1. Course Logistics
2. Review of Vector + Matrix Algebra

$\mathbb{C}^n$

$\mathbb{R}$

$\vec{v} \in \mathbb{R}^n$ $\vec{v}$ $\underset{\sim}{v}$ $\mathbf{v}$

$$\mathbb{R}^n = \{ (a_1, \dots, a_n) \mid a_i \in \mathbb{R} \ \forall i \}$$

<u>vector space</u>

- closed under + and scalar mult

$\vec{v}, \vec{w} \in \mathbb{R}^n$ $\vec{v} + \vec{w} \in \mathbb{R}^n$

$\alpha \in \mathbb{R}$ $\alpha \vec{v} \in \mathbb{R}^n$

$(u + v) + w = u + (v + w)$ associat.

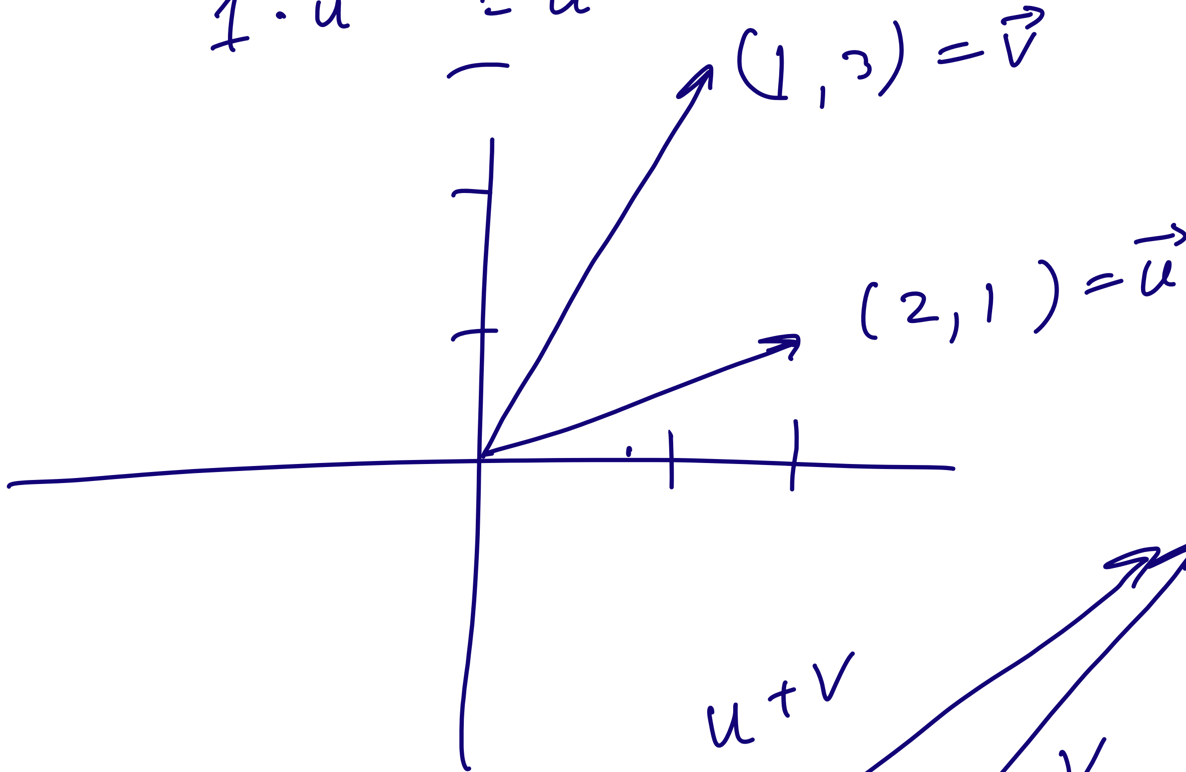
$u + v = v + u$ commut.

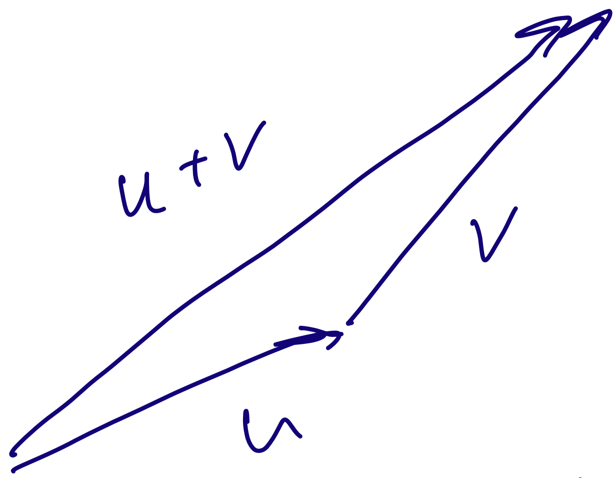
$\alpha(\vec{u} + \vec{v}) = \alpha \vec{u} + \alpha \vec{v}$

