Oblig2 STK-IN Bernhard

November 17, 2019

```
[1]: import pandas as pd # For dataframe
     import numpy as np # For matrix operations
     import sklearn.preprocessing as sklpre # For preprocessing (scaling)
     import sklearn.linear model as skllm # For OLS
     import sklearn.model_selection as sklms # For train_test_split
     from scipy import stats # To calc p-value
     import matplotlib.pyplot as plt # For plotting
     import pygam # For generalized additive models
     import sklearn.ensemble as sklen # For ensemble methods
     import sklearn.neighbors as sklne # For nearest neighbors
     import sklearn.tree as skltree # For trees
     import sklearn.neural_network as sklnn # For neural networks
     import warnings
     from sklearn.exceptions import ConvergenceWarning
     # Ignoring annoying convergence warnings from Scikit-Learn, we can't expect
     → every neural network to converge
     warnings.filterwarnings("ignore", category=ConvergenceWarning)
     np.random.seed(len("jeff")) # Setting seed
     # For automatic formatting of code, sparing you from my usually horrible looking_
     \hookrightarrow code
     %load_ext nb_black
```

<IPython.core.display.Javascript object>

1 Task 1.1

I have chosen to one-hot encode the SEX-category, as neither male nor female should be considered adifferent numbers. The rest of the categorical values are just true/false, so those aren't encoded. Then I scale all the scalar features, not touching the categorical features.

```
[2]: # Reading the data into dataframe
df = pd.read_csv("data_task_1.txt", header=0, sep=" ")
# Onehot-encoding gender
```

```
onehot_gender = pd.get_dummies(df["SEX"]).set_axis(
    ["Male", "Female"], axis=1, inplace=False
# Replacing old gender column
df = df.join(onehot_gender)
df.drop("SEX", axis=1, inplace=True)
# List of boolean categories
categorical = [
    "ADHEU",
    "HOCHOZON",
    "AMATOP",
    "AVATOP",
    "ADEKZ",
    "ARAUCH",
    "FSNIGHT",
    "FSPT",
    "FSATEM",
    "FSAUGE".
    "FSPFEI",
    "FSHLAUF",
    "Male",
    "Female",
]
# A loop that splits the data and tries again until there is no split where only
→ one modality is in one split
first = True
while (
   first
    or np.any(
        np.logical_or(X_train.sum(axis=0) == 0, X_train.sum(axis=0) == X_train.
\rightarrowshape[0])
    )
    or np.any(
        np.logical_or(X_test.sum(axis=0) == 0, X_test.sum(axis=0) == X_test.
\rightarrowshape [0])
    )
):
    first = False
    # Splitting over and over until splits are good, stratifying the most biased
    X_train, X_test, y_train, y_test = sklms.train_test_split(
        df.loc[:, df.columns != "FFVC"],
        df["FFVC"],
        test_size=0.5,
        stratify=df["FSATEM"],
    )
```

```
# Scaling scalar features based on train set
scaler = sklpre.StandardScaler()
X_train_continous = scaler.fit_transform(
   X_train.loc[:, np.logical_not(np.isin(X_train.columns, categorical))].values
X_test_continous = scaler.transform(
   X_test.loc[:, np.logical_not(np.isin(X_test.columns, categorical))].values
# Putting all scalar and categorical features together
X train.loc[
    :, np.logical_not(np.isin(X_train.columns, categorical))
] = X_train_continous
X_test.loc[:, np.logical_not(np.isin(X_test.columns, categorical))] = 

→X_test_continous

# Need to specify datatypes because it chose a dumb datatype by default
X_train = X_train.astype({"Male": "int32", "Female": "int32"})
X_test = X_test.astype({"Male": "int32", "Female": "int32"})
# All preprocessing done!
```

2 Task 1.2

Running OLS, calculating uncertainties and p-values

```
[3]: def get_summary_linear_model(model, X_train, y_train):
         Scikit-learn has no built in support for confidence intervals and p-values,,,
      \hookrightarrowso I
         made this to calculate it for me after fitting the model. Put into a_{\sqcup}
      \hookrightarrow function for reuse.
         11 11 11
         # Combining intercept and coefficients in same array
         coefficients = np.append(model.intercept_, model.coef_)
         # Predicting y
         y_hat = model.predict(X_train)
         # Calculating RSS to get variance for use when calculating stddev of coeffs
         residuals = y_train.values - y_hat
         rss = residuals.reshape(-1, 1).T @ residuals.reshape(-1, 1)
         var = rss[0, 0] / (len(X_train) - len(X_train.columns) - 1)
         # Adding intercept to X train, as sklearn usually does not need the column
      \hookrightarrow of 1's
         X_with_intercept = np.append(
```

```
np.ones(X_train.shape[0]).reshape(-1, 1), X_train, axis=1
)
# Stddev of coefficients
stddev = np.sqrt(
    (np.diag(var * np.linalg.pinv(X_with_intercept.T @ X_with_intercept)))
labels = ["Intercept"] + X_train.columns.tolist()
coef_over_std = coefficients / stddev
p values = [
    2 * (1 - stats.t.cdf(np.abs(i), (len(X_with_intercept) - 1)))
    for i in coef_over_std
]
# Putting results into table
coeffs_table = pd.DataFrame(zip(labels, coefficients, stddev, p_values))
# Giving nice names with TeX formatting
coeffs table.rename(
    columns={0: "Feature", 1: r"$\beta_i$", 2: r"$\pm$", 3: "p-values"},
    inplace=True,
)
return coeffs_table
```

