# Advanced Statistics

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Class 3, October 16, 2014

### Your last serving of R basics

- Working with files
- R can open all kinds of data files
- You do have to tell it where to find them
- R's working directory
- This is where R looks for files to open
- You change it using 'setwd("C:/My\_example\_R\_directory")
- No backslashes \ allowed! Use forward slashes instead /

### Getting more practice with R

• Here's a free introduction to R which might be helpful to you. It repeats a lot of the things I have showed you only briefly.

### Setting your working directory

- Don't know where your working directory should be?
- Start by putting your script files in a location that is sensible (to you)
  - $\ e.g. \ \texttt{C:} \\ \texttt{Some\_Document\_Folder} \\ \texttt{AdvStats} \\ \texttt{Class3} \\ \text{or /home/My\_Name/Documents/MastersProject/Data} \\$
  - That directory should also be your working directory for the current session
  - In RStudio, open your script file, then use the menu bar and go to Session -> Set Working Directory -> To Source File Location
  - RStudio will write a setwd command to the console.
  - Copy and paste this command to your script, and you're set. Every time you run the script, the
    working directory will be automatically set
- When you use R Markdown, Knitr sets your working directory to the location of the Rmd file automatically.

# Opening files and importing data

- If you have data in Excel:
- Save them as comma-separated values (CSV)
- If you are using Excel formulas and other bells and whistles, check that your data are exported correctly
- Make sure the first row of every column contains its column name
- Save this file in your working directory (remember where that was?)
- Now you can import the file using new\_object\_with\_a\_sensible\_name <- read.csv(file =
   "my\_filename.csv")</li>
- If there is an error/warning message, check that there are no missing cells
  - Same number of data points in each row

#### **Data Frames**

- A Data Frame is a combination of a list and a matrix
- It gets its row/column structure from the matrix
- But you can use columns of any data type (numeric, logical, factor (discrete), logical, character)
- Perfect for the kind of analyses we would like to do
- Very similar to SPSS data files
- Get a column by using object\_name\$column\_name
- Get a column by slicing: object\_name[,1]
- Get a row by slicing: object\_name[1,]
- Get a cell by slicing: object\_name[1,1]

### **Packages**

- Many authors have worked on extending R and teaching it new tricks
- Packages are collections of functions that help you perform a certain task
- For example, the ez package has all kinds of useful functions for performing ANOVAs
- To install a package, type install.packages("package\_name") at the Console.
- e.g. install.packages("ez")
- You can ask R to install multiple packages at once:
- Run this line in the console now: 'install.packages(c("ez","reshape","ggplot2","lme4", "plyr"))

# Loading packages

- Installed packages are not loaded automatically
- If you want to use a command from a package, you need to load it: library(package\_name)
- e.g. library(ez)
- Once it is loaded, the package and all of its commands are ready to go until you restart R.

# Starting a script

Good practice when starting a script:

```
# My_testscript.R
# Author: Bernhard Angele
# Description: Demonstrates how to start a script
##############################

rm(list = ls()) # clear the workspace to avoid problems caused by old objects
library(ez) # load (only) those libraries that you need
?library(reshape)
library(ggplot2)
# Set your working directory
setwd("C:/I_know_my_working_directory_and_this_is_it/")
# Now load your data, do your analyses and win fame and fortune!
```

### Starting an Rmd file

• Fill in the header:

```
title: "My fantastic results section"
output: pdf_document
---
```

- Then do the same things as above in the first code chunk
- Hint: you can tell R to hide this boring code by adding hide = TRUE to the header. It will still get evaluated

```
Start your code chunk like this: ```{r, hide = TRUE}
```

### Comparing multiple groups

- t-tests are nice if you only have two groups that you want to compare.
- But maybe you have more groups

Example: A researcher wants to find out if there is a systematic difference in intelligence between MSc students from different universities. She performs intelligence tests on 10 students each from BU, University of Southampton and Oxford University and records the results.

