

(1)

a) transfer function in s domain

$$H(s) = \frac{K s^3}{\left(1 + \frac{s}{2f_{cl}}\right)^3 \left(1 + \frac{s}{2\pi f_{ch}}\right)^3}$$

slope decreasing  
points  
 $f_{cl}$  and  $f_{ch}$

written in terms of s and omega

$$\omega = 2\pi f$$

to write the pseudo code, we need to perform a bilinear transformation to convert to z-domain then do direct programming

For bilinear transform, we substitute

$$s = \frac{2}{T} \frac{(z-1)}{(z+1)}$$

$$T = \frac{1}{f} = \frac{1}{44100} \quad (\text{given})$$

we acquire

$$\frac{K \left( \frac{2}{T} \frac{(z-1)}{(z+1)} \right)^3}{\left( 1 + \frac{\frac{2}{T} \frac{(z-1)}{(z+1)}}{2\pi f_{cl}} \right)^3 \left( 1 + \frac{\frac{2}{T} \frac{(z-1)}{(z+1)}}{2\pi f_{ch}} \right)^3}$$

For ease of calculation, I will use many coefficients

Let's say

$$A = \pi T f_{cl}$$

$$B = \pi T f_{ch}$$

K calculation

$$H(s) = \frac{Ks^3}{\left(1 + \frac{s}{2\pi f_{CL}}\right)^3 \left(1 + \frac{s}{2\pi f_{CH}}\right)^3}$$

$$\begin{aligned} |H(j\omega)| &= 20 \log_{10} K + 60 \log_{10} |\omega| - 60 \log_{10} \left| 1 + \frac{j\omega}{2\pi f_{CL}} \right| - 60 \log_{10} \left| 1 + \frac{j\omega}{2\pi f_{CH}} \right| \\ &= 20 \log_{10} K + 60 \log_{10} (\omega) - 60 \log_{10} \sqrt{1 + \left(\frac{\omega}{2\pi f_{CL}}\right)^2} - 60 \log_{10} \sqrt{1 + \left(\frac{\omega}{2\pi f_{CH}}\right)^2} \end{aligned}$$

$$\omega = 2\pi f_{CL}$$

$$20 \log_{10} K = 60 \log_{10} \sqrt{2} + 60 \log_{10} \sqrt{1 + \left(\frac{f_{CL}}{f_{CH}}\right)^2} - 60 \log_{10} 2\pi f_{CL}$$

$$K^{\frac{2}{3}} = \frac{(f_{CL}^2 + f_{CH}^2)}{2\sqrt{2}(f_{CL} f_{CH} \pi)^3}$$



$$\frac{8K}{T^3} A^3 B^3 \frac{(z-1)^3 (z+1)^3}{\left( \frac{(A(z+1) + (z-1))^3 (B(z+1) + (z-1))^3}{(z-1)^3 (z+1)} \right)} \quad (2)$$

$$= \frac{AB(z^2 + 2z + 1) + \underbrace{A(z+1)(z-1)}_{Az^2 - A} + \underbrace{B(z+1)(z-1)}_{Bz^2 - B} + \underbrace{(z-1)(z-1)}_{z^2 - 2z + 1}}{z^2 - 1}$$

$$= \frac{ABz^2 + 2ABz + AB + Az^2 - A + Bz^2 - B + z^2 - 2z + 1}{(z^2 - 1)} \quad \frac{8K(AB)^3}{T^3}$$

$$= \frac{(AB + A + B + 1)z^2 + (2AB - 2)z + (AB - A - B + 1)}{(z^2 - 1)^3}$$

$$AB + A + B + 1 = D$$

$$2AB - 2 = E$$

$$AB - A - B + 1 = F$$

$$= \frac{8KA^3B^3}{T^3} \frac{(z^2 - 1)^3}{(Dz^2 + Ez + F)^3}$$



(3)

$$\begin{aligned}
 & D^2 z^4 + \cancel{DE z^3} + \cancel{DF z^2} \\
 & \cancel{DE z^3} + \cancel{E^2 z^2} + EF z \\
 & \cancel{DE z^2} + EF z + F^2
 \end{aligned}$$

