# Quick review of Deep Learning

EE 5178
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# Machine learning

**Goal:** Learning from the data with minimal intervention from the user

**Supervised learning:**

- Learns a mapping b/w *input* and *output* pairs $(x_i, y_i)$ e.g. image classification

$\xrightarrow{f(x)}$ Coffee mug

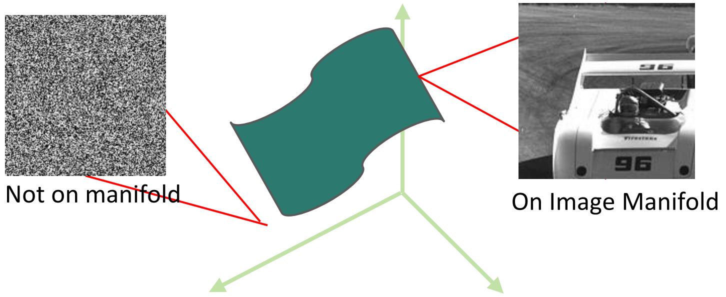
$\xrightarrow{f(x)}$ tiger

$\xrightarrow{f(x)}$ car

- *Applications*: image classification, object detection, scene recognition

**Unsupervised learning:**

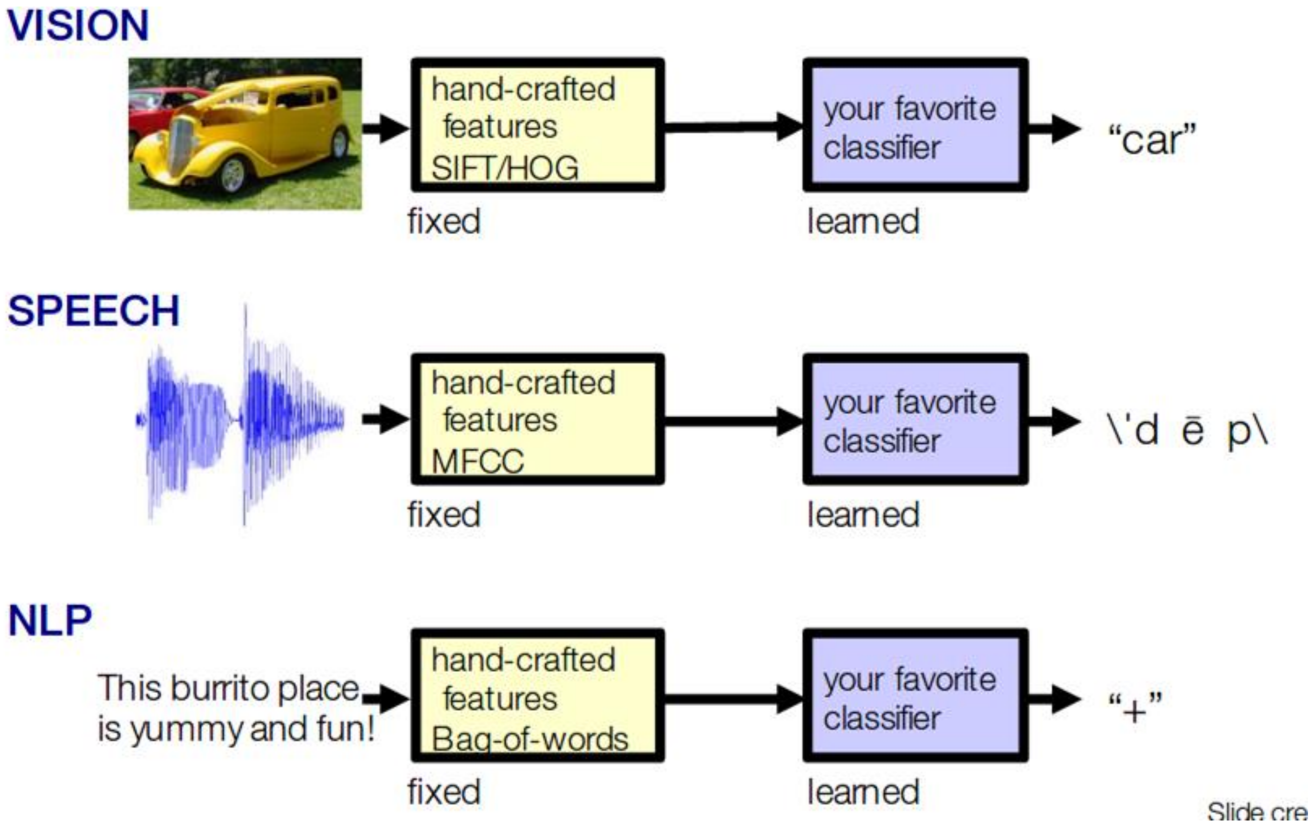
- Given only data '*x*' learn the inherent underlying structure
- Consider a 64x64 binary image



- *Applications*: clustering, dimensionality reduction, density estimation

# Traditional approaches

- [Manual] feature extraction (SIFT/HOG)
- Classifier is learned independent of feature extraction

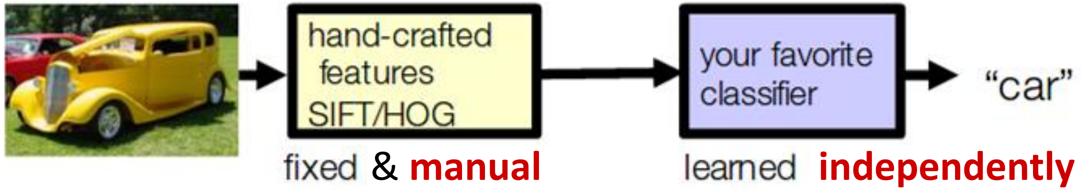


*slide courtesy, Yoshua Bengio and Yahn Lecun

# Traditional approaches vs Deep learning

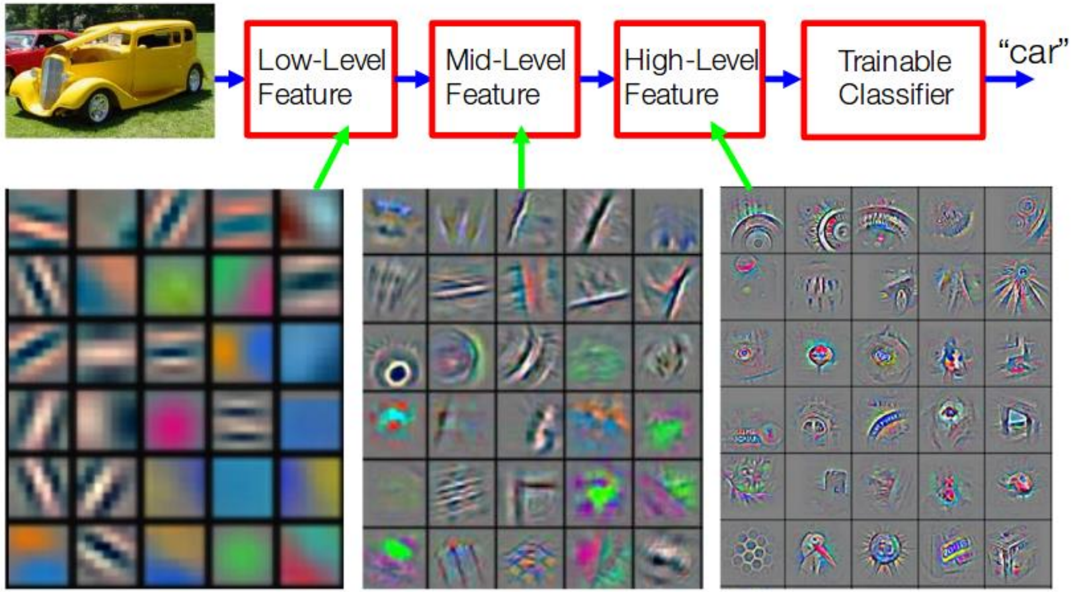
What's wrong with the traditional approaches?

- Compositional feature abstraction is missing



3 key ideas of deep learning

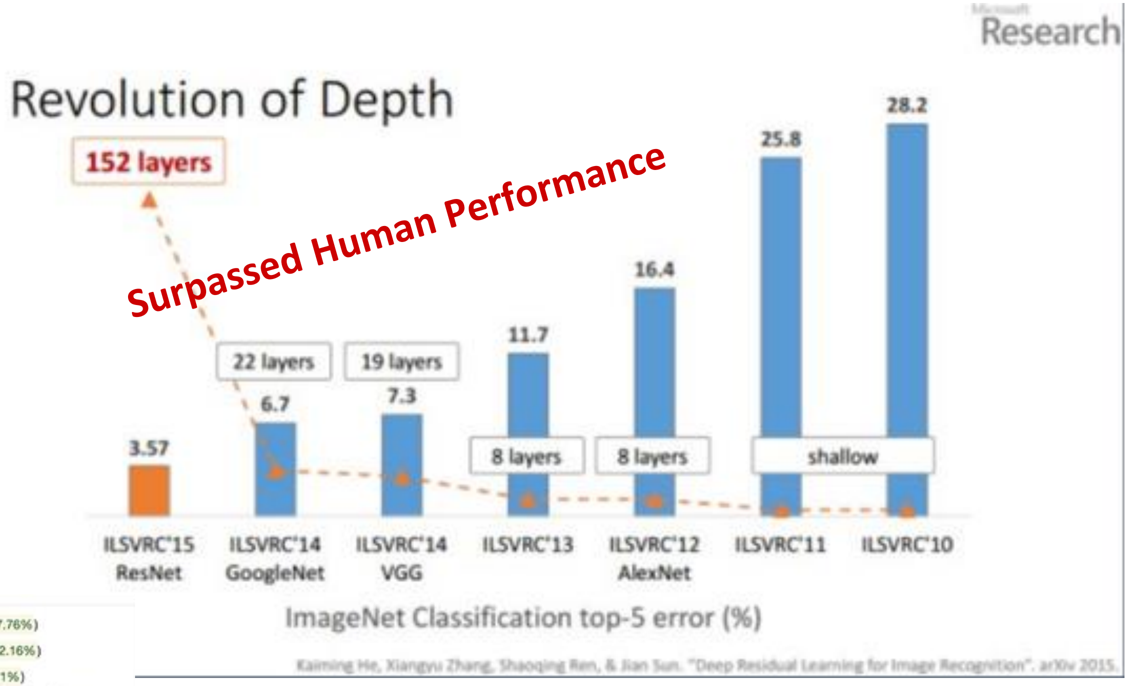
- (Hierarchical) Compositionality
  - Cascade of nonlinear functions
  - Multiple layers of abstractions
- End-to-End learning
  - Learning (task-driven) representations
  - Learning to extract features
- Distributed Representation
  - No single neuron encodes everything
  - Group of neurons work together



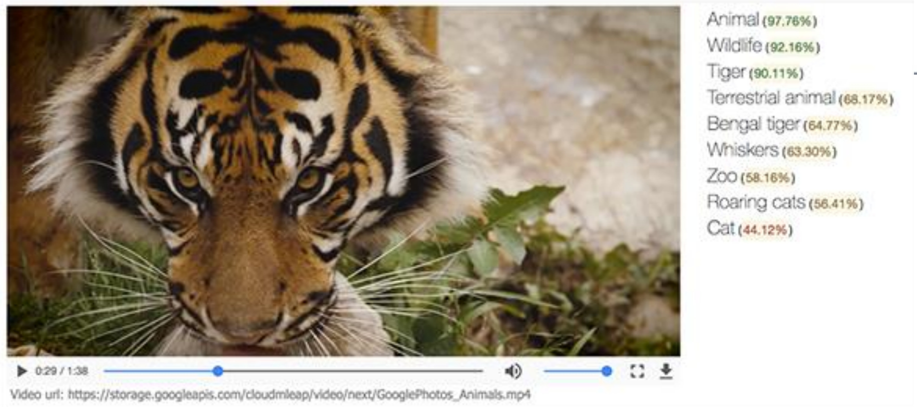
*slide courtesy, Yoshua Bengio and Yahn Lecun

# Image Classification

LeNet by Lecun et al. 1998 (MNIST)
AlexNet by Krish et al. NIPS' 12
VGGNet by Simonyan et al. ICLR' 15
GoogLeNet by Szegedy et al. CVPR' 15
ResNet by He et al., CVPR' 17 best paper

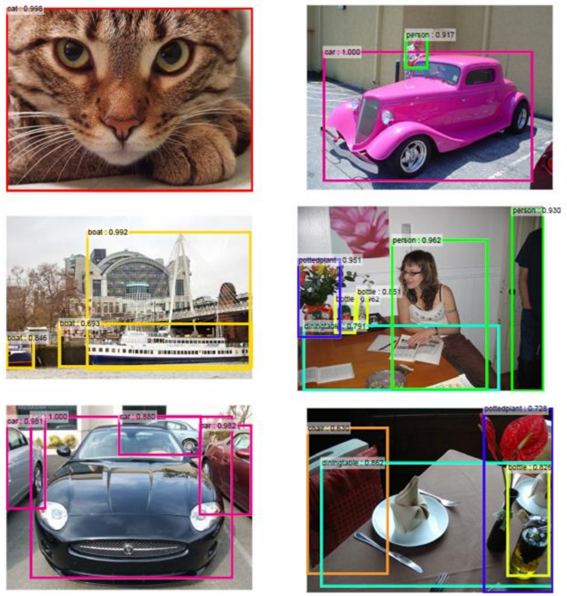


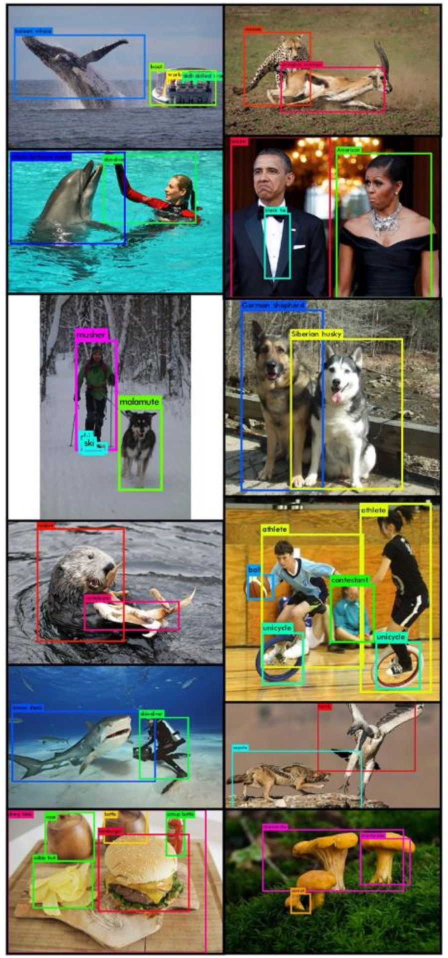
*pic courtesy: Kaiming He



# Object Detection

OverFeat by  Sermanet et al., ICLR' 14 (NYU)
R-CNN by Girshick et al., CVPR' 14 (UCB)
SPP by He et al., ECCV' 14 (MSR)
Fast R-CNN by Girshick et al., arxiv (MSR)
Faster R-CNN by Ren et al., NIPS' 15 (MSR)
YOLO by Redmon et al., arxiv 2015
YOLO 9000 by Redmon et al., CVPR' 17