### Making fake data for our example

- Let's assume that the true state of affairs is that there is no difference in intelligence
- In that case, all intelligence scores would come from the same distribution: a normal distribution with mean = 100 and sd = 15
- Let's generate 3 data sets according to this criterion:

```
# The following line sets the random number generator to a specific state
# and ensures that you get the same numbers that I did.
set.seed("16102014")
bu <- rnorm(n = 10, mean = 100, sd = 15)
soton <- rnorm(n = 10, mean = 100, sd = 15)
oxford <- rnorm(n = 10, mean = 100, sd = 15)</pre>
```

#### What if we use t-tests?

- We could simply perform t-tests on these data
- How many would we need?
- 3: BU vs. Soton, BU vs. Oxford, Soton vs. Oxford

#### BU vs Soton

```
##
## Welch Two Sample t-test
##
## data: bu and soton
## t = -0.7242, df = 17.09, p-value = 0.4787
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -19.924 9.738
## sample estimates:
## mean of x mean of y
## 94.61 99.70
```

#### BU vs Oxford

```
##
## Welch Two Sample t-test
##
## data: bu and oxford
## t = 0.5039, df = 17.34, p-value = 0.6207
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -11.44 18.63
## sample estimates:
## mean of x mean of y
## 94.61 91.01
```

#### Soton vs Oxford

```
t.test(soton, oxford)
```

```
##
## Welch Two Sample t-test
##
## data: soton and oxford
## t = 1.383, df = 17.98, p-value = 0.1837
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
## -4.514 21.890
## sample estimates:
## mean of x mean of y
## 99.70 91.01
```

### Anything wrong with that?

- We are doing three independent t-tests
- Each t-test has a 5% chance of producing a spurious result  $(\alpha)$
- What is the chance that we get at least one spuriously significant result?
- It's 1 the chance that we get no spurious results
- 1 .95 \* .95 \* .95 = .14
- We have a problem: our  $\alpha$  is almost three times as high as it should be.
- SPSS calls this LSD (least significant differences) don't use it!

### Exercise: Give it a try

- Instead of running the simulation on my computer, I'll run it on you
- What I mean by that:
- Generate three data sets like I've just shown you
- Don't use set.seed() now, or we'll all get the exact same data
- Make sure that all three data sets are samples from the same normal distribution
  - Same mean and sd, of course same n as well
- Run three two-sample t-tests comparing the means
- Afterwards, I will ask you and count how many of you found at least one significant result.
- If the  $\alpha$  level isn't inflated, only one or two of you should find one

#### Solutions

- We can adjust the  $\alpha$  level of each t-test:
- If we divide the alpha level by the number of tests, we get .05/3 = .0167
- Our total  $\alpha$  is then:  $1 (1 .05/3)^3 = .049$
- This is called a Bonferroni correction
- Problem solved?
- Yes, but the lower the  $\alpha$  level, the lower the power.
- Better ways (but still lowering power):
- Holm-Bonferroni (same principle as Bonferroni, but better power)
- Tukey's HSD (honestly significant differences)
- Maybe we just want to know if there is a difference at all between any of these three means
- One-way ANOVA

# Putting our fake data in a different format

- Usually, we aren't getting our data neatly in different objects
- Instead, we'll have a big table with our data
- If we are going to put our data in this format, we need a variable (or column) identifying the group
- Let's add that to each of our data sets

```
bu <- data.frame(iq = bu, group = "BU")
soton <- data.frame(iq = soton, group = "Soton")
oxford <- data.frame(iq = oxford, group = "Oxford")</pre>
```

### Looking at these data now

```
bu
##
          iq group
## 1
     123.53
## 2
      107.44
      102.43
## 3
## 4
       88.58
                 BU
       76.41
       80.20
## 6
                BU
      113.23
                BU
## 8
       96.15
                BU
## 9
       90.62
                 BU
## 10 67.46
                 BU
```

## Putting it all together

# Running an ANOVA – by hand!

• One-way ANOVAs are so simple, all you need is a calculator (or R, even better)

- Numeric variables are continuous (interval or ratio scale)

