

<sup>1</sup> Fairness-aware Reweighting in Federated Learning

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<sup>4</sup> **Abstract**

We address the problem of enforcing group fairness in federated learning. To this end, we first adapt FairGrad, a recently proposed fairness method in centralized learning, to the federated setting. This comes with a large communication cost as it requires sharing the fairness level of the learned models after each gradient descent step. Unfortunately, using the usual federated learning trick consisting in taking several local steps before communicating with the server leads to models with low utility. To tackle this issue, we propose an alternative approach that combines ideas from FairGrad and cost-sensitive fairness methods. Our preliminary experiments show that this new method is either competitive or better than several state-of-the-art approaches.

<sup>13</sup> **Keywords:** Fairness, Federated Learning.

<sup>14</sup> **1 Introduction**

<sup>15</sup> Machine Learning approaches are nowadays used to solve a wide range of problems. Based on historical data, they often reach human-like performances. Unfortunately, the datasets used to train these models sometimes reflect biases present in our society or may be under-representative of some subgroups of the population. Together with potentially inadequate algorithmic choices, this often leads to models that tend to unfairly disadvantage some individuals. For example, De-Arteaga et al. (2019) show that a model trained to predict the professional occupation of a person from its biography may perpetuate existing gender stereotypes. When human lives get impacted by automated decision-making (Chouldechova and Roth, 2020), it is of the utmost importance to ensure that such biases do not arise. Thus, fair machine learning has gained a lot of attention in the recent years (Barocas et al., 2023).

<sup>24</sup> Defining fairness in Machine Learning is challenging as different problems usually call for different notions of non-discrimination. Two main kind of notions became prevalent in the recent literature. On the one hand, individual fairness advocates that similar individual should be treated similarly (Dwork et al., 2012). On the other hand, group fairness seeks to equate the performance of machine learning models across different sensitive groups, that is groups based on sensitive attributes such as gender or ethnicity (Hardt et al., 2016; Zafar et al., 2017). In this work, we consider a generic group fairness formulation encompassing several of these notions (Maheshwari and Perrot, 2023).

<sup>31</sup> The goal of fair machine learning is to learn models that maximize utility while satisfying a given fairness definition. Many approaches have been proposed, that fall into several categories (Caton and Haas, 2023). Pre-processing techniques aim to debias data before training (Kamiran and Calders, 2012; Zemel et al., 2013; Calmon et al., 2017), in-processing approaches steer the training procedure towards directions that maintain fairness within the model (Maheshwari and Perrot, 2023; Celis et al., 2019; Cotter et al., 2019), and post-processing methods adjust the predictions made after training to achieve fairness (Hébert-Johnson et al., 2018; Hardt et al., 2016; Pleiss et al., 2017). Most fairness approaches have been proposed in the centralized setting where the data can be accessed at will, and only a few of them consider the federated setting (Kairouz et al., 2021), where data is partitioned across many clients and a server orchestrates them to learn a global model while only accessing the

41 data through aggregate statistics. The goal of this paper is to bridge this gap. More precisely, we  
42 propose new algorithms to learn fair models in cross-silo federated learning, where clients have enough  
43 communication and computation resources to be responsive to every query of the server. Furthermore,  
44 we assume that the clients are honest and provide accurate information (gradients, fairness-levels, . . . ).

45 Learning fair models in federated learning is challenging. Indeed, merely applying debiasing methods  
46 locally does not ensure the composition of these local models to be fair (Dwork and Ilvento, 2019).  
47 Furthermore, even with recurrent synchronization of these models, the aggregated model is generally  
48 not fair because of the data heterogeneity across the clients (Wang et al., 2023). Nevertheless, several  
49 works have addressed fairness in federated learning from different angles. For instance, some approaches  
50 propose to balance the local datasets in a pre-processing step (Abay et al., 2020) or strive to minimize  
51 the loss of the most disadvantaged groups (Papadaki et al., 2022). Similarly, several recent approaches  
52 propose to calculate adaptive sensitive group weights (Ezzeldin et al., 2023; Zeng et al., 2021). In this  
53 work, we follow the latter trend and propose two extensions of FairGrad (Maheshwari and Perrot,  
54 2023), a recent reweighting approach for fairness in the centralized setting that has been shown to be  
55 easy to implement and applicable to a wide range of settings.

56 **Contributions** In this work, we study new reweighting approaches to learn fair models in federated  
57 learning. First, a method called Fed-FairGrad straightforwardly extends FairGrad (Maheshwari and  
58 Perrot, 2023) by computing the weights for each sensitive group in a federated way. This method  
59 requires an additional communication cost of  $K$  floats, one for each sensitive group, between each  
60 training round. Its strict application would also require rounds to consist of a single step, which  
61 can be quite expensive. To alleviate this issue, we use a standard approach in federated learning  
62 (McMahan et al., 2017). Unfortunately, we show that this leads to models with reduced utility. Based  
63 on these observations, we propose a second approach, called Fed-FairGrad-Convex, that combines ideas  
64 from cost-sensitive fairness methods (Agarwal et al., 2018) and FairGrad (Maheshwari and Perrot,  
65 2023). Through a series of experiments with varying levels of heterogeneity, we demonstrate that  
66 Fed-FairGrad-Convex tends to learn fair models with better utility than state-of-the-art methods.

67 **Related Work** Fairness in federated learning can be seen from two main perspectives. On the one  
68 hand, client parity ensures that the clients receive models with performances proportional to their  
69 contributions (Mohri et al., 2019; Li et al., 2019; Yue et al., 2023). On the other hand, the goal in  
70 group fairness, the problem investigated in this paper, is to ensure that the global model learned in  
71 a federated way is fair on the overall data distribution formed by a mixture of the local, potentially  
72 different, distributions. Among the different works proposed in this literature (Abay et al., 2020; Gálvez  
73 et al., 2021; Zhang et al., 2020; Padala et al., 2021; Papadaki et al., 2022; Su et al., 2024), the ones  
74 closest to ours are the ones that use some kind of reweighting scheme to balance the impact that each  
75 sensitive group has on the learned model. First, Du et al. (2021) propose to learn group-wise weights  
76 based on relaxations of the fairness constraints under consideration. Our solution also uses reweighting  
77 but with respect to the true fairness constraints, avoiding the pitfall of relaxations that may lead to  
78 unfair models (Lohaus et al., 2020). Similarly, FairFed (Ezzeldin et al., 2023) is an approach that  
79 explores the idea of assigning adaptive weights to the updates received from each client to achieve  
80 fairness. More precisely, it proposes to increase the weight of clients with updates with fairness levels  
81 closer to the global fairness level and to decrease the weight of the others. In our work, rather than  
82 altering the importance of each client update directly, we modify the importance of each sensitive group,  
83 implicitly reducing the impact of clients that only have access to examples from the advantaged groups.  
84 Finally, the closest work to ours is FedFB (Zeng et al., 2021) which proposes to alter, at each iteration  
85 of gradient descent, the sampling probabilities used to generate minibatches of sensitive groups based  
86 on the fairness level of the current model. This is an extension of the FairBatch method, designed for  
87 the centralized setting (Roh et al., 2021). Our approach is also an extension of a centralized approach  
88 to the federated setting but with a different basis, namely FairGrad (Maheshwari and Perrot, 2023).

