# Model complexity breeds contempt: using non-linear models to compare basic properties of species' distributions

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## 1 Abstract

2 We know a lot about the factors that could theoretically influence species' distributions,
3 and a rapidly growing body of research have been primarily focused on trying to untangle
4 some of such biotic and abiotic predictors—with an increasing effort placed in improving
5 the predictive power of statistical models. However, much less is known about how species'
6 distributions compare to each other. Here, we use a conceptually more conservative ap-
7 proach to instead understand and compare basic aspects regarding the shape of species'
8 distribution along environmental gradients.

## 9 Introduction

10 One of the central goals of ecology is to understand the ways species are distributed across
11 space and time (ref). Over the last two decades, ecologists have developed multiple distri-
12 bution models to try to untangle the factors that play a role in defining such distributions
13 (Guisan & Zimmermann, 2000). These models estimate species' realized niches using sev-
14 eral covariates, including environmental variables (?), species ecological traits' (Pollock
15 *et al.*, 2012) and phylogenetic relations (Ives & Helmus, 2011). More recently, some of
16 the focus have shifted towards approaches that estimate and account for biotic factors,
17 such as competitive or facilitative relationships between species (Ovaskainen *et al.*, 2017).
18 The idea is that by untangling the ways in which such biotic and abiotic factors shape
19 species' distributions, we can gain a mechanistic understanding on how ecological commu-
20 nities are established and change over time. However, while these factors can increase the

predictive performance of some of the models (Norberg *et al.*, 2019), the interpretation of the corresponding parameter estimates has been often questioned (Gotelli & Ulrich, 2010; Harris, 2016; Thurman *et al.*, 2019). This was best illustrated by Blanchet *et al.* (2020), who used basic statistical arguments to highlight the artefactual nature of the link between co-occurrence and species' ecological interactions drawn by some distribution models.

The value of gaining a mechanistic understanding of species' distributions is unquestionable (ref), with several studies highlighting the importance of factors such as biotic interactions and dispersal ability in setting species' range limits (Wisz *et al.*, 2013; Pollock *et al.*, 2014; Neuschulz *et al.*, 2018). That said, a lot can be learned from taking a phenomenological approach, focussing instead on the description of basic properties of species' realized niches. For example, the study of species' range sizes along environmental gradients can reveal general biodiversity patterns that are crucial from a conservation and management perspective (Stevens, 1992). Differences in species' responses to the environment could shed light on how climatic processes and historical contingencies have shaped their distributions (Rohde, 1992; **?**). Other properties, such as the skewness of species' distributions, can also reveal general underlying processes regarding species' physiological tolerance to different environmental conditions (Kaufman, 1995). More generally, understanding the shape of species' realized niches and the extend to which these vary across species is a crucial issue in ecology and biogeography (ref); however, we do not have an effective way to parsimoniously compare the realized niches of many species. Indeed, there is no general agreement on the shape of species' distributions (ref).

Many ecological textbooks (Krebs, 1972) assume the shape of species distributions to be unimodal and symmetric, but some have warned that empirical distributions can take many different forms (Austin, 1987; **?**). In practice, distribution frameworks often use logistic regressions with a linear relationship between covariates (but see XX and YY). This is useful because it simplifies the optimization process, but it comes with some statistical shortcomings. First and foremost, such response curve and the linear relationship between covariates often comes with a set of implicit mathematical constrains that might not be biologically justified. From a purely statistical perspective, if all that we are willing to assume is that species occupy finite geographic ranges—i.e. their probability distributions have fi-