- First, we need an estimate of the total variance in the data: calculate the total sum of squared deviations from the mean (or short, sum of squares, SS)
- $SS_{total} = \sum (x_i \bar{x})^2$ , where  $x_i$  is each individual value and  $\bar{x}$  is the grand mean of the data.

```
(SS_tot <- sum((iqdata$iq - mean(iqdata$iq))^2))
```

## [1] 6674

### Running an ANOVA – by hand! (2)

- Now we need an estimate of the variance explained by group
- $SS_{model} = n_k(\bar{x}_k \bar{x})^2$ , where  $\bar{x}_k$  denotes the mean for each group,  $n_k$  is the number of subjects in each group, and  $\bar{x}$  is the grand mean of the data.

```
bu_mean <- mean(subset(iqdata, group == "BU")$iq)
soton_mean <- mean(subset(iqdata, group == "Soton")$iq)
oxford_mean <- mean(subset(iqdata, group == "Oxford")$iq)
n <- 10

(SS_model <- sum(n*(bu_mean - mean(iqdata$iq))^2, n*(soton_mean - mean(iqdata$iq))^2, n*(oxford_mean -
## [1] 381.1</pre>
```

### Running an ANOVA – by hand! (3)

- Now get an estimate of the variance that is *not* explained by group, i.e. the error.
- Easy: Just subtract  $SS_{model}$  from  $SS_{total}$ .
- $SS_{error} = SS_{total} SS_{model}$

```
(SS_error <- SS_tot - SS_model)
```

## [1] 6293

# Running an ANOVA – by hand! (4)

- With  $SS_{model}$  and  $SS_{E}rror$ , we can compute the ratio of explained variance to error (or unexplained) variance.
- First, we need to take into account the number of measurements which went into each SS
- These are called the degrees of freedom
- This is equivalent to calculating the variance as a descriptive statistic (SS/df)
- $df_{total} = n * k 1$ , where n is the number of observations per group (10) and k is the number of groups
- Why -1? Very good question. Short answer: Whenever we want to make conclusions about the population, we use n-1 instead of n.
- Long answer (if you are REALLY interested and won't shut up about it): Here

# Running an ANOVA – by hand! (5)

- $df_{total} = n \cdot k 1$
- $df_{model} = k 1$ , where k is the number of groups.
- $df_{error} = df_{total} df_{error} = n \cdot k k$

```
k <- 3
(df_total <- n * k - 1)
```

```
## [1] 29
```

```
(df_model \leftarrow k - 1)
```

## [1] 2

```
(df_error \leftarrow n * k - k)
```

## [1] 27

## Running an ANOVA – almost done!

- Now we compute the mean squares (MS) as an estimate of the variance
- $MS_{model} = \frac{SS_{model}}{df_{model}}$   $MS_{error} = \frac{SS_{error}}{df_{error}}$

```
(MS_model <- SS_model/df_model)
```

## [1] 190.6

```
(MS_error <- SS_error/df_error)
```

## [1] 233.1

# Running an ANOVA – final steps!

- Finally, we take the ratio of the two.
- $F(df_{model}, df_{error}) = \frac{MS_{model}}{MS_{error}}$

```
(F_value <- MS_model/MS_error)</pre>
```

## [1] 0.8177

#### What to do with this F-value

- It turns out that the ratio between model and error variance follows a specific distribution
- If there is no actual effect (!) and
- As long as certain assumptions are valid (more on that later)
- This distribution is called the F-distribution
- Occasionally you will get a high  $MS_{model}$  simply by chance, but such occurrences are quite rare
- The F-distribution is the probability density function for different values of the variance ratio, i.e. the
- We essentially want to test if the F-value we get is extreme enough that it could only have occurred by chance 5% of the time (our  $\alpha$  level)

