1.

$$\sigma(n) = 0.05T$$
$$\varphi(n) = 0.95T$$
$$p = 10$$
$$\Psi(n,p) \le \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{10}} \approx 6.897$$

2.

$$\sigma(n) = 0.06T$$
$$\varphi(n) = 0.94T$$
$$\Psi_{max}(n,p) = \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{p}} \ge 10$$
$$\therefore p \ge 23.5$$

So, we at least need 24 processors.

3.

$$\Psi_{max}(n,p) = \frac{\sigma(n) + \varphi(n)}{\sigma(n)} = \frac{T}{\sigma(n)} \ge 50$$
$$\sigma(n) \le 0.02T$$

So, the maximum fraction is 2%.

4.

$$\Psi(n,p) = 9 \le \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{10}} = \frac{T}{0.9\sigma(n) + 0.1T}$$
$$\sigma(n) \le 0.0123T$$

The maximum fraction is 1.23%.

5.

$$\sigma(n) = 9$$
$$\varphi(n) = 233 \cdot 16 = 3728$$
$$\Psi(n,p) = \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{16}} = 15.44$$

6.

$$\frac{\frac{\varphi(n)}{40}}{\sigma(n) + \frac{\varphi(n)}{40}} = 0.99$$
$$\sigma(n) = \frac{T}{3961}$$
$$\varphi(n) = \frac{3960T}{3961}$$
$$\Psi(n,p) = \frac{\sigma(n) + \varphi(n)}{\sigma(n) + \frac{\varphi(n)}{40}} = 39.61$$

7.

$$e_{p=10} = \frac{\frac{1}{\Psi(n,10)} - \frac{1}{10}}{1 - \frac{1}{10}} = \frac{1}{81}$$

If $\Psi(n, 100) = 90$,

$$e_{p=100} = \frac{\frac{1}{\Psi(n,100)} - \frac{1}{100}}{1 - \frac{1}{10}} = \frac{1}{891}$$

Since $e(100) < e(10)$, the assumption '$\Psi(n, 100) = 90$' is incorrect.

8.

$$e_{p=4} = \frac{\frac{1}{\frac{1000}{500}} - \frac{1}{4}}{1 - \frac{1}{4}} = \frac{1}{3}$$

$$e_{p=16} = \frac{\frac{1}{\Psi(n,16)} - \frac{1}{16}}{1 - \frac{1}{16}} \geq e_{p=4} = \frac{1}{3}$$

$$\Psi(n,16) \leq \frac{8}{3}$$

So the running time is at least $\frac{1000}{\Psi(n,16)} = 375s$

9.

(a) $\frac{M(f(p))}{p} = C^2 p$

(b) $\frac{M(f(p))}{p} = C^2 \log^2 p$

(c) $\frac{M(f(p))}{p} = C^2$

(d) $\frac{M(f(p))}{p} = C^2 p \log^2 p$

(e) $\frac{M(f(p))}{p} = C$

(f) $\frac{M(f(p))}{p} = p^{C-1}$, $1 < C < 2$

(g) $\frac{M(f(p))}{p} = p^{C-1}$, $C > 2$

So, the order is e=c>b>f>a>d>g.

10. We need to maximize $\Psi(n,p) = \frac{2n^3}{\frac{2n^3}{p} + 16n^2 \log_2 p}$ with constraints $1 \leq p \leq 1024$ and $24n^2 \leq 2^{30} \cdot p$. We can prove that $\frac{\partial \Psi(n,p)}{\partial n} > 0$ and $\frac{\partial \Psi(n,p)}{\partial p} \begin{cases} > 0 & p < \frac{n \ln 2}{8} \\ = 0 & p = \frac{n \ln 2}{8} \\ < 0 & p > \frac{n \ln 2}{8} \end{cases}$. So when $n = 214039$ and $p = 1024$, we can get the maximum $\Psi(n,p) = 740.56$.

When $p = \frac{n \ln 2}{8}$, $\Psi(n, p) = \frac{n \ln 2}{8 + 8 \ln\left(\frac{n \ln 2}{8}\right)}$, which is the maximum possible speed up given n.

We can choose $n = 25734$ to get the maximum $\Psi\left(n, \frac{n \ln 2}{8}\right) = 256.002$. However, here $p = 2229.68$, which exceeds the boundary.

When $p = 1024$, $\Psi(n, p) = \frac{1}{\frac{1}{1024} + \frac{80}{n}}$. We can choose $n = 27307$ and $\Psi(n, 1024) = 256.002$.