# Backpropagation Examples

Original Slide Credits: Andrej Karpathy

Modified by Amit Roy-Chowdhury

$$f(x,y,z) = (x+y)z$$
  
e.g.  $x = -2$ ,  $y = 5$ ,  $z = -4$ 

$$f(x,y,z)=(x+y)z$$
e.g.  $x=-2$ ,  $y=5$ ,  $z=-4$ 

$$q=x+y \qquad \frac{\partial q}{\partial x}=1, \frac{\partial q}{\partial y}=1$$

Want: 
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

$$f(x,y,z)=(x+y)z$$
e.g.  $x=-2$ ,  $y=5$ ,  $z=-4$ 
 $q=x+y$ 
 $\frac{\partial q}{\partial x}=1$ ,  $\frac{\partial q}{\partial y}=1$ 
 $\frac{\partial f}{\partial q}=z$ ,  $\frac{\partial f}{\partial q}=z$ 

Want: 
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$$f(x,y,z)=(x+y)z$$
e.g.  $x=-2$ ,  $y=5$ ,  $z=-4$ 
 $q=x+y$   $\frac{\partial q}{\partial x}=1$ ,  $\frac{\partial q}{\partial y}=1$ 
 $f=qz$   $\frac{\partial f}{\partial q}=z$ ,  $\frac{\partial f}{\partial z}=q$ 

Want: 
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

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 $\frac{\partial f}{\partial z}$ 

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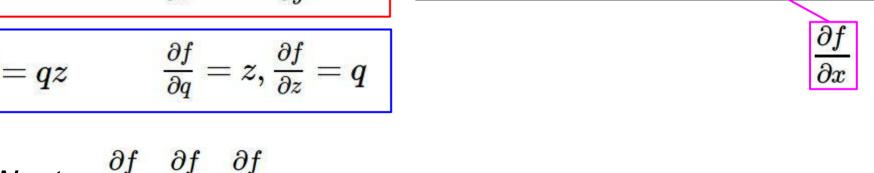
$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

$$f=qz$$
  $rac{\partial f}{\partial q}=z,rac{\partial f}{\partial z}=q$  Chain rule:  $rac{\partial f}{\partial y}=rac{\partial f}{\partial q}rac{\partial f}{\partial z}$ 

$$f(x,y,z) = (x+y)z$$
e.g.  $x = -2$ ,  $y = 5$ ,  $z = -4$ 

$$q = x + y \qquad \frac{\partial q}{\partial x} = 1, \frac{\partial q}{\partial y} = 1$$

Want: 
$$\frac{\partial f}{\partial x}$$
,  $\frac{\partial f}{\partial y}$ ,  $\frac{\partial f}{\partial z}$ 



