Task 2

The transformation T_G is defined as $T_G : g \mapsto f$ with $f(x,y) = c * g^{\gamma}(x,y)$, where $f, g \in [0, 255]$, $\gamma \in [0.5, 2]$. The image used for this task were the following:

Figure 1: Initial Image

Two cases are investigated: $\gamma = 0.5$, $\gamma = 2$. For both cases, it is assumed that $c \in \mathbb{R}$.

Case I. $\gamma = 0.5$

In this case, $f(x,y) = T_G = c * \sqrt{g(x,y)}$, indicating that the tansformer becomes a square root function.

$$g(x,y) \in [0, 255] \implies \sqrt{g(x,y)} \in [0, 15]$$

If we want the condition $f(x,y) \in \{0, 1, ..., 255\}$ to hold, then c can take the following values:

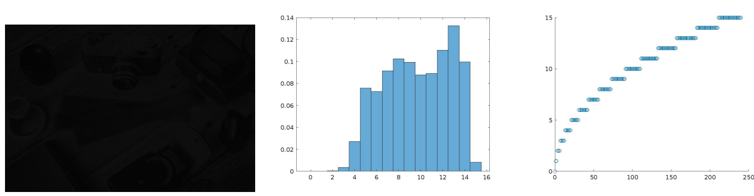
$$c \in \{0, 1, 2, ..., 15\}$$

Using `Matlab`, the following plots are generated for a given c:

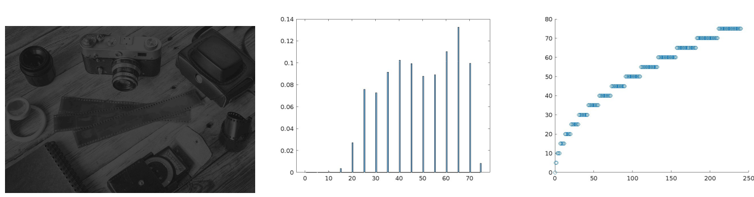
1. Transformed Image
2. Histogram of intensity values. In x axis, there is pixel intensity. In y axis, there is frequency of intensity obtained from pixels of transformed image.
3. Scatterplot in which x axis represents the initial image's intensity values, and y axis represents the transformed image's intensity values.

For $c \in \{1, 5, 15\}$, the plots are presented below:

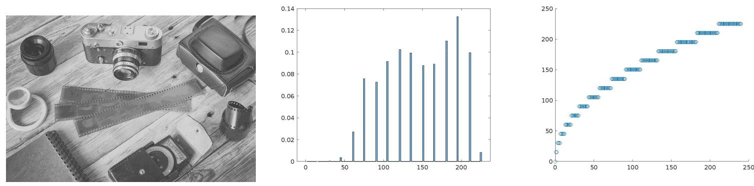
- $c = 1$



- $c = 5$



- $c = 15$



Evidenced by the plots, increasing c value would have the following effects

1. Rescaling intensity values to higher ones.

2. Increasing variance of values, i.e., the distance between intensity values of transformed image becomes higher than that between initial image.

3. Rescaling to higher values leads to brighter images. Let us we exclude $c = 0$ that outputs constant 0 values for all pixels. Among c values, $c = 1$ results in the darkest image, and $c = 15$ results to the brightest image.

Case II. $\gamma = 2$

In this case, $T_G = f(x, y) = c * g^2(x, y)$, indicating the tansformer becomes a square function scaled by value of c. Let $g^2(x, y) = k$. Therefore, k can take the following values $\{1, 4, 8, 16, ..., 255^2\}$ It can be said that $c = \frac{f(x,y)}{k}$. If we want the condition $f(x, y) \in \{0, 1, ..., 255\}$ to hold, then based on value of k, we have the following cases:

- $1 < k < 255$

$$c \in \{0, 1, 2, , int(\frac{255}{k})\} \cup \{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, ..., \frac{1}{k}\}$$

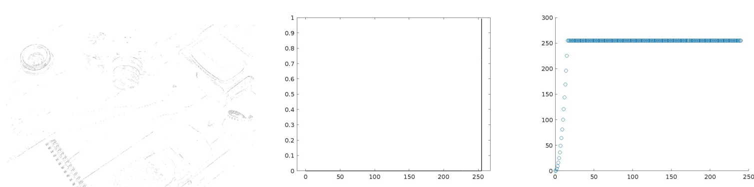
- $k \geq 255$

$$c \in \{0\} \cup \{\frac{1}{k}, \frac{2}{k}, ... \frac{255}{k}\}$$

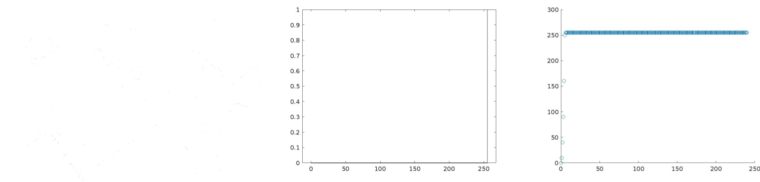
Since the value of c should work for all values of $g(x, y)$ and equivalently for those of k, then the possible values of c at intersection of the aforementioned cases of k should be considered, which are $c \in \{0, \frac{1}{k}\}$. But the intensity value of pixels are dependent on k as well, therefore unless an image has only one intensity value for all its pixels, there is no value of c that keeps all the pixels' intensities in the valid range of $\{0, 1, ..., 255\}$.

In the following, the same plots introduced in case I are provided. However, as there is no c that keeps all the intensity values in the valid range (except the trivial case of $c = 0$), there will be intensity values outside the valid range. To omit them, the values greater than 255 are replaced with 255, and the values less than 0 are replaced with 0.

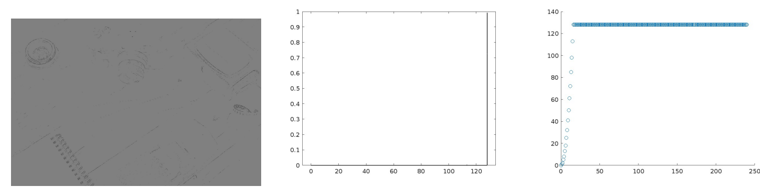
- $c = 1$



- $c = 10$



- $c = \frac{1}{2}$



- $c = \frac{1}{10}$