Name: _____ Student ID: _____

HW 1

CS 205 - 2024 Winter

Natural Deduction — Prove the following 4 tautologies. Label ALL inference rules (such as ¬E or →I) using the provided inference system, use assumption elimination marks appropriately. Do not use an alternative deduction system from another class (e.g., PHIL 124).

$\neg\,(A \wedge \neg A)$	$(A \wedge B) \rightarrow (B \wedge A)$
$\neg\,(\neg A) \rightarrow A$	$(A \vee (B \wedge A)) \rightarrow A$

For extra practice (ungraded):

$A \rightarrow ((A \rightarrow B) \rightarrow B)$	$(A \vee \bot) \rightarrow A$	$A \vee \neg A$

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$$\dfrac{\dfrac{}{A}\,1}{\begin{array}{c}\vdots\\ B\end{array}}{\scriptstyle 1}\ \to\mathrm{I} \qquad \dfrac{A \to B \qquad A}{B}\ \to\mathrm{E}$$

Conjunction:

$$\dfrac{A \qquad B}{A \wedge B}\ \wedge\mathrm{I} \qquad \dfrac{A \wedge B}{A}\ \wedge\mathrm{E_l} \qquad \dfrac{A \wedge B}{B}\ \wedge\mathrm{E_r}$$

Negation:

$$\dfrac{\dfrac{}{A}\,1}{\begin{array}{c}\vdots\\ \bot\end{array}}{\scriptstyle 1}\ \neg\mathrm{I} \qquad \dfrac{\neg A \qquad A}{\bot}\ \neg\mathrm{E}$$

Reductio ad absurdum (proof by contradiction):

$$\dfrac{\dfrac{}{\neg A}\,1}{\begin{array}{c}\vdots\\ \bot\end{array}}{\scriptstyle 1}\ \mathrm{RAA}$$

Disjunction:

$$\dfrac{A}{A \vee B}\ \vee\mathrm{I_l} \qquad \dfrac{B}{A \vee B}\ \vee\mathrm{I_r} \qquad \dfrac{A \vee B \qquad \begin{array}{c}\dfrac{}{A}\,1\\ \vdots\\ C\end{array} \qquad \begin{array}{c}\dfrac{}{B}\,1\\ \vdots\\ C\end{array}}{C}\ 1\ \vee\mathrm{E}$$

Truth and falsity:

$$\dfrac{\bot}{A}\ \bot\mathrm{E} \qquad \dfrac{}{\top}\ \top\mathrm{I}$$

Bi-implication:

$$\dfrac{\begin{array}{cc}\dfrac{}{A}\,1 & \dfrac{}{B}\,1\\ \vdots & \vdots\\ B & A\end{array}}{A \leftrightarrow B}\ 1\ \leftrightarrow\mathrm{I} \qquad \dfrac{A \leftrightarrow B \qquad A}{B}\ \leftrightarrow\mathrm{E}_l \qquad \dfrac{A \leftrightarrow B \qquad B}{A}\ \leftrightarrow\mathrm{E}_r$$