Problema 1 (Ayudantía anterior)

Suponga que $X_1, \ldots, X_{n_1} \overset{\text{iid}}{\sim} N(\mu_1, \sigma_1^2)$ e $Y_1 \ldots, Y_{n_2} \overset{\text{idd}}{\sim} N(\mu_2, \sigma_2^2)$ son 2 muestras aleatorias de tamaños n_1 y n_2 respectivamente.

(a) Suponga que $\mu_1, \mu_2, \sigma_1^2, \sigma_2^2$ son desconocidos. Construya un intervalo de confianza $100(1-\alpha)\%$ para $\dfrac{\sigma_1^2}{\sigma_2^2}$.

$$\frac{(n_1-1)S_1^2}{\sigma_1^2} \sim \chi^2_{(n_1-1)} \qquad \frac{(n_2-1)S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}$$

$$\underbrace{\frac{\overbrace{\frac{(n_1-1)S_1^2}{\sigma_1^2}}^{\chi^2_{n_1-1}} \cdot \frac{1}{n_1-1}}{\underbrace{\frac{(n_2-1)S_2^2}{\sigma_2^2}}_{\chi^2_{n_2-1}} \cdot \frac{1}{n_2-1}}}_{\text{Por hint}} \sim F_{n_1-1,\, n_2-1}$$

$$\frac{\frac{S_1^2}{\sigma_1^2}}{\frac{S_2^2}{\sigma_2^2}} = \frac{S_1^2}{S_2^2} \cdot \frac{\sigma_2^2}{\sigma_1^2} \sim F_{n_1-1, n_2-1}$$

Pivote

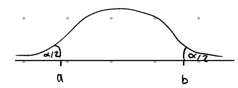
$$\left\{ a \leq \frac{S_1^2}{S_2^2} \frac{\sigma_2^2}{\sigma_1^2} \leq b \right\}$$

Con colas iguales

$$P\left(a \leq \frac{S_1^2}{S_2^2} \cdot \frac{\sigma_2^2}{\sigma_1^2} \leq b\right) = 1-\alpha$$

$$P\left(a \frac{S_2^2}{S_1^2} \leq \left(\frac{\sigma_1^2}{\sigma_2^2}\right)^{-1} \leq b \frac{S_2^2}{S_1^2}\right) = 1-\alpha$$

$$P\left(\frac{1}{b} \frac{S_1^2}{S_2^2} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{1}{a} \frac{S_1^2}{S_2^2}\right) = 1-\alpha$$



$$P(F_{n_1-1, n_2-1} \leq b) = 1 - \alpha/2$$

$$P(F_{n_1-1, n_2-1} \leq a) = \frac{\alpha}{2}$$

$$\left.\begin{array}{l} b = F_{n_1-1, n_2-1, 1-\frac{\alpha}{2}} \\ a = F_{n_1-1, n_2-1, \alpha/2} \end{array}\right\} IC_\alpha\left(\frac{\sigma_1^2}{\sigma_2^2}\right) = \left[\frac{1}{F_{n_1-1, n_2-1, 1-\alpha/2}} \cdot \frac{S_1^2}{S_2^2}, \frac{1}{F_{n_1-1, n_2-1, \alpha/2}} \cdot \frac{S_1^2}{S_2^2}\right]$$

Si repito el experimento 100 veces, 95 veces estaría acá

Problema 2 (Casella y Berger, 2002)

Suponga que tiene dos muestras independientes $X_1, \ldots, X_n \sim$ Exponencial(θ), e $Y_1, \ldots, Y_m \sim$ Exponencial(μ).

(a) Encuentre el TRV de $H_0 : \theta = \mu$ vs. $H_1 : \theta \neq \mu$

(b) Muestre que el test de la parte (a) puede basarse en el estadístico

$$T = \frac{\sum_{i=1}^n X_i}{\sum_{i=1}^n X_i + \sum_{j=1}^m Y_j}$$

(c) Encuentre la distribución de T cuando H_0 es cierta.

Nota: Si $X \sim$ Exponencial(θ), entonces

$$f_\theta(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}}$$

$H_0: \theta = \mu \qquad H_1: \theta \neq \mu$

De la hipótesis nula.

$$\lambda(\underset{\sim}{x}) = \frac{\sup_{\theta \in \Theta_0} L(\theta, \mu, \underset{\sim}{x})}{\sup_{\theta \in \Theta} L}$$

