# CHAPTER 8

## QUANTITATIVE CONCEPTS

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## LEARNING OUTCOMES

After completing this chapter, you should be able to do the following:

**a** Define the concept of interest;

**b** Compare simple and compound interest;

**c** Define present value, future value, and discount rate;

**d** Describe how time and discount rate affect present and future values;

**e** Explain the relevance of net present value in valuing financial investments;

**f** Describe applications of time value of money;

**g** Explain uses of mean, median, and mode, which are measures of frequency or central tendency;

**h** Explain uses of range, percentile, standard deviation, and variance, which are measures of dispersion;

**i** Describe and interpret the characteristics of a normal distribution;

**j** Describe and interpret correlation.

## INTRODUCTION 1

Knowledge of quantitative (mathematically based) concepts is extremely important to understanding the world of finance and investing. Quantitative concepts play a role in financial decisions, such as saving and borrowing, and also form the foundation for valuing investment opportunities and assessing their risks. The time value of money and descriptive statistics are two important quantitative concepts. They are not directly related to each other, but we combine them in this chapter because they are key quantitative concepts used in finance and investment.

The time value of money is useful in many walks of life: it helps savers to know how long it will take them to afford a certain item and how much they will have to put aside each week or month, it helps investors to assess whether an investment should provide a satisfactory return, and it helps companies to determine whether the profit from investing will exceed the cost.

Statistics are also used in a wide range of business and personal contexts. As you attempt to assess the large amount of personal and work-related data that are part of our everyday lives, you will probably realise that an efficient summary and description of data is helpful to make sense of it. Most people, for instance, look at summaries of weather information to make decisions about how to dress and whether to carry an umbrella or bring rain gear. Summary statistics help you understand and use information in making decisions, including financial decisions. For example, summary information about a company's or market's performance can help in investment decisions.

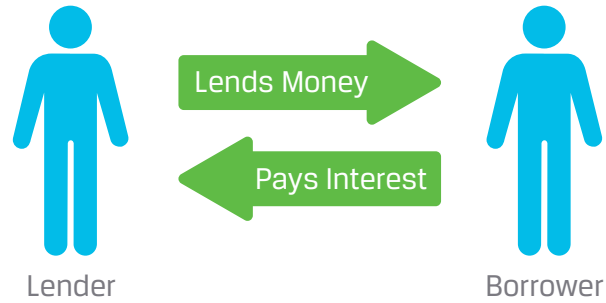
In short, quantitative concepts are fundamental to the investment industry. For anyone working in the industry, familiarity with the concepts described in this chapter is critical. **As always, you are *not* responsible for calculations, but the presentation of formulae and illustrative calculations may enhance your understanding.**

## TIME VALUE OF MONEY 2

Valuing cash flows, which occur over different periods, is an important issue in finance. You may be concerned with how much money you will have in the future (the future value) as a result of saving or investing over time. You may want to know how much you should save in a certain amount of time to accumulate a specified amount in the future. You may want to know what your expected return is on an investment with specified cash flows at different points in time. These types of problems occur every day in investments (e.g., in buying a bond), personal finance (e.g., in arranging an automobile loan or a mortgage), and corporate finance (e.g., in evaluating whether to build a factory). These problems are known as "time value of money" problems because their solutions reflect the principle that the timing of a cash flow affects the cash flow's value.

## 2.1 Interest

Borrowing and lending are transactions with cash flow consequences. Someone who needs money borrows it from someone who does not need it in the present (a saver) and is willing to lend it. In the present, the borrower has money and the lender has given up money. In the future, the borrower will give up money to pay back the lender; the lender will receive money as repayment from the borrower in the form of **interest**, as shown below. The lender will also receive back the money lent to the borrower. The money originally borrowed, which interest is calculated on, is called the **principal**. Interest can be defined as payment for the use of borrowed money.




Interest is all about timing: someone needs money now while someone else is willing and able to give up money now, but at a price. The borrower pays a price for not being able to wait to have money and to compensate the lender for giving up potential current consumption or other investment opportunities; that price is interest. Interest is paid by a borrower and earned by the lender to compensate the lender for opportunity cost and risk. **Opportunity cost**, in general, is the value of alternative opportunities that have been given up by the lender, including lending to others, investing elsewhere, or simply spending the money. Opportunity cost can also be seen as compensation for deferring consumption. Lending delays consumption by the term of the loan (the time over which the loan is repaid). The longer the consumption is deferred, the more compensation (higher interest) the lender will demand.

The lender also bears risks, such as the risk of not getting the money back if the borrower defaults (fails to make a promised payment). The riskier the borrower or the less certain the borrower's ability to repay the loan, the higher the level of interest demanded by the lender. Another risk is that as a result of inflation (an increase in prices of goods and services), the money received may not be worth as much as expected. In other words, a lender's purchasing power may decline even if the money is repaid as promised. The greater the expected inflation, the higher the level of interest demanded by the lender.

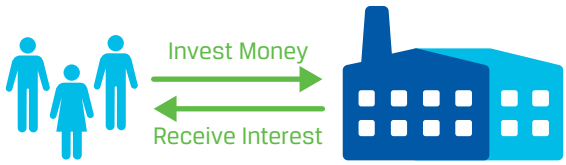
From the borrower's perspective, interest is the cost of having access to money that they would not otherwise have. An interest rate is determined by two factors: opportunity cost and risk. Even if a loan is viewed as riskless (zero likelihood of default), there still has to be compensation for the lender's opportunity cost and for expected inflation. Exhibit 1 shows examples of borrowers and lenders.

**Exhibit 1 Examples of Borrowers and Lenders**

Borrowers and lenders can be people, companies, financial institutions, and so on. Here are some examples of borrowers and lenders that you may be familiar with.


If you get money from the bank and pay interest, ***you are the borrower*** and ***the bank is the lender.***


If you put money into a savings account and earn interest, ***you are a saver*** who lends your savings to the bank and ***the bank is the borrower.***




If people invest in a company and earn interest by buying bonds, ***they are the lenders*** and ***the company is the borrower***.

### 2.1.1 *Simple Interest*

A **simple interest rate** is the cost to the borrower or the rate of return to the lender, per period, on the original principal (the amount borrowed). Conventionally, interest rates are stated as annual rates, so the period is assumed to be one year unless stated otherwise. The cost or return is stated as a percentage rate of the original principal so the rates can then be compared, regardless of the amount of principal they apply to. For example, a loan with a 5% interest rate is more expensive to the borrower than a loan with a 3% interest rate. Similarly, a loan with a 5% interest rate provides a higher promised return to the lender than a loan with a 3% interest rate.

The actual amount of interest earned or paid depends on the simple interest rate, the amount of principal lent or borrowed, and the number of periods over which it is lent or borrowed. We can show this mathematically as follows:

$$\text{Simple interest} = \text{Simple interest rate} \times \text{Principal} \times \text{Number of periods}$$

If you put money in a bank account and the bank offers a simple interest rate of 10% per annum (or annually), then for every £100 you put in, you (as a lender to the bank) will receive £10 in the course of the year (assume at year end to simplify calculations):

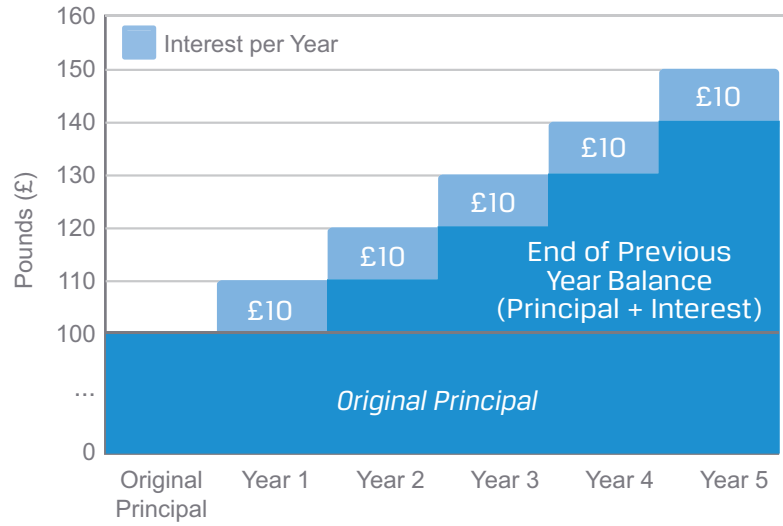
$$\text{Interest} = 0.10 \times £100 \times 1 = £10$$

If your money is left in the bank for two years, the interest paid will be £20:

$$\text{Interest} = 0.10 \times £100 \times 2 = £20$$

Simple interest is not reinvested and is applied only to the original principal, as shown in Exhibit 2.

**Exhibit 2 Simple Interest of 10% on £100 Original Principal**




If the interest earned is added to the original principal, the relationship between the original principal and its future value with simple interest can be described as follows:

Future value = Original principal × [1 + (Simple interest rate × Number of periods)]

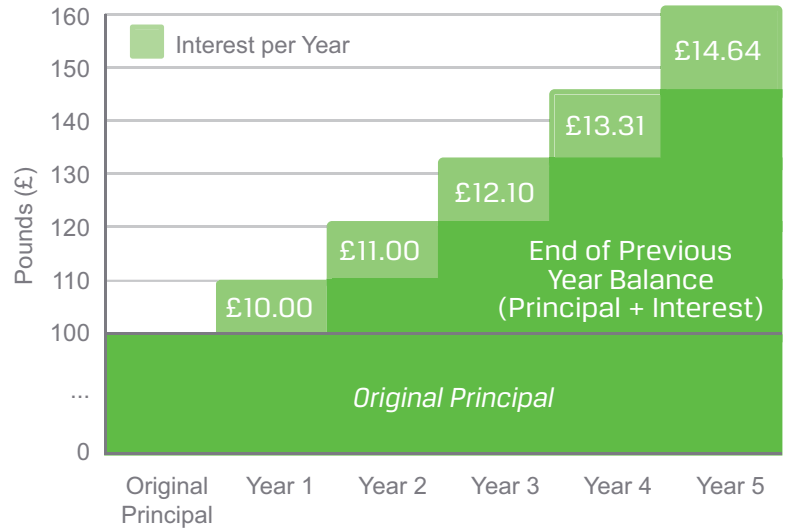
To extend our deposit example: £100 × [1 + (0.10 × 2)] = £100 × (1.20) = £120. The value at the end of two years is £120.

### 2.1.2 *Compound Interest*

Interest compounds when it is added to the original principal. **Compound interest** is often referred to as "interest on interest". As opposed to simple interest, interest is assumed to be reinvested so future interest is earned on principal and reinvested interest, not just on the original principal.

If a deposit of £100 is made and earns 10% and the money is reinvested (remains on deposit), then additional interest is earned in the course of the second year on the £10 of interest earned in the first year. The interest is being compounded. Total interest after two years will now be £21; £10 (= £100 × 0.10) for the first year, plus £11 (= £110 × 0.10) for the second year. The second year's interest is calculated on the original £100 principal plus the first year's interest of £10. As shown in Exhibit 3, the total interest after two years is £21 rather than £20 as in the case of simple interest shown in Exhibit 2.

**Exhibit 3 Compound Interest of 10% on £100 Original Principal**




The relationship between the original principal and its future value when interest is compounded can be described as follows:

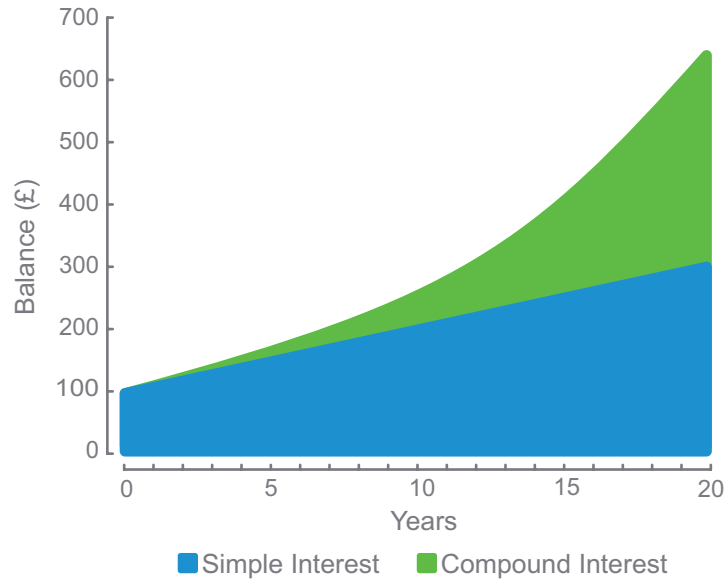
$$\text{Future value} = \text{Original principal} \times (1 + \text{Simple interest rate})^{\text{Number of periods}}$$

In the deposit example, £100 × $(1 + 0.10)^2$ = £100 × $(1.10)^2$ = £121. With compounding, the value at the end of two years is £121.

### 2.1.3 *Comparing Simple Interest and Compound Interest*

Compound interest is extremely powerful for savers; reinvesting the interest earned on investments is a way of growing savings. Somebody who invests £100 at 10% for two years will end up with £1 more by reinvesting the interest (£121) than with simple interest (£120). This amount may not look very impressive, but over a longer time period, say 20 years, £100 invested at 10% for 20 years becomes £300 with simple interest {£100 × [1 + (0.10 × 20)] = £100 × 3 = £300} but £673 with compound interest [£100 × $(1 + 0.10)^{20}$ = £100 × $(1.10)^{20}$ = £673]. This concept is illustrated in Exhibit 4.

**Exhibit 4 Effects on Savings of Simple and Compound Interest**




### 2.1.4 *Annual Percentage Rate and Effective Annual Rate*

Unless stated otherwise, interest rates are stated as annual rates. The rate quoted is often the **annual percentage rate** (APR), which is a simple interest rate that does not involve compounding. Another widely used rate is the **effective annual rate** (EAR). This rate involves annualising, through compounding, a rate that is paid more than once a year—usually monthly, quarterly, or semi-annually. The following equation shows how to determine the EAR given the APR.

$$\text{EAR} = \left[\left(1 + \frac{\text{APR}}{\text{Number of periods per year}}\right)^{\text{Number of periods per year}}\right] - 1$$

Example 1 shows a few types of financial products and their simple interest rates (APRs) and their compound rates (EARs).

**EXAMPLE 1. SIMPLE AND COMPOUND INTEREST RATES**

A credit card charges interest at an APR of 15.24%, compounded daily. A bank pays 0.2% monthly on the average amount on deposit over the month. A loan is made with a 6.0% annual rate, compounded quarterly. The following table shows what the expected annual rate is for each of these situations. The rate is higher than the APR because of compounding.

| | Simple Interest Rate or APR | Compound Interest Rate or EAR |
|---|---|---|
| Credit card | 15.24% | $16.46\% = \left[\left(1 + \frac{0.1524}{365}\right)^{365}\right] - 1$ |
| Bank deposit | 2.4% (= 0.2% × 12) | $2.43\% = \left[\left(1 + \frac{0.024}{12}\right)^{12}\right] - 1$ |
| Loan | 6.0% | $6.14\% = \left[\left(1 + \frac{0.06}{4}\right)^{4}\right] - 1$ |

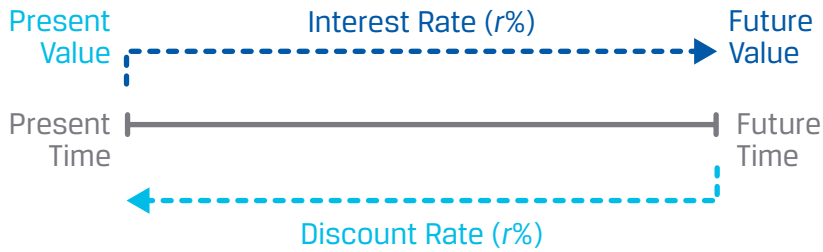
As can be seen in Example 1, in general, whenever an interest rate compounds more often than annually, the EAR is greater than the APR. In other words, more frequent compounding leads to a higher EAR.

## 2.2 Present Value and Future Value

Two basic time value of money problems are finding the value of a set of cash flows now (present value) and the value as of a point of time in the future (future value).

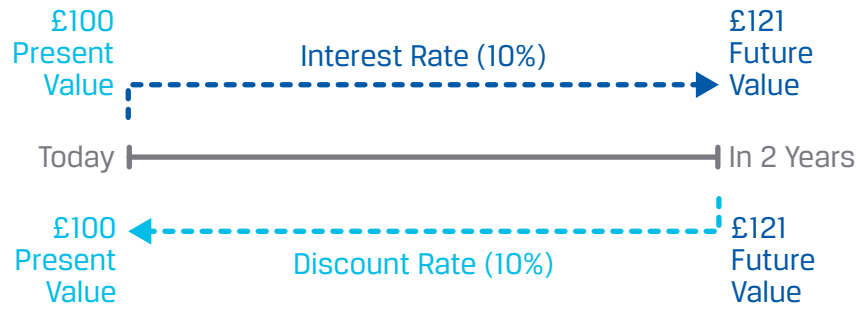
### 2.2.1 *Present Value and Future Value*

If you are offered £1 today or £1 in a year's time, which would you choose? Most people say £1 today because it gives them the choice of whether to spend or invest the money today and avoid the risk of never getting it at all. The £1 to be received in the future is worth less than £1 received today. The £1 to be received in the future is today worth £1 minus the opportunity cost and the risk of being without it for one year. The present value is obtained by discounting the future cash flow by the interest rate. The rate of interest in this context can be called the **discount rate**.




Time affects the value of money because delay creates opportunity costs and risk. If you earn a return of $r$% for waiting one year, £1 × (1 + $r$%) is the **future value** after one year of £1 invested today. Put another way, £1 is the **present value** of £1 × (1 + $r$%) received in a year's time.

A saver may want to know how much money is needed today to produce a certain sum in the future given the rate of interest, $r$. In the example in Exhibit 3, today's value is £100 and the interest rate is 10%, so the future value after two years is £100 × $(1 + 0.10)^2$ = £121. The present value—the equivalent value today—of £121 in two years, given that the annual interest rate is 10%, is £100.




Before you can calculate present or future values, you must know the appropriate interest or discount rates to use. The rate will usually depend on the overall level of interest rates in the economy, the opportunity cost, and the riskiness of the investments under consideration. The following equations generalise the calculation of future and present values:

$$\text{Future value} = \text{Present value} \times (1 + \text{Interest rate})^{\text{Number of periods}}$$

$$\text{Present value} = \frac{\text{Future value}}{(1 + \text{Discount rate})^{\text{Number of periods}}}$$

Note that the interest and discount rates are the same percentage rates, but the terminology varies based on context. Calculating present values allows investors and analysts to translate cash flows of different amounts and at different points in the future into sums in the present that can be compared with each other. Likewise, the cash flows can be translated into the values they would be equivalent to at a common future point.

