## Problema 1

Calcule la masa de una curva $C$ cuya función de densidad está dada por $\rho(x, y, z) = \sqrt{x^2 + y^2 + 2|z|}$.
Si $C$ está parametrizada por el vector:

$$r(t) = \left(\cos(t), \sin(t), \frac{t^2}{2}\right) \quad t \in [-2, 2]$$

$r'(t) = (-\text{sen}(t), \cos(t), t)$

$\|r'(t)\| = \sqrt{1+t^2}$

$\Rightarrow m = \int_C \rho \, ds = \int_a^b \rho(r(t)) \, \|r'(t)\|$

$= \int_{-2}^{2} \sqrt{1+t^2} \cdot \sqrt{1+t^2} \, dt$

$= \int_{-2}^{2} 1+t^2 \, dt = \left[t + \frac{t^3}{3}\right]_{-2}^{2}$

$= 2 + \frac{8}{3} - \left(-2 - \frac{8}{3}\right) = 4 + \frac{16}{3} = \frac{28}{3}$

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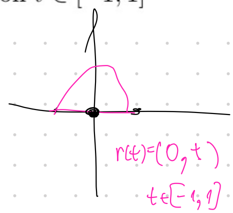
## Problema 2

Calcule

$$\int_\gamma \underbrace{(-2xy\sin(x^2) - y^2)}_{P}dx + \underbrace{(\cos(x^2) - 2xy)}_{Q}dy$$

Donde $\gamma$ es el segmento de la curva parametrizada por $r(t) = (t^3 - t, t+1)$ con $t \in [-1, 1]$

$r'(t) = (3t^2 - 1, 1)$




$Q_x = 2x \cdot -\text{sen}(x^2) - 2y$
$P_y = -2x\,\text{sen}(x^2) - 2y$
} son iguales y D es una región abierta y simplemente conexa
$\Rightarrow$ Campo conservativo

$r(t) = (0, t)$ $\quad t \in [-1, 1]$

$r'(t) = (0, 1)$

$F = (-t^2, 1)$

$= \int_{-1}^{1} (-t^2, 1)(0, 1) \, dt = \int_{-1}^{1} 1 \, dt = t\Big|_{-1}^{1} = 1 - -1 = 2.$

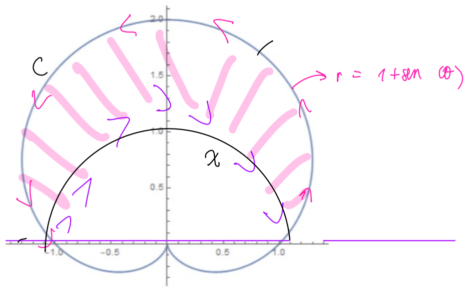
$\int P dx = \int$

$f = \int Q \, dy = y\cos(x^3) - y^2 x + c(x) \Rightarrow f_x = y \cdot 2x - \text{sen}(x^2) - y^2 + c'(x) = -2xy\,\text{sen}(x^2) - y^2$

$\Rightarrow f = -2xy\,\text{sen}(x^2) - y^2.$

## Problema 3

Calcule el trabajo que realiza el campo de fuerza $\mathbf{F} = \left( y - \dfrac{y}{x^2+y^2}, 2x + \dfrac{x}{x^2+y^2} \right)$ sobre una partícula que se mueve a lo largo de la parte superior del cardioide del dibujo, desde $(1,0)$ hasta $(-1,0)$.




$F = (0, x)$

$r(t) = (\cos(\theta) + \text{sen}(2\theta)$

$$P = y - \frac{y}{x^2+y^2} \qquad Q = 2x \cancel{+ \frac{x}{x^2+y^2}} \; 1$$

$$Q_x = 2 + \frac{(x^2+y^2) - x(2x)}{(x^2+y^2)^2} = 2 + \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$P_y = 1 - \frac{(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = 1 + \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\Rightarrow Q_x - P_y = 1$$

$$\int_C F\, dr + \overbrace{\int_x F\, dr}^{1°} = \overbrace{\iint_D Q_x - P_y \, dA}^{2°}$$

$P, Q$ son de Clase $C_1$ en $D$.

$$\int_C F dr = \int_{-x} F dr + \iint 1 \, dA$$

$-x$: $r(t) = (\cos(t), \text{sen}(t))$ $\quad t \in [0, \pi]$

$$F = \left( y - \underbrace{\frac{y}{x^2+y^2}}_{\text{radio es 1.}}, 2x + \frac{x}{x^2+y^2} \right)$$

$$F(r(t)) = \left( \text{sen}(t) - \text{sen}(t), 2\cos(t) + \cos(t) \right)$$

$$r'(t) = (-\text{sen}(t), \cos(t))$$

$$\Rightarrow \int \left( 2\cos^2(t) + \cos^2(t) \right) dt = 3 \int \cos^2(t) = \frac{3}{2} \int 1 + \cos(2t) = \frac{3\pi}{2}$$

2° $$\iint 1 \, dA = \int_0^{\pi} \int_1^{1+\text{sen}(\theta)} 1 \cdot r \, dr \, d\theta$$