## 89 2 Problem setting and Notations

90 In this work, we consider a binary classification problem in federated learning, where data is distributed  
 91 across  $m$  clients. The goal is to accurately predict a label  $y \in \{0, 1\}$  based on features  $x \in \mathcal{X}$  using a  
 92 model  $h_\theta : \mathcal{X} \rightarrow [0, 1]$  while staying fair with respect to some discrete sensitive attribute  $s \in \mathcal{S}$  such as  
 93 gender or ethnicity. We denote  $\theta \in \mathbb{R}^d$  the learnable model parameters. We assume that there exists  
 94 an underlying distribution  $\mathcal{T}$  over  $\mathcal{X} \times \{0, 1\} \times \mathcal{S}$  and that our data can be partitioned into  $K$  different  
 95 groups  $\{g_1, \dots, g_K\}$  based on their sensitive attributes and their labels (Maheshwari and Perrot, 2023).  
 96 We note  $\mathcal{T}_k$  the distribution of samples for a group  $g_k$ . We also note  $\mathcal{T}^{(i)}$  the distribution of samples  
 97 held by a client  $(i)$ , and  $\mathcal{T}_k^{(i)}$  that of samples that belong to both client  $(i)$  and group  $g_k$ . We implicitly  
 98 assume that  $\mathcal{T}$  is a convex combination of local distributions:

$$99 \quad \exists(p^{(1)}, \dots p^{(m)}) \geq 0 : \mathcal{T} = \sum_{i=1}^m p^{(i)} \mathcal{T}^{(i)} \text{ such that } \sum_{i=1}^m p^{(i)} = 1. \quad (1)$$

100 where  $\forall i, p^{(i)} = \mathbb{P}(\mathcal{T}^{(i)})$ , which is usual in federated learning, but not ubiquitous (Mohri et al., 2019).  
 101 In practice, the true distributions are unknown and each client holds an empirical dataset  $D^{(i)}$  of  $n^{(i)}$   
 102 samples drawn i.i.d. from  $\mathcal{T}^{(i)}$ . We note  $n_k^{(i)}$  the number of samples in  $D^{(i)}$  belonging to group  $g_k$   
 103 such that  $n = \sum_{i=1}^m n^{(i)}$  and  $n_k = \sum_{i=1}^m n_k^{(i)}$ . We use these empirical sample counts to approximate  
 104 probabilities of belonging to an underlying distribution, that is  $\mathbb{P}(\mathcal{T}^{(i)}) \approx \frac{n^{(i)}}{n}$  and  $\mathbb{P}(\mathcal{T}^{(i)} | \mathcal{T}_k) \approx \frac{n_k^{(i)}}{n_k}$ .

105 To learn a model, we consider empirical risk minimization approaches, and we denote by  $L(h_\theta)$   
 106 the loss function that is to be minimized over  $\mathcal{T}$ . We further note  $L_k(h_\theta)$  the loss over  $\mathcal{T}_k$  and  $L_k^{(i)}(h_\theta)$   
 107 that over  $\mathcal{T}_k^{(i)}$ . All of these are in practice estimated on batches of samples drawn from (subsets of)  
 108 the empirical datasets.

109 **Fairness Measure** To measure fairness, we use a canonical form (Maheshwari and Perrot, 2023)  
 110 that allows us to simultaneously reason about several definitions that were proposed in the literature:

$$111 \quad F_k(h_\theta, \mathcal{T}) = C_k^0 + \sum_{k'=1}^K C_k^{k'} \mathbb{P}(h_\theta(x) \neq y | \mathcal{T}_{k'}) \quad (2)$$

112 where  $C_k^{k'}$  are constants that depend on the fairness definition and can be estimated from empirical  
 113 sample counts, and  $\mathbb{P}(h_\theta(x) \neq y | \mathcal{T}_k)$  denotes the misclassification probability on samples from  $\mathcal{T}_k$ . A  
 114 positive value of  $F_k$  means that the group  $g_k$  is advantaged, while a negative one implies that the group  
 115 is disadvantaged. We refer to Maheshwari and Perrot (2023); Mangold et al. (2023) for explicit values  
 116 of  $C_k^{k'}$  for Accuracy Parity (Zafar et al., 2017), Demographic Parity (Calders et al., 2009), Equality of  
 117 Opportunity and Equalized Odds (Hardt et al., 2016).

118 **FairGrad for exact fairness** Our algorithms extend FairGrad (Maheshwari and Perrot, 2023) to  
 119 federated learning, hence we recall here its formulation in the centralized setting. It is an iterative  
 120 fairness algorithm that learns adaptive sample weights based on whether they belong to an advantaged  
 121 or disadvantaged group. To do so, FairGrad introduces a reweighted loss:  $\mathcal{L}(h_\theta, w^t) = \sum_{k=1}^K w_k^t L_k(h_\theta)$   
 122 where  $w^t = (w_1^t, \dots, w_K^t)$  are group-wise weights that are defined and updated based on current fairness  
 123 measures, using a group-fairness definition that follows the form previously introduced in Equation (2):

$$124 \quad w_k^t = \mathbb{P}(\mathcal{T}_k) + \sum_{k'=1}^K C_k^{k'} \sum_{\tau=0}^{t-1} \eta F_{k'}(\theta^\tau, \mathcal{T}). \quad (3)$$

125 FairGrad uses stochastic optimization and an alternating approach, where at each step fairness is  
 126 estimated, weights are updated, and finally model parameters are updated using the reweighted loss

124  $\mathcal{L}$  as  $\theta^{t+1} = \theta_t - \gamma \nabla_{\theta} \mathcal{L}(h_{\theta}, w^t)$ . In these formulas,  $\gamma$  is the usual learning rate for stochastic gradient  
125 descent (SGD), while  $\eta$  is a FairGrad-specific learning rate for the weights. At each step, the loss is  
126 estimated on a batch of samples. As for  $\{F_k\}_{k=1}^K$ , the estimates of the fairness of the current model, the  
127 authors suggest to compute them over the same training batch for computational efficiency, resulting  
128 in updating both weights and model parameters at every training step with a single forward pass.

## 129 3 FairGrad in Federated Learning

130 In this section, we first show how to evaluate group-fairness measures in a federated setting. Then, we  
131 propose two extensions of FairGrad (Maheshwari and Perrot, 2023). First, we introduce Fed-FairGrad a  
132 straightforward extension that we adapt to reduce its inflated communication cost. Second, we present  
133 Fed-FairGrad-Convex, a variant that aims at tackling utility issues encountered with Fed-FairGrad.