Example 2 compares two investments with the same initial outflow (investment) but with different future cash inflows at different points in time.

## EXAMPLE 2. COMPARING INVESTMENTS

**1** You are choosing between two investments of equal risk. You believe that given the risk, the appropriate discount rate to use is 9%. Your initial investment (outflow) for each is £500. One investment is expected to pay out £1,000 three years from now; the other investment is expected to pay out £1,350 five years from now. To choose between the two investments, you must compare the value of each investment at the same point in time.

Present value of £1,000 in three years discounted at 9%

$$= \frac{£1{,}000}{(1.09)^3} = \frac{£1{,}000}{1.295} = £772.18$$

Present value of £1,350 in five years discounted at 9%

$$= \frac{£1{,}350}{(1.09)^5} = \frac{£1{,}350}{1.5386} = £877.41$$

As you can see, the investment with a payout of £1,350 five years from now is worth more in present value terms, so it is the better investment.

**2** You are choosing between the same two investments but you have reassessed their risks. You now consider the five-year investment to be more risky than the first and estimate that a 15% return is required to justify making this investment.

Present value of £1,350 in five years discounted at 15%

$$= \frac{£1{,}350}{(1.15)^5} = \frac{£1{,}350}{2.0114} = £671.19$$

The investment paying £1,000 in three years (discounted at 9%) is, in this case, preferable to the investment paying £1,350 in five years (discounted at 15%) in present value terms. Its present value of £771.18 is higher than the present value of £671.19 on the five-year investment.

Example 2 shows three elements that must be considered when comparing investments:

- the cash flows each investment will generate in the future,
- the timing of these cash flows, and
- the risk associated with each investment, which is reflected in the discount rate.

Present value considers the joint effect of these three elements and provides an effective way of comparing investments with different risks that have different future cash flows at different points in time.

### 2.2.2 *Net Present Value*

Present value is appropriate for comparing investments when the initial outflow for each investment is the same, as in Example 2. But investments may not have the same initial cash outflow, and outflows may occur at times other than time zero (the time of the initial outflow). The **net present value** (NPV) of an investment is the present value of future cash flows or returns minus the present value of the cost of the investment (which often, but not always, occurs only in the initial period). Using NPV rather than present value to evaluate investments is especially important when the investments have different initial costs. Example 3 below illustrates this.

**EXAMPLE 3. COMPARING INVESTMENTS USING NET PRESENT VALUE**

The NPV of the investment in Example 2 that is paying £1,350 in five years (discounted at 15%) if it initially cost £500 is:

£671.19 – £500.00 = £171.19

The NPV of the investment paying £1,000 in three years discounted at 9% if it initially cost £700 is:

£772.18 – £700 = £72.18.

This amount is less than £171.19, making the investment paying £1,350 in five years discounted at 15% worth more in present value terms. This conclusion differs from that reached when present value only was used.

If costs were to occur at times different from time zero, then they would also be discounted back to time zero for the purposes of comparison and calculation of the NPV. If the NPV is zero or greater, the investment is earning at least the discount rate. An NPV of less than zero indicates that the investment should not be made.

Calculating the NPV allows an investor to compare different investments using their projected cash flows and costs. The concepts of present value and net present value have widespread applications in the valuation of financial assets and products. For example, equities may pay dividends and/or be sold in the future, bonds may pay interest and principal in the future, and insurance may lead to future payouts.

Estimating values by using cash flows is also important to companies considering a range of investment opportunities. For example, should the sales team be supplied with tablets or laptops, or should the company open a new office in Asia or carry on visiting from the company's European headquarters? In order to choose, decision makers estimate the expected future cash flows of the alternatives available. The decision makers then discount the estimated cash flows by an appropriate discount rate that reflects the riskiness of these cash flows. They work out the discounted cash flows for each opportunity to estimate the value of the cash flows at the current time (the present value) and to arrive at the net present value. They then compare the net present values of all the opportunities and choose the opportunity or combination of opportunities with the largest positive net present value.

### 2.2.3 *Application of the Time Value of Money*

The time value of money concept can help to solve many common financial problems. If you save in a deposit account, it can tell you by how much your money will grow over a given number of years. Time value of money problems can involve both positive cash flows (inflows or savings) and negative cash flows (outflows or withdrawals). Example 4 illustrates, with two different sets of facts, how cash inflows and outflows affect future value.

**EXAMPLE 4. FUTURE VALUE**

**1** You place £1,000 on deposit at an annual interest rate of 10% and make regular contributions of £250 at the end of each of the next two years. How much do you have in your account at the end of two years?

| | | |
|---|---|---|
| The initial £1,000 becomes £1,000 × $(1 + 0.10)^2$ | = | £1,210 |
| The first annual £250 payment becomes £250 × (1+ 0.10) | = | £275 |
| The second annual payment is received at the end and earns no interest | = | £250 |
| The total future value | = | £1,735 |

**2** You place £1,000 on deposit and withdraw £250 at the end of the first year. The balance on deposit at the beginning of the year earns an annual interest rate of 10%. How much do you have in your account at the end of two years?

| | | |
|---|---|---|
| At the end of the first year, you have £1,000 × (1 + 0.10) | = | £1,100 |
| You withdraw £250 and begin the second year with an amount | = | £850 |
| At the end of the second year, you have £850 × (1 + 0.10) | = | £935 |

Time value of money can also help determine the value of a financial instrument. It can help you work out the value of an annuity or how long it will take to pay off the mortgage on your home.

**2.2.3.1 Present Value and the Valuation of Financial Instruments** People invest in financial products and instruments because they expect to get future benefits in the form of future cash flows. These cash flows can be in the form of income, such as dividends and interest, from the repayment of an amount lent, or from selling the financial product or instrument to someone else. An investor is exchanging a sum of money today for future cash flows, and some of these cash flows are more uncertain than others. The value (amount exchanged) today of a financial product should equal the value of its expected future cash flows. This concept is shown in Example 5.

## EXAMPLE 5. VALUE OF A LOAN

Consider the example of a simple loan that was made three years ago. Two years from today, the loan will mature and the borrower should repay the principal value of the loan, which is £100. The investor who buys (or owns) this loan should also receive from the borrower two annual interest payments at the originally promised interest rate of 8%. The interest payments will be £8 (= 8% × £100), with the first interest payment received a year from now and the second two years from now.

How much would an investor pay today to secure these two years of cash flow if the appropriate discount rate is 10% (i.e. $r = 0.10$)? Note that the rate used for discounting the future cash flows should reflect the risk of the investment and interest rates in the market. In practice, it is unlikely that the discount rate will be equal to the loan's originally promised interest rate because the risk of the investment and interest rates in the market may change over time.

The first year's interest payment is worth $\frac{£8}{1.10^1} = £7.27$.

The second year's interest payment is worth $\frac{£8}{1.10^2} = £6.61$.

The repayment of the loan's principal value in two years is worth $\frac{£100}{1.10^2} = £82.64$.

So today, the cash flows returned by the loan are worth £7.27 + £6.61 + £82.64 = £96.52. So this loan is worth £96.52 to the investor. In other words, if the original lender wanted to sell this loan, an investor would pay £96.52.

Through the understanding of present value and knowing how to calculate it, investors can assess whether the price of a financial instrument trading in the marketplace is priced cheaply, priced fairly, or overpriced.

**2.2.3.2 Time Value of Money and Regular Payments** Many kinds of financial arrangements involve regular payments over time. For example, most consumer loans, including mortgages, involve regular periodic payments to pay off the loan. Each period, some of the payment covers the interest on the loan and the rest of the payment pays off some of the principal (the loaned amount). A pension savings scheme or pension plan may also involve regular contributions.

Most consumer loans result in a final balance of money equal to zero. That is, the loan is paid off. Two time value of money applications that require the final balance of money to be zero are annuities and mortgages.

An **annuity** involves the initial payment of an amount, usually to an insurance company, in exchange for a fixed number of future payments of a certain amount. Each period, the insurance company makes payments to the annuity holder; these payments are equivalent to the annuity holder making withdrawals. These withdrawals can be viewed as negative cash flows because they reduce the annuity balance. The initial payment to the insurer is called the value of the annuity and the final value is equal to zero.

A repayment or amortising mortgage involves a loan and a series of fixed payments. The initial amount of the loan is referred to as the principal. Although the payment amounts are fixed, the portion of each payment that is interest is based on the remaining principal at the beginning of each period. As some of the principal is repaid each period, the amount of interest decreases over time, and thus the amount of principal repaid increases with each successive payment until the value of the principal is reduced to zero. At this point, the loan is said to mature.

Example 6 illustrates the reduction of an annuity to zero over time and the reduction of a mortgage to zero over time. To simplify the examples, the assumption is that the annuity and the mortgage each mature in five years and entail a single withdrawal or payment each of the five years.

**EXAMPLE 6. ANNUITY AND MORTGAGE**

**1** A retired French man pays an insurance company €10,000 in exchange for a promise by the insurance company to pay him €2,375 at the end of each of the next four years and €2,370 at the end of the fifth year. The insurance company is in effect paying him 6.0% interest on the annuity balance.

| Year | Annuity Balance at Beginning of Year | Balance at End of Year before Withdrawal | Withdrawal (Payment by Insurance Company) |
|---|---|---|---|
| 1 | €10,000 | €10,600 (= 10,000 × 1.06) | €2,375 |
| 2 | €8,225 | €8,719 (= 8,225 × 1.06) | €2,375 |
| 3 | €6,344 | €6,725 (= 6,344 × 1.06) | €2,375 |
| 4 | €4,350 | €4,611 (= 4,350 × 1.06) | €2,375 |
| 5 | €2,236 | €2,370 (= 2,236 × 1.06) | €2,370 |
| 6 | €0 | | |

**2** You borrow £60,000 to buy a small cottage in the country. The interest rate on the mortgage is 4.60%. Your payment at the end of each year will be £13,706.

| Year | Mortgage Outstanding at Beginning of Year | Total Mortgage Payment | Interest Paid | Principal Reduced |
|---|---|---|---|---|
| 1 | £60,000 | £13,706 | £2,760 (= 60,000 × 0.046) | £10,946 |
| 2 | £49,054 | £13,706 | £2,257 (= 49,054 × 0.046) | £11,449 |
| 3 | £37,605 | £13,706 | £1,730 (= 37,605 × 0.046) | £11,976 |
| 4 | £25,630 | £13,706 | £1,179 (= 25,630 × 0.046) | £12,527 |
| 5 | £13,103 | £13,706 | £603 (= 13,103 × 0.046) | £13,103 |
| 6 | £0 | | | |

As you can see in Example 6, both the annuity and mortgage balances decline to zero over time.

## DESCRIPTIVE STATISTICS 3

As the name suggests, descriptive statistics are used to describe data. Often, you are confronted by data that you need to organise in order to understand it. For example, you get the feeling that the drive home from work is getting slower and you are thinking of changing your route. How could you assess whether the journey really is getting slower? Suppose you calculated and compared the average daily commute time each month over a year. The first question you need to address is, what is meant by average? There are a number of different ways to calculate averages that are described in Section 3.1, each of which has advantages and disadvantages.

In general, descriptive statistics are numbers that summarise essential features of a data set. A data set relates to a particular variable—the time it takes to drive home from work in our example. The data set includes several observations—that is, observed values for the variable. For example, if you keep track of your daily commute time for a year, you will end up with approximately 250 observations. The **distribution** of a variable is the values a variable can take and the number of observations associated with each of these values.

We will discuss two types of descriptive statistics: those that describe the central tendency of a data set (e.g., the average or mean) and those that describe the dispersion or spread of the data (e.g., the standard deviation). In addition to knowing whether the drive to work is getting slower (by comparing monthly averages), you might also want to find a way to measure how much variation there is between journey times from one day to another (by using standard deviation).

Similar needs to summarise data arise in business. For example, when comparing the time taken to process two types of trades, a sample of the times required to process each trade would need to be collected. The average time it takes to process each type of trade could be calculated and the average times could then be compared. Descriptive statistics efficiently summarise the information from large quantities of data for the purpose of making comparisons. Descriptive statistics may also help in predicting future values and understanding risk. For example, if there was little variation in the times taken to process a trade, then presumably you would be confident that you had a good idea of the average time it takes to process a trade and comfortable with that as an estimate of how long it will take to process future trades. But if the time taken to process trades was highly variable, you would have less confidence in how long it would take on average to process future trades.

## 3.1 Measures of Frequency and Average

The purpose of measuring the frequency of outcomes or "central tendency" is to describe a group of individual data scores with a single measurement. The value used to describe the group will be the single value considered to be most representative of all the individual scores.

Measures of central tendency are useful for making comparisons between groups of individuals or between sets of figures. Such measures reduce a large number of measurements to a single figure. For instance, the mean or average temperature in country X in July from 1961 to 2011 is calculated to be 16.1°C. Over the same period in September, the average temperature is 13.6°C. Because it is a long time series, you can reasonably conclude that it is usually warmer in July than September in country X.

Common measures of central tendency are

- arithmetic mean,
- geometric mean,
- median, and
- mode.

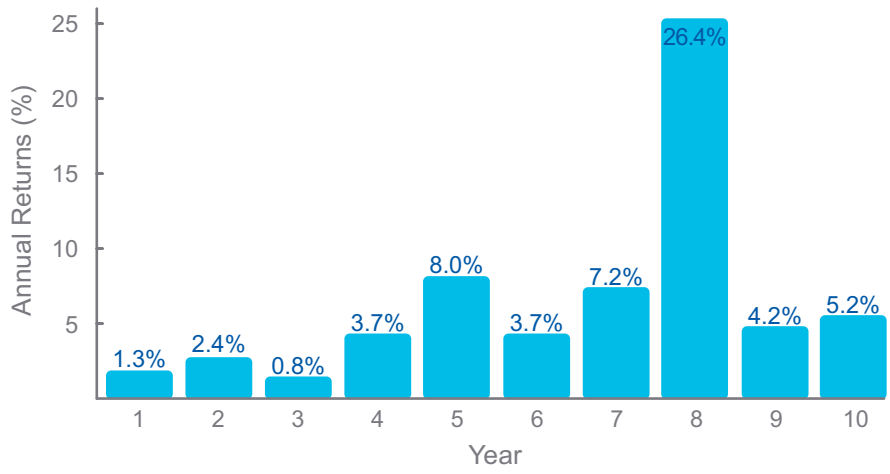
The appropriate measure for a given data set depends on the features of the data and the purpose of your calculation. These measures are examined in the following sections.

### 3.1.1 *Arithmetic Mean*

The **arithmetic mean** is the most commonly used measure of central tendency and is familiar to most people. It is usually shortened to just "mean" or "average". To calculate the mean, you add all the numbers in the data set together and divide by the number of observations (items in the data set). The arithmetic mean assumes that each observation is equally probable (likely to occur). If each observation is not equally probable, you can get a weighted mean by multiplying the value of each observation by its probability and then summing these values. The sum of the probabilities always equals 1.

Exhibit 5 shows the annual returns earned on an investment over a 10-year period. The information contained in Exhibit 5 will be used in examples throughout this section.

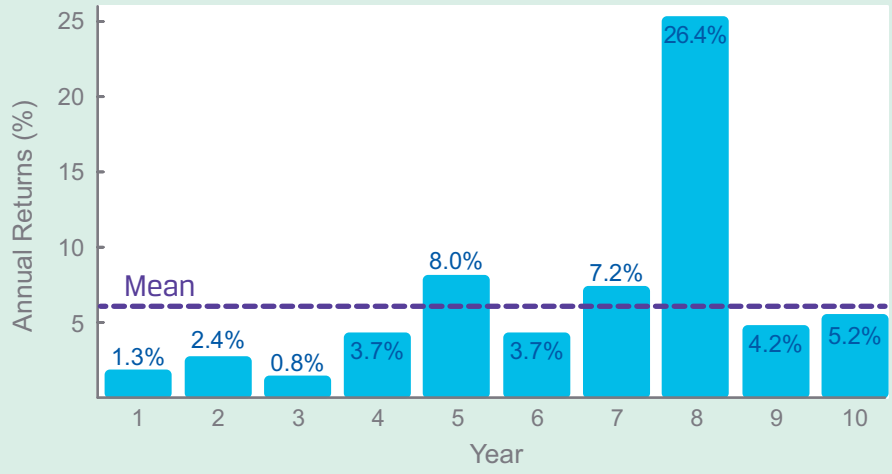
**Exhibit 5 Ten Years of Annual Returns**




Example 7 shows the calculation of the arithmetic mean.

**EXAMPLE 7. ARITHMETIC MEAN**

An investment earns the returns shown in Exhibit 5 over a 10-year period.




$$\frac{(1.3 + 2.4 + 0.8 + 3.7 + 8.0 + 3.7 + 7.2 + 26.4 + 4.2 + 5.2)}{10} = \textbf{6.3\% Mean}$$

The arithmetic mean return or average annual return over the 10-year period is 6.3%. The weighted mean return (shown in the following equation) is the same as the arithmetic return because the probability assigned to each return is the same: 10% or 0.1.

Weighted mean annual return

$$= (0.1 \times 1.3) + (0.1 \times 2.4) + (0.1 \times 0.8) + (0.1 \times 3.7) + (0.1 \times 8.0) + (0.1 \times 3.7) + (0.1 \times 7.2) + (0.1 \times 26.4) + (0.1 \times 4.2) + (0.1 \times 5.2)$$

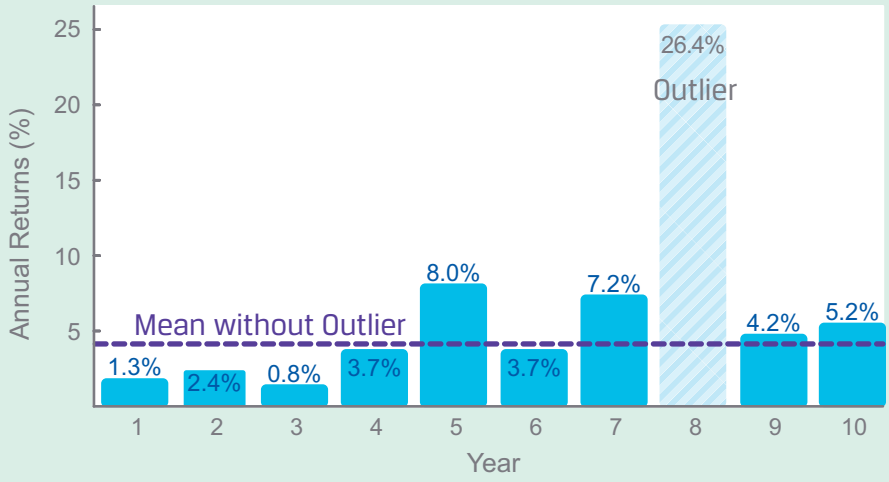
$$= 6.3\%$$

The arithmetic mean annual return is 6.3%.