nite variance—the most conservative statistical approach is to model these as a Gaussian distributions (Frank, 2009). This is rarely the starting point in most statistical frameworks that study general biodiversity patterns (but see ref), choosing to use instead Gaussian-logit response curves (refs). Other factors might then condition species distributions to showcase fat-tails or a skewed shapes, revealing interesting ecological processes shaping biodiversity patterns (Austin, 1976; Minchin, 1987). The starting point, nevertheless, should be the one that makes the fewest assumptions (i.e. the maximum entropy distribution; Frank 2009), and every new shape will imply a hypotheses on how communities are distributed (D'Amen *et al.*, 2017). Second, the aforementioned structural constrains also limit our ability to include any prior information to our parameter estimates. Observations on species' geographic variation and optimal climatic conditions have long been documented, with extensive databases compiled by botanists and field ecologists documenting basic knowledge on species' realized niches (e.g. Landolt *et al.* 2010). That said, this information is rarely accounted for in most modelling approaches, potentially because there is not a straightforward way to feed this information into the parameters of a linear model (Scherrer & Guisan 2019; but see ter Braak & Looman 1986). Finally, and perhaps most importantly, a direct biological interpretation of parameter estimates in linear models becomes increasingly difficult as one moves from unimodal and symmetric distributions (ter Braak & Looman, 1986; Jamil & ter Braak, 2013) to skewed distributions (Huisman *et al.*, 1993), making the tests of hypothesis on global biodiversity patterns particularly challenging. For example, Huisman *et al.* (1993) proposed several non-linear models to characterize several features of species' response curves; however, species' environmental indicator values, range size or distribution skewness are difficult to understand altogether following these model structures.

The field of ecology has quickly moved towards mechanistic and process-based approaches to understand species' distributions (Warton *et al.*, 2015). This has resulted in a plethora of models accounting for several biotic and abiotic factors into the predictions of species co-occurrence. Here, we instead rethink traditional modelling approaches and develop a conceptually simple—and yet statistical and computationally complex—statistical framework to revisit some classic hypothesis in ecology and biogeoraphy. In particular, we develop a Bayesian hierarchical model that accounts for all prior information that we have regarding the distribution of alpine plant species along an elevation gradient in the Swiss Alps, in-

cluding expert knowledge on species environmental indicator values, range sizes, and plant physiology. We start by considering species' response curves as Gaussian distributed, and then we adapt our model to allow for skewed and long-tailed distributions. Using this statistical framework, we are able to compare the basic properties of the realized niches of multiple species, testing for the existence of general biogeographical patterns. First, we test for the Rapopor's rule, which predicts a positive relationship between range size and elevation (Stevens, 1992). While this pattern has been largely studied for multiple systems and across gradients (McCain & Knight, 2013); contrasting evidence suggests this rule not to be pervasive across species (Ribas & Schoereder, 2006; Bhattarai & Vetaas, 2006; McCain & Knight, 2013). Our results not only allow us to properly test the existence of this geographical pattern, but they also showcase variation in how different types of species, such as native or neophytes, might respond to an environmental gradient. Second, we study whether or not species' distributions show steeper declines towards stressful conditions, testing the so-called abiotic stress limitation hypothesis (ref). Normand *et al.* (2009) tested this for vegetation data using Huisman *et al.*'s statistical models for several independent species, finding no clear support for such a hypothesis. Our results are able to shed light on this geographical pattern as well as to highlight the degree to which different species will showcase different levels of decline towards stressful conditions. Specifically, we are able to link plant physiological traits to the skewness of their distributions. Overall, we use models that are solely constrained by the empirical information that we truly have regarding our system, relaxing as much as possible the structural constrains of the statistical framework. Using these models, we are able uncover the approximate shape of empirical plant distributions and answer fundamental questions regarding the way systems of many species are distributed along environmental gradients.

# Methods

## Empirical data

We studied the distribution of alpine plant communities along an elevation gradient. To do so, we combined two different datasets: i) one describing the co-occurrence of species across

multiple open grasslands in the Swiss Alps, and ii) an extensive floristic database containing environmental and physiological traits for all vegetation across Switzerland (Landolt *et al.*, 2010).