Got an R^2 score of 0.65 for the test set. The most important feature (lowest p-value) is FLGROSS.

```
[4]: Feature $\beta_i$ $\pm$ p-values
0 Intercept 2.314333 0.028610 0.000000e+00
1 ALTER 0.021497 0.016804 2.020056e-01
2 ADHEU -0.086719 0.065251 1.850741e-01
3 HOCHOZON -0.113580 0.042905 8.637147e-03
4 AMATOP 0.044360 0.034373 1.980698e-01
```

```
5
      AVATOP
              -0.040302 0.034556 2.446285e-01
6
       ADEKZ
                        0.037623
                                  7.002223e-01
              -0.014503
7
      ARAUCH
              -0.029416 0.031876
                                 3.569903e-01
8
     AGEBGEW
               0.000868
                        0.015130 9.543048e-01
9
     FSNIGHT
               0.061645 0.051301
                                  2.306612e-01
10
     FLGROSS
               0.176560 0.021877
                                  3.064216e-14
11
       FMILB
              -0.036336 0.021645 9.446940e-02
12
      FNOH24
             -0.045788 0.019995 2.286794e-02
13
       FTIER
               0.008533 0.016562 6.068483e-01
14
       FPOLL
              -0.045686 0.027496
                                 9.788058e-02
15
    FLTOTMED
              -0.031583 0.014946
                                  3.559735e-02
16
      F03H24
               0.068101 0.029741 2.287646e-02
17
        FSPT
               0.075925 0.069903 2.784689e-01
18
      FTEH24
              -0.045609 0.027235 9.526779e-02
19
      FSATEM
               0.079570 0.080771 3.255187e-01
20
      FSAUGE
               0.035503 0.046510
                                 4.459981e-01
21
               FLGEW
22
      FSPFEI
               0.067144
                        0.076172
                                  3.789117e-01
23
     FSHLAUF
              -0.004718
                        0.056847
                                  9.339289e-01
                                  2.067764e-06
24
               0.100242
        Male
                        0.020617
             -0.100242 0.020536
25
      Female
                                 1.893804e-06
```

The most important (lower p-value) feature seems to be FLGROSS. Some other important features seem to be gender. Male and female seem to completely cancel each other, implying that men are of higher risk?

3 Task 1.3

Scikit-learn for some reason doesn't have built in forward and backward selection, so I will create my own functions!

```
[5]: def backward_elimination(regressor, X_train, y_train, max_p_limit):
    """
    Takes a regressor, training set and a max p-value, runs backward
    elimination and returns the regresson fitted on the reduced
    features, the reduced feature matrix, a table of betas,
    standard deviations and p-values and the removed features
    """
    # Fitting regressor on full model
    regressor.fit(X_train, y_train)
    # Getting table of p-values to find what to eliminate
    result_table = get_summary_linear_model(regressor, X_train, y_train)
    p_values = result_table["p-values"].values
```

```
p_val_max, p_val_max_pos = p_values[1:].max(), p_values[1:].argmax() + 1
# Getting name of feature with highest p-val to make list of removed features
feature_max_p_val = result_table["Feature"][p_val_max_pos]
removed_features = [feature_max_p_val]
# Dropping feature with highest p-val
X_reduce = X_train.drop(columns=feature_max_p_val, inplace=False)
# Running backwards elimination until all p-values are below limit
while p_val_max > max_p_limit:
    # Fitting on reduced model
   regressor.fit(X_reduce, y_train)
   result_table = get_summary_linear_model(regressor, X_reduce, y_train)
   p_values = result_table["p-values"].values
   p_val_max, p_val_max_pos = p_values[1:].max(), p_values[1:].argmax() + 1
   feature_max_p_val = result_table["Feature"][p_val_max_pos]
    # If one or more features have to high p-value, remove
    if p_val_max > max_p_limit:
        # Append name to list that keeps track of removed features
        removed_features.append(feature_max_p_val)
        # Dropping feature
        X_reduce.drop(columns=feature_max_p_val, inplace=True)
        # Sorting features
        X_reduce.sort_index(axis=1, inplace=True)
        # Fitting reduced model
        regressor.fit(X_reduce, y_train)
return regressor, X_reduce, result_table, removed_features
```

```
[6]: # Model to send in to function. It is an OLS regressor.
     base_regressor = skllm.LinearRegression()
     # Running backwards elimination
         regressor reduced,
         X_reduce_train,
         result table reduced,
         removed_features,
     ) = backward elimination(base regressor, X train, y train, max p_limit=1e-2)
     # Using list of reduced features to also create test feature matrix with same_{\sqcup}
     \hookrightarrow features
     X_reduce_test = X_test.drop(columns=removed_features).sort_index(axis=1)
     # Printing results
     print(
         f"R^2 score of full model: {ols_reg.score(X_test, y_test):.2f} Backward⊔
      →Model: {regressor_reduced.score(X_reduce_test, y_test):.2f}"
     )
```

result_table_reduced R^2 score of full model: 0.65 Backward Model: 0.66 [6]: Feature \$\beta_i\$ \$\pm\$ p-values 0 Intercept 2.267274 0.009212 0.000000e+00 1 FLGEW 0.059817 0.019108 1.954982e-03 2 FLGROSS 0.193067 0.019126 0.000000e+00 3 Female -0.097007 0.014604 1.948932e-10 4 0.097007 0.014675 2.343874e-10 Male <IPython.core.display.Javascript object> [7]: def forward_selection(regressor, X_train, y_train, max_p_limit): Takes a regressor, training set and a max p-value, runs forward selection and returns the regresson fitted on the reduced features, the reduced feature matrix, a table of betas, standard deviations and p-values and the removed features X_null = pd.DataFrame({"null": np.zeros_like(y_train)}) regressor.fit(X_null, y_train) # The p-value for the O-column is invalid, but also not used, so I ignore_ \rightarrow the warnings with np.errstate(invalid="ignore"): # Getting results for null-model result_table = get_summary_linear_model(regressor, X_null, y_train) # p-value for intercept p_val_max = result_table["p-values"][0] # Dataframe used for incresing X_increased = pd.DataFrame() # List of features features = X_train.columns.values # while max p-val is below threshold, repeat while p val max < max p limit:</pre> # Set best p to infinity so that all values are less best_p = np.inf # Looping over features for feature in features: # Creating new column with feature in loop new_col = pd.DataFrame({feature: X_train[feature].values}) # If null model we need to append to the dataframe differently than $\rightarrow usual$

if len(X_increased.values) == 0:

