Plano tangente normal es $r_u \times r_v$

$$\iint_S f(x,y,z)\,dS = \iint f(r(u,v))\,\|r_u \times r_v\|\,du\,dv$$

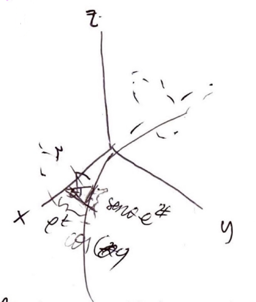
$$\iint_S F(x,y,z)\cdot d\vec{S} = \iint F(r(u,v))\cdot(r_u \times r_v)\,du\,dv$$

Stokes F con componentes de clase C^1 en S. $\int_C F\,d\vec{r} = \iint_S (\vec{\nabla}\times F)\cdot d\vec{S}$

Problema 1

Sea S la superficie obtenida al rotar la gráfica de la función $f(x)=e^{2x}$ en el plano XY en torno al eje x. Determine el conjunto de puntos en S tal que su plano tangente pasa por $(0,0,0)$

Ayudantía P2



superficie al rotar $f(x)=e^{2x}$ en torno al eje x

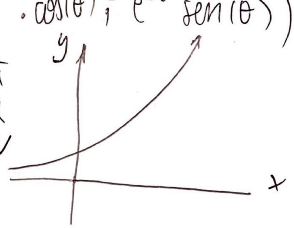
$r(t) = (t, e^{2t}, 0)$ se queda igual

$\Downarrow$ rotas

$= (t, e^{2t}\cdot\cos(\theta), e^{2t}\,\mathrm{sen}(\theta))$

$0 \le \theta \le 2\pi$

$t \in \mathbb{R}$



Buscan todos los puntos $\rightarrow$ $\&$ N tales que su plano tangente pasan por $(0,0,0)$

$r_t = (1, 2e^{2t}\cos(\theta), 2e^{2t}\,\mathrm{sen}(\theta))$

$r_\theta = (0, -e^{2t}\,\mathrm{sen}(\theta), e^{2t}\cos(\theta))$

$r_t \times r_\theta = (2e^{4t}(\cos^2(\theta)+\mathrm{sen}^2(\theta)) - \hat{j}\,(e^{2t}\cos(\theta)) + \hat{k}(-e^{2t}\,\mathrm{sen}\theta)$

$= e^{2t}(2e^{2t}, -\cos(\theta), -\mathrm{sen}(\theta))$ } vector normal.

$(P_1 - P_0)\cdot N = 0$

$r(t,\theta)$ $\quad$ $(0,0,0)$

$[(t, e^{2t}\cos\theta, e^{2t}\,\mathrm{sen}\theta) - (0,0,0)]\cdot(e^{2t}(2e^{2t}, -\cos\theta, -\mathrm{sen}\theta)) = 0$

$2te^{2t} - e^{2t}\cos^2(\theta) - e^{2t}\,\mathrm{sen}^2(t) = 0$

$2te^{2t} - e^{2t} = 0$

$2t - 1 = 0 \Rightarrow t = 1/2$

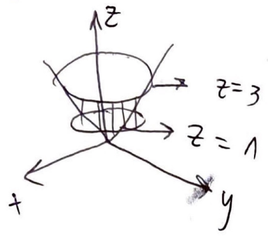
$\Rightarrow (1/2, e\cos\theta, e\,\mathrm{sen}(\theta)) \mid \theta \in [0, 2\pi) \rightarrow$ circunferencia.

