3b) $x'' + x = \text{sen}(at)$  $x(0) = x'(0) = 0$

$\mathcal{L}\{x''\} = s^2 X(s) - sX(0) - x'(0) \to \text{pvi} = s^2 X(s)$

$\mathcal{L}\{f^{(n)}(t)\} = s^n F(s) - \sum_{i=1}^{n} s^{n-i} f^{(i-1)}(0)$

$\mathcal{L}\{x^{(4)}\} = s^4 X(s) - s^3 x(0) - s^2 X'(0) - sX''(0) - x'''(0)$

$\mathcal{L}\{x'' + x\} = \{\sin(2t)\}$

$s^2 X(s) - sX(0) - X'(0) + X(s) = \frac{2}{s^2+4}$

$X(s)(s^2+1) = \frac{2}{s^2+4} \Rightarrow X(s) = \frac{2}{(s^2+4)(s^2+1)} \;/\; \mathcal{L}^{-1}\{\}$

$x(t) = \mathcal{L}^{-1}\left\{\frac{2}{(s^2+4)(s^2+1)}\right\} \to$ fracciones parciales

FP. $\frac{as+b}{s^2+4} + \frac{cs+d}{s^2+1} = \frac{2}{(s^2+4)(s^2+1)}$

$(as+b)(s^2+1) + (cs+d)(s^2+4) = 2$

$as^3 + as + bs^2 + b + cs^3 + 4cs + ds^2 + 4d = 2$

$$\left.\begin{aligned} a+c &= 0 \\ b+d &= 0 \\ a+4c &= 0 \\ b+4d &= 2 \end{aligned}\right\} \quad a=0 \quad c=0 \quad d = 2/3 \quad b = -2/3$$

$x(t) = \mathcal{L}^{-1}\left\{-\frac{2}{3}\cdot\frac{1}{s^2+4} + \frac{2}{3}\cdot\frac{1}{s^2+1}\right\}$

$= -\frac{2}{3}\,\frac{\text{sen}(2t)}{2} + \frac{2}{3}\,\text{sen}(t)$

5b) $f(t) = e^{-2t}\sin(3\pi t)$

$\mathcal{L}\{e^{at} f(t)\} = F(s-a)$

$F(s) = \mathcal{L}\{f(t)\}$

$F(s+2) \longrightarrow F(s) = \mathcal{L}\{\sin(3\pi t)\}$

$\frac{3\pi}{(s+2)^2 + 9\pi^2} \qquad = \frac{3\pi}{s^2+9\pi^2}$

3.b

$$\left.\begin{aligned} x' &= 2x + y \\ y' &= 6x + 3y \end{aligned}\right| \quad \begin{aligned} x(0) &= 1 \\ y(0) &= -2 \end{aligned}$$

$sX(s) - X(0) = 2X(s) + Y(s)$

$sY(s) - y(0) = 6X(s) + 3Y(s)$

$(s-2)X(s) - 1 = Y(s)$

$(s-3)Y(s) = 6X(s) - 2$

$(s-3)((s-2)X(s) - 1) = 6X(s) - 2$

$(s^2 - 5s + 6)X(s) - s + 3 = 6X(s) - 2$

$s(s-5)X(s) = s - 5$

$X(s) = \frac{1}{s} \;/\; \mathcal{L}^{-1}\{\} \longrightarrow x(t) = 1$

$(s-2)\frac{1}{s} - 1 = Y(s)$

$Y(s) = \frac{-2}{s} \xrightarrow{\mathcal{L}^{-1}\{\}} y(t) = -2$

4.a) $\mathcal{L}\{\cosh(kt)\}$  $\mathcal{L}\{\sinh(kt)\} = \frac{k}{s^2-k^2}$

$f(t) = \sinh(kt)$

$f'(t) = k\cosh(kt)$

$\mathcal{L}\{f'(t)\} = \mathcal{L}\{\cosh(kt)\,k\} = sF(s) - f(0)$

$\cancel{k}\,\mathcal{L}\{\cosh(kt)\} = \frac{\cancel{s}\,k}{s^2-k^2}$

5d) $F(s) = \frac{3s+5}{s^2-6s+25} = \frac{3s+5}{(s-3)^2+16} = \frac{3s+9-9+5}{(s-3)^2+16} = \frac{3(s-3)+14}{(s-3)^2+16}$

$= \frac{3(s-3)}{(s-3)^2+16} + \frac{14}{(s-3)^2+16} \Rightarrow F(s) = \frac{3s}{s^2+16} + \frac{14}{s^2+16}$

$e^{3t}\,\mathcal{L}^{-1}\left\{\frac{3s}{s^2+16} + \frac{14}{s^2+16}\right\}$

$f(t) = e^{3t}\left(3\cos(4t) + \frac{14}{4}\,\text{sen}(4t)\right)$

6a) $X^{(3)} + x'' - 6X' = 0$

$x(0) = 0 \qquad X'(0) = X''(0) = 1$

$s^3 X(s) - s^2 \cancel{X(0)} - sX'(0) - X''(0) + s^2 X(s) - s\cancel{X(0)} - X'(0) - 6sX(s) + 6\cancel{X(0)} = 0$