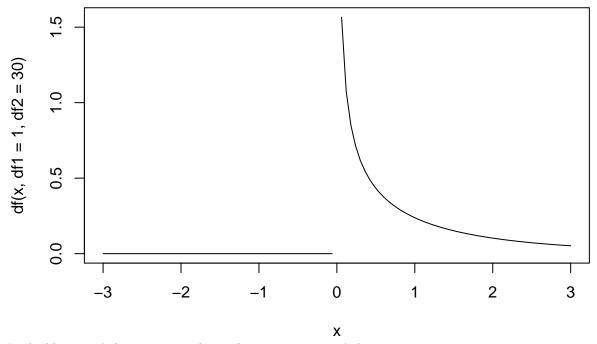
### The F-distribution

- Like the t-distribution, the shape of the F-distribution varies depending on sample size (degrees of freedom).
- Remember that F is a ratio of two variances.
- Because of this, the F distribtion has two degrees of freedom parameters
  - dfnumerator
  - $-df_{denominator}$
- Guess what the R functions for the F distribution are called
- That's right: df, pf, qf, rf

# Plotting it

• Let's take a look

$$curve(df(x, df1 = 1, df2 = 30), from = -3, to = 3)$$

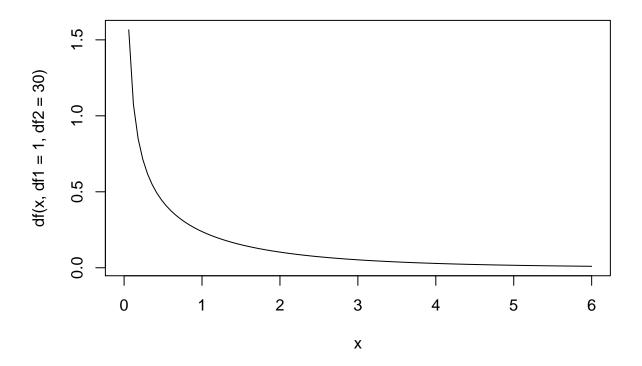


Looks like x can't be negative - this makes sense: you can't have negative variance

# Plotting it (2)

• Adjusting the x-axis

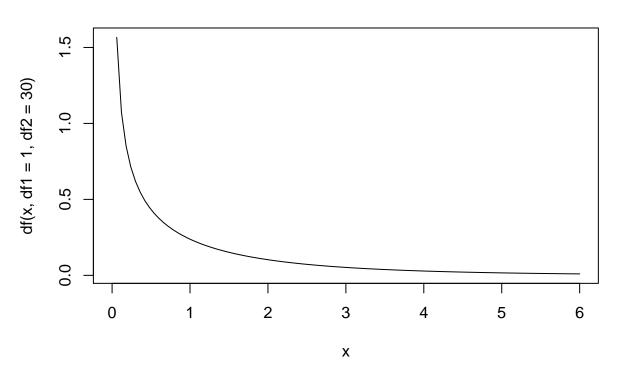
```
curve(df(x, df1 = 1, df2 = 30), from = 0, to = 6)
```



The F distribution

```
curve(df(x, df1 = 1, df2 = 30), from = 0, to = 6, main = "F(1,30)")
```

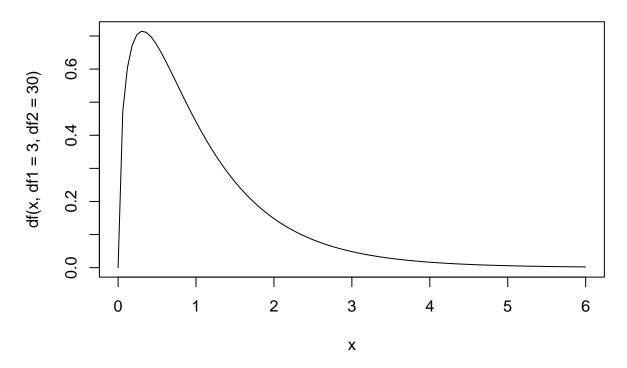




# The F distribution (2)

```
curve(df(x, df1 = 3, df2 = 30), from = 0, to = 6, main = F(3,30)")
```

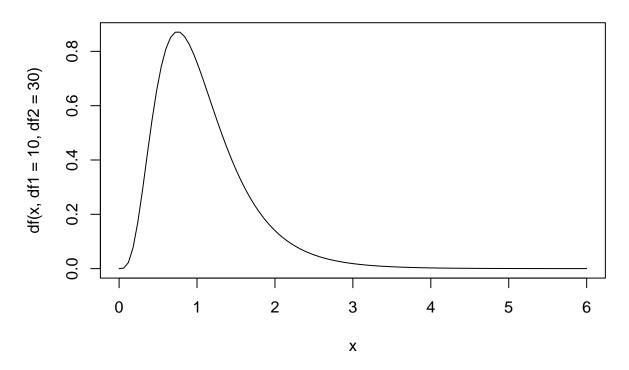
# F(3,30)



