Project Assignment: Educational Robotic Arm

Robotics 34753

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PURPOSE

The purpose of this project assignment is to practice the topics learned in the Robotics course 34753 on a realistic robotic application.

INTRODUCTION

In this project assignment, a DIY 4-joint robotic arm is considered, made of 4 Dynamixel servos. The robot is shown in the picture on the front page of this document and is used for educational and research purposes. It can be equipped with a camera and stylus on its end-effector in order to perform a variety of tasks. The project assignment treats tasks which involve direct and inverse kinematics, dynamics, singularities, simulation, trajectory planning and computer vision.

ABOUT THIS MATERIAL:

This document defines a project assignment which consists of **11 problems** organized in 5 different parts. Parts 1-4 are theoretical and can be solved without access to the actual hardware, while part 5 is practical and requires access to a robot arm in the lab and programming the actual robotic arm. Annex A contains a description of the 'independent joint control principle' which can be useful for improving the control of the robot arm.

CONTENTS:

Part 1 contains problems 1-5, all concerning different aspects of forward, inverse and velocity kinematics. Part 2 contains problems 6 and 7 which deal with trajectory planning. In part 3 you are asked to solve problems 8 an 9, which concern singularities and static loads, respectively. Part 4 contains problem 10 which is about the dynamics modelling of the robotic arm.
The last part 5, contains four different choices for problem 11, but you have to choose only one. This last problem is much more open than the previous 10, and requires a significant amount of effort and research for solving.

SOLVING AND REPORTING THE PROJECT ASSIGNMENT:

A project assignment hand-in is considered complete if all 11 problems are solved. The answers shall be supported by a sufficient amount of intermediate calculations and explanatory text, so that the principles and methods used are clear. All plots, tables, and equations must be numbered consecutively.

The report is assessed as a whole based on the quality of the explanatory text and the correctness of the answers. The last page of the report should be signed by the participant(s). Remember your 'study registration number'. The project assignment needs to be carried out by a team of four persons and the individual contribution to the project work must be clearly indicated for each of the 11

Problem solutions contained in the handed in report. Specify the work contribution to each problem by percentages, e.g. Elisabeth 30% / Peter 30% / Jack 40%.

DEADLINES:

The project report must be submitted via DTU Learn, NO LATER THAN 5pm, 6.Dec.2022.