### 134 3.1 Evaluating Fairness in Federated Learning

135 Fairness functions that follow the canonical form in Equation (2) are straightforward to compute in a  
136 federated way. Indeed, using the law of total probabilities, we can rewrite the initial formula as:

$$F_k(\theta, \mathcal{T}) = C_k^0 + \sum_{k'=1}^K C_k^{k'} \mathbb{P}(h_{\theta}(x) \neq y | \mathcal{T}_{k'}) = C_k^0 + \sum_{k'=1}^K C_k^{k'} \left( \sum_{i=1}^m \mathbb{P}(\mathcal{T}_{k'}^{(i)} | \mathcal{T}_{k'}) \mathbb{P}(h_{\theta}(x) \neq y | \mathcal{T}_{k'}^{(i)}) \right) \quad (4)$$

137 In practice, this gives  $F_k(\theta, \mathcal{T}) = C_k^0 + \sum_{k'=1}^K C_k^{k'} \left( \frac{1}{n_{k'}} \sum_{i=1}^m n_{k'}^{(i)} \mathbb{P}(h_{\theta}(x) \neq y | \mathcal{T}_{k'}^{(i)}) \right)$ , since we assume  
138  $\mathbb{P}(\mathcal{T}_k^{(i)} | \mathcal{T}_k) \approx \frac{n_k^{(i)}}{n_k}$ . It means that the clients only need to estimate and share the group-wise misclassifi-  
139 cation rate of the global model on their local data, weighted by local sample counts. This corresponds  
140 to  $K$  scalar values for each client, that are then aggregated by the server to estimate the fairness level of  
141 the global model on the overall distribution. This also requires the one-time sharing and aggregation of  
142 group-wise sample counts, so that the server can compute the  $C_k^{k'}$  constants and access the  $n_k$  values.

### 143 3.2 Fed-FairGrad

144 In this section, we explain how to derive Fed-FairGrad from FairGrad at the expense of a large  
145 communication cost. We then consider practical issues and reduce this cost using standard techniques.

146 **Theoretical update rules** We use the total law of probabilities to decompose the reweighted loss  
147 function  $\mathcal{L}$  and model parameters update rule of FairGrad into client-wise terms:

$$\mathcal{L}(h_{\theta}, w^t) = \sum_{k=1}^K w_k^t L_k(h_{\theta}) = \sum_{i=1}^m \sum_{k=1}^K \mathbb{P}(\mathcal{T}_k^{(i)} | \mathcal{T}_k) w_k^t L_k^{(i)}(h_{\theta}) = \sum_{i=1}^m \mathcal{L}(h_{\theta}, w^{(i),t}) \quad (5)$$

$$\theta^{t+1} = \theta_t - \gamma \nabla_{\theta} \mathcal{L}(h_{\theta}, w^t) = \theta_t - \gamma \sum_{i=1}^m \nabla_{\theta} \mathcal{L}(h_{\theta}, w^{(i),t}) \quad (6)$$

148 where  $w^{(i),t} := (w_1^t \mathbb{P}(\mathcal{T}_1^{(i)} | \mathcal{T}_1), \dots, w_K^t \mathbb{P}(\mathcal{T}_K^{(i)} | \mathcal{T}_K))$  are client-adjusted weights. In practice, given our  
149 assumptions,  $w_k^t \mathbb{P}(\mathcal{T}_k^{(i)} | \mathcal{T}_k) \approx w_k^t \frac{n_k^{(i)}}{n_k}$ . Hence, having received  $\frac{w_k^t}{n_k}$  for all groups ( $K$  scalars) from the  
150 server, the clients refine them based on their sample counts, run a SGD step using the reweighted  
151 loss, and finally send back the resulting gradient, similar to what is done in fairness-agnostic federated  
152 learning. The update rule for the weights remains the same as in Equation (3), but applying it requires  
153 that, before each local gradient step, the clients evaluate and share their local misclassification rates,

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**Algorithm 1** Fed-FairGrad

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**Require:**  $m$  clients, an initial model  $h_\theta$ , a learning rate  $\gamma$ , a fairness rate  $\eta$  and a fairness definition  $F$ .

- 1:  $\triangleright$  Initialize fairness functions and constraints:
- 2:   Clients share their group-wise sample counts  $\{n_k^{(i)}\}_{k=1}^K$ .
- 3:   Server aggregates all  $n_k$  counts, calculates  $C_k^{k'}$  constants, and initializes  $w_k^0$  values.
- 4: **for** each round **do**
- 5:    $\triangleright$  Estimate fairness and update weights:
- 6:     Clients compute and share group-wise misclassification rates on their training data.
- 7:     Server computes  $\{F_k\}_{k=1}^K$  estimates as per Equation (4).
- 8:     Server updates loss weights  $w^t$  as per Equation (3) and shares them with clients.
- 9:    $\triangleright$  Update model parameters:
- 10:    Clients run local SGD steps as per Equation (7) and send back the resulting  $\theta^{(i),t+1}$ .
- 11:    Server aggregates updates into  $\theta^{t+1}$  as per Equation (8) and shares it with clients.

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154 so that the server can estimate the fairness of the model as explained in Section 3.1, introducing some  
155 extra communication and synchronization costs compared with fairness-agnostic federated learning.

156 This adaptation of FairGrad is straightforward mathematically, but costly. Indeed, its strict  
157 application requires two rounds of communication per gradient step: one to update fairness estimates  
158 and share the new weights, the other to aggregate the updated model parameters.

159 **Reducing the communication cost** A first possible improvement, similar to what is done in  
160 FairFed (Ezzeldin et al., 2023), would consist in computing parameter updates using weights derived  
161 from fairness measures that are one-step-behind (that is, replacing  $w_k^t$  with  $w_k^{t-1}$  in Equation (6)).  
162 This reduces the number of exchanges per round as the parameter and fairness updates can then be  
163 conducted at once, with a single synchronisation step instead of two. However, it would not affect the  
164 amount of information shared. In particular, it would not address the main communication bottleneck,  
165 which is that model parameters need to be shared back and forth after every single local gradient step.

166 To alleviate this issue, a standard approach is to have the clients run multiple local gradient steps  
167 between each synchronization step. This was first proposed in FedAvg (McMahan et al., 2017), and  
168 has since been shown to be grounded theoretically (Khaled et al., 2020; Mishchenko et al., 2022). In  
169 our case, this involves that, at each round, clients iteratively update local model parameters using  
170 fixed weights  $w_k^t \mathbb{P}(\mathcal{T}_k^{(i)} | \mathcal{T}_k)$  based on the estimated fairness level of initial parameters  $\theta^t$ . Formally,  
171 with  $B$  the number of local SGD steps,  $\tau$  a local time index, and  $\theta^{(i),t,0} := \theta^t$ , we have:

$$\theta^{(i),t,\tau+1} = \theta^{(i),t,\tau} - \gamma \nabla_{\theta} \mathcal{L} \left( h_{\theta^{(i),t,\tau}}, w^{(i),t} \right), \forall \tau \in \{0, \dots, B-1\}. \quad (7)$$

172 Clients then send the resulting parameters  $\theta^{(i),t+1} := \theta^{(i),t,B}$  to the server, which sum-aggregates them:

$$\theta^{t+1} = \theta^t - \sum_{i=1}^m (\theta^t - \theta^{(i),t+1}) \quad (8)$$

173 In this case, the fairness of the model and the weights are only updated once per training round and  
174 are then used for all the gradient steps in that round. As a consequence, we propose computing them  
175 based on the entire training datasets of clients without the one-step-behind trick to produce more  
176 robust estimates. This is a major difference with the centralized version of FairGrad, where fairness is  
177 estimated in a less robust fashion (based on a single training batch) but way more frequently (at each  
178 training step). We summarize the whole Fed-FairGrad approach in Algorithm 1.