$$f(x,y,z)=(x+y)z$$
e.g.  $x=-2$ ,  $y=5$ ,  $z=-4$ 

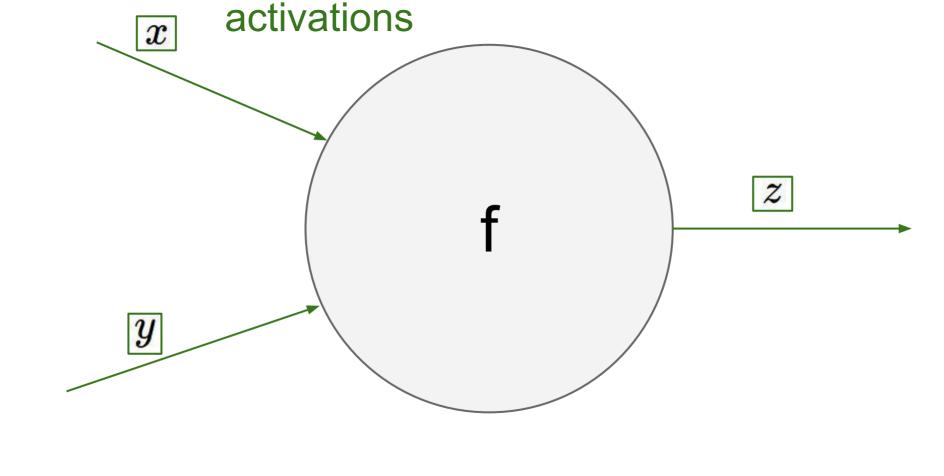
$$q=x+y$$

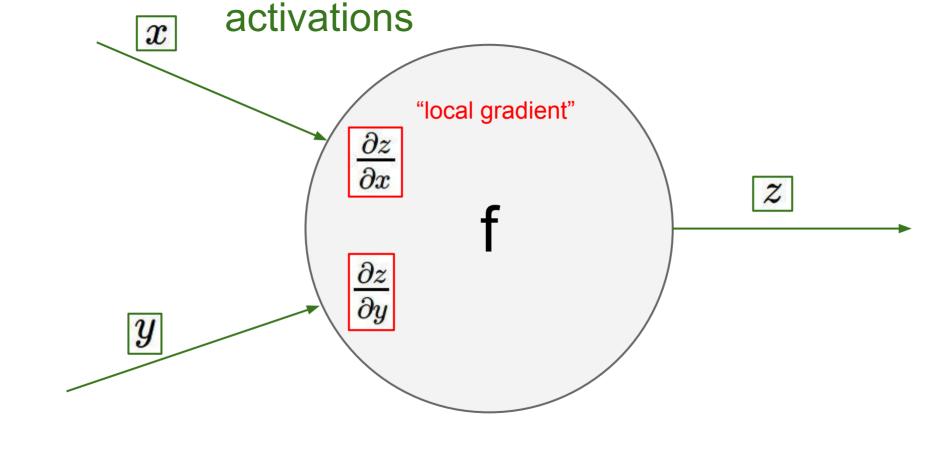
$$\frac{\partial q}{\partial x}=1, \frac{\partial q}{\partial y}=1$$

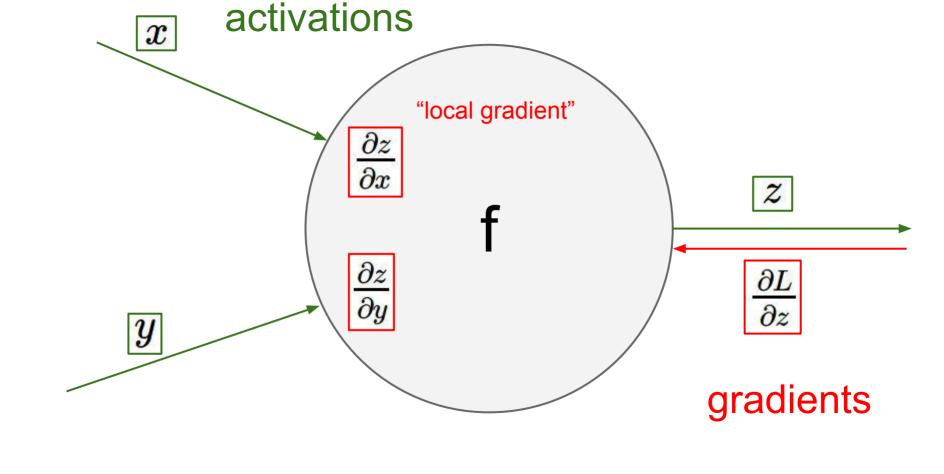
Chain rule:

