CS218, Winter 2024, **Due: 11:59pm, Wed 03/13, 2024**
Assignment #5 [1] **Training Programming Due: 11:59pm, Wed 03/06, 2024**
Name: **Student ID:**

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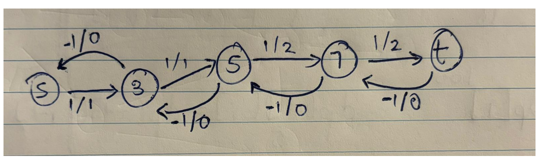
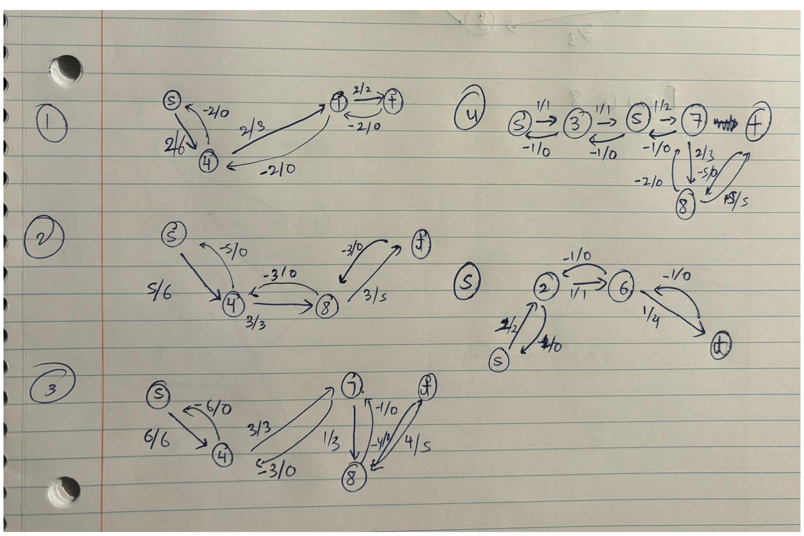
1 (1.5 pt) Union-Find

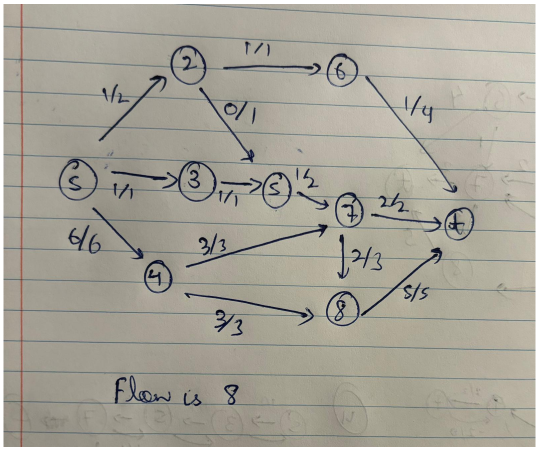
- We can solve this by induction.
- Base case is trivial and easily proven.
- Assume we have 2 trees of nodes l and m.
- Then when these 2 trees are joined the new height will be $max(1{+}log(l), log(m)) <= max(log(2l), log(m) <= log(l+m)$, base here is 2
- Since it still remains less than or equal to $log(l+m)$ hence it is still O(logn).

2 (1.5 pt) Network Flow

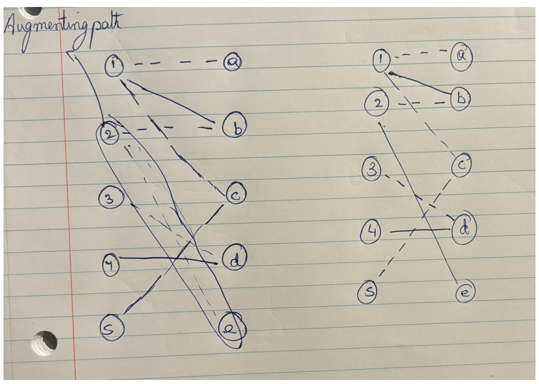
1. (0.3pts) Augmenting path S, 3, 5, 7, t, the flow in this path is 1 at max.
2. (0.6pts) 

3. (0.6pts) 

[1] Some of the problems are adapted from existing problems from online sources. Thanks to the original authors.

4. 

3 (0.8 pt) Bipartite Graph Matching

1. (0.4pts) 

2. above image is for both of the problems

3. (0.4pts)

4 (1.2 pt) Strongly Connected Components

1. (0.2 pt) A SCC of a graph is a subset of nodes such that every node in this subset is reachable to every other node through a path.

2. (0.3 pt) A, B, C

3. (0.5 pt)

4. (0.2 pt)

5 Challenge Problems

5.1 Catch the theft

1. Since we have to find minimum number of police cars on the roads we can treat roads as edges and crossroads at nodes.

2. The two points a and b are both at crossroads.
3. If we treat a and b as source and sink respectively as the thief has to start from a and reach b. We can apply the ford fulkerson method assuming capacity of each edge to be the number of police cars given.
4. This will give us the min cut and the flow (which will be a set of edges), placing that much police cars on each edge should be the final answer.
5. Submission id 250912816 (for some cases it is giving memory exceed error, I suppose that is due to the adjacency matrix representation in my code).