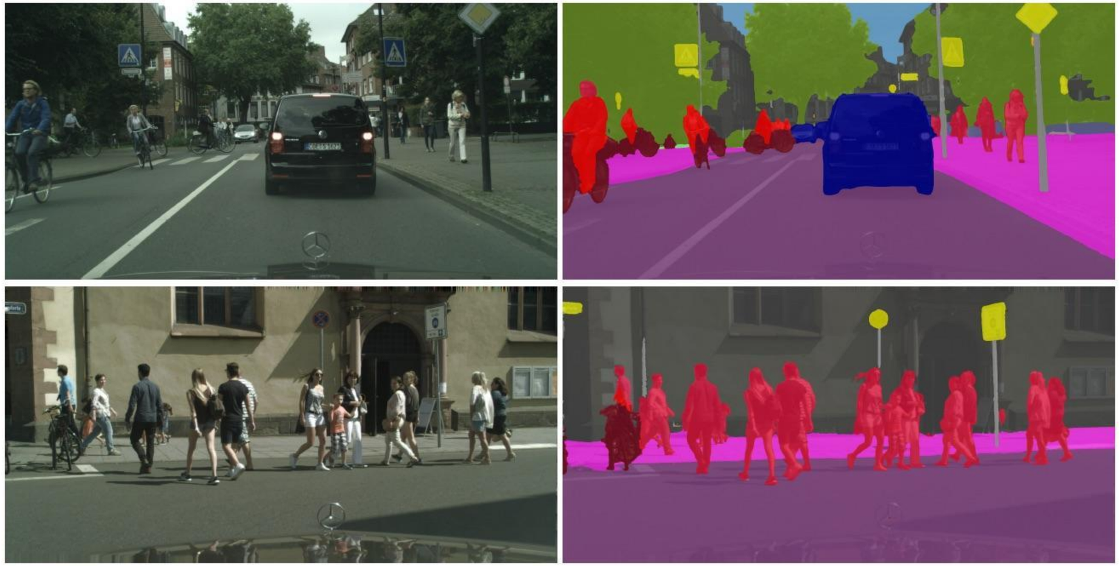

Faster RCNN detections


YOLO 9000 detections

pic courtesy: http://mrulafi.blogspot.in/2016_05_01_archive.html

## Image Segmentation

FCN by Long et al. CVPR’ 15
DeepLab by Chen et al. arxiv 2015
CRFS as RNNs by Zheng et al. ICCV’ 15

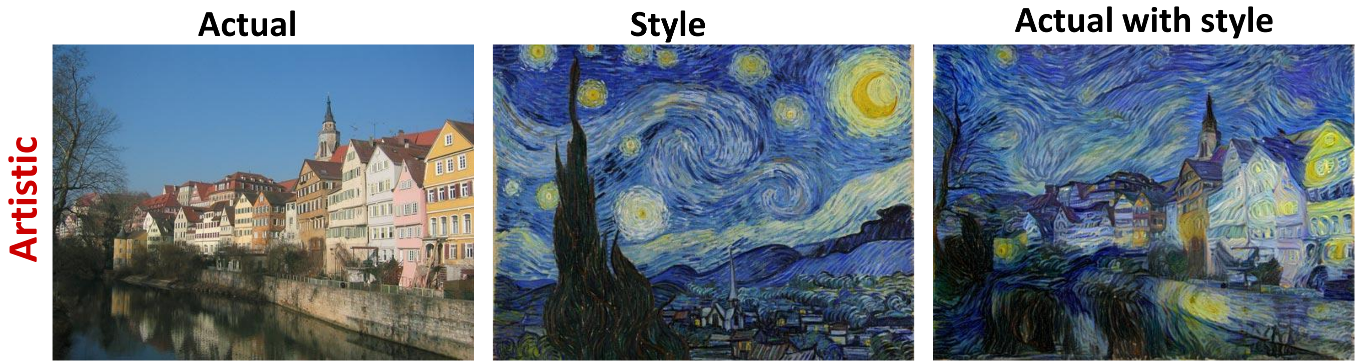


*Pic courtesy: Kundu et al. CVPR 2016 on City scapes dataset

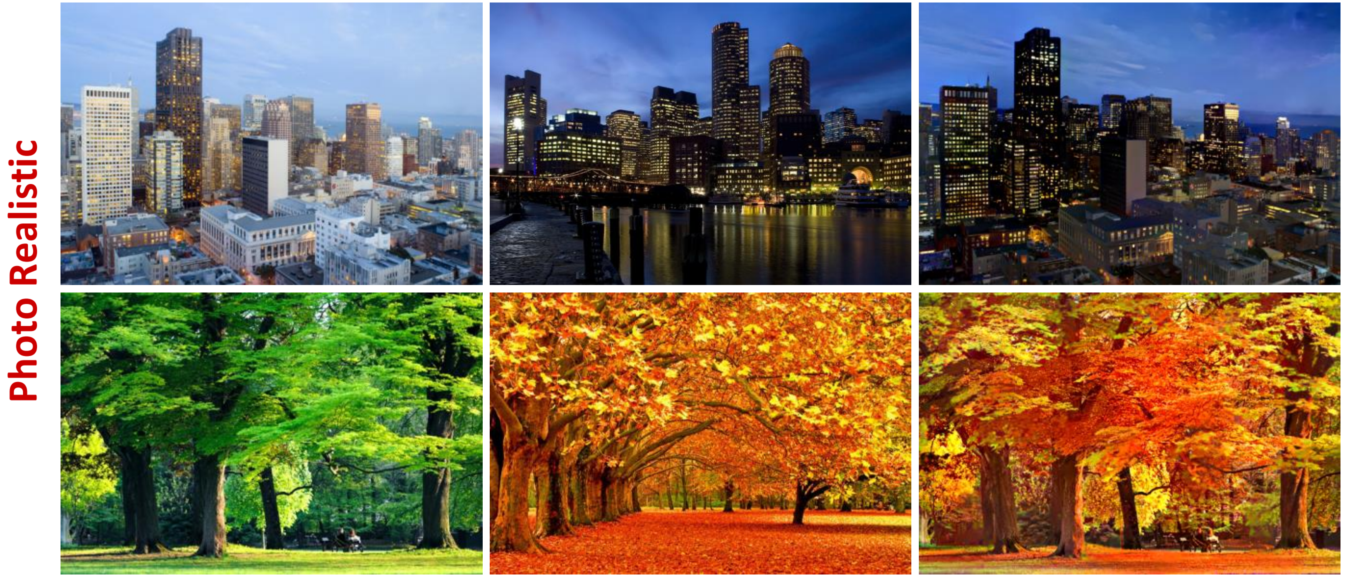
# Style transfer

Neural style transfer by Gatys et al., CVPR’ 16

Deep Photo Style Transfer by Luan et al., CVPR’ 17



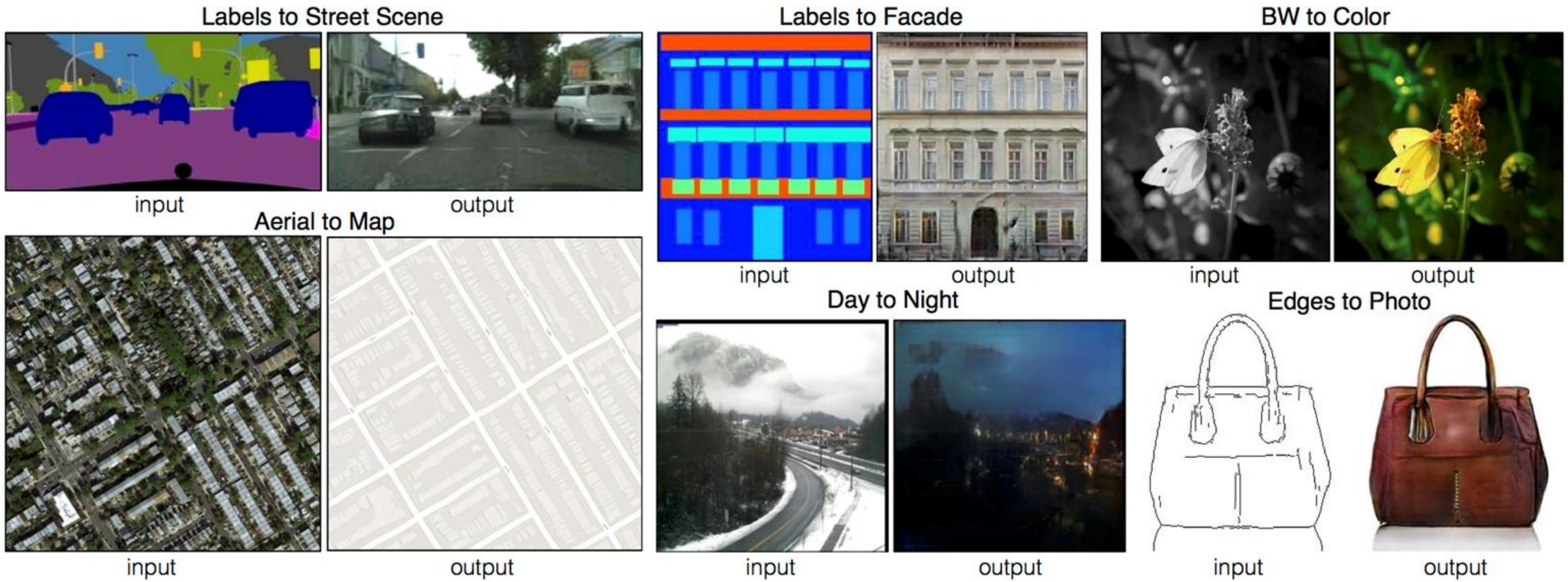
*Pic courtesy: http://deepart.io



*Pic courtesy: Deep Photo Style Transfer, Luan et al. CVPR 2017

## Artistic applications

**Image-to-Image Translation** with Conditional Adversarial Nets by Isola et al., CVPR’ 17



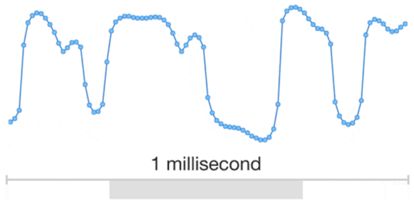
# Image, Video and Audio Generation

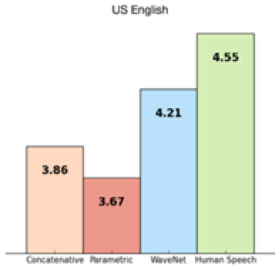
**PixelRNN/CNN** by Gregor et al.,
ICML' 16 Best paper (Samples from ImageNet)

**DCGAN** Chintala et al. ICLR' 16
Sample bedroom images

Generating **videos** with scene dynamics, by Vondrick et al., NIPS' 16

**WaveNet** for **audio** synthesis by Oord et al. 2016, Deepmind





*Check out wavenet's generated music*
*piano clips*

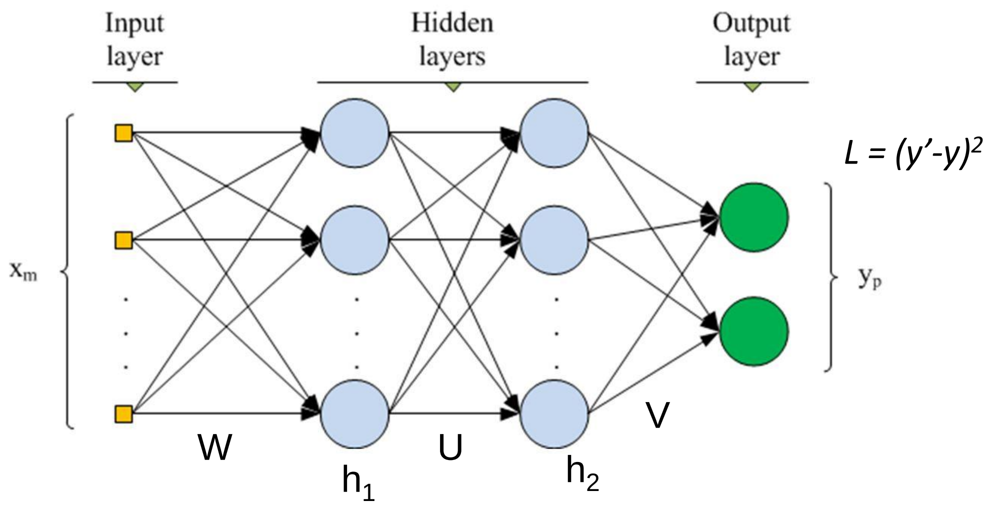
# 1. Basic Neural Networks

Multi-layer perceptrons (MLP) is a feed forward neural network with hidden layers

$h_1 = f(Wx + b_1)$, $f$ is an activation function

$h_2 = g(Uh_1 + b_2)$

$y' = Vh_2 + b_3$



$L = (y'-y)^2$

Hidden layers *increase* abstraction

– hence, better to have more hidden layers than a single layer with large number of neurons

Universal Approximation Theorem:

– simple neural networks can *represent* a wide variety of interesting functions when given appropriate parameters

*pic courtesy, Sancho course

# 1. Basic Neural Networks

What we will learn in the course about Neural Networks?

## Introduction

- McCulloch and Pits model
- Rosenblatt's perceptron

## Perceptrons

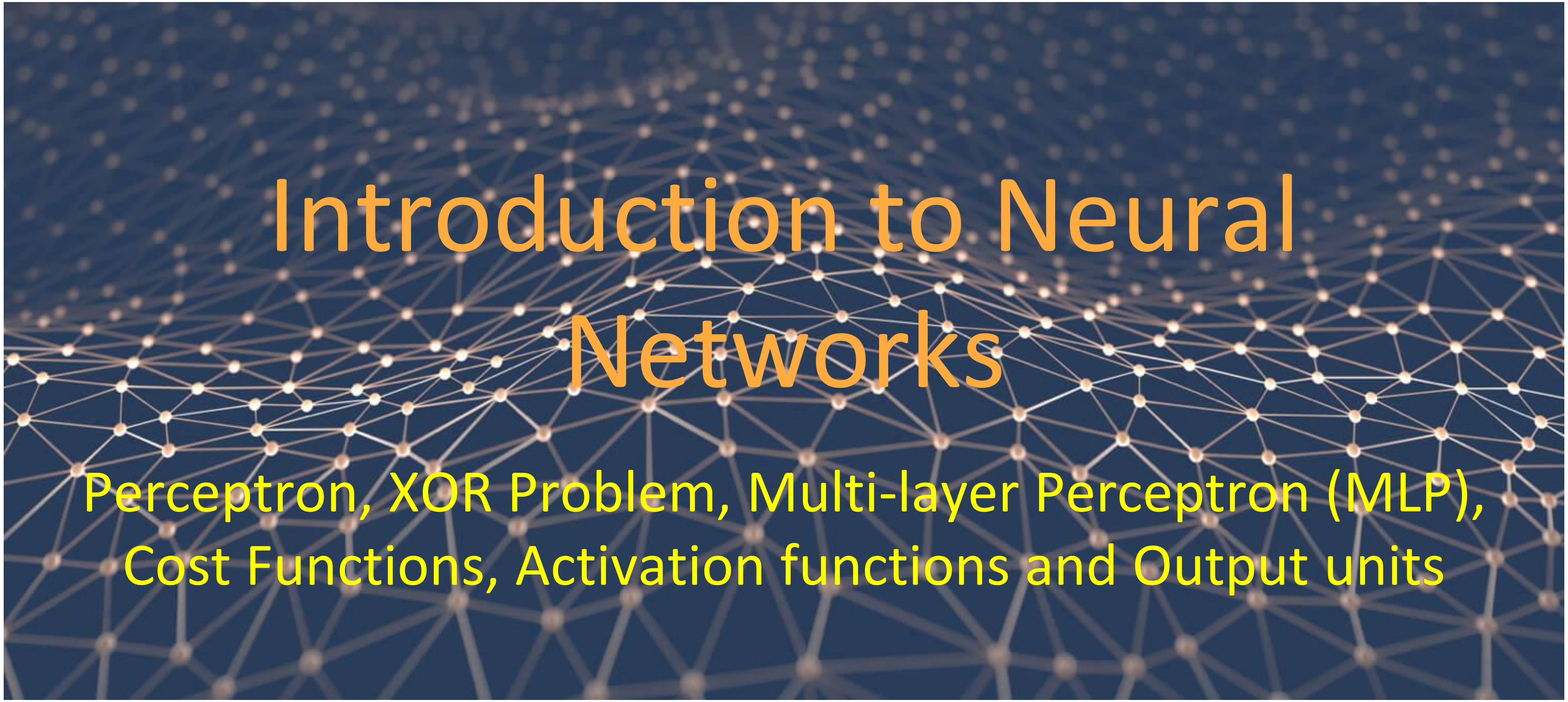
- Geometry and linear separability
- XoR problem
- Multi-layer perceptron (MLP)