# The F distribution (3)

```
curve(df(x, df1 = 10, df2 = 30), from = 0, to = 6, main = F(10,30))
```

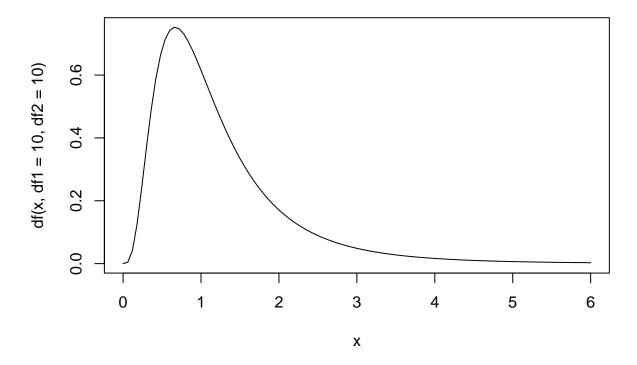
F(10,30)



# The F distribution (4)

```
curve(df(x, df1 = 10, df2 = 10), from = 0, to = 6, main = F(10,10))
```

F(10,10)



# Finally finishing that ANOVA

- So, how extreme is our F-value given the  $H_0$ ?
- Let's get the p-value for our F-value

```
(p_value <- 1 - pf(F_value, df1 = df_model, df2 = df_error))</pre>
```

## [1] 0.4521

## Residuals

It's 1 - pf since pf gives you the probability for an F value to be smaller than the value given. We want the opposite. If  $p \le .05$ , the effect of group is significant (spuriously in this case, since we know there was no effect).

# Can we just let R do it?

• Remember the formula format for the t-test command:

233

• summary() is necessary here to get the p-value

6293

• This is just a convention in r

27

### A different way, using a package

```
library(ez)
# ezANOVA wants a subject identifier, so let's generate one
# every row is a different subject, so we just need to make a new variable that numbers the rows
iqdata$subnum <- 1:30</pre>
```

#### ezANOVA

```
(iq_anova <- ezANOVA(iqdata, dv = iq, wid = subnum, between = group))

## Warning: Converting "subnum" to factor for ANOVA.

## $ANOVA
## Effect DFn DFd F p p<.05 ges
## 1 group 2 27 0.8177 0.4521 0.05711

##

## $`Levene's Test for Homogeneity of Variance`
## DFn DFd SSn SSd F p p<.05
## 1 2 27 52.68 1887 0.3769 0.6895</pre>
```

• Nice. What's ges and what is that second output?

#### Effect size

- $\bullet$  ges = generalised eta squared
- $\eta^2 = \frac{SS_{model}}{SS_{total}}$
- $\eta_G^2$  is  $\eta^2$  with a correction for repeated measures designs (if applicable; we'll talk about those later)
- $\eta^2$  is an estimate of the relationship between variance explained by the ANOVA model and total variance in the data
- Compare this to Cohen's d, another estimate of effect size:
- $d = \frac{\bar{x_1} \bar{x_2}}{s}$
- Cohen's d is an estimate of how large a difference in means is (in standard deviations)
- What is s? The standard deviation of the difference in means (from your sample)

# Levene's test for homogeneity of variance

```
iq_anova[2] # get the second element of the list iq_anova

## $`Levene's Test for Homogeneity of Variance`
## DFn DFd SSn SSd F p p<.05
## 1 2 27 52.68 1887 0.3769 0.6895</pre>
```

- When you perform an ANOVA, you make the assumption that the variances within each group are similar
- For example, the IQs within the BU group should not be more variable than the IQs within the Soton and the Oxford groups
- Levene's test compares the group variances
  - What's the test statistic for a test that compares variances? It's F!
  - The p-value tells you if the variances are significantly different between groups.