THE PROJECT ASSIGNMENT

PART 1: KINEMATICS

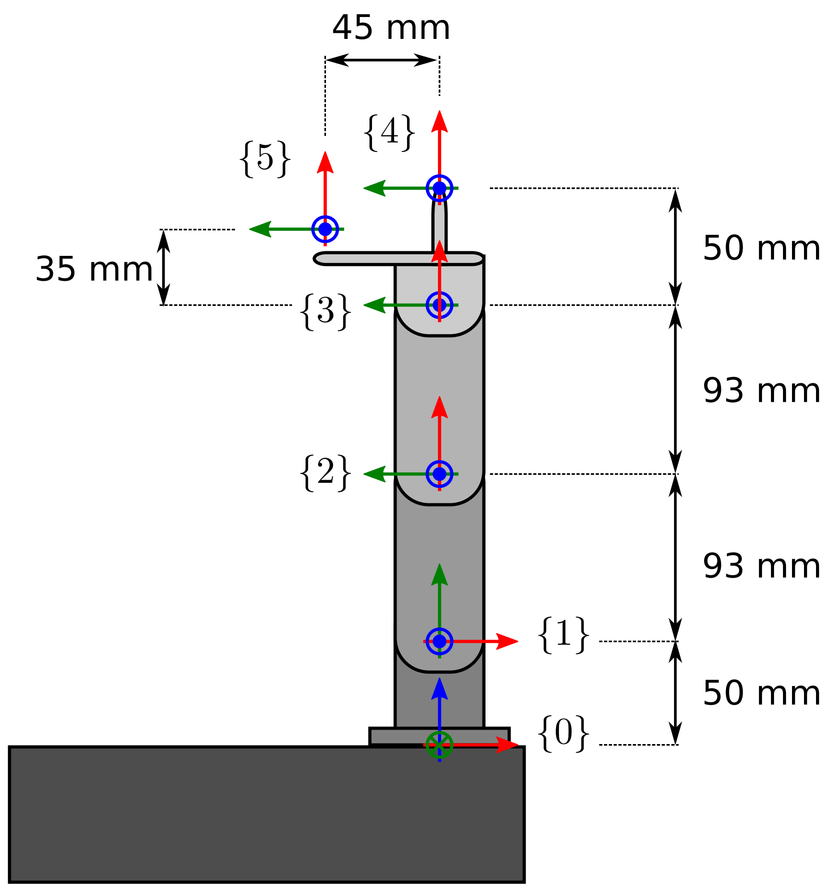

Figure 1: Robot model in its default configuration and fundamental dimensions.

•**Problem 1:** Find, by the use of Figure 1, the direct kinematic transformations, T_4^0 for the robot stylus, and T_5^0 for the robot camera, as function of all joint angles.

•**Problem 2:** Determine the inverse kinematic transformation

$$q = [q_1, q_2, q_3, q_4]^T = f\left(x_4^0, o_4^0\right)$$

where x_4^0 are the first 3 components of the first column of T_4^0, and o_4^0 are the first 3 components of the last column of T_4^0, respectively. Satisfy all position components in o_4^0 and as many components in x_4^0 as possible.

The robot manipulator is now supposed to track, with the stylus tip, a circle with center p_c

and radius R. The circle is defined by the equation:

$$p^0(\varphi) = p_c^0 + R \begin{bmatrix} 0 \\ \cos(\varphi) \\ \sin(\varphi) \end{bmatrix}$$

for $0 \leq \varphi \leq 2\pi$.

•**Problem 3:** Find a sequence of 37 robot configurations

$$q^{(j)} = [q_1^{(j)}, q_2^{(j)}, q_3^{(j)}, q_4^{(j)}]^T = f\left(x_4^0 = [?, ?, 0]^T,\ o_4^0 = p^0(\varphi_j)\right), \quad j = 0, 1, \ldots, 36$$

that are necessary for the stylus tip to track 36 equidistant points on a circle with $R = 32\,\text{mm}$ and $p_c^0 = [150, 0, 120]^T$ mm. The tracked points start at $\varphi_0 = 0$ and end at $\varphi_{36} = 2\pi$, while the stylus remains horizontal at all configurations.

•**Problem 4:** Determine the Jacobian of the manipulator for the robot end-effector and the Jacobian for the robot camera (as a function of the joint configuration q). Report the numerical results for the two Jacobians at $\varphi = 0$, $\varphi = \pi/2$, $\varphi = \pi$, and $\varphi = 3\pi/2$ along the path studies in Problem 3.

•**Problem 5:** Compute the joint velocities $\dot{q}$ at $\varphi = \pi/2$, along the path from Problem 3, so that the stylus tip velocity is $v_4^0 = [0, 0, 3]$ mm/s and $\dot{x}_4 = [?, ?, 0]$.
Hint: the last quantity $\dot{x}_4$, you have not seen it as such in the course before, so you need to think about how to interpret it in terms of angular velocities.

PART 2: TRAJECTORY PLANNING

In this part, the goal is to plan a trajectory which approximates the circular path from Problem 3 by means of 5 knot-points at φ_0, φ_9, φ_{18}, φ_{27}, φ_{36}.

•**Problem 6:** Use the inverse computed joint configurations $q^{(0)}$, $q^{(9)}$, $q^{(18)}$, $q^{(27)}$, $q^{(36)}$ from Problem 3, to find suitable interpolation polynomials for the following segments:

Segment A: $q^{(0)} \to q^{(9)}$, $0 \le t_A \le 2$ s

$$\begin{aligned}
q_1(t_A) &= A_{15} \cdot t_A^5 + A_{14} \cdot t_A^4 + A_{13} \cdot t_A^3 + A_{12} \cdot t_A^2 + A_{11} \cdot t_A + A_{10} \\
q_2(t_A) &= A_{25} \cdot t_A^5 + A_{24} \cdot t_A^4 + A_{23} \cdot t_A^3 + A_{22} \cdot t_A^2 + A_{21} \cdot t_A + A_{20} \\
q_3(t_A) &= A_{35} \cdot t_A^5 + A_{34} \cdot t_A^4 + A_{33} \cdot t_A^3 + A_{32} \cdot t_A^2 + A_{31} \cdot t_A + A_{30} \\
q_4(t_A) &= A_{45} \cdot t_A^5 + A_{44} \cdot t_A^4 + A_{43} \cdot t_A^3 + A_{42} \cdot t_A^2 + A_{41} \cdot t_A + A_{40}
\end{aligned}$$