## Training MLPs

- Error back propagation
- Loss functions

## Regularization and optimization

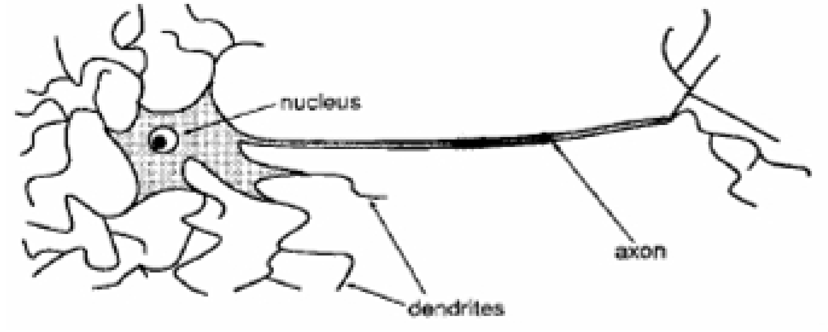
- Over fitting / Under fitting
- Regularization
- Various algorithm

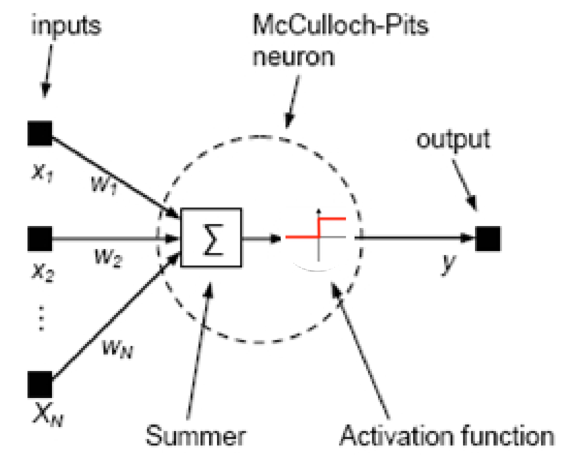


# Introduction to Neural Networks

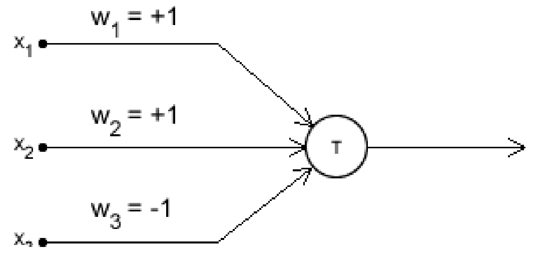
Perceptron, XOR Problem, Multi-layer Perceptron (MLP), Cost Functions, Activation functions and Output units

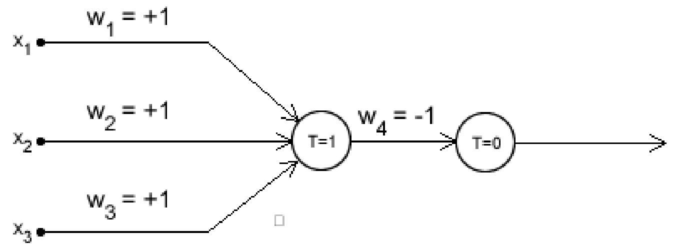
# McCulloch - Pits model





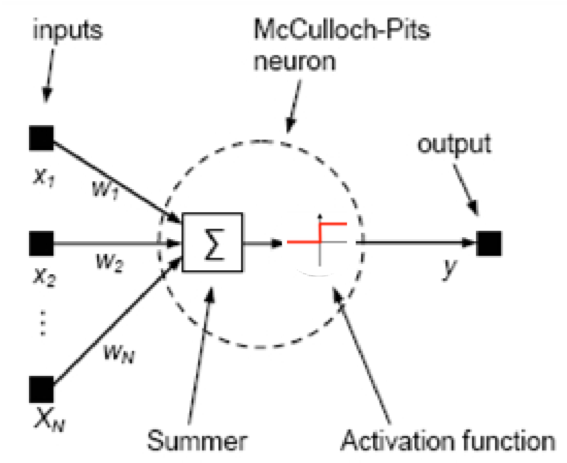
$$\sum_{i=1}^{n} w_i x_i > \mu$$



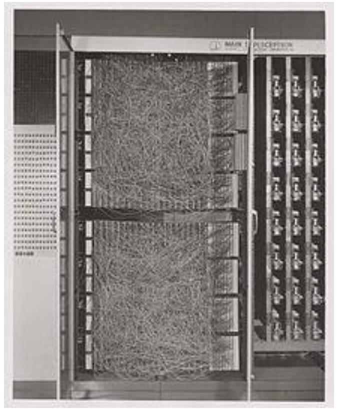


NOR gate

# The Perceptron - Rosenblatt (1953)



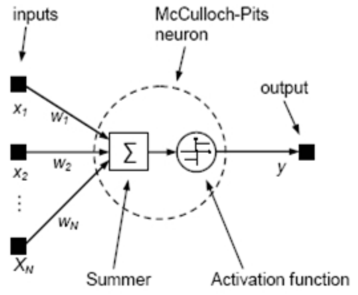
$$\sum_{i=1}^{n} w_i x_i > \mu$$



*Pic courtesy, wikipedia

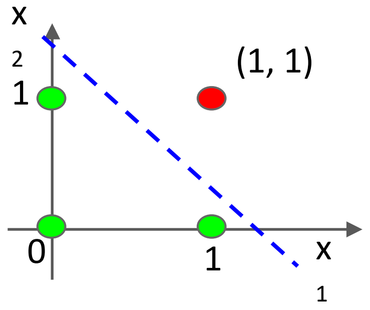
The Mark 1 Perceptron
By Rosenblat
for digit recognition

# Perceptron - geometrical interpretation



$$\sum_{i=1}^{n} w_i x_i > \mu$$ , What does this inequality imply in 2D case? Half plane

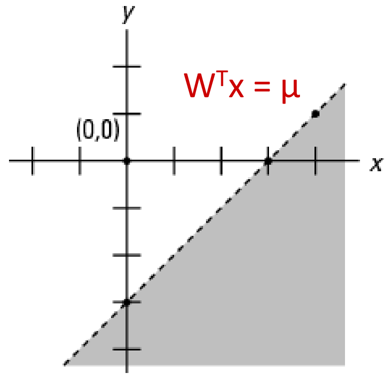
| $\mathbb{X}$ | *AND* |
|---|---|
| (0, 0) | 0 |
| (0, 1) | 0 |
| (1, 0) | 0 |
| (1, 1) | 1 |



Solve for W, μ:

$x_1 + x_2 > 1.5$

$w_1 = 1, w_2 = 1$ and $\mu = 1.5$



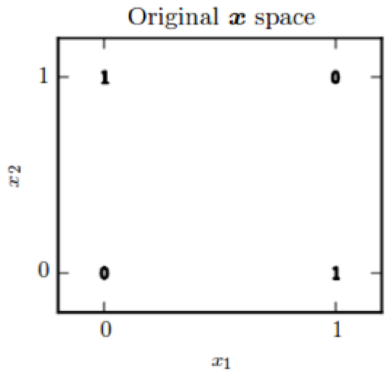
*Pic courtesy, cliffsnotes

Any function that is linearly separable can be computed by a perceptron

# Perceptron - Limitations

Goal: learn the XoR function (*f**)

| $\mathbb{X}$ | *f** |
|---|---|
| (0, 0) | 0 |
| (0, 1) | 1 |
| (1, 0) | 1 |
| (1, 1) | 0 |



**The data is not linearly separable**

How to tackle this problem?

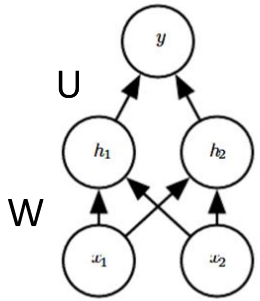
- Can we use more than one line?
- Yes, but how?

*Slide courtesy, Ian Goodfellow et al., deep learning book

# Perceptron - Limitations

How to tackle this problem?

– Add a hidden layer with two units



$y = f^{(2)}(h; U, c)$

$y = f^{(2)}(\ f^{(1)}(x)\ )$

$h = f^{(1)}(x; W, b)$

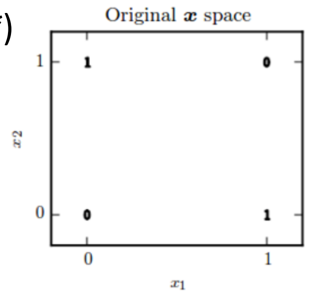
What should $f^{(1)}$ compute?

– If its linear again the composition still remains linear

$f^{(2)}(h) = U^T h$ and $h = Wx$

$y = U^T W x = W'x$

Goal: learn the XoR function ($f^*$)

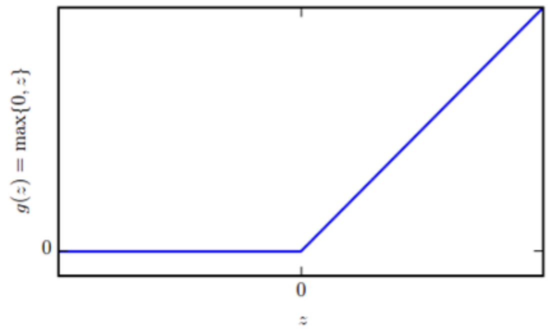


– $f^{(1)}$ should be nonlinear to extract useful features

$h = f^{(1)}(x; W, b) = g(Wx+b)$

– $g$ is referred as activation function commonly

– We will use ReLU here
  - ❑ Rectified Linear Unit (widely used)
  - ❑ $g(z) = \max\{0,z\}$



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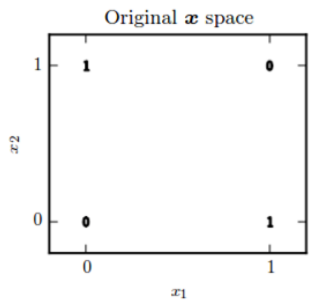
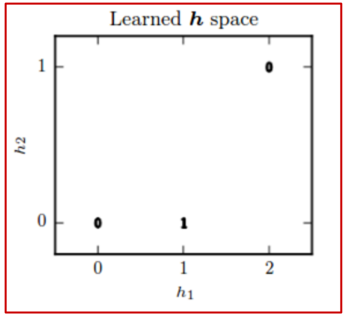
# Perceptron - Limitations

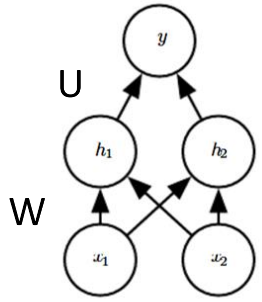
Goal: learn the XoR function ($f^*$)





How to tackle this problem?

- Add a hidden layer with two units
- Use ReLU activation in 1st layer



$y = U^T h + c$ ; $y = U^T \underbrace{\max\{0, Wx+b\}}_{\text{ReLU}} + c$

$h = g(Wx+b) = \max\{0, Wx+b\}$

Let,

$$W = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 \\ -2 \end{bmatrix},$$

c = 0

$$X = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad WX = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

$$WX + b = \begin{bmatrix} 0 & 1 & 1 & 2 \\ -1 & 0 & 0 & 1 \end{bmatrix} \qquad \boxed{h = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{Upon ReLU}}$$

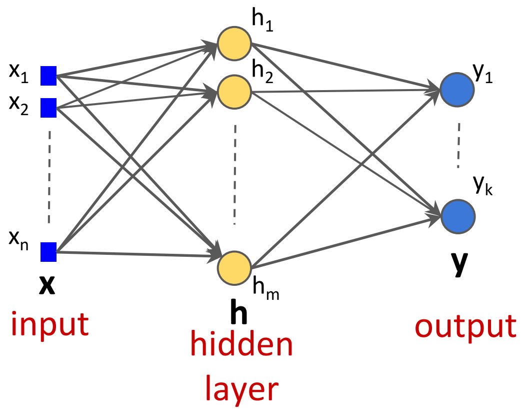
After layer2

$[0\ 1\ 1\ 0]$

*Slide courtesy, Ian Goodfellow et al., deep learning book

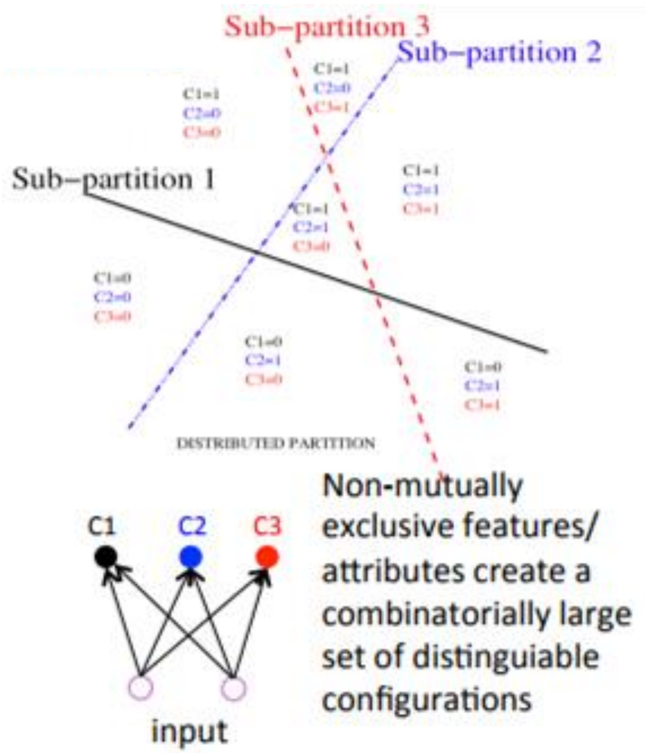
# Multi-layer Perceptrons (MLP)

A typical feed forward neural network



$$\mathbf{h} = f(W\mathbf{x} + \mathbf{b}_1); \quad \mathbf{y} = g(U\mathbf{h} + \mathbf{b}_2)$$

With more hidden units network is more expressible

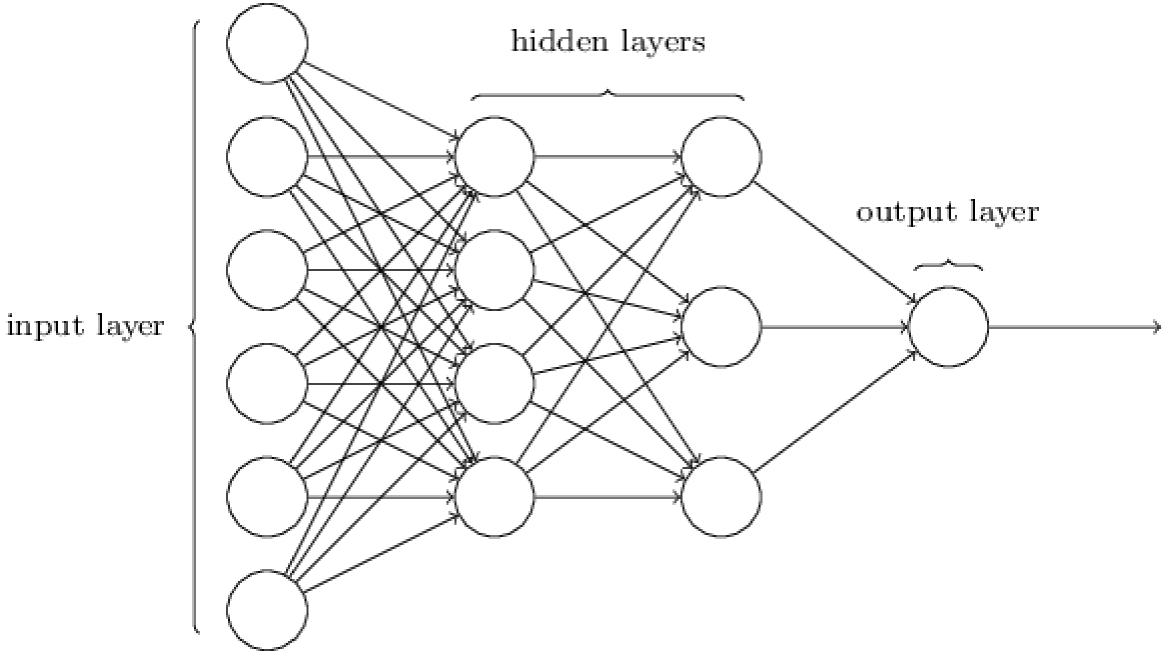


Non-mutually exclusive features/ attributes create a combinatorially large set of distinguiable configurations

