Classification Examples

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Linear Classification Example

[32x32x3]
array of numbers 0...1
(3072 numbers total)

image parameters

$f(\mathbf{x},\mathbf{W})$ $\longrightarrow$

10 numbers, indicating class scores

Linear Classification Example

$$f(x, W) = Wx$$

$\longrightarrow$

10 numbers,
indicating class
scores

[32x32x3]
array of numbers 0...1

Linear classification example

$$\boxed{f(x,W)} = \boxed{W}\boxed{x}$$

3072x1

10x1

10x3072

[32x32x3]
array of numbers 0...1

10 numbers,
indicating class
scores

parameters, or “weights”

Linear classification example

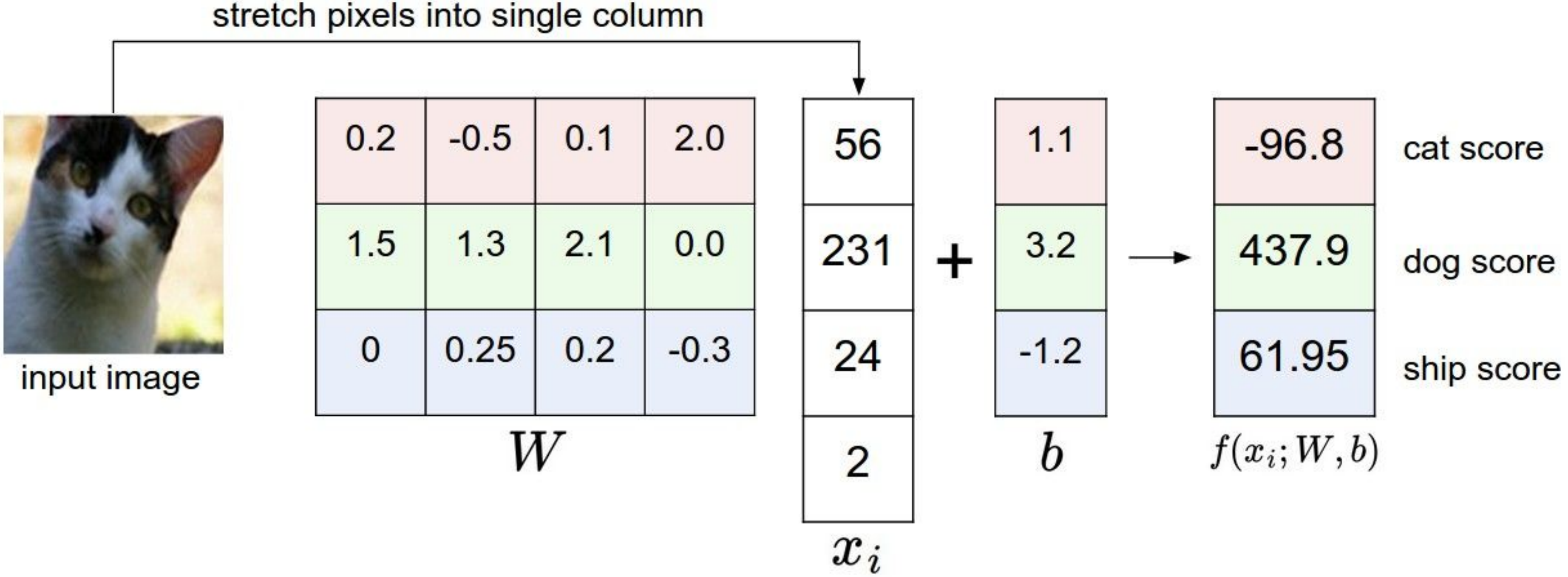
$$\underset{\text{10x1}}{\boxed{f(x,W)}} = \underset{\text{10x3072}}{\boxed{W}}\,\boxed{x} \quad \text{3072x1} \qquad \boxed{(+b)}\ \text{10x1}$$

[32x32x3]
array of numbers 0...1

10 numbers,
indicating class
scores

parameters, or “weights”

Example with an image with 4 pixels, and 3 classes (cat/dog/ship)