The mean has one main disadvantage: it is particularly susceptible to the influence of **outliers**. These are values that are unusual compared with the rest of the data set by being especially small or large in numerical value. The arithmetic mean is not very representative of the whole set of observations when there are outliers. Example 8 shows the effect of excluding an outlier from the calculation of the arithmetic mean.

## EXAMPLE 8. EFFECT OF OUTLIER ON ARITHMETIC MEAN

In the case of the annual returns in Exhibit 5, there is one value—26.4%—that is much larger than the others. If this value is included, the mean is 6.3%, but excluding this value reduces the mean to 4.1%.



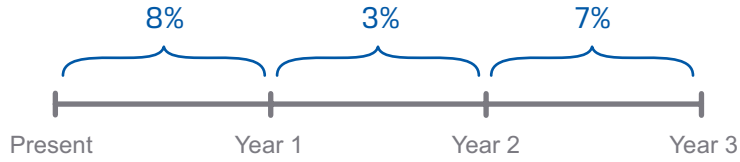

$$\frac{(1.3 + 2.4 + 0.8 + 3.7 + 8.0 + 3.7 + 7.2 + 4.2 + 5.2)}{9} = \textbf{4.1\% Mean}$$

The arithmetic mean excluding the outlier is 4.1%.

Including the outlier, the mean is dragged in the direction of the outlier. When there are one or more outliers in a set of data in one direction, the data are said to be **skewed** in that direction. In Example 7, ordering data so larger numbers are to the right of smaller numbers, 26.4% lies to the right of the other data. Thus, the data are said to be right skewed (or positively skewed). Other measures of central tendency may better accommodate outliers.

### 3.1.2 *Geometric Mean*

An alternative average to the arithmetic mean is the **geometric average** or **geometric mean**. Applied to investment returns, the geometric mean return is the average return assuming that returns are compounding. To illustrate how the geometric mean is calculated, let us start with the example of a three-year investment that returns 8% the first year, 3% the second year, and 7% the third year.




$$[(1 + 8\%) \times (1 + 3\%) \times (1 + 7\%)]^{1/3} - 1 \approx \textbf{6.0\%}$$

The first step to calculate the geometric mean return is to multiply 1 plus each annual return and add them together, which gives you the amount you would have accumulated at the end of the three years per currency unit of investment: [(1 + 8%) × (1 + 3%) × (1 + 7%) ≈ 1.1903]. This value of 1.1903 reflects three years of investment, but the geometric mean return should capture an average rate of return for each of the three years. So, the second step requires moving from three years to one by raising the accumulation to the power of "one over the number of periods held," three in this particular case; this calculation can also be described as taking "the number of

periods held" root of the value ($1.1903^{1/3} \approx 1.060$). This value of 1.060 includes both the original investment and the average yearly return on the investment each year (1 plus the geometric mean return). The last step is, therefore, to subtract 1 from this value to arrive at the return that would have to be earned on average each year to get to the total accumulation over the three years ($1.060 - 1 \approx 0.060$ or 6.0%). The geometric mean return is 6.0%, which in this case is the same as the arithmetic mean return. Geometric mean is frequently the preferred measure for the investment industry.

The following formula is used to arrive at the geometric mean return:

$$\text{Geometric mean return} = \left[\left(1 + r_1\right) \times \ldots \left(1 + r_t\right)\right]^{1/t} - 1$$

where

$r_i$ = the return in period $i$ expressed using decimals

$t$ = the number of periods

Example 9 shows the calculation of the geometric mean return for the investment of Exhibit 5.

**EXAMPLE 9. GEOMETRIC MEAN RETURN**

If 1 currency unit was invested, you would have 1.8 currency units at the end of the 10 years.

Total accumulation after 10 years

= [(1 + 1.3%) × (1 + 2.4%) × (1 + 0.8%) × (1 + 3.7%) × (1 + 8.0%) × (1 + 3.7%) × (1 + 7.2%) × (1 + 26.4%) × (1 + 4.2%) × (1 + 5.2%)]

= [(1.013) × (1.024) × (1.008) × (1.037) × (1.08) × (1.037) × (1.072) × (1.264) × (1.042) × (1.052)]

= 1.8

Average accumulation per year = 10th root of 1.8 = $(1.8)^{1/10}$ = 1.061

Geometric mean annual return = 1.061 − 1 = 0.061 = 6.1%

This can also be done as one calculation:

Geometric mean annual return

= {[(1 + 1.3%) × (1 + 2.4%) × (1 + 0.8%) × (1 + 3.7%) × (1 + 8.0%) × (1 + 3.7%) × (1 + 7.2%) × (1 + 26.4%) × (1 + 4.2%) × (1 + 5.2%)]$^{(1/10)}$} − 1

= 6.1%

The geometric mean annual return is 6.1%. One currency unit invested for 10 years and earning 6.1% per year would accumulate to approximately 1.8 units.

An important aspect to notice is that the geometric mean is lower than the arithmetic mean even though the annual returns over the 10-year holding period are identical. This result is because the returns are compounded when calculating the geometric

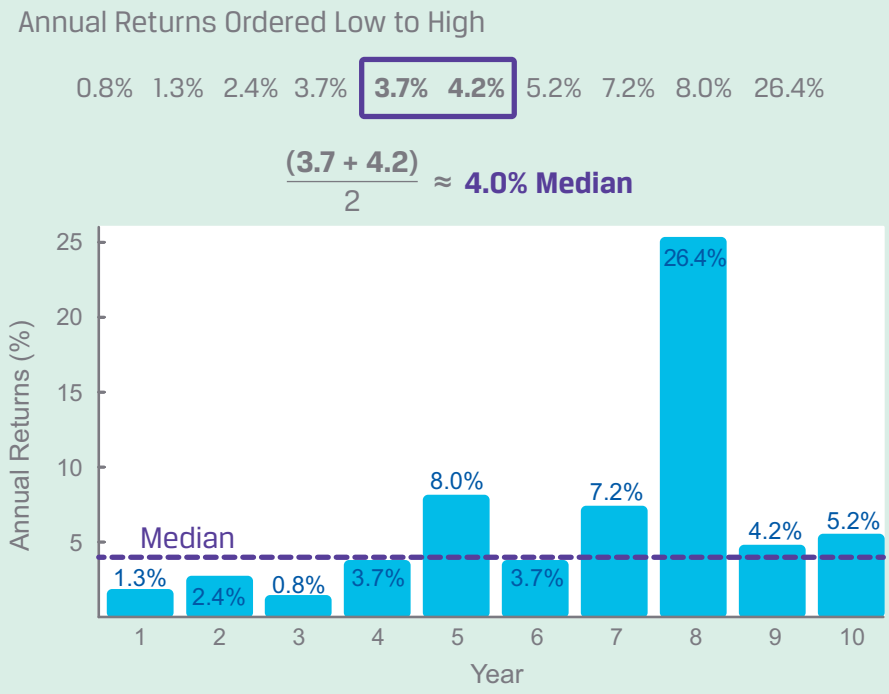
mean return. Recall that compounding will result in a higher value over time, so a lower rate of return is required to reach the same amount. In fact, if the same set of numbers is used to calculate both means, the geometric mean return is never greater than the arithmetic mean return and is normally lower.

### 3.1.3 *Median*

If you put data in ascending order of size from the smallest to the largest, the **median** is the middle value. If there is an even number of items in a data set, then you average the two middle observations. Hence, in many cases (i.e., when the sample size is odd or when the two middle-ranked items of an even-numbered data set are the same) the median will be a number that actually occurs in the data set. Example 10 shows the calculation of the median for the investment of Exhibit 5.

**EXAMPLE 10. MEDIAN**

When the returns are ordered from low to high, the median value is the arithmetic mean of the fifth and sixth ordered observations.




The median investment return over the 10-year period is 4.0%.

An advantage of the median over the mean is that it is not sensitive to outliers. In the case of the annual returns shown in Exhibit 5, the median of close to 4.0% is more representative of the data's central tendency. This 4.0% median return is close to the 4.1% arithmetic mean return when the outlier is excluded. The median is usually a better measure of central tendency than the mean when the data are skewed.

### 3.1.4 *Mode*

The **mode** is the most frequently occurring value in a data set. Example 11 shows how the mode is determined for the investment of Exhibit 5.

**EXAMPLE 11. MODE**

Looking at Exhibit 5, we see that one value occurs twice, 3.7%. This value is the mode of the data.

Annual Returns Ordered Low to High

0.8% 1.3% 2.4% **3.7% 3.7%** 4.2% 5.2% 7.2% 8.0% 26.4%

**3.7% Mode**

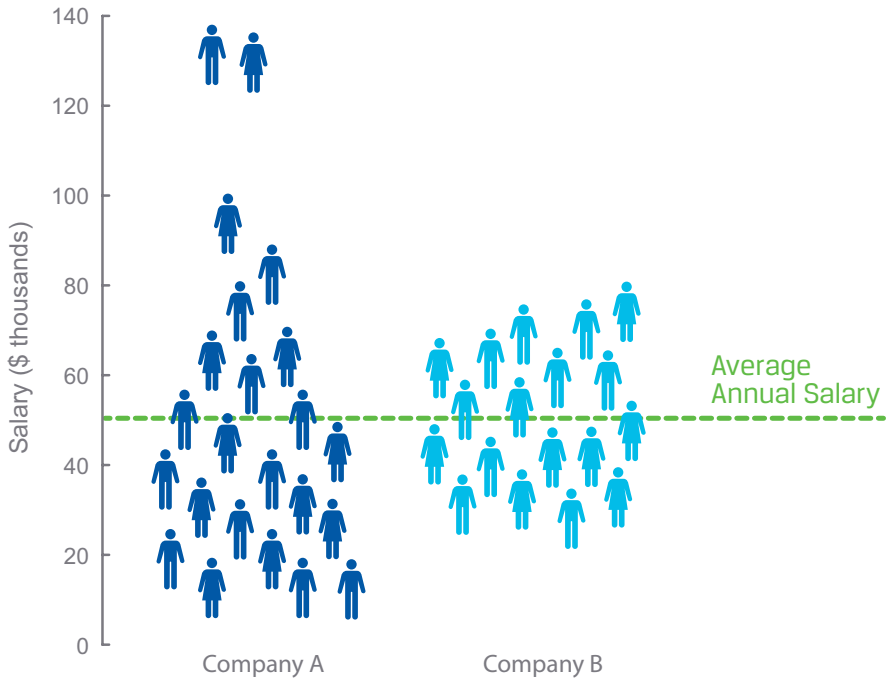
The mode can be used as a measure of central tendency for data that have been sorted into categories or groups. For example, if all the employees in a company were asked what form of transportation they used to get to work each day, it would be possible to group the answers into categories, such as car, bus, train, bicycle, and walking. The category with the highest number would be the mode.

A problem with the mode is that it is often not unique, in which case there is no mode. If there are two or more values that share the same frequency of occurrence, there is no agreed method to choose the representative value. The mode may also be difficult to compute if the data are continuous. **Continuous data** are data that can take on an infinite number of values between whole numbers—for example, weights of people. One person may weigh 62.435 kilos and another 62.346 kilos. By contrast, **discrete data** show observations only as distinct values—for example, the number of people employed at different companies. The number of people employed will be a whole number. For continuous data, it is less likely that any observation will occur more frequently than once, so the mode is generally not used for identifying central tendency for continuous data.

Another problem with the mode is that the most frequently occurring observation may be far away from the rest of the observations and does not meaningfully represent them.

## 3.2 Measures of Dispersion

Whereas measures of central tendency are used to estimate representative or central values of a set of data, measures of dispersion are important for describing the spread of the data or its variation around a central value. Two data sets may have the same mean or median but completely different levels of variability, or vice versa. A description of a data set should include both a measure of central tendency, such as the mean, and a measure of dispersion. Suppose two companies both have an average annual salary of $50,000, but in one company most salaries are clustered close to the average, whereas in the second they are spread out with many people earning very little and some earning a lot. It would be useful to have a measure of dispersion that can help identify such differences between data sets.




Another reason why measures of dispersion are important in finance is that investment risk is often measured using some measure of variability. When investors are considering investing in a security, they are interested in the likely (expected) return on that investment as well as in the risk that the return could differ from the expected return (its variability). A risk-averse investor considering two investments that have similar expected returns but very different measures of variability (risk) around those expected returns, typically prefers the security with the lower variability.

Two common measures of dispersion of a data set are the range and the standard deviation.

### 3.2.1 *Range*

The **range** is the difference between the highest and lowest values in a data set. It is the easiest measure of dispersion to calculate and understand, but it is very sensitive to outliers. Example 12 explains the calculation of the range of returns for the investment of Exhibit 5.

**EXAMPLE 12. RANGE**

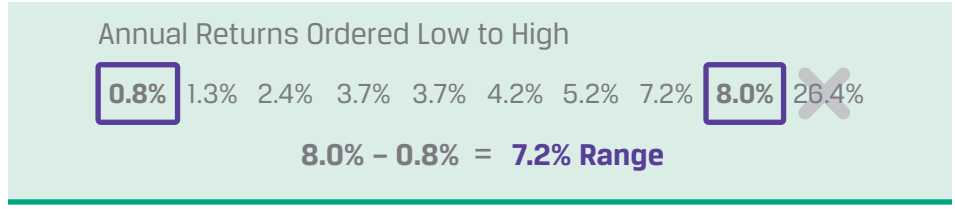
In Exhibit 5 we see that the highest annual return is 26.4% and the lowest annual return is 0.8%.

Annual Returns Ordered Low to High

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **0.8%** | 1.3% | 2.4% | 3.7% | 3.7% | 4.2% | 5.2% | 7.2% | 8.0% | **26.4%** |

**26.4% – 0.8% = 25.6% Range**

If the extreme value at the upper end of the range is excluded, the next highest value, 8.0%, is used to estimate the range, and the range is reduced significantly.




Clearly, the range is affected by extreme values and, if there are outliers, it says little about the distribution of the data between those extremes.

If there are a large number of observations ranked in order of size, the range can be divided into 100 equal-sized intervals. The dividing points are termed percentiles. The 50th percentile is the median and divides the observations so that 50% are higher and 50% are lower than the median. The 20th percentile is the value below which 20% of observations in the series fall. So, the dispersion of the observations can be described in terms of percentiles. Observations can be divided into other equal-sized intervals. Commonly used intervals are quartiles (the observations are divided into four equal-sized intervals) and deciles (the observations are divided into 10 equal-sized intervals)

### 3.2.2 *Standard Deviation*

A commonly used measure of dispersion is the **standard deviation**. It measures the variability or volatility of a data set around the average value (the arithmetic mean) of that data set. Although, as mentioned before, you are not responsible for any calculations, you may find it helpful to look at the formula for how standard deviation is calculated.

$$\text{Standard deviation} = \sqrt{\frac{[X_1 - E(X)]^2 + [X_2 - E(X)]^2 + \ldots + [X_n - E(X)]^2}{n}}$$

where

$X_i$ = observation $i$ (one of $n$ possible outcomes for $X$)

$n$ = number of observations of $X$

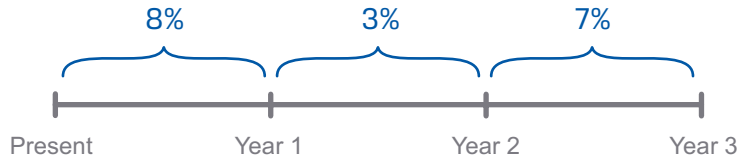
$E(X)$ = the mean (average) value of $X$ or the expected value of $X$

$[X_i - E(X)]$ = difference between value of observation $X_i$ and the mean value of $X$

The differences between the observed values of $X$ and the mean value of $X$ capture the variability of $X$. These differences are squared and summed. Note that because the differences are squared, what matters is the size of the difference not the sign of the difference. The sum is then divided by the number of observations. Finally, the square root of this value is taken to get the standard deviation.

The value before the square root is taken is known as the **variance**, which is another measure of dispersion. The standard deviation is the square root of the variance. The standard deviation and the variance capture the same thing—how far away from the mean the observations are. The advantage of the standard deviation is that it is expressed in the same unit as the mean. For example, if the mean is expressed as minutes of journey time, the standard deviation will also be expressed as minutes, whereas the variance will be expressed as minutes squared, making the standard deviation an easier measure to use and compare with the mean.

To illustrate the calculation of the standard deviation, let us return to the example of a three-year investment that returns 8% or 0.08 the first year, 3% or 0.03 the second year, and 7% or 0.07 the third year. The arithmetic mean return is 6% or 0.06. The standard deviation is approximately 2.16%.




$$\text{Standard deviation} = \sqrt{\frac{(0.08 - 0.06)^2 + (0.03 - 0.06)^2 + (0.07 - 0.06)^2}{3}}$$

$$= \sqrt{\frac{(0.02)^2 + (-0.03)^2 + (0.01)^2}{3}}$$

$$= \sqrt{\frac{(0.0004) + (0.0009) + (0.0001)}{3}}$$

$$= \sqrt{\frac{(0.0014)}{3}} = 0.0216 = 2.16\%$$

Example 13 shows the calculation of the standard deviation for the investment in Exhibit 5.

**EXAMPLE 13. STANDARD DEVIATION**

The arithmetic mean annual return, as calculated in Example 7, is 6.3%.

Standard deviation

= square root of {[(0.013 – 0.063)$^2$ + (0.024 – 0.063)$^2$ + (0.008 – 0.063)$^2$ + (0.037 – 0.063)$^2$ + (0.08 – 0.063)$^2$ + (0.037 – 0.063)$^2$ + (0.072 – 0.063)$^2$ + (0.264 – 0.063)$^2$ + (0.042 – 0.063)$^2$ + (0.052 – 0.063)$^2$] ÷ 10}

= square root of [(0.0025 + 0.0015 + 0.0030 + 0.0007 + 0.0003 + 0.0007 + 0.0001 + .0404 + 0.0004 + 0.0001) ÷ 10]

= square root of 0.00497

= 0.0705, rounded to the nearest 10th percent = 7.1% (this value is used in Example 14).

The standard deviation is 7.1%.

Larger values of standard deviation relative to the mean indicate greater variation in a data set. Also, by using standard deviation, you can determine how likely it is that any given observation will occur based on its distance from the mean. Example 14 compares the returns of the investment shown in Exhibit 5 and the returns on another investment over the same period using mean and standard deviation.

