Combining Gaussian Processes

immediate

Imagine that you have the "the varying slopes model" case described Richard McElreath book (Statistical Rethinking, second edition). In that example, you had a coffee robot that was programmed to move among cafés, order coffee, and record the waiting time in the morning and the afternoon. In short, it was an example of pooling in multilevel models and covariance among parameters. Given the waiting times W_i for all cafés and a binary variable A_i indicating whether or not the waiting time was recorded in the afternoon, the model for this example was the following:

$$W_i \sim \text{Normal}\left(\mu_i, \sigma\right)$$

$$\mu_i = \alpha_{\text{CAFE}_i} + \beta_{\text{CAFE}_i} A_i$$

$$r_n = x_n - \lambda$$

$$\lambda \sim \text{MVNormal}\left(\hat{\lambda}, E\right)$$

$$\epsilon \sim \text{MVNormal}\left(\hat{\epsilon}, V\right)$$

$$\alpha \sim \text{Normal}\left(\hat{\alpha}, \sigma_\alpha\right)$$

$$\hat{\lambda} \sim \text{Normal}\left(0, 1\right)$$

$$\hat{\epsilon} \sim \text{Normal}\left(0, 1\right)$$

$$\hat{\alpha} \sim \text{Normal}\left(0, 1\right)$$

$$\sigma_\alpha \sim \text{Exponential}\left(1\right)$$

1 Imagine now that we have additional information on the different cafés. For example,
2 suppose that we know how far are these cafés from each other. One could think that
3 different neighbourhoods might have different waiting times (i.e. fancy cafés from posh
4 neighbourhoods might have similar waiting times, or cafés from non-residential areas might
5 have longer morning times). To incorporate this prior information, one could use a Gaussian
6 processes for characterizing the different parameters. That is, $alpha_i$ could be characterized

7   by a multivariate normal distribution $N(\mu_i, \Sigma)$, where Σ is a covariance matrix such that

8   $\Sigma_{ij} = \rho \exp(-\gamma * (D_{ij}^2)) + \delta_{ij}\sigma^2$