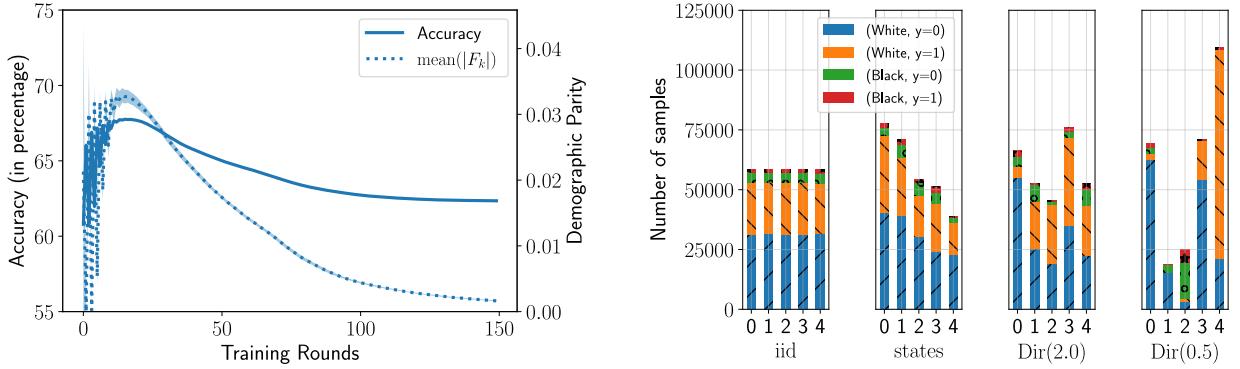


Figure 1: (Left) Example of Fed-FairGrad performance metrics. (Right) Training samples partitions. (Left) The model was trained using Fed-FairGrad ( $\eta = 0.1$ ) for Demographic Parity on i.i.d.-partitioned data (see Section 4 for details). The metrics were computed on validation data at the end of each round and only the first 150 out of 500 rounds are included for readability. The results are averaged across 20 replicas, with the mostly negligible standard deviations rendered as surfaces around the lines. (Right) The height of the bars indicates the number of samples in a local training set. Bars are made of stacked sub-bars indicating sensitive groups, as defined by ethnicity and target labels.

179 **Reduced utility** Our experiments (detailed in Section 4) show that Fed-FairGrad tends to learn  
 180 models that are fair but inaccurate. More precisely, their utility and fairness oscillates for a few  
 181 iterations before dropping to levels that are close to a majority-label-predicting baseline as shown  
 182 in Figure 1 (Left). We hypothesize that this is due to the fact that the weights in FairGrad can be  
 183 negative, a necessary property to enforce fairness. Indeed, we believe that this creates cases where  
 184 the local weighted losses become non-convex and thus may steer the local updates in vastly different  
 185 directions from one training round to the other. This creates the oscillations and pushes the weights  
 186 towards a solution where this does not happen: all weights are 0 except for the majority label groups.  
 187 The learned model is then almost constant. To tackle this issue, we present, in the following subsection,  
 188 an alternative algorithm that builds upon Fed-FairGrad by modifying its loss function to ensure that  
 189 the local losses remain convex and thus that the local updates steer the updates towards similar  
 190 directions from one training round to the other.

### 191 3.3 Fed-FairGrad-Convex: a convex alternative to Fed-FairGrad

192 In this section, we introduce Fed-FairGrad-Convex, an alternative that follows the same logic as  
 193 Fed-FairGrad, updating parameters based on a reweighted loss that reflects the current group fairness  
 194 measures, but is designed so that the local weighted losses are convex for a fixed set of weights. To  
 195 do so, we replace the weighted loss in Equation (5) with a new loss inspired by cost-sensitive fairness  
 196 approaches (Agarwal et al., 2018) that affect different cost-functions to different sensitive groups:

$$\mathcal{L}(h_\theta, w^t) = \sum_{k=1}^K \left( L_k(h_\theta) \frac{|w_k^t| + w_k^t}{2\|w^t\|_2} + L_k(-h_\theta) \frac{|w_k^t| - w_k^t}{2\|w^t\|_2} \right) \quad (9)$$

197 where  $L$  is the raw loss function, which we assume to be convex, and  $w^t = (w_1^t, \dots, w_K^t) \in \mathbb{R}^K$  are group-  
 198 wise weights. Here,  $-h_\theta$  denotes a model that predicts opposite labels, meaning  $-h_\theta(x) = 1 - h_\theta(x)$   
 199 (as  $h_\theta(x) \in [0, 1]$ ). Hence, if  $w_k$  is negative for an unfairly advantaged group, the loss is computed using  
 200 opposite labels, so that the performance on that group is reduced. On the opposite, if  $w_k$  is positive,  
 201 performance on that group improves with training. Apart from these changes in terms of loss, the  
 202 decomposition across clients is similar to Fed-FairGrad, with client-adjusted weights  $w_k^{(i),t} \approx w_k^t \frac{n_k^{(i)}}{n_k}$ .

203 Regarding the weights, they are initialized as  $w_k^0 = \frac{1}{\sqrt{K}}$  and updated as follows:

$$w_k^{t+1} = w_k^t - \eta F_k(\theta^t, \mathcal{T}) \quad \forall k \in \{1, \dots, K\} \quad (10)$$

204 The resulting algorithm is almost identical to Algorithm 1, with the only differences being that the  
205 weights are updated using Equation (10) and the loss considered is the one in Equation (9). The  
206 computation and communication costs remain unchanged.

## 207 4 Experiments

208 In this section, we empirically evaluate the performance of Fed-FairGrad and Fed-FairGrad-Convex.  
209 We compare them to 3 baselines on Folktale (Ding et al., 2021), a well-known fairness dataset.

210 **Methods** We compare Fed-FairGrad (FedFG) and Fed-FairGrad-Convex (FedFG-C) with three  
211 baselines. FedAvg McMahan et al. (2017) is a standard federated learning approach that does not  
212 consider fairness constraints. FairFed (Ezzeldin et al., 2023) calculates adaptive aggregation weights  
213 based on the differences between local and global fairness levels. FedFB (Zeng et al., 2021) is a  
214 federated extension of FairBatch Roh et al. (2021) that calculates minibatch sampling probabilities  
215 based on fairness. We use linear models with sigmoid activation and cross-entropy loss. In Appendix A,  
216 we provide details on hyper-parameters, that were selected based on preliminary experiments and  
217 recommendations from the original papers.

218 **Dataset** We conduct our experiments on Folktale (Ding et al., 2021), a US census dataset where  
219 the goal is to predict the income of individuals ( $\leq 50k$  or  $\geq 50k$  dollars yearly). We split our data in  
220 three subsets (60 % for training, 20 % for validation and 20 % for testing). To simulate a federated  
221 context, we partition the training data among five clients using different strategies to create various  
222 degrees of heterogeneity: i.i.d. splits, states-based splits that reflect some heterogeneity due to real-life  
223 sociohistorical factors, and artificial heterogeneity based on the Dirichlet distribution (Yurochkin et al.,  
224 2019) as  $\text{Dir}(2.0)$  and  $\text{Dir}(0.5)$ . The resulting sample counts are represented in Figure 1 (Right). The  
225 pre-processing and data partitioning details are provided in Appendix A.

