The Slide Rule

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1 First the nitty gritty

To understand the action of a slide rule we have to start from Logarithms. I'm using the term Logarithm and log interchangeably. Logarithm may be the correct term, but everyone I know always calls them logs.

Consider the two equations:

$$y = \log_b x$$

and

$$x = b^y$$

These say the same thing, one gives y in terms of x (and b).
The other gets x in terms of y (and b)
In this case b is called the base of the log,
and putting $x = b^y$ into that first equation:

$$y = \log_b b^y$$

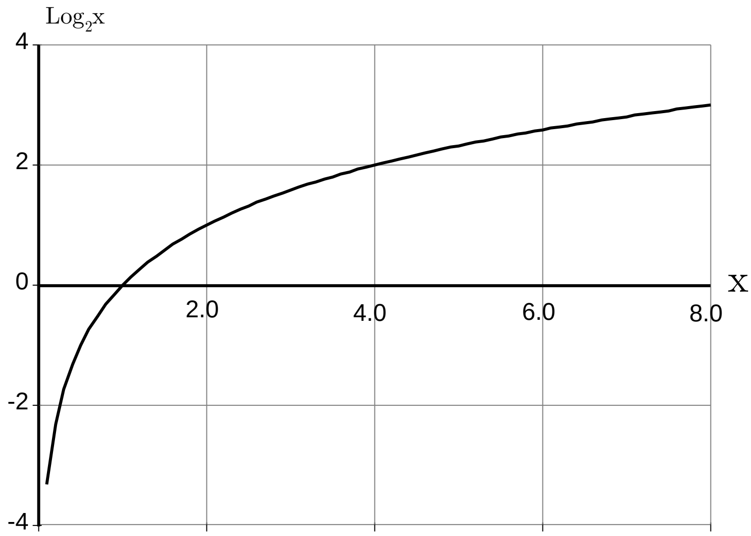
So an example:

$$8 = 2^3$$

and

$$3 = \log_2 8$$

To illustrate this, here's a graph of $\log_2 x$ against x.



A point to notice is that no negative x values are plotted. Logarithms are not defined for negative x, but the logs themselves do take negative values as x becomes less than 1.0

$$0.5 = 2^{-1}$$

$$-1 = \log_2 0.5$$

1.1 The heart of the matter

Here's the magic, take two numbers $c = b^p$ and $d = b^q$, so therefore:

$$p = \log_b c$$

and

$$q = \log_b d$$

We know

$$cd = b^p b^q = b^{p+q}$$

To illustrate:

$$8 \times 4 = 2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2 = 2^5 = 32$$

Taking the log of both sides, and noting that $y = \log_b b^y$

$$\log_b b^p b^q = \log_b b^{p+q} = p + q$$

therefore

$$\log_b cd = p + q$$

But we know $p = \log_b c$ and $q = \log_b d$ so lets go all the way

$$\log_b cd = \log_b c + \log_b d$$

So that means we are mapping the multiplication of two numbers, to an addition! Which is the basis of how a slide rule works.

The problem seems to be twofold, we need to find the logs of any two numbers, and after adding them, we get the log of their product - so we need to get back to the desired product from its log - that is, we need to get the anti-log.

So how do we get logs and antilogs? There are techniques such as power series that converge on the logarithm of a value, however for this article, we don't bother, a slide rule has logarithms built-in, the length along a slide rule is the log_{10} of the number inscribed on the rule. More of this later.

As a school child everyone over a certain age had a booklet of 'Logarithmic Tables' which also included anti-log and trigonometric 'four-figure' tables. Ask your granddad.

To continue, how about division:

$$\frac{c}{d} = \frac{b^p}{b^q} = b^p b^{-q} = b^{p-q}$$

And that leads to

$$\log_b \frac{c}{d} = \log_b b^{p-q} = p - q = \log_b c - \log_b d$$

So division maps to subtraction, that's neat.

Now let's look at another sweet property. Starting again from:

$$x = b^y$$

$$x^n = b^{ny}$$

Take the log of both sides:

$$\log_b x^n = \log_b b^{ny}$$

$$\log_b x^n = ny$$

$$\log_b x^n = n \log_b x$$

We've reduced finding a power to a multiplication! The schoolboy would have to look up the log of a number x, do the n multiplication, and then look up the anti-log to get the final result x^n, but that's a powerful technique. As n can be fractional, this includes roots as well.