## *Distribution data*

We used data describing the distribution of 798 species across 912 sites covering most of the mountain region of the Western Alps in the Canton de Vaud (Switzerland; Scherrer & Guisan 2019). Each of these sites is a $8 \times 8$ m plot placed somewhere along an elevation range from 375 m to 3210 m. In all sites, presence/absence data as well as Braun-Blanquet abundance-dominance classes were recorded for all species. Additionally, we used meteorological data provided by Scherrer & Guisan (2019), containing multiple variables characterizing the climate in each site at high spatial resolution (25 m). This dataset was compiled based on 30 years (1961–1990) of records from national weather stations. Since most of the data is highly correlated, we calculated the main axes of variation of the following variables: daily minimum, maximum and average temperature; sum of growing degree-days above 5°C; mean temperature of wettest quarter; annual precipitation, precipitation seasonality, and precipitation of driest quarter (see Supplementary Methods; Supplementary Fig. 1).

## *Floristic data*

To complement the aforementioned distribution data, we used a floristic database of most vegetation across Switzerland. This database was build based on expert knowledge and field experience of botanists and ecologists, and contains information regarding species' environmental preferences and physiological traits. Species' environmental preferences in this database can be used to inform distribution models—e.g. as an informative prior in a Bayesian framework. These are characterized following the ecological indicator values developed by Landolt *et al.* (2010), providing both an estimate of the average conditions in which a species can be found as well as a broad description of their range of variation. These values are provided for a range of 10 climatic variables, including temperature, continentality, light conditions, as well as moisture, acidity and nutrient content of the soil (see

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a full list and description of the ecological indicators in the Supplementary Table 1; Landolt *et al.* 2010). On the other hand, the information regarding species' physiological traits represents general descriptions of species' growth and life strategies—examples include their growth forms, nature of the storage organs, dispersal ability and pollinator agents. In total, we identify more than 120 binary traits that characterize the physiology of species (see a full list and description of the ecological indicators in the Supplementary Table 1; Landolt *et al.* 2010). Finally, and in addition to species' environmental preferences and physiological traits, the floristic data also contains information on species types (e.g. identifying those species that are neophytes)—and growth tendency (e.g. indicating species that have shown decline or increase in their populations over the recent years). We describe this information in more detail in the Supplementary Table 1.

## Baseline model

There is a long list of model structures well suited to characterize species' distributions (see Norberg *et al.* 2019). As a baseline model, however, we were interested in a hierarchical model that does not make any assumptions regarding the shape of the distributions, and yet explicitly incorporates all information that we have regarding plant's environmental preferences. More specifically, we wanted to account for the climatic indicator values and range of variation registered in the floristic database for all plants in our dataset. These two values provide basic information regarding plant's optimal environmental conditions and width of their distributions.

### *Response curve*

To choose an appropriate response curve, we first need to agree on what we truly know about the system. Given the prior information that we have about the system, we know that species occupy specific geographic ranges; therefore, we know that their distributions have finite variance. While we could also assume that many other factors might influence species' presence in a given site—e.g. the biotic interactions among specie in the site—we do not necessarily have an *a priori* expectation of how exactly these factors will influence the shape of species' distributions. Therefore, for this baseline model, if all that we are

willing to assume about species' realized niches is that these have finite variance, the most conservative assumption and the safest bet—i.e. the one with the largest entropy—is that they follow a Gaussian distribution. That is, given the presence/absence or abundance $y_{ij}$ of any species $i$ in any given site $j$, and an environmental variable $x_j$, we define can species' responses to the environment as

$$y_{ij} \sim F\left(p_{ij}\right)$$
$$\log\left(p_{ij}\right) = -\alpha_i - \gamma_i\left(x_j - \beta_i\right)^2, \tag{1}$$

where $F$ is the likelihood function, and $\alpha_i$, $\beta_i^k$, and $\gamma_i$ describe amplitude of the probability $p_{ij}$, species' average climatic suitability and range of variation along the an environmental gradient, respectively. Notice that $F$ characterizes a Bernoulli distribution when considering binary data, and it characterizes an ordered categorical likelihood function when we consider Braun-Blanquet abundance-dominance classes as response variables (see the full description of both models in the Supplementary Methods). For the sake of simplicity, we use only one environmental variable to characterize species' probability distribution. That said, this model can easily be generalized to account for multiple predictors (see Supplementary Methods).