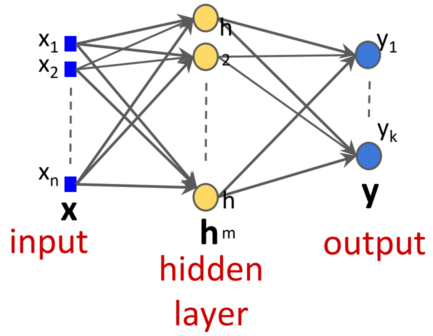
*Pic courtesy, Yoshua Bengio

## Specification of a MLP

- ❖ Number of hidden layers and units in each layer
- ❖ Activation function for
  - ➢ Hidden layers
  - ➢ Output layers
- ❖ Cost function



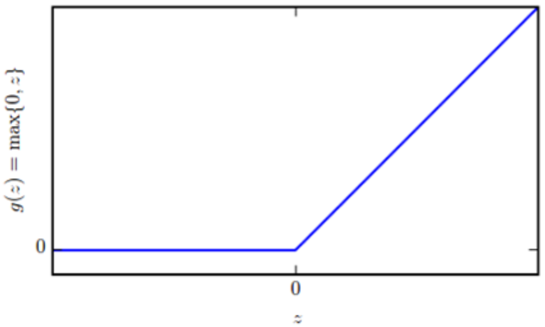
# Activation functions for hidden layers



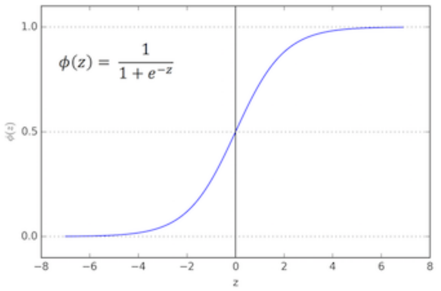
h = *g*(Wx+b); Affine transformation followed by activation function, *g*

- Very important factor in learning features

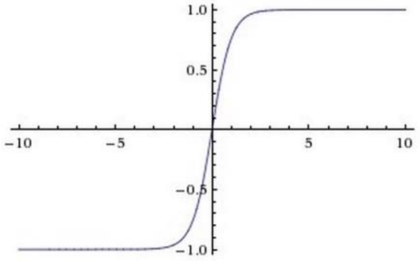
*g*(z) = max{0,z}, ReLU



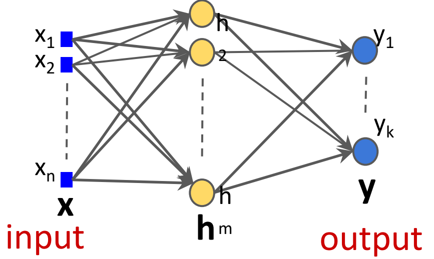
*g*(z) = σ(z), sigmoid



*tanh*(z) = 2σ(2z) - 1



# Activation functions for Output units



- Linear units for real valued outputs
  - Activation function is left to be linear
  - Given features h,

    y' = Wh+b

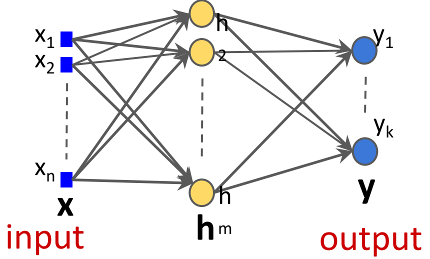
  - Most commonly used with regression tasks

- Say you want to do binary classification
  - What kind of distribution describes output?
    **Bernouli**
  - How to constrain the output - valid probability?
    Can you use linear activation?

$$P(y = 1 \mid \boldsymbol{x}) = \max\left\{0, \min\left\{1, \boldsymbol{w}^\top \boldsymbol{h} + b\right\}\right\}.$$

  - What is the problem? *Not amenable for gradient based learning*
  - Instead, use sigmoid unit - output ∊ [0,1]

$$\hat{y} = \sigma\left(\boldsymbol{w}^\top \boldsymbol{h} + b\right)$$



# Activation functions for Output units

– Now, say we want to do multi-class classification (K classes)
  - ❑ Output should be K probabilities,
    $p_k = p(class = k|x)\ \forall\ k = 1$ to K
  - ❑ Can we use K sigmoid units?
    Won't be sufficient, since probabilities are not constrained to sum to 1
    $\sum_k p_k = 1$
  - ❑ We will look at softmax unit for this
    Idea is to convert a vector of real values to valid probabilities,
    How?
    - ❑ Make all the elements positive
    - ❑ Normalize the values

– Let, $\mathbf{z} = [z_1, \dots, z_K]^T$;  $\mathbf{z} = W\mathbf{h} + \mathbf{b}$

$$\text{softmax}(\boldsymbol{z})_i = \frac{\exp(z_i)}{\sum_j \exp(z_j)}.$$

# Cost functions

For regression,

$$J(\theta) = \frac{1}{2}\mathbb{E}_{\mathbf{x},\mathbf{y}\sim\hat{p}_{\text{data}}} ||\boldsymbol{y} - f(\boldsymbol{x};\boldsymbol{\theta})||^2$$

$$\frac{1}{2}\sum_{\{x_i,y_i\}} ||y_i - f(x_i,\theta)||^2$$

For classification,

- Typically outputs a probability vector $q(c = k|\mathrm{x})\ \forall\ k$
- How do you compare two distributions?
  - ❑ KL divergence, KL($p\|q$)

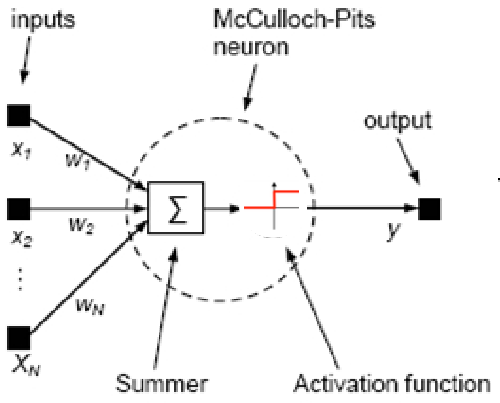
$$D_{KL}(p(x)||q(x)) = \sum_{x\in X} p(x)\ln\frac{p(x)}{q(x)}$$

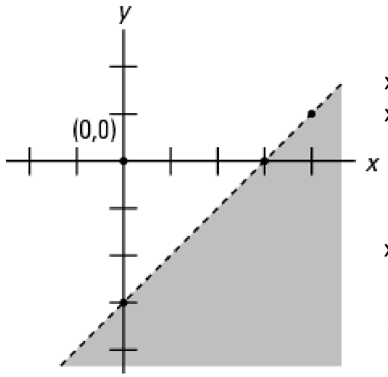
$$= \sum p(\mathrm{x}) \ln p(\mathrm{x}) - p(\mathrm{x}) \ln q(\mathrm{x})$$

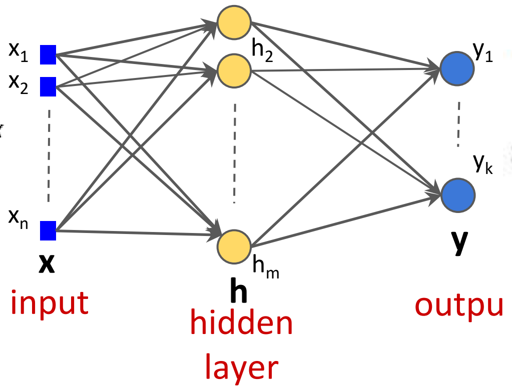
$$= \underbrace{- H(p)}_{\text{Entropy}} + \underbrace{H(p,q)}_{\text{cross-entropy}}$$

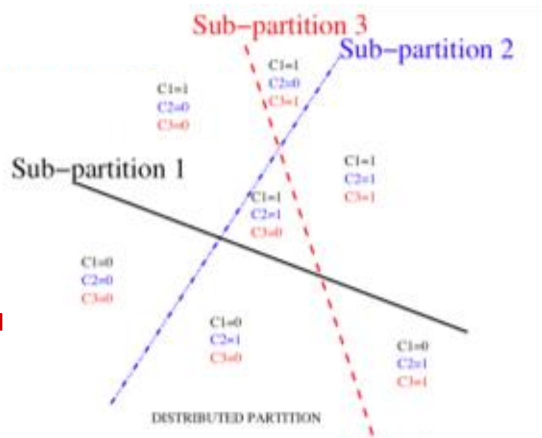
$$J(\theta) = \sum_{x_i,y_i} H(p(x_i),q(x_i))$$

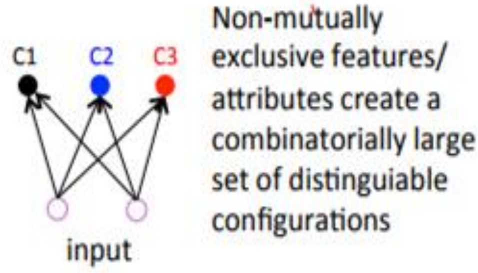
# What we learnt till now:







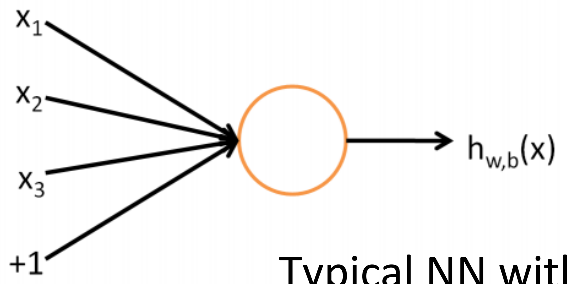



C1
C2
C3
input
Non-mutually exclusive features/ attributes create a combinatorially large set of distinguiable configurations

- ❖ Network specification
  - ➢ Number of hidden layers and units in each layer
  - ➢ Activation function for
    - ■ Hidden layers
    - ■ Output layers
  - ➢ Cost function

## How to learn the network parameters?
## Error back propagation

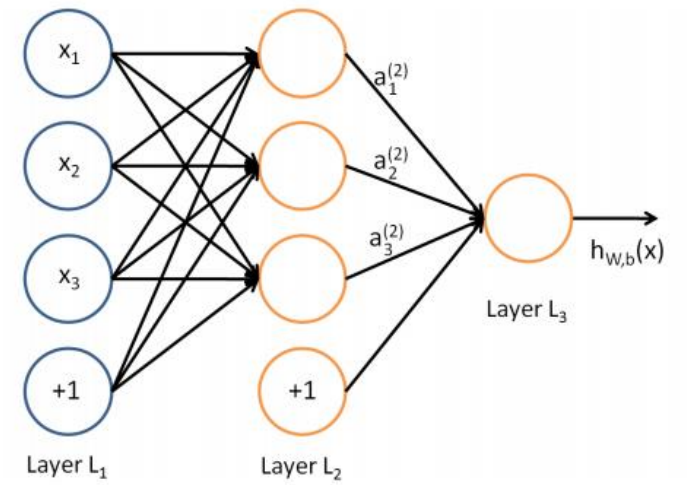
## How to learn the parameters?



**Bias**

$$h_{W,b}(x) = f(W^T x) = f(\textstyle\sum_{i=1}^{3} W_i x_i + b)$$

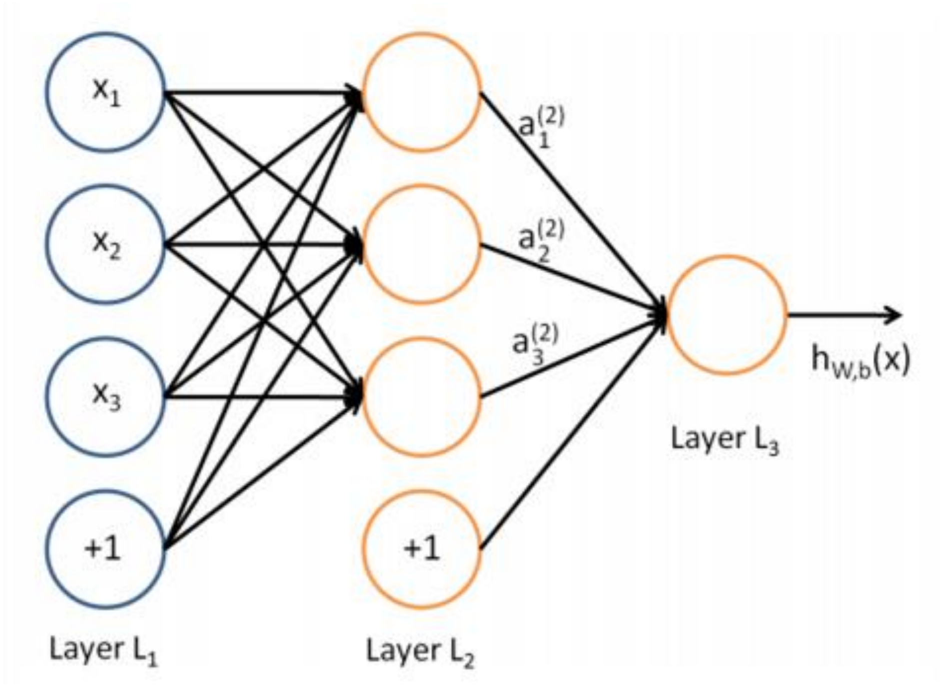
Typical NN with many such units



- One hidden layer
  - 3 neuron units
- One output

*Slide courtesy, sparse autoencoder by Andrew Ng

## How to learn the parameters?



$W^{(1)}, b^{(1)}$ $W^{(2)}, b^{(2)}$

| | | |
|---|---|---|
| $L_l$ | – | Layer *l* |
| $a_i^{(l)}$ | – | activation of unit *i* in layer *l* |
| $W_{ij}^{(l)}$ | – | Weight from *j*th unit in *l* to *i*th unit in *l+1* |
| $b_i^{(l)}$ | – | bias to unit i in layer *l+1* |

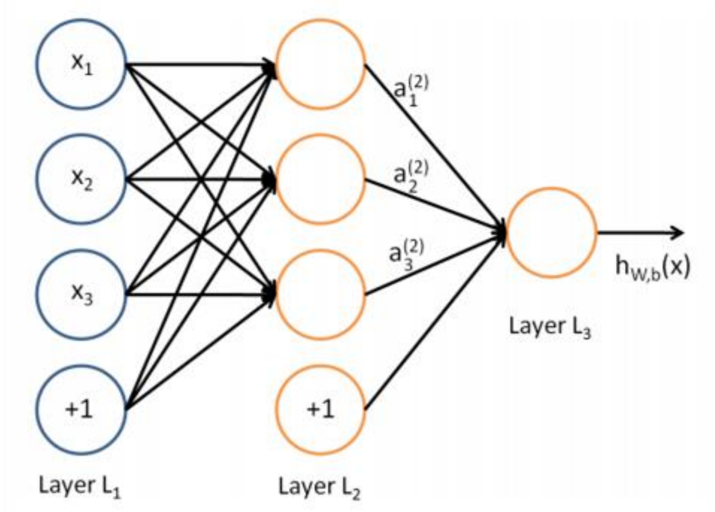
**Parameters**:

$$(W^{(1)}, b^{(1)}, W^{(2)}, b^{(2)})$$

$$W^{(1)} \in \mathbb{R}^{3\times3}, W^{(2)} \in \mathbb{R}^{1\times3}$$

*Slide courtesy, sparse autoencoder by Andrew Ng

# How to learn the parameters?



**Layer 2,**

$$\begin{aligned} a_1^{(2)} &= f(W_{11}^{(1)}x_1 + W_{12}^{(1)}x_2 + W_{13}^{(1)}x_3 + b_1^{(1)}) \\ a_2^{(2)} &= f(W_{21}^{(1)}x_1 + W_{22}^{(1)}x_2 + W_{23}^{(1)}x_3 + b_2^{(1)}) \\ a_3^{(2)} &= f(W_{31}^{(1)}x_1 + W_{32}^{(1)}x_2 + W_{33}^{(1)}x_3 + b_3^{(1)}) \end{aligned}$$