And if we really want to push the boat out, we can do that n times multiplication with logs:

$$\log_a \log_b x^n = \log_a n + \log_a \log_b x$$

I've introduced another base a here, just to show that the multiplication is not necessarily in base b again.

So there's a few things to notice:

- The multiplication to addition property works for any base b.
- $\log_b 1 = 0$ for any base, since $b^0 = 1$
- $\log_b b = 1$ for any base, since $b^1 = b$
- $\log_b x^n = n \log_b x$

If you check the earlier graph, you can see these points in action. For further information about logarithms, a good description is given at [1]

1.2 All about bases

So lets dive a bit deeper into that base number b, especially with respect to slide rules. First we'll show that changing bases is surprisingly easy.

Starting from these two again:

$$y = \log_b x$$

$$x = b^y$$

Take the log (of base c) of both sides

$$\log_c x = \log_c b^y$$

$$\log_c x = y \log_c b$$

$$\frac{\log_c x}{\log_c b} = y$$

$$\frac{\log_c x}{\log_c b} = \log_b x$$

So if someone has kindly created a table of logarithms to base c, then creating a new table of logarithms to base b is simply a matter of dividing by a constant $\log_c b$

Logs to base e, the exponential constant, are used all over the place by physicists and mathematicians. However the base we are particularly interested in is base 10, and we'll now explore why. From the previous section we now use 10 instead of b:

- $x = 10^y$
- $y = \log_{10} x$
- $\log_{10} 1 = 0$
- $\log_{10} 10 = 1$
- $\log_{10} x^n = n \log_{10} x$
- $\log_{10} 10^n = n \log_{10} 10 = n$

I'll just use log from now on, rather than $\log_{10}$

We see that the values $\log 1$ to $\log 10$ will span the range 0 to 1.

Or put another way, $1 = 10^0$ and $10 = 10^1$ and an intermediate value $5 = 10^{0.7}$(approx).

Given an x value in this range of 1 to 10:

$$0 \leqq \log x \leqq 1$$

So what about $10x$?

$$\log 10x = \log 10 + \log x = 1 + \log x$$

$$1 \leqq \log 10x \leqq 2$$

and it follows

$$3 \leqq \log 100x \leqq 4$$

So in general, if you have calculated the logs of all values between 1 and 10, by simply adding unit values you get all logs for any number greater than 1

The logs of the numbers 10 to 100 are simply one added to the logs of the numbers 1 to 10.

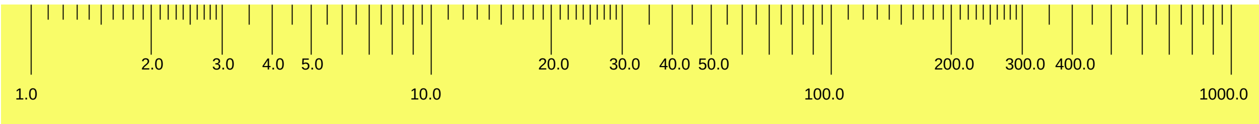
The logs of the numbers 100 to 1000 are simply two added to the logs of the numbers 1 to 10.

It works the other way as well, the logarithms of 0.1 to 1 are one subtracted from the logarithms of the numbers 1 to 10.

The basis of a slide rule is that x values are inscribed, with the distance from the left of each point being the $\log x$.

The x numbers shown on the rule are therefore the antilogs of the distance along the rule.

To get every number inscribed would take an infinitely long ruler, which could be unwieldy. Below is an image of x values from 1.0 to 1000.0



You might notice there is a level of redundancy here, values from 10.0 to 100.0 do not give any extra information, other than the power of 10, and similarly 100.0 to 1000.0 is just another repeat.

So slide rules can take advantage of this, by just having the numbers 1 to 10 inscribed along the rule.

There is a disadvantage; the slide rule can be used to add (or subtract) the Mantissa (the fractional part) of the logarithm, and ignore the exponent (the units). This means that the result does not give the powers of ten information.