*Model priors*

The model structure described above allows us to explicitly incorporate all prior knowledge that we have regarding species' distributions contained in the floristic database. To do so, we define the prior distributions for the parameters in model (1) as:

$$\beta_i \sim \text{MVNormal}\left(\hat{\beta}, \Sigma^\beta\right)$$
$$\log(\gamma_i) \sim \text{MVNormal}\left(\hat{\gamma}, \Sigma^\gamma\right)$$
$$\log(\alpha_i) \sim \text{Normal}\left(\hat{\alpha}, \sigma_\alpha\right)$$
$$\hat{\beta}, \hat{\gamma}, \hat{\alpha} \sim \text{Normal}\left(0, 1\right)$$
$$\sigma_\alpha \sim \text{Exponential}\left(1\right) \tag{2}$$

where parameters $\gamma_i$ and $\beta_i$ are expressed as multivariate normal distributions—i.e. Gaussian processes—such that $\Sigma^\beta$ and $\Sigma^\gamma$ are variance-covariance matrices describing species' similarity in terms of their average climatic suitability and range of variation along the different environmental gradients, respectively. We define these variance-covariance matrices as follows:

$$\Sigma_{ij} = \eta \exp\left(-\rho D_{ij}{}^2\right) + \delta_{ij}\sigma, \tag{3}$$

where $\Sigma_{ij}$ characterizes the covariance between any pair of species $i$ and $j$. Notice that such a covariance structure declines exponentially with the square of a distance matrix $D_{ij}$, which characterize differences between species computed using our prior information. In the floristic database, this information is represented by the set of ordinal specified for the different species. While there are many different ways to turn ordinal data into distance matrices, we choose to use a mixed-membership stochastic block model because it allows us to deal with cases of missing data (see Supplementary Methods for extended details; Godoy-Lorite *et al.* 2016). In each covariance matrix, the hyperparameter $\rho$ determines the rate of decline of the covariance between any two species, and $\eta$ defines its maximum value. The hyperparameter $\sigma$ describes the additional covariance between the different observations for any given species. For all these hyperparameters, we choose weekly informative priors such that $\sigma, \eta \sim \text{Exponential}(1)$ and $\rho \sim \text{Exponential}(0.5)$.

## *Alternative variance-covariance structures*

The model structure defined above allows us to test how different sources of information characterize each of the different parameters. Specifically, we can do this by modifying Eq. (3). For example, imagine that we have multiple matrices $D^k$ characterizing species' differences along different axis of variation—e.g. two matrices characterizing physiological and environmental traits. One can modify Eq. (3) for a particular parameter—e.g. $\beta_i$—such that

$$\Sigma_{ij} = \eta \exp\left(-\sum_k \rho_k D_{ij}^{k}{}^2\right) + \delta_{ij}\sigma, \tag{4}$$

where now $\rho_k$ are separate relevance hyperparameters for each distance matrix in the total variance of $\beta_i$.

*Sampling the posterior*

We generated the posterior samples for the Bayesian models with the Hamiltonian Monte Carlo algorithm implementation provided by the R package 'rstan' to (Stan Developent Team, 2021). Sampling models like the ones described above can be computationally very expensive. This is especially true when using ordered categorical likelihood functions (see Stan Development Team 2021). Therefore, we focus on those species for which we have more than 30 occurrences when modelling ordinal data, which is the case for the majority of the results of this work. When using presence/absence data, we limit our study to those species for which have more than 10 occurrences.

To test the performance of the model as well as our choice of prior distributions, we modelled simulated data and compared the sampled posterior distributions to the data-generating parameters (see Supplementary Methods; Supplementary Fig. 2). Notice that using the link function in Eq. (1) could cause problems when sampling the model, and some adjustments need to be made when specifying the model (see Supplementary Methods and the Code Availability section).