**EXAMPLE 14. COMPARISON OF INVESTMENTS**

An investment earns the returns shown in Exhibit 5 over a 10-year period:

Number of observations = 10

Mean = 6.3%

Standard deviation = 7.1%

Another investment over the same time period has the following characteristics:

Number of observations = 10

Mean = 6.5%

Standard deviation = 2.6%

Based on mean and standard deviation, the second investment is better than the first investment. It has a higher mean return and less variability, which implies less risk, in its returns.

### 3.2.3 *Normal Distribution*

The arithmetic mean and standard deviation are two powerful ways of describing many distributions of data. A distribution is simply the set of values that a variable can take, showing their observed or theoretical frequency of occurrence. For example, consider the distribution of salaries earned by employees in two companies as shown in Exhibit 6. The observations in these distributions are grouped into different salary ranges.

**Exhibit 6 Number of Employees in Various Salary Ranges**

| | Number of Employees | |
|---|---|---|
| Salary ($) | Company X | Company Y |
| 15,000–20,000 | 5 | 1 |
| 20,001–25,000 | 8 | 1 |
| 25,001–30,000 | 20 | 3 |
| 30,001–35,000 | 30 | 8 |
| 35,001–40,000 | 22 | 10 |
| 40,001–45,000 | 12 | 15 |
| 45,001–50,000 | 6 | 20 |
| 50,001–55,000 | 2 | 9 |
| 55,001–60,000 | 1 | 7 |

Sometimes it is helpful to look at a picture of the distribution to understand it. The shape of the distribution has a bearing on how you interpret the summary measures of the distribution. This data can be shown pictorially using a **histogram**—a bar chart with bars that are proportional to the frequency of occurrence of each group of observations—as shown in Exhibits 7A and 7B.

**Exhibit 7A Salaries of Employees at Company X**

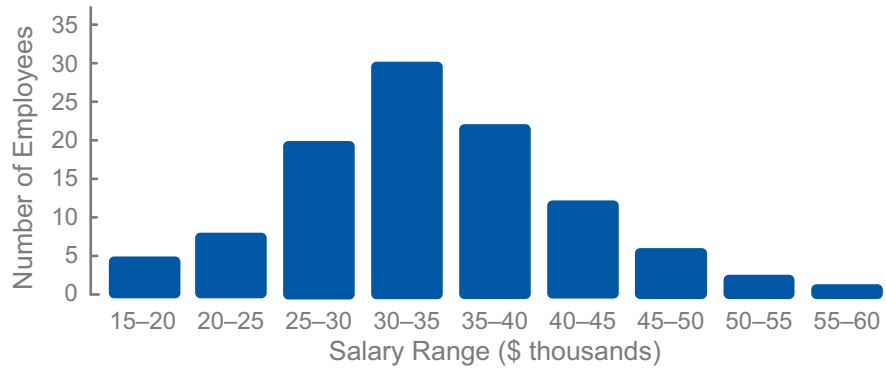
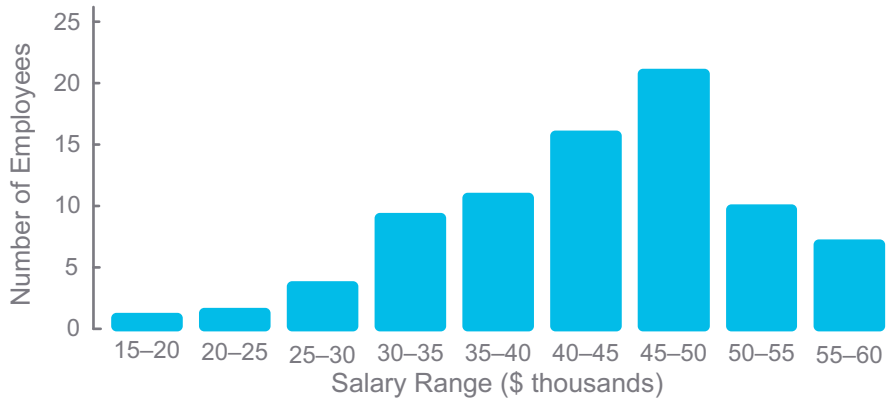



**Exhibit 7B Salaries of Employees at Company Y**




Note that the two distributions are not symmetrical. A symmetrical distribution would have observations falling off fairly evenly on either side of the centre of the range of salaries ($35,001–$40,000). Instead, in each of these distributions, the bulk of the observations are stacked towards one end of the range and tail off gradually towards the other end. The two distributions are different in that each is stacked towards a different end. Such distributions are considered skewed; the distribution for Company X is positively skewed (i.e., the majority of the observations are on the left and the skew or tail is on the right), whereas the distribution for Company Y is negatively skewed (left skewed).

Although the range of the observations is the same in each case, the mean for each is very different. Company X's mean is approximately $35,000, whereas Company Y's mean is approximately $44,000.

For a perfectly symmetrical distribution, such as a normal distribution (see Exhibit 8), the mean, median, and mode will be identical. If the distribution is skewed, these three measures of central tendency will differ. Looking again at Company X's salary data, for instance, we do not have enough detailed information to identify a mode. The mean

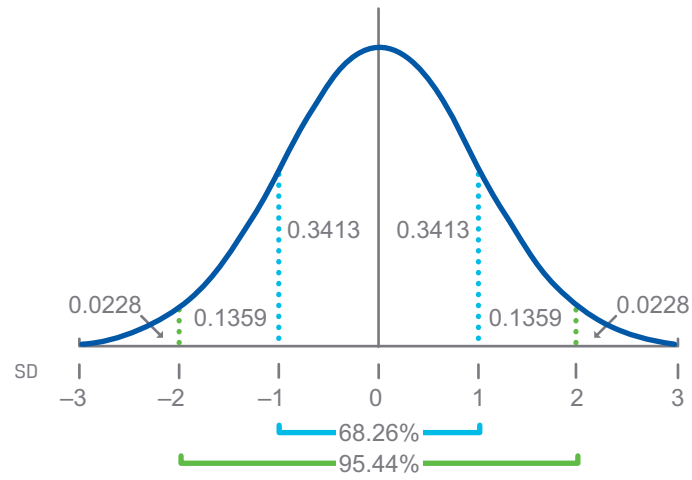
is larger than the median because the mean is more affected by extreme values than the median. The distribution is skewed to the right, so the mean is dragged towards the extreme positive values. The reverse is true for distributions that are negatively skewed, such as in Company Y's salary data. In this case, the mean is smaller than the median because the mean is pulled left in the direction of the skew.

A **normal distribution** is represented in a graph by a bell curve; an example of a bell-shaped curve is shown in Exhibit 8. The shape of the curve is symmetrical with a single central peak at the mean of the data and the graph falling off evenly on either side of the mean; 50% of the distribution lies to the left of the mean, and 50% lies to the right of the mean. The shape of a normal distribution depends on the mean and the standard deviation. The mean of the distribution determines the location of the centre of the curve, and the standard deviation determines the height and width of the curve. When the standard deviation is large, the curve is short and wide; when the standard deviation is small, the curve is tall and narrow.

A normal distribution has special importance in statistics because many variables have the approximate shape of a normal distribution—for example, height, blood pressure, and lengths of objects produced by machines. This distribution is often useful as a description of data when there are a large number of observations.

A normal distribution is a distribution of a continuous random variable (i.e., a variable that can take on an infinite number of values). The vertical axis for the normal distribution is the probability or likelihood of occurrence. By contrast, on the histogram shown earlier, the vertical axis was frequency of occurrence. The mean (and median) is the centre of the distribution and has the highest probability of occurrence. Half of the observations lie on one side of the mean and half on the other. Approximately two-thirds of the observations are within one standard deviation of the mean, and 95% of observations are within two standard deviations of the mean. Exhibit 8 shows a normal distribution.

**Exhibit 8 Standard Deviation (SD) and Normal Distribution**



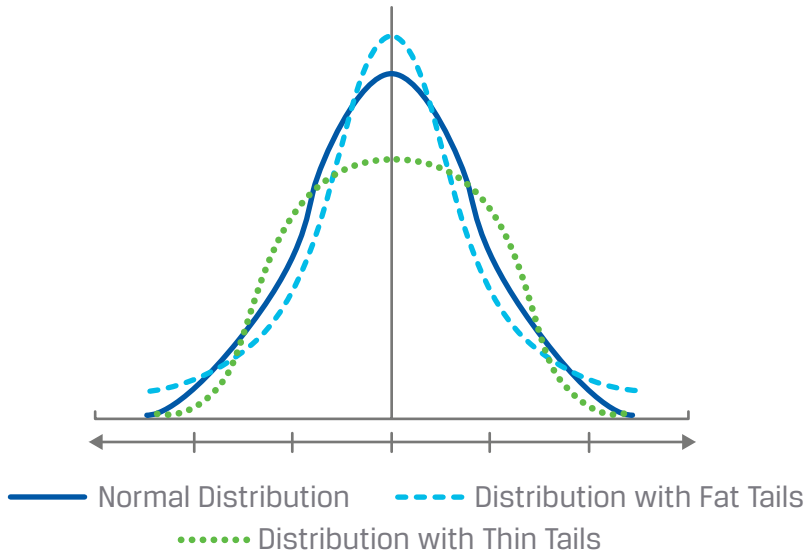
The total area under the curve or bell is 100% of the distribution. The area under the curve that is within one standard deviation of the mean is about 68% of all the observations. In other words, given a mean of 0 and a standard deviation of 1, about 68% of the observations fall between –1 and +1, and 32% of the observations are more than one standard deviation from the mean. The area under the curve that is within 2 standard deviations of the mean is about 95% of the observations. Given a mean of 0 and a standard deviation of 1, about 95% of the observations fall between –2 and +2, and 5% of the observations are more than two standard deviations from the mean. The area under the curve that is within three standard deviations of the mean represents about 99% of the observations. Given a mean of 0 and a standard deviation of 1, about 99% of the observations fall between –3 and +3, and less than 1% of the observations occur more than three standard deviations away from the mean.

The observations that are more than a specified number of standard deviations from the mean can be described as lying in the tails of the distribution. Assuming that returns on a portfolio of stocks are normally distributed, the chance of extreme losses (a return more than three standard deviations lower than the mean return) is relatively small. The chance of the return being in the left tail more than two standard deviations from the mean (which would be an extreme loss under typical circumstances) is just 2.5%. In other words, out of 200 days, 5 days are expected to have observations that are more than two standard deviations from the mean. But during the financial crisis of 2008, the losses that were incurred by some banks over several days in a row were 25 standard deviations below the mean.

To put this in perspective, if returns are normally distributed, a return that is 7.26 standard deviations below the mean would be expected to occur once every 13.7 billion years. That is approximately the age of the universe. The frequency of extreme events during the financial crisis of 2008 was, therefore, much higher than predicted by the normal distribution. This inconsistency is often referred to as the distribution having "fat tails", meaning that the probability of observing extreme outcomes is higher than that predicted by a normal distribution.

Exhibit 9 gives examples of different types of bell-shaped distributions. How would you describe each curve? What does each tell you about the likelihood of extreme outcomes?

**Exhibit 9 Bell-Shaped Distributions with Fat and Thin Tails**




In Exhibit 9, the curve with the solid line represents the normal distribution. The curve with the dotted line is an example of distribution with thinner tails than the normal distribution, indicating a reduced probability of extreme outcomes. By contrast, the curve with the dashed line is an example of a distribution with fatter tails than the normal distribution, indicating increased likelihood of extreme outcomes.

## 3.3 Correlation

Another way of using and understanding data is identifying connections between data sets. The strength of a relationship between two variables, such as growth in gross domestic product (GDP) and stock market returns, can be measured by using **correlation**. Essentially, two variables are correlated when a change in one variable helps predict change in another variable.

When both variables change in the same direction, the variables are positively correlated. If we take the example of traders at an investment bank, salary and age are positively correlated if salaries increase as age increases. If the variables move in the opposite direction, then they are negatively correlated. For example, the size of a transaction and the fees expressed as a percentage of the transaction are negatively correlated if the larger the transaction, the smaller the associated fees. When there is no clear tendency for one variable to move in a particular direction (up or down) relative to changes in the other variable, then the variables are close to being uncorrelated. In practice, it is difficult to find two variables that have absolutely no relationship, even if just by chance.

Correlation is measured by the **correlation coefficient**, which has a scale of –1 to +1. When two variables move exactly in step with each other in the same direction—if one goes up, the other goes up in the same proportion—the variables are said to be perfectly positively correlated. In that case, the correlation coefficient is at its maximum

of +1. When the two variables move exactly in step in opposite directions, they are perfectly negatively correlated and the correlation coefficient is –1. Variables with no relationship to each other will have a correlation coefficient close to 0.

Correlation measures both the direction of the relationship between two variables (negative or positive) and the strength of that relationship (the closer to +1 or –1, the stronger the relationship). In practice, it is unusual to find variables that are perfectly positively or perfectly negatively correlated. The stronger the relationship between two variables—the higher the degree of correlation—the more confidently one variable can be predicted given the other variable. For example, there may be a high correlation between stock market index returns and expected economic growth. In that case, if economic growth in the future is expected to be high then returns on the stock market index are likely to be high too.

It is important, however, to realise that correlation does not imply causation. For example, historically in the United States, stock market returns and snowfall are both higher in January, and from that you may assume a correlation. But obviously snowfall does not cause an increase in stock market returns, and an increase in stock market returns clearly does not cause snowfall. There may be situations in which a correlation implies some causal relationship. For example, a high correlation has been found between power production and job growth. It may follow that the more workers there are, the more power is consumed, but it does not necessarily follow that an increase in power generation will create jobs.

Correlation is important in investing because the rise or fall in value of a variable may help predict the rise or fall in value of another variable. It is also important because when two or more securities that are not perfectly correlated are combined together in a portfolio, there is normally a reduction in risk (measured by the portfolio's standard deviation of returns). As long as the returns on the securities do not have a correlation of +1 (that is, they are less than perfectly correlated), then the risk of the portfolio will be less than the weighted average of the risks of the securities in the portfolio because it is not likely that all the securities will perform poorly at the same time.

The practice of combining securities in a portfolio to reduce risk is known as **diversification**. An extreme example of an undiversified portfolio is one holding only one security. This approach is risky because it is not unusual for a single security to go down in value by a large amount in one year. It is much less common for a diversified portfolio of 20 different securities to go down by a large amount, even if they are selected at random. If the securities are selected from a variety of sectors, industries, company sizes, asset classes, and markets, it is even less likely. One caveat is that the benefits of diversification are much reduced in periods of financial crisis. In such periods, the correlation between returns on different securities (and different asset classes) tends to increase towards +1.

## SUMMARY

The better your understanding of quantitative concepts, the easier it will be for you to make sense of the financial world. Knowledge of quantitative concepts, such as time value of money and descriptive statistics, is important to the understanding of many of the key products in the financial industry. Understanding the time value of money allows you to interpret cash flows and thus value them. Meanwhile, knowledge of statistical concepts will help in identifying the important information in a large amount of data, as well as in understanding what statistical measures reported by others mean. It is easy to misinterpret or be misled by statistics, such as mean and correlation, so an understanding of their uses and limitations is crucial.

- Interest is return earned by a lender that compensates for opportunity cost and risk. For the borrower, it is the cost of borrowing.
- The simple interest rate is the cost to the borrower or the rate of return to the lender, per period, on the original principal borrowed. A commonly quoted simple interest rate is the annual percentage rate (APR).
- Compound interest is the return to the lender or the cost to the borrower when interest is reinvested and added to the original principal.
- The effective annual rate (EAR) of interest is calculated by annualising a rate that is compounded more than once a year. The EAR is equal to or greater than the annual percentage rate.
- The present value of a future sum of money is found by discounting the future sum by an appropriate discount rate. (The present value of multiple cash flows is the sum of the present value of each cash flow.)
- Three elements must be considered when comparing investments: the cash flows each investment will generate in the future, the timing of these cash flows, and the risk associated with each investment. The discount rate reflects the riskiness of the cash flows.
- All else being equal (in other words, only one of the three elements differs):
  - the higher the cash flows, the higher the present and future values.
  - the earlier the cash flows, the higher the present and future values.
  - the lower the discount rate, the higher the present value.
  - the higher the interest rate, the higher the future value.
- The net present value is the present value of future cash flows net of the investment required to obtain them. It is useful when comparing alternatives that require different initial investments.
- Financial instruments can be valued as the present value of their expected future cash flows.

- An annuity involves an initial payment (outflow) in exchange for a fixed number of future receipts (inflows), each of an equal amount. Mortgages are amortising loans; the periodic payment is fixed, and in each period some of the payment covers the interest on the loan and the rest of the payment pays off some of the principal. Over time, the portion of the payment that reduces the principal increases.

- The role of descriptive statistics is to summarise the information given in large quantities of data for the purpose of making comparisons, predicting future values, and better understanding the data.

- The purpose of measures of frequency or central tendency is to describe a group of individual data scores with a single measurement. This measure is intended to be representative of the individual scores. Measures of central tendency include arithmetic mean, geometric mean, median, and mode. Different measures are appropriate for different types of data.

- The arithmetic mean is the most commonly used measure. It represents the sum of all the observations divided by the number of observations. It is an easy measure to understand but may not be a good representative measure when there are outliers.

- The geometric mean return is the average compounded return for each period—that is, the average return for each period assuming that returns are compounding. It is frequently the preferred measure of central tendency for returns in the investment industry.

- When observations are ranked in order of size, the median is the middle value. It is not sensitive to outliers and may be a more representative measure than the mean when data are skewed.

- The mode is the most frequently occurring value in a data set. A data set may have no identifiable unique mode. It may not be a meaningful representative measure of central tendency.

- Measures of dispersion are important for describing the spread of the data, or its variation around a central value. Two common measures of dispersion are range and standard deviation.

- Range is the difference between the highest and lowest values in a data set. It is easy to measure, but it is sensitive to outliers.

- Standard deviation measures the variability of a data set around the mean of the data set. It is in the same unit of measurement as the mean.

- A distribution is simply the values that a variable can take, showing its observed or theoretical frequency of occurrence.

- For a perfectly symmetrical distribution, the mean, median, and mode will be identical.

- A common symmetrical distribution is the normal distribution, a bell-shaped curve that is represented by its mean and standard deviation. In a normal distribution, 68% of all the observations lie within one standard deviation of the mean and about 95% of the observations are within two standard deviations.

- The strength of a relationship between two variables can be measured by using correlation.
- Correlation is measured by the correlation coefficient on a scale from –1 to +1. When two variables move exactly in tandem with each other, the variables are said to be perfectly positively correlated and the correlation coefficient is +1. When two variables move exactly in opposite directions, they are perfectly negatively correlated and the correlation coefficient is –1. Variables with no relationship to each other will have a correlation coefficient close to 0.
- It is important to realise that correlation does not imply causation.

