Controlled) Intersections

Kang, N., and H. Nakamura. An Estimation Method of Roundabout Entry Capacity Considering Pedestrian Impact.

Cowan, R. J. Useful Headway Models. Transportation Research, Vol. 9, No. 6, 1975, pp. 371375.

Siegloch, W. Die Leistungsermittlung an Knotenpunkten Ohne Lichtsig-nalsteuerung. Schriftenreihe Strassenbau und Strassenverkehrstechnik

## Matlabcode

### Implementation „distance“

```
%main skript
%Initialization
% x:= vector which contains values for n_a which is defined as the
%distance between the crosswalk and the yieldline measured in cars
x = [1 2 3 4 5];
%Z := general entry-capacity of the roundabout stored in a vector matrix with the results of
c_s(x,q) [cars/hour]
Z = zeros(11,5);
%q := circulating flow in front of the subject entrance [cars/sec]
q = [0.1 100 200 300 400 500 600 700 800 900 1000];
%z := general entry-capacity of the roundabout stored in a vector with the results of c_s(x,800)
[cars/hour]
z2 = zeros(5);

%Determ cs(n_a) for q_c = 800
for i=1:(5)

    z12 = x(i);
    z22 = (1/3600)*800;
    z2(i)= cs(z12, z22);
end

%Plot of cs(n_a)
figure(2);
plot(x,z2,'LineWidth', 2);
box on;
grid on;
set(gca, 'FontSize', 16);
xlabel('n_a [cars]', 'FontSize', 16);
ylabel('cs [cars/hour]', 'FontSize', 16);

%Determ cs(n_a,q_cir)
for i=1:(5)
    for j=1:11
        z1 = x(i);
        z2 = (1/3600)*q(j);
        Z(j,i)= cs(z1, z2);
    end
end

%Plot of cs(n_a,q_cir)
figure(1);
surface(x,q,Z,'LineWidth', 2);
box on;
grid on;
set(gca, 'FontSize', 16);
xlabel('n_a [cars]', 'FontSize', 16);
ylabel('q_c [cars/hour]', 'FontSize', 16);
zlabel('cs [cars/hour]', 'FontSize', 16);

fid=fopen('cs1','w+')
fprintf(fid,'n_a:      ',x(i));
for i = 1:(5)
    fprintf(fid,'%e      ',x(i));
end

% Save cs(n_a,q_cir) to cs1
fprintf(fid,'\n');
fprintf(fid,'q_cir:      ',x(i));
for i = 1:(11)
    fprintf(fid,'%e      ',q(i));
end

fprintf(fid,'\n');
fprintf(fid,'\n');
fprintf(fid,'Cs(n_a,q_cir):      ');
fprintf(fid,'\n');

for i = 1:(11)
    for j = 1:(5)
        fprintf(fid,'%e      ',Z(i,j));
    end
    fprintf(fid,'\n');
end

fclose(fid);
%End main skript
```

```

%Calculation of ca (Entrycapacity when the circulatingflow is not blocked)
function [y] = ca(n_a,q_c)
parameters;
temp =75000 * g(q_c);
y= (n_a/(n_a+1))*f(t_fc,q_c,tau_c,t_Cc)+(1/(n_a+1))*temp;

%Returns the general Entrycapacity of the roundabout
function[y]=cs(n_a,q_c)
p_Q = P_q();
Ca = ca(n_a,q_c);
y = (1-p_Q)*Ca;

%Calculation of ca (Entrycapacity when the circulatingflow is not blocked)
function [y] = ca(n_a,q_c)
parameters;
temp =75000 * g(q_c);
y= (n_a/(n_a+1))*f(t_fc,q_c,tau_c,t_Cc)+(1/(n_a+1))*temp;

%Probability of the queue in thecirculating roadqy reaching up to the front
%of the subject entry
function [y]= P_q()
parameters;
x_pprime = (q_mprime*exp(t_Cp-t_fp)*(exp(t_fp*q_pprime)-1))/q_pprime;
lambda = lambda_x(alpha_x, q_cprime);

%mue_p := service rate whooch is the reciprocal value of the aberage service time
mue_p = 1-(1-x_pprime)*(1-q_pprime*tau_p);

y = 150*(1-(lambda/mue_p))*(lambda/mue_p)^(n_b);

%Returns the maximum entry flow considering only the circulating flow q_c
function[y]= f(t_fc,q_cir,tau_c,t_Cc)
y = 3600*(1/t_fc)*(1-tau_c*(q_cir))*exp((-q_cir)*(t_Cc-(t_fc/2)-tau_c));

function [y] = g(q_c)
parameters;
%Maximum entryflow considering pedestrians and circulating flow without
%stopping and storage space
x_p = x(q_m,t_Cp,t_fp,q_p);
x_c = x(q_m,t_Cc,t_fc,q_c);
t_f = (t_fp + t_Cp)/2;

%traffic intenistiy with the portion of free traffic
q_fpl = q_fp(tau_p, q_p);
q_fcl = q_fc(tau_c,q_c);

y = ((q_fpl+q_fcl)/(1-exp(-(q_fpl+q_fcl)*t_f)))*(1-x_p) * (1- q_p * tau_p)*exp(-
q_fpl*(t_Cp*tau_p))*(1-x_c) * (1- q_c * tau_c)*exp(-q_fcl*(t_Cc*tau_c));

%Arrival rate of vehicles exiting from the main exit
function [y]= lambda_x(alpha_x, q_cprime)
y = (alpha_x)* q_cprime;

%traffic intenistiy with the portion of free traffic
function [y] = q_fc(tau_c,q_cir)

phi = 1-q_cir*tau_c;
y = (phi*q_cir)/(1-q_cir*tau_c);

%traffic intenistiy with the portion of free traffic
function [y]= q_fp(tau_p, q_p)
phi = 1-q_p*tau_p;
y = (phi*q_p)/(1-q_p*tau_p);

%saturation degree of the queuing sytem
function y = x(q_m,t_c,t_f,q)
y = (q_m*exp(t_c-t_f)*(exp(t_f*q)-1))/q;

%Start parameters skript

% Min. Headway for pedestrians: The time interval [sec] between of two successive
% pedestrians
tau_p = 2;

% Min. Headway for cars: The time interval between the arrivals of two successive
% the vehicles in the Majorflow(from front to front);
tau_c = 2.2;

%For pedestrians
%Follow-up time: Time between the departure of one vehicle from the minor s
%treet and the departure of the next vehicle using the same gap under a