So 9 x 7 may give 63, but the slide rule will not tell you if it is 6.3, or 63, or 630 etc.,

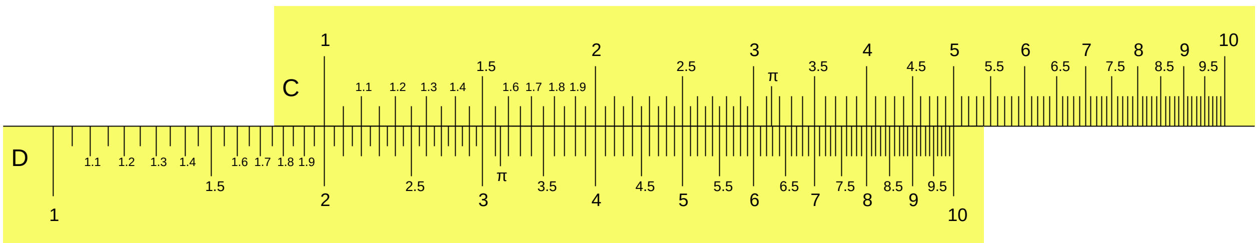
And again for 90 x 7, the slide rule will not tell you if it is 6.3, or 63, or 630 etc.,

So to use a slide rule you also need to use the mark 1 brain to figure out where that decimal point lies.

2 Using the Slide rule

2.1 Multiplication

To multiply two numbers using a slide rule, you add the two associated lengths together - the length of a slide rule being the log of the number. The two lengths together give another length (the log of the product), and from that resultant length you can just read off the result. So you need two slide rules, or, as I'm sure you know, the slide rule is made up of sliding parts.



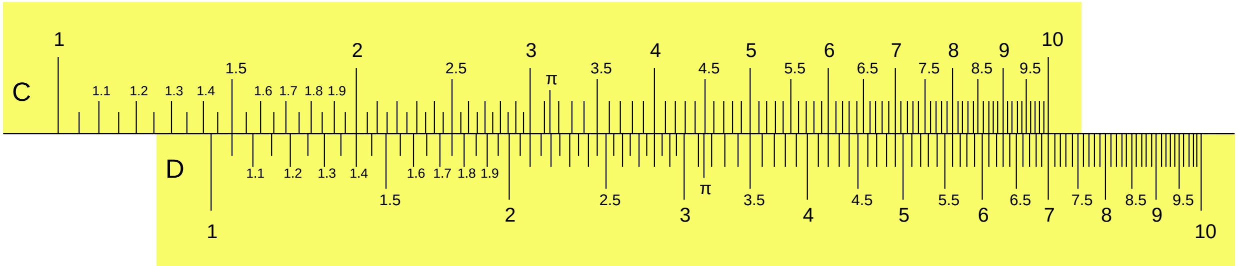
By aligning the 1 of scale C above the 2 of scale D, we have added the $\log 2$ distance to scale C.

Remember that though we have put a '1' over the 2, $\log 1 = 0$, and so in terms of the lengths, this is putting a zero over the 2 position.

By moving along C to any number, say 3, we have added the log of 3. Then reading down onto scale D - which is now the length of $\log 2 + \log 3$,we can see the value of the product, which in this case is 6.

You would do the same thing if you were multiplying 20 by 30. As discussed previously, it is up to you to work out the decimal place.

You may have spotted a problem with this. What if we wanted 2x7? The 7 on the C scale is beyond the end of the D scale. So here's another magic trick:



In this case the 10 of the C scale is positioned over the 7. Then move to the 2 of the C scale, and below it you can see the result 1.4

As the slide rule is indifferent to powers of ten, we have to figure the result is actually 14.

So how are the logarithms being added in this method?

By setting the 10 on to 7, we have the length of the D scale, which is $\log 7$, but then traveling left on the C, from 10 right back to 1, we have subtracted $\log 10$. Then traveling right on the C from 1 to 2, we have added $\log 2$. So we have actually done:

$$\log 7 - \log 10 + \log 2 = \log \frac{7 \times 2}{10}$$

By moving the C scale left we are dividing by 10, but we know not to worry about this, and so we have the significant digits of the result.

In practice, the usage is simple; put either the 1 or the 10 of the sliding C scale over one number, go to the other number on the sliding scale, and read the result on the D scale.

Earlier we mentioned '9 x 7 may give 63', just for fun, on the scale above, you can see the 9 on C is above 6.3 on D.