Adding new feature to null model

X_candidate = X_increased.append(new_col)

```
else:
               # Adding new feature to model
               new_col_names = np.append(
                   X_increased.columns.values, new_col.columns.values
               X_candidate = pd.DataFrame(
                   np.append(X_increased.values, new_col.values, axis=1),
                   columns=new_col_names,
               )
           # Fitting increased model to find p-value
           regressor.fit(X candidate, y train)
           result_table = get_summary_linear_model(regressor, X_candidate,_
→y_train)
           p_i = result_table["p-values"].values[-1]
           # This if-statement is used to find the minimum p-value of the
→potential features to add
           if p_i < best_p:</pre>
               best_p = p_i
               best_new_feature = feature
       # Now that we have the best feature to add, we add it properly
       new_col = pd.DataFrame({best_new_feature: X_train[best_new_feature].
→values})
       if len(X increased.values) == 0:
           X_candidate = X_increased.append(new_col)
       else:
           new_col_names = np.append(
               X_increased.columns.values, new_col.columns.values
           X_candidate = pd.DataFrame(
               np.append(X_increased.values, new_col.values, axis=1),
               columns=new_col_names,
           )
       # Get results for new model
       result_table = get_summary_linear_model(regressor, X_candidate, y_train)
       p_val_max = result_table["p-values"].values.max()
       # Sorting features
       X increased = X candidate.sort index(axis=1)
       # Removing added feature from list of potential features so that we can't
\rightarrow add it again next iteration
       features = features[features != best_new_feature]
   # List of omitted features
   omitted features = features
   # Fitting increased model
   regressor.fit(X_increased, y_train)
   # Table of results for best model
```

```
result_table_best = get_summary_linear_model(regressor, X_increased, y_train)
return regressor, X_increased, result_table_best, omitted_features
```

```
[8]: # Model to send in to function. It is an OLS regressor.
     base_regressor = skllm.LinearRegression()
     # Running forward selection
     (
         regressor_increased,
         X_increased_train,
         result_table_increased,
         omitted_features_increased,
     ) = forward_selection(base_regressor, X_train, y_train, max_p_limit=1e-2)
     # Using list of reduced features to also create test feature matrix with same,
     X_increased_test = X_test.drop(columns=omitted_features_increased).
      \rightarrowsort_index(axis=1)
     # Printing results
     print(
         f"R^2 score of forward model: {regressor_increased.score(X_increased_test,_
      \rightarrowy_test):.2f}"
     result_table_increased
```

R^2 score of forward model: 0.66

```
[8]:
         Feature $\beta_i$
                                $\pm$
                                          p-values
       Intercept
                   2.267274 0.009212 0.000000e+00
                   0.059817 0.019108 1.954982e-03
    1
           FLGEW
    2
         FLGROSS
                   0.193067 0.019126 0.000000e+00
    3
          Female -0.097007 0.014604 1.948932e-10
            Male
                   0.097007 0.014675 2.343874e-10
```

<IPython.core.display.Javascript object>

For the reduced models with $p_{\text{max}} = 0.01$, both forward and backward selection give the exact same model, and therefore the same features. They also get a slightly better R^2 -score, possibly because they have less features, and are therefore less likely to overfit on the training data. I chose to look at R^2 instead of MSE as I feel it is a more intuitive value. However, higher R^2 also implies lower MSE, so the models are better.

Next I will test with a less strict $p_{\text{max}} = 0.1$ and see how the models perform:

```
[9]: # Model to send in to function. It is an OLS regressor.
base_regressor = skllm.LinearRegression()
# Running backwards elimination
```

```
(
        regressor_reduced_less_strict,
        X_reduce_train_less_strict,
        result_table_reduced_less_strict,
        removed_features_less_strict,
    ) = backward_elimination(base_regressor, X_train, y_train, max_p_limit=1e-1)
     # Using list of reduced features to also create test feature matrix with same_
     \rightarrow features
    X_reduce_test_less_strict = X_test.drop(
        columns=removed_features_less_strict
    ).sort_index(axis=1)
    # Printing results
    print(
        f"R^2 score of full model: {ols_reg.score(X_test, y_test):.2f}"
        + f" Backward Model: {regressor_reduced_less_strict.
     ⇔score(X_reduce_test_less_strict, y_test):.2f}"
    result_table_reduced_less_strict
    R^2 score of full model: 0.65 Backward Model: 0.67
[9]:
          Feature $\beta_i$
                                 $\pm$
                                            p-values
    0
        Intercept
                   2.325300 0.020509 0.000000e+00
    1
            FLGEW 0.061588 0.018903 1.278942e-03
    2
          FLGROSS 0.193914 0.019226 0.000000e+00
    3
         FLTOTMED -0.028568 0.014493 4.982569e-02
           FNOH24 -0.043130 0.019478 2.772511e-02
    4
    5
           F03H24 0.065094 0.028925 2.530011e-02
            FPOLL -0.033717 0.014188 1.824031e-02
    6
    7
           FSPFEI 0.127272 0.059145 3.237879e-02
           FTEH24 -0.048164 0.026385 6.914913e-02
    8
    9
           Female -0.100690 0.017013 1.079644e-08
    10
         HOCHOZON -0.098319 0.040697 1.642320e-02
             Male 0.100690 0.017617 3.142613e-08
    11
    <IPython.core.display.Javascript object>
```

```
[10]: # Model to send in to function. It is an OLS regressor.
base_regressor = skllm.LinearRegression()
# Running forward selection
(
    regressor_increased_less_strict,
    X_increased_train_less_strict,
    result_table_increased_less_strict,
    omitted_features_increased_less_strict,
) = forward_selection(base_regressor, X_train, y_train, max_p_limit=1e-1)
```

R^2 score of forward model: 0.66

```
[10]:
          Feature $\beta_i$
                                 $\pm$
                                            p-values
                    2.267274 0.009212 0.000000e+00
        Intercept
     1
            FLGEW
                    0.059817 0.019108 1.954982e-03
     2
          FLGROSS
                    0.193067 0.019126 0.000000e+00
     3
           Female -0.097007 0.014604 1.948932e-10
     4
                    0.097007 0.014675 2.343874e-10
             Male
```

<IPython.core.display.Javascript object>

Now the models are not the same anymore. This is to be expected, as the p-values estimated are not the same for each feature independent of the other features. The backward elimination model seems to give a better R^2 -score this time.

4 Task 1.4

CV is easily implemented in Scikit-Learn. Bootstap on the other hand... I need to create my own class (Maybe there is a better way of doing this than what I'm doing...)

```
[11]: # 5-fold cross validation, n-jobs=-1 is for parallelisation (use multiple cpu

cores)

lasso_cv = skllm.LassoCV(n_jobs=-1, cv=5).fit(X_train, y_train)

# List of hyperparameters

lambdas_lasso_cv = lasso_cv.alphas_

# List of validation mean squared errors. Need to average them over axis 1 to

get average across all 5 folds

mses_lasso_cv = lasso_cv.mse_path_.mean(axis=1)

print(

f"Best hyperparameter is {lasso_cv.alpha_:.2e}, giving a test R^2 score of

{lasso_cv.score(X_test, y_test):.2f}"

)
```