$1 \leq r \leq 1 + \text{sen}(\theta)$

$0 \leq \theta \leq \pi$

Jac $= r$

$$= \int_0^{\pi} \frac{r^2}{2} \Big|_1^{1+\text{sen}(\theta)} d\theta$$

$$= \frac{1}{2}\int_0^{\pi} \cancel{1} + 2\,\text{sen}(\theta) + \text{sen}^2(\theta) - \cancel{1}\ d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 2\,\text{sen}(\theta) + \frac{1}{2} - \frac{\cos(2\theta)}{2}$$

$$= \frac{1}{4}\int_0^{\pi} 4\,\text{sen}(\theta) + 1 - \cos(2\theta)\ d\theta$$

$$= \frac{1}{4}\left[-\cos(\theta)\cdot 4 + \theta - \frac{\text{sen}(2\theta)}{2}\right]_0^{\pi}$$

$$= \frac{1}{4}\left[4 + \pi - 0 - (-4)\right] = \frac{1}{4}\left[8+\pi\right] = 2 + \frac{\pi}{4}$$

$$\int_C F\,dr = \frac{3\pi}{2} + 2 + \frac{\pi}{4}$$

## Problema 4

Considere el campo

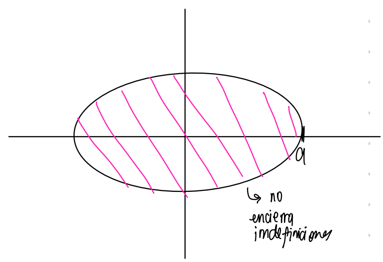
$$\mathbf{F}(x,y) = (-x^2y + \cos(x^2), y^2 + e^y)$$

(a) Muestre que la integral de línea sobre una elipse $C$ de semieje en $x$ igual a $a$ y semieje en $y$ igual a $b$, centrada en el origen y recorrida en sentido antihorario es:

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = \frac{a^3b\pi}{4}$$

(b) En base a la información de (a), calcule la integral de línea sobre $\partial D$ orientada de manera positiva, donde $D$ es la región dada por las inecuaciones:

$$\frac{x^2}{4} + \frac{y^2}{1} \geq 1$$
$$\frac{x^2}{16} + \frac{y^2}{4} \leq 1$$




$$r(t) = (a\cos(t),\ b\,\text{sen}(t))$$
$$r'(t) = (-a\,\text{sen}(t),\ b\cos(t))$$

$$F = (-x^2y + \cos(x^2),\ y^2 + e^y)$$
$t \in [0, 2\pi]$

$$F = (-a^2\cos^2(t)\cdot b\,\text{sen}(t) + \cos(a^2\cos^2(t)),\ b^2\text{sen}^2(t) + e^{b\,\text{sen}(t)}$$

$x = a\cos$

$Q_x = 0$ $\quad P_y = -x^2$

$$\oint_C F\,dr = \iint Q_x - P_y\ dA$$

$$\begin{vmatrix} X_\theta & X_r \\ y_\theta & y_r \end{vmatrix} =$$

$$= \iint x^2\ dA$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$x = a\,r\cos(\theta)$
$y = b\,r\ \text{sen}(\theta)$

$$J = \begin{vmatrix} a\cos(\theta) & -ar\,\text{sen}(\theta) \\ b\,\text{sen}(\theta) & br\cos(\theta) \end{vmatrix} = ab \cdot r$$

$r^2\cos^2(\theta) + r^2\,\text{sen}^2(t) \leq 1$

$r^2 \leq 1$

$0 \leq r \leq 1$

$0 \leq \theta \leq 2\pi$

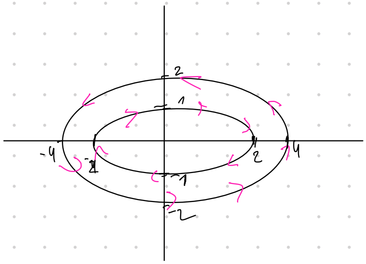
//.

$$\int_0^{2\pi} \int_0^1 \underbrace{a^2 r^2 \cos^2(\theta)}_{x^2} \cdot abr \; dr \; d\theta = a^3 b \int_0^{2\pi} \frac{1+\cos(2\theta)}{2} \cdot r^3$$

$$= \frac{a^3 b}{2} \left( \theta \; \frac{+\text{sen}(2\theta)}{2} \right) \Bigg|_0^{2\pi} \cdot \frac{r^4}{4} \Bigg|_0^{1}$$

$$= \frac{a^3 b}{2} (2\pi) \frac{1}{4} = \frac{a^3 b \pi}{4}$$

b)




$$\frac{x^2}{16} + \frac{y^2}{4} \leq 4$$

$$\int_{C_1} F\,dr + \int_{C_2} F\,dr$$

$$\rightarrow \int_{C_1} F\,dr - \int_{-C_2} F\,dr$$

$$\int_{C_1} F\,dr - \int_{-C_2} F\,dr$$

$$\underbrace{\frac{4 \cdot 2\pi}{4}}_{\frac{a^3 b \pi}{4}} - \frac{2^3 \cdot 1}{4} \pi = 32\pi - 2\pi = 30\pi$$