# CHAPTER REVIEW QUESTIONS

1 Interest is paid by the borrower to compensate the lender:

A for opportunity cost and risk.

B for forgoing future consumption.

C for increases in future purchasing power.

2 A company obtains a loan from a local bank for $50 million. From the company's perspective, interest is *best* defined as the:

A risk of default.

B cost of borrowing.

C value of the next best alternative.

3 The greater the risk associated with a borrower's ability to repay a loan, the greater the:

A opportunity cost for the borrower.

B interest rate demanded by the lender.

C risk of purchasing power increasing over the life of the loan.

4 To maintain purchasing power, lenders demand an interest rate that reflects the:

A likelihood of default.

B current rate of inflation.

C expected rate of inflation.

5 If interest is paid and compounded annually, the compound interest rate is *most likely* to be:

A higher than the simple interest rate.

B the same as the simple interest rate.

C lower than the simple interest rate.

6 Compared with compound interest, simple interest assumes that interest is:

A paid annually.

B calculated using only the original amount invested.

C reinvested and added to the original amount invested.

**7** Which of the following is associated with the concept of interest on interest?

**A** Simple interest

**B** Compound interest

**C** Annual percentage rate

**8** If $1,000 is deposited to an account with an annual interest rate of 3% and is left on deposit for three years, the amount of money in the account at the end of three years will be:

**A** lower using simple interest compared with using compound interest.

**B** the same using either simple interest or compound interest.

**C** greater using simple interest compared with using compound interest.

**9** The interest rate used to determine the present value of future cash flows is called the:

**A** discount rate.

**B** effective annual rate.

**C** annual percentage rate.

**10** The most effective way to compare investments with the same initial outflow that have different cash flows at different points in time is to determine each investment's:

**A** discount rate.

**B** present value.

**C** future cash flows.

**11** The present value of €100 that will be received two years from today is:

**A** less than €100.

**B** equal to €100.

**C** more than €100.

**12** All else being equal, given a choice of when to pay for a purchase, an individual would *most likely* prefer to pay £100:

**A** today.

**B** one year from today.

**C** two years from today.

**13** Assuming a discount rate of 10%, which of the following projects will have the highest present value?

**A** A €10,000 lump-sum payment received today

**B** A €10,000 lump-sum payment received in one year

**C** A €5,000 payment received today plus €5,000 to be received in one year

**14** Given an interest rate of 10% and assuming that interest is reinvested, which of the following will have the highest future value?

**A** €10,000 invested today for 5 years.

**B** €10,000 invested today for 10 years.

**C** €10,000 invested today for 15 years.

**15** When evaluating an investment, if the discount rate increases while holding all other factors constant, the present value will:

**A** increase.

**B** decrease.

**C** remain unchanged.

**16** When choosing among investments that have different initial costs and future cash flows, the *best* choice is the investment with the highest:

**A** discount rate.

**B** net present value.

**C** present value of future cash flows.

**17** Which of the following investments is unacceptable? An investment with a net present value of:

**A** negative $5.

**B** $0.

**C** positive $5.

**18** If an individual makes an initial payment to an insurance company in exchange for a fixed number of future payments of a certain amount from the insurance company, the individual has:

**A** received a loan.

**B** obtained a mortgage.

**C** purchased an annuity.

**19** In a mortgage transaction, the amount of each fixed payment made by the borrower that represents interest:

**A** decreases over time.

**B** remains the same over time.

**C** increases over time.

**20** Which of the following is a measure of central tendency?

**A** Mean

**B** Range

**C** Standard deviation

**21** If the data in a set are continuous and skewed, which of the following gives the *best* measure of central tendency?

**A** Mean

**B** Mode

**C** Median

**22** The preferred measure of central tendency for investment returns is the:

**A** mode.

**B** arithmetic mean.

**C** geometric mean.

**23** An analyst is comparing the returns of two investment portfolios. The two portfolios have the same mean return. The portfolio with the higher standard deviation *most likely*:

**A** is less risky.

**B** is more risky.

**C** has a smaller range.

**24** Which of the following is a measure of dispersion?

**A** Mode

**B** Range

**C** Median

**25** Which of the following is a measure of dispersion used to assess the risk of an investment?

**A** Arithmetic mean

**B** Geometric mean

**C** Standard deviation

**26** Which of the following characteristics *most likely* represents a normal distribution?

**A** The values of the mean and median are identical.

**B** There are more observations to the right of the mean than to the left.

**C** There are more observations to the left of the mean than to the right.

**27** A characteristic of a normal distribution is that the distribution of data is:

**A** symmetrical.

**B** positively skewed.

**C** negatively skewed.

**28** For a normal distribution, the height and width of the distribution is determined by the distribution's:

**A** mean.

**B** median.

**C** standard deviation.

**29** If there is no relationship between two variables, the correlation coefficient is *closest* to:

**A** +1.

**B** 0.

**C** −1.

**30** Assume the correlation between the unemployment rate and the inflation rate is close to −1. Based on this information, if the unemployment rate is expected to increase, then the inflation rate will *most likely*:

**A** increase.

**B** decrease.

**C** remain unchanged.

# ANSWERS

**1** A is correct. Interest is a payment paid by the borrower and earned by the lender that compensates for opportunity cost and risk. B is incorrect because the lender is forgoing *current,* not future, consumption. C is incorrect because interest is paid by the borrower to compensate the lender for potential *decreases* in future purchasing power.

**2** B is correct. From the company's perspective, interest is defined as payment for the use of borrowed money or the cost of borrowing. A and C are incorrect because from the lender's perspective, interest is earned to compensate for opportunity cost (value of the next best alternative) and risk, including default.

**3** B is correct. The lower the certainty a borrower will make the promised payments on the loan in a timely manner, the higher the level of interest required by the lender to compensate for the increased risk of default. A is incorrect because the opportunity cost for the lender, not the borrower, is unrelated to the risk of the borrower. C is incorrect because the lender bears the risk that purchasing power will *decrease* not increase.

**4** C is correct. To maintain purchasing power, lenders demand an interest rate that takes into account expected inflation. As a result of inflation, the money received from a loan may not be worth as much as expected even if the loan is repaid as promised. To compensate for the risk of inflation and a decline in purchasing power, the interest rate demanded by the lender includes expected inflation over the life of the loan. The higher the expected inflation, the higher the interest rate demanded. A is incorrect because even if the loan is repaid as promised, purchasing power will decline if inflation is greater than expected over the life of the loan. B is incorrect because the expected rate of inflation is reflected in the interest rate, not in the current rate of inflation.

**5** B is correct because the number of compounding periods is one. Therefore, the simple and compound interest rates are the same. More frequent compounding leads to a higher compound interest rate than the simple interest rate.

**6** B is correct. Simple interest assumes that interest is not reinvested; consequently, simple interest is only calculated on the original principal amount, or the original amount invested. C is incorrect because compound interest (not simple interest) assumes that, over time, interest is earned on the original amount invested as well as on the interest earned. A is incorrect because both simple and compound interest may be paid annually or more frequently.

**7** B is correct. Compound interest is based on the assumption that interest earned is added to the original amount invested. Over time, interest is earned on the original amount invested as well as on the interest earned. As a result, compound interest is often referred to as interest on interest. A is incorrect because simple interest assumes that interest is always calculated based on the original amount invested. C is incorrect because the annual percentage rate is a simple interest rate and does not involve compounding.

**8** A is correct. The amount of money in the account at the end of three years will be lower using simple interest compared with compound interest. Compound interest will result in interest being earned on the original deposit as well as interest on interest.

**9** A is correct. The present value of future cash flows is obtained by discounting the future cash flows by the interest rate. The interest rate in this context is called the discount rate. C is incorrect because the annual percentage rate is a quoted simple interest rate. B is incorrect because the effective annual rate is used to annualise a rate that is paid more than once a year.

**10** B is correct. Present value considers the three elements that should be used when comparing investments; the amount of the future cash flows, the timing of the future cash flows, and the risk associated with each investment as reflected by the discount rate. A and C are incorrect because they are elements needed to determine present value.

**11** A is correct. The present value of any amount is less than the same amount received in the future. How much less depends on the discount rate used to determine the present value. B and C are incorrect because any amount received in the future is worth less than the same amount received today.

**12** C is correct. For any given amount of money, most individuals would prefer to pay the amount later compared with paying the same amount of money today because money has a time value. The present value of the payment of £100 is lower the further in the future that amount is paid. A and B are incorrect because if the individual postpones payment, the £100 could be invested for two years rather than for one year or not at all.

**13** A is correct. A lump-sum payment received today has a higher present value than the same amount received in the future or in instalments.

**14** C is correct. The longer the compounding period, the greater the future value.

**15** B is correct. The higher the discount rate, the lower the present value.

**16** B is correct. Net present value is the difference between the present value of future cash inflows and the cost of that investment. The investment with the highest positive net present value is the best choice if only one investment will be chosen. A is incorrect because the investment with the largest discount rate is the one with the most risk. It may or may not have the highest net present value, depending on expected cash flows and their timing and initial cost. C is incorrect because the present value of future cash flows must be compared with the cost before a choice can be made.

**17** A is correct. A net present value of negative $5 means that the cost of the investment is $5 more than the present value of the future cash flows from the investment and the investment should not be made. B and C are incorrect because a net present value of zero or greater means that the investment is earning at least the discount rate and is acceptable.

**18** C is correct. The time value of money application described is an annuity. After the initial payment, the insurance company will make payments to the individual which can also be considered withdrawals by the individual. These withdrawals reduce the balance of the annuity over time. A is incorrect because

a simple loan requires interest payments and the repayment of the loan amount when the loan matures. B is incorrect because the individual obtaining the mortgage must make the fixed payments of a certain amount to the lender.

**19** A is correct. Although the payment amount is fixed, the portion of each payment that is interest is based on the remaining principal at the beginning of each period. As the principal declines, so does the amount of the fixed payment that constitutes interest. B is incorrect because the payment remains fixed, but the portion allocated to interest decreases over time while the portion allocated to principal increases over time. C is incorrect because the portion of the fixed payment allocated to principal increases over time.

**20** A is correct. Mean is a measure of central tendency. B and C are incorrect because both the range and standard deviation are measures of dispersion.

**21** C is correct. The median is the middle value in a data set when the items in that data set are ordered from smallest to largest. Thus, the median is not affected by outliers (extremely high or low value items in the data set). A is incorrect because the mean is affected by outliers. B is incorrect because with continuous data, it is less likely that any observation will appear more than once; thus, the mode may not be identifiable.

**22** C is correct. The investment industry prefers the geometric mean because the geometric mean is the average return earned each year to get the total accumulation assuming that returns are compounding. A is incorrect because investment returns are continuous data; the mode is not a useful measure of central tendency when data are continuous. B is incorrect because the arithmetic mean does not assume compounding.

**23** B is correct. Higher standard deviation indicates higher variability and risk. A and C are incorrect because the portfolio with the higher standard deviation is more dispersed, has a larger range, and is more risky.

**24** B is correct. Measures of dispersion are used to measure the spread associated with a dataset or its variation around a central value. The range is a measure of dispersion that is calculated by taking the difference between the highest and lowest values of the dataset. A and C are incorrect because the mode and the median are measures of central tendency.

**25** C is correct. The standard deviation is a measure of dispersion in a dataset. The higher the standard deviation, the greater the risk associated with an investment. A and B are incorrect because they are measures of central tendency.

**26** A is correct. The mean and median are identical in a normal distribution. B and C are incorrect because in a normal distribution, like any symmetrical distribution, there are the same number of observations to the left and to the right of the mean.

**27** A is correct. A normal distribution is a distribution in which 50% of the distribution lies to the left of the mean and 50% of the distribution lies to the right of the mean; the shape of the distribution is symmetrical. B and C are incorrect because if the distribution is skewed, more than 50% of the distribution lies to either the left (negatively skewed) or right (positively skewed) of the mean.

**28** C is correct. The height and width of a normal distribution is determined by the standard deviation. If the standard deviation is large, the curve is short and wide; if the standard deviation is small, the curve is tall and narrow. A and B are incorrect because the mean and median are the same in a normal distribution; they determine the centre of the curve in a normal distribution.

**29** B is correct. Correlation measures the strength of a relationship between two variables. A correlation close to 0 indicates that there is no relationship between the variables. A is incorrect because a correlation of +1 indicates that the two variables are perfectly positively correlated. C is incorrect because a correlation of –1 indicates that the two variables are perfectly negatively correlated.

**30** B is correct. Because the unemployment rate and the inflation rate are negatively correlated, if unemployment is expected to increase, then the inflation rate is likely to decrease

# CHAPTER 10

## EQUITY SECURITIES

by Lee M. Dunham, PhD, CFA, and Vijay Singal, PhD, CFA

## LEARNING OUTCOMES

After completing this chapter, you should be able to do the following:

**a** Describe features of equity securities;

**b** Describe types of equity securities;

**c** Compare risk and return of equity and debt securities;

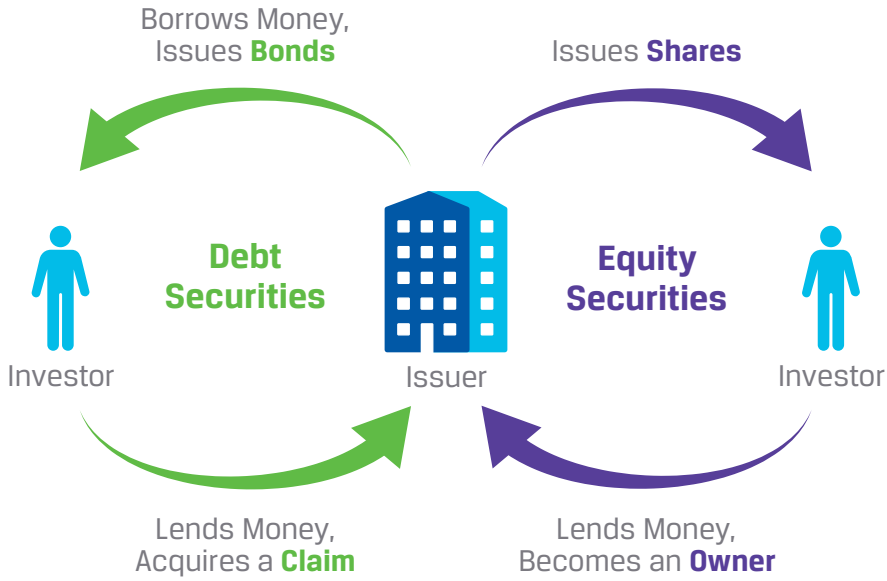
**d** Describe approaches to valuing common shares;

**e** Describe company actions that affect the company's shares outstanding.

# INTRODUCTION 1

At some point in their lives, many people participate in the stock market either directly, such as by buying shares, or indirectly, perhaps by contributing to a retirement plan or by investing through a mutual fund.[1] Whether or not they participate in the stock market, most people tend to be aware of shares and stock markets because stock market information, such as stock market indices, is widely reported. As discussed in the Macroeconomics chapter, stock market indices, which represent the performance of a group of shares, are useful indicators of the state of the economy.

In addition to borrowing funds, companies may raise external capital to finance their operations by issuing (selling) equity securities. Issuing shares (also called stock and shares of stock) is a company's main way of raising equity capital and shares are the primary equity securities discussed in this chapter.[2]




This chapter also describes other basic types of equity securities available in the market and features of these securities. There is some discussion of debt securities in order to make some basic comparisons between debt securities and equity securities.

Given the importance of equity securities in the investment industry, an understanding of what they are and how they are valued is likely to help you in your role. Some approaches that investment professionals use to value common shares are discussed. Some company actions that affect a company's number of shares are also described. Examples intended to enhance your understanding are included. Some of these examples include calculations but, as always, you are not responsible for calculations.

1 Recall from the Investment Industry: A Top-Down View chapter that a mutual fund is a professionally managed investment vehicle that has investments in a variety of securities. Mutual funds are discussed further in the Investment Vehicles chapter.

2 Security market indices are discussed further in the Investment Vehicles chapter.

## 2 FEATURES OF EQUITY SECURITIES

Companies may issue different types of equity securities. The types of equity securities, or equity-like securities, that companies typically issue are common stock (or common shares), preferred stock (or preferred shares), convertible bonds, and warrants. Each of these types is discussed more extensively in the next section. Each type of equity security has different features attached to it. These features affect a security's expected return, risk, and value.

There are four features that characterise and vary among equity securities:

- Life
- Par value
- Voting rights
- Cash flow rights

***Life.*** Many equity securities are issued with an infinite life. In other words, they are issued without maturity dates. Some equity securities are issued with a maturity date.

***Par Value.*** Equity securities may or may not be issued with a par value. The **par value** of a share is the stated value, or face value, of the equity security. In some jurisdictions, issuing companies are required to assign a par value when issuing shares.

***Voting Rights.*** Some shares give their holders the right to vote on certain matters. Shareholders do not typically participate in the day-to-day business decisions of large companies. Instead, shareholders with voting rights collectively elect a group of people, called the **board of directors**, whose job it is to monitor the company's business activities on behalf of its shareholders. The board of directors is responsible for appointing the company's senior management (e.g., chief executive officer and chief operating officer), who manage the company's day-to-day business operations. But decisions of high importance, such as the decision to acquire another company, usually require the approval of shareholders with voting rights.

***Cash Flow Rights.*** Cash flow rights are the rights of shareholders to distributions, such as dividends, made by the company. In the event of the company being liquidated, assets are distributed following a priority of claims, or **seniority ranking**. This priority of claims can affect the amount that an investor will receive upon liquidation.

# TYPES OF EQUITY SECURITIES

# 3

Companies may issue different types and classes of equity securities. The two main types of equity securities are common shares (also called common stock or ordinary shares) and preferred shares (also known as preferred stock or preference shares). In addition, companies may issue convertible bonds and warrants. Depositary receipts are not issued by a company, but they give the holder an equity interest in the company.

## 3.1 Common Stock

**Common stock** (also known as common shares, ordinary shares, or voting shares) is the main type of equity security issued by companies. A common share represents an ownership interest in a company. Common shares have an infinite life; in other words, they are issued without maturity dates. Common stock may or may not be issued with a par value. When common shares are issued with par values, companies typically set their par value extremely low, such as 1 cent per share in the United States. It is important to note that the par value of a common share may have no connection to its market value, even at the time of issue. For instance, a common share with a par value of 1 cent may be issued to a shareholder for $50.

Common shares represent the largest proportion of equity securities by market value. Large companies often have many common shareholders, each of whom owns a portion of the company's total shares. Investors may own common stock of public or private companies. Shares of public companies typically trade on stock exchanges that facilitate trading of shares between buyers and sellers. Private companies are typically much smaller than public companies, and their shares do not trade on stock exchanges. The ability to sell common shares of public companies on stock exchanges offers potential shareholders the ability to trade when they want to trade and at a fair price.