Problema 2

Calcule la masa de la parte del cono $z^2 = x^2 + y^2$ que está entre los planos $z = 1$ y $z = 3$ si su función densidad es $x^2 z^2$

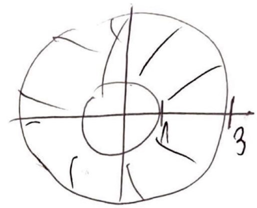
masa   cono $z^2 = x^2 + y^2$ entre $z = 1$ y $z = 3$

$f(x, z) = x^2 z^2$



$x = r\cos(\theta)$

$y = r\,sen(\theta)$



$0 \le \theta \le 2\pi \quad 1 < r < 3$

$\Rightarrow r(\theta, r) = (r\cos\theta, r\,sen\theta, r) \quad z = \sqrt{r^2} = r$

$r_r(t) = (\cos(\theta), sen(\theta), 1) \qquad r_\theta(t) = (-r\,sen(\theta), r\cos(\theta), 0)$

$r_r \times r_\theta = \hat{i}(-r\cos(\theta)) - \hat{j}(r\,sen(\theta)) + \hat{k}(r)$

$= r(-\cos(\theta), -sen(\theta), 1)$

$\| r_r \times r_\theta \| = |r| \sqrt{\cos^2(\theta) + sen^2(\theta) + 1} = \sqrt{2}\, r.$

$\int_0^{2\pi} \int_1^3 r^2 \cos^2(\theta)\, r^2 \,(\sqrt{2}\, r)\, dr\, d\theta$

$\int_0^{2\pi} \cos^2(\theta)\, d\theta \cdot \int_1^3 r^5 = \int_0^{2\pi} \frac{1 + \cos(2\theta)}{2} d\theta \cdot \int_1^3 r^5 dr.$

$= \frac{\sqrt{2}}{2} \left(\theta + \frac{sen(2\theta)}{2} \Big|_0^{2\pi} \cdot \frac{r^6}{6} \Big|_1^3 \right)$

$= \frac{\sqrt{2}}{12} \cdot 2\pi \left(3^6 - 1\right) = \frac{\sqrt{3}\,\pi}{6} \left(3^6 - 1\right).$

4

27·27

1x9   729

Problema 3

La carga total Q dentro de una superficie cerrada S se relaciona con el campo eléctrico $\mathbf{E}$ (campo vectorial) y la misma superficie de la siguiente manera:

$$\oint_S \mathbf{E} \cdot \mathbf{n} \, dS = \frac{Q}{\epsilon_0}$$

Donde ϵ_0 es una constante y $\mathbf{n}$ apunta "hacia afuera" de la superficie. Calcular la carga total dentro de la región delimitada por el paraboloide $z = 4 - x^2 - y^2$ y el plano XY cuando el campo eléctrico es:

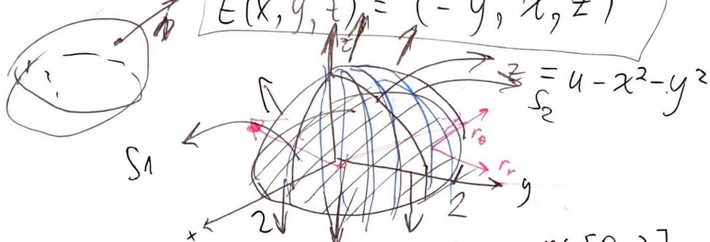
$$\mathbf{E}(x, y, z) = (-y, x, z)$$

$$\oint E \cdot n \, dS = \frac{Q}{\varepsilon_0}$$

n apunta hacia afuera

$z = 4 - x^2 - y^2$

$$\boxed{E(x, y, z) = (-y, x, z)}$$



$r(t) = (r\cos(\theta),\ r\,\text{sen}(\theta),\ 4 - r^2)$ $\quad r \in [0, 2]$

$r_r \times r_\theta = (\cos(\theta), \text{sen}(\theta), -2r) \times (-r\,\text{sen}\theta, r\cos(\theta), 0)$ $\quad \theta \in [0, 2\pi]$

$= \hat{i}(2r^2\cos\theta) - \hat{j}(-2r^2\,\text{sen}(\theta)) + \hat{k}(r)$

$$\int_0^{2\pi}\int_0^2 (-r\,\text{sen}\theta, r\cos\theta, 4 - r^2) \cdot (2r^2\cos\theta, 2r^2\,\text{sen}\theta, r)\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^2 \cancel{-2r^3\,\text{sen}\theta\cos(\theta)} + \cancel{2r^3\cos(\theta)\,\text{sen}(\theta)} + 4r - r^3\, dr\, d\theta$$

$$2\pi\int_0^2 4r - r^3\, dr = 2\pi\left(2r^2 - \frac{r^4}{4}\right)\Big|_0^2 = 2\pi\left(8 - \frac{16}{4}\right) = 8\pi.$$

