

For matrix of size $n \times n$ multiplying with other matrix of the same dimension.

Total floating point operation = $(n+n-1) * n^2$

$N = 64 \rightarrow 520192$

$N = 128 \rightarrow 4117920$

$N = 256 \rightarrow 33488896$

$N = 512 \rightarrow 268173312$

$N = 1024 \rightarrow 2146435072$

$N = 2048 \rightarrow 17175674880$

Problem 1.1

Here we are assuming no cache

Clock frequency 2GHz

For **dgemmo** we have (per innermost iteration)

- 2 load (200 cycles)
- 1 store (100 cycles)
- 2 floating point operations (1 cycle)

For $n=1000$ total iterations we will have is n^3 .

Hence, it will take $(200 + 100 + 1) * (10^9)$ cycles = $(301 * 10^9)$

Time spent on reading/writing (Converting this to seconds)

$$(300 * 10^9) / (2 * 10^9) = 150 \text{ sec}$$

For **dgemm1** we have ($n = 1000$)

per innermost iteration

- 2 load (200 cycles)
- 2 floating point operations (1 cycle)

2nd loop

- One load (100 cycle)
- One Store (100 cycle)

Inner loop runs n^3 times = $(200 + 1) * 10^9$

2nd loop runs n^2 times = $200 * (10^6)$

Time spent on reading/writing (Converting this to seconds)

$$((200) * (10^9) / 2 * 10^9) \text{ for inner loop}$$

$$(200 * 10^6) / (2 * 10^9) \text{ for 2nd loop}$$

$$\text{Total} = 100 + 0.1 = 100.1 \text{ sec}$$

Problem 1.2

For dgemm0 we try the values of n given there and we get the following output

...

```
n=64
time=0.002019s
n=128
time=0.017137s
n=256
time=0.202748s
n=512
time=2.310106s
n=1024
time=17.662696s
n=2048
time=252.926354s
```

...

Problem 2

For dgemm2 the output is

...

```
n=64
time=0.000262s
n=128
time=0.002180s
n=256
time=0.020356s
n=512
time=0.229089s
n=1024
time=2.095926s
n=2048
time=44.806179s
dgemm2
```

...

Converting to Gflops respectively it will be

- 1.985465649
- 1.888954128
- 1.645160935
- 1.170607546
- 1.024098691
- 0.3833327292

Problem 3

For dgemm3 the output is

...

```
n=64
time=0.000273s
n=128
time=0.002223s
n=256
time=0.023768s
n=512
time=0.242686s
n=1024
time=1.879007s
n=2048
time=46.796065s
```

...

Converting to Gflops respectively it will be

- 1.905465201
- 1.852415655
- 1.408990912
- 1.105021765
- 1.142324149
- 0.3670324605

Somewhat unexpected decrease in performance as compared to dgemm3 that is because I decreased one register named “size” which was used to store the value of n and use every time instead of integer n, so overall register usage decreased but performance slightly downgraded. **Implemented dgemm3 in 10 registers**

Problem 4

Here “total cache miss” == “total read cache miss”