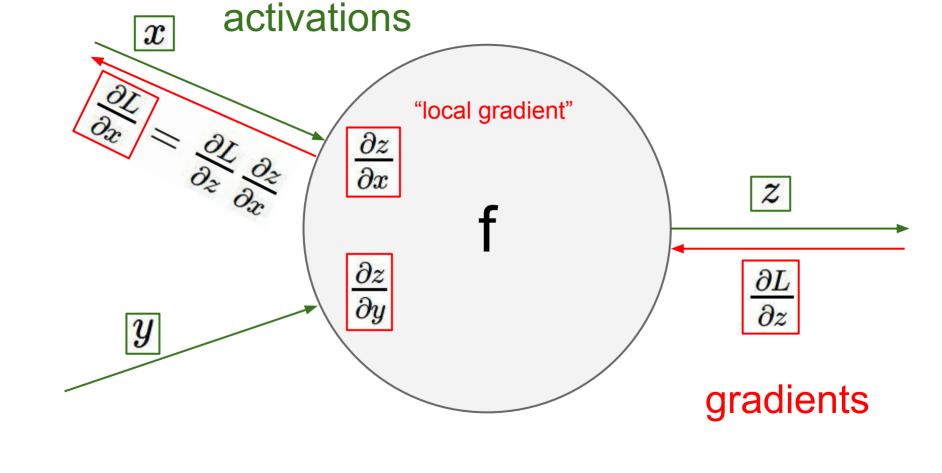
$$=qz$$
  $rac{\partial f}{\partial q}=z, rac{\partial f}{\partial z}=q$ 

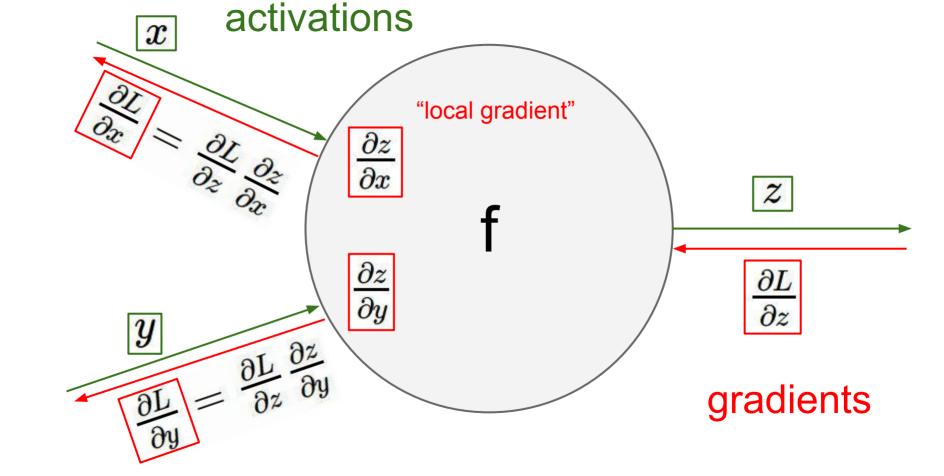
Want: 
$$\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$$