## Modifying the baseline model

We proposed a baseline model that is naive regarding how the data is distributed, and yet accounts for all prior information that we have about the system. Now, we want to modify this model to test the extent to which empirical species' distributions showcase different properties, while preserving both the interpretation of the parameter estimates and the prior information. More specifically, to propose new species' response curves, we follow three key criteria: (i) the probability distribution must have a defined variance and mean, (ii) the Gaussian shape must be a special case of the probability distribution, and (iii) there must be a re-parametrization of the model that allows us to keep the same prior information and interpretable parameters.

*Fat-tail response curve*

Distributions with fat-tails are very common across fields, as they can capture processes such as seasonality (e.g. in communications patterns; Malmgren *et al.* 2008) or some stochastic events (e.g. in the spread of infectious diseases; Wong & Collins 2020). Indeed, fat-tail distributions are pervasive in ecology; for example, species' dispersal patterns have been shown to have fat-tails due to natural variability among individuals (Petrovskii *et al.*, 2009). Therefore, one might expect these properties to also emerge in alpine communities, where seasonality and dispersal patterns are crucial factors determining species' distributions (**?**). To accommodate this feature into our baseline model, we could consider response curves that follow a generalized error or a non-standardized Student's t-distributions. In both cases, the normal distribution is a special case of them, and both contain parameters that regulate the kurtosis of the probability distribution (Supplementary Figure XX).

Starting with the generalized error distribution, we can adapt Eq. (1) to present this non-linear form as follows:

$$\log\left(p_{ij}\right) = -\alpha_i - \gamma_i' \left|x_j - \beta_i\right|^{\nu_i}, \tag{5}$$

where $\gamma_i' = g(\gamma_i, \nu_i)$, and $\nu_i$ is a parameter that describes the kurtosis of the distribution, which we define as $\nu_i \in (1, \infty)$. Following this, we choose an adaptive prior for this set of new parameter such that $\log\left(\nu_i - 1\right) \sim \mathrm{Normal}\left(\hat{\nu}, \sigma_\nu\right)$, where $\hat{\nu} \sim \mathrm{Normal}\left(0, 1\right)$ and $\sigma_\nu \sim \mathrm{Exponential}\left(2\right)$. Given the relationship between $\gamma_i'$ and $\gamma_i$, we can re-parametrize the model and follow Eq. (2) to define the corresponding prior distributions (see Supplementary Table 2; Code Availability section; **?**). Notice that the Gaussian distribution will naturally emerge when $\nu_i = 2$.

In a similar fashion, we can use a non-standardized Student's t-distribution by modifying Eq. (1). In this case, the response curve can be characterized as follows:

$$p_{ij} = \alpha_i' \left(1 + \frac{\gamma_i'}{\nu_i} \left(x_j - \beta_i\right)^2\right)^{-\frac{\nu_i + 1}{2}}, \tag{6}$$

where $\alpha_i' = f_1\left(\alpha_i, \gamma_i', \nu_i\right)$, $\gamma_i' = f_2\left(\gamma_i, \nu_i\right)$, and $\nu_i$ is a parameter that describes the kurtosis of

<sub>259</sub> the distribution, which we define as $\nu_i \in (2, \infty)$. These boundary conditions are necessary

<sub>260</sub> in order for the distributions to have finite variance. Similar to the description above, we

<sub>261</sub> choose an adaptive prior such that $\log (\nu_i - 2) \sim \mathrm{Normal} (\hat{\nu}, \sigma_\nu)$, where $\hat{\nu} \sim \mathrm{Normal} (0, 1)$

<sub>262</sub> and $\sigma_\nu \sim \mathrm{Exponential} (1)$. Again, we can re-parametrize the model following $f_1$ and $f_2$,

<sub>263</sub> and set the rest of the prior distributions as above (see Supplementary Table 2; Code

<sub>264</sub> Availability section). In this case, the Gaussian distribution is a special case of Eq. (6)

<sub>265</sub> when $\nu_i \to \infty$ (?).

<sub>266</sub> *Skewed response curve*

<sub>267</sub> When species experience abiotic or biotic pressures that increase or decrease along an en-

<sub>268</sub> vironmental gradient, one might expect their distributions to be skewed in one direction.

