High Diminsional Statistics-Sheet 3-Exercise 4

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Prepare

First of all we import the dataset:

```r
url <- "https://hastie.su.domains/ElemStatLearn/datasets/prostate.data"
df <- read.table(url, sep = '\t', header = TRUE)
df %>% head(10)
```

```
##     X     lcavol  lweight age       lbph svi       lcp gleason pgg45       lpsa
## 1   1 -0.5798185 2.769459  50 -1.3862944   0 -1.386294       6     0 -0.4307829
## 2   2 -0.9942523 3.319626  58 -1.3862944   0 -1.386294       6     0 -0.1625189
## 3   3 -0.5108256 2.691243  74 -1.3862944   0 -1.386294       7    20 -0.1625189
## 4   4 -1.2039728 3.282789  58 -1.3862944   0 -1.386294       6     0 -0.1625189
## 5   5  0.7514161 3.432373  62 -1.3862944   0 -1.386294       6     0  0.3715636
## 6   6 -1.0498221 3.228826  50 -1.3862944   0 -1.386294       6     0  0.7654678
## 7   7  0.7371641 3.473518  64  0.6151856   0 -1.386294       6     0  0.7654678
## 8   8  0.6931472 3.539509  58  1.5368672   0 -1.386294       6     0  0.8544153
## 9   9 -0.7765288 3.539509  47 -1.3862944   0 -1.386294       6     0  1.0473190
## 10 10  0.2231436 3.244544  63 -1.3862944   0 -1.386294       6     0  1.0473190
##    train
## 1   TRUE
## 2   TRUE
## 3   TRUE
## 4   TRUE
## 5   TRUE
## 6   TRUE
## 7  FALSE
## 8   TRUE
## 9  FALSE
## 10 FALSE
```

Question 1

Build the regression model for the variable prostate antigen (lpsa)

```
##
## Call:
## lm(formula = df$lpsa ~ ., data = features)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.76644 -0.35510 -0.00328  0.38087  1.55770
##
## Coefficients:
```

```
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.181561   1.320568   0.137  0.89096
## lcavol       0.564341   0.087833   6.425 6.55e-09 ***
## lweight      0.622020   0.200897   3.096  0.00263 **
## age         -0.021248   0.011084  -1.917  0.05848 .
## lbph         0.096713   0.057913   1.670  0.09848 .
## svi          0.761673   0.241176   3.158  0.00218 **
## lcp         -0.106051   0.089868  -1.180  0.24115
## gleason      0.049228   0.155341   0.317  0.75207
## pgg45        0.004458   0.004365   1.021  0.31000
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6995 on 88 degrees of freedom
## Multiple R-squared:  0.6634, Adjusted R-squared:  0.6328
## F-statistic: 21.68 on 8 and 88 DF,  p-value: < 2.2e-16
```

The estimators of b_0 and $b_j \in \{1, ..., 8\}$ are respectively:

```
coef(model_lm)
```

```
##  (Intercept)       lcavol      lweight          age         lbph          svi
##  0.181560845  0.564341280  0.622019788 -0.021248185  0.096712522  0.761673402
##          lcp      gleason        pgg45
## -0.106050939  0.049227934  0.004457512
```

Question 2

Build the regression model with L1-constraint on the parameters. Estimate then the co- efficients and plot them.

```
#model_lasso <- l1ce(df$lpsa ~ . , data = features)
model_lasso <- glmnet(features, df$lpsa, alpha = 1)
model_lasso
```