```

```

%condition of continuous queuing.
t_fp = 3.2;

%For cars
%Follow-up time: Time between the departure of one vehicle from the minor s
%treet and the departure of the next vehicle using the same gap under a
%condition of continuous queuing.
t_fc = 3.2;

%Critical gap: The minimum major-stream headway during which a minor-street
%vehicle can make a maneuver.
%%t_Cc=Critical gap for cars
t_Cc= 4.5;

%t_Cp: Critical gap for pedestrians
t_Cp= 6.2;

%Intercept gap for cars: Mimimum physical gap when one car from the
%minorflow can merge into the majorflow
t_0c= t_Cc-(0.5*t_fc);

%Intercept gap for pedestrians: Mimimum physical gap when one car
%from the minorflow can cross the majorflow(pedestrians)
t_0p= t_Cp-(0.5*t_fp);

%q_p := pedestrians crossing the crosswalk at the subject entrance [pedestrians/sec]
q_p=(1/3600)*100;

%q_m: Constant representing the minorflow at the entrance
q_m= (1/3600)*50;

%q_cprime := circulating flow infront of the main exit [cars/sec]
q_cprime= (1/3600)*400;

%q_pprime := pedestrians crossing the crosswalk at the main entrance [pedestrians/sec]
q_pprime=(1/3600)*50;

%q_m: Constant representing the minorflow at the main exit
q_mprime= (1/3600)*50;

%alpha_x: Propoation of demand of vehicles exiting the main exit X in
%circulating flow q_c
alpha_x = 0.2;

%Number of cars which can be stored between the entrance and the following
%exit. It defines the size of the roundabout
n_b= 2;

%End parameters skript