Segment B: $q^{(9)} \to q^{(18)}$, $0 \le t_B \le 2$ s

$$\begin{aligned}
q_1(t_B) &= B_{15} \cdot t_B^5 + B_{14} \cdot t_B^4 + B_{13} \cdot t_B^3 + B_{12} \cdot t_B^2 + B_{11} \cdot t_B + B_{10} \\
q_2(t_B) &= B_{25} \cdot t_B^5 + B_{24} \cdot t_B^4 + B_{23} \cdot t_B^3 + B_{22} \cdot t_B^2 + B_{21} \cdot t_B + B_{20} \\
q_3(t_B) &= B_{35} \cdot t_B^5 + B_{34} \cdot t_B^4 + B_{33} \cdot t_B^3 + B_{32} \cdot t_B^2 + B_{31} \cdot t_B + B_{30} \\
q_4(t_B) &= B_{45} \cdot t_B^5 + B_{44} \cdot t_B^4 + B_{43} \cdot t_B^3 + B_{42} \cdot t_B^2 + B_{41} \cdot t_B + B_{40}
\end{aligned}$$

Segment C: $q^{(18)} \to q^{(27)}$, $0 \le t_C \le 2$ s

$$\begin{aligned}
q_1(t_C) &= C_{15} \cdot t_C^5 + C_{14} \cdot t_C^4 + C_{13} \cdot t_C^3 + C_{12} \cdot t_C^2 + C_{11} \cdot t_C + C_{10} \\
q_2(t_C) &= C_{25} \cdot t_C^5 + C_{24} \cdot t_C^4 + C_{23} \cdot t_C^3 + C_{22} \cdot t_C^2 + C_{21} \cdot t_C + C_{20} \\
q_3(t_C) &= C_{35} \cdot t_C^5 + C_{34} \cdot t_C^4 + C_{33} \cdot t_C^3 + C_{32} \cdot t_C^2 + C_{31} \cdot t_C + C_{30} \\
q_4(t_C) &= C_{45} \cdot t_C^5 + C_{44} \cdot t_C^4 + C_{43} \cdot t_C^3 + C_{42} \cdot t_C^2 + C_{41} \cdot t_C + C_{40}
\end{aligned}$$

Segment D: $q^{(27)} \to q^{(36)}$, $0 \le t_D \le 2$ s

$$\begin{aligned}
q_1(t_D) &= D_{15} \cdot t_D^5 + D_{14} \cdot t_D^4 + D_{13} \cdot t_D^3 + D_{12} \cdot t_D^2 + D_{11} \cdot t_D + D_{10} \\
q_2(t_D) &= D_{25} \cdot t_D^5 + D_{24} \cdot t_D^4 + D_{23} \cdot t_D^3 + D_{22} \cdot t_D^2 + D_{21} \cdot t_D + D_{20} \\
q_3(t_D) &= D_{35} \cdot t_D^5 + D_{34} \cdot t_D^4 + D_{33} \cdot t_D^3 + D_{32} \cdot t_D^2 + D_{31} \cdot t_D + D_{30} \\
q_4(t_D) &= D_{45} \cdot t_D^5 + D_{44} \cdot t_D^4 + D_{43} \cdot t_D^3 + D_{42} \cdot t_D^2 + D_{41} \cdot t_D + D_{40}
\end{aligned}$$