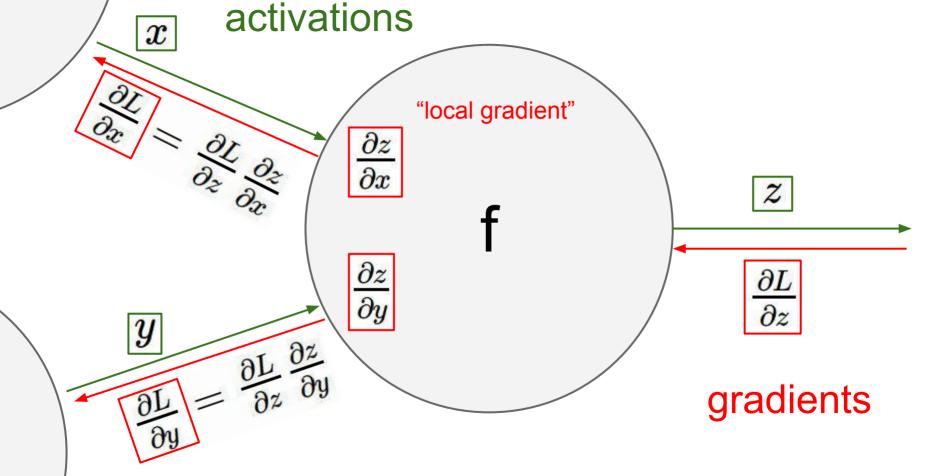




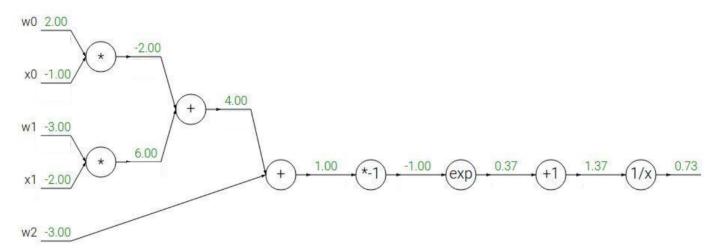




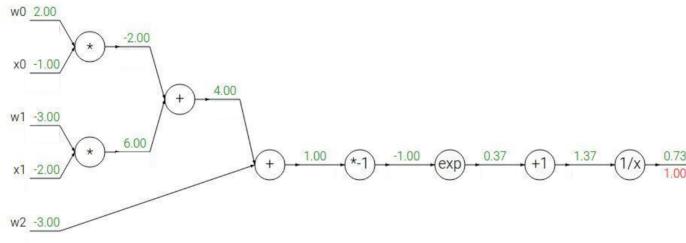




$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_1 x_2 + w_2 x_1 + w_2 x_2 + w_2 x_2$$

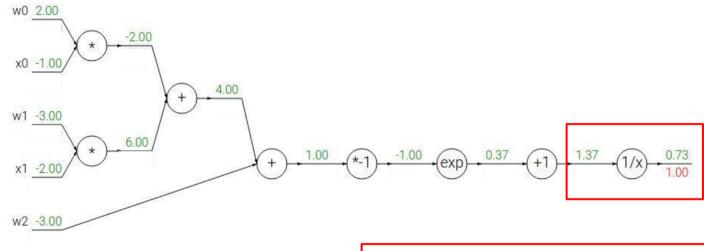


ole: 
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^{x} \qquad \rightarrow \qquad \frac{df}{dx} = e^{x} \qquad f(x) = \frac{1}{x} \qquad \rightarrow \qquad \frac{df}{dx} = a \qquad f_{c}(x) = c + x \qquad \rightarrow$$