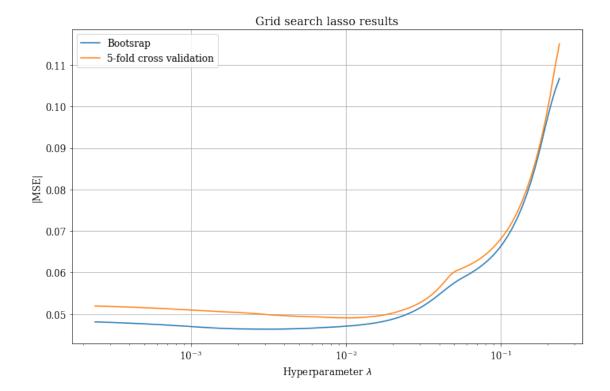
Best hyperparameter is 1.04e-02, giving a test R^2 score of 0.65

This seems to be very similar to forward and backward selection. Now I need to make a new class for bootstrap manually implemented.

```
[12]: class Bootstrap:
          def __init__(self, y):
               I was sort of confused on how to this, so I just made a class and tried,
       \hookrightarrow stuff.
              Now it works, so I won't change it anymore.
               This class takes y_train and saves its length.
              self.len_y = len(y)
          @property
          def get_bootstrap(self):
              This method returns one train-validation bootstrap split of the training
       \rightarrow data (indices of the data).
               The Oproperty is just so that i can call get\_bootstrap without the ()_{\sqcup}
       \rightarrow after (like in get_bootstrap()).
              This splits train and validation into 2/3, 1/3 of the length of the data.
       \rightarrow Not sure if that is the correct
              method.
               11 11 11
               # All indices (0-lenght of y)
              indices = np.arange(self.len_y)
               # Picking random indices with replacement.
              indices_train = np.random.choice(
                   indices, replace=True, size=int(self.len_y * 0.67)
               indices_validate = np.random.choice(
                   indices, replace=True, size=int(self.len_y * 0.37)
               # Returns a list of lists
              return [indices_train.tolist(), indices_validate.tolist()]
      # Creating instance of Bootstrap class
      bootstrap = Bootstrap(y_train)
      splits = []
      # This loops creates 100 different bootstrap samples
      for i in range(100):
          splits.append(bootstrap.get_bootstrap)
      # Running LassoCV with bootstrap instead of CV.
      lasso_bootstrap = skllm.LassoCV(n_jobs=-1, cv=splits).fit(X_train, y_train)
```

Best hyperparameter is 3.39e-03, giving a test R^2 score of 0.66 <IPython.core.display.Javascript object>

```
[13]: # Fixing font sizes of plots
      fonts = {
          "font.family": "serif",
          "axes.labelsize": 12,
          "font.size": 12,
          "legend.fontsize": 12,
          "xtick.labelsize": 12,
          "ytick.labelsize": 12,
      }
      plt.rcParams.update(fonts)
      # Log plot
      plt.figure(figsize=(10, 6.7))
      plt.semilogx(lambdas_lasso_bootstrap, mses_lasso_bootstrap, label="Bootsrap")
      plt.semilogx(lambdas_lasso_cv, mses_lasso_cv, label="5-fold cross validation")
      plt.xlabel(r"Hyperparameter $\lambda$")
      plt.ylabel(r"|MSE|")
      plt.title("Grid search lasso results")
      plt.legend()
      plt.grid()
      plt.tight_layout()
      plt.show()
```

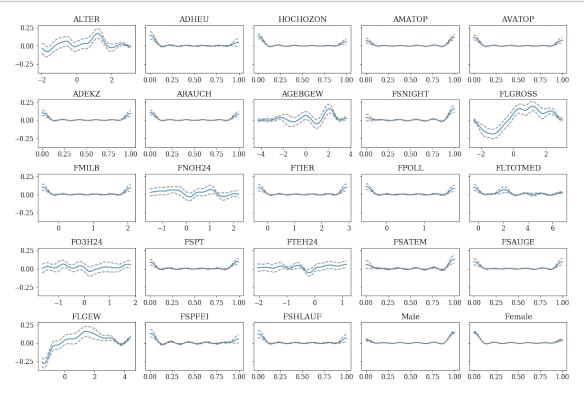


Bootstrap and cross validation seem to estimate the error quite similarily. CV is probably a bit more realistic, as the error is a bit higher, but that i not important. The important thing is where the minimum is, and it seems to be almost the same for both methods.

5 Task 1.5

To determine which features are nonlinear, I plot them all:

```
fig.tight_layout()
plt.show()
```



FLGEW, and FLGROSS to me seem nonlinear.

```
[44]: # Splines and betas. pygam.s is a spline, pyham.l is a linear feature.
features_gam = pygam.l(0)

nonlinear_features = ["FLGEW", "FLGROSS"]

# Setting up splines for all non-linear values.
for i, feature in enumerate(X_train.columns.values[1:]):
    if feature in nonlinear_features:
        features_gam += pygam.s(i)
    else:
        features_gam += pygam.l(i)

# Lambdas for ramdomized search of penalties.
lambdas = np.exp(np.random.rand(100, len(features_gam)) * 3 - 3)
# Fitting a GAM-model and searching for best hyperparameters lambda
gam = pygam.LinearGAM(features_gam).gridsearch(
```

```
X_train.values, y_train.values, lam=lambdas
)
```

100% (100 of 100) | ################## Elapsed Time: 0:00:11 Time: 0:00:11 < IPython.core.display.Javascript object>

```
[45]: # R^2 score. There was a bug with pygam so I had to calculate it like this.

r2_test_gam = gam._estimate_r2(X_test, y_test)["explained_deviance"]

r2_train_gam = gam._estimate_r2(X_train, y_train)["explained_deviance"]

print(f"R^2 score test: {r2_test_gam:.2f}, R^2 score train: {r2_train_gam:.2f}")
```

```
R^2 score test: 0.65, R^2 score train: 0.64
<IPython.core.display.Javascript object>
```

This seems to make no improvement. I am wondering if something is implemented wrongly or if this was to be expected. Could perheps try many more penalties. The increased complexity may have lead to overfitting as seen on the difference between train and test R^2 .