```

## Implementation „daytime“

```

%Start main skript
%Initialisation

%q_c := circulating flow in front of the subject entrance [cars/sec]
q_c = (1/3600) * 0.8 * [550 450 350 200 100 50 50 50 50 100 200 300 600 700 800 1000 1200 1400
1150 900 750 700 650 700 850 950 900 1000 1000 850 750 700 750 850 950 1150 1350 1450 1100 950 900
850 800 700 700 700 600 550];
%q_cprime := circulating flow in front of the main exit [cars/sec]
q_cprime = (1/3600) * [550 450 350 200 100 50 50 50 50 100 200 300 600 700 800 1000 1200 1400 1150
900 750 700 650 700 850 950 900 1000 1000 850 750 700 750 850 950 1150 1350 1450 1100 950 900 850
800 700 700 700 600 550];
%q_p := pedestrians crossing the crosswalk at the subject entrance [pedestrians/sec]
q_p = (1/3600) * 0.6 * [25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25 25
225 250 275 300 300 275 275 275 275 275 275 275 275 275 275 250 225 225 175 150 150 125 125 100 75 50 50
50];
%q_pprime := pedestrians crossing the crosswalk at the main entrance [pedestrians/sec]
q_pprime = (1/3600) * [25 25 25 25 25 25 25 25 25 25 25 25 50 50 75 100 125 175 175 200 225 225 225
250 275 300 300 275 275 275 275 275 275 275 275 275 250 225 225 175 150 150 125 125 100 75 50 50
50];
%t := time of the day;
t = [0.0 0.5 1 1.5 2 2.5 3 3.5 4 4.5 5 5.5 6 6.5 7 7.5 8 8.5 9 9.5 10 10.5 11 11.5 12 12.5 13 13.5
14 14.5 15 15.5 16 16.5 17 17.5 18 18.5 19 19.5 20 20.5 21 21.5 22 22.4 23 23.5];
%c_s := general entry-capacity of the roundabout for n_a= 1 [cars/hour]
cs_n1 = zeros(length(t));
%c_s := general entry-capacity of the roundabout for n_a= 2 [cars/hour]
cs_n2 = zeros(length(t));

```

```

%Settings for the plot
figure(3);
hold on;
box on;
grid on;
title('Entrance-Capacity over a day')
set(gca, 'FontSize', 16);
xlabel('Daytime','FontSize', 16);
ylabel('cs [cars/hour]','FontSize', 16);
axis([0.0,23.5,0,1200])

%Calculation and plotting of cs(for n_a = 2)
for i = 1: (length(t)-1)
    c1_temp = cs(2,q_c(i),q_cprime(i),q_p(i),q_pprime(i));
    c2_temp = cs(2,q_c(i+1),q_cprime(i+1),q_p(i+1),q_pprime(i+1));
    t1_temp = t(i);
    t2_temp = t(i+1);
    v_t = [t1_temp t2_temp];
    v_cs = [c1_temp c2_temp];
    cs_n2(i)=c1_temp;
    if i == 36
        cs_n2(i+1) = c2_temp;
    end

figure(3);
color = 'r';
p2 = plot(v_t,v_cs,'b','LineWidth', 2);

end

%Calculation and plotting of cs(for n_a = 1)
for i = 1: (length(t)-1)
    c1_temp = cs(1,q_c(i),q_cprime(i),q_p(i),q_pprime(i));
    c2_temp = cs(1,q_c(i+1),q_cprime(i+1),q_p(i+1),q_pprime(i+1));
    t1_temp = t(i);
    t2_temp = t(i+1);
    v_t = [t1_temp t2_temp];
    v_cs = [c1_temp c2_temp];
    cs_n1(i)=c1_temp;
    if i == 36
        cs_n1(i+1) = c2_temp;
    end

figure(3);
color = 'b';
p1 = plot(v_t,v_cs,'r','LineWidth', 2);

end
hold off
legend([p1,p2],'n_a = 1','n_a = 2');

%Calculation of the average advantage a_p of n_a = 1 to n_a= 2
temp_c1 = 0;
temp_c2 = 0;
for i=1:length(t)
    temp_c1 = temp_c1 + cs_n1(i);
    temp_c2 = temp_c2 + cs_n2(i);
end
a_p = (100/temp_c1)*temp_c2-100;
disp(a_p);

%Storing data to of cs to "Daytime_cs" and a_p
fid=fopen('Daytime_cs','w+');
fprintf(fid,'Daytime:      ');
fprintf(fid,'cs(n_a=1):    ');
fprintf(fid,'cs(n_a=2):    ');
fprintf(fid,'\n');

for i = 1:length(t)
    fprintf(fid,'%e      ',t(i));
    fprintf(fid,'%e      ',cs_n1(i));
    fprintf(fid,'%e      ',cs_n2(i));
    fprintf(fid,'\n');
end
fprintf(fid,'\n');
fprintf(fid,'\n');
fprintf(fid,'a_p= ');
fprintf(fid,'%e      ',a_p);

fclose(fid);

%End main skript

%Returns the general Entrycapacity of the roundabout
function[y]=cs(n_a,q_c,q_cprime,q_p,q_pprime)