### Levene's test for homogeneity of variance

- You do not need Levene's test if you only have two independent groups (and ezANOVA won't do it then)
- If Levene's test is not significant, all is well
- If Levene's test is significant, you're violating the homogeneity of variance assumption
- Not a big issue if the sample sizes are equal for all groups (balanced design)
- If sample sizes aren't equal (unbalanced design) and the larger groups have higher variance, your ANOVA loses power
- If sample sizes aren't equal and the larger groups have lower variance, your ANOVA becomes anticonservative ( $\alpha$  increases)!

#### Alternative: Bartlett's test

```
bartlett.test(formula = iq ~ group, data = iqdata)

##

## Bartlett test of homogeneity of variances

##

## data: iq by group

## Bartlett's K-squared = 0.5702, df = 2, p-value = 0.7519
```

- Bartlett's test is more sensitive to deviations from normality than Levene's test
- It's up to you which one you want to use (with ezANOVA, Levene's test is more convenient)

# What to do if Levene's or Bartlett's tests are significant

- If your group sizes are equal, nothing to worry about.
- If not:
- Calculate the variance for each group and see if you're dealing with just a power issue or an  $\alpha$  issue
- If the largest group variance is less than 4 times the smallest group variance, you're good.
- If you have huge variance differences and there might be an  $\alpha$  issue:
  - Easiest solution: Fix the sample size issue (e.g. run more participants)!
  - Use linear mixed models (LMMs; more on that later)
  - Use specialised tests (this is SPSS's approach):
  - Welch's t-test (R has this test as oneway.test())
  - Brown-Forsythe instead of ANOVA (R has this test in the lawstats package)

- Post-hoc tests:
  - \* Games-Howell for unequal variance
  - \* Hochberg's GT2 for non-equal sample sizes

### The SPSS approach to statistics

- Throw as many obscure tests at the problem as you can
- This is a sales strategy: "We need to buy SPSS since no other program has the Games-Howell test!"
- In reality, the standard ANOVA is remarkably robust to all but the most extreme violations of its assumptions
- Specialised tests often come at a huge cost in terms of power
- This doesn't mean that you shouldn't test the assumptions
- But a simplistic strategy where you run one type of test if the assumption test is significant and another
  one if it isn't is not helpful
- Take a good look at your data
  - Be aware of potential issues
  - Interpret the data accordingly.
  - Only use specialised and non-parametric tests as a last resort if your data massively violate the assumptions

#### Just so we're clear

- Inflated  $\alpha$  is not harmless
- But "researcher degrees of freedom" inflate  $\alpha$  much more than all but the most extreme assumption violations
- Stopping rules (test after every X participants, then stop as soon as you have a significant result)
- Failing to report non-significant conditions
- Failing to correct for multiple comparisons
- Don't let SPSS (or over-cautious textbooks) discourage you from running plain, simple ANOVAs
- Be honest and transparent about your data and how you collected them and you'll be fine.

# Reporting a one-way ANOVA

- Let's walk through it together
- I stole this data set from Andy Johnson
- We are investigating the effect of swearing on pain tolerance (see Stephens et al., 2009)
- Three groups: continuous use of swear word, neutral word, or no word whilst hand in cold water (DV = time until participant can't stand the pain and pulls hand from water)

```
# open the data file (available on GitHub; don't forget to use setwd to set the working directory to wh
# setwd("C:/my_data/")
pain <- read.csv("pain.csv")</pre>
```

