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HW 2

CS 205 - 2024 Winter

First Order Logic (FOL)

1. Write the following statement in FOL, using <u>only</u> the predicates listed, "you can fool some people all of the time, and all people some of the time, but not all of the people all of the time."

person(x) - x is a person
time(t) - t is a time
fools(x, y, t) - x fools y at time t
you - a constant, a person

~~$\exists Y\, \forall t [(person(Y) \land time(t))] \land fools(x, y, t) \land time(t)$~~

$\exists Y, \forall t \; person(Y) \land fools(x, y, t) \land time(t)$

$\land \exists t, \forall Y \; person(Y) \land fools(x, y, t) \land time(t)$

$\land \neg \; \forall Y, \forall t \; person(Y) \land time(t) \rightarrow fools(x, y, t)$

2. Use the following predicates (and standard equality constructs) to write "97 is prime" in FOL.

natural(x) - x is a natural number
product(x,y) - a function that yields the product of x and y
1, 97 - constants

$\forall x, \forall y \left(natural(x) \land natural(y) \land product(x, y) \right) \rightarrow (x=1, y=97) \lor (x=97, y=1)$

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3. Negate the following statement and then simplify it such that you only have negations of simple statements*. That is, you may not have negations in front of quantifiers or complex statements. Show your work, step by step.

$\forall x. (p(x) \rightarrow \exists y. (q(x) \wedge r(y)))$

$\forall x. (p(x) \rightarrow \exists y. (q(x) \wedge r(y)))$

$\downarrow$

$\forall x (\neg p(x) \vee \exists y. (q(x) \wedge r(y)))$

$\neg \forall x \neg p(x) \vee \exists y. (q(x) \wedge r(y))$

$\exists x (\neg\neg p(x) \wedge \neg (\exists y. q(x) \wedge r(y)))$

$\exists x (p(x) \wedge \forall y. \neg(q(x) \wedge r(y)))$

$\exists x . (p(x) \wedge \forall y. (\neg q(x) \vee \neg r(y)))$

*For example, ¬(∃y. (q(x) ∧ r(y)) is not simplified enough, whereas, ∀x. (q(x) ∧ ¬r(y)) is. There are some great slides and tutorials on how to simplify FOL expressions, as well as the textbook.

4. Next to each FOL formula, write the corresponding statement in natural language.

$\neg\exists x. \forall y. sees(x,y)$	No x sees all of the y.
$\forall x. \exists y. \neg sees(x,y)$	~~All of the x can't see any y~~. For all x there exists at least one y which x can't see
$\forall x. \neg\exists y. sees(x,y)$	All of the x can't see all of the y.

5. Very briefly state what's wrong with the following attempt for the statement "there are exactly two solutions" and then provide the correct one?

$\exists x. \exists y. solution(x) \wedge solution(y)$

Wrong: if $x=y$, then this ~~expression~~ statement'd be "there ~~are~~ is 1 solution".

the correct one: $\exists x, \exists y, solution(x) \wedge solution(y) \wedge (x \neq y \wedge \forall z, solution(z) \rightarrow (z = x \vee z=y))$

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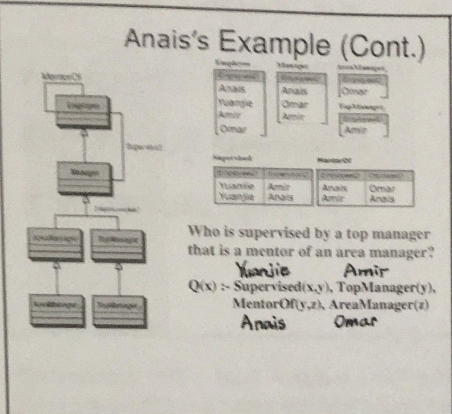
6. Explain very briefly and concisely why the substitution {x\Yuanjie} is a valid solution for Anai's example in this fictional object-relational model. You should mention the open world assumption.



The open world assumption (OWA): what's not known to be true or false might be true, or absence of information is interpreted as unknown information, not as negative information.

Case I Anais is Top Manager

Yuanjie is x
Anais is y [Top Manager]
Anais is mentor of Omar
Omar is z [Area Manager]

Case II Anais is Area Manager

Yuanjie is x
Anais is z [Area Manager]
Amir is y [Top Manager]
Amir is mentor of Anais

Both cases hold true

7. Explain very briefly and concisely why the following entailment is true, given the facts provided. That is, IOKASTE is such a person who satisfies the query. You may want to read more about the famous story of Oedipus, to help you understand this situation better.

hasChild(IOKASTE, OEDIPUS) hasChild(IOKASTE, POLYNEIKES) hasChild(OEDIPUS, POLYNEIKES) hasChild(POLYNEIKES, THERSANDROS) Patricide(OEDIPUS) ¬Patricide(THERSANDROS) Fig. 2.5. The Oedipus ABox $\mathcal{A}_{oe}$. $\mathcal{A}_{oe} \models (\exists hasChild.(Patricide \sqcap \exists hasChild.\neg Patricide))(IOKASTE)$?	Query: Is the one who has a child that satisfy the following: 1. is a patricide, 2. has a not-patricide child, IOKASTE? Reason: 1. We can see from the left that only ~~one~~ OEDIPUS is Patricide, and he's the child of IOKASTE. V. ~~2. We can OEDIPUS has 1 child (mentioned from the given information)~~ 2. THERSANDROS is the only given not patricide, and he's the child of IOKASTE. V 3. OEDIPUS and THERSANDROS don't have any other common parent, so IOKASTE is the only answer.

8. Every student but Samir smiles. Explain briefly why the following FOL is wrong and then fix it.

$\forall x\ ((student(x)\ \&\ x \neq Samir) \rightarrow smile(x))$

why wrong: we don't know if Samir smiles. Correct one:

$\forall x\ student(x) \rightarrow (x \neq Samir \leftrightarrow smile(x))$

(only students who are not Samir smile.)

another version for the correct one: $\forall x (student(x)) \rightarrow smile(x) \wedge (x \neq Samir)$