```

```

p_Q = P_q(q_cprime,q_pprime);
Ca = ca(n_a,q_c,q_p);
y = (1-p_Q)*Ca;

%Calculation of ca (Entrycapacity when the circulatingflow is not blocked)
function [y] = ca(n_a,q_c,q_p)
parameters;
temp =75000 * g(q_c,q_p);
y= (n_a/(n_a+1))*f(t_fc,q_c,tau_c,t_Cc)+(1/(n_a+1))*temp;

%Maximum entryflow considering pedestrians and circulating flow without
%stopping and storage space
function [y] = g(q_c,q_p)
parameters;
x_p = x(q_m,t_Cp,t_fp,q_p);
x_c = x(q_m,t_Cc,t_fc,q_c);
t_f = (t_fp + t_Cp)/2;

%traffic intenistiy with the portion of free traffic
q_fpl = q_fp(tau_p, q_p);
q_fcl = q_fc(tau_c,q_c);

y = ((q_fpl+q_fcl)/(1-exp(-(q_fpl+q_fcl)*t_f)))*(1-x_p) * (1- q_p * tau_p)
*exp(-q_fpl*(t_Cp*tau_p))*(1-x_c) * (1- q_c * tau_c)*exp(-q_fcl*(t_Cc*tau_c));

%Arrival rate of vehicles exiting from the main exit
function [y]= lambda_x(alpha_x, q_cprime)
y = (alpha_x)* q_cprime;

%Probability of the queue in thecirculating roadqy reaching up to the front
%of the subject entry
function [y]= P_q(q_cprime,q_pprime)
parameters;
x_pprime = (q_mprime*exp(t_Cp-t_fp)*(exp(t_fp*q_pprime)-1))/q_pprime;
lambda = lambda_x(alpha_x, q_cprime);

%mue_p := service rate whoch is the reciprocal value of the aberage service time
mue_p = 1-(1-x_pprime)*(1-q_pprime*tau_p);

y = 150*(1-(lambda/mue_p))*(lambda/mue_p)^(n_b);

%traffic intenistiy with the portion of free traffic
function [y] = q_fc(tau_c,q_c)
%phi := portion of free traffic in the mahor stream
phi = 1-q_c*tau_c;
y = (phi*q_c)/(1-q_c*tau_c);

%traffic intenistiy with the portion of free traffic
function [y]= q_fp(tau_p, q_p)
%phi := portion of free traffic in the mahor stream
phi = 1-q_p*tau_p;
y = (phi*q_p)/(1-q_p*tau_p);

%saturation degree of the queuing sytem
function y = x(q_m,t_c,t_f,q)
y = (q_m*exp(t_c-t_f)*(exp(t_f*q)-1))/q;

%Returns the maximum entry flow considering only the circulating flow q_c
function[y]= f(t_fc,q_c,tau_c,t_Cc)
y = 3600*(1/t_fc)*(1-tau_c*(q_c))*exp((-q_c)*(t_Cc-(t_fc/2)-tau_c));

%Start parameters skript
% Min. Headway for pedestrians: The time interval between of two successive
% pedestrians
tau_p = 2;

% Min. Headway for cars: The time interval between the arrivals of two successive
% the vehicles in the Majorflow(from front to front);
tau_c = 2.2;

%For pedestrians
%Follow-up time: Time between the departure of one vehicle from the minor s
%treet and the departure of the next vehicle using the same gap under a
%condition of continuous queuing.
t_fp = 3.2;

%For cars
%Follow-up time: Time between the departure of one vehicle from the minor s
%treet and the departure of the next vehicle using the same gap under a
%condition of continuous queuing.
t_fc = 3.2;

```

```

%Critical gap: The minimum major-stream headway during which a minor-street
%vehicle can make a maneuver.
%%t_Cc=Critical gap for cars
t_Cc= 4.5;

%t_Cp: Critical gap for pedestrians
t_Cp= 6.2;

%Intercept gap for cars: Mimimum physical gap  when one car from the
%minorflow can merge into the majorflow
t_0c= t_Cc-(0.5*t_fc);

%Intercept gap for pedestrians: Mimimum physical gap when one car
%from the minorflow can cross the majorflow(pedestrians)
t_0p= t_Cp-(0.5*t_fp);

%q_m: Constant representing the minorflow at the entrance
q_m= (1/3600)*50;

%q_m: Constant representing the minorflow at the main exit
q_mprime= (1/3600)*50;

%alpha_x: Propoation of demand of vehicles exiting the main exit X in
%circulating flow q_c
alpha_x = 0.2;

%Number of cars which can be stored between the entrance and the following
%exit. It defines the size of the roundabout
n_b= 2;

%End parameters skript