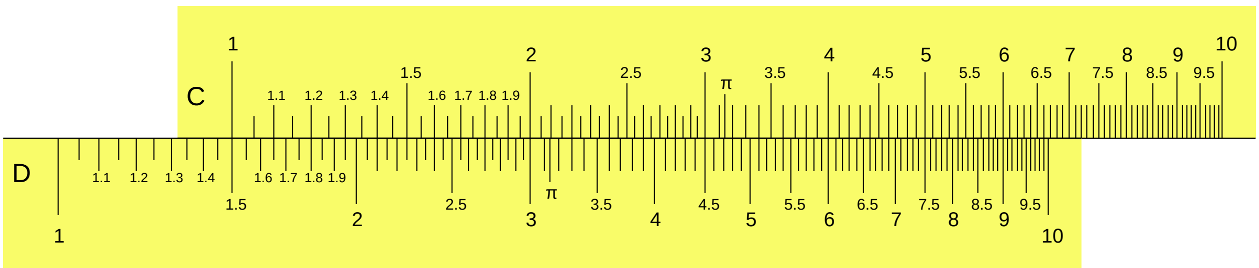
2.2 Division

We know division maps to subtraction with logs.

$$\log \frac{c}{d} = \log c - \log d$$

So how to subtract distance on a slide rule? Lets try six divided by four, which is 1.5

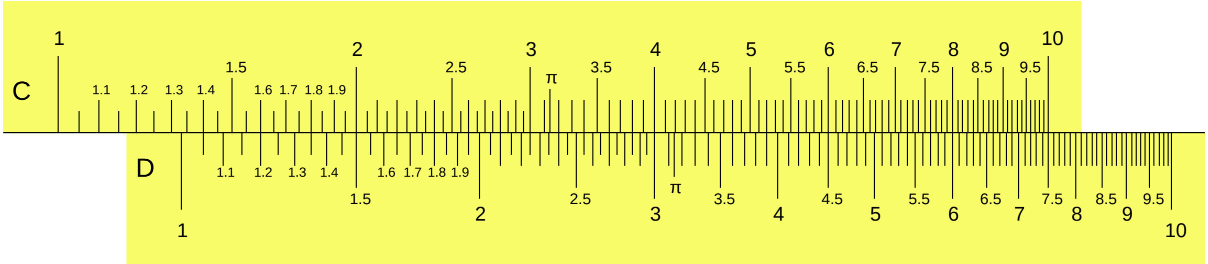
The distance to 6 of the D scale is $\log 6$, move the 4 of the C scale above it, and traveling back along the C scale to the 1 is equivalent to subtracting $\log 4$.



So the distance on the D scale is $\log 6 - \log 4 = \log 1.5$ and by reading the inscribed value, which is the equivalent of reading the antilog, we find 1.5.

Notice, though we see division as $\frac{6}{4}$ the 6 is on the bottom scale, and the four is above it.

Again we have a problem, what about $\frac{6}{8}$? Moving the 8 above the 6 leaves the 1 on the C scale beyond the D scale.



However the 10 on C scale shows 7.5 on D, and 0.75 is the right answer, so how does that come about?

6 on the D scale is equivalent to $\log 6$, subtracting $\log 8$ takes you leftwards to C1, then adding $\log 10$ takes you to C10.

$$\log 6 - \log 8 + \log 10 = \log \frac{6 \times 10}{8}$$

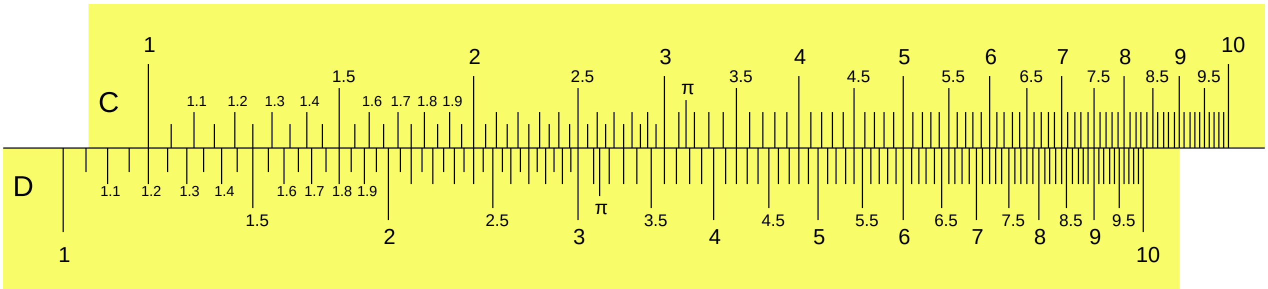
Powers of ten do not bother us, so reading the antilog on the D scale gives us 7.5, which we realize means 0.75

We can be confident that reading right along a scale from the leftmost 1 is adding logs (multiplying) and moving left to the leftmost 1 is subtracting logs (dividing). If necessary to get a result, add or subtract the entire scale, which is the equivalent of multiplying or dividing by 10.