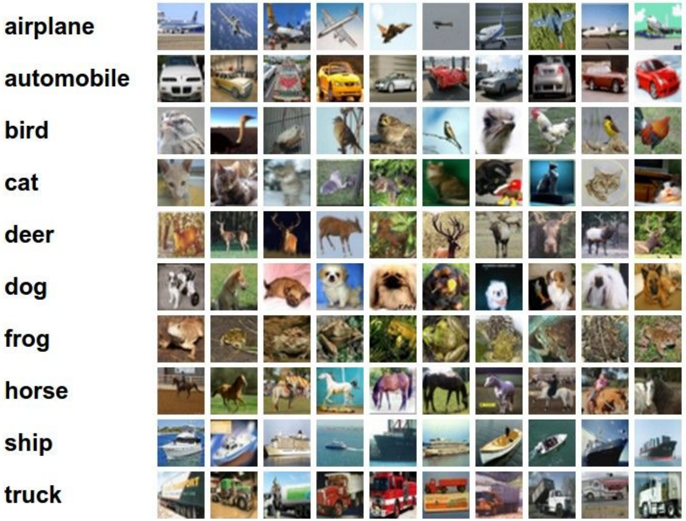


stretch pixels into single column

input image

$$\underset{W}{\begin{bmatrix}0.2 & -0.5 & 0.1 & 2.0\\ 1.5 & 1.3 & 2.1 & 0.0\\ 0 & 0.25 & 0.2 & -0.3\end{bmatrix}}\underset{x_i}{\begin{bmatrix}56\\ 231\\ 24\\ 2\end{bmatrix}}+\underset{b}{\begin{bmatrix}1.1\\ 3.2\\ -1.2\end{bmatrix}}\rightarrow\underset{f(x_i;W,b)}{\begin{bmatrix}-96.8\\ 437.9\\ 61.95\end{bmatrix}}\begin{matrix}\text{cat score}\\ \text{dog score}\\ \text{ship score}\end{matrix}$$

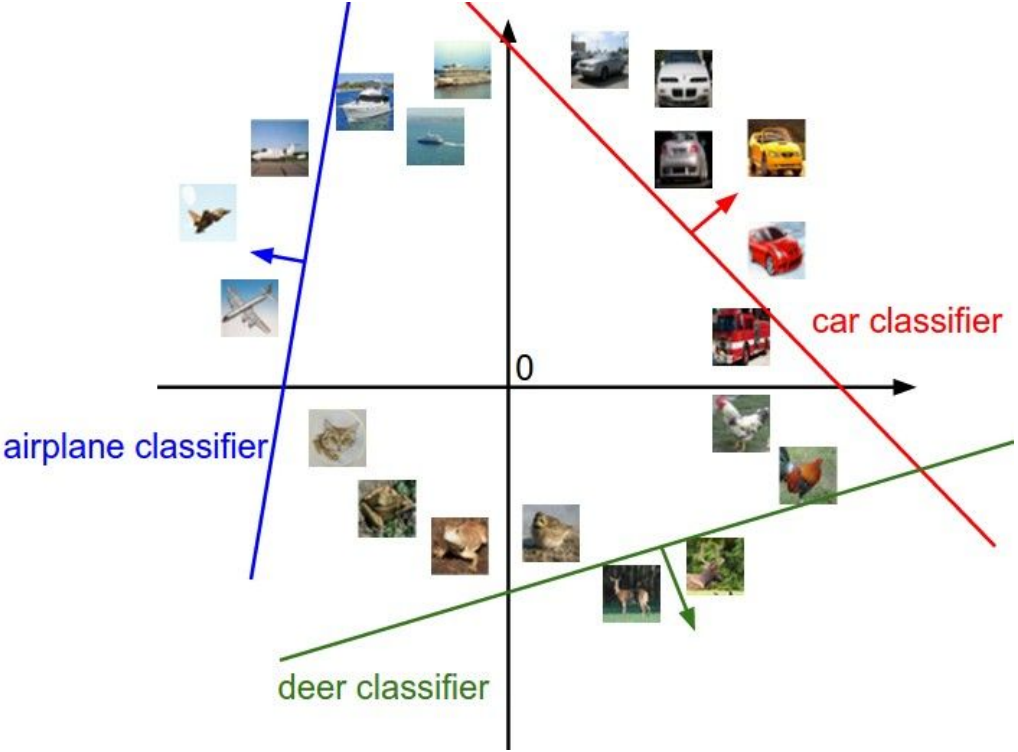
Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

Example trained weights of a linear classifier trained on CIFAR-10:

Interpreting a Linear Classifier



$$f(x_i, W, b) = Wx_i + b$$

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Multinomial Logistic Regression

