AI 4

1.2

$$L_{w\,simple} = (\sigma(w,[1,0]) - 1)^2 + (\sigma(w,[0,1]))^2 + (\sigma(w,[1,1]) - 1)^2$$

$$\frac{\partial}{\partial w_1}\sigma(w,[0,0]) = \frac{\partial}{\partial w_1}\frac{1}{1+e^{-w_1}} = \frac{\partial}{\partial w_1}(1+e^{-w_1})^{-1}$$

$$= +1 \cdot e^{-w_1}(1+e^{-w_1})^{-2}$$

$$= \frac{e^{-w_1}}{(1+e^{-w_1})^2} = e^{-w_1}\cdot\sigma^2[1,0]$$

similarly

$$\frac{\partial}{\partial w_2}\sigma(w,[0,1]) = e^{-w_2}\sigma^2[0,1]$$

$$\frac{\partial}{\partial w_1}\sigma[w,[1,1]] = \frac{\partial}{\partial w_1}\left(\frac{1}{1+e^{-w_1-w_2}}\right) = \frac{\partial}{\partial w_1}(1+e^{-w_1-w_2})^{-1}$$

$$= +1\cdot(e^{-w_1-w_2})$$

$$\frac{}{(1+e^{-w_1-w_2})^2}$$

similarly

$$= e^{-w_1-w_2}\,\sigma^2[1,1]$$

$$\frac{\partial}{\partial w_2}\sigma[w,[1,1]] = e^{-w_1-w_2}\,\sigma^2[1,1]$$

$$\frac{\partial L_{simple}(w)}{\partial w_1} = 2(\sigma[1,0]-1)\cdot\frac{\partial}{\partial w}\sigma[1,0] + 0$$

$$+ 2(\sigma[1,1]-1)\cdot\frac{\partial}{\partial w_1}\sigma[1,1]$$

$$= 2(\sigma[1,0]-1)\, e^{-w_1}\, \sigma^2[1,0]$$

$$+ 2(\sigma[1,1]-1)\, e^{-w_1-w_2}\, \sigma^2[1,1]$$

$$\frac{\partial L_{simple}(w)}{\partial w_2} = 0 + 2\sigma[0,1]\cdot\frac{\partial}{\partial w_2}\sigma[0,1] +$$

$$2(\sigma[1,1]-1)\cdot\frac{\partial}{\partial w_2}\sigma[1,1]$$

$$= 2\sigma[0,1]\, e^{-w_2}\, \sigma^2[0,1] +$$

$$2(\sigma[1,1]-1)\cdot e^{-w_1-w_2}\, \sigma^2[1,1]$$

$$\nabla_w L_{simple}(w) = \left[\frac{\partial L_{simple}}{\partial w_1}, \frac{\partial L_{simple}}{\partial w_2}\right]$$