Building the model

```
##
## Call:  glmnet(x = features, y = df$lpsa, alpha = 1)
##
##    Df  %Dev  Lambda
## 1   0  0.00 0.84340
## 2   1  9.16 0.76850
## 3   1 16.76 0.70020
## 4   1 23.07 0.63800
## 5   1 28.32 0.58130
## 6   1 32.67 0.52970
## 7   1 36.28 0.48260
## 8   1 39.28 0.43980
## 9   2 42.21 0.40070
## 10  2 44.90 0.36510
## 11  3 48.01 0.33270
## 12  3 50.66 0.30310
## 13  3 52.85 0.27620
## 14  3 54.68 0.25160
```

```
## 15  3 56.19 0.22930
## 16  3 57.45 0.20890
## 17  3 58.49 0.19040
## 18  3 59.36 0.17350
## 19  3 60.08 0.15800
## 20  3 60.67 0.14400
## 21  4 61.23 0.13120
## 22  5 61.75 0.11960
## 23  5 62.24 0.10890
## 24  5 62.64 0.09926
## 25  5 62.98 0.09044
## 26  5 63.25 0.08240
## 27  5 63.49 0.07508
## 28  5 63.68 0.06841
## 29  6 63.89 0.06234
## 30  6 64.21 0.05680
## 31  6 64.48 0.05175
## 32  6 64.70 0.04715
## 33  6 64.88 0.04297
## 34  6 65.03 0.03915
## 35  7 65.16 0.03567
## 36  7 65.27 0.03250
## 37  7 65.36 0.02961
## 38  7 65.44 0.02698
## 39  7 65.50 0.02459
## 40  7 65.55 0.02240
## 41  8 65.67 0.02041
## 42  8 65.78 0.01860
## 43  8 65.88 0.01695
## 44  8 65.95 0.01544
## 45  8 66.02 0.01407
## 46  8 66.07 0.01282
## 47  8 66.12 0.01168
## 48  8 66.16 0.01064
## 49  8 66.19 0.00970
## 50  8 66.21 0.00884
## 51  8 66.23 0.00805
## 52  8 66.25 0.00734
## 53  8 66.27 0.00668
## 54  8 66.28 0.00609
## 55  8 66.29 0.00555
## 56  8 66.30 0.00506
## 57  8 66.30 0.00461
## 58  8 66.31 0.00420
## 59  8 66.32 0.00382
## 60  8 66.32 0.00348
## 61  8 66.32 0.00317
## 62  8 66.33 0.00289
## 63  8 66.33 0.00264
## 64  8 66.33 0.00240
## 65  8 66.33 0.00219
## 66  8 66.33 0.00199
## 67  8 66.33 0.00182
## 68  8 66.33 0.00166
```

```
## 69  8 66.34 0.00151
## 70  8 66.34 0.00138
```

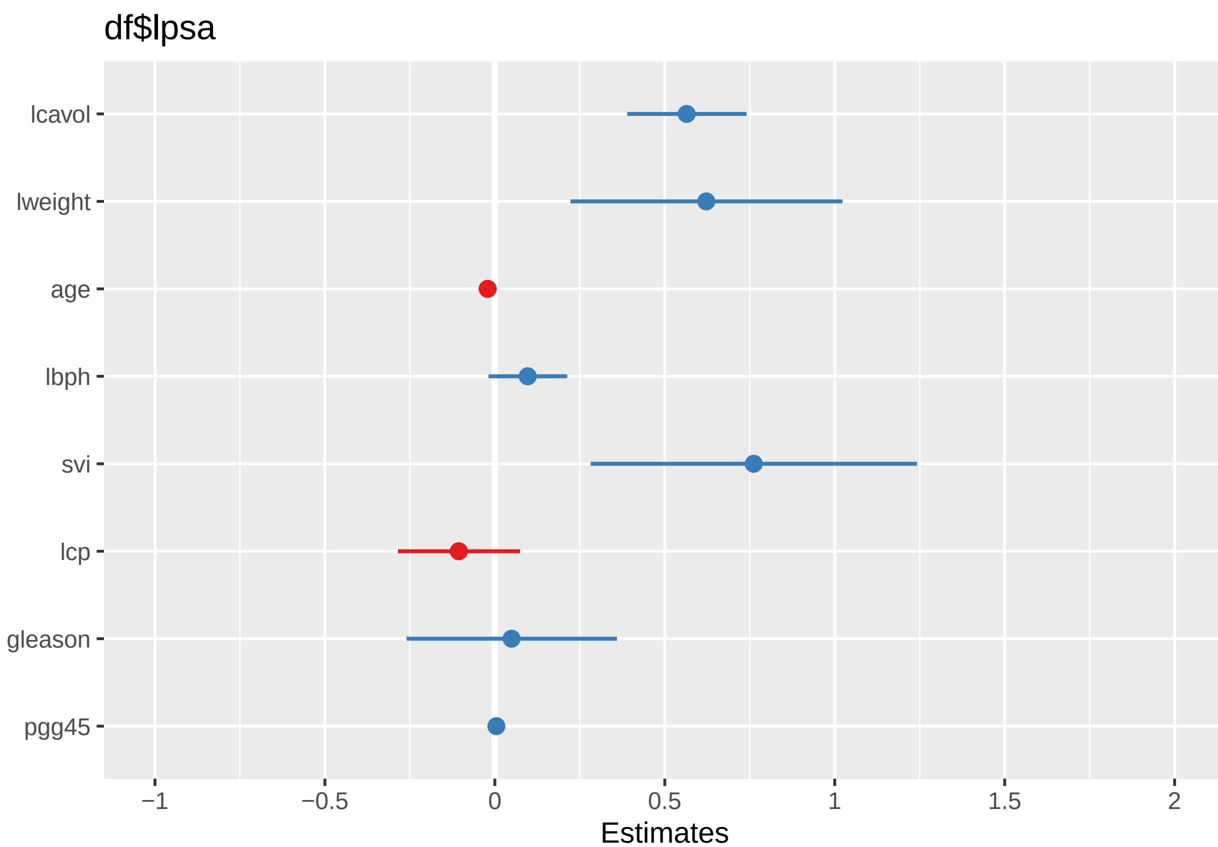
```
plot_model(model_lm)
```

Plotting the estimations of the coefficients for both linear model and lasso model