Determine the coefficients A_{ij}, B_{ij}, C_{ij}, D_{ij} so that

$$\begin{aligned}
v(t_A{=}0) &= [0,0,0]^T \text{mm/s} & \ddot{q}(t_A{=}0) &= [0,0,0,0]^T \text{ rad/s}^2 \\
v(t_A{=}2) = v(t_B{=}0) &= [0,-4,0]^T \text{mm/s} & \ddot{q}(t_A{=}2) = \ddot{q}(t_B{=}0) &= [0,0,0,0]^T \text{ rad/s}^2 \\
v(t_B{=}2) = v(t_C{=}0) &= [0,0,-4]^T \text{mm/s} & \ddot{q}(t_B{=}2) = \ddot{q}(t_C{=}0) &= [0,0,0,0]^T \text{ rad/s}^2 \\
v(t_C{=}2) = v(t_D{=}0) &= [0,4,0]^T \text{mm/s} & \ddot{q}(t_C{=}2) = \ddot{q}(t_D{=}0) &= [0,0,0,0]^T \text{ rad/s}^2 \\
v(t_D{=}2) &= [0,0,0]^T \text{mm/s} & \ddot{q}(t_D{=}2) &= [0,0,0,0]^T \text{ rad/s}^2
\end{aligned}$$

Hint: Note that end-effector velocities v need to be converted to joint velocities $\dot{q}$ and that requiring zero joint acceleration at knot points is maybe not the best but the simplest option.

•**Problem 7:** Plot the actual path of the end-effector as a function of time for the interpolated trajectory from Problem 6 and its deviation from the desired circular path. Try to improve the approximation either by using more knot-points or by using different interpolation functions than those found in Problem 6.

PART 3: SINGULARITIES AND STATICS

•**Problem 8:** Plot the condition number of the Jacobian matrix of the manipulator along the path from Problem 3 as well as along the actual path from Problem 6 or 7, and evaluate if the path includes any singularities.

•**Problem 9:** Neglecting the own mass of the robot arm, and assuming a weight of 1 N along the negative z_0 direction, calculate and plot all joint torques τ_1, τ_2, τ_3, τ_4 as a function of the position $\varphi \in [0, 2\pi]$. Hint: Neglect friction and other losses.

PART 4: DYNAMICS

Figure 2 shows the positions of the centers of mass for the four links and defines the corresponding principal axes of inertia.

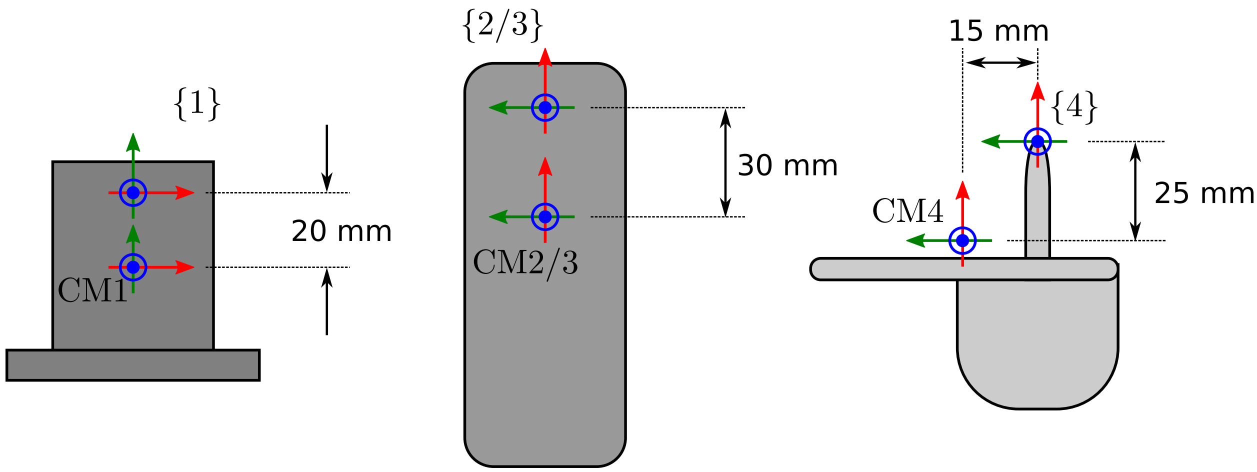

Figure 2: Centers of mass and principal axes of inertia for links 1, 2, 3 and 4.