ljk

Total cache miss for A in one iteration of the innermost loop = $n/10$

Total cache miss for B in one iteration of the innermost loop = $n/10$

Total write request = n^2

Total Read attempts = $2 \cdot (n^3)$

Hence total cache miss = $(2n/10) \cdot (n^2)$

Jki

Total cache miss for A in one iteration of the innermost loop = $n/10$

Total cache miss for B in one iteration of the innermost loop = 0

Total cache miss for C in one iteration of the innermost loop = $n/10$

Total write request = n^3

Hence total cache miss = $(2n/10) \cdot (n^2)$

Total Read attempts = $2 \cdot (n^3) + n^2$

Kji

Total cache miss for A in one iteration of the innermost loop = $n/10$

Total cache miss for B in one iteration of the innermost loop = 0

Total cache miss for A in one iteration of the innermost loop = $n/10$

Total write request = n^3

Hence total cache miss = $(2n/10) * (n^2)$

Total read attempts = $2*(n^3) + n^2$

lkj

Total cache miss for A in one iteration of the innermost loop = 0

Total cache miss for B in one iteration of the innermost loop = $n/10$

Total cache miss for C in one iteration of the innermost loop = $n/10$

Total write request = n^3

Total read attempts = $2*(n^3) + n^2$

Hence total cache miss = $(2n/10) * (n^2)$

Kij

Total cache miss for A in one iteration of the innermost loop = 0

Total cache miss for B in one iteration of the innermost loop = $n/10$

Total cache miss for C in one iteration of the innermost loop = $n/10$

Total write requests = n^3

Total read attempts = $2*(n^3) + n^2$

Hence total cache miss = $(2n/10) * (n^2)$

Jik

Total cache miss for A in one iteration of the innermost loop = $n/10$

Total cache miss for B in one iteration of the innermost loop = $n/10$

Total Write request = n^2

Total read attempts = $2(n^3)$

Hence total cache miss = $(2n/10) * (n^2)$

If $n/10$ has quotient 0, then just round it of to 1 (no remainder)

Problem 5

Box size = 100

Matrix size = 10000^2

Cache line = 10 double

Cache size = 60 lines

We can easily see that $3*100 < 60 * 100$

Whole box will fit inside the cache

Total boxes in one iteration = $2 * (n/B)$

For ijk

Miss in one cycle = $(B^2) / 10$ for each block (box A and box B)

Miss in one cycle in box C = 0

Hence total = $((B^2) / 10) * (2n/B)$

For jki

Miss in box A = $B^2/10$

Miss in Box C = $B^2/10$

Miss in Box B = 0

Hence total = $((B^2) / 10) * (2n/B)$

For kji

Miss in box A = $B^2/10$

Miss in Box C = $B^2/10$

Miss in Box B = 0

Hence total = $((B^2) / 10) * (2n/B)$

For ikj

Miss in box A = 0

Miss in box B = $B/10$

Miss in box C = $B/10$

Hence total = $((B^2) / 10) * (2n/B)$

For kij

Miss in box A = 0

Miss in box B = $B/10$

Miss in box C = $B/10$

Hence total = $((B^2) / 10) * (2n/B)$

For jik

Miss in box A = $B/10$

Miss in Box B = $B/10$

Miss in box C = 0

Hence total = $((B^2) / 10) * (2n/B)$

Problem 6

For non matrix operations we have the following output

...

```
n=2048
time=186.109614s
ijk
n=2048
time=265.962940s
jki
n=2048
time=153.756097s
jik
n=2048
time=255.656537s
kji
n=2048
time=26.420366s
kij
n=2048
time=26.246056s
lkj
```

...

We see very less time for the last 2 (kij and ikj) which is natural as they had the least number of load/store operations

For matrix operations we have the following output for box size 256 * 256

...

```
n=2048
time=59.165108s
ijk2
n=2048
time=69.910760s
jki2
n=2048
time=82.598219s
jik2
n=2048
time=75.923451s
kji2
n=2048
time=27.186330s
kij2
n=2048
time=36.770345s
ikj2
```

...
We see a lot of improvement obviously.

Now if we change the box size to 512*512 we get increased time duration

...

```
n=2048
time=89.242343s
ijk2
n=2048
time=143.001556s
jki2
n=2048
time=102.204625s
jik2
n=2048
time=116.107644s
kji2
n=2048
time=47.113896s
kij2
n=2048
time=43.177031s
ikj2
```

...

Problem 7

For using block size = 2 we get

...

```
n=2048
time=7.465826s
```

...

For using block size = 512 we get

...

```
n=2048
time=3.614114s
```

...

I did not go beyond this as if we go beyond this we will need to introduce padding as our parameter and padding will increase the computation unnecessarily