ole: 
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



$$f(x)=e^x$$

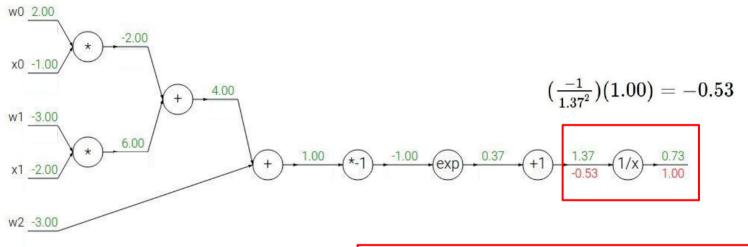
$$rac{df}{dx}=e^x \ df$$

$$f(x) = rac{1}{x}$$

$$rac{df}{dx}=-1/x^2$$

$$ightarrow rac{d}{dx} = e^{x}$$
  $ightarrow rac{df}{dx} = a$ 

ole: 
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x)=e^x$$

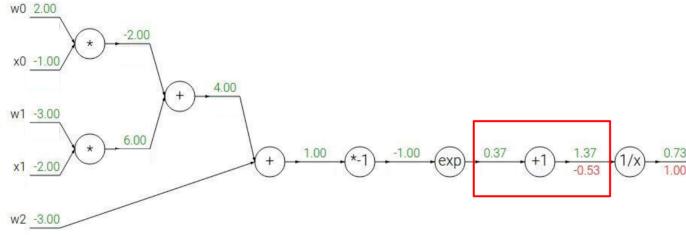
$$rac{df}{dx}=e^x$$

$$rac{df}{dx} = -1/df$$

$$\rightarrow$$

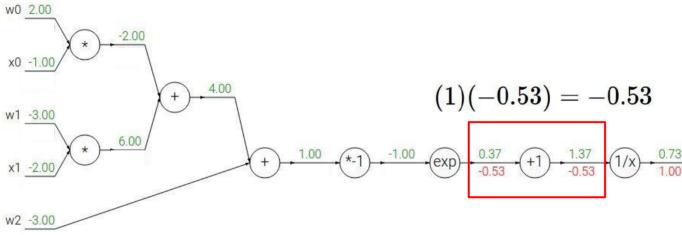
 $f(x) = e^x$ 

ole: 
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$egin{aligned} f(x) &= e^x & 
ightarrow & rac{df}{dx} &= e^x & f(x) &= rac{1}{x} & 
ightarrow & rac{df}{dx} &= -1/x \ f_a(x) &= ax & 
ightarrow & rac{df}{dx} &= a & f_c(x) &= c + x & 
ightarrow & rac{df}{dx} &= -1/x \ \hline \end{pmatrix}$$

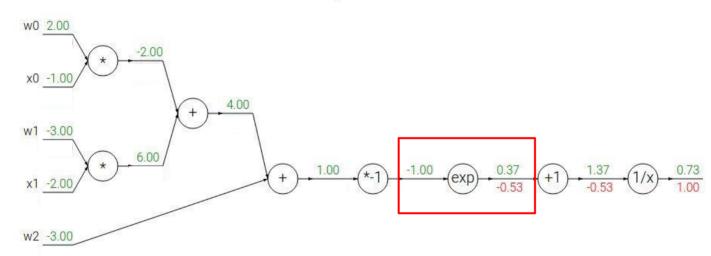
ole: 
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



$$f(x) = e^{x} \qquad \rightarrow \qquad \frac{df}{dx} = e^{x} \qquad f(x) = \frac{1}{x} \qquad \rightarrow \qquad \frac{d}{dx}$$

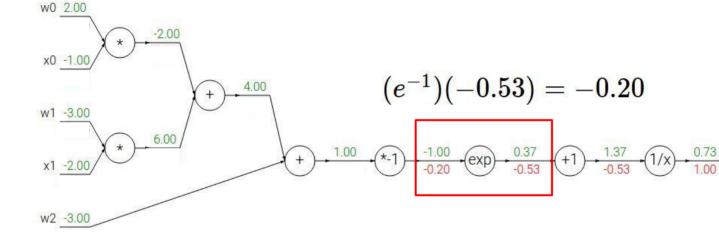
$$egin{aligned} f(x) = e^x & 
ightarrow & rac{df}{dx} = e^x & f(x) = rac{1}{x} & 
ightarrow & rac{df}{dx} = -1/x^2 \ f_a(x) = ax & 
ightarrow & rac{df}{dx} = a & f_c(x) = c + x & 
ightarrow & rac{df}{dx} = 1 \end{aligned}$$

ole: 
$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



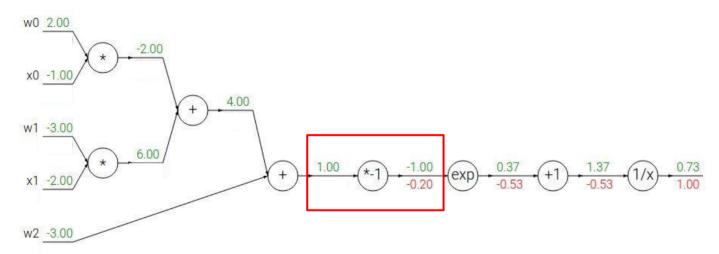
$$f(x)=e^x \qquad \qquad o \qquad \qquad rac{df}{dx}=e^x \ f_a(x)=ax \qquad \qquad o \qquad \qquad rac{df}{dx}=a$$

ole: 
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_2)}}$$



$$egin{aligned} f(x) = e^x & 
ightarrow & rac{df}{dx} = e^x \ f_a(x) = ax & 
ightarrow & rac{df}{dx} = a \end{aligned}$$