226 **Randomness and repetitions** We consider 16 different settings resulting from the combination of  
227 our 4 dataset partitions and 4 fairness metrics. We run 20 replicas per setting and algorithm to measure  
228 the sensitivity of the results to training-time randomness factors. These affect the choice of initial  
229 model parameters, and shuffling of the local datasets of each client at the start of each round. They  
230 are controlled by seeding pseudo-random number generators, with an arbitrary base seed (20231127)  
231 that is shared across settings, and incremented when passing to the next replica of the same setting.  
232 Note that the data partition itself does not change across replicas as the study of outside-of-training  
233 randomness factors is left for future work.

234 **Results** We present representative results here and defer the others, that follow similar trends,  
235 to Appendix C, along with an expanded analysis in Appendix B. Figure 2 illustrates results for  
236 Demographic Parity with i.i.d. partition (Left) and for Equalized Odds with  $\text{Dir}(0.5)$  partition (Right).

237 In all our experiments, FedAvg sets a baseline for quick convergence towards relatively accurate but  
238 mostly unfair models. It becomes less accurate in more heterogeneous settings, in line with previous  
239 studies (Karimireddy et al., 2020). Second, FedFair and FedFB obtain disparate results; they either  
240 learn unfair but accurate models or learn fair but very inaccurate predictors, close to the constant  
241 model. For instance, FairFed tends to learn unfair models on the i.i.d. partition and fair models on the  
242 more heterogeneous ones. It is probably due to the fact that it operates based on fairness discrepancies  
243 between clients, which are bound to be negligible with lower levels of heterogeneity. The more erratic

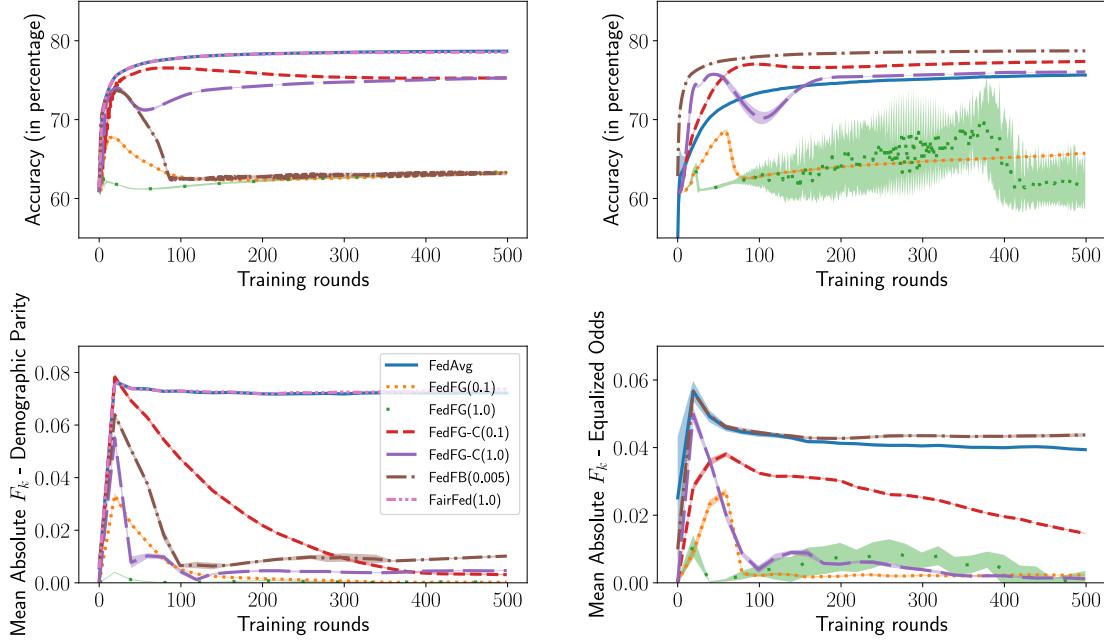


Figure 2: Settings: (Left) Demographic Parity with i.i.d. partition; (Right) Equalized Odds with  $\text{Dir}(0.5)$  partition. Each plot represents the evolution throughout training of a given metric. Each line represents the average of models trained with a given algorithm, with a surface around it denoting the standard deviation across replicas. Accuracy is computed at the end of each round, while fairness is computed at the end of rounds 1 and  $\{20, 40, \dots, 500\}$ . Both are computed on the validation subsets. For reference, a model predicting the majority label would achieve 60.79 % accuracy and 0.0 mean  $|F_k|$ .

244 behaviour of FedFB remains unexplained and requires more investigation. Third, FedFG reliably learns  
 245 fair models but tends to converge to very inaccurate solutions. As mentioned in Section 3.2, this is  
 246 probably related to the non-convexity of the local losses. Finally, FedFG-C is the only approach that  
 247 consistently manages to learn fair models with a limited decrease of utility as compared with FedAvg.

## 248 5 Conclusion And Future Work

249 In this work, we addressed the problem of group fairness in federated learning. We proposed two  
 250 algorithms, Fed-FairGrad and Fed-FairGrad-Convex. Both methods use a reweighting scheme to achieve  
 251 a better balance between the sensitive groups. Empirically, Fed-FairGrad learns fair but inaccurate  
 252 models while Fed-FairGrad-Convex remains more accurate while remaining fair.

253 The preliminary results presented in this paper are encouraging and we envision several future works.  
 254 First, we want to evaluate our methods in a wider range of settings, for example taking into account the  
 255 outside-of-training randomness, considering more datasets, more complex model architectures or other  
 256 heterogeneity scenarios. Second, we want to further integrate our implementation to the DecLearn  
 257 open-source framework for federated learning, to provide reproducible baselines and an extendable  
 258 playground for fair federated learning solutions. Third, our algorithms are compatible with Secure  
 259 Aggregation (Bonawitz et al., 2016), hence we would like to implement it. Fourth, we would like to  
 260 further study the trade-offs between communication and computation costs, for example by considering  
 261 other communication reduction techniques, such as partial client participation or gradient compression  
 262 (Wang and Ji, 2022; Haddadpour et al., 2021). Finally, longer-term perspectives include studying  
 263 asynchronous training (Sprague et al., 2018), where client information is not necessarily up-to-date,  
 264 and handling malicious clients that may share untrustworthy information (Blanchard et al., 2017).

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## 390 A Details on the experimental setting

391 In this appendix section, we provide with additional details as to the experimental setting, including  
 392 dataset pre-processing, data partition schemes and hyper-parameter choices for compared algorithms.

393 **Dataset pre-processing** We conduct our experiments on Folktbles (Ding et al., 2021), a US  
 394 census dataset containing explanatory and dependent variables such as income or employment. We  
 395 use data from the year 2018, restricted to the 5 highest-population states: California, Texas, Florida,  
 396 New-York and Pennsylvania. All the experiments were conducted to predict the income of individuals  
 397 (either  $\leq 50k$  or  $\geq 50k$  dollars yearly) using a linear model with sigmoid activation and binary cross-  
 398 entropy loss. We use 9 raw features, out of which 3 are continuous, 1 is binary, and 6 are categorical.  
 399 We apply min-max normalization to continuous features, and encode categorical ones into sets of  
 400 dummy variables, sometimes using high-level groups from the US census technical documentation. In  
 401 the end, 66 pre-processed features are used as predictors. The sensitive attribute is ethnicity. We select  
 402 recordings belonging to white and black individuals and drop the rest (Zafar et al., 2017) to make the  
 403 attribute binary, both for simplicity and to enable FairFed.

