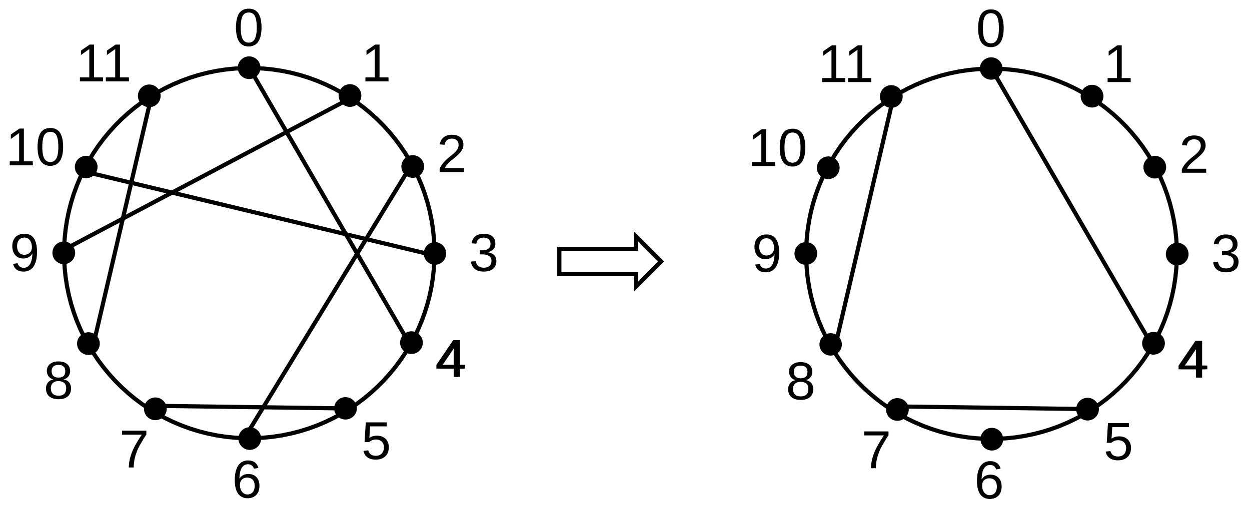
# Maximum Planar Subset of Chords

# Supowit's Algorithm for Finding MPSC

- Supowit, "Finding a maximum planar subset of a set of nets in a channel," *IEEE TCAD*, 1987.
- Problem: Given a set of $n$ chords $C$ and assume no two chords of $C$ share an endpoint, find a maximum planar subset of chords.
  - Label the vertices on the circle $0$ to $2n-1$.
  - Compute $MIS(i,j)$: size of maximum independent set between vertices $i$ and $j$, $i \le j$.
  - Answer = $MIS(0,2n-1)$



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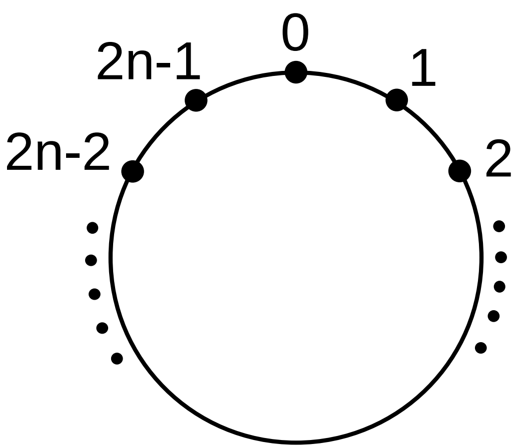
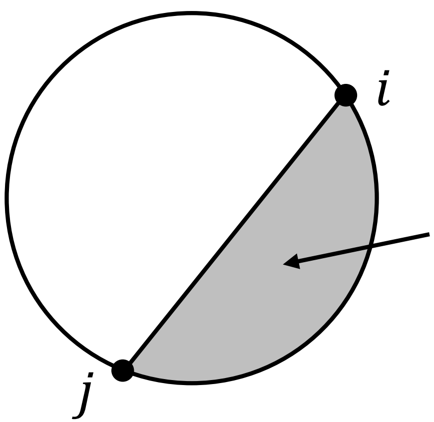