#### Get an idea of what's in the file

```
head(pain)
    X Swear.Condition Time.In.Cold.Water
## 1 1
           Swear Word
## 2 2
           Swear Word
                                      64
## 3 3
          Swear Word
                                      70
## 4 4
           Swear Word
                                      60
## 5 5
           Swear Word
                                      54
## 6 6
           Swear Word
                                      66
str(pain)
## 'data.frame':
                   60 obs. of 3 variables:
                       : int 1 2 3 4 5 6 7 8 9 10 ...
## $ Swear.Condition : Factor w/3 levels "Neutral Word",..: 3 3 3 3 3 3 3 3 3 ...
## $ Time.In.Cold.Water: int 55 64 70 60 54 66 48 72 63 65 ...
```

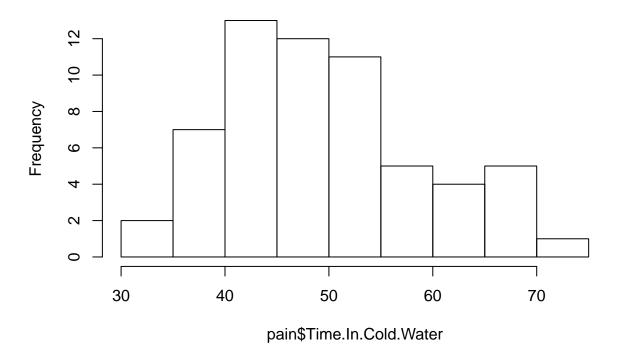
# Is the design balanced?

```
##
## Neutral Word Quiet Swear Word
## 20 20 20
```

### How is the DV distributed?

```
hist(pain$Time.In.Cold.Water)
```

# Histogram of pain\$Time.In.Cold.Water



#### Is the DV normal?

- Note: The raw DV does not have to be perfectly normal!
- In fact, your alternative hypothesis is that it's not, since it is influenced by the treatment effect!
- The error should be normal, though.
- That is, the residual data that you get when you take the treatment effect out should be normal.
- I'll show you how to test that in a second.

#### Calculate means and SDs

```
## Swear.Condition N Mean SD FLSD
## 1 Neutral Word 20 44.70 6.906 4.231
## 2 Quiet 20 45.75 5.711 4.231
## 3 Swear Word 20 60.45 7.323 4.231
```

### Calculate means and SEs

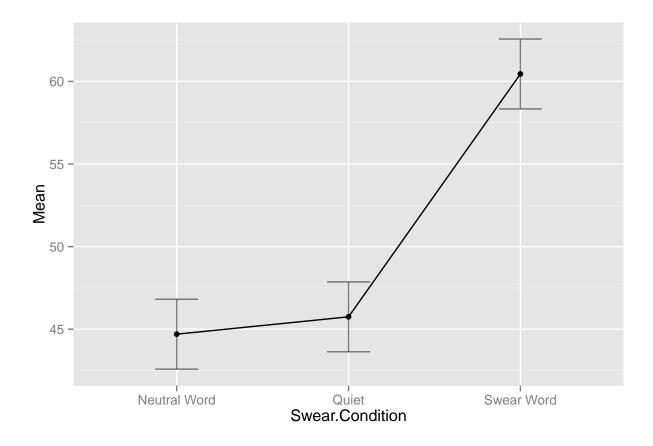
```
pain_stats
```

```
## Swear.Condition N Mean SD FLSD
## 1 Neutral Word 20 44.70 6.906 4.231
## 2 Quiet 20 45.75 5.711 4.231
## 3 Swear Word 20 60.45 7.323 4.231
```

- This gives you a nice overview of the group sizes, group means, and group variance/
- N is the group size. It's the same for each group, so your design is balanced.
- The mean is almost the same for the neutral and the quiet conditions, but higher in the swear word condition.
- SDs are quite similar, so it doesn't seem like the homogeneity of variance assumption is an issue
- Fisher's Least Significant Difference (FLSD): gives you an idea of the minimum effect size that would be significant in a t-test between any of these groups. This is just an estimate, not a replacement for the ANOVA and a post-hoc test.
- Still useful to see which differences might be significant and which definitely aren't.

