Practice Problem Set 3 - Basics of Estimation Theory

1. Find the MLE for μ and Σ for the n-dimensional multivariate Gaussian distribution with a covariance matrix that is a scaling of the identity matrix:

$$p(x;\mu,\sigma) = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{\frac{-1}{2\sigma^2}(x-\mu)^\top (x-\mu)}$$

   Assume m IID data points to compute the MLE from.

2. Murphy 4.7

3. Find the MLE for the Poisson distribution (a distribution over non-negative integers, generally counts):

$$p(n;r) = e^{-r}\frac{r^n}{n!}$$

   Assume m IID data points to compute the MLE from.

4. Consider the exponential density function $p(y;\theta) = \theta e^{-\theta y}$, if $y \geq 0$, and zero otherwise. $\Theta = \theta$ is a random variable also with an exponential density function $w(\theta) = \alpha e^{-\alpha\theta}$ for $\theta \geq 0$, and zero elsewhere and $\alpha > 0$ is known. Find the MAP estimate of θ given m IID data points.

5. Consider sample data of n observed pairs $(x_1, y_1), \ldots.., (x_n, y_n)$. Assume that $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$, $i = 1, \cdots, n$. Write down the least squares error criteria for estimating β_0 and β_1 and compute their estimates.

6. Consider the model $Y_k = N_k + \mu s_k$, $k = 1, \ldots, n$. $N_1, \ldots, N_n$ are iid $\mathcal{N}(0, \sigma^2)$ noise samples, $\mathbf{s} = (s_1, \ldots, s_n)^T$ is a known signal, and μ is the signal amplitude parameter. Assume σ^2 is known. Find the maximum likelihood estimate of μ.

7. Assume $x_i \in \Re, i = 1, \ldots., m$ is a set of IID observations and we model $p(x)$ as a Gaussian with known (fixed, not a random variable) σ^2, but unknown (parameter) μ. Find the MAP estimate of μ if $p(\mu)$ is also a Gaussian, with mean η and variance τ^2.