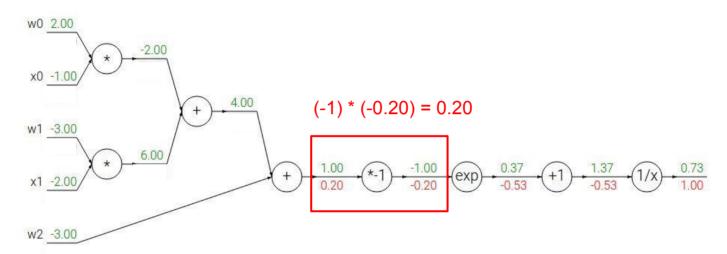
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$$egin{aligned} f(x) = e^x & 
ightarrow & rac{df}{dx} = e^x \ \hline f_a(x) = ax & 
ightarrow & rac{df}{dx} = a \ \hline \end{aligned}$$

$$f(x) = rac{1}{x} \qquad o \qquad rac{df}{dx} = -1/c$$
  $f(x) = a \qquad f_c(x) = c + x \qquad o \qquad rac{df}{dx} = -1/c$ 

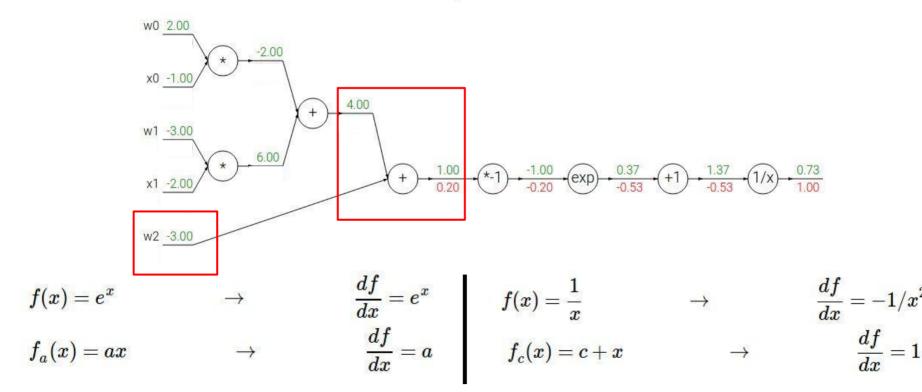
ole: 
$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_1 x_2 + w_2 x_2 + w_2$$



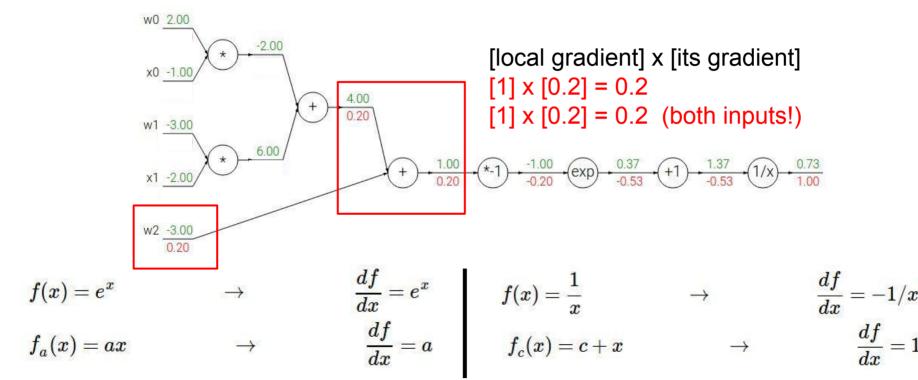
$$f(x) = e^x \qquad o \qquad rac{af}{dx} = e^x \ f_a(x) = ax \qquad o \qquad rac{df}{dx} = a$$

