## Handmatige uitwerking

Gegeven:

|  |  |  |
| --- | --- | --- |
| 5 |  |  |
|  |  | 4 |
|  |  | 6 |

Een magisch vierkant heeft de volgende eigenschappen:

* Alle rijen en diagonalen hebben dezelfde uitkomst.
* Het middelste getal heeft dezelfde uitkomst als deze rijen en diagonalen.

Op basis hiervan kan je de volgende vergelijkingen opstellen:



Dit geeft de volgende matrix van coëfficiënten.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -4 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -10 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Vervolgens kan dit stelsel met de Gauss-eliminatie opgelost worden als volgt:

R5 = R5 – 1 \* R1

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -4 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| **0** | **0** | **-1** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **5** |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -10 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Eerste coëfficiënt omhoog zetten:

R2 en R5 wisselen.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| **0** | **0** | **-1** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **5** |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| **0** | **0** | **0** | **1** | **1** | **0** | **0** | **0** | **0** | **-1** | **-4** |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -10 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 1 | 0 | 1 | 0 | 1 | 0 | 0 | -1 | 0 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Enen in rij 6 en 8 wegwerken:

R6 = R6 + 1 \* R2

R8 = R8 + 1 \* R2

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -4 |
| **0** | **0** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **-1** | **-5** |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| **0** | **0** | **0** | **0** | **2** | **0** | **1** | **1** | **0** | **-1** | **5** |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Eerste coëfficiënt omhoog zetten:

R3 en R4 wisselen.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| **0** | **0** | **0** | **1** | **0** | **0** | **1** | **0** | **0** | **-1** | **-5** |
| **0** | **0** | **0** | **0** | **0** | **0** | **1** | **1** | **0** | **-1** | **-6** |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -4 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | -1 | 5 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Nullen in onderstaande rijen wegwerken:

R5 = R5 – 1 \* R3

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| **0** | **0** | **0** | **0** | **1** | **0** | **-1** | **0** | **0** | **0** | **1** |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | -1 | 5 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Eerste coëfficiënt omhoog zetten:

R5 wisselen voor R4

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| **0** | **0** | **0** | **0** | **1** | **0** | **-1** | **0** | **0** | **0** | **1** |
| **0** | **0** | **0** | **0** | **0** | **0** | **1** | **1** | **0** | **-1** | **-6** |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | -1 | -11 |
| 0 | 0 | 0 | 0 | 2 | 0 | 1 | 1 | 0 | -1 | 5 |
| 0 | 0 | 0 | 0 | 3 | 0 | 0 | 0 | 0 | -1 | 0 |

Nullen in onderstaande rijen wegwerken:

R6 = R6 – 1 \* R4

R7 = R7 – 1 \* R4

R8 = R8 – 2 \* R4

R9 = R9 – 3 \* R4

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| **0** | **0** | **0** | **0** | **0** | **0** | **1** | **1** | **0** | **-1** | **-6** |
| **0** | **0** | **0** | **0** | **0** | **0** | **1** | **0** | **0** | **-1** | **-12** |
| **0** | **0** | **0** | **0** | **0** | **0** | **3** | **1** | **0** | **-1** | **3** |
| **0** | **0** | **0** | **0** | **0** | **0** | **3** | **0** | **0** | **-1** | **-3** |

Nullen in onderstaande rijen wegwerken:

R6 = R6 – 1 \* R5

R7 = R7 – 1 \* R5

R8 = R8 – 3 \* R5

R9 = R9 – 3 \* R5

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **-1** | **0** | **0** | **-6** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **-2** | **0** | **2** | **21** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **-3** | **0** | **2** | **15** |

Eerste coëfficiënt omhoog zetten:

R7 en R6 wisselen

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **-1** | **0** | **0** | **-6** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -2 | 0 | 2 | 21 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -3 | 0 | 2 | 15 |

Nullen in onderstaande rijen wegwerken:

R8 = R8 – 2 \* R6

R9 = R9 – 3 \* R6

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | -6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **2** | **33** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **2** | **33** |

Eerste coëfficiënt omhoog zetten:

R8 wisselen met R7

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | -6 |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **2** | **33** |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 33 |

Nullen in onderstaande rijen wegwerken:

R9 = R9 – 1 \* R7

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | -5 |
| 0 | 0 | -1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 5 |
| 0 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 0 | -1 | -5 |
| 0 | 0 | 0 | 0 | 1 | 0 | -1 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | -1 | -6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | -6 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 2 | 33 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** | **0** |

Wat betekent dat:

1. x2 + x3 – T = -5

2. -x3 + x5 + x8 = 5

3. x4 + x7 – T = -5

4. x5 – x7 = 1

5. x7 + x8 – T = -6

6. -x8 = -6

7. 2 \* T = 33

Dus:

T = 33 / 2 **=** **16.5**

x8 = -1 \* -6 **=** **6**

Dus:

x7 = -x8 + T – 6 = -6 + 16.5 – 6 **= 4.5**

Dus:

x5 = x7 + 1 = 4.5 + 1 **= 5.5**

x4 = -x7 + T – 5 = -4.5 + 16.5 – 5 **= 7**

Dus:

x3 = x5 + x8 – 5 = 5.5 + 6 – 5 **= 6.5**

Dus:

x2 = -x3 + T – 5 = -6.5 + 16.5 – 5 **= 5**

Dus:

x1 = 5 (gegeven)

x2 = 5

x3 = 6.5

x4 = 7

x5 = 5.5

x6 = 4 (gegeven)

x7 = 4.5

x8 = 6

x9 = 6 (gegeven)

T = 16.5

Uitkomst:

|  |  |  |
| --- | --- | --- |
| 5 | 5 | 6.5 |
| 7 | 5.5 | 4 |
| 4.5 | 6 | 6 |

## Uitleg methode

De methode die gebruikt is in de handmatige uitwerking is de Gauss-methode. Het doel van de Gauss-methode is om een matrix in echolonvorm te zetten.

De echolonvorm heeft de eigenschap dat elke volgende rij met meer nullen begint dan de vorige rij, dit wordt ook wel een bovendriehoeks-matrix genoemd.

Om de matrix in echolonvorm te zetten kunnen er de volgende operaties op het stelsel uitgevoerd worden:

* Rijen verwisselen
* Een rij vermenigvuldigen voor elk gewoon getal ongelijk aan 0.
* Een rij (of een vermenigvuldiging daarvan) aftrekken of optellen bij een andere rij.

Hier kunnen vervolgens nieuwe vergelijkingen uit opgesteld worden, deze kunnen dan met de balansmethode stuk voor stuk opgelost worden. Wanneer de matrix in correcte echolonvorm is er 100% zekerheid dat dit op te lossen is. Als de balansmethode vanaf onderaan uitgevoerd wordt, is het altijd zo dat onderstaande rijen bekende variabelen bevat, zoals bij de handmatige uitwerking ook te zien is, de laatste kolom (T) heeft alleen de bekende (33 in dit geval) nodig. De laatste kolom (T) is nodig om de op één na laatste kolom (x9) te bereken, enzovoort.

## Bronnen

<https://matrixcalc.org/>

<https://nl.wikipedia.org/wiki/Gauss-eliminatie>

## Python implementatie GitHub

<https://github.com/berryhijwegen/AC_HU/tree/master/linear_algebra/magic_square>