Next I test with adding higher orders of the nonlinear features.

```
[17]: # Reversing scale of training features, need this to get the second order
      → features. Will scale them again after.
      X_train_nonlinear = scaler.inverse_transform(
          X_train.loc[:, np.logical_not(np.isin(X_train.columns, categorical))]
      # Reversing scale of testing features, need this to get the second order
      → features. Will scale them again after.
      X_test_nonlinear = scaler.inverse_transform(
          X_test.loc[:, np.logical_not(np.isin(X_test.columns, categorical))]
      # Lists to save the nonlinear features and names
      nonlinear columns train = []
      nonlinear_columns_test = []
      nonlinear columns name = []
      # Looping over features
      for i, name in enumerate(
          X_train.loc[:, np.logical_not(np.isin(X_train.columns, categorical))]
      ):
          # If feature is in nonlinear features, add it to list of nonlinear features
          if name in nonlinear_features:
              nonlinear_columns_train.append(X_train_nonlinear[:, i] ** 2)
              nonlinear_columns_test.append(X_test_nonlinear[:, i] ** 2)
              nonlinear_columns_name.append(name + "^2")
      # Creating columns with right dimensions out of second order features
      nonlinear_columns_train = np.array(nonlinear_columns_train).T
      nonlinear_columns_test = np.array(nonlinear_columns_test).T
```

```
# Scaling the nonlinear features
     scaler_nonlinear = sklpre.StandardScaler()
     nonlinear_columns_train = scaler_nonlinear.

→fit_transform(nonlinear_columns_train)
     nonlinear_columns_test = scaler_nonlinear.transform(nonlinear_columns_test)
      # Creating new dataframes containing the nonlinear features
     X_train_nonlinear = pd.DataFrame(
         nonlinear_columns_train, columns=nonlinear_columns_name, index=X_train.
      →index.values
     X_test_nonlinear = pd.DataFrame(
         nonlinear columns test, columns=nonlinear columns name, index=X test.index.
     )
      # Creating new dataframes with all features, ready for fit
     X_train_nonlinear = X_train.join(X_train_nonlinear)
     X_test_nonlinear = X_test.join(X_test_nonlinear)
     # Fitting OLS model
     regressor_nonlinear = skllm.LinearRegression().fit(X_train_nonlinear, y_train)
     # R2 scores
     r2 score nonlinear train = regressor nonlinear.score(X train nonlinear, y train)
     r2_score_nonlinear_test = regressor_nonlinear.score(X_test_nonlinear, y_test)
     print(
         f"R2 score test:{r2_score_nonlinear_test:.2f}, train:
      →{r2 score nonlinear train:.2f}"
      # Printing table of p-values etc.
     get_summary_linear_model(regressor_nonlinear, X_train_nonlinear, y_train)
     R2 score test:0.62, train: 0.65
[17]:
           Feature $\beta_i$
                                  $\pm$
                                             p-values
         Intercept 2.333132 0.028541 0.000000e+00
     0
     1
             ALTER 0.023352 0.016571 1.600366e-01
     2
             ADHEU -0.110205 0.064810 9.031186e-02
     3
          HOCHOZON -0.124566 0.042453 3.658687e-03
     4
            AMATOP
                    0.040683 0.033883 2.310271e-01
     5
            AVATOP -0.049731 0.034209 1.472911e-01
     6
             ADEKZ 0.000444 0.037452 9.905588e-01
     7
            ARAUCH -0.028092 0.031537 3.739126e-01
     8
           AGEBGEW 0.000352 0.014916 9.811785e-01
           FSNIGHT 0.081877 0.051221 1.112064e-01
     10
           FLGROSS 1.476849 0.522406 5.083228e-03
     11
             FMILB -0.029861 0.021719 1.704188e-01
     12
            FNOH24 -0.044203 0.019784 2.636298e-02
```

```
13
        FTIER
                0.010317
                          0.016345 5.285007e-01
14
        FPOLL
               -0.033570
                           0.027581
                                     2.247053e-01
15
     FLTOTMED
               -0.033735
                          0.014749
                                     2.302437e-02
16
       F03H24
                0.063638
                          0.029419
                                     3.148700e-02
17
         FSPT
                0.044646 0.070165
                                     5.251699e-01
18
       FTEH24
               -0.038890
                          0.026988
                                     1.508564e-01
19
                0.095268 0.080177
                                     2.358902e-01
       FSATEM
20
       FSAUGE
                0.025336 0.045944 5.818227e-01
21
        FLGEW
                0.114865 0.124844
                                     3.584316e-01
22
                0.069122 0.075962
       FSPFEI
                                    3.637302e-01
23
      FSHLAUF
              -0.021575
                          0.056372
                                     7.022573e-01
24
         Male
                0.103132 0.020657
                                     1.125685e-06
25
       Female
               -0.103132 0.020270
                                     7.173637e-07
26
   FLGROSS<sup>2</sup>
               -1.309999
                           0.522009
                                     1.272994e-02
27
      FLGEW<sup>2</sup>
               -0.041754 0.121355
                                     7.310923e-01
```

Seems to perform about equally to the GAM model. As we can see the p-values aren't very impressive for the new nonlinear features. They also have rather high uncertainties.

6 Task 1.6

I can't find an implementation of component-wise boosting, so I will load R into my notebook and try my best there! I feel like R still has many more features than Python when it comes to lesser known statistical learning methods, so I think it is still good to learn the best of both worlds.

```
[18]: %load_ext rpy2.ipython
    train_for_r = X_train.join(y_train)
    test_for_r = X_test.join(y_test)
```

<IPython.core.display.Javascript object>

First I fit a component_wise boost with splines!

```
[19]: %%R -i train_for_r -i test_for_r
library(compboost)
library(ggplot2)
spline_boost = boostSplines(data = train_for_r, target = "FFVC", loss = LossQuadratic$new(), trace=10)
```

```
1/100: risk = 0.054
10/100: risk = 0.038
20/100: risk = 0.03
30/100: risk = 0.026
40/100: risk = 0.024
```

```
50/100: risk = 0.022
60/100: risk = 0.021
70/100: risk = 0.02
80/100: risk = 0.019
90/100: risk = 0.019
100/100: risk = 0.018

Train 100 iterations in 0 Seconds.
Final risk based on the train set: 0.018

<IPython.core.display.Javascript object>
```

```
[20]: %%R
table(spline_boost$getSelectedBaselearner())
```

```
AGEBGEW_spline ALTER_spline FLGEW_spline FLGROSS_spline FNOH24_spline 7 11 21 24 2 FO3H24_spline FTEH24_spline Female_spline Male_spline 5 12 14 4
```

As we can see, not all features are chosen. The FLGROSS is chosen the most, which makes sense, as it had the lowest p-value in our original OLS model as well.

Next I will do the same for linear regressors.

```
[21]: %%R
linear_boost = boostLinear(data = train_for_r, target = "FFVC", loss = L

→LossQuadratic$new(), trace=10)
```

```
1/100: risk = 0.054

10/100: risk = 0.039

20/100: risk = 0.032

30/100: risk = 0.029

40/100: risk = 0.027

50/100: risk = 0.025

60/100: risk = 0.025

70/100: risk = 0.024

80/100: risk = 0.024

90/100: risk = 0.023

100/100: risk = 0.023
```

Train 100 iterations in 0 Seconds.