$$(D^2 z^4 + 2DEF z^3 + (2DF + E^2) z^2 + 2EF z + F^2)$$

$$\begin{aligned}
 & D^3 z^6 + 2D^2 E z^5 + (2D^2 F + DE^2) z^4 + 2DEF z^3 + DF z^2 \\
 & D^2 E z^5 + 2DE^2 z^4 + (2DEF + E^3) z^3 + 2E^2 F z^2 + EF z^2 \\
 & D^2 F z^4 + 2DEF z^3 + (2DF^2 + E^2 F) z^2 + 2EF^2 z + F^3
 \end{aligned}$$

simplifies to:

$$\begin{aligned}
 & D^3 z^6 + 3D^2 E z^5 + (3D^2 F + 3DE^2) z^4 \\
 & (6DEF + E^3) z^3 + (3E^2 F + 3DF^2) z^2 \\
 & 3EF^2 z + F^3
 \end{aligned}$$

we acquire

$$\frac{8KA^3 B^3}{T^3} \frac{(z^6 - 3z^4 + 3z^2 - 1)}{\left[ D^3 z^6 + 3D^2 E z^5 + (3D^2 F + 3DE^2) z^4 + (6DEF + E^3) z^3 + (3E^2 F + 3DF^2) z^2 + 3EF^2 z + F^3 \right]}$$

 $\pi$  taken as 3.14

$$A = \pi T f_{CL} = 7.12 \times 10^{-5} f_{CL}$$

$$T = \frac{1}{f} = \frac{1}{44100} \text{ s}$$

$$B = \pi T f_{CH} = 7.12 \times 10^{-5} f_{CH}$$

$$D = AB + A + B + 1 = 5.06 \times 10^{-9} f_{CL} f_{CH} + 7.12 \times 10^{-5} (f_{CL} + f_{CH}) + 1$$

$$E = 2AB - 2 = 10^{-8} f_{CL} f_{CH} - 2$$

$$F = AB - A - B + 1 = 5.06 \times 10^{-9} f_{CH} f_{CL} - 7.12 \times 10^{-5} (f_{CL} + f_{CH}) + 1$$

$$\frac{8KA^3 B^3}{T^3} = 2.36$$

$$\frac{8KA^3 B^3}{T^3} = C$$



Pseudo code

④

$$C(1 - 3z^{-2} + 3z^{-4} - z^{-6})$$

$$D^3 + 3D^2Ez^{-1} + (3D^2F + 3DE^2)z^{-2} + (6DEF + E^3)z^{-3} + (3E^2F + 3DF^2)z^{-4} + 3EF^2z^{-5} + F^3z^{-6}$$