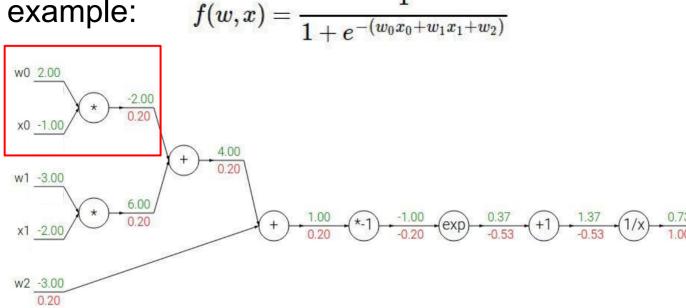
$$f(x)=rac{1}{x} \qquad \qquad 
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ightarrow \qquad rac{df}{dx}=$$

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$$f(w,x) = rac{1}{1 + e^{-(w_0 x_0 + w_1 x_1 + w_1 x_2 + w_2 x_1 + w_2 x_2 + w_2 x_2$$





 $f(x) = e^x$ 

$$x1 -2.00$$
 $w2 -3.00$ 
 $0.20$ 

$$rac{df}{dx}=e^x \ df$$

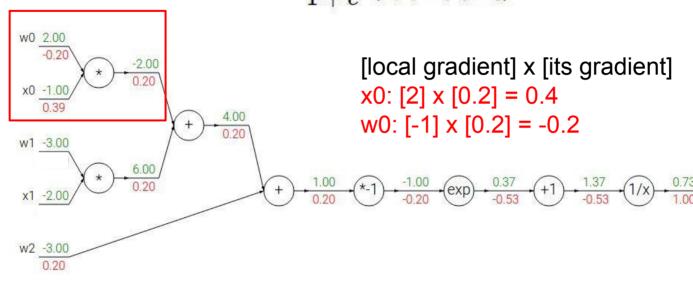
$$egin{array}{c} x & & & \\ a & & & \end{array}$$

$$=\frac{1}{x}$$

$$\rightarrow$$

$$rac{df}{dx} = -1/x$$
  $rac{df}{dx} = -1/x$ 

$$\frac{df}{dx} = \epsilon$$



 $f(x) = e^x$ 

$$\rightarrow$$

$$rac{f}{x}=e^x \ df$$

$$(x) = \frac{1}{x}$$

$$\rightarrow$$

$$rac{df}{dx} = -1/x^2$$

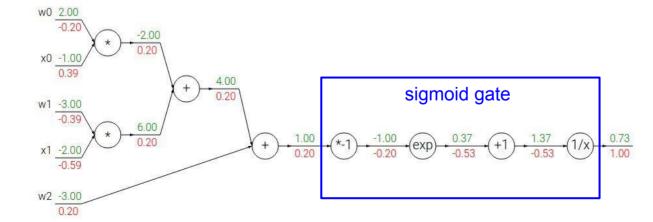
$$rac{dx}{df} = a$$

$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\sigma(x) = rac{1}{1+e^{-x}}$$

sigmoid function

$$rac{d\sigma(x)}{dx} = rac{e^{-x}}{(1+e^{-x})^2} = \left(rac{1+e^{-x}-1}{1+e^{-x}}
ight) \left(rac{1}{1+e^{-x}}
ight) = \left(1-\sigma(x)
ight)\sigma(x)$$



$$f(w,x)=rac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$f(x)=rac{1}{1+e^{-x}}$$
 sigmoid function

$$rac{d\sigma(x)}{dx} = rac{e^{-x}}{\left(1 + e^{-x}
ight)^2} = \left(rac{1 + e^{-x} - 1}{1 + e^{-x}}
ight) \left(rac{1}{1 + e^{-x}}
ight) = \left(1 - \sigma(x)
ight)\sigma(x)$$