404 **Dataset partitions** We split our data in three subsets. Hence, we sampled uniformly from each  
 405 State’s data to attribute 60 % of samples for training, 20 % for validation and 20 % for testing. To  
 406 simulate a federated context, we further partition the training data among five clients using different  
 407 strategies to create heterogeneity. First, the data is shuffled and split evenly, resulting in a *i.i.d.*  
 408 setting. Second, the five *states*’ datasets are attributed to clients, resulting in some heterogeneity  
 409 due to real-life sociohistorical factors. Finally, some artificial heterogeneity is introduced using the  
 410 Dirichlet distribution Yurochkin et al. (2019): for each sensitive group, the proportion of its samples  
 411 attributed to each client is sampled from  $\text{Dir}(\alpha)$ . We use either  $\text{Dir}(2.0)$  or  $\text{Dir}(0.5)$ , the latter being  
 412 more heterogeneous than the former. The resulting subset-wise and client-wise sample counts are  
 413 represented in Figure 3.

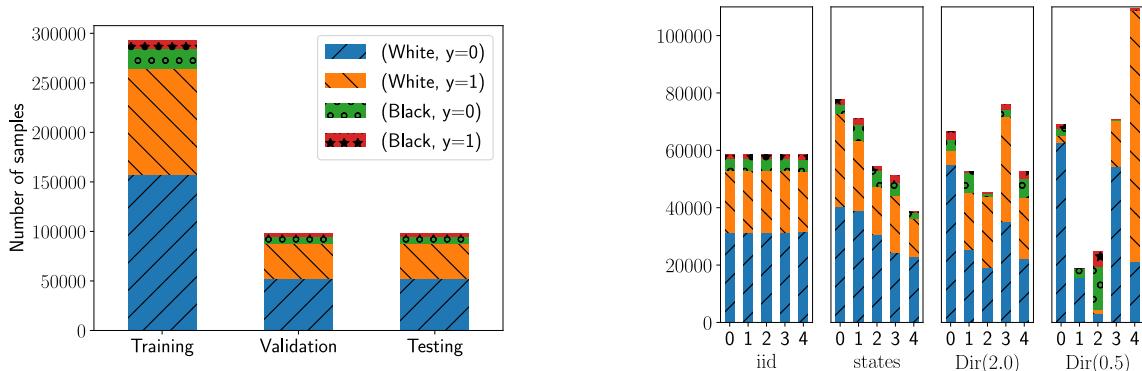


Figure 3: Distribution of samples across subsets (Left) and clients (Right). The height of the bars indicates the total number of samples in a global subset or local training sets. Bars are made of stacked sub-bars indicating sensitive groups, as defined by ethnicity and target labels.

414 **Hyper-parameters** We set a number of common hyper-parameters for all our experiments based on  
415 preliminary results. We run 500 rounds of federated training. During each round, clients first randomly  
416 shuffle their training data, then run a full epoch of training with 512-samples batches. They use 0.05  
417 as learning rate, and clip gradients so that their global L2-norm cannot exceed 0.05.

418 Apart from FedAvg, each algorithm has a differently-named hyper-parameter that impacts the  
419 update rule associated with fairness constraints, acting as a sort of learning rate. For FairFed, the  
420  $\beta$  parameter impacts the update rule for client-updates-averaging weights; we set it to  $\beta = 1.0$  as  
421 advised in the original paper. For FedFB, the  $\alpha$  parameter impacts the update rule for group-wise  
422 sampling weights; we set it to  $\alpha = 0.005$  based on preliminary results. For Fed-FG and FedFG-C, the  
423  $\eta$  parameter is the learning rate for updating the fairness weights; we set it either to  $\eta = 0.1$  or  $\eta = 1.0$ .  
424 The former would be our choice in the centralized setting, whereas we hypothesized the latter to be  
425 more suitable for the federated setting due to weights being updated only once per epoch.

426 **A precision on settings** Some baselines are only applicable to a subset of fairness definitions.  
427 FairFed only applies to Demographic Parity and Equality of Opportunity, while FedFB applies to all  
428 of our definitions but Accuracy Parity. As for FedAvg, since it is agnostic to fairness, we only run it  
429 once per data partition (with 20 replicas) and evaluate the resulting models' fairness by all definitions.

430 **Implementation** Our experiments were implemented using the DecLearn open-source Python  
431 package for federated learning. Both our contributed algorithms and baselines from the literature were  
432 implemented on a dedicated software branch. Federated learning was simulated, running concurrent  
433 routines for the clients and server with network communications on the localhost.

## 434 B Expanded commentary of results

435 In this appendix section, we provide a more detailed overview of our experimental results than that  
436 exposed in Section 4.

437 **FedAvg sets a baseline for quick convergence towards a relatively accurate unfair model.**  
438 In most settings, it achieves more than 78 % accuracy and 71 % precision and recall, with negligible  
439 variance. Metrics are slightly lower on the Dir(0.5) partition, which is expected given client heterogeneity  
440 (Karimireddy et al., 2020). The resulting models are unfair by all definitions, save for Accuracy Parity.

441 **FedFG-C is the only algorithm that achieves fairness in all settings, with a limited decrease  
442 of utility as compared with FedAvg.** It does so using  $\eta = 1.0$ , usually converging in less than 150  
443 rounds. In 14 out of 16 settings, it achieves more than 75 % accuracy and more than 67 % precision  
444 and recall, with negligible variance. With  $\eta = 0.1$ , models are much slower to converge, sometimes not  
445 being done after 500 rounds, resulting in a less fair (albeit sometimes more accurate) model than when  
446 using  $\eta = 1.0$ . We note that accuracy usually stabilizes early in the training process, and appears to  
447 be sparingly degraded as fairness is being optimized. We observe a similar pattern for Fed-FairGrad,  
448 that starts with optimizing utility (although to a lower point) prior to enforcing fairness (albeit in an  
449 utility-degrading manner).

450 **Most other algorithms converge to solutions that are fair but close to a constant-prediction  
451 model.** As exposed at the end of Section 3.2, this is the case for FedFG, which save for accuracy  
452 parity settings achieves at most 66 % accuracy, with around 90 % precision and below 15 % recall.  
453 This is better than a constant-prediction baseline, but still far from the utility of FedAvg. Indeed, a  
454 model that always predicts the majority label  $y = 0$  would achieve perfect fairness ( $\forall k, F_k = 0$ ) as per  
455 all definitions but accuracy parity, 60.79 % accuracy and 0 % precision and recall. For Demographic

456 Parity, FedFB also quickly converges to a model with less than 64 % accuracy and more than 93 %  
 457 precision, which is fairer than FedAvg baselines but not as fair as models obtained using Fed-FG  
 458 and Fed-FG-C. Additional experiments on the i.i.d. partition with distinct values of the  $\alpha$  hyper-  
 459 parameter (0.001, 0.0025, 0.01, 0.05, 0.1) all converged to similar results, save for some being fairer than  
 460 others. On non-i.i.d. partitions, FairFed, which is only defined for Demographic Parity and Equality  
 461 of Opportunity, mostly converges to constant-prediction models, outputting either the majority or  
 462 minority label depending on settings.