<sub>269</sub> Likewise, this same skewed shape can also be the result of asymmetric environmental tol-

<sub>270</sub> erance. One way to accommodate this feature to our models is by considering a skewed

<sub>271</sub> normal distribution (Supplementary Figure XX). As for the two cases described above, the

<sub>272</sub> Gaussian is a special case of this distribution, and it contains a parameter that controls

<sub>273</sub> for the level and direction of 'skewness'. Importantly, this distribution presents normal-like

<sub>274</sub> tails; therefore, the added skewness does not make additional assumptions regarding how

<sub>275</sub> species' are distribution along the gradient. To test for the existence of this feature, we

<sub>276</sub> modified the species' response curve in Eq. (1) as

$$p_{ij} = \hat{p}_{ij} \left[ 1 + \mathrm{erf} \left( \lambda_i \left( x_j - \beta_i' \right) \sqrt{\frac{\gamma_i'}{2}} \right) \right]$$

$$\log (\hat{p}_{ij}) = -\alpha_i' - \gamma_i' \left( x_j - \beta_i' \right)^2, \tag{7}$$

<sub>277</sub> where $\alpha_i' = q_1 (\alpha_i, \gamma_i', \lambda_i)$, $\beta_i' = q_2 (\gamma_i', \lambda_i)$, $\gamma_i' = q_3 (\gamma_i, \lambda_i)$, and $\lambda_i$ is a parameter that

<sub>278</sub> describes the skewness of the distribution, which we define as $\lambda_i \in (-10, 10)$. Notice that

<sub>279</sub> these boundary conditions for $\lambda_i$ are arbitrarily set for computational purposes, as this

<sub>280</sub> parameter is theoretically not bounded. The function 'erf' is the error function (see ?).

<sub>281</sub> We choose an adaptive prior such that $\lambda_i \sim \mathrm{Normal} \left( \hat{\lambda}, \sigma_\lambda \right)$, where $\hat{\lambda} \sim \mathrm{Normal} (0, 1)$ and

<sub>282</sub> $\sigma_\lambda \sim \mathrm{Exponential} (1)$. This model can be re-parametrized following $q_1$, $q_2$ and $q_3$, and set

<sub>283</sub> the rest of the prior distributions as above (see Supplementary Table 2; Code Availability

<sub>284</sub> section). In this case, the Gaussian distribution is a special case of Eq. (7) when $\lambda_i = 0$

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285 (**?**).

286     One way to test the extend to which species' physiological traits inform the skewness

287 of the distribution of any species $i$ is by characterizing $\lambda_i$ as a Gaussian process. That

288 is, we can consider the prior distribution for $\lambda_i$ as a multivariate normal with a variance

289 covariance matrix $\Sigma^\lambda$. As described before, this variance-covariance matrix can be estimated

290 by the model following Eq. (3), and its structure can shed light on how informative the

291 physiological traits are.