#### Plot means

```
ezPlot(data = pain,
    dv = Time.In.Cold.Water,
    wid = subject,
    between = Swear.Condition,
    x = Swear.Condition)
```



# Notes about the plot

- This plot is pretty good (better than anything SPSS gives you), but it is not perfect
- Discrete factors should be plotted as bars, not lines
- The error bars are Fisher's LSD, which is helpful in interpreting the differences, but unusual
- In a plot for publication, you would also want to change the axis labels
- I will give you a better function to use later
- For now, this is perfectly fine

### Perform the ANOVA

- Note the return\_aov = TRUE.
- We need that because we want to look at the residuals later.

#### Look at the results

```
pain_anova$ANOVA
```

```
## Effect DFn DFd F p p<.05 ges
## 1 Swear.Condition 2 57 34.74 1.367e-10 * 0.5493
```

• Looks like there is a significant effect of swear condition.

```
pain_anova$"Levene's Test for Homogeneity of Variance"
```

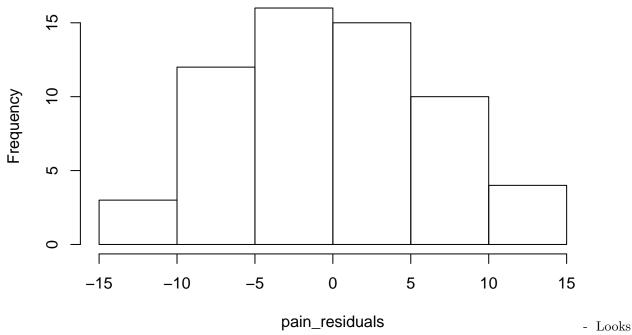
```
## DFn DFd SSn SSd F p p<.05
## 1 2 57 33.23 817.7 1.158 0.3213
```

• No issues with homogeneity of variance.

# Visually examine residuals

```
pain_residuals <- resid(pain_anova$aov)
hist(pain_residuals)</pre>
```

# Histogram of pain\_residuals

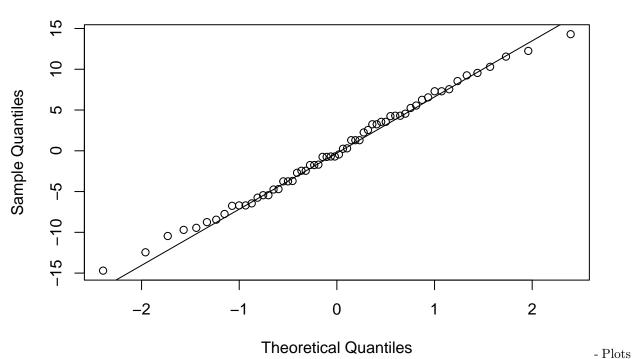


quite normal.

# Quantile-Quantile Plot

```
qqnorm(pain_residuals)
qqline(pain_residuals)
```

### Normal Q-Q Plot



the distribution of the variable in question against a hypothetical perfect normal distribution - Deviations from the line are deviations from normality - Our residuals are very slightly deviated. Is this an issue? Probably not.

# Formally test normality of residuals

• Shapiro-Wilk normality test

shapiro.test(pain\_residuals)

```
##
## Shapiro-Wilk normality test
##
## data: pain_residuals
## W = 0.9918, p-value = 0.9596
```

• Not significant at all. No issues with non-normality.

### Perform post-hoc pairwise comparisons

```
##
## Pairwise comparisons using t tests with non-pooled SD
##
## data: Time.In.Cold.Water and Swear.Condition
##
## Neutral Word Quiet
## Quiet 1 -
## Swear Word 7.6e-08 7.9e-08
##
## P value adjustment method: bonferroni
```

• These are the p-values for all the possible t-tests (Bonferroni correction applied)

## Perform post-hoc pairwise comparisons

• Better: Use Holm's correction instead of Bonferroni's correction for higher power

# Perform post-hoc pairwise comparisons

• If you have no issues with homogeneity of variance, you can use the same pooled sd for all the tests:

# Alternative: Tukey's HSD

```
TukeyHSD(x = pain_anova$aov)
```

• Another way of correcting for multiple comparisons. Assumes equal variance.

# Now: report it

I'll show you how in Homework 3.