$(s^3 + s^2 - 6s)X(s) - s - 1 - 1 = 0 \Rightarrow X(s) = \frac{s+2}{s(s+3)(s-2)}$

$\frac{a}{s} + \frac{b}{s+3} + \frac{c}{s-2} = \frac{s+2}{s(s-2)(s+3)}$

$as^2 + as - 6a + bs^2 - 2bs + cs^2 + 3cs = s+2$

$a+b+c = 0$

$a - 2b + 3c = 1$

$-6a = 2$

$a = -\frac{1}{3} \quad b = -\frac{1}{15} \quad c = \frac{2}{5}$

$X(s) = -\frac{1}{3}\cdot\frac{1}{s} - \frac{1}{15}\cdot\frac{1}{s+3} + \frac{2}{5}\cdot\frac{1}{s-2}$

$x(t) = -\frac{1}{3} - \frac{1}{15}e^{-3t} + \frac{2}{5}e^{2t}$

7.a) $F(s) = \frac{1}{(s^2+9)^2}$

$f * g = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$

$\mathcal{L}\{f*g\} = F(s)G(s)$

$\mathcal{L}^{-1}\left\{\frac{1}{s^2+9}\cdot\frac{1}{s^2+9}\right\} = \mathcal{L}^{-1}\{F(s)\cdot G(s)\}$

$f(t) = \frac{\sin(3t)}{3} = f(t) * g(t) \to$ convolución

$g(t) = \frac{\sin(3t)}{3}$

$$\int_0^t \frac{1}{3}\sin(3\tau)\,\frac{1}{3}\sin(3t-3\tau)\,d\tau = \frac{1}{9}\int_0^t \sin(3\tau)\left(\sin(3t)\cos(3\tau)-\sin(3\tau)\cdot\cos(3t)\right)d\tau$$

suma de ángulos

$$= \frac{1}{9}\left(\sin(3t)\int_0^t \sin(3\tau)\cos(3\tau)\,d\tau - \cos(3t)\int_0^t \sin^2(3\tau)\,d\tau\right)$$

$u = \sin(3t)$
$du = 3\cos(3t)\,dt$

$$\frac{1}{9}\left(\sin(3t)\int_0^{\sin(3t)} \frac{u}{3}\,du - \cos(3t)\int_0^t \frac{1}{2}-\frac{\cos(6t)}{2}\,d\tau\right)$$

$$\frac{1}{9}\left(\frac{u^2}{6}\Big|_0^{\sin(3t)} \sin(3t) - \cos(3t)\left(\frac{1}{2}\tau - \frac{\sin(6\tau)}{12}\Big|_0^t\right)\right)$$

$$\frac{1}{9}\left(\frac{\sin^3(3t)}{6}\left(\sin^2(3t)+\cos^2(3t)\right) - \frac{t}{2}\cos(3t)\right)$$

$$f(t) = \frac{\sin(3t)}{54} - \frac{t\cos(3t)}{18}$$

8.a) $f(t) = te^{2t}\cos(3t)$

$$\mathcal{L}\{t^n f(t)\} = (-1)^n F^{(n)}(s)$$

$$\mathcal{L}\{te^{2t}\cos(3t) = -1F'(s)$$

$F(s) = \mathcal{L}\{e^{2t}\cos(3t)\}$ $\neq (s-2)$

$$= \frac{(s-2)}{(s-2)^2+9}$$

$\mathcal{L}\{f(t)\} = -1\cdot F'(s)$

$$F(s) = \frac{s-2}{(s-2)^2+9} \qquad F'(s) = \frac{(s-2)^2+9-2(s-2)^2}{((s-2)^2+9)^2}$$

$$\mathcal{L}\{te^{2t}\cos(3t)\} = \frac{(s-2)^2-9}{((s-2)^2+9)^2}$$

8b) $f(t) = \frac{\sin(t)}{t}$ $\qquad \mathcal{L}\left\{\frac{f(t)}{t}\right\} = \int_s^\infty \mathcal{L}\{f(t)\}\,dt$

$$\mathcal{L}\left\{\frac{\sin(t)}{t}\right\} = \int_0^\infty \frac{1}{t^2+1}\,dt = \tan^{-1}(t)\Big|_s^\infty$$

$$= \pi/2 - \tan^{-1}(s)$$

8d) $\mathcal{L}^{-1}\left\{\operatorname{Arctan}\left(\frac{3}{s+2}\right)\right\}$

$$\mathcal{L}\{tf(t)\} = -1\cdot F'(s)$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\{F'(s)\}$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{1}{1+\left(\frac{3}{s+2}\right)^2} - \frac{3}{(s+2)}\right\}$$

$$f(t) = -\frac{1}{t}\mathcal{L}^{-1}\left\{\frac{-3}{s^2+4s+13}\right\} = -1$$