## 292 Results

## 293 Discussion
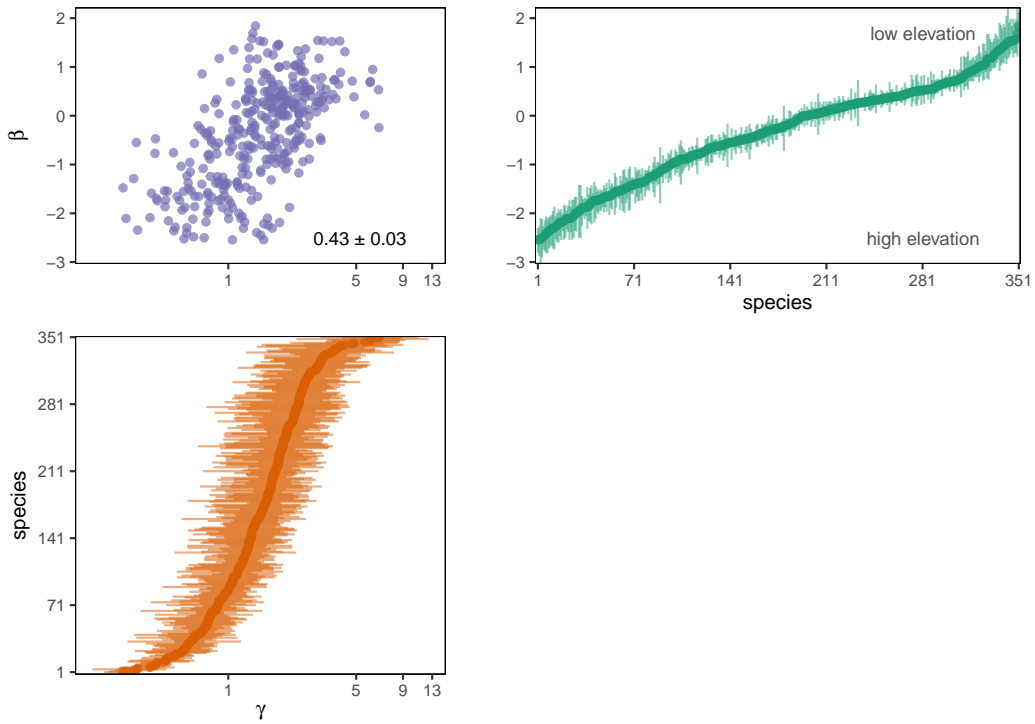


**Figure 1**: Relationship between mean and variance of species' distributions. These are the results for the main axis of variation for the climatic data (results for the second axis of variation presented in the Supplementary Fig. 2).
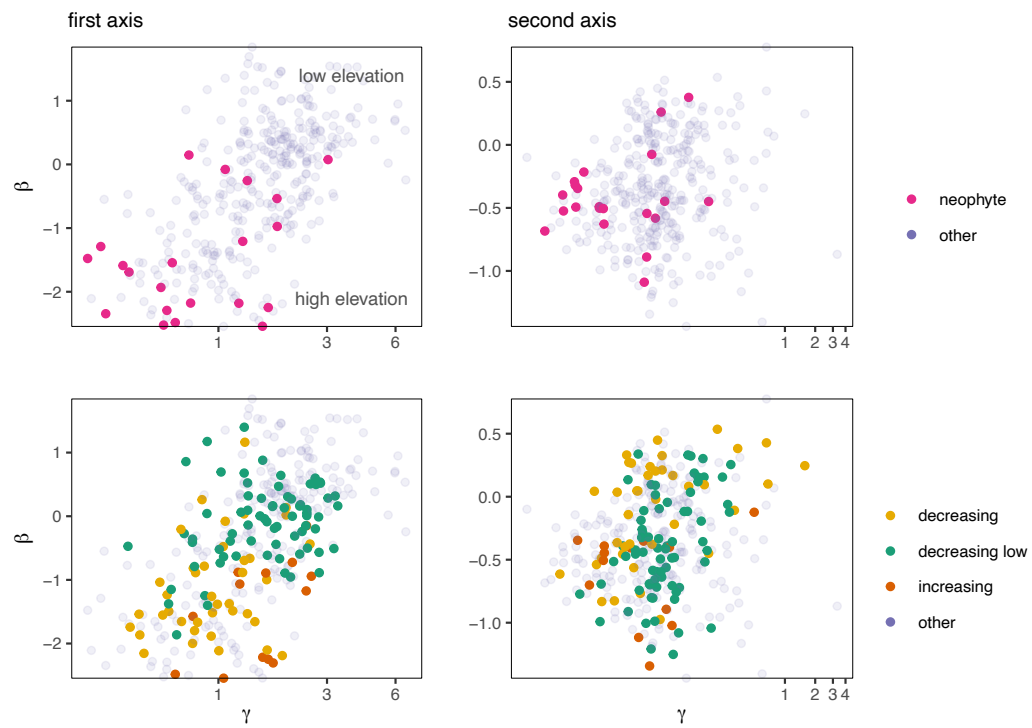
**Figure 2**: Are there clear geographical patterns for neophytes and for species with decreasing or increasing abundance?

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