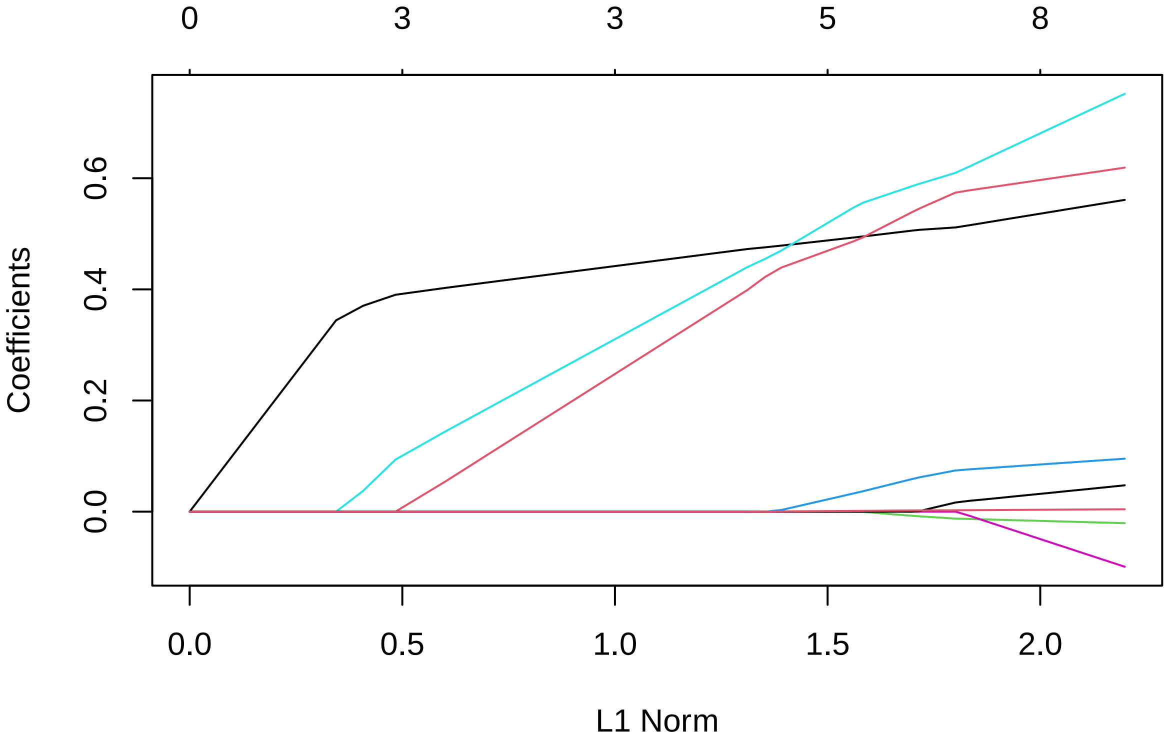
```
## Warning: Using `$` in model formulas can produce unexpected results. Specify your model
##   using the `data` argument instead.
##   Try: lpsa ~ lcavol + lweight + age + lbph +
##   svi + lcp + gleason + pgg45, data =

## Warning: Using `$` in model formulas can produce unexpected results. Specify your model
##   using the `data` argument instead.
##   Try: lpsa ~ lcavol + lweight + age + lbph +
##   svi + lcp + gleason + pgg45, data =

## Warning: Using `$` in model formulas can produce unexpected results. Specify your model
##   using the `data` argument instead.
##   Try: lpsa ~ lcavol + lweight + age + lbph +
##   svi + lcp + gleason + pgg45, data =
```




```
plot(model_lasso)
```