$(\alpha + \beta)\vec{u} = \alpha \vec{u} + \beta \vec{u}$

$\vec{0}$ $\qquad \vec{0} + \vec{u} = \vec{u}$ additive 0

$1 \cdot \vec{u} = \vec{u}$ identity scalar mult





linear combinations of u + v :

$$\alpha \vec{u} + \beta \vec{v}$$

<u>span</u> $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_k\}$

$$\mathrm{span}(v_1, \dots, v_k) = \{\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k \mid \forall \alpha_i \in \mathbb{R}\}$$

<u>$\mathbb{R}^3$</u> subspaces of $\mathbb{R}^3$

0

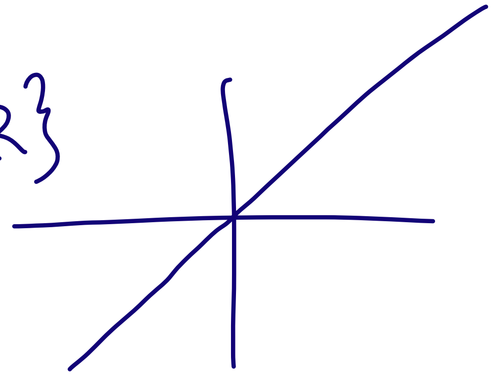
1

2

3

dim 0 $\{\vec{0}\}$ given $\vec{u} \in \mathbb{R}^3$

dim 1 $\{\alpha \vec{u} \mid \alpha \in \mathbb{R}\}$



$\vec{u}$ $0 \cdot \vec{u} = \vec{0}$

dim 2 $\{\underbrace{\alpha \vec{u}_1 + \beta \vec{u}_2} \mid \forall \alpha, \beta \in \mathbb{R}\}$

$\vec{u}_1, \vec{u}_2$ $\vec{u}_1 + \vec{u}_2$

linearly dependent: "linearly independent"

$u_1 = u_2$

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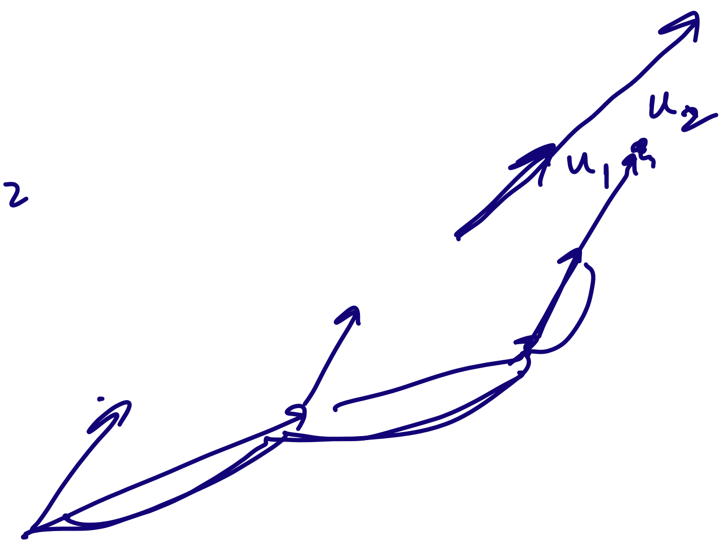


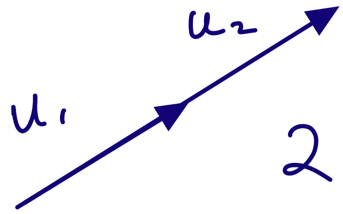
Linear Dependence

$\vec{v}_1, \ldots, \vec{v}_k \in \mathbb{R}^n$

linearly dependence: $\exists$ $\alpha_1, \ldots, \alpha_k \in \mathbb{R}$ not all 0