463 **Some baseline algorithms fail to enforce fairness altogether.** FairFed does so on the i.i.d.  
 464 partition, producing nearly the same results as FedAvg. This is explained by the fact that it operates  
 465 based on fairness discrepancies between clients, which are bound to be negligible in the i.i.d. setting.  
 466 This may also explain why FairFed exposes higher variance than other algorithms and sometimes fails to  
 467 enforce fairness in heterogeneous settings without a clear pattern to it. FedFB fails to enforce Equality  
 468 of Opportunity or Equalized Odds fairness, converging to a model that is similar to FedAvg in accuracy,  
 469 but as bad or even worse as to fairness. This remains to be explained. On the Dir(0.5) partition,  
 470 FedFB notably achieves better utility than any other method, but worse fairness than FedAvg. This  
 471 may be explained by the reweighting of the loss by sensitive group making the problem homogeneous  
 472 across clients, since they are only made heterogeneous as to these sensitive groups' distribution.

## 473 C Additional Results

474 In this appendix section, we provide exhaustive results of our experiments. For each setting, defined  
 475 by a data partition and a fairness definition, we present a table with metrics from the models resulting  
 476 from the compared algorithms after the full 500 rounds of training. All metrics are computed on the  
 477 validation dataset. Accuracy, precision and recall are presented as percentages with  $10^{-2}$  precision; the  
 478 higher the more useful the model, provided precision and recall are somewhat balanced. The average,  
 479 minimum and maximum values of  $F_k$  fairness measures are provided with  $10^{-4}$  precision; the lower the  
 480 better. Values are averaged across 20 replicas. We omit standard deviations for readability, noting  
 481 that they are almost always negligible. In each table, the first line indicates results for a model that  
 482 always predicts the majority label, which are the same across settings.

### 483 C.1 Results for Accuracy Parity

Table 1: Results for Accuracy Parity fairness on i.i.d. partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0676	0.0135	0.1218
FedAvg	78.68	73.52	71.30	0.0075	0.0015	0.0135
FedFG( $\eta = 0.1$ )	71.97	70.99	64.56	0.0196	0.0039	0.0353
FedFG( $\eta = 1.0$ )	72.71	60.10	90.44	0.0154	0.0031	0.0278
FedFG-C( $\eta = 0.1$ )	78.57	73.18	71.56	0.0076	0.0015	0.0137
FedFG-C( $\eta = 1.0$ )	76.82	71.85	70.90	0.0079	0.0016	0.0142

Table 2: Results for Accuracy Parity fairness on states partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0676	0.0135	0.1218
FedAvg	78.63	73.34	71.50	0.0069	0.0014	0.0124
FedFG( $\eta = 0.1$ )	72.73	70.43	67.06	0.0181	0.0036	0.0327
FedFG( $\eta = 1.0$ )	72.44	59.78	90.75	0.0163	0.0032	0.0294
FedFG-C( $\eta = 0.1$ )	78.58	72.88	72.25	0.0074	0.0015	0.0133
FedFG-C( $\eta = 1.0$ )	76.02	69.72	74.48	0.0095	0.0019	0.0171

Table 3: Results for Accuracy Parity fairness on Dir(2.0) partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0676	0.0135	0.1218
FedAvg	78.52	72.11	73.73	0.0065	0.0013	0.0118
FedFG( $\eta = 0.1$ )	78.39	72.86	71.51	0.0039	0.0008	0.0070
FedFG( $\eta = 1.0$ )	78.36	73.61	69.82	0.0032	0.0006	0.0058
FedFG-C( $\eta = 0.1$ )	78.31	70.44	76.96	0.0043	0.0009	0.0078
FedFG-C( $\eta = 1.0$ )	78.20	69.84	78.16	0.0027	0.0005	0.0049

Table 4: Results for Accuracy Parity fairness on Dir(0.5) partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0676	0.0135	0.1218
FedAvg	75.65	64.10	86.09	0.0038	0.0008	0.0068
FedFG( $\eta = 0.1$ )	65.95	86.70	15.62	0.0392	0.0078	0.0706
FedFG( $\eta = 1.0$ )	65.50	92.76	13.03	0.0493	0.0098	0.0888
FedFG-C( $\eta = 0.1$ )	77.83	75.11	64.98	0.0057	0.0011	0.0103
FedFG-C( $\eta = 1.0$ )	77.80	74.57	65.83	0.0046	0.0009	0.0083

## 484 C.2 Results for Demographic Parity

Table 5: Results for Demographic Parity fairness on i.i.d. partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.68	73.52	71.30	0.0722	0.0144	0.1300
FairFed( $\beta = 1.0$ )	78.55	73.61	70.61	0.0737	0.0147	0.1328
FedFB( $\alpha = 0.005$ )	63.17	93.69	6.51	0.0102	0.0020	0.0183
FedFG( $\eta = 0.1$ )	63.27	90.35	7.07	0.0002	0.0000	0.0003
FedFG( $\eta = 1.0$ )	63.37	90.19	7.39	0.0003	0.0001	0.0005
FedFG-C( $\eta = 0.1$ )	75.28	69.03	67.03	0.0031	0.0006	0.0056
FedFG-C( $\eta = 1.0$ )	75.28	69.22	66.54	0.0047	0.0009	0.0084

Table 6: Results for Demographic Parity fairness on states partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.63	73.34	71.50	0.0727	0.0145	0.1309
FairFed( $\beta = 1.0$ )	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedFB( $\alpha = 0.005$ )	63.36	93.55	7.02	0.0112	0.0022	0.0201
FedFG( $\eta = 0.1$ )	63.59	90.19	8.01	0.0003	0.0001	0.0006
FedFG( $\eta = 1.0$ )	63.74	90.04	8.46	0.0003	0.0001	0.0006
FedFG-C( $\eta = 0.1$ )	75.31	67.85	70.34	0.0038	0.0008	0.0069
FedFG-C( $\eta = 1.0$ )	75.24	67.97	69.70	0.0044	0.0009	0.0079

Table 7: Results for Demographic Parity fairness on Dir(2.0) partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.52	72.11	73.73	0.0748	0.0149	0.1348
FairFed( $\beta = 1.0$ )	39.21	39.21	100.00	0.0000	0.0000	0.0000
FedFB( $\alpha = 0.005$ )	63.21	93.51	6.62	0.0104	0.0021	0.0187
FedFG( $\eta = 0.1$ )	64.34	89.06	10.31	0.0002	0.0000	0.0004
FedFG( $\eta = 1.0$ )	64.53	88.88	10.89	0.0003	0.0001	0.0005
FedFG-C( $\eta = 0.1$ )	75.09	66.49	73.53	0.0047	0.0009	0.0085
FedFG-C( $\eta = 1.0$ )	75.10	66.62	73.15	0.0064	0.0013	0.0116