S_2: $x = r\cos\theta$ $\quad r \in (0, 2)$ $\quad \theta \in [0, 2\pi]$

$y = r\,\text{sen}\theta$

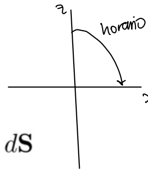
$z = 0$ $\quad r_r = (\cos\theta, \text{sen}\theta, 0)$ | $r_r \times r_\theta = \hat{i}(0) - \hat{j}(0) + \hat{k}\, r \rightarrow \begin{pmatrix} 0 \\ 0 \\ -r \end{pmatrix}$ $r_\theta \times r_r$

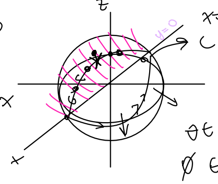
$$\int_0^{2\pi}\int_0^2 (-r\,\text{sen}(\theta), r\cos(\theta), 0) \cdot (0, 0, -r)\, dr\, d\theta = 0 \;//$$

$Q = 8\pi\,\varepsilon_0$

Problema 4

$r(t) (r\cos\theta, r\sin\theta)$ antihorario





$\theta \in (0, \pi)$

$\phi \in (0, \pi)$

Ocupe el teorema de Stokes para calcular

$$\oint_C \vec{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

$$-\oint_{-C} F \cdot dr =$$

para el campo $\mathbf{F}(x, y, z) = (y, z, x)$ y S el hemisferio $x^2 + y^2 + z^2 = \boxed{1}$, $\boxed{y \geq 0}$ orientado en la dirección de y positivo

$\sin\phi \cos\theta$ $\quad$ $\sin\phi \sin\theta$ $\quad$ $\cos\phi$

$-C: \quad r(t) = (\cos(t), 0, \sin(t))$

$F(r(t)) = (0, \sin(t), \cos(t))$

$r'(t) = (-\sin(t), 0, \cos(t))$

$F \cdot r' = \cos^2(t)$

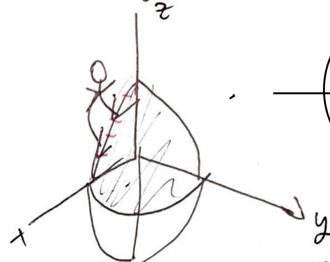
$2\int_0^{2\pi} 1 + \cos(2t)$

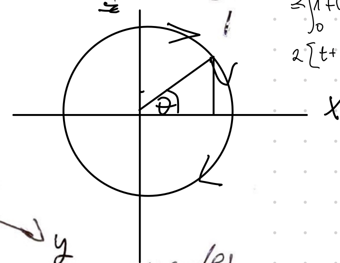
$2\left[t + \frac{\sin(2t)}{2}\right]_0^{2\pi}$

$\boxed{4}$

$$F = (y, z, x)$$

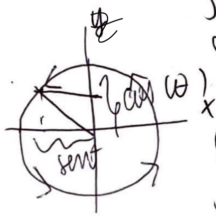
$$x^2 + y^2 + z^2 = 1, \quad y \geq 0.$$





$$\iint_S (\nabla \times F)\, d\vec{S}$$

Recuerdo = Por Stokes $\int_C F\, d\vec{r}$ $\quad$ F de tiene clase componentes C^1.



$C: \quad r(\theta) = (\sin(\theta), 0, \cos(\theta))$

$r'(\theta) = (\cos(\theta), 0, -\sin(\theta))$

$$\int_0^{2\pi} \underbrace{(0, \cos(\theta), \sin(\theta))}_{F(r(\theta))} \cdot \underbrace{(\cos\theta, 0, -\sin\theta)}_{r'(\theta)}\, d\theta$$

$= -\pi$ ✓