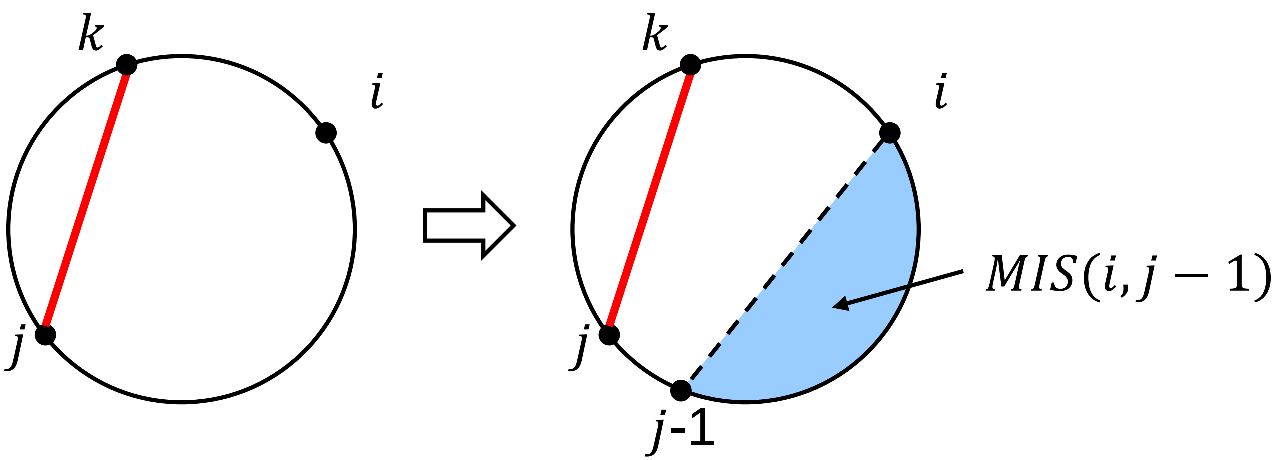



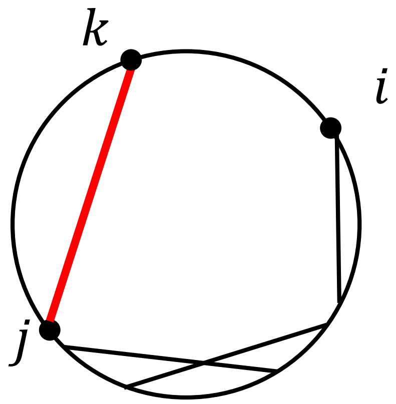

$MIS(i,j)$: #chords of the maximum planar subset in the shaded region

# Dynamic Programming in Supowit's Algorithm

- Apply dynamic programming to compute $MIS(i, j)$.
  - **Case 1:** $kj \in C, k \notin [i, j] \Rightarrow MIS(i,j) = MIS(i, j-1)$
  - **Case 2:** $kj \in C, k \in [i, j]$
  $\Rightarrow MIS(i,j) = \max\big(MIS(i, j-1), MIS(i, k-1) + 1 + MIS(k+1, j-1)\big)$
  - **Case 3:** $ij \in C \Rightarrow MIS(i,j) = MIS(i+1, j-1) + 1$