Common stock typically provides its owners with **voting rights** and **cash flow rights** in proportion to the size of their ownership stake. Common shareholders usually have the right to vote on certain matters. Companies often pay out a portion of their profits each year to their shareholders as dividends; the rights to such distributions are the shareholders' cash flow rights. Dividends are typically declared by the board of directors and vary according to the company's performance, its reinvestment needs, and the management's view on paying dividends. As owners of the underlying company, common shareholders participate in the performance of the company and have a residual claim on the company's liquidated assets after all liabilities (debts) and other claims with higher seniority have been paid.

Many companies have a single class of common stock and follow the rule of "one share, one vote". But some companies may issue different classes of common stock that provide different cash flow and voting rights. In general, an arrangement in which a company offers two classes of common stock (e.g., Class A and Class B) typically provides one class of shareholders with superior voting and/or cash flow rights.

Example 1 describes the two classes of common stock of Berkshire Hathaway and their cash flow and voting rights.

**EXAMPLE 1. DIFFERENT SHARE CLASSES**

As of May 2012, Berkshire Hathaway, a US company, has two classes of common stock: Class A (NYSE: BRK.A)[3] and Class B (NYSE: BRK.B). In terms of cash flow rights, one Class A share is equivalent to 1,500 Class B shares. But the ratio of the voting rights of Class A shares to the voting rights of Class B shares is not 1,500:1. Voting rights for 1 Class A share are equivalent to the voting rights of 10,000 Class B shares.

| | BRK.A | | BRK.B |
|---|---|---|---|
| Cash flow rights | 1 | = | 1,500 |
| Voting rights | 1 | = | 10,000 |

The reason for having multiple share classes is usually that the company's original owner wants to maintain control, as measured by voting power, while still offering cash flow rights to attract shareholders. In general, for large public companies in which nearly all shareholders hold small ownership positions, the difference in voting rights may not be important to shareholders.

## 3.2 Preferred Stock

Companies may also issue **preferred stock** (also known as preferred shares or preference shares). These shares are called preferred because owners of preferred stock will receive dividends before common shareholders. They also have a higher claim on the company's assets compared with common shareholders if the company ceases operations. In other words, preferred shareholders receive preferential treatment in some respects. Generally, preferred shareholders are not entitled to voting rights and have no ownership or residual claim on the company.

Preferred shares are typically issued with an assigned par value. Along with a stated dividend rate, this par value defines the amount of the annual dividend promised to preferred shareholders. Preferred share terms may provide the issuing company with the right to buy back the preferred stock from shareholders at a pre-specified price, referred to as the redemption price. In general, the pre-specified redemption price equals the par value for a preferred share. The par value of a preferred share also typically represents the amount the shareholder would be entitled to receive in a liquidation, as long as there are sufficient assets to cover the claim.

Preferred shareholders usually receive a fixed dividend, although it is not a legal obligation of the company. The preferred dividend will not increase if the company does well. If the company is performing poorly, the board of directors is often reluctant to reduce preferred dividends.

3 These are ticker symbols, which are used to identify a particular stock, share class, or issue on a particular stock exchange.

Preferred shares differ with respect to the policy on missed dividends, depending on whether the preferred stock is cumulative or non-cumulative. Cumulative preferred stock requires that the company pay in full any missed dividends (dividends promised, but not paid) before paying dividends to common shareholders. In comparison, non-cumulative preferred stock does not require that missed dividends be paid before dividends are paid to common shareholders. In a liquidation, preferred shareholder may have a claim for any unpaid dividends before distributions are made to common shareholders.

Example 2 provides a variety of the features that can characterise a preferred share issue. It shows the features of two different issues of Canadian preferred stock.

**EXAMPLE 2. PREFERRED STOCK**

| Issue | Cumulative/ Non-Cumulative | Par Value (Canadian dollars) | Annual Dividend Rate | Redeemable |
|---|---|---|---|---|
| Royal Bank of Canada, Series B | Non-cumulative | C$25.00 | 6.25%, reset after five years and every five years thereafter to 3.50% over the five-year Government of Canada bond yield | Yes, redeemable on or after 24 February 2014 at par |
| Canadian Utilities Limited, Series AA | Cumulative | C$25.00 | 4.90% | Yes, redeemable after 1 September 2017, redemption price begins at C$26.00 and declines over time to C$25.00 |

Some companies have more than a single issue of preferred stock. Multiple preferred stock issues (or rounds) are referred to by series. Each preferred stock issue by a company usually carries its own dividend, based on stated par value and dividend rate, and may differ with respect to other features as well.

## 3.3 Convertible Bonds

To raise capital, companies may issue convertible bonds. A **convertible bond** is a bond issued by a company that offers the bondholder the right to convert the bond into a pre-specified number of common shares. Although a convertible bond is actually a debt security prior to conversion, the fact that it can be converted to common shares makes its value somewhat dependant on the price of common shares. Thus, convertible bonds are known as hybrid securities. Hybrid securities have features of and relationships with both equity and debt securities.

The number of common shares that the bondholder will receive from converting the bond is known as the conversion ratio. The conversion ratio may be constant for the security's life, or it may change over time. The conversion value (or parity value) of a convertible bond is the value of the bond if it is converted to common shares. The conversion value is equal to the conversion ratio times the share price. At conversion, the bonds are retired (cease to exist) and common shares are issued.

Because the conversion feature is a benefit to the bondholder, a convertible bond typically offers the bondholder a lower fixed annual coupon rate than that of a comparable bond without a conversion feature (a straight bond). Convertible bonds have a maturity date. If the bonds are not converted to common stock prior to maturity, they will be paid off like any other bond and retired at the maturity date.

When a convertible bond is issued, the conversion ratio is set so that its value as a straight bond (i.e., the value of the bond if it were not convertible) is higher than its conversion value. If the share price of the company significantly increases, the conversion value of the bond will rise and may become greater than the value of the convertible bond as a straight bond. If this happens, converting the bond becomes attractive. In general, if the conversion value is low relative to the straight bond value, the convertible bond will trade at a price close to its straight bond value. But if the conversion value is greater than the straight bond value, the convertible bond will trade at a value closer to its conversion value.

Because a convertible bond should not trade below its conversion value, bondholders may choose not to convert into common shares even if the conversion value is higher than the par (principal) value of the bond. Often, a convertible bond includes a redemption (buyback) option. The redemption (buyback) option gives the issuing company the right to buy back (redeem) the convertible bonds, usually at a prespecified redemption price and only after a certain amount of time. Convertible bond issues typically include redemption options so that the issuing company can force conversion into common shares.

Example 3 describes a convertible bond issue of Navistar International Corp. The Navistar bond issue illustrates the typical features of a convertible bond.

**EXAMPLE 3. CONVERTIBLE BONDS**

On 22 October 2009, Navistar, a US company, issued convertible bonds. The bond issue pays interest semiannually (twice a year) at a rate of 3.0% per year and has a maturity date of 15 October 2014. Owners of this convertible bond issue may convert each $1,000 bond into 19.891 common shares. The owners may unconditionally convert at any time on or after 15 April 2014 up to the maturity date and may convert the bond prior to that date under certain conditions. No redemption right is included as part of the bond issue. On 9 October 2012, the company's common shares closed at $22.26 and, therefore, each $1,000 bond's conversion value was $442.77 (= $22.26 × 19.891). The bond price in the market was $912. In this case, the bond is trading at close to its straight bond value, rather than at its conversion value.

Similar to convertible bonds, some preferred shares include a convertible feature. The convertible feature provides the shareholder with the option to convert the preferred share into a specified number of common shares. With this option, a preferred shareholder may be able to participate in the performance of the company. That is, if the company is doing well, it may be to a preferred shareholder's advantage to convert the preferred share into the specified number of common shares. Also, similar to convertible bonds, convertible preferred shares typically include a redemption option.

## 3.4 Warrants

A **warrant** is an equity-like security that entitles the holder to buy a pre-specified amount of common stock of the issuing company at a pre-specified per share price (called the exercise price or strike price) prior to a pre-specified expiration date. A company may issue warrants to investors to raise capital or to employees as a form of compensation. The holders of warrants may choose to exercise the rights prior to the expiration date. A warrant holder will exercise the right only when the exercise price is equal to or lower than the price of a common share. Otherwise, it would be cheaper to buy the stock in the market. When a warrant holder exercises the right, the company issues the pre-specified number of new shares and sells them to the warrant holder at the exercise price.

Warrants typically have expiration dates several years into the future. In some cases, companies may attach warrants to a bond issue or a preferred stock issue in an effort to make the bond or preferred stock more attractive. When issued in this manner, warrants are known as sweeteners because the inclusion of the warrants typically allows the issuer to offer a lower coupon rate (interest rate) on a bond issue or a lower annual fixed dividend on a preferred stock issue.

Companies may also issue warrants to employees as a form of compensation, in which case they are referred to as employee stock options. When warrants are used as employee compensation, the goal is to align the objectives of the employees with those of the shareholders. Many companies compensate their senior management with salaries and some form of equity-based compensation, which may include employee stock options.

Example 4 describes the use of warrants to make a deal more attractive to an investor.

**EXAMPLE 4. WARRANTS**

On 25 August 2011, Bank of America, a US company, announced it had reached an agreement with Berkshire Hathaway, another US company; Berkshire Hathaway would invest $5 billion in Bank of America in exchange for preferred stock and warrants. Berkshire Hathaway received $5 billion in preferred stock, offering a fixed dividend of 6% per year, redeemable by Bank of America at any time at a 5% premium to the $5 billion par value. In addition to the preferred stock, Berkshire Hathaway received warrants to purchase 700 million shares of Bank of America common stock at an exercise price of $7.142857 per share. The warrants can be exercised at any time during the 10 years following the closing date of the transaction. In this example, the warrants served as a sweetener to the preferred stock issue. It is likely that the annual dividend of 6% on the preferred stock would have been higher in the absence of the warrants.

## 3.5 Depositary Receipts

A **depositary receipt** is a security representing an economic interest in a foreign company that trades like a common share on a domestic stock exchange. For investors buying shares of foreign companies, the transaction costs associated with purchasing depositary receipts are significantly lower than the costs of directly purchasing the stock on a foreign country's stock exchange.

Depositary receipts are not issued by the company and do not raise capital for the company, but rather, they are issued by financial institutions. Depositary receipts facilitate trading of a company's stock in countries other than the country where the company is located. Depositary receipts are often referred to as global depositary receipts (GDRs), but may be called by different names in different countries. In the United States, GDRs are known as American Depositary Receipts (ADRs) or American depositary shares. Depositary receipts are generally similar globally but may vary slightly because of different laws.

Now we will consider how depositary receipts are created and work, using the example of Sony and Mexican investors. Mexican investors may want to invest in the stock of Sony, a Japanese company, but Sony's stock is not listed on the Mexican Stock Exchange. Buying Sony stock on the Tokyo Stock Exchange is expensive and inconvenient for Mexican investors. To make this process easier, a financial institution in Mexico, such as a bank, can buy Sony's stock on the Tokyo Stock Exchange and make it available to Mexican investors. Rather than making the shares directly available for trading on the Mexican Stock Exchange, the bank holds the shares in custody and issues GDRs against the shares held. The Sony GDRs issued by the custodian bank are listed on the Mexican Stock Exchange for trading. In essence, the Sony GDRs trade like the stock of a domestic company on the Mexican Stock Exchange in the local currency (Mexican peso).

Depositary receipts, like the shares they are based on, have no maturity date (i.e., they have an infinite life). Depositary receipts typically do not offer their owners any voting rights even though they essentially represent common stock ownership; the custodian financial institution usually retains the voting rights associated with the stock.

Example 5 describes the depositary receipt of Vodafone Group in the United States.

**EXAMPLE 5. DEPOSITARY RECEIPTS**

The ordinary shares (common stock) of Vodafone, a UK company, trade on the London Stock Exchange. The company's stock trades on the NASDAQ exchange in the United States in the form of an American Depositary Receipt (ADR). The Bank of New York Mellon (BNY Mellon) is the financial institution that holds the ordinary shares in custody and issues ADRs of Vodafone against the ordinary shares of Vodafone held in custody. The ADRs of Vodafone are available for US and international investors. The ADRs are quoted in US dollars, and each one is equivalent to 10 ordinary shares. Unusually, BNY Mellon does not retain the voting rights associated with the shares, and ADR shareholders can instruct BNY Mellon on the exercise of voting rights relative to the number of ordinary shares represented by their holding of ADRs.

# RISK AND RETURN OF EQUITY AND DEBT SECURITIES 4

There are significant risk and return differences between debt and equity securities because of differences in cash flow, voting rights, and priority of claims.

Exhibit 1 shows the three main types of securities and their typical cash flow and voting rights.

**Exhibit 1 Cash Flow and Voting Rights by Security Type**

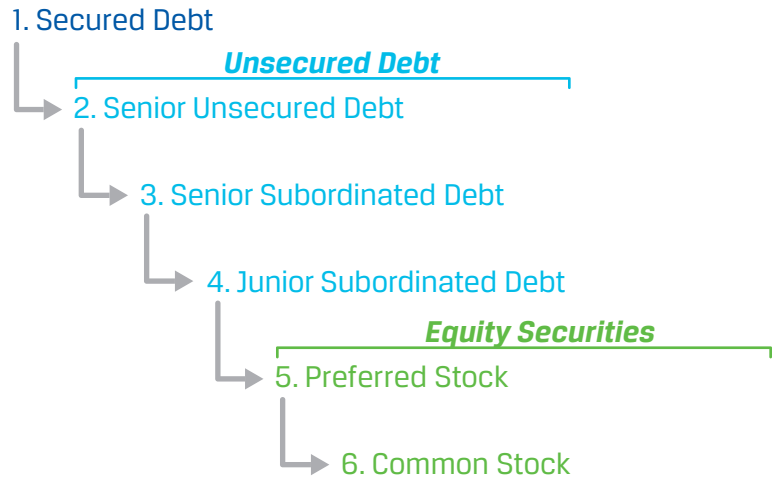
| Type of Security | Cash Flow Rights | Voting Rights |
|---|---|---|
| Common stock | Right to dividends if declared by the board of directors | Proportional to ownership |
| Preferred stock | Right to promised dividends if declared by the board of directors; board does not have a legal obligation to declare the dividends | None |
| Debt security | Legal right to promised cash flows | None |

The return potential for both debt securities and preferred stock is limited because the cash flows (interest, dividends, and repayment of par value) do not increase if the company performs well. The return potential to common shareholders is higher because the share price rises if the company performs well. Relative to holders of debt securities and preferred stock, common shareholders expect a higher return but must accept greater risk. The voting rights of common shareholders may give them some influence over the company's business decisions and thereby somewhat reduce risk.

Debt securities are the least risky because the cash flows are contractually obligated. Preferred stock is less risky than common stock because it ranks higher than common stock with respect to the payment of dividends. The risk of preferred stock is also reduced to some degree by the expectation of a dividend each year. Although the dividend is not a contractual obligation, companies are reluctant to omit dividends on preferred shares. Common stock is considered the riskiest of the three because it ranks last with respect to the payment of dividends and distribution of net assets if the company is liquidated.

In the event of the company being liquidated, assets are distributed following a priority of claims, or **seniority ranking**. This priority of claims can affect the amount that an investor will receive upon liquidation. Exhibit 2 illustrates the priority of claims.

**Exhibit 2 Priority of Claims**




Debt capital is borrowed money and represents a contractual liability of the company. Debt investors thus have a higher claim on the company's assets than equity investors.[4] After the claims of debt investors have been satisfied, preferred stock investors are next in line to receive what they are due. Common shareholders are last in line and known as the **residual claimants** in a company. Common shareholders share proportionately in the remaining assets after all other claims have been satisfied. If funds are insufficient to pay off all claims, equity investors will likely receive only a fraction of their investment back or may even lose their entire investment. Accordingly, investing in equity securities is riskier than investing in corporate debt securities.

Equity investors are at least protected by **limited liability**, which means that higher claimants, particularly debt investors, cannot recover money from other assets belonging to the shareholders if the company's assets are insufficient to fully cover their claims.[5] Because a company is a legal entity separate from its shareholders, it is responsible, at the corporate level, for all company liabilities. By legally separating the shareholders from the company, an individual shareholder's liability is limited to the amount he or she invested. So, shareholders cannot lose more money than they have invested in the company.

It is important to note that limited liability of shareholders can actually increase the losses of debt investors as the company approaches bankruptcy. As a company moves closer to a bankruptcy filing, shareholders do not have any incentive to maintain or upgrade the assets of the company because doing so might require additional capital, which they might be unwilling to invest. The consequent deterioration in asset quality hurts debt investors because the liquidation value of the company decreases. Debt investors are thus motivated to closely monitor the company's actions to ensure that the company operates in accordance with the debt contract.

**4** The priority of claims of debtholders is discussed in the Debt Securities chapter.
**5** An exception is cases of fraud and wilful negligence; in such situations, management and the board of directors may be held personally liable.

Given the fact that equity securities are riskier than debt securities, shareholders expect to earn higher returns on equity securities over the long term. Because equity is riskier than debt, risk-averse investors may prefer debt securities to equity securities. However, although debt is safer than equity for a given entity, debt securities are not risk-free; they are subject to many risk factors, which are discussed in the Debt Securities chapter.

Exhibit 3 shows annualised historical return and risk data on various equity and debt indices for the 1980–2010 period. Recall from the Quantitative Concepts chapter that the standard deviation of returns is often used as a measure of risk. The shaded rows in Exhibit 3 present return and risk data (based on standard deviation of returns) for six equity indices. The non-shaded rows present return and risk data for three bond indices.