$$b_0 = C$$

$$a_0 = D^3$$

$$a_4 = 3E^2F + 3DF^2$$

$$b_2 = -3C$$

$$a_1 = 3D^2E$$

$$a_5 = 3EF^2$$

$$b_4 = 3C$$

$$a_2 = 3D^2F + 3DE^2$$

$$a_6 = F^3$$

$$b_6 = -C$$

$$a_3 = 6DEF + E^3$$

$$b_1, b_3, b_5 = 0$$

$$\frac{b_0}{a_0} = \frac{C}{D^3}$$

$$\frac{1}{a_0} = \frac{1}{D^3}$$

$$\frac{b_1}{a_0} = 0$$

$$\frac{-a_1}{a_0} = -\frac{3E}{D}$$

$$\frac{b_2}{a_0} = -\frac{3C}{D^3}$$

$$\frac{-a_2}{a_0} = -\frac{3DF + 3E^2}{D^2}$$

$$\frac{b_3}{a_0} = 0$$

$$\frac{-a_3}{a_0} = -\frac{(6DEF + E^3)}{D^3}$$

$$\frac{b_4}{a_0} = \frac{3C}{D^3}$$

$$\frac{-a_4}{a_0} = -\frac{(3E^2F + 3DF^2)}{D^3}$$

$$\frac{b_5}{a_0} = 0$$

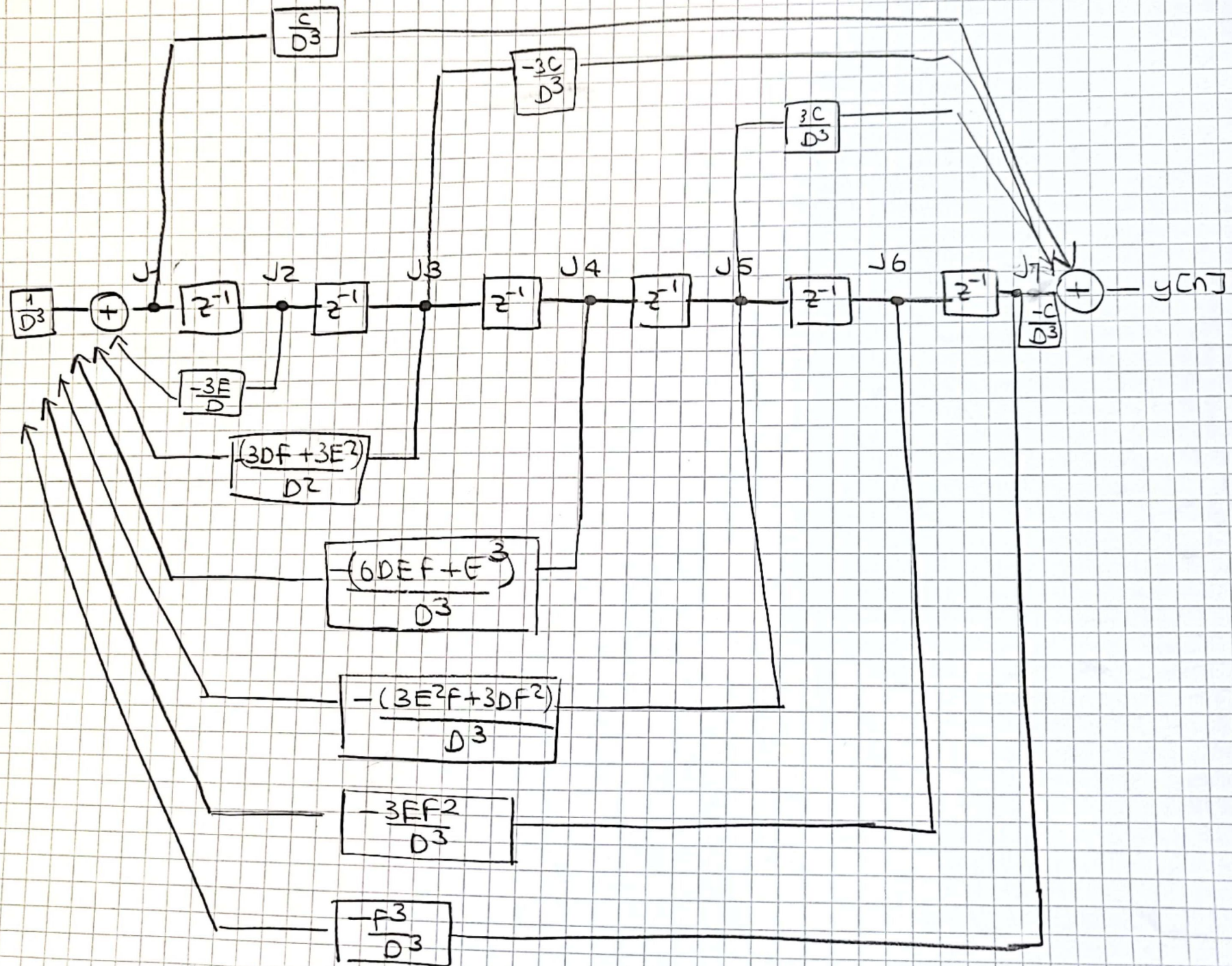
$$\frac{-a_5}{a_0} = -\frac{3EF^2}{D^3}$$

$$\frac{b_6}{a_0} = -\frac{C}{D^3}$$

$$\frac{-a_6}{a_0} = -\frac{F^3}{D^3}$$



5)





Pseudo code @TS t=n.S

⑥

input x

$$J1 = \frac{x}{D^3} - \frac{3E}{D} J2 - \frac{(3DF + 3E^2)}{D^2} J3 - \frac{(-6DEF + E^3)}{D^3} J4 \\ - \frac{(3E^2F + 3DF^2)}{D^3} J5 - \frac{3EF^2}{D^3} J6 - \frac{F^3}{D^3} J7$$

$$y = \frac{C}{D^3} J1 - \frac{3C}{D^3} J3 + \frac{3C}{D^3} J5 - \frac{C}{D^3} J7$$

output y

$$J7 = J6$$

$$J6 = J5$$

$$J5 = J4$$

$$J4 = J3$$

$$J3 = J2$$

$$J2 = J1$$

Return