**Layer 3,**

$$h_{W,b}(x) = a_1^{(3)} = f(W_{11}^{(2)}a_1^{(2)} + W_{12}^{(2)}a_2^{(2)} + W_{13}^{(2)}a_3^{(2)} + b_1^{(2)})$$

**Simplification**

Let, $z_i^{(l)}$ denote weighted sum for the activation $a_i^{(l)}$

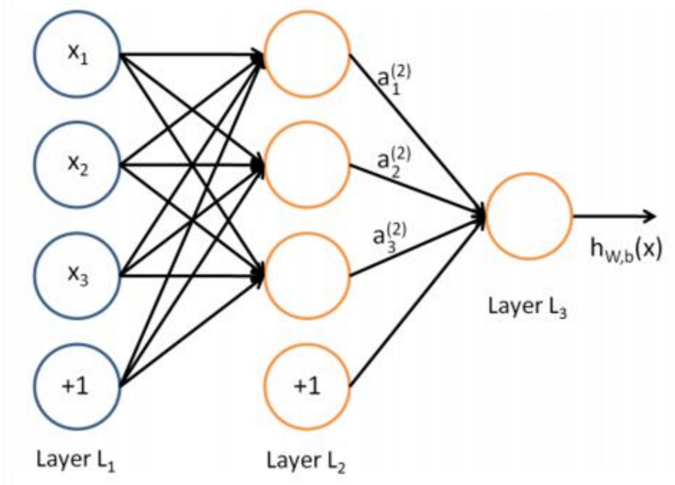
$a_i^{(l)} = f(z_i^{(l)})$ *f*(.) applies the function point wise

*Slide courtesy, sparse autoencoder by Andrew Ng

## How to learn the parameters?

$$\begin{aligned}
a_1^{(2)} &= f(W_{11}^{(1)}x_1 + W_{12}^{(1)}x_2 + W_{13}^{(1)}x_3 + b_1^{(1)}) \\
a_2^{(2)} &= f(W_{21}^{(1)}x_1 + W_{22}^{(1)}x_2 + W_{23}^{(1)}x_3 + b_2^{(1)}) \\
a_3^{(2)} &= f(W_{31}^{(1)}x_1 + W_{32}^{(1)}x_2 + W_{33}^{(1)}x_3 + b_3^{(1)})
\end{aligned}$$

$$h_{W,b}(x) = a_1^{(3)} = f(W_{11}^{(2)}a_1^{(2)} + W_{12}^{(2)}a_2^{(2)} + W_{13}^{(2)}a_3^{(2)} + b_1^{(2)})$$



Let, $z_i^{(l)}$ denote weighted sum for the activation $a_i^{(l)}$

$$\begin{aligned}
z^{(2)} &= W^{(1)}x + b^{(1)} \\
a^{(2)} &= f(z^{(2)}) \\
z^{(3)} &= W^{(2)}a^{(2)} + b^{(2)} \\
h_{W,b}(x) &= a^{(3)} = f(z^{(3)})
\end{aligned}$$

$$\begin{aligned}
z^{(l+1)} &= W^{(l)}a^{(l)} + b^{(l)} \\
a^{(l+1)} &= f(z^{(l+1)})
\end{aligned}$$

*Slide courtesy, sparse autoencoder by Andrew Ng

# How to learn the parameters?

Given *m* training examples

$$\{(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})\}$$

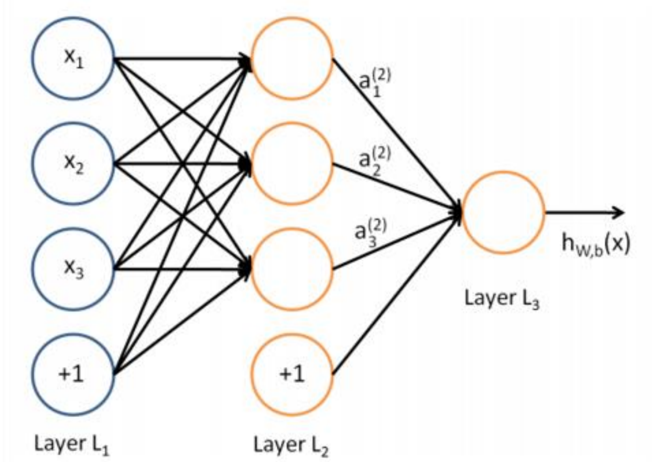
Minimize:

$$J(W,b;x,y) = \frac{1}{2}\left\|h_{W,b}(x) - y\right\|^2$$

Assume we are solving a regression problem

$$J(W,b) = \left[\frac{1}{m}\sum_{i=1}^{m} J(W,b;x^{(i)},y^{(i)})\right]$$

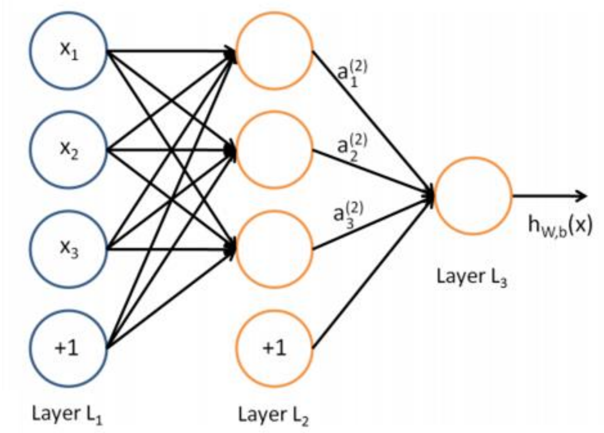
$$= \left[\frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{2}\left\|h_{W,b}(x^{(i)}) - y^{(i)}\right\|^2\right)\right]$$



*Slide courtesy, sparse autoencoder by Andrew Ng

## How to learn the parameters?

Minimize: $$J(W,b) = \left[\frac{1}{m}\sum_{i=1}^{m} J(W,b;x^{(i)},y^{(i)})\right] = \left[\frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{2}\left\|h_{W,b}(x^{(i)}) - y^{(i)}\right\|^2\right)\right]$$



Gradient descent:

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha\frac{\partial}{\partial W_{ij}^{(l)}}J(W,b)$$

$$b_i^{(l)} := b_i^{(l)} - \alpha\frac{\partial}{\partial b_i^{(l)}}J(W,b)$$

How to evaluate these partial derivatives?

*Error back-propagation*

*Slide courtesy, sparse autoencoder by Andrew Ng

# Error back propagation

Gradient descent:

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b) \qquad J(W,b) = \left[\frac{1}{m}\sum_{i=1}^{m} J(W,b;x^{(i)},y^{(i)})\right]$$

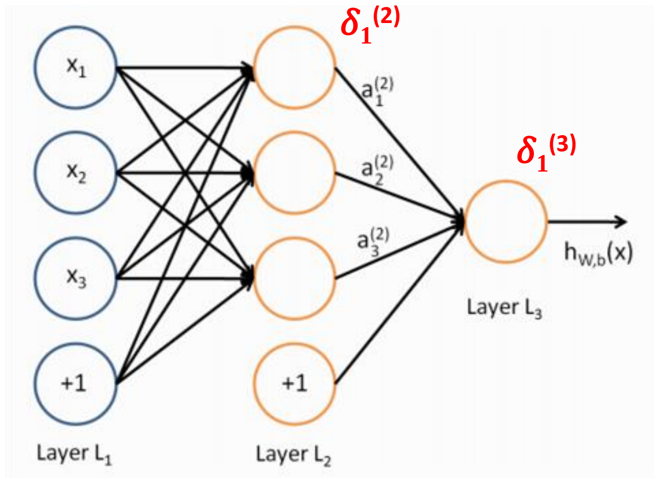
$$b_i^{(l)} := b_i^{(l)} - \alpha \frac{\partial}{\partial b_i^{(l)}} J(W,b)$$

$$= \left[\frac{1}{m}\sum_{i=1}^{m}\left(\frac{1}{2}\left\|h_{W,b}(x^{(i)}) - y^{(i)}\right\|^2\right)\right]$$

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b) = \frac{\partial}{\partial W_{ij}^{(l)}} \cdot \left[\frac{1}{m}\sum_{i=1}^{m} J(W,b;x^{(i)},y^{(i)})\right]$$

- ❖ Overall gradient can be computed by computing gradients wrt individual data terms
- ❖ Perform back propagation for computing individual data gradients
- ❖ Average them to get the overall gradient

# Back-propagation algorithm



Gradient descent:

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b)$$

**Idea**:

First, forward pass the data to calc. all responses

In backward pass, for each unit $i$ in layer $l$ calculate **error** term $\delta_i^{(l)}$ - measures how much unit $i$ is responsible for output error

- For output unit in last layer ($n_l$), this is easy

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \left\|y - h_{W,b}(x)\right\|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

- How to measure $\delta_i^{(l)}$ for hidden units?

*Slide courtesy, sparse autoencoder by Andrew Ng

# Preview of back-propagation

1. Perform a feedforward pass
   - Computing activations $L_1$, $L_2$ and so on ...

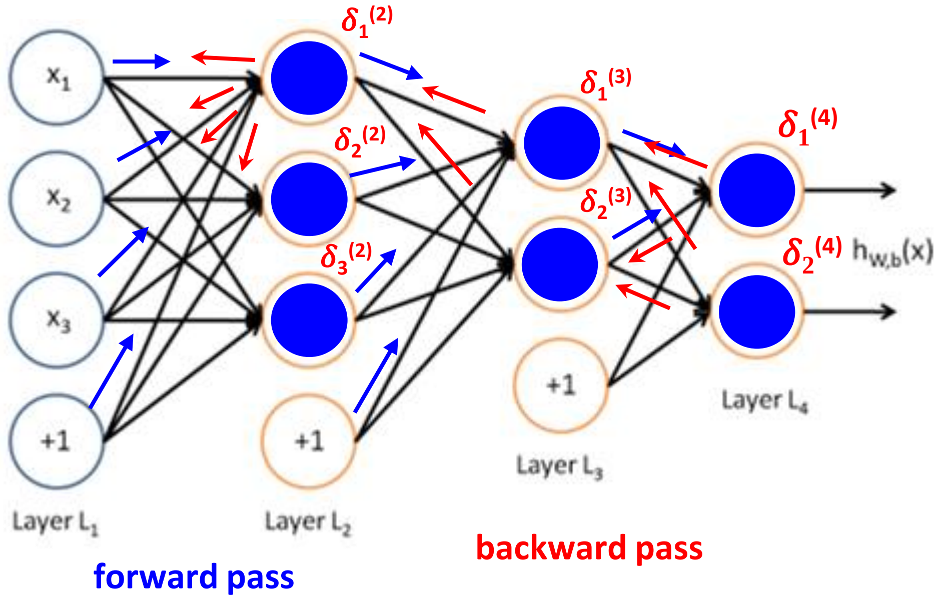
2. For each output unit $i$ in layer $L_4$ (output layer), set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \|y - h_{W,b}(x)\|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

3. Starting from last but one layer to 2nd layer;
   $l = n_l - 1, n_l - 2, \ldots\ldots, 2$
   - For each node $i$ in layer $l$, set $\delta_i^{(l)} = \left( \sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$

4. Compute the desired partial derivatives, as:

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b;x,y) = a_j^{(l)} \delta_i^{(l+1)} \qquad \frac{\partial}{\partial b_i^{(l)}} J(W,b;x,y) = \delta_i^{(l+1)}.$$



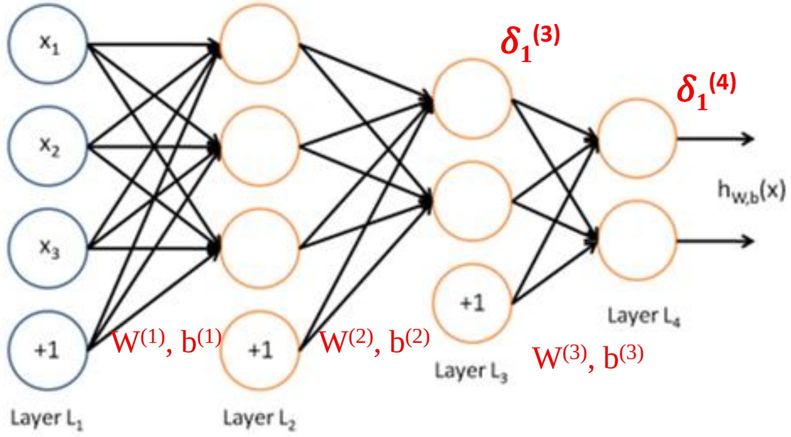
*Slide courtesy, sparse autoencoder by Andrew Ng

# Back-propagation algorithm

**Gradient descent:**

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b)$$

$$J(W,b;x,y) = \frac{1}{2}\left\| h_{W,b}(x) - y \right\|^2$$



$$h_{W,b}(x) = a^{(4)} = f(z^{(4)}); \; z^{(4)} = W^{(3)}a^{(3)} + b^{(3)}$$

**For last layer:**

$$\frac{\partial J}{\partial W_{ij}^{(3)}} = \underbrace{\boxed{\frac{\partial J}{\partial z_i^4}}}_{\delta_i^{(4)}} \underbrace{\boxed{\frac{\partial z_i^4}{\partial W_{ij}^{(3)}}}}_{\boxed{a_j^{(3)}}}$$

$$\boxed{\frac{\partial J}{\partial z_i^{\,4}} = -(y_i - a_i^{(4)}) \;.f'(z_i^4)}$$

$\delta_i^{(4)}$ **error term**

$$\boxed{\frac{\partial z_i^4}{\partial W_{ij}^3} = a_j^{(3)}}$$

$$\frac{\partial J}{\partial W_{ij}^{(l)}} = \delta_i^{(l+1)} a_j^{(l)} \qquad \frac{\partial J}{\partial b_i^{(l)}} = \delta_i^{(l+1)}$$

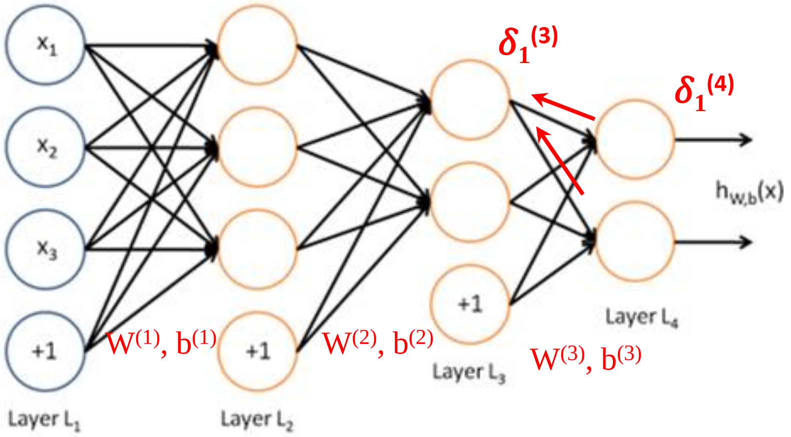
*Slide courtesy, sparse autoencoder by Andrew Ng