Final risk based on the train set: 0.023

<IPython.core.display.Javascript object>

```
[22]: \%\R
      linear_boost$getEstimatedCoef()
     $ALTER_linear
                    [,1]
     [1,] -5.781440e-18
     [2,] 8.443146e-03
     $FLGEW_linear
                   [,1]
     [1,] 8.923426e-17
     [2,] 4.644789e-02
     $FLGROSS_linear
                   [,1]
     [1,] 4.881892e-16
     [2,] 1.803422e-01
     $FPOLL_linear
                    [,1]
     [1,] -9.331164e-18
     [2,] -1.952278e-03
     $FSNIGHT_linear
                   [,1]
     [1,] -0.002378388
     [2,] 0.021065720
     $Female_linear
     [1,] 0.08103014
     [2,] -0.16076380
     $offset
     [1] 2.266492
```

<IPython.core.display.Javascript object>

More or less the same features seem to be selected here as well. FLGROSS having the largest coefficient (Male and female have higher, but they are binary and therefore act differently.)

I didn't manage to get component-wise boosted trees working, sorry!

7 Task 1.7

Calculating the train and test MSE for all models:

```
[23]: def mean_squared_error(sklearn_model, X, y):
    """
    Function for computing MSE of scikit-learn models.
    """
    return ((sklearn_model.predict(X) - y) ** 2).mean()
```

<IPython.core.display.Javascript object>

```
[24]: # MSE train and test of OLS
     print(
         f"OLS: train: {mean_squared_error(ols_reg, X_train, y_train):g}"
         + f" test: {mean_squared_error(ols_reg, X_test, y_test):g}"
     # MSE train and test of strict forward selection
     print(
         f"Increased: train: {mean_squared_error(regressor_increased,_
      + f" test: {mean squared error(regressor_increased, X_increased_test,__

y_test):g}"
     # MSE train and test of less strict forward selection
     print(
         f"Increased less strict: train:
      → {mean_squared_error(regressor_increased_less_strict, __
      {\scriptstyle \mathrel{\hookrightarrow}} X\_increased\_train\_less\_strict, \ y\_train):g}"
         + f" test: {mean_squared_error(regressor_increased_less_strict,__
      # MSE train and test of strict backward elimination
     print(
         f"Reduced: train: {mean_squared_error(regressor_reduced, X_reduce_train, __
      + f" test: {mean_squared_error(regressor_reduced, X_reduce_test, y_test):g}"
     # MSE train and test of less strict backward elimination
     print(
         f"Reduced less strict: train:
      →{mean_squared_error(regressor_reduced_less_strict,__
      →X_reduce_train_less_strict, y_train):g}"
         + f" test: {mean_squared_error(regressor_reduced_less_strict,_
      →X_reduce_test_less_strict, y_test):g}"
```

```
# MSE train and test for LASSO CV
print(
    f"LASSO CV: train: {mean_squared_error(lasso_cv, X_train, y_train):g}"
    + f" test: {mean_squared_error(lasso_cv, X_test, y_test):g}"
# MSE train and test for LASSO bootstrap
print(
    f"LASSO bootstrap: train: {mean_squared_error(lasso_bootstrap, X_train,_
 + f" test: {mean squared error(lasso_bootstrap, X_test, y_test):g}"
# MSE train and test for GAM
print(
    f"GAM: train: {mean_squared_error(gam, X_train, y_train):g}"
    + f" test: {mean_squared_error(gam, X_test, y_test):g}"
OLS: train: 0.0411125 test: 0.0460966
Increased: train: 0.0463959 test: 0.0448621
Increased less strict: train: 0.0463959 test: 0.0448621
Reduced: train: 0.0463959 test: 0.0448621
```

OLS: train: 0.0411125 test: 0.0460966
Increased: train: 0.0463959 test: 0.0448621
Increased less strict: train: 0.0463959 test: 0.0448621
Reduced: train: 0.0463959 test: 0.0448621
Reduced less strict: train: 0.043149 test: 0.0437728
LASSO CV: train: 0.0461212 test: 0.0460014
LASSO bootstrap: train: 0.0431835 test: 0.0447356
GAM: train: 0.0411193 test: 0.0460086
<IPython.core.display.Javascript object>

```
[25]: %%R
      library(MLmetrics)
      print("Element-wise spline boosting:")
      print("Train: ")
      print(MSE(y_pred=data.matrix(spline_boost$predict(train_for_r)), y_true=data.
      →matrix(train_for_r["FFVC"])))
      print("Test: ")
      print("N/A")
      # The commented line under crashes everything!
      #print(MSE(y_pred=data.matrix(spline_boost$predict(test_for_r)), y_true=data.
      →matrix(test_for_r["FFVC"])))
      print("Element-wise linear boosting:")
      print("Train: ")
      print(MSE(y_pred=data.matrix(linear_boost$predict(train_for_r)), y_true=data.
      →matrix(train_for_r["FFVC"])))
      print("Test: ")
      print(MSE(y_pred=data.matrix(linear_boost$predict(test_for_r)), y_true=data.
       →matrix(test_for_r["FFVC"])))
```

```
R[write to console]:
Attaching package: 'MLmetrics'

R[write to console]: The following object is masked from 'package:base':
    Recall

[1] "Element-wise spline boosting:"
[1] "Train: "
[1] 0.03643842
[1] "Test: "
[1] "N/A"
[1] "Element-wise linear boosting:"
[1] "Train: "
[1] 0.04649712
[1] "Test: "
[1] 0.04679446

<IPython.core.display.Javascript object>
```

Unfortunately comploost crashes when I try to use it to predict with data different from what it was fit on. Makes no sense if you ask me, but therefore I couldn't get it to print the test MSE.

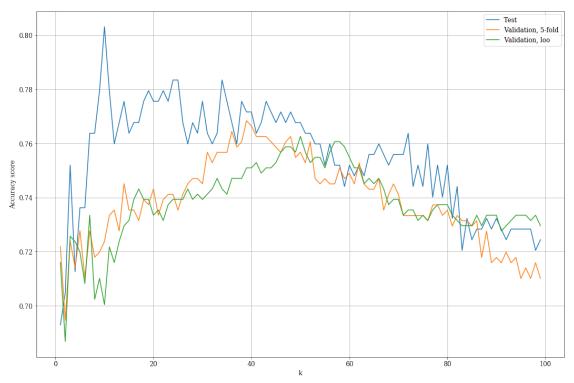
The best models actually seem to be the forward and backward OLS-models, surprisingly!