```

## txt-Files

### Daytime\_cs

Daytime:	cs(n_a=1):	cs(n_a=2):
0.000000e+00	4.620623e+02	5.180568e+02
5.000000e-01	5.637577e+02	6.181445e+02
1.000000e+00	6.789727e+02	7.270239e+02
1.500000e+00	8.799614e+02	9.072704e+02
2.000000e+00	1.034383e+03	1.038773e+03
2.500000e+00	1.118018e+03	1.107832e+03
3.000000e+00	1.118018e+03	1.107832e+03
3.500000e+00	1.118018e+03	1.107832e+03
4.000000e+00	1.118018e+03	1.107832e+03
4.500000e+00	1.034383e+03	1.038773e+03
5.000000e+00	8.799614e+02	9.072704e+02
5.500000e+00	7.103802e+02	7.640819e+02
6.000000e+00	4.049145e+02	4.650511e+02
6.500000e+00	3.191786e+02	3.779749e+02
7.000000e+00	2.493616e+02	3.030777e+02
7.500000e+00	1.436944e+02	1.803000e+02
8.000000e+00	7.004107e+01	9.030283e+01
8.500000e+00	1.670396e+01	2.174793e+01
9.000000e+00	8.857285e+01	1.144371e+02
9.500000e+00	1.893605e+02	2.424414e+02
1.000000e+01	2.627142e+02	3.326648e+02
1.050000e+01	2.894267e+02	3.649480e+02
1.100000e+01	3.123836e+02	3.958648e+02
1.150000e+01	2.821973e+02	3.620065e+02
1.200000e+01	2.095237e+02	2.727370e+02
1.250000e+01	1.674821e+02	2.188229e+02
1.300000e+01	1.884245e+02	2.443341e+02
1.350000e+01	1.473042e+02	1.918359e+02
1.400000e+01	1.473042e+02	1.918359e+02
1.450000e+01	2.104645e+02	2.722580e+02
1.500000e+01	2.573763e+02	3.311568e+02
1.550000e+01	2.821973e+02	3.620065e+02
1.600000e+01	2.573763e+02	3.311568e+02
1.650000e+01	2.104645e+02	2.722580e+02
1.700000e+01	1.673624e+02	2.175075e+02
1.750000e+01	9.188347e+01	1.198062e+02
1.800000e+01	3.283443e+01	4.295881e+01
1.850000e+01	1.175581e+01	1.542800e+01
1.900000e+01	1.048432e+02	1.343473e+02
1.950000e+01	1.670720e+02	2.102033e+02



2.000000e+01	1.912184e+02	2.394405e+02
2.050000e+01	2.184382e+02	2.697454e+02
2.100000e+01	2.465544e+02	3.026368e+02
2.150000e+01	3.133062e+02	3.753319e+02
2.200000e+01	3.191786e+02	3.779749e+02
2.240000e+01	3.254414e+02	3.808163e+02
2.300000e+01	4.049145e+02	4.650511e+02
2.350000e+01	0.000000e+00	0.000000e+00

a\_p= 1.123363e+01

## cs1

n\_a: 1.000000e+00 2.000000e+00 3.000000e+00 4.000000e+00 5.000000e+00  
q\_cir: 1.000000e-01 1.000000e+02 2.000000e+02 3.000000e+02 4.000000e+02 5.000000e+02 6.000000e+02 7.000000e+02  
8.000000e+02 9.000000e+02 1.000000e+03

Cs(n\_a,q\_cir):

8.032027e+02	8.780855e+02	9.155269e+02	9.379917e+02	9.529683e+02
6.911126e+02	7.762520e+02	8.188217e+02	8.443636e+02	8.613915e+02
5.969198e+02	6.872404e+02	7.324007e+02	7.594969e+02	7.775611e+02
5.170390e+02	6.086629e+02	6.544748e+02	6.819620e+02	7.002868e+02
4.485330e+02	5.385366e+02	5.835383e+02	6.105394e+02	6.285401e+02
3.890808e+02	4.752901e+02	5.183947e+02	5.442575e+02	5.614993e+02
3.368539e+02	4.176810e+02	4.580945e+02	4.823426e+02	4.985080e+02
2.904155e+02	3.647286e+02	4.018851e+02	4.241791e+02	4.390417e+02
2.486397e+02	3.156600e+02	3.491702e+02	3.692763e+02	3.826804e+02
2.106464e+02	2.698669e+02	2.994772e+02	3.172433e+02	3.290874e+02
1.757497e+02	2.268707e+02	2.524312e+02	2.677675e+02	2.779917e+02