# Back-propagation algorithm

**Gradient descent:**

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b)$$

$$J(W,b;x,y) = \frac{1}{2} \left\| h_{W,b}(x) - y \right\|^2$$



$$h_{W,b}(x) = a^{(4)} = f(z^{(4)});\; z^{(4)} = W^{(3)}a^{(3)} + b^{(3)}$$

$$a^{(3)} = f(z^{(3)});\; z^{(3)} = W^{(2)}a^{(2)} + b^{(2)}$$

**For layers other than last:**

$$\frac{\partial J}{\partial W_{ij}^{(2)}} = \boxed{\frac{\partial J}{\partial z_i^{(3)}}} \boxed{\frac{\partial z_i^{(3)}}{\partial W_{ij}^{(2)}}} \rightarrow a_j^{(2)}$$

$\delta_i^{(3)}$ **error term**

$$\boxed{\begin{aligned} \frac{\partial J}{\partial z_i^{(3)}} &= \frac{\partial J}{\partial a_i^{(3)}} \frac{\partial a_i^{(3)}}{\partial z_i^{(3)}} \\ &= \left( \sum_j \underbrace{\frac{\partial J}{\partial z_j^{(4)}}}_{\delta_j^{(4)}} \underbrace{\frac{\partial z_j^{(4)}}{\partial a_i^{(3)}}}_{W_{ji}^{(3)}} \right) f'(z_i^{(3)}) \end{aligned}}$$

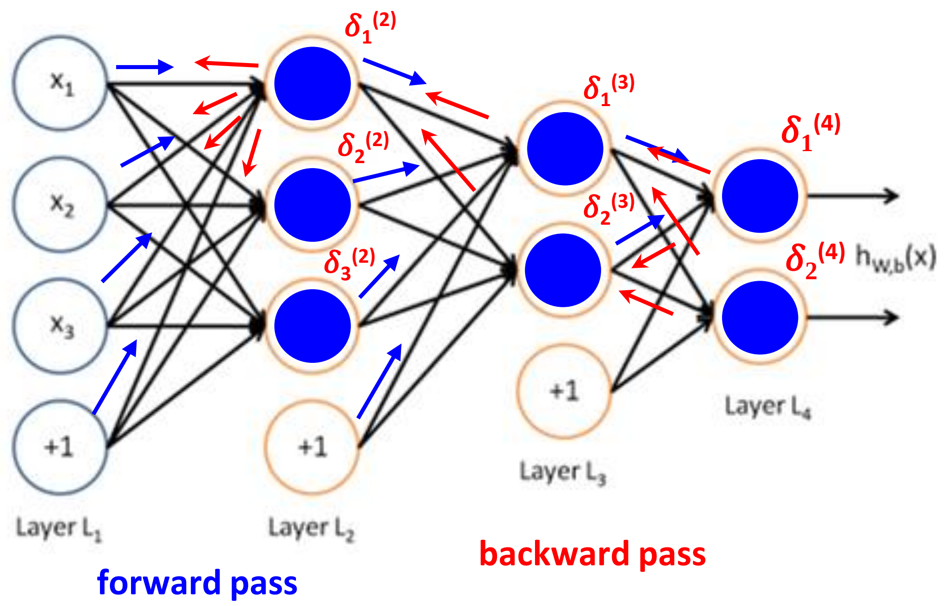
$\delta_j^{(4)}$ Layer - $(l+1)$

$$\delta_i^{(l)} = \left( \sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$$

$$\frac{\partial J}{\partial W_{ij}^{(l)}} = \delta_i^{(l+1)} a_j^{(l)} \qquad \frac{\partial J}{\partial b_i^{(l)}} = \delta_i^{(l+1)}$$

*Slide courtesy, sparse autoencoder by Andrew Ng

# Back-propagation algorithm



1. Perform a feedforward pass
   - Computing activations $L_1$, $L_2$ and so on …

2. For each output unit $i$ in layer $L_4$ (output layer), set

$$\delta_i^{(n_l)} = \frac{\partial}{\partial z_i^{(n_l)}} \frac{1}{2} \|y - h_{W,b}(x)\|^2 = -(y_i - a_i^{(n_l)}) \cdot f'(z_i^{(n_l)})$$

3. Starting from last but one layer to 2nd layer;
   $l = n_l\text{-}1, n_l\text{-}2, \ldots\ldots, 2$
   - For each node $i$ in layer $l$, set $\delta_i^{(l)} = \left( \sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$

4. Compute the desired partial derivatives, as:

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W,b;x,y) = a_j^{(l)} \delta_i^{(l+1)} \qquad \frac{\partial}{\partial b_i^{(l)}} J(W,b;x,y) = \delta_i^{(l+1)}.$$

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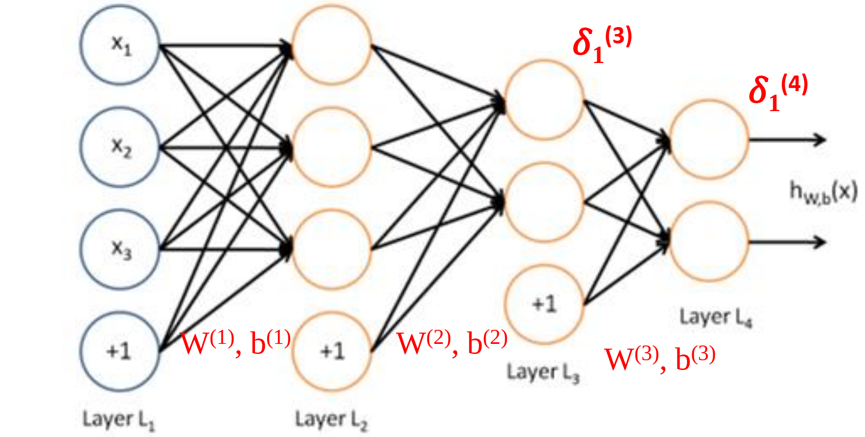
# Back-propagation algorithm



$$h_{W,b}(x) = a^{(4)} = f(z^{(4)}); \; z^{(4)} = W^{(3)}a^{(3)} + b^{(3)}$$

**Gradient descent:**

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W,b)$$

$$J(W,b;x,y) = \frac{1}{2} \left\| h_{W,b}(x) - y \right\|^2$$

**Partial derivatives:**

$$\delta_i^{(l)} = \left( \sum_{j=1}^{s_{l+1}} W_{ji}^{(l)} \delta_j^{(l+1)} \right) f'(z_i^{(l)})$$

$$\frac{\partial J}{\partial W_{ij}^{(l)}} = \delta_i^{(l+1)} a_j^{(l)} \qquad \frac{\partial J}{\partial b_i^{(l)}} = \delta_i^{(l+1)}$$

**Matrix notation:**

$$\delta^{(l)} = \left( (W^{(l)})^T \delta^{(l+1)} \right) \bullet f'(z^{(l)})$$

$$\frac{\partial J}{\partial W^{(l)}} = \delta^{(l+1)} (a^{(l)})^T \qquad \frac{\partial J}{\partial b^{(l)}} = \delta^{(l+1)}$$

*Slide courtesy, sparse autoencoder by Andrew Ng

# Back-propagation algorithm

1. Perform a feedforward pass
   - Computing activations $L_1$, $L_2$ and so on ...

2. For each output unit i in layer $L_4$ (output layer), set

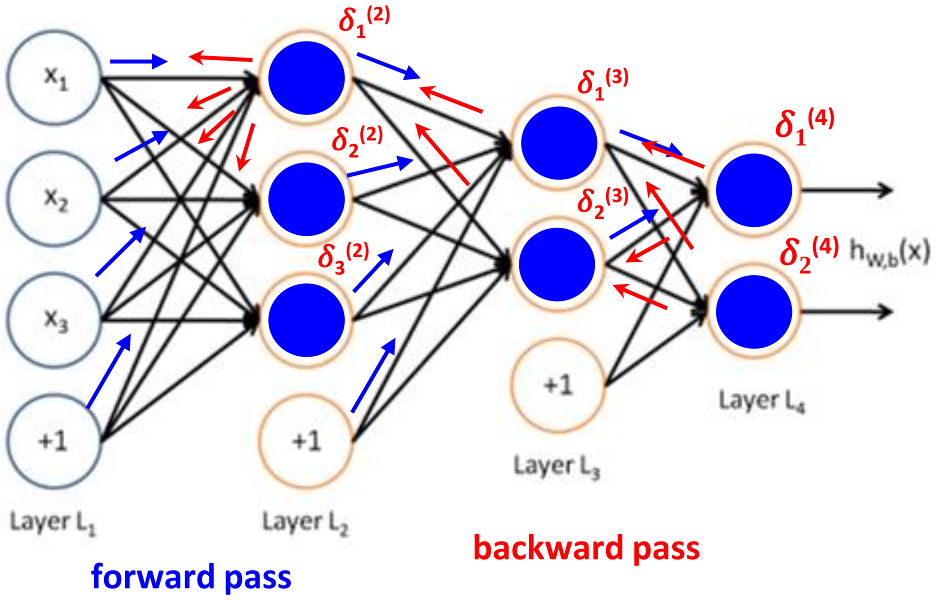
$$\delta^{(n_l)} = -(y - a^{(n_l)}) \bullet f'(z^{(n)})$$

3. Starting from last but one layer to 2nd layer;
   $l = n_l\text{-}1, n_l\text{-}2, \ldots\ldots, 2$

$$\delta^{(l)} = \left((W^{(l)})^T \delta^{(l+1)}\right) \bullet f'(z^{(l)})$$

4. Compute the desired partial derivatives, as:

$$\begin{aligned} \nabla_{W^{(l)}} J(W,b;x,y) &= \delta^{(l+1)} (a^{(l)})^T, \\ \nabla_{b^{(l)}} J(W,b;x,y) &= \delta^{(l+1)}. \end{aligned}$$



*Slide courtesy, sparse autoencoder by Andrew Ng

## Summary: Error back propagation

Gradient descent:

$$W_{ij}^{(l)} := W_{ij}^{(l)} - \alpha \frac{\partial}{\partial W_{ij}^{(l)}} J(W, b)$$

$$b_i^{(l)} := b_i^{(l)} - \alpha \frac{\partial}{\partial b_i^{(l)}} J(W, b)$$

$$\frac{\partial}{\partial W_{ij}^{(l)}} J(W, b) = \frac{\partial}{\partial W_{ij}^{(l)}} \left[ \frac{1}{m} \sum_{i=1}^{m} J(W, b; x^{(i)}, y^{(i)}) \right]$$

- ❖ Perform back propagation for computing individual gradient wrt each data
- ❖ Average them to get the overall gradient

## Basic Neural Networks

What we will learn in the course about Neural Networks?

### Introduction

- McCulloch and Pits model
- Rosenblatt’s perceptron

### Perceptrons

- Geometry and linear separability
- XoR problem
- Multi-layer perceptron (MLP)

### Training MLPs

- Error back propagation
- Loss functions

### Regularization and optimization

- Over fitting / Under fitting
- Regularization
- Various algorithm

## Regularization

Overfitting and underfitting, L2 and L1 norm regularizations, Bagging, Dropout

## Generalization

The central challenge of machine learning is to perform well on the - *unseen test* data, not just the *training data*

*While training the model*

**Train err** $$\frac{1}{m^{(\text{train})}}||X^{(\text{train})}w - y^{(\text{train})}||_2^2,$$

*What we actually want*

**Test err (or) Generalization err** $$\frac{1}{m^{(\text{test})}}||X^{(\text{test})}w - y^{(\text{test})}||_2^2.$$

How can we say something about the test data by only seeing the train data?

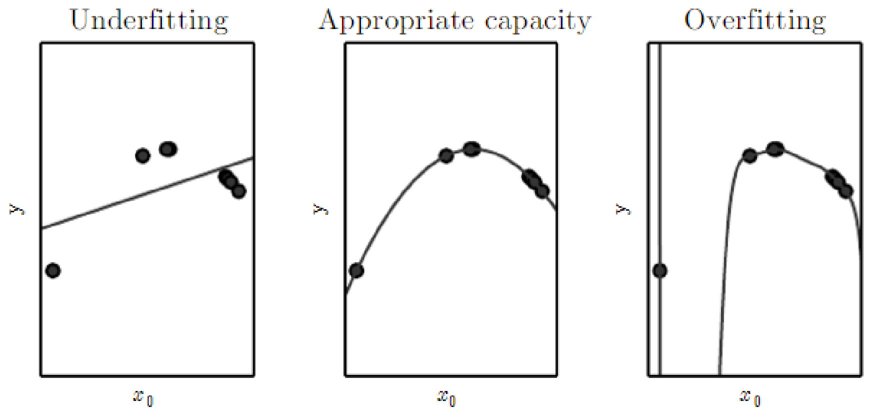
Statistical learning theory

- Training and Test sets are not arbitrary
- Underlying *data generating distribution* is same

*Slide courtesy, Ian Goodfellow et al., deep learning book

# Capacity, Overfitting and Underfitting

The central challenge of machine learning is to perform well on the *unseen test* data, not just the *training data*



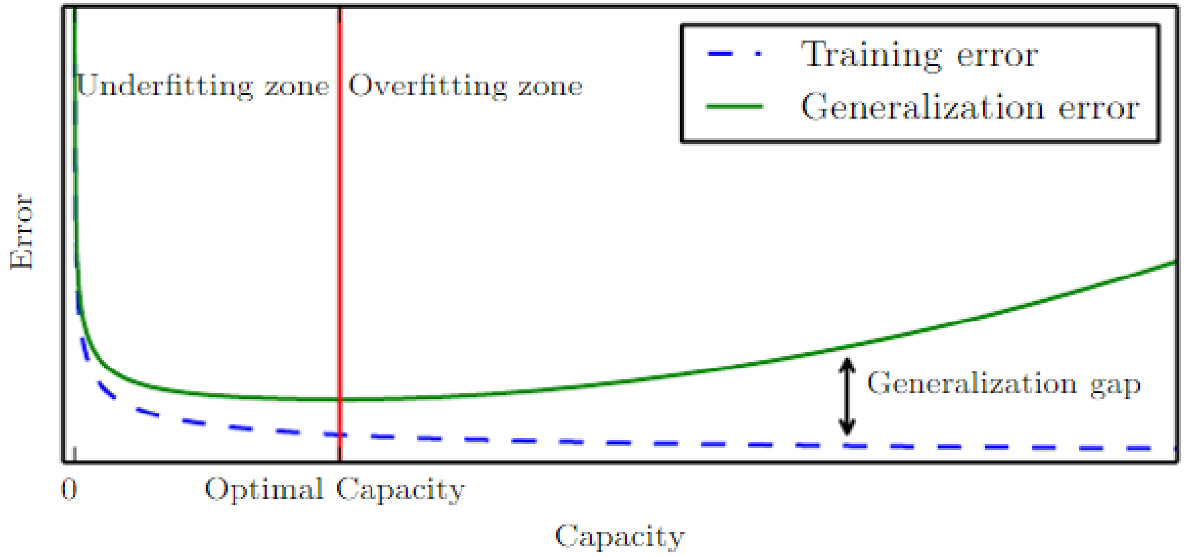
$\hat{y} = b + wx.$ $\hat{y} = b + w_1 x + w_2 x^2.$ $\hat{y} = b + \sum_{i=1}^{9} w_i x^i.$

Occam's razor: This principle states that among competing hypotheses that explain known observations equally well, one should choose the "simplest" one.

