Problem 1.1

Dgemm0:

For each inner loop, there are 4 memory access and 2 operations.

Cycle/loop=4*100+1

Total time=401*n^3/(2*10^9)=200.5s

Time spend on memory=200s

Dgemm1:

For each j loop, there are two memory access.

For each k loop, there are two memory access and 2 operations.

Total cycle=2*100*n^2+(2*100+1)*n^3=2.012*10^11.

Total time=100.6s.

Time spend on memory=100.1s.

Problem 3

Matrix A block into 3*1, B into 1*3, C into 3*3.

```
void dgemm3(double *C,double *A,double *B,int n)
{
    for (int i=0;i<n;i+=3)
    {
        for (int j=0;j<n;j+=3)
        {
            register double c00,c01,c02,c10,c11,c12,c20,c21,c22;
            c00=C[i*n+j]; c01=C[i*n+j+1]; c02=C[i*n+j+2];
            c10=C[(i+1)*n+j]; c11=C[(i+1)*n+j+1]; c12=C[(i+1)*n+j+2];
            c20=C[(i+2)*n+j]; c21=C[(i+2)*n+j+1]; c22=C[(i+2)*n+j+2];
            register double a0,a1,a2;
            register double b0,b1,b2;
            for (int k=0;k<n;k++)
            {
                a0=A[i*n+k]; a1=A[(i+1)*n+k]; a2=A[(i+2)*n+k];
                b0=B[k*n+j]; b1=B[k*n+j+1]; b2=B[k*n+j+2];
                c00+=a0*b0; c01+=a0*b1; c02+=a0*b2; c10+=a1*b0; c11+=a1*b1; c12+=a1*b2; c20+=a2*b0; c21+=a2*b1; c22+=a2*b2;
            }
            C[i*n+j]=c00; C[i*n+j+1]=c01; C[i*n+j+2]=c02;
            C[(i+1)*n+j]=c10; C[(i+1)*n+j+1]=c11; C[(i+1)*n+j+2]=c12;
            C[(i+2)*n+j]=c20; C[(i+2)*n+j+1]=c21; C[(i+2)*n+j+2]=c22;
        }
    }
}
```

Problem 4

n=10000:

Loop order	Cache miss on each element			Cache read			Miss rate
	A[i][j]	B[i][j]	C[i][j]	A	B	C	
ijk or jik	$\begin{cases} n & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	n	1	n^3	n^3	n^2	$\approx 55\%$
ikj or kij	1	$\begin{cases} n & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	$\begin{cases} n & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	n^2	n^3	n^3	$\approx 10\%$
jki or kji	n	1	n	n^3	n^2	n^3	100%

n=10:

Loop order	Cache miss on each element			Cache read			Miss rate
	A[i][j]	B[i][j]	C[i][j]	A	B	C	
ijk or jik	$\begin{cases} 1 & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	$\begin{cases} 1 & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	$\begin{cases} 1 & j\%10=0 \\ 0 & j\%10\neq 0 \end{cases}$	n^3	n^3	n^2	$\approx 1.43\%$

ikj or kij	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	n^2	n^3	n^3	$\approx 1.43\%$
jki or kji	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	n^3	n^2	n^3	$\approx 1.43\%$

Problem 5

Loop order	Cache miss on each element			Cache read			Miss rate
	A[i][j]	B[i][j]	C[i][j]	A	B	C	
ijk or jik	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	n^3	n^3	$\frac{n^3}{b}$	$\approx 0.95\%$
ikj or kij	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\frac{n^3}{b}$	n^3	n^3	$\approx 0.95\%$
jki or kji	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}1 & j\%10=0\\0 & j\%10\neq 0\end{cases}$	$\begin{cases}\frac{n}{b} & j\%10=0\\0 & j\%10\neq 0\end{cases}$	n^3	$\frac{n^3}{b}$	n^3	$\approx 0.95\%$

Problem 7

```
void dgemm7(double *C,double *A,double *B,int n)
{
    int b=60;
    for (int i=0;i<n;i+=b)
        for (int k=0;k<n;k+=b)
            for (int j=0;j<n;j+=b)
                for (int ii=i;ii<i+b;ii+=3)
                    for (int jj=j;jj<j+b;jj+=3)
                    {
                        register double c00,c01,c02,c10,c11,c12,c20,c21,c22;
                        c00=C[ii*n+jj]; c01=C[ii*n+jj+1]; c02=C[ii*n+jj+2];
                        c10=C[(ii+1)*n+jj]; c11=C[(ii+1)*n+jj+1]; c12=C[(ii+1)*n+jj+2];
                        c20=C[(ii+2)*n+jj]; c21=C[(ii+2)*n+jj+1]; c22=C[(ii+2)*n+jj+2];
                        register double a0,a1,a2;
                        register double b0,b1,b2;
                        for (int kk=k;kk<k+b;kk++)
                        {
                            a0=A[ii*n+kk]; a1=A[(ii+1)*n+kk]; a2=A[(ii+2)*n+kk];
                            b0=B[kk*n+jj]; b1=B[kk*n+jj+1]; b2=B[kk*n+jj+2];
                            c00+=a0*b0; c01+=a0*b1; c02+=a0*b2; c10+=a1*b0; c11+=a1*b1; c12+=a1*b2; c20+=a2*b0; c21+=a2*b1; c22+=a2*b2;
                        }
                        C[ii*n+jj]=c00; C[ii*n+jj+1]=c01; C[ii*n+jj+2]=c02;
                        C[(ii+1)*n+jj]=c10; C[(ii+1)*n+jj+1]=c11; C[(ii+1)*n+jj+2]=c12;
                        C[(ii+2)*n+jj]=c20; C[(ii+2)*n+jj+1]=c21; C[(ii+2)*n+jj+2]=c22;
                    }
}
```