2.3 CF and DF

Another twist to the tale of multiplication. Depending on the model, some sliderules have a selection of different scales on both sides which are often useful.

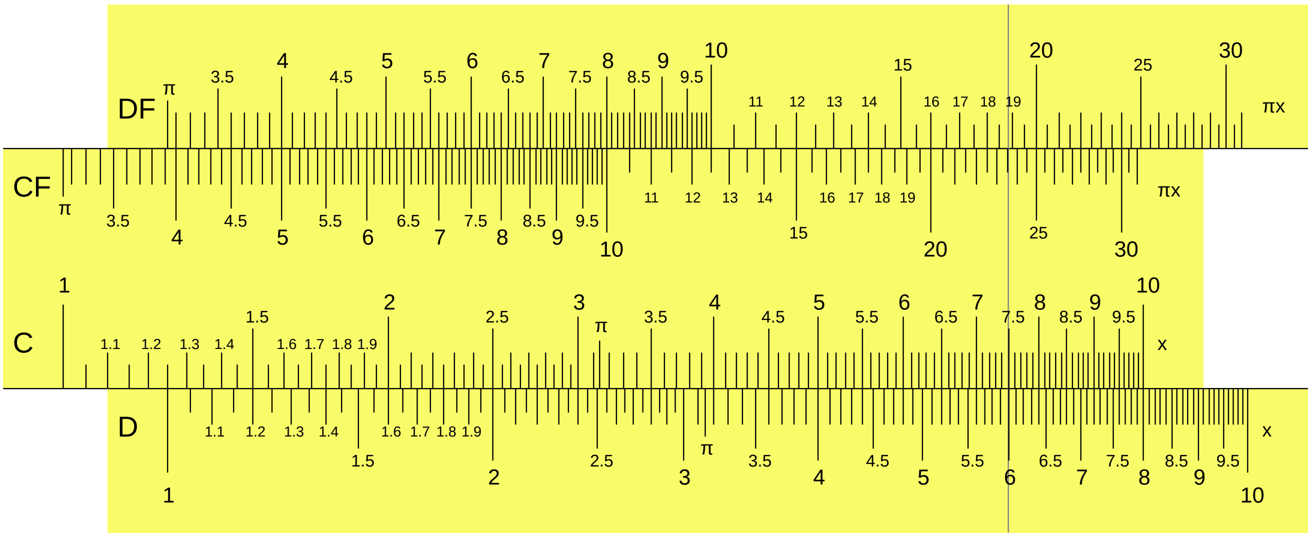
Consider the multiplication of 12×8, or in other words $\log 12 + \log 8$:



So the 1 of the C scale has been set over 1.2 (12) of D, and under the 8 of the C scale you can read the result on the D scale of 9.6 (or 96). Nothing wrong with that, unfortunately with a real sliderule that's where your thumb is!

Physically you are holding the slide rule with a hand at either end, and for numbers at the ends, that can be slightly awkward.

Most sliderules also have the scales CF and DF, which are the same as C and D but 'Folded', by the scales being offset by $x = \pi$, and which puts a 10 value more or less in the centre of the scale. See below:



In this case, we have done the multiplication using the central sliding portion and the CF and DF scales. CF does not have a 1, but we know 10 does as good a job, so we have set CF 10 against DF 8, which in terms of distance is $\log 8$ along DF from the DF 1 which would be off to the left if it had one. Then we want to add $\log 12$.

So go along CF to the 12, which adds the $\log 12$,(give or take a factor of 10), and then read the result on DF of 9.6 (96). All conveniently done in the centre of the ruler.

The DF scale has another use, as its starting point of $\pi$is directly over the D value of 1, we can add log distances along the DF scale just by moving along the D scale. Using the hairline slider over D6 we can view $6 \times \pi = 18.8$ approx by reading it from the DF scale.

References

[1] https://en.wikipedia.org/wiki/Logarithm