**Exhibit 3 Historical Annual Returns on Equity and Debt Securities, 1980–2010**

| Index | Annual Return | Standard Deviation of Returns | |
|---|---|---|---|
| S&P 500 | 10.80% | 15.60% | Equity |
| Russell 2000 | 10.35 | 19.94 | |
| MSCI Europe | 10.81 | 17.80 | |
| MSCI Pacific Basin | 7.89 | 21.13 | |
| FTSE All World | 7.26 | 15.98 | |
| MSCI EAFE | 7.09 | 17.71 | |
| Lehman Brothers Corporate Bond | 8.82 | 7.23 | Debt |
| Barclays Capital Government Bond | 8.15 | 5.51 | |
| Merrill Lynch World Government Bond | 7.88 | 7.04 | |

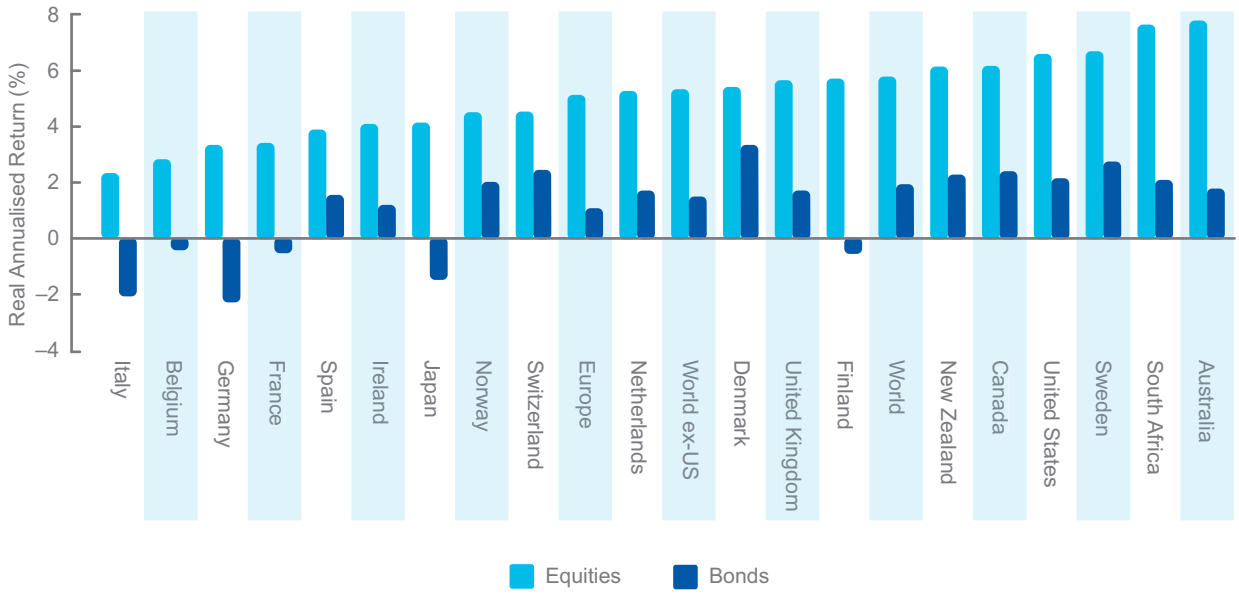
*Source:* Frank K. Reilly and Keith C. Brown, *Investment Analysis and Portfolio Management*, 10th ed. (Mason, OH: South-Western Cengage Learning, 2012).

The data are generally consistent with the expectation that riskier investments should generate higher returns over the long term. For the United States and Europe, annual equity returns (first three shaded indices) were higher than annual bond returns (non-shaded indices). Annual equity returns exhibited higher risk than annual debt returns. Note that for the three indices that include emerging economies (the last three shaded indices), however, annual equity returns were marginally lower than annual bond returns but more risky.

Exhibit 4 presents annual real returns (returns adjusted for inflation) on equity securities and government long-term bonds for 19 countries, Europe, the world, and the world excluding the United States (ex-US) for 1900–2010. Equity returns over the period are higher than government bond returns within every country and region. The real return (return adjusted for inflation) of equity securities ranged from approximately 2% to 7%. The real returns of government bonds ranged from approximately –2% (that is, they failed to cover inflation) to +3%. On average, government bonds have

beaten inflation, earning a modest positive real return per year. But in some countries, the return to bondholders was not sufficient to cover inflation, so bondholders lost purchasing power.

**Exhibit 4 Real Annualised Returns on Equities vs. Bonds Internationally, 1900–2010**




*Source:* E. Dimson, P. Marsh, and M. Staunton, *Credit Suisse Global Investment Returns Sourcebook 2011* (Zurich: Credit Suisse Research Institute, 2011).

# 5 VALUATION OF COMMON SHARES

Valuing common shares is a complex process because of their infinite life and the difficulty of estimating future company performance. There are three basic approaches to valuing common shares:

- Discounted cash flow valuation
- Relative valuation
- Asset-based valuation

Analysts frequently use more than one approach to estimate the value of a common share. Once an estimate of value has been determined, it can be compared with the current price of the share, assuming that the share is publicly traded, to determine whether the share is overvalued, undervalued, or fairly valued.

## 5.1 Discounted Cash Flow Valuation

The discounted cash flow (DCF) valuation approach takes into account the time value of money. This approach estimates the value of a security as the present value of all future cash flows that the investor expects to receive from the security. This valuation approach applied to common shares relies on an analysis of the characteristics of the company issuing the shares, such as the company's ability to generate earnings, the expected growth rate of earnings, and the level of risk associated with the company's business environment.

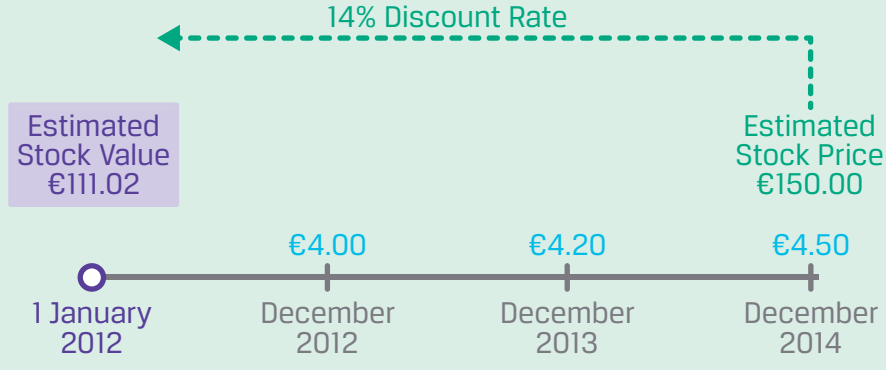
Common shareholders expect to receive two types of cash flows from investing in equity securities: dividends and the proceeds from selling their shares. Example 6 illustrates the application of the DCF approach, using estimates of dividends and selling price, for a common share of Volkswagen.

**EXAMPLE 6. DISCOUNTED CASH FLOW APPROACH**

On 1 January 2012, an investor expects Volkswagen, a German company, to generate dividends of €4.00 per share at the end of 2012, €4.20 per share at the end of 2013, and €4.50 per share at the end of 2014. Furthermore, the investor estimates that the stock price of Volkswagen will trade at €150.00 per share at the end of 2014. Note that, under the DCF valuation approach, the expected price of Volkswagen stock at the end of 2014 (€150.00 per share) represents the present value of cash flows to investors expected to be generated by the company beyond 2014. The investor considers all risks and concludes that a discount rate of 14% is appropriate. In other words, the investor wants to earn at least an annual rate of return of 14% by investing in Volkswagen.

The estimated value of a Volkswagen share using the DCF valuation approach is equal to the present value of the cash flows the investor expects to receive from the equity investment. The investor computes the present value of the expected cash flows as follows:

$$\text{Value} = \frac{4.00}{(1+0.14)^1} + \frac{4.20}{(1+0.14)^2} + \frac{4.50}{(1+0.14)^3} + \frac{150.00}{(1+0.14)^3} = €111.02$$



So, the investor's estimated value of Volkswagen on a per share basis is €111.02. If shares of Volkswagen are priced at less than €111.02 on 1 January 2012, the investor may conclude that the stock is undervalued and decide to buy it. Alternatively, if the stock is priced at more than €111.02, the investor may conclude that the stock is overvalued and decide not to buy.

The DCF valuation approach can also be used to value preferred shares. Valuing preferred shares is typically easier than for common shares because the expected dividends are specified and do not change over time. The value of a preferred share, with a fixed dividend and no maturity date, is the discounted value of the future dividends, which is equal to the dividend divided by the discount rate.

## 5.2 Relative Valuation

The relative valuation approach estimates the value of a common share as the multiple of some measure, such as earnings per share (EPS) or revenue per share. The multiple is determined based on price and the relevant measure for publicly traded, comparable equity securities. The key assumption of the relative valuation approach is that common shares of companies with similar risk and return characteristics should have similar values. Relative valuation relies on the use of price multiples of comparable, publicly traded companies or an industry average.

One multiple commonly used in relative valuation is the **price-to-earnings ratio** (P/E), which is the ratio of a company's stock price to its EPS. For instance, a publicly traded company that generates annual earnings per share of $1.00 and is trading at $12 per share has a P/E (or price-to-earnings multiple) of 12. Example 7 illustrates two applications of the relative valuation approach.

**EXAMPLE 7. RELATIVE VALUATION**

1. An investor is estimating the value of an airline's common stock on a per share basis. The airline in question generates annual EPS of €2.00. The investor finds that the average price-to-earnings multiple or P/E for the industry is 9. Using relative valuation, the investor estimates that the value of the airline's stock, on a per share basis, is €18.00 (= €2.00 × 9).

2. An investor is estimating the value of the common stock of Ford Motor Company, a US automobile manufacturing company, on a per share basis. Analysts estimate that Ford will generate EPS of $1.60 next year. The investor gathers information, shown in the second and third columns of the following table, on three competing automobile makers: General Motors, Toyota, and Honda. The investor calculates the P/E (shown in the fourth column) for each of the three companies. The investor then calculates the average P/E for the three companies as 9 [= (8 + 10 + 9)/3].

| Company | Current Stock Price | Next Year's Estimated EPS | P/E |
|---|---|---|---|
| General Motors | $40.00 | $5.00 | $40.00/$5.00 = 8 |
| Toyota | $85.00 | $8.50 | $85.00/$8.50 = 10 |
| Honda | $36.00 | $4.00 | $36.00/$4.00 = 9 |
| Average | | | (8 + 10 + 9)/3 = 9 |

The investor estimates the value of Ford common stock, on a per share basis, is $14.40 (= $1.60 × 9). It is important to note that even though the P/E is 9 in both examples, this does not mean that 9 is a typical P/E.

One issue with the use of the relative valuation approach is that price multiples change with investor sentiment. Companies trade at higher multiples and as a result of higher market prices when investors are optimistic and at lower multiples and prices when investors are pessimistic.

## 5.3 Asset-Based Valuation

The asset-based valuation approach estimates the value of common stock by calculating the difference between the value of a company's total assets and its outstanding liabilities. In other words, the asset-based valuation approach estimates the value of common equity by calculating a company's net asset value. The asset-based valuation approach implicitly assumes that the company is liquidated, sells all its assets, and then pays off all its liabilities. The residual value after paying off all liabilities is the value to the shareholders.

The difference between total assets and total liabilities on a company's balance sheet represents shareholders' equity, or the book value of equity. But the values of some assets on the balance sheet are based on historical cost (the cost when they were purchased), and the actual market value of these assets may be very different. For instance, the value of land on a company's balance sheet, typically carried at historical cost, may be quite different from its current market value. As a result, estimating the value of the equity of a company using asset values taken directly from the balance sheet may provide a misleading estimate. To improve the accuracy of the value estimate, current market values can be estimated instead.

Also, some assets may not be included on the balance sheet because of financial reporting rules. For instance, some internally developed intangible assets, such as a brand or reputation, are not listed in the financial reports. It is important that analysts using asset-based valuation estimate reasonable values for *all* of a company's assets, which can be very challenging to do.

### 5.4 Implicit Assumptions of Valuation Approaches

The DCF valuation approach relies solely on estimates of a company's future cash flows and implicitly assumes that the company will continue to operate forever. In contrast, the asset-based valuation approach implicitly assumes that the company will stop operating and essentially provides a liquidation value.

The relative valuation approach does not estimate future cash flows but instead uses price multiples of other comparable, publicly traded companies to arrive at an estimate of equity value. These price multiples rely on performance measures, such as EPS or revenue per share, to estimate value. The relative valuation approach implicitly assumes that common shares of companies with similar risk and return characteristics should have similar price multiples.

## 6 COMPANY ACTIONS THAT AFFECT EQUITY OUTSTANDING

Companies undertake major changes as they grow, evolve, mature, or merge with another company. Some of these changes result in changes to the number of common shares outstanding—the number of common shares currently held by shareholders. Various corporate actions can affect equity outstanding:

- Selling shares to the public for the first time (when a private company becomes a public company), referred to as an **initial public offering** (IPO)
- Selling shares to the public in an offering subsequent to the initial public offering, referred to as a **seasoned equity offering** or **secondary equity offering**
- Buying back existing shares from shareholders, referred to as a **share repurchase** or **share buyback**
- Issuing a **stock dividend** or conducting a **stock split**
- Issuing new stock after the exercise of warrants
- Issuing new stock to finance an acquisition
- Creating a new company from a subsidiary in a process referred to as a spinoff

Each of these actions and their effects are discussed in the following sections.

### 6.1 Initial Public Offering

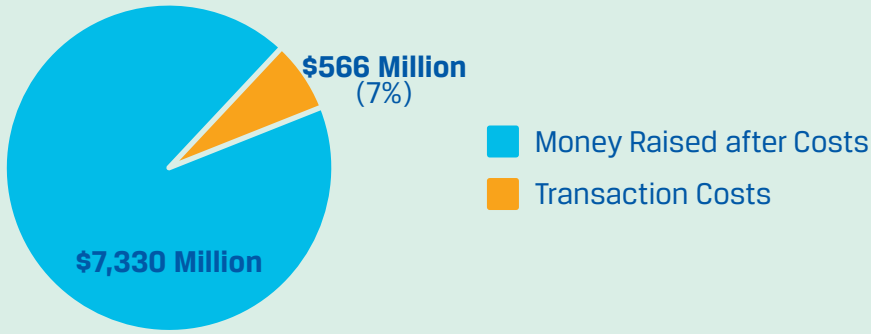
The main difference between a private company and a publicly traded company is that the shares of a private company are available only to select investors and are not traded on a public market. A private company becomes a publicly traded company through an IPO, which is the first time that it sells new shares to investors in a public market.

Private companies become publicly traded companies for a number of reasons. First, it gives the company more visibility, which makes it easier to raise capital to fund growth opportunities. It also helps attract talented staff, raise brand awareness, and gain credibility with trading partners. In addition, it provides greater liquidity for shareholders who want to sell their shares or buy additional shares. At or after the IPO, some of the original shareholders may choose to sell some of their shares. The fact that the shares now trade in a public market makes the shares more liquid and thus easier to sell.

A disadvantage to becoming a public company is increased regulatory and disclosure requirements. IPOs are also expensive; their cost can be as much as 10% of the proceeds. Example 8 gives an example of how costly an IPO can be.

**EXAMPLE 8. INITIAL PUBLIC OFFERING**

Glencore International, a Swiss company founded in 1974, announced in April 2011 its intention to become a publicly traded company. The shares were to trade on both the London Stock Exchange (LSE) and the Hong Kong Stock Exchange (HKSE). The company raised $7,896 million, but had to pay transaction costs of $566 million (about 7% of the entire proceeds of the IPO).




## 6.2 Seasoned Equity Offering

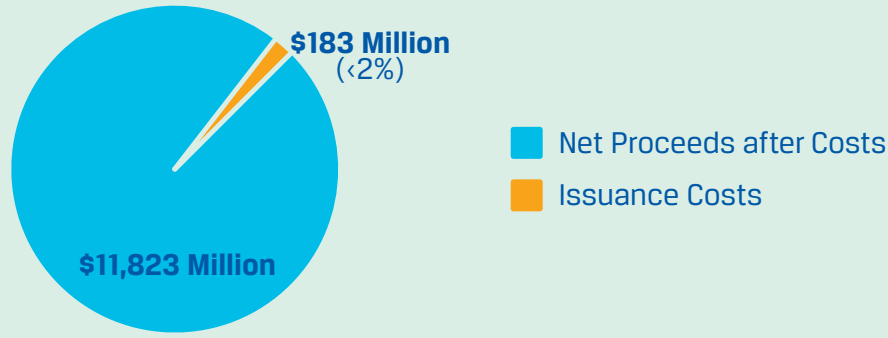
After an IPO, publicly traded companies may sell additional shares to raise more capital. The selling of new shares by a publicly traded company after an IPO is referred to as a seasoned or secondary equity offering. A seasoned equity offering typically has far lower costs associated with it compared with an IPO.

A typical seasoned equity offering increases the number of shares outstanding by 5%–20%. For an existing investor who does not buy additional shares in the seasoned equity offering, the increase in shares outstanding dilutes the investor's ownership percentage.

Example 9 gives an example of a seasoned equity offering and the associated costs.

**EXAMPLE 9. SEASONED EQUITY OFFERING**

On 1 October 2008, General Electric, a US company that has traded publicly since 1896, announced it would sell additional shares to the public in a seasoned equity offering. According to the 2008 annual report, 547.8 million shares were issued at $22.25 share (= $12,189 million = 547.8 million × $22.25). The net proceeds were $12,006 million, which implies issuance costs of $183 million (= $12,189 million – $12,006 million, less than 2% of the proceeds). The issuance costs for this seasoned offering are much lower than the costs of the IPO in Example 8.




## 6.3 Share Repurchases

Companies may choose to return cash to shareholders by repurchasing shares rather than paying dividends. Assuming that the company's net income is unaffected by the repurchase, the share repurchase will increase the company's earnings per share because net income will be divided by a smaller number of shares. Repurchased shares are either cancelled or kept and reported as treasury stock in the shareholders' equity account on the company's balance sheet. Treasury shares are not included in the number of shares outstanding.

To buy back shares, a company can buy shares on the open market just like other investors or it can make a formal offer for repurchase directly to shareholders. Shareholders may choose to sell their shares or to remain invested in the company. For an existing investor who does not sell shares, the decrease in the number of shares outstanding effectively increases that investor's ownership percentage.

Example 10 compares a share repurchase and a dividend distribution.

**EXAMPLE 10. SHARE REPURCHASE**

A company with 2 million common shares outstanding and a current stock price of $50 wants to distribute $1 million to its shareholders. The company could pay a dividend of 50 cents per share ($1 million/2 million shares) or buy back 20,000 shares from shareholders willing to sell their shares (20,000 shares

× $50 = $1,000,000), assuming that the company can buy the shares at their current market value. After the repurchase, the number of shares outstanding would decrease to 1.98 million (2 million – 20,000).

## 6.4 Stock Splits and Stock Dividends

Companies may, on occasion, conduct stock splits or issue stock dividends. A stock split is when a company replaces one existing common share with a specified number of common shares. A stock dividend is a dividend in which a company distributes additional shares to its common shareholders. Stock splits and stock dividends both *increase* the number of shares outstanding, but they do not change any single shareholder's proportion of ownership.

When a company splits its stock or issues a stock dividend, the number of shares outstanding increases and additional shares are issued proportionally to existing shareholders based on their current ownership percentages. The overall value of the company should not change, so the price of each share should decrease. But the value of any single shareholder's total shares should not change in value. Example 11 illustrates the effects of a stock split and a stock dividend on the stock price, number of shares, and total shareholder value.

