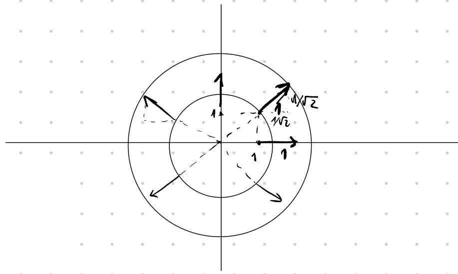
1] $f = \sqrt{x^2+y^2}$

Problema 1

(a) Encuentre el gradiente ∇f del campo escalar $f(x,y) = \sqrt{x^2+y^2}$. Dicho gradiente corresponde a un campo vectorial, haga un bosquejo de él en el plano.

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \left(\frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}, \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}\right)$$

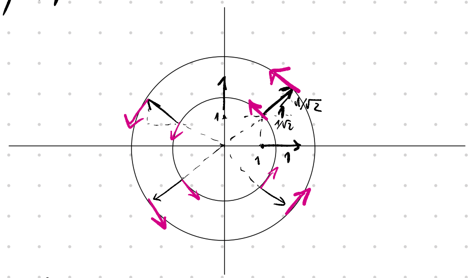
$$\nabla f = \frac{1}{\sqrt{x^2+y^2}}(x,y) \Rightarrow \nabla f(1,1) = \frac{(1,1)}{\sqrt{2}}$$



(b) Ocupe (a) para representar el campo vectorial:

$$\mathbf{F}(x,y) = \frac{1}{\sqrt{x^2+y^2}}(-y,x)$$

$$(x,y) \rightarrow \frac{1}{\sqrt{x^2+y^2}}(-y,x)$$



2] a) $f = e^{xy} + 2\arctan(y)$

$$\nabla f = \left(ye^{xy},\ xe^{xy} + \frac{2}{1+y^2}\right) \Rightarrow f \text{ función escalar) } \mathbb{R}^3 \rightarrow \mathbb{R}.$$

$F: \mathbb{R}^3 \longrightarrow \mathbb{R}^3$

b) $f = e^{yz}\,\text{sen}(xy)$

$$\nabla f = \left(e^{yz}\cos(xy)\cdot y,\ e^{yz}\cdot z\ \text{sen}(xy) + e^{yz}\cdot\cos(xy)\cdot x,\ y\cdot e^{yz}\ \text{sen}(xy)\right)$$

Problema 3

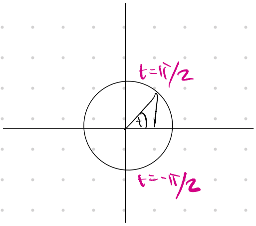
Calcule las siguientes integrales de línea, siendo C la curva dada en cada caso

(a) $\int_C xy^4\,ds$ donde C es la mitad derecha de la circunferencia $x^2+y^2=16$

$r = 4$

$$\int_C f(x,y,z)\,ds, \quad t\in[a,b] \quad = \int_a^b f(r(t))\,\underbrace{\|r'(t)\|\,dt}_{ds}.$$

$$\int_C xy^4 \, ds \qquad x^2+y^2=16$$



$r=4$

$x = 4\cos\theta$

$y = 4\,\text{sen}\,\theta$

$$r'(t) = (-4\,\text{sen}(t),\, 4\cos(t))$$

$s = 4t. \qquad s/4 = t$

$$\int_C f\,ds = \int_{-\pi/2}^{\pi/2} 4\cos(t)\, 4^4 \text{sen}^4(t) \cdot \underbrace{4}_{\|r'\|} \, dt$$

$$= 4^6 \int_{-\pi/2}^{\pi/2} \cos(t)\,\text{sen}^4(t)\, dt \qquad \begin{aligned} u &= \text{sen}(t) \\ du &= \cos(t)\,dt \end{aligned}$$

$$= 4^6 \int_{-1}^{1} u^4 \, du$$

$$= 4^6 \left[\frac{u^5}{5}\right]_{-1}^{1} = 4^6\left(\frac{1}{5} - -\frac{1}{5}\right) = \frac{2}{5}\cdot 4^6 = \frac{2\cdot 4^6}{5}$$

b) $\int_C x e^{yz} \, ds \qquad C: (0,0,0) \to (1,2,3)$

$t \in [0,1]$

$$r(t) = \overset{P_0}{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}(1-t) + \overset{A}{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}} t.$$

$$r(t) = (t, 2t, 3t)$$

$$r'(t) = (1, 2, 3) \Rightarrow \|r'(t)\| = \sqrt{1+4+9} = \sqrt{14}$$

$$\int_C f\,ds = \int_0^t \underbrace{te^{6t^2}}_{f(r(t))} \underbrace{\sqrt{14}}_{\|r'(t)\|} \, dt = \sqrt{14} \int_0^1 te^{6t^2} \, dt =$$

$u = 6t^2$

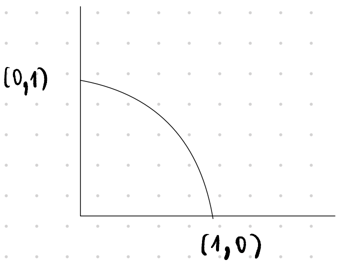
$du = 12t\,dt$

$\frac{du}{12} = t\,dt$

$$\sqrt{14}\int_0^6 \frac{e^u}{12}\,du = \frac{\sqrt{14}}{12}\left[e^u\right]_0^6 = \frac{\sqrt{14}}{12}\left[e^6 - 1\right]$$

Problema 4

Encuentre la masa de un cable delgado que se encuentra estirado sobre un arco de la circunferencia unitaria centrada en el origen, el arco va desde el punto $(1,0)$ hasta $(0,1)$, si la densidad del cable está dada por la función $\rho(x,y) = xy$. Usando la masa encuentre las coordenadas del centro de masa



$$\rho = xy.$$

$$\bar{x} = \frac{1}{m}\int_c x\,\|r'(t)\|\,dt =$$

$$r(t) = (\cos(t), \operatorname{sen}(t))$$

$$r'(t) = (-\operatorname{sen}(t), \cos(t))$$

$$\|r'(t)\| = \sqrt{1} = 1$$

$$\boxed{m = \int_C \rho\;ds} = \int_0^{\pi/2} \cos(t)\operatorname{sen}(t)\cdot 1\,dt$$

$$= \int_0^{\pi/2} \frac{\operatorname{sen}(2t)}{2}\;1\,dt$$

$$= \left[-\frac{\cos(2t)}{2}\right]_0^{\pi/2}$$

$$= \frac{1}{2}\left(\frac{1}{2} - -\frac{1}{2}\right) = \frac{1}{2}\cdot\frac{2}{2} = \frac{1}{2}.$$

$$C = (\bar{x}, \bar{y})$$

$$\bar{x} = \frac{1}{m}\int_C x\,\rho\,ds = 2\int_0^{\pi/2} \cos^2(t)\operatorname{sen}(t)\,1\cdot dt$$

$$= 2 \cdot \frac{-\cos^3(t)}{3}\Big|_0^{\pi/2} = 2/3.$$

$$\bar{y} = \frac{1}{m}\int_C y\,\rho\, ds$$

$$= 2\int_0^{\pi/2} \underbrace{\cos(t)\,\text{sen}^2(t)}_{y\rho}\ \underbrace{1\,dt}_{ds} = \frac{2\,\text{sen}^3(t)}{3}\Big|_0^{\pi/2} = 2/3.$$

$$c = (2/3,\ 2/3)$$