The mass of link 1 is 60 g and its matrix of inertia in the frame CM1 is in the form:

$$\bar{D}_1 = \begin{bmatrix} I_0 & 0 & 0 \\ 0 & 0.4I_0 & 0 \\ 0 & 0 & 0.9I_0 \end{bmatrix}$$

The masses of links 2 and 3 are equal to 80 g and their matrices of inertia respectively in the frames CM2 and CM3 are in the form:

$$\bar{D}_2 = \bar{D}_3 = \begin{bmatrix} 0.45I_0 & 0 & 0 \\ 0 & 1.4I_0 & 0 \\ 0 & 0 & 1.2I_0 \end{bmatrix}$$

Finally, the mass of link 4 (including the camera) is 40 g and its matrix of inertia in the frame CM4 is in the form:

$$\bar{D}_4 = \begin{bmatrix} 0.5I_0 & 0 & 0 \\ 0 & 0.5I_0 & 0 \\ 0 & 0 & 0.5I_0 \end{bmatrix}$$

•**Problem 10:** Provide a rough but realistic estimate for I_0 based on the dimensions and masses of the links. Then derive the dynamic system for the robot arm in its standard form

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

and plot the required joint torques τ for the trajectory of Problem 6 or 7.

PART 5: COMPUTER VISION AND CONTROL

The robot arm is implemented with 4 robotis servos of type AX-12A which can be controlled from Matlab or Python through a usb port serial communication protocol. The servo specifications are available at: `https://emanual.robotis.com/docs/en/dxl/ax/ax-12a/` and the communication API is based on a Port Handler object for Matlab) or Python and a Packet Handler object for Matlab) or Python. You will receive some code template for communicating with the robot.

In this part of the assignment you are expected to use the actual robot arm, and a camera mounted at the position of frame 5, in order to perform some complex tasks that involve image recognition.

Choose and implement one of Problems 11a, 11b, 11c, 11d, or a problem of similar complexity after agreement with the course responsibles.

•**Problem 11a:** Create a software which enables the robot arm to recognize the keys of a keyboard that is placed in front of it and it types "hello world".

•**Problem 11b:** Create a software which enables the robot arm to recognize all red "smarties" among a cluster of colourful "smarties" on a surface in front of it, and to probe all of them with its stylus.

•**Problem 11c:** Create a software which enables the robot arm to track a black line drawn on an A4 page, placed in front of the robot, with the tip of its stylus.

•**Problem 11d:** Create a software which enables the robot arm to enter its stylus through a ring that you place in front of it.

For any of the problems that you choose to solve, you must provide details about the theory behind all necessary calculations, the code implementation (in Matlab or Python) and a video recording demonstrating how well the robot is performing the desired task.

Hint: for extracting depth information with only 1 camera, you can combine the information from images from multiple poses of the end-effector.

Annex A: Independent joint control

Independent joint control is a well-proven control principle for robot control. By this kind of control, the control signal for link i is generated exclusively by considering the position, (and sometimes also the derivative of the position for link i i.e. the speed of link i), that is $u_i = u_i(q_i)$.

The independent joint control is often implemented by the use of PD and PID control combined with a proper pre-filtering and feed-forward control. These classic control concepts have proven to be sufficient for use with large gear reduction ratios whereby the nonlinear and coupled structure of the robot become of minor importance.

There exist methods to handle the nonlinear closed control loop systems (e.g. Lyaponov functions, hyper stability). However, these methods for handling the non-linearities are typically concentrated on the stability of the system rather than on the system performance.

A very much-applied method is to use classic linear single input single output control theory (known from a basic control theory courses) on a linearized or otherwise simplified control object.

A simplified control scheme will be examined in the following. The dynamics model of the robot manipulator can be described by the following equation:

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

This equation may be rewritten to:

$$\hat{D}(q)\ddot{q} = \tau + (\hat{D}(q) - D(q))\ddot{q} - C(q,\dot{q})\dot{q} - g(q) = \tau - T_L$$

where

$$\hat{D}(q) = \mathrm{diag}(I_{1\mathrm{max}}, \ldots, I_{m\mathrm{max}})$$

is a constant diagonal matrix determined by

$$I_{i\mathrm{max}} = \sup_q(d_{ii}),$$

$I_{i\mathrm{max}}$ is the largest moment of inertia for joint axis i. A critical damped or over-damped closed loop system is secured which is desirable for robot control by choosing these maximum moments of inertia. Critically damped or over-damped closed loop systems are preferred for robot control because overshoots cannot be accepted.

T_L can be considered as a disturbance and is considered independent of q when used in simple stability analysis.