1.- $\rho = 3y^2 - 2x$

$x^{2/3} + y^{2/3} = 1$ $\quad (x^{1/3})^2 + (y^{1/3})^2 = 1^2$

$m = \int \rho \, ds$

$x^{1/3} = x^*$

$x^{1/3} = \cos(\theta)$

$y^{1/3} = \operatorname{sen}(\theta)$

$\Rightarrow r(\theta) = (\cos^3(\theta), \operatorname{sen}^3(\theta))$

$r'(\theta) = (3\cos^2(\theta)\cdot -\operatorname{sen}(\theta), 3\operatorname{sen}^2(\theta)\cdot\cos(\theta))$

$\|r'(\theta)\| = \sqrt{9\cos^4(\theta)\operatorname{sen}^2(\theta) + 9\operatorname{sen}^4(\theta)\cdot\cos^2(\theta)} = 3\,|\cos(\theta)\cdot\operatorname{sen}(\theta)|$

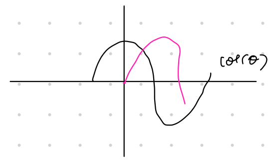
$\cos^4(\theta)\operatorname{sen}^2(\theta) + \operatorname{sen}^4(\theta)\cos^2(\theta)$

$\cos^2(\theta)\cdot\operatorname{sen}^2(\theta)\,(\cos^2(\theta) + \operatorname{sen}^2(\theta))$

Problema 1

Calcule la masa de la curva $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$ si la densidad lineal viene dada por $\rho(x,y) = 3y^2 - 2x$

	0	$\pi/2$	π	$3\pi/2$	2π
$\cos(\theta)$	+	−	−	+	
$\operatorname{sen}(\theta)$	+	+	−	−	
	+	−	+	−	



$$3\int_0^{2\pi}\left(3\operatorname{sen}^6(\theta) - 2\cos^3(\theta)\right)\cdot|\cos(\theta)\operatorname{sen}(\theta)|\,d\theta$$

$$9\int_0^{2\pi}\operatorname{sen}^6(\theta)\cdot|\cos(\theta)\cdot\operatorname{sen}(\theta)| - 6\int_0^{2\pi}\cos^3(\theta)\,|\cos(\theta)\operatorname{sen}(\theta)|\,d\theta$$

$$\int_0^{\pi/2}\operatorname{sen}^6(\theta)\cdot\cos(\theta)\cdot\operatorname{sen}(\theta) + \int_{\pi/2}^{\pi}$$

$$\int_0^{\pi/2}\cos^4(\theta)\operatorname{sen}(\theta)\,d\theta - \int_{\pi/2}^{\pi}\cos^4(\theta)\operatorname{sen}(\theta) + \int_{\pi}^{3\pi/2}\cos^4(\theta)\operatorname{sen}(\theta) - \int_{3\pi/2}^{2\pi}\cos^4(\theta)\operatorname{sen}(\theta) = 0$$

1/5 $\qquad$ 1/5.

$$\int_0^{\pi/2}\underbrace{\operatorname{sen}^7(\theta)\cos(\theta)}_{1/8}\,d\theta - \underbrace{\int_{\pi/2}^{\pi}\operatorname{sen}^7(\theta)\cos(\theta)}_{-1/8} + \int_{\pi}^{3\pi/2}\operatorname{sen}^7(\theta)\cos(\theta)\,d\theta - \int_{3\pi/2}^{2\pi}\operatorname{sen}^7(\theta)\,d\theta = \frac{1}{2}.$$