Table 8: Results for Demographic Parity fairness on Dir(0.5) partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	75.65	64.10	86.09	0.0723	0.0144	0.1302
FairFed( $\beta = 1.0$ )	39.26	39.22	99.86	0.0056	0.0011	0.0101
FedFB( $\alpha = 0.005$ )	64.05	93.29	8.95	0.0129	0.0026	0.0233
FedFG( $\eta = 0.1$ )	65.52	87.92	13.98	0.0003	0.0001	0.0005
FedFG( $\eta = 1.0$ )	65.93	87.70	15.26	0.0010	0.0002	0.0017
FedFG-C( $\eta = 0.1$ )	74.81	78.01	49.78	0.0146	0.0029	0.0264
FedFG-C( $\eta = 1.0$ )	72.24	79.19	39.61	0.0030	0.0006	0.0054

485 C.3 Results for Equality of Opportunity

Table 9: Results for Equality of Opportunity fairness on i.i.d. partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.68	73.52	71.30	0.0321	0.0000	0.1196
FairFed( $\beta = 1.0$ )	78.09	73.19	69.63	0.0385	0.0000	0.1436
FedFB( $\alpha = 0.005$ )	78.59	73.56	70.85	0.0414	0.0000	0.1541
FedFG( $\eta = 0.1$ )	64.01	90.64	9.16	0.0004	0.0000	0.0014
FedFG( $\eta = 1.0$ )	64.01	90.61	9.17	0.0003	0.0000	0.0009
FedFG-C( $\eta = 0.1$ )	77.28	72.17	68.44	0.0081	0.0000	0.0303
FedFG-C( $\eta = 1.0$ )	75.83	71.14	64.51	0.0003	0.0000	0.0011

Table 10: Results for Equality of Opportunity fairness on states partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.63	73.34	71.50	0.0328	0.0000	0.1223
FairFed( $\beta = 1.0$ )	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedFB( $\alpha = 0.005$ )	78.57	73.48	70.96	0.0426	0.0000	0.1588
FedFG( $\eta = 0.1$ )	64.33	90.37	10.11	0.0007	0.0000	0.0026
FedFG( $\eta = 1.0$ )	64.34	90.28	10.14	0.0005	0.0000	0.0018
FedFG-C( $\eta = 0.1$ )	77.28	71.22	70.56	0.0099	0.0000	0.0368
FedFG-C( $\eta = 1.0$ )	75.90	70.14	67.11	0.0011	0.0000	0.0040

Table 11: Results for Equality of Opportunity fairness on Dir(2.0) partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.52	72.11	73.73	0.0324	0.0000	0.1205
FairFed( $\beta = 1.0$ )	69.00	88.03	24.23	0.0089	0.0000	0.0333
FedFB( $\alpha = 0.005$ )	78.61	73.56	70.94	0.0423	0.0000	0.1575
FedFG( $\eta = 0.1$ )	64.81	90.24	11.48	0.0006	0.0000	0.0024
FedFG( $\eta = 1.0$ )	64.81	90.16	11.49	0.0006	0.0000	0.0021
FedFG-C( $\eta = 0.1$ )	77.23	70.44	72.23	0.0092	0.0000	0.0342
FedFG-C( $\eta = 1.0$ )	75.94	69.28	69.43	0.0015	0.0000	0.0057

Table 12: Results for Equality of Opportunity fairness on  $\text{Dir}(0.5)$  partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	75.65	64.10	86.09	0.0243	0.0000	0.0907
FairFed( $\beta = 1.0$ )	70.68	61.74	85.01	0.0163	0.0000	0.0607
FedFB( $\alpha = 0.005$ )	78.70	73.32	71.80	0.0367	0.0000	0.1368
FedFG( $\eta = 0.1$ )	65.25	90.31	12.72	0.0001	0.0000	0.0003
FedFG( $\eta = 1.0$ )	65.74	89.06	14.45	0.0061	0.0000	0.0228
FedFG-C( $\eta = 0.1$ )	76.54	66.10	82.48	0.0160	0.0000	0.0597
FedFG-C( $\eta = 1.0$ )	76.12	69.84	68.79	0.0016	0.0000	0.0060

#### 486 C.4 Results for Equalized Odds

Table 13: Results for Equalized Odds fairness on i.i.d. partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.68	73.52	71.30	0.0442	0.0058	0.1196
FedFB( $\alpha = 0.005$ )	78.65	73.85	70.53	0.0423	0.0052	0.1171
FedFG( $\eta = 0.1$ )	64.14	90.56	9.55	0.0022	0.0003	0.0043
FedFG( $\eta = 1.0$ )	56.96	13.05	19.98	0.0003	0.0000	0.0007
FedFG-C( $\eta = 0.1$ )	77.25	72.33	67.99	0.0103	0.0015	0.0268
FedFG-C( $\eta = 1.0$ )	75.67	71.48	63.16	0.0017	0.0002	0.0038

Table 14: Results for Equalized Odds fairness on states partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.63	73.34	71.50	0.0450	0.0058	0.1223
FedFB( $\alpha = 0.005$ )	78.62	73.68	70.72	0.0432	0.0053	0.1198
FedFG( $\eta = 0.1$ )	64.48	90.38	10.54	0.0020	0.0002	0.0048
FedFG( $\eta = 1.0$ )	53.24	13.72	35.00	0.0000	0.0000	0.0000
FedFG-C( $\eta = 0.1$ )	77.29	71.37	70.27	0.0107	0.0013	0.0296
FedFG-C( $\eta = 1.0$ )	75.83	70.83	65.23	0.0014	0.0001	0.0033

 Table 15: Results for Equalized Odds fairness on  $\text{Dir}(2.0)$  partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	78.52	72.11	73.73	0.0459	0.0065	0.1205
FedFB( $\alpha = 0.005$ )	78.69	73.85	70.67	0.0431	0.0053	0.1193
FedFG( $\eta = 0.1$ )	64.96	89.92	11.97	0.0022	0.0002	0.0055
FedFG( $\eta = 1.0$ )	57.37	16.01	22.95	0.0002	0.0000	0.0006
FedFG-C( $\eta = 0.1$ )	77.27	70.54	72.17	0.0107	0.0015	0.0282
FedFG-C( $\eta = 1.0$ )	75.99	70.20	67.38	0.0025	0.0002	0.0075

Table 16: Results for Equalized Odds fairness on  $\text{Dir}(0.5)$  partition

algorithm	Accuracy $\uparrow$	Precision $\uparrow$	Recall $\uparrow$	$ F_k $ -avg $\downarrow$	$ F_k $ -min $\downarrow$	$ F_k $ -max $\downarrow$
Constant	60.79	0.00	0.00	0.0000	0.0000	0.0000
FedAvg	75.65	64.10	86.09	0.0394	0.0067	0.0907
FedFB( $\alpha = 0.005$ )	78.72	74.04	70.41	0.0437	0.0052	0.1223
FedFG( $\eta = 0.1$ )	65.74	89.74	14.24	0.0023	0.0002	0.0056
FedFG( $\eta = 1.0$ )	62.11	12.48	4.21	0.0009	0.0000	0.0029
FedFG-C( $\eta = 0.1$ )	77.37	68.65	77.84	0.0146	0.0006	0.0501
FedFG-C( $\eta = 1.0$ )	76.06	73.09	61.63	0.0012	0.0001	0.0035