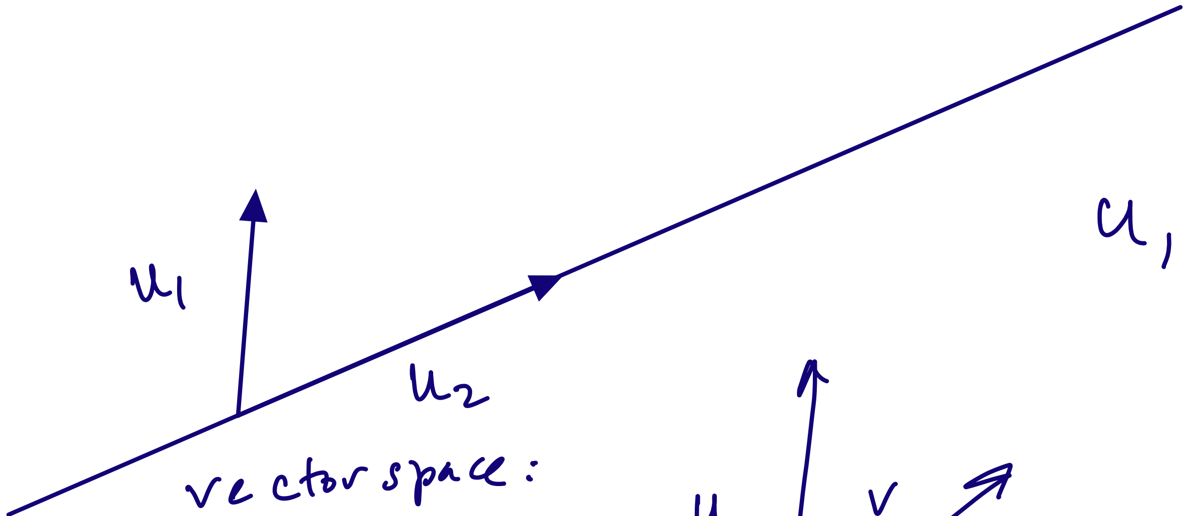
s.t.

$$\alpha_1 \vec{v}_1 + \alpha_2 v_2 + \ldots + \alpha_k \vec{v}_k = \sum_{i=1}^{k} \alpha_i \vec{v}_i = 0$$



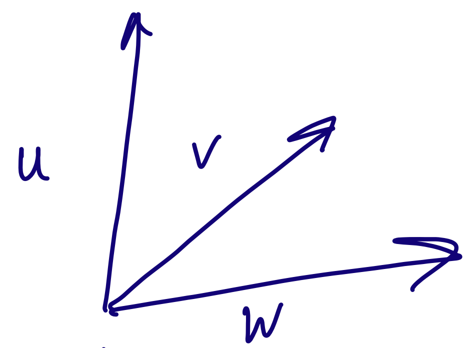
$2u_1 = u_2$

$2u_1 - u_2 = 0$



$u, v, w \in \mathbb{R}^3$

vector space: S



$\dim(S) =$ # of elements in a minimal set of linearly indep. vectors that span S.

Such a set of vectors is a <u>basis</u> for S.

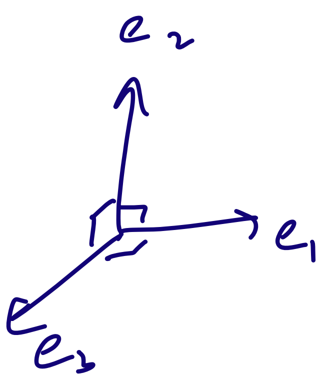
$\mathbb{R}^n$ basis

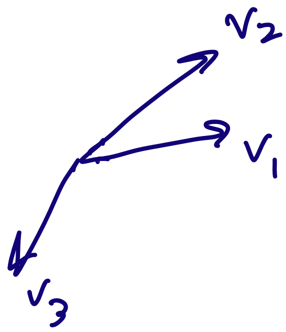
canonical basis

standard basis

$$\vec{e}_1 = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \vec{e}_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \vec{e}_n = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ -1.1 \\ 0 \end{pmatrix} = 2\vec{e}_1 - 1.1\vec{e}_2$$





$\vec{u}, \vec{v} \in \mathbb{R}^n$

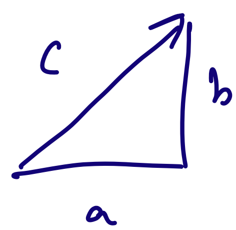
dot product / inner product

$$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + \dots + u_n v_n = \sum_{i=1}^{n} u_i v_i$$

$$\vec{u} = \begin{pmatrix} u_1 \\ \vdots \\ u_n \end{pmatrix} \qquad \vec{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$$

Euclidean length

$$\underbrace{\|\vec{u}\|}_{}{}_2 = \sqrt{\vec{u} \cdot \vec{u}}$$



$$\vec{u} \cdot \vec{v} = \|\vec{u}\|_2 \, \|\vec{v}\|_2 \cos\theta$$

$c = \sqrt{a^2 + b^2}$

$c = (a, b)$

$\|\vec{c}\| = (\vec{c} \cdot \vec{c})^{1/2}$