$\Rightarrow 9/2$

Problema 2

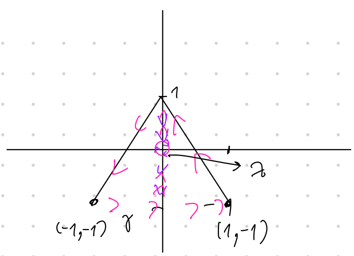
Calcule la integral de linea: $\int_C P dx + Q dy$

$$P(x,y) = \frac{y}{x^2+y^2} \qquad Q(x,y) = -\frac{x}{x^2+y^2}$$

Y C es la curva cuyos segmentos rectos pasan por los vertices (1,-1),(0,1) y (-1,-1) de forma secuencial

$$Q_x = \frac{-(x^2+y^2) + 2x^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2} \Rightarrow Q_x - P_y = 0$$

$$P_y = \frac{(x^2+y^2) - y(2y)}{(x^2+y^2)^2} = \frac{x^2-y^2}{(x^2+y^2)^2}$$



$$\int_C P dx + Q dy + \int_{\lambda} P dx + Q dy + \int_{\gamma} P dx + Q dy = 0$$

$$\int_C P dx + Q dy = \int_{-\lambda} P dx + Q\,dy - \int_{\gamma} P dx + Q\,dy$$

$-\lambda$: $r(t) = (r\cos(t), \; r\,sen(t))$ $\quad F = \left(\frac{r\,sen(t)}{r^2}, \; -\frac{r\cos(t)}{r^2} \right)$

$r'(t) = (-r\,sen(t), \; r\cos(t))$

$$\int_0^{2\pi} -sen^2(t) - \cos^2(t)\, dt = -\int_0^{2\pi} 1\, dA = -2\pi$$

γ: $r(t) = (-1,-1)(1-t) + (1,-1)\,t$ $\quad F = \left(\frac{y}{x^2+y^2}, \; \frac{-x}{x^2+y^2} \right)$

$r(t) = (t, -1) \quad t \in [-1,1]$

$r'(t) = (1,0)$

$$\int_\gamma F \cdot r'(t) = \int_{-1}^{1} \left(\frac{-1}{t^2+1}, \; \frac{-t}{t^2+1} \right)(1,0)\, dt = \int_{-1}^{1} -\frac{1}{t^2+1}\, dt = -\left[\arctan(t)\right]_{-1}^{1}$$

$$= -\left[\arctan(1) - \arctan(-1)\right]$$

$$= -\left[\frac{\pi}{4} + \frac{\pi}{4}\right] = -\frac{\pi}{2}$$

$\frac{-3\pi}{2}$ //

Problema 3

Div F = $\nabla \cdot f$ $\quad$ rot(F) = $\nabla \times F$

Considere $\mathbf{F}(x,y,z) = (y^2cos(x) + z^3, 2ysen(x), 3xz^2 + 2z)$ Demuestre que es conservativo y encuentre f

$f_x \qquad f_y \qquad f_z$

Despues calcule $\int_C \mathbf{G} \cdot d\vec{l}$, donde $G(x,y,z) = F(x,y,z) + (z^3, 0, 0)$ y C es la curva cuyos segmentos rectos pasan por los vertices $(0,0,0)$, $(1,1,0)$ y $(0,0,1)$ de forma secuencial

$$f = \int f_z\, dz = xz^3 + z^2 + C(x,y)$$

$r(t) = \underbrace{(0,0,0)}_{P_i}(1-t) + \underbrace{(1,1,0)}_{P_f}(t)$

$$f_x = z^3 + C_x(x,y) = y^2\cos(x) + z^3$$

$C_x(x,y) = y^2\cos(x)$

$C(\;\; = y^2\,sen(x)$

$\Rightarrow f = xz^3 + z^2 + y^2\,sen(x) + C$

$\Rightarrow$ Entonces, F es conservativo dado que $\exists\, f: F = \nabla f$. //

$G = \left(y^2\cos(x) + 2z^3, \; 2y\,sen(x), \; 3xz^2 + 2z \right)$

Problema 4

Utilizando el teorema de Green calcule el area de la region limitada por la siguiente curva:

$$(ax)^{\frac{2}{3}} + (by)^{\frac{2}{3}} = (a^2 - b^2)^{\frac{2}{3}}$$