**Case 1**






$$MIS(i,j) = MIS(i,j-1) = 2$$

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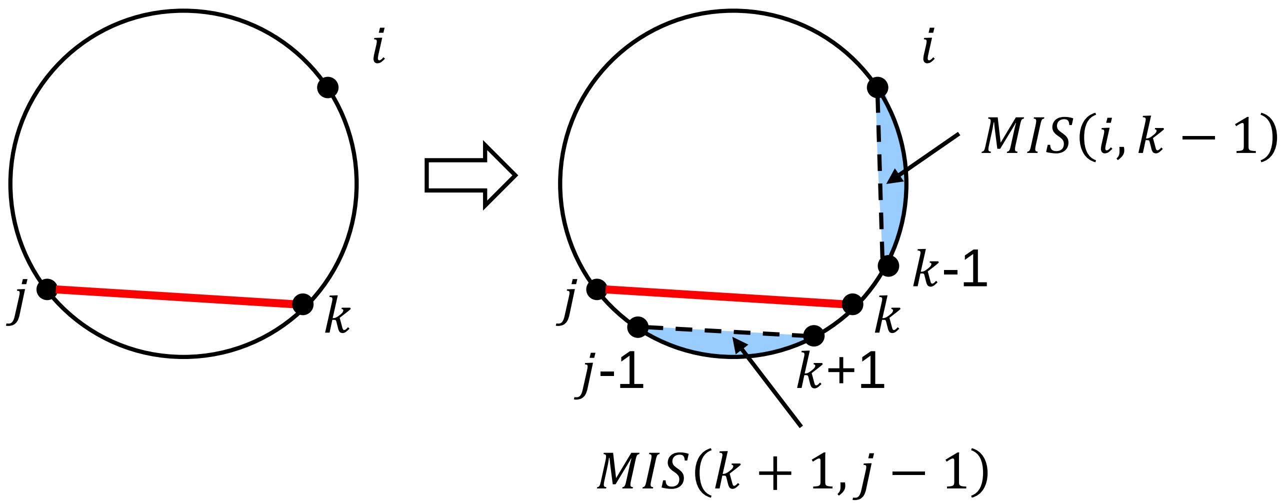
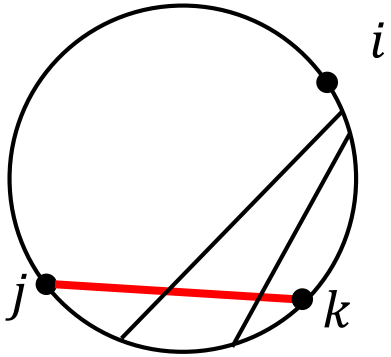
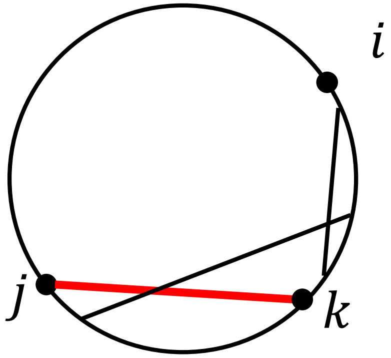
**Case 2**






$$MIS(i, j-1) = 2$$
$$MIS(i, k-1) + 1 + MIS(k+1, j-1) = 1$$




$$MIS(i, j-1) = 1$$
$$MIS(i, k-1) + 1 + MIS(k+1, j-1) = 2$$

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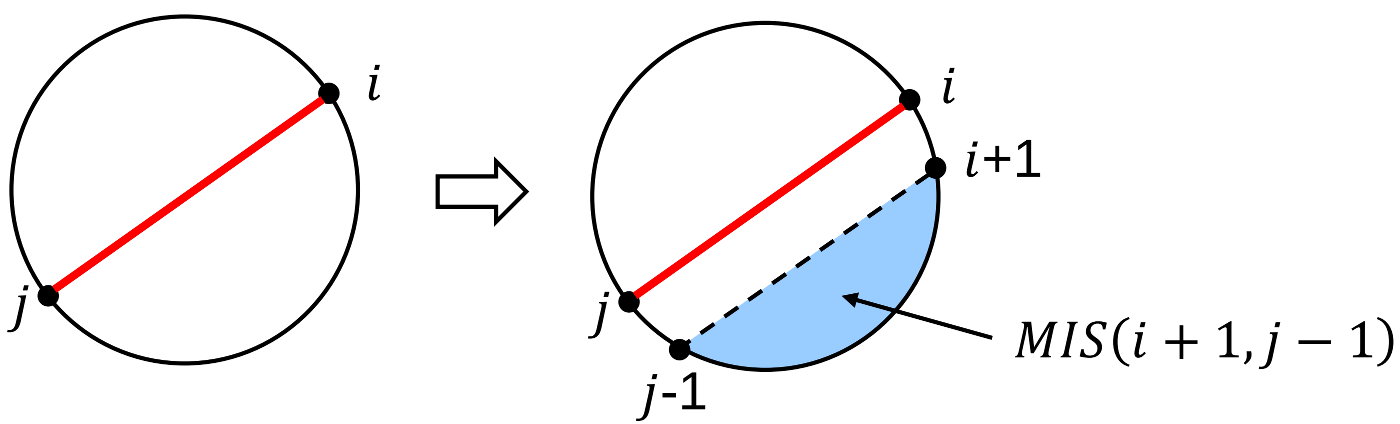
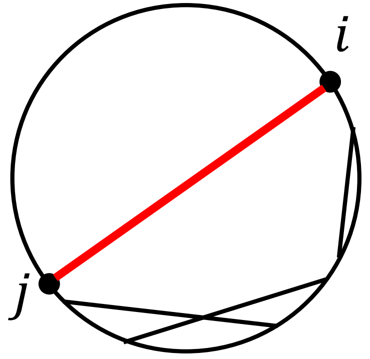
**Case 3**






$$MIS(i,j) = MIS(i+1, j-1) + 1 = 2 + 1 = 3$$