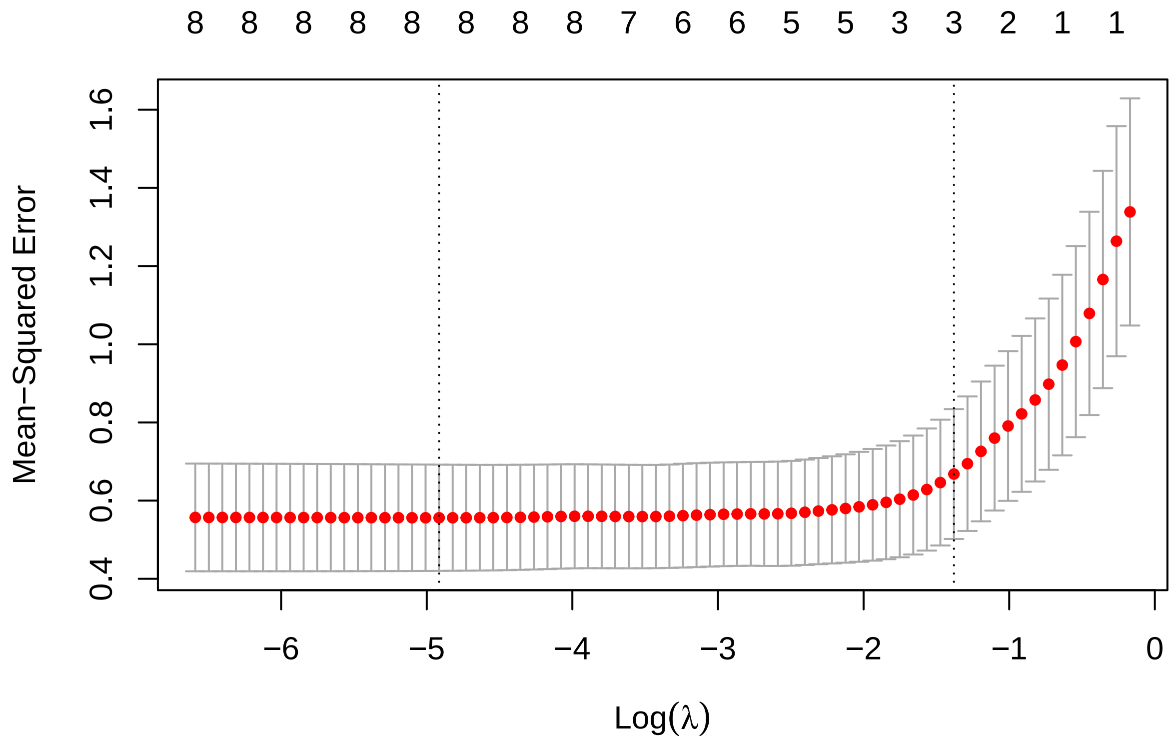
For $j \in 1, \ldots, 8$ ith curve corresponds to jth variable. It shows the path of b_j against the ℓ_1-norm of the whole coefficient vector b as λ varies. The axis above indicates the number of nonzero coefficients at the current λ.

Question 3:

Report two values for λ : "lambda.min" and "lambda.1se", where "lambda.min" is the λ at which the smallest mean squared error (MSE) is achieved and "lambda.1se" is the largest λ at which the MSE is within one standard error of the smallest MSE (default). Report the number of nonzero coefficients for the selected values of λ and the corresponding estimated coefficients.

Perform k-fold cross-validation to find optimal lambda value

```
X <- data.matrix(features)
y <- df$lpsa
cv_lasso <- cv.glmnet(X, y, alpha = 1)
plot(cv_lasso)
```




```
lambda_min = cv_lasso$lambda.min
lambda_1se = cv_lasso$lambda.1se
print(paste( "lambda.min = ",lambda_min))
```

```
## [1] "lambda.min =  0.00733570173207768"
```

```
print(paste( "lambda.1se = ",lambda_1se))
```

```
## [1] "lambda.1se =  0.251648994854596"
```

```
#lasso_model_min <- glmnet(features, y, alpha  = 1,lambda = #lambda_min)
#obain number of non-zero coefficients
#lasso_model_min$beta
#lasso_model_se <- glmnet(features, y=y, alpha = 1, lambda #=lambda_1se)
#obain number of non-zero coefficients
#lasso_model_se$beta
#predict(lasso_model_min,type="coef")
```

```
#coef.exact <- coef(model_lasso, s = c(lambda_min, lambda_1se), exact = TRUE)
#predict(model_lasso, newx = X, s = c(lambda_min, lambda_1se))
#coef.apprx <- coef(model_lasso, s = c(lambda_min, lambda_1se), exact = FALSE, x=X, y=y)
#coef.apprx[which(coef.apprx != 0)]
#coef.exact[which(coef.exact != 0)]

coeffs <- predict(model_lasso, s = c(lambda_min, lambda_1se), type="coef")

coeffs_s1 = coeffs[,1]
coeffs_s2 = coeffs[,2]


n1 <- coeffs_s1[which(coeffs_s1 != 0)] %>% length()
n2 <- coeffs_s2[which(coeffs_s2 != 0)] %>% length()
```

```r
print(paste( "Number of non-zero coefficients for model with  lambdal.min = ",n1))
```

```
## [1] "Number of non-zero coefficients for model with  lambdal.min =  9"
```

```r
print(paste( "Number of non-zero coefficients for model with  lambda.1se = ",n2))
```

```
## [1] "Number of non-zero coefficients for model with  lambda.1se =  4"
```