**EXAMPLE 11. EFFECTS OF A STOCK SPLIT AND A STOCK DIVIDEND**

A company has 24,000 shares outstanding and each share trades at €75.00. An investor owns 900 shares.

***Stock Split***

The company announces a three-for-two stock split. This means for every two shares the investor currently owns, she will receive three shares in replacement. So, she will have 1,350 shares after the stock split.

(900/2) × 3 = 1,350 shares

***Stock Dividend***

The company declares a 50% stock dividend—that is, for every share the investor currently owns, she will receive an additional 0.5 shares. In other words, she will have 1,350 shares.

900 × 1.5 = 1,350 shares

The effects of the stock split and stock dividend are shown in the following table.

| | Stock Price | Number of Shares Outstanding | Total Value |
|---|---|---|---|
| *Before Stock Split* | | | |
| Company | €75.00 | 24,000 | €1,800,000 |
| Investor | 75.00 | 900 | 67,500 |
| *After Stock Split* | | | |
| Company | €50.00 | 36,000 | €1,800,000 |
| Investor | 50.00 | 1,350 | 67,500 |
| *Before Stock Dividend* | | | |
| Company | €75.00 | 24,000 | €1,800,000 |
| Investor | 75.00 | 900 | 67,500 |
| *After Stock Dividend* | | | |
| Company | €50.00 | 36,000 | €1,800,000 |
| Investor | 50.00 | 1,350 | 67,500 |

As Example 11 illustrates, a stock split or stock dividend does not change each shareholder's proportional ownership of the company. Shareholders do not invest any additional money for the increased number of shares, and the stock split or stock dividend does not have any effect on the company's operations. The total value of the company's shares and an investor's shares are unchanged by the stock split or stock dividend.

Given that stock splits and stock dividends do not have any effect on company operations or value, why do you think companies take these actions? One explanation is that as a company does well and its assets and profits increase, the stock price is likely to increase. At some point, the stock price may get so high that shares become unaffordable to some investors and liquidity decreases. A stock split or stock dividend will have the effect of lowering a company's stock price, making the stock more affordable to investors, and thereby improving liquidity.

It is important to note that the affordability of a company's stock is different from whether the stock is undervalued or overvalued. That is, a company with a stock price of $500 per share may be unaffordable to some investors, but may still be considered undervalued when the price per share is compared with the estimated value per share. Similarly, a company with a stock price of $5 per share may be affordable to most investors yet still be overvalued.

Companies with very *low* stock prices may conduct a reverse stock split to increase their stock price. In this case, the company *reduces* the number of shares outstanding. The primary reason for a reverse stock split is that a company may face the risk of having its shares delisted from a public exchange if its stock price falls below a minimum level dictated by the exchange. After the reverse stock split, shareholders will still own the same proportion of the shares they originally owned. In other words, a reverse stock split reduces the number of shares outstanding but does not affect a shareholder's proportional ownership of the company. After a reverse stock split, the stock price should increase by the same multiple as the reverse stock split. Example 12 describes a 1-for-10 reverse stock split by Citigroup.

**EXAMPLE 12. REVERSE STOCK SPLIT**

On 21 March 2011, Citigroup, a US company, announced a 1-for-10 reverse stock split effective after the close of trading on 6 May 2011. Before the split, Citigroup had approximately 29 billion shares outstanding. The closing stock price of Citigroup on 6 May was $4.52. After the reverse split, the number of shares outstanding decreased to approximately 2.9 billion. On the next trading day after the reverse stock split took effect, which was 9 May, the opening stock price was $44.89; this price is about ten times the pre-split price of $4.52.

## 6.5 Exercise of Warrants

Companies that issue warrants as a form of additional or bonus compensation to employees may have to increase shares outstanding if the warrants are exercised. If an investor exercises warrants, the issuing company's number of shares outstanding increases and all other existing shareholders of the company's stock will see their ownership percentage decrease. Given that there may be numerous employees who exercise warrants on a recurring basis, companies that issue warrants to employees as a form of compensation will typically experience an increase in shares outstanding every year. To mitigate the dilution effect on existing shareholders, these companies may repurchase a small amount of shares each year to offset the additional shares issued when warrants are exercised.

## 6.6 Acquisitions

One company may acquire another by agreeing to buy all of its shares outstanding. All of the outstanding shares of the acquired company are redeemed for cash, for stock in the acquiring company, or for a combination of cash and stock of the acquiring company. Shareholders of the acquiring company and the target company (the company to be acquired) are typically asked to vote on a proposed acquisition. If the company being acquired is small and the acquirer has sufficient cash, there is no need to issue new shares.

For larger acquisitions, the acquiring company may pay for the purchase by issuing new shares. The amount of new shares issued depends on the purchase price and the ratio of the two companies' stock prices. An acquisition in which the company uses its stock to finance the transaction results in an increase in the acquiring company's shares outstanding. For existing shareholders in the acquiring company, the increased shares outstanding effectively dilutes their ownership percentage.

## 6.7 Spinoffs

A company may create a new company from an existing subsidiary in a process referred to as a **spinoff**. Shares of the new entity are distributed to the parent company's existing shareholders. After the spinoff, the value of the shares of the parent company initially declines as the assets of the parent company are reduced by the amount allocated to the new company. But shareholders receive the shares of the newly formed company to compensate them for the decrease in value.

A company's management may conduct a spinoff in an effort to create value for its shareholders by splitting the company into two separate businesses. The rationale behind a spinoff is that the market may assign a higher valuation to two separate but more specialised companies compared with the value assigned to these entities when they were part of the parent company.

## SUMMARY

Equity securities are an important way for companies to raise financing to fund their activities. They are also popular assets among investors, who are attracted by their potential returns. However, equities are riskier than debt securities and must be analysed with care and skill.

The following points recap what you have learned in this chapter about equity securities:

- Companies often issue different types or classes of equity securities. The types of equity securities, or equity-like securities, that companies may issue include common shares, preferred shares, convertible bonds, and warrants.
- Equity securities are typically characterised by four main features: specified life (infinite or with a maturity date), par value, voting rights, and cash flow rights.
- Debt securities include contractual obligations to pay a return to the debt providers. Equity securities, however, contain no such contractual obligations. A company does not have to repay the amounts contributed by the shareholders or pay a dividend.
- The board of directors, elected by the common shareholders, plays an important role in monitoring the company's business activities and management on behalf of its shareholders. The board is also responsible for declaring dividends on shares of the company.
- Common stock is the main type of equity security issued by a company. Common shares have an infinite life and may or may not have a par value. A common share represents an ownership interest in a company. Common shareholders have a residual claim on the net assets of the company and typically have voting rights.
- Preferred shares typically offer fixed dividends, based on stated par values and dividend rates. Generally, preferred shareholders have no voting rights or ownership claim on the company.
- A convertible bond is a bond issued by a company that offers the bondholder the right to convert the bond into a specified number of common shares. It has features of and relationships with both equity and debt securities.
- A warrant is an equity-like security that entitles the holder to buy a specified amount of common stock of the issuing company at a specified price per share prior to the warrant's expiration date.

- A depositary receipt is a security representing an interest in a foreign company that trades like a common share on a domestic stock exchange. It is not issued by the foreign company.
- In the event of liquidation, priority of claims states that debt investors rank higher than preferred shareholders and preferred shareholders rank higher than common shareholders.
- Relative to preferred stock, common stocks offer the potential for a higher return but with greater investment risk.
- Equity securities are riskier than debt securities, and empirical data suggest that equity securities earn higher returns than debt securities, thereby compensating investors for the higher risk.
- Common approaches used to value common shares include discounted cash flow valuation, relative valuation, and asset-based valuation approaches.
- The discounted cash flow approach estimates the value of a security as the present value of its expected future cash flows to its holder.
- The relative valuation approach estimates the value of a common share as the multiple of some measure, such as earnings per share. This approach implicitly assumes that common shares of companies with similar risk and return characteristics should have similar price multiples.
- The asset-backed valuation approach estimates the value of common stock of a company as the difference between the value of its total assets and liabilities, in other words, as its net asset value.
- Some corporate actions result in changes to the number of common shares outstanding. Such actions include initial public offerings, seasoned equity offerings, share repurchases, stock splits, stock dividends, acquisitions, and spinoffs.

# CHAPTER REVIEW QUESTIONS

1 Which of the following securities *most likely* provides voting rights to investors?

A Common shares

B Preferred shares

C Depositary receipts

2 The right to elect members of the board of directors of a company belongs to that company's:

A senior management.

B common shareholders.

C preferred shareholders.

3 Which of the following is *most likely* an advantage of owning common stock?

A Low risk

B Finite life

C Limited liability

4 The key difference between cumulative preferred stock and non-cumulative preferred stock relates to:

A voting rights.

B the treatment of missed dividends.

C the company's ability to buy back the preferred shares.

5 Compared with a preferred shareholder, a common shareholder *most likely* has:

A voting rights.

B limited liability.

C cash flow rights.

6 All else being equal, the fixed coupon rate on a convertible bond compared with a straight bond is *most likely*:

A lower.

B the same.

C higher.

**7** Compared with preferred shareholders, the ranking of common shareholders in the priority of claims on the company's net assets upon liquidation is:

**A** equal.

**B** lower.

**C** higher.

**8** A security representing an economic interest in a foreign company that trades like a common stock on a local stock exchange is *most likely* a:

**A** warrant.

**B** convertible bond.

**C** depositary receipt.

**9** Depositary receipts are issued by:

**A** governments.

**B** financial institutions.

**C** the company whose shares are represented by the depositary receipts.

**10** If the price of a company's common shares increases significantly, the conversion value of a convertible bond issued by that company *most likely*:

**A** increases.

**B** decreases.

**C** remains unchanged.

**11** Stock options issued by a company to its employees as a form of compensation are an example of:

**A** warrants.

**B** convertible bonds.

**C** depositary receipts.

**12** Compared with common shares, an investment in preferred shares is *most likely* to be:

**A** less risky.

**B** more risky.

**C** equally risky.

**13** Compared with the expected return on an investment in preferred shares, the expected return on an investment in common shares is *most likely* to be:

**A** equal.

**B** lower.

**C** higher.

**14** The discounted cash flow approach to valuation of a company's common shares *most likely* considers the:

**A** expected dividends on the shares.

**B** current value of the company's assets.

**C** price-to-earnings ratios of comparable companies.

**15** The approach to valuing common shares that uses price multiples of other comparable, publicly traded companies *best* describes:

**A** relative valuation.

**B** asset-based valuation.

**C** discounted cash flow valuation.

**16** A company that needs to raise capital in a public market for the first time would *most likely*:

**A** repurchase shares.

**B** conduct an initial public offering.

**C** conduct a seasoned equity offering.

**17** The process of a publicly traded company raising additional capital by selling new shares to the public *best* describes a:

**A** stock dividend.

**B** share repurchase.

**C** seasoned equity offering.

**18** Which of the following corporate actions would decrease a company's number of outstanding shares?

**A** Share repurchase

**B** Exercise of warrants

**C** Seasoned equity offering

**19** After a company conducts a stock split, a common shareholder's proportional ownership will *most likely*:

**A** increase.

**B** decrease.

**C** remain unchanged.

**20** The process of a company creating a new company from an existing subsidiary *best* describes a:

**A** spinoff.

**B** stock split.

**C** reverse stock split.

**21** The corporate action *most likely* taken to mitigate the effects of exercised warrants is:

**A** a stock dividend.

**B** an issuance of new shares.

**C** a share repurchase program.

# ANSWERS

**1** A is correct. Common shareholders usually have the right to vote on certain matters. B is incorrect because preferred shareholders are not generally entitled to voting rights. C is incorrect because depositary receipts are securities that represent an economic interest in a foreign company, are issued by a custodian financial institution, and trade like common stock on a local stock exchange. Although they essentially represent common stock ownership, they typically do not offer their owners any voting rights because the custodian financial institution usually retains the voting rights associated with the stock.

**2** B is correct. Common shareholders collectively elect members of the board of directors, whose job it is to monitor the company's business activities on behalf of its shareholders. A is incorrect because senior management is appointed/hired by the board of directors, not the other way around. C is incorrect because preferred shareholders are usually not entitled to voting rights.

**3** C is correct. By legally separating the shareholders from the company, an individual shareholder's liability is limited to the amount he or she invested. A is incorrect because investing in common stock carries relatively high risk. B is incorrect because common stock is issued without maturity dates. Thus, it has an infinite life.

**4** B is correct. Cumulative preferred stock requires that the company pay in full any missed dividends (dividends promised but not paid in prior years) before paying dividends to common shareholders. By comparison, non-cumulative (or straight) preferred stock does not require that missed dividends from prior years be paid before dividends are paid to common shareholders. A is incorrect because preferred shareholders are usually not entitled to voting rights, irrespective of whether the preferred stock is cumulative or non-cumulative. C is incorrect because the redemption feature (that is, the company's ability to buy back the preferred shares) is unrelated to the distinction between cumulative and non-cumulative preferred stock.

**5** A is correct. Except in rare circumstances, preferred shareholders do not possess voting rights. By contrast, common shareholders receive voting rights. B and C are incorrect because both preferred and common shareholders have limited liability and possess cash flow rights to declared dividends and in liquidation.

**6** A is correct. Because the conversion feature represents a benefit to the bondholder, a convertible bond typically offers the bondholder a lower fixed annual coupon rate than that of a comparable bond without a conversion feature (a straight bond).

**7** B is correct. Common shareholders are last in line if the company is liquidated; they are the residual claimants in a company. Thus, common shareholders have a lower claim on the company's net assets (that is, the difference between a company's total assets and its outstanding liabilities) upon liquidation than preferred shareholders.

**8** C is correct. A depositary receipt is a security representing an economic interest in a foreign company that trades like a common stock on a local stock exchange. A is incorrect because a warrant is an equity-like security that entitles the holder to buy a pre-specified amount of common stock of the issuing company at a pre-specified share price prior to a pre-specified expiration date. B is incorrect because a convertible bond is a type of debt security issued by a company that offers the holder the right to convert the bond into a pre-specified number of common shares.

**9** B is correct. Depositary receipts are securities representing an economic interest in a foreign company that trade like common stock on a local stock exchange. They are issued by a custodian financial institution that is located in the domestic country. The financial institution buys the shares in the foreign country, holds them in custody, issues depositary receipts against the shares held, and sells the depositary receipts to domestic investors who can trade them on the local stock exchange. A and C are incorrect because governments and the company whose shares are represented by the depositary receipts do not issue depositary receipts.

**10** A is correct. If the price of a company's common shares increases significantly, the conversion value of a convertible bond issued by that company increases.

**11** A is correct. Stock options issued by a company to its employees as a form of compensation are an example of warrants. A warrant is an equity-like security that entitles the holder to buy a pre-specified amount of common stock of the issuing company at a pre-specified share price prior to a pre-specified expiration date. By issuing stock options to its employees, the company's goal is to align the objectives of the employees (such as senior management) with those of the shareholders. B is incorrect because a convertible bond is a bond issued by a company that offers the bondholder the right to convert the bond into a pre-specified number of common shares. C is incorrect because a global depository receipt is a security representing an economic interest in a foreign company that trades like a common share on a local stock exchange.

**12** A is correct. Preferred shares are less risky than common shares because they rank higher than common shares with respect to the payment of dividends and distribution of net assets upon liquidation. The risk of preferred shares is also reduced to some degree by the expectation of a fixed dividend each year.

**13** C is correct. Common shares are considered riskier than preferred shares, but they offer a higher expected return. If a company does very well, common shareholders stand to benefit greatly whereas preferred shareholders only receive the fixed dividend.

**14** A is correct. The discounted cash flow approach to valuation estimates the value of a security as the present value of all future cash flows that the investor expects to receive from the security. Common shareholders expect to receive two types of cash flows: dividends and the proceeds from selling their shares. Thus, the expected dividends on the shares are an important component of the discounted cash flow valuation approach. B is incorrect because the valuation approach that considers the current value of the company's assets is the asset-based valuation approach. C is incorrect because the valuation approach that considers the price-to-earnings ratios of comparable companies is the relative

valuation approach. In such a valuation approach, the value of a common share is estimated by using multiples based on market prices and some other measure for comparable, publicly traded companies.

**15** A is correct. The relative valuation approach estimates the value of a common share by using multiples based on prices and some other measure for comparable, publicly traded companies. One multiple commonly used is the price-to-earnings ratio, which is the ratio of a company's share price to its earnings per share. B is incorrect because the asset-based valuation approach estimates the value of common shares by calculating the company's net asset value—that is, the difference between a company's total assets and its outstanding liabilities. C is incorrect because the discounted cash flow valuation approach estimates the value of a common share as the present value of all future cash flows that the investor expects to receive from the common share.

**16** B is correct. An initial public offering (IPO) is a way for a company to raise capital in a public market for the first time. In the process, the company becomes a publicly traded company. A is incorrect because share repurchases require the company to use capital to buy back (or repurchase) shares from existing shareholders. C is incorrect because publicly traded companies may raise additional capital by selling additional shares in a seasoned (or secondary) equity offering subsequent to the IPO.

**17** C is correct. The selling of new shares to the public by a publicly traded company to raise additional capital is referred to as a seasoned (or secondary) equity offering. A and B are incorrect because stock dividends and share repurchases do not raise additional capital for the company. In a stock dividend, the company distributes new shares at no cost to existing shareholders. In a share repurchase, the company buys back (or repurchases) shares from existing shareholders.

**18** A is correct. In a share repurchase, the company buys back (or repurchases) shares from existing shareholders, which decreases the number of shares outstanding. B and C are incorrect because the exercise of warrants and seasoned equity offerings increase the number of shares outstanding.

**19** C is correct. A stock split replaces one existing common share with a specified number of common shares. It increases the number of shares outstanding but does not change any single shareholder's proportion of ownership.

**20** A is correct. The process of a company creating a new company from an existing subsidiary is called a spinoff. Shares of the new entity are distributed to the parent company's existing shareholders. B is incorrect because a stock split simply replaces one existing common share with a specified number of common shares, which increases the number of shares outstanding. C is incorrect because a reverse stock split reduces the number of shares outstanding. Stock splits and reverse stock splits do not change any single shareholder's proportion of ownership.

**21** C is correct. Whenever warrants are exercised, there is an increase in the total number of shares outstanding of a company. To mitigate the dilution effect on existing shareholders, companies typically buy back or repurchase shares in the open market to offset the shares issued when warrants are exercised. A is incorrect because issuing a stock dividend increases the number of shares

outstanding but does not change any single shareholder's proportional ownership. A stock dividend does nothing to mitigate the dilution effect created by the exercise of warrants. B is incorrect because issuing new shares compounds the effect of the exercised warrants, increasing the dilution effect on existing shareholders.