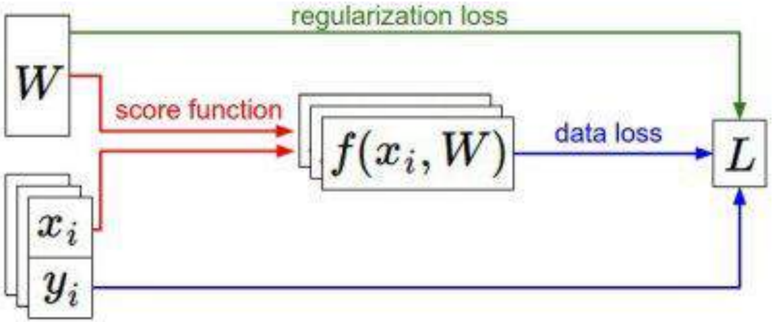
- We have some dataset of (x,y)
- We have a **score function:** $s = f(x; W) \overset{\text{e.g.}}{=} Wx$
- We have a **loss function**:

Softmax

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

$$L = \frac{1}{N} \sum_{i=1}^{N} L_i + R(W)$$

Full loss



Softmax Classifier (Multinomial Logistic Regression)

cat	**3.2**
car	5.1
frog	-1.7

Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$$\boxed{s = f(x_i; W)}$$

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car	5.1
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Softmax Classifier (Multinomial Logistic Regression)

scores = unnormalized log probabilities of the classes.

$$\boxed{P(Y=k|X=x_i)=\frac{e^{s_k}}{\sum_j e^{s_j}}}$$ where $$\boxed{s=f(x_i;W)}$$

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Softmax function

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Softmax Classifier (Multinomial Logistic Regression)

cat **3.2**

car 5.1

frog -1.7

scores = unnormalized log probabilities of the classes.

$$P(Y=k|X=x_i) = \frac{e^{s_k}}{\sum_j e^{s_j}}$$ where $$s = f(x_i; W)$$

Want to maximize the log likelihood, or (for a loss function) to minimize the negative log likelihood of the correct class:

$$L_i = -\log P(Y=y_i|X=x_i)$$

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in summary: $$L_i=-\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

Softmax Classifier (Multinomial Logistic Regression)

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unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

exp

cat	**3.2**	**24.5**
car	5.1	164.0
frog	-1.7	0.18

unnormalized log probabilities

Softmax Classifier (Multinomial Logistic Regression)

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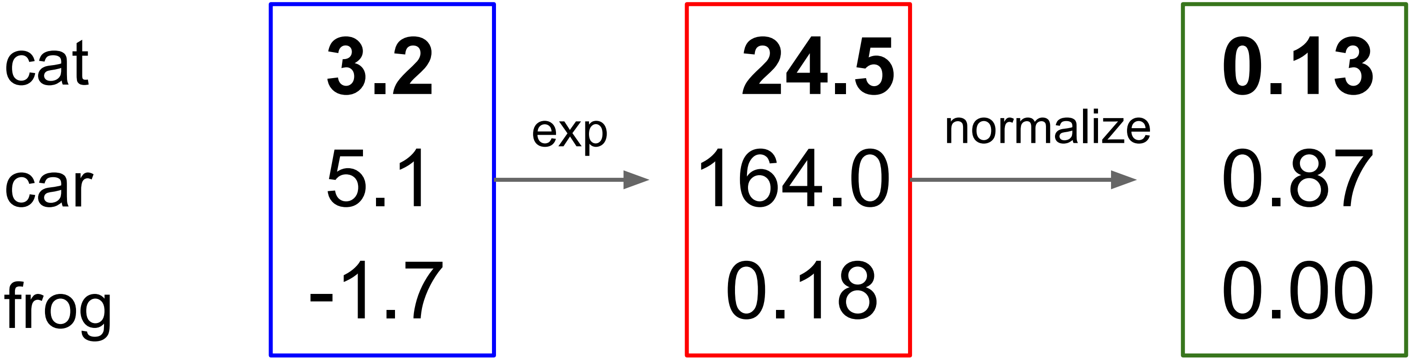
unnormalized probabilities



cat	**3.2**	**24.5**	**0.13**
car	5.1	164.0	0.87
frog	-1.7	0.18	0.00

unnormalized log probabilities

probabilities

Softmax Classifier (Multinomial Logistic Regression)

$$L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$$

unnormalized probabilities

cat	**3.2**	exp →	**24.5**	normalize →	**0.13**
car	5.1		164.0		0.87
frog	-1.7		0.18		0.00

→ L_i = -log(0.13)
= **0.89**

unnormalized log probabilities

probabilities