$\underset{\sim}{\theta} = (\theta, \mu) \qquad \Theta = \mathbb{R}_+^2$

$$L(\theta, \mu, \underset{\sim}{x}) = \prod_{i=1}^n \frac{1}{\theta} \exp\left\{\frac{-x_i}{\theta}\right\} \prod_{i=1}^n \frac{1}{\mu} \exp\left\{-\frac{y_i}{\mu}\right\}$$

$$= \frac{1}{\theta^n} \exp\left\{\frac{1}{\theta} \Sigma x_i\right\} \cdot \frac{1}{\mu^n} \exp\left\{-\frac{1}{\mu^n} \cdot \Sigma y_i\right\}$$

$$\sup_{\substack{\theta \in \Theta \\ \theta = \mu}} L(\theta, \mu, \underset{\sim}{x})$$

$$= \sup_{\theta = \mu} \frac{1}{\theta^n} \exp\left\{-\frac{\Sigma x_i}{\theta}\right\} \frac{1}{\theta^m} \exp\left(\frac{-\Sigma y_i}{\theta}\right)$$

$$= \sup_{\underbrace{\theta = \mu}_{\theta \in \Theta_0}} \frac{1}{\theta^{n+m}} \exp\left\{-\frac{1}{\theta}\left(\Sigma x_i + \Sigma y_i\right)\right\}$$

Aplicando log:

$$-(n+m) \log \theta - \frac{1}{\theta}(\Sigma x_i + \Sigma y_i) \Big/ \partial/\partial\theta$$

$$\frac{-n+m}{\theta} + \frac{1}{\theta^2}(\Sigma x_i + \Sigma y_i) = 0$$

$$\hat{\theta}_0 = \frac{\Sigma x_i + \Sigma y_i}{n+m}$$

Sacar $\frac{\partial^2}{\partial\theta^2}$ y verificar máximo.

$\hat{\theta} = \bar{x}, \quad \hat{\mu} = \bar{y}$

$$\lambda(\underset{\sim}{x}) = \frac{\left(\frac{n+m}{\Sigma x_i + \Sigma y_j}\right)^{n+m} \exp\left\{-\left(\frac{n+m}{\Sigma x_i + \Sigma y_j}\right)(\Sigma x_i + \Sigma y_j)\right\}}{\left(\frac{n}{\Sigma x_i}\right)^n \exp\left\{-\left(\frac{n}{\Sigma x_i}\right) \cdot \Sigma x_i\right\} \left(\frac{n}{\Sigma y_j}\right)^m \exp\left\{\frac{m}{\Sigma y_j} \cdot \Sigma y_j\right\}}$$

$$= \frac{\left(\frac{n+m}{\Sigma x_i + \Sigma y_j}\right)^{n+m} \exp\{-(n+m)\}}{\left(\frac{n}{\Sigma x_i}\right)^n e^{-n} e^{-m} \left(\frac{m}{\Sigma y_j}\right)^m}$$

$$= \frac{(n+m)^{n+m}}{n^n m^m} \cdot \frac{(\Sigma x_i)^n (\Sigma y_j)^m}{(\Sigma x_i + \Sigma y_j)^{n+m}}$$

→ lo voy a separar

b) $\underset{\text{supremo}}{\underline{\lambda(\underset{\sim}{x}, \underset{\sim}{y})}} = c\underbrace{\left(\frac{\Sigma x_i}{\Sigma x_i + \Sigma y_j}\right)^n}_{T^n} \underbrace{\left(\frac{\Sigma y_j}{\Sigma x_i + \Sigma y_j}\right)^m}_{(1-T)^m}$

$$\frac{\Sigma x_i + \Sigma y_j}{\Sigma x_i + \Sigma y_j} - \frac{\Sigma x_i}{\Sigma x_i + \Sigma y_j} = \frac{\Sigma y_j}{\Sigma x_i + \Sigma y_j}$$

$$R = \{(\underset{\sim}{x}, \underset{\sim}{y}) : \lambda(\underset{\sim}{x}, \underset{\sim}{y}) \leq k\}$$

$$\frac{\partial\lambda}{\partial T} = c\left(n T^{n-1} (1-T)^m - T^n m (1-T)^{m-1}\right)$$

$$= c\left(T^{n-1} (1-T)^{n-1} \left(n(1-T) - mT\right)\right) = 0$$

$$n(1-T) - nT = 0$$

$$\Rightarrow n - nT - mT = 0$$

$$T = \frac{n}{n+m}$$

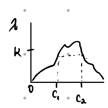
$$\frac{\partial\lambda}{\partial T} \geq 0$$

$$\Leftrightarrow (n(1-T) - mT) \geq 0$$

$$\Leftrightarrow -(n+m)T + n \geq 0$$

$$\Leftrightarrow \frac{n}{n+m} \geq T$$

$$\frac{\partial\lambda}{\partial T} \leq 0 \Leftrightarrow T \geq \frac{n}{n+m}$$



$$R = \{(x, y) : T \leq C_1 \text{ ó } T \geq C_2\}$$