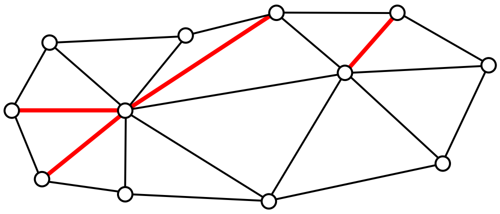
Mesh Triangulation

- **Triangle mesh**: 2-manifold with all polygons triangles
- Any polygon can be triangulated
- Vertices co-planar
- Efficient data structures and algorithms
- Barycentric interpolation



Barycentric Coordinates

- Concept proposed by A. Möbius (1827)
- For non-collinear distinct vertices $\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{n-1}$ and weights $\lambda_0, \lambda_1, \ldots, \lambda_{n-1}$ vertex $\mathbf{q}$ is **barycenter** iff

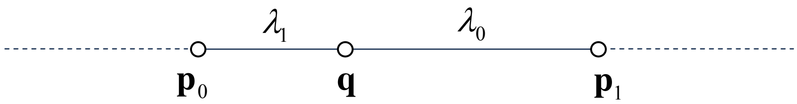
$$\mathbf{q} = \frac{\lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 + \cdots + \lambda_{n-1} \mathbf{p}_{n-1}}{\lambda_0 + \lambda_1 + \cdots + \lambda_{n-1}}$$

- Weights λ_i **barycentric coordinates** of $\mathbf{q}$ with respect to $\mathbf{p}_0, \mathbf{p}_1, \ldots, \mathbf{p}_{n-1}$
- Note: $k\lambda_0, k\lambda_1, \ldots, k\lambda_{n-1}$, $k \neq 0$ also barycentric coordinates of $\mathbf{q}$

Barycentric Coordinates

- Typical to employ **normalized barycentric coordinates** $\lambda_0 + \lambda_1 + \cdots + \lambda_{n-1} = 1$
- Note: for $\mathbf{q} = \mathbf{p}_i$ we have $\lambda_i = 1$ and $\lambda_{j \neq i} = 0$
- Example: barycentric coordinates on a line

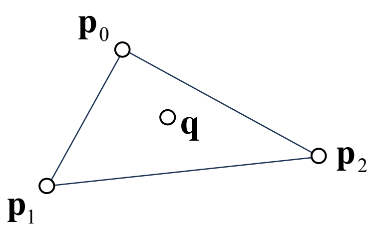
$$\mathbf{q} = \lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 = \lambda_0 \mathbf{p}_0 + (1 - \lambda_0)\mathbf{p}_1$$



Barycentric Coordinates in Triangles

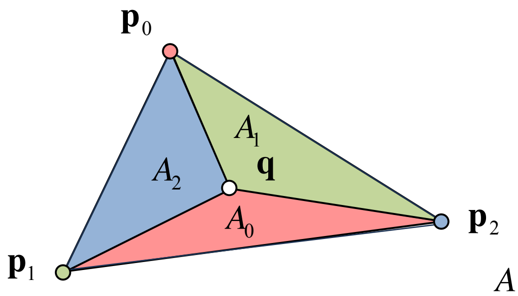
- Barycentric coordinates of vertex $\mathbf{q}$ in triangle given by vertices $\mathbf{p}_0$, $\mathbf{p}_1$, $\mathbf{p}_2$

$$\begin{aligned}\mathbf{q} &= \lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 + \lambda_2 \mathbf{p}_2 \\ &= \lambda_0 \mathbf{p}_0 + \lambda_1 \mathbf{p}_1 + (1 - \lambda_0 - \lambda_1)\mathbf{p}_2\end{aligned}$$



Barycentric Coordinates in Triangles

- Normalized barycentric coordinates unique in triangle
- In triangles also known as areal coordinates
- Directly related to ratio of (signed) areas of subtriangles formed by **q** to total triangle area



$$\lambda_0 = \frac{A_0}{A}$$

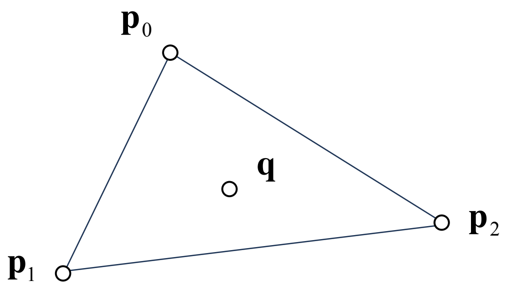
$$\lambda_1 = \frac{A_1}{A}$$

$$\lambda_2 = \frac{A_2}{A}$$

$$A = A_0 + A_1 + A_2$$

Barycentric Coordinates in Triangles

- Vertex **q** inside triangle iff barycentric coordinates $\lambda_i > 0$
- Vertex **q** at centroid iff $\lambda_0 = \lambda_1 = \lambda_2 = 1/3$
- Vertex **q** on line, e.g. between $\mathbf{p}_0$ and $\mathbf{p}_2$ iff $\lambda_1 = 0$



Barycentric Interpolation

- Barycentric coordinates useful for linear interpolation of function f over triangle area (e.g. during shading)
- Assume function values at triangle vertices

$$f(\mathbf{p}_i) = \mathbf{f}_i, \quad i = 0,1,2$$

- Interpolation function

$$g(\mathbf{x}) = \lambda_0 \mathbf{f}_0 + \lambda_1 \mathbf{f}_1 + \lambda_2 \mathbf{f}_2$$

  with barycentric coordinates λ_0, λ_1, λ_2 of vertex $\mathbf{x}$
- Note: interpolation also possible outside of triangle

Stored Properties in Data Structure

- **Geometry**: vertex coordinates
- **Topology**: faces, adjacency relationships
- **Attributes**: normals, colors, texture coordinates