*Slide courtesy, Ian Goodfellow et al., deep learning

# Capacity, Overfitting and Underfitting

The central challenge of machine learning is to perform well on the *unseen test* data, not just the *training data*

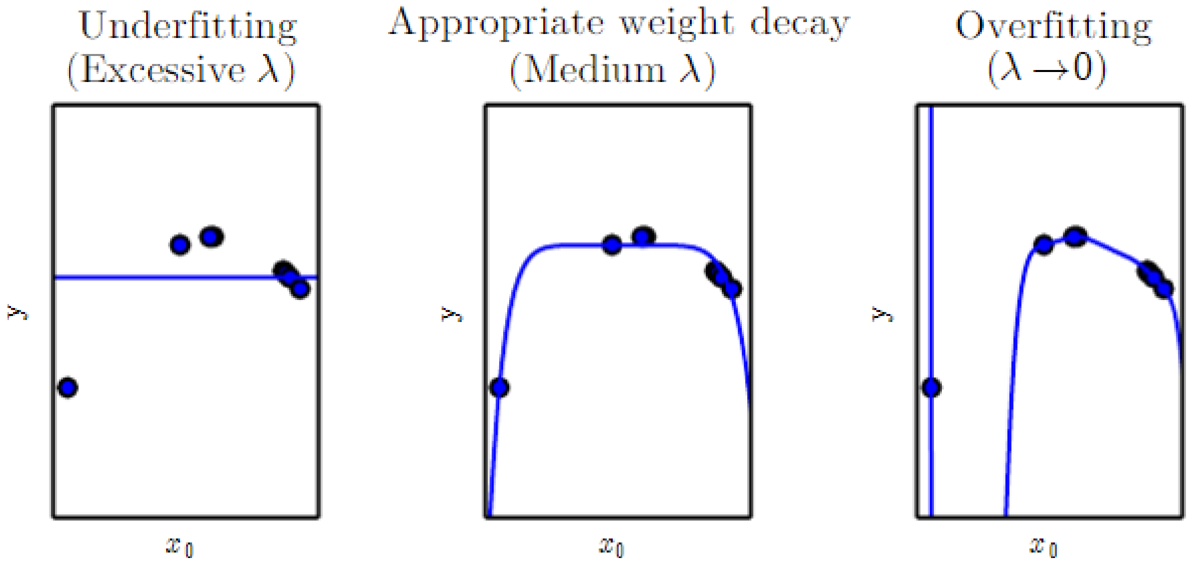


*Slide courtesy, Ian Goodfellow et al., deep learning book

# Regularization

*Regularization* is any modification we make to a learning algorithm that is intended to reduce its generalization error but not its training error

$$J(\boldsymbol{w}) = \text{MSE}_{\text{train}} + \lambda \boldsymbol{w}^{\top}\boldsymbol{w},$$



*Slide courtesy, Ian Goodfellow et al., deep learning book

## Regularization - parameter norm penalties

**$L_2$ norm regularization**
(weight decay, ridge regression )

$$\tilde{J}(\boldsymbol{\theta}; \boldsymbol{X}, \boldsymbol{y}) = J(\boldsymbol{\theta}; \boldsymbol{X}, \boldsymbol{y}) + \alpha\Omega(\boldsymbol{\theta})$$

$$\tilde{J}(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}) = \frac{\alpha}{2}\boldsymbol{w}^\top \boldsymbol{w} + J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}),$$

Parameter update:

$$\nabla_{\boldsymbol{w}}\tilde{J}(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}) = \alpha\boldsymbol{w} + \nabla_{\boldsymbol{w}} J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}).$$

$$\boldsymbol{w} \leftarrow \boldsymbol{w} - \epsilon\left(\alpha\boldsymbol{w} + \nabla_{\boldsymbol{w}} J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y})\right).$$

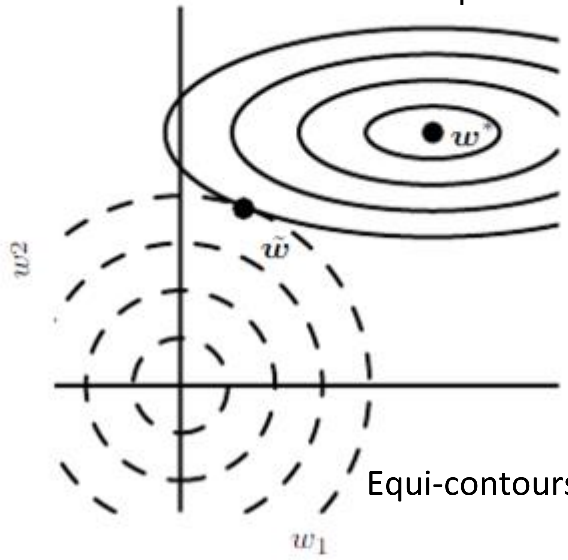
$$\boldsymbol{w} \leftarrow (1 - \epsilon\alpha)\boldsymbol{w} - \epsilon\nabla_{\boldsymbol{w}} J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}).$$

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Illustration of Weight decay

$$\tilde{J}(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}) = \frac{\alpha}{2} \boldsymbol{w}^\top \boldsymbol{w} + J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}),$$

Equi-contours of unregularized objective $\boldsymbol{J}$



Equi-contours of $\boldsymbol{L_2}$ regularizer

# Regularization - parameter norm penalties

***$L_1$* norm regularization**

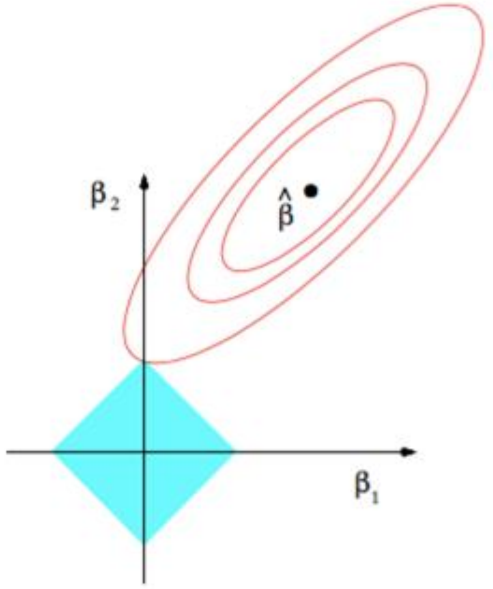
$$\tilde{J}(\boldsymbol{\theta}; \boldsymbol{X}, \boldsymbol{y}) = J(\boldsymbol{\theta}; \boldsymbol{X}, \boldsymbol{y}) + \alpha\Omega(\boldsymbol{\theta})$$

$$\Omega(\boldsymbol{\theta}) = ||\boldsymbol{w}||_1 = \sum_i |w_i|,$$

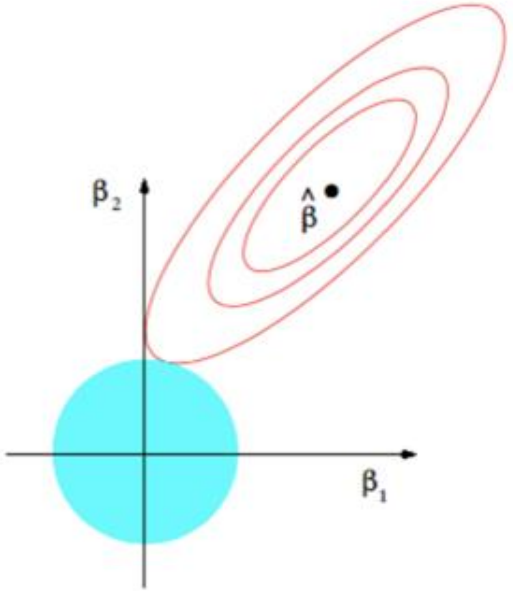
$$\tilde{J}(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y}) = \alpha||\boldsymbol{w}||_1 + J(\boldsymbol{w}; \boldsymbol{X}, \boldsymbol{y})$$

L1 regularization results in sparser solution

- ❖ Feature selection



L1 regularization

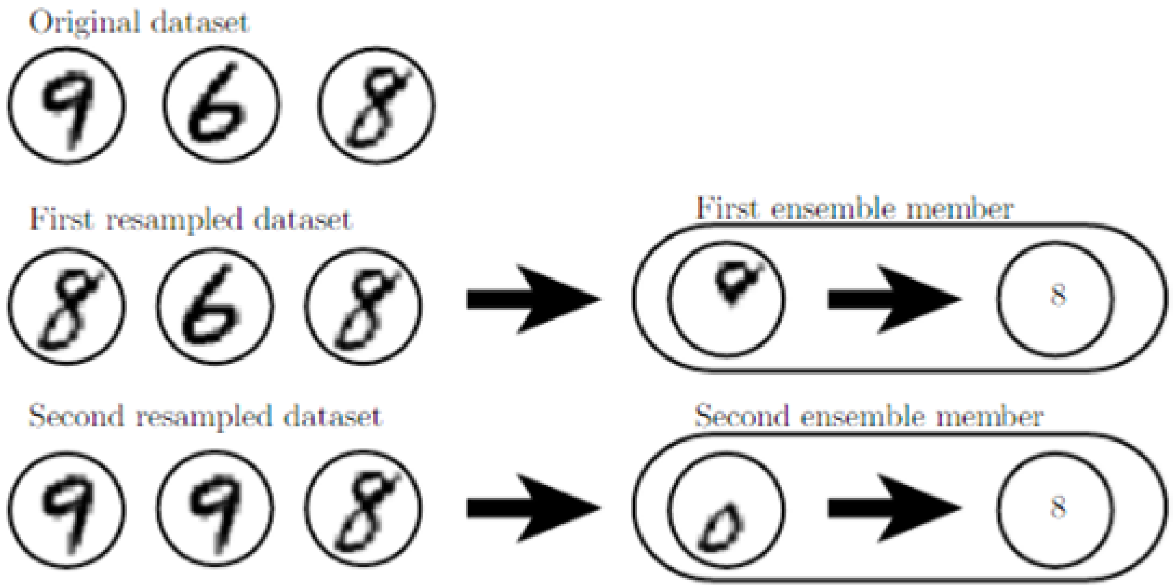


L2 regularization

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Regularization - other methods

**Bagging - bootstrap aggregating** (Breiman, 1994)



# Regularization - other methods

**Bagging - bootstrap aggregating**

- *Say **k** regression models*
  - Say each of them makes $\epsilon_i$ error
  - Error is drawn from Multivariate normal distribution

$$\mathbb{E}[\epsilon_i^2] = v; \quad \mathbb{E}[\epsilon_i \epsilon_j] = c$$

- *Avg. error by **k** models*

$$(1/k) \sum_i \epsilon_i$$

- *Expected squared error of the ensemble predictor*

$$\mathbb{E}\left[\left(\frac{1}{k}\sum_i \epsilon_i\right)^2\right] = \frac{1}{k^2}\mathbb{E}\left[\sum_i\left(\epsilon_i^2 + \sum_{j\neq i}\epsilon_i\epsilon_j\right)\right]$$
$$= \frac{1}{k}v + \frac{k-1}{k}c.$$

  - If perfectly uncorrelated, *$c$ = 0, error reduces to $v/k$*
  - If the models are perfectly correlated and $c = v$, error remains at $v$

Disadvantage: Need to train multiple networks. Not very feasible for deep learning

# Regularization - other methods

**Drop-out** (Srivastava et al., 2014)

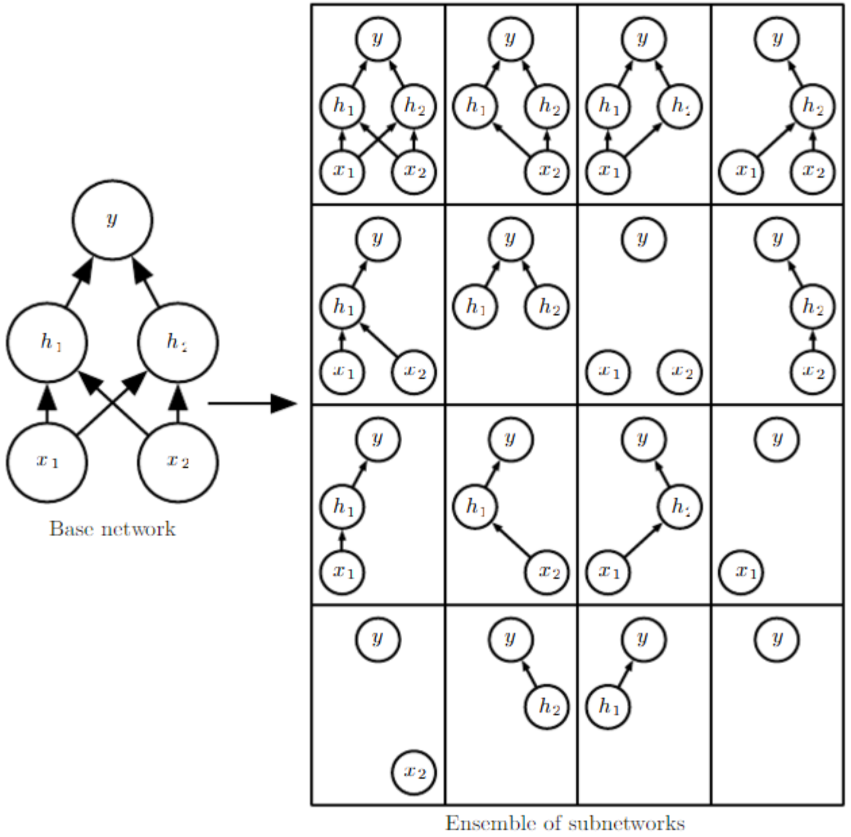
Stochastically turn the activation of the hidden unit off with a probability, ***p***

$$h^{(k)} = f(Wh^{(k-1)} + b^{(k-1)})$$
$$\hat{h}^{(k)} = \mu^{(k)} \odot h^{(k)}$$

– How to train it? Each batch of data sees a sampled sub-network

**Inference: Weight rescaling** (Hinton et al., 2012)

- Multiply weights going out of unit $i$ with probability of including unit $i$



*Slide courtesy, Ian Goodfellow et al., deep learning book

## Regularization - other methods

**Dataset augmentation**

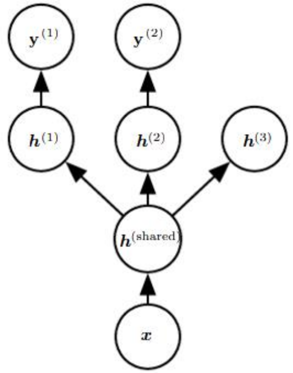
Flipping the image for classification

*pic courtesy, web

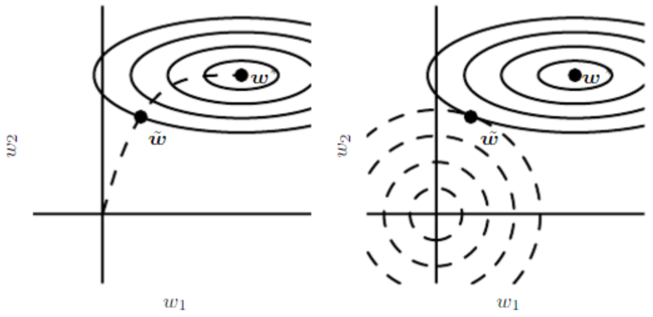
**Parameter sharing and tying**

Most extensively employed with Convolutional Neural Nets (CNN)

**Multi-task learning**



**Early stopping**



*Slide courtesy, Ian Goodfellow et al., deep learning book

## What we covered in Regularization

- Overfitting and underfitting
- L2 and L1 norm regularization
- Bagging
- Drop-out
- Other methods

# Optimization

Minibatch optimization, Stochastic Gradient Descent, Momentum, Algorithms (AdaGrad, RMSProp, Adam)

## Cost function to optimize

*While training the model*

$$J(\boldsymbol{\theta}) = \mathbb{E}_{(\boldsymbol{x},\mathrm{y})\sim\hat{p}_{\text{data}}} L(f(\boldsymbol{x};\boldsymbol{\theta}), y),$$

$\hat{p}_{\,data}$ distribution of training data

*What we actually want*

$$J^{*}(\boldsymbol{\theta}) = \mathbb{E}_{(\boldsymbol{x},\mathrm{y})\sim p_{\text{data}}} L(f(\boldsymbol{x};\boldsymbol{\theta}), y).$$

$p_{data}$ distribution of actual data

Empirical risk minimization

$$\mathbb{E}_{\boldsymbol{x},\mathrm{y}\sim\hat{p}_{\text{data}}(\boldsymbol{x},y)}[L(f(\boldsymbol{x};\boldsymbol{\theta}), y)] = \frac{1}{m}\sum_{i=1}^{m} L(f(\boldsymbol{x}^{(i)};\boldsymbol{\theta}), y^{(i)})$$

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Batch and Minibatch algorithms

*Loss function*

$$J(\boldsymbol{\theta}) = \mathbb{E}_{(\boldsymbol{x},y)\sim\hat{p}_{\text{data}}} L(f(\boldsymbol{x};\boldsymbol{\theta}), y),$$

*Training by backpropagation*

$$\nabla_\theta J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta L(f(x_i;\theta), y_i)$$

It requires you to evaluate gradients w.r.t all the training examples for gradient estimation

**Is this efficient?**

– Variance in the estimation with *m* samples - $\sigma / \sqrt{m}$

– By calculating grads over all samples, we get only sub-linear performance

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Batch and Minibatch algorithms

*Loss function*

$$J(\boldsymbol{\theta}) = \mathbb{E}_{(\boldsymbol{x},y)\sim\hat{p}_{\text{data}}} L(f(\boldsymbol{x};\boldsymbol{\theta}), y),$$

*Training by backpropagation*

$$\nabla_\theta J(\theta) = \frac{1}{m} \sum_{i=1}^{m} \nabla_\theta L(f(x_i;\theta), y_i)$$

By calculating grads over all samples, we get only **sub-linear** performance

**What is the alternative?**

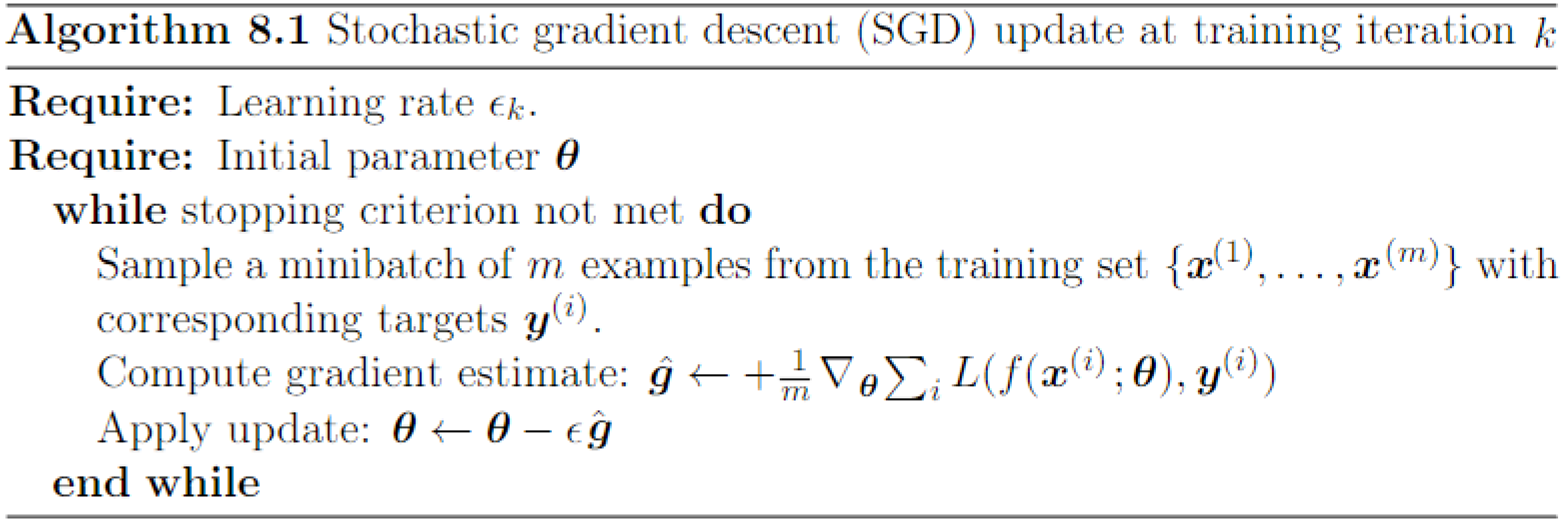
- Simple solution, don't use all the samples for gradient estimation
- At each update iteration, randomly chose $B$ samples and use them for estimating gradients **Minibatch training**
- Also, does as unbiased estimate of gradients

$$\nabla_\theta J(\theta) = \frac{1}{B} \sum_{i=1}^{B} \nabla_\theta L(f(x_i;\theta), y_i)$$

*Slide courtesy, Ian Goodfellow et al., deep learning book

# Algorithms for optimization

## Stochastic Gradient Descent (SGD)



**Algorithm 8.1** Stochastic gradient descent (SGD) update at training iteration $k$

```
Require: Learning rate $\epsilon_k$.
Require: Initial parameter $\boldsymbol{\theta}$
  while stopping criterion not met do
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient estimate: $\hat{\boldsymbol{g}} \leftarrow +\frac{1}{m}\nabla_{\boldsymbol{\theta}}\sum_i L(f(\boldsymbol{x}^{(i)};\boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \epsilon\hat{\boldsymbol{g}}$
  end while
```

*Slide courtesy, Ian Goodfellow et al., deep learning book

# Algorithms for optimization

## Stochastic Gradient Descent (SGD) with momentum

*Parameter update step of SGD*

$$\text{Apply update: } \boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \epsilon \hat{\boldsymbol{g}}$$

- Depending on $\boldsymbol{\epsilon}$, learning can be very slow or have drastic oscillations
- Momentum is designed to accelerate SGD
- The momentum algorithm accumulates a weighted avg. of past gradients and continues to move in their direction.

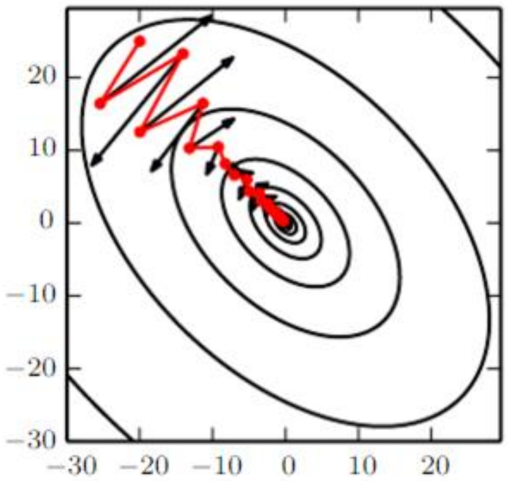


Figure showing effect of momentum
----- path with momentum
→ direction that SGD would take

$$\boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \epsilon \nabla_{\boldsymbol{\theta}} \left( \frac{1}{m} \sum_{i=1}^{m} L(\boldsymbol{f}(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)}) \right),$$

Velocity $\boldsymbol{v}$ accumulates the past gradients

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}.$$

The larger $\alpha$ is relative to $\boldsymbol{\epsilon}$, the effect of past gradients is more

*Slide courtesy, Ian Goodfellow et al., deep learning book

# Algorithms for optimization

## Stochastic Gradient Descent (SGD) with momentum

*Parameter update step now*

$$v \leftarrow \alpha v - \epsilon \nabla_{\theta} \left( \frac{1}{m} \sum_{i=1}^{m} L(f(x^{(i)}; \theta), y^{(i)}) \right),$$

$$\theta \leftarrow \theta + v.$$

- In SGD, update step size was $\epsilon$ ||g||
- With momentum, depends on how large and how aligned a *sequence* of gradients are
- Its largest, when successive gradients are same

If momentum repeatedly observes gradient as ***g***, it accelerates by a factor of $\frac{1}{1-\alpha}$, resulting in $\frac{\epsilon ||g||}{1-\alpha}$.

For $\alpha$ = 0.9, the descent is 10 times normal SGD

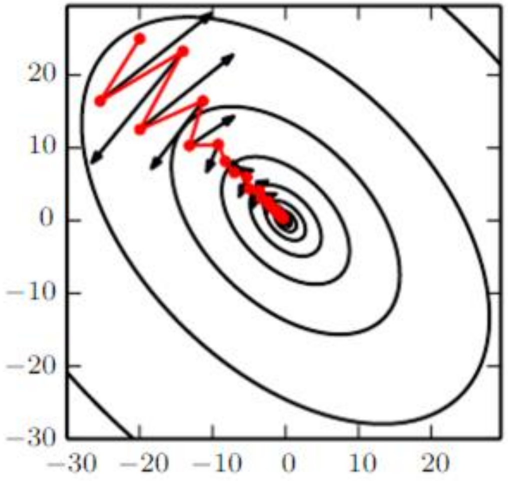
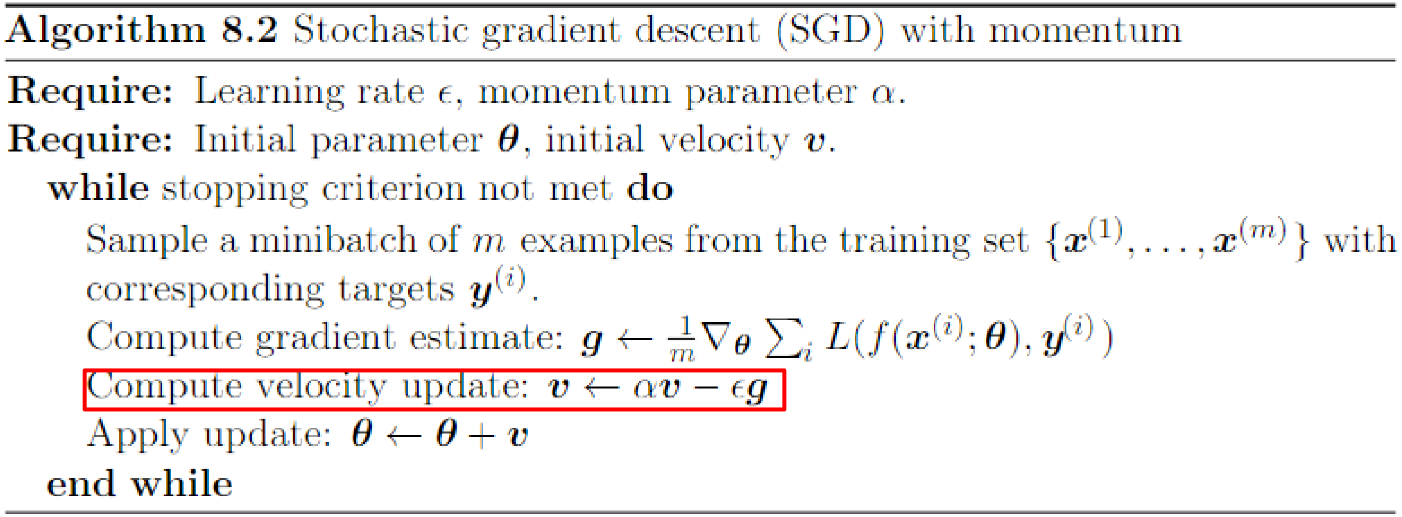


Figure showing effect of momentum
----- path with momentum
→ direction that SGD would take

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Algorithms for optimization

Stochastic Gradient Descent (SGD) with momentum



**Algorithm 8.2** Stochastic gradient descent (SGD) with momentum

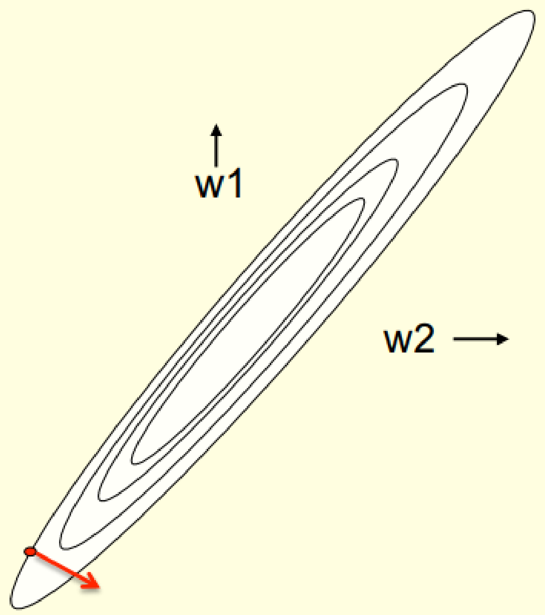
```
Require: Learning rate $\epsilon$, momentum parameter $\alpha$.
Require: Initial parameter $\boldsymbol{\theta}$, initial velocity $\boldsymbol{v}$.
  while stopping criterion not met do
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with
    corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient estimate: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    $\boxed{\text{Compute velocity update: } \boldsymbol{v} \leftarrow \alpha \boldsymbol{v} - \epsilon \boldsymbol{g}}$
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \boldsymbol{v}$
  end while
```

*Slide courtesy, Ian Goodfellow et al., deep learning book

## Algorithms for optimization

### Why learning can be slow

- If the ellipse is very elongated, the direction of steepest descent is almost perpendicular to the direction towards the minimum!
  - The red gradient vector has a large component along the short axis of the ellipse and a small component along the long axis of the ellipse.
  - This is just the opposite of what we want.



*Slide courtesy, G. Hinton’s course on cousera week 3

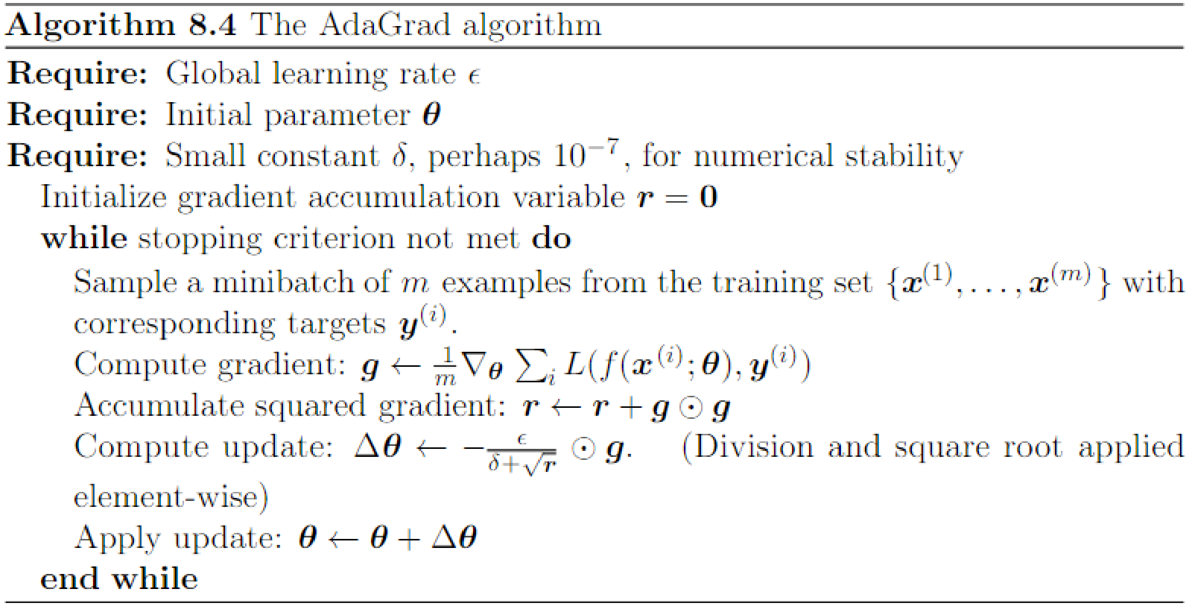
# Algorithms for optimization - adaptive learning rate

## AdaGrad (Duchi et al., 2011)

*Parameter update*

Scales the learning rate with square root of sum of past gradients

– Larger partial derivatives - reduced learning rates (viceversa)



**Algorithm 8.4** The AdaGrad algorithm