8 Task 2.1

```
[27]: loo = sklms.LeaveOneOut()
      base_knn = sklne.KNeighborsClassifier()
      k_grid = np.arange(1, 100)
      hyperparams = {"n_neighbors": k_grid}
      grid_search_knn_5_fold = sklms.GridSearchCV(
          base_knn, param_grid=hyperparams, cv=5, iid=False
      grid search knn 5 fold.fit(
          X_train_2.values, y_train_2.values.ravel(),
      grid_search_knn_loo = sklms.GridSearchCV(
          base_knn, param_grid=hyperparams, cv=loo, iid=False
      grid_search_knn_loo.fit(
          X_train_2.values, y_train_2.values.ravel(),
      test_scores = np.zeros_like(k_grid).astype(np.float)
      for i, k in enumerate(k_grid):
          knn_tmp = sklne.KNeighborsClassifier(n_neighbors=k).fit(
              X_train_2.values, y_train_2.values.ravel()
          test_scores[i] = knn_tmp.score(X_test_2.values, y_test_2.values.ravel())
```

<IPython.core.display.Javascript object>

```
ax.set_ylabel("Accuracy score")
ax.legend()
fig.tight_layout()
plt.show()
```



It seems like 5-fold and loo perform very similarily. The difference is that 5-fold is much less computationally expensive, as it has fewer splits. Both of them seem to miss the maximum point of the train set, and will therefore lead to a slight overfit.

9 Task 2.2

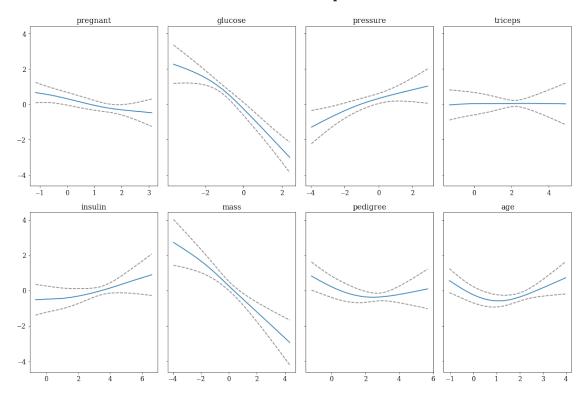
```
[29]: splines = pygam.s(0)

for i in range(1, len(X_train_2.columns)):
    splines += pygam.s(i)

with np.errstate(invalid="ignore", divide="ignore"):
    gam_test_2 = pygam.LogisticGAM(splines).gridsearch(
         X_train_2.values, y_train_2.values
)
```

```
fig, axes = plt.subplots(2, 4, figsize=[15, 10], sharey=True)
for i, ax in enumerate(axes.ravel()):
    XX = gam_test_2.generate_X_grid(i)
    pdep, confi = gam_test_2.partial_dependence(
        term=i, X=XX, width=0.95, meshgrid=False
    )
    ax.plot(XX[:, i], pdep)
    ax.plot(XX[:, i], confi[:, 0], c="grey", ls="--")
    ax.plot(XX[:, i], confi[:, 1], c="grey", ls="--")
    ax.set_title(X_train_2.columns.values[i])
fig.tight_layout()
plt.show()
```

100% (11 of 11) | #################### Elapsed Time: 0:00:02 Time: 0:00:02



<IPython.core.display.Javascript object>

I couldn't get subset selection to work with p-values here, as pyGAM has a bug with p-values. I'm guessing it's just better to use R for this task right now, but I couldn't figure out how. Sorry...

10 Task 2.3

Tree first:

Best depth: {'max_depth': 2} Test Score: 0.73
<IPython.core.display.Javascript object>

I expect trees to be the worst estimator, especially since the max depth is only 2!

Bagged trees (probability):

```
Best amount of trees: {'n_estimators': 22} Test score: 0.72
<IPython.core.display.Javascript object>
```

Bagged trees with probability perform slightly worse than a single tree, meaning it probably overfit slightly.

Next up is concencus bagged trees. I expect these to perform slightly worse or equally to probability bagged trees. Unfortunately Scikit-Learn has no way to actually implement this, and I don't have

time to learn how to do it in R. Sorry! I know from the lectures however that we expect slightly better result from probability than from voting.

Next up is a random forest:

```
[32]: forest = sklen.RandomForestClassifier()
      hyperparameters_forest = {
          "n_estimators": candidate_n_estimators,
          "max_depth": max_depth_list,
      # Will take long time for grid search here, so I use random search instead.
      random_search_forest_1 = sklms.RandomizedSearchCV(
          estimator=forest,
          cv=5,
          iid=False,
          param_distributions=hyperparameters_forest,
          n_iter=100,
      ).fit(X_train_2.values, y_train_2.values.ravel())
      print(
          f"Best hyperparameters: {random_search_forest_1.best_params_} "
          + f"Test score: {random_search_forest_1.score(X_test_2.values, y_test_2.
       ⇔values.ravel()):.2f}"
```

```
Best hyperparameters: {'n_estimators': 23, 'max_depth': 31} Test score: 0.74 
<IPython.core.display.Javascript object>
```

This seems slightly better or equal to the bagging classifiers. As we have two hyperparameters, we use random search instead of grid search, and to get a better result we could have used more samples, or used distributions instead of grids to sample from.

Next I will use neural networks. Ideally we should use Tensorflow in Python, but I will stay within Scikit-Learn and use their more basic implementation with a sigmoid activation function in each layer.

I choose to test different values of L_2 penalty, batch size and learning rate.

```
[33]: penalty_nn_candidates = np.logspace(-3, 0, 100)
  batch_size_nn_candidates = np.arange(10, 200)
  learning_rate_nn_candidates = np.logspace(-4, 0, 100)
  hyperparameters_nn = {
    "alpha": penalty_nn_candidates,
    "batch_size": batch_size_nn_candidates,
    "learning_rate_init": learning_rate_nn_candidates,
}
```

```
Best hyperparameters: {'learning_rate_init': 0.006579332246575682, 'batch_size': 74, 'alpha': 0.026560877829466867} Test score: 0.78
<IPython.core.display.Javascript object>
```

The neural network performs a bit better, but not by much, and we also have to tune way more hyperparameters here. We should ideally also try with different layer sizes and more layers, different activation functions, etc...

Next up is AdaBoost:

Best hyperparameters: {'n_estimators': 55, 'learning_rate': 0.12328467394420659}
Test score: 0.78
<IPython.core.display.Javascript object>

Similar performance to neural network.

11 Task 2.4

I would use AdaBoost, as it performs similarly to a neural network, and has much fewer hyperparameters. I am fairly convinced that you could make the neural network perform even better here with more tuning, but that is sort of an argument against neural networks as well, as that would be much more computationally expensive.

The single tree and bagged tree models perform noticably worse, but then again there are few cases where one would want to use these models instead of a random forest or boosted model. The one argument I can see for a decision tree is that it is the most interpretable and portable model, though.

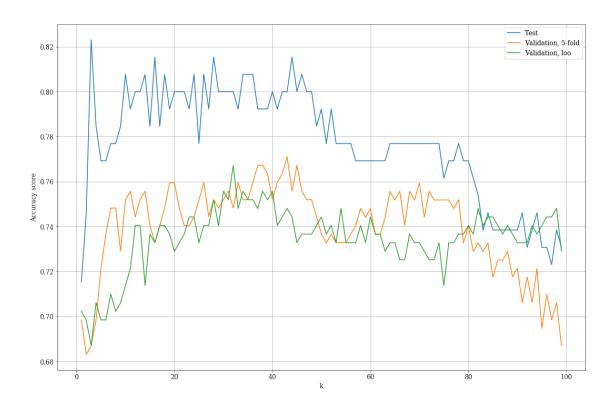
12 Task 2.5

Here I will quickly run all the models on the new dataset after downloading it. (Please forgive me for copy/pasting the code instead of just making functions!)

```
[35]: # Loading dataframe
      %R data(PimaIndiansDiabetes2)
      # Now saving the dataframe in a Pandas dataframe in Python instead.
      # This is actually really cool, just found out how to do it. Using
      # Python together with R this way is probably very useful in data science!
      df 3 = %R PimaIndiansDiabetes2
      # Dropping all rows with NaN
      df_3.dropna(inplace=True)
      X_3 = df_3.loc[:, df_3.columns != "diabetes"]
      y_3 = df_3["diabetes"]
      y_3 = (y_3.values == "neg").astype(np.int)
      y_3 = pd.DataFrame(y_3, columns=["diabetes"])
      X_train_3, X_test_3, y_train_3, y_test_3 = sklms.train_test_split(
         X_3, y_3, stratify=y_3, test_size=0.33
      )
      scaler_3 = sklpre.StandardScaler()
      X_train_3 = pd.DataFrame(scaler_3.fit_transform(X_train_3), columns=X_train_3.
      X_test_3 = pd.DataFrame(scaler_3.transform(X_test_3), columns=X_test_3.columns)
```

<IPython.core.display.Javascript object>

```
[37]: fig, ax = plt.subplots(figsize=[15, 10])
    ax.plot(k_grid, test_scores_2, label="Test")
    ax.plot(
        k_grid,
        grid_search_knn_5_fold_2.cv_results_["mean_test_score"],
        label="Validation, 5-fold",
))
    ax.plot(
        k_grid,
        grid_search_knn_loo_2.cv_results_["mean_test_score"],
        label="Validation, loo",
))
    ax.grid()
    ax.set_xlabel("k")
    ax.set_ylabel("Accuracy score")
    ax.legend()
    fig.tight_layout()
    plt.show()
```

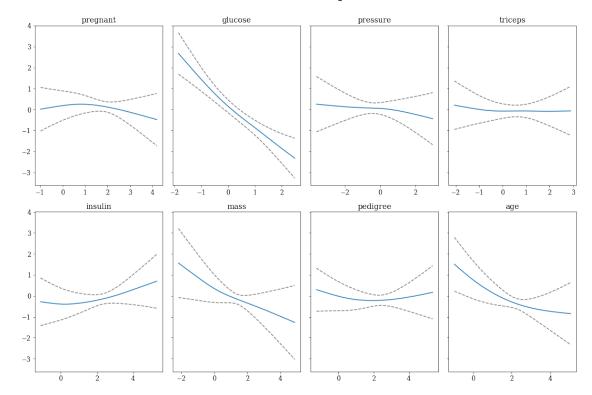


The maximum of the test score is placed differently here than with the wrong values. Loo and 5-fold seem to find the maximum more properly in this case, should lead to a better fit.

Next up would be the GAM models, but I couldn't get the stepwise selection to work for GAM, so I cannot do this part. I will plot the partial dependencies and see if they differ for this data though.

```
ax.plot(XX[:, i], confi[:, 0], c="grey", ls="--")
ax.plot(XX[:, i], confi[:, 1], c="grey", ls="--")
ax.set_title(X_train_3.columns.values[i])
fig.tight_layout()
plt.show()
```

100% (11 of 11) | ################### Elapsed Time: 0:00:02 Time: 0:00:02



The partial dependencies seem much more linear now (although with higher uncertainties). Maybe the 0s in the data made the relationsips seem more non-linear.

Next I will do Task 2.3 again:

```
Best depth: {'max_depth': 3} Test Score: 0.79
<IPython.core.display.Javascript object>
```

Better performance than before somehow. Implies more biased outcomes maybe. The best max_depth is higher than before too.

Next is bagged trees:

```
Best amount of trees: {'n_estimators': 48} Test score: 0.78 
<IPython.core.display.Javascript object>
```

Similar to before, but now outperformed by single tree.

Next is random forest:

```
Best hyperparameters: {'n_estimators': 14, 'max_depth': 59} Test score: 0.80
<IPython.core.display.Javascript object>
```

Much better than on wrong data, but still only slightly better than a single tree of depth 3...

Next is neural network:

```
Best hyperparameters: {'learning_rate_init': 0.001023531021899027, 'batch_size': 108, 'alpha': 0.004641588833612782} Test score: 0.81 
<IPython.core.display.Javascript object>
```

Performance is better for the neural network.

AdaBoost:

```
Best hyperparameters: {'n_estimators': 5, 'learning_rate': 0.5336699231206312}
Test score: 0.84
<IPython.core.display.Javascript object>
```

With the improved model, AdaBoost still seems to be the better model.

Another big difference here is that the tree performed better. As we can see though, the best tree depth is very low, and the reason the tree has a score that seems high is that the dataset it biased. This implies also that most of the people with invalid reported do not have positive outcome(diabetes), which makes sense as healthy people probably aren't as concerned about properly reporting information about a disease they do not have.

It is important to pay attention to whether your dataset is biased or not, and in reality we should have used a more robust metric than the accuracy score. As we are dealing with a disease, the confusion matrix would have been a much better metric, preferably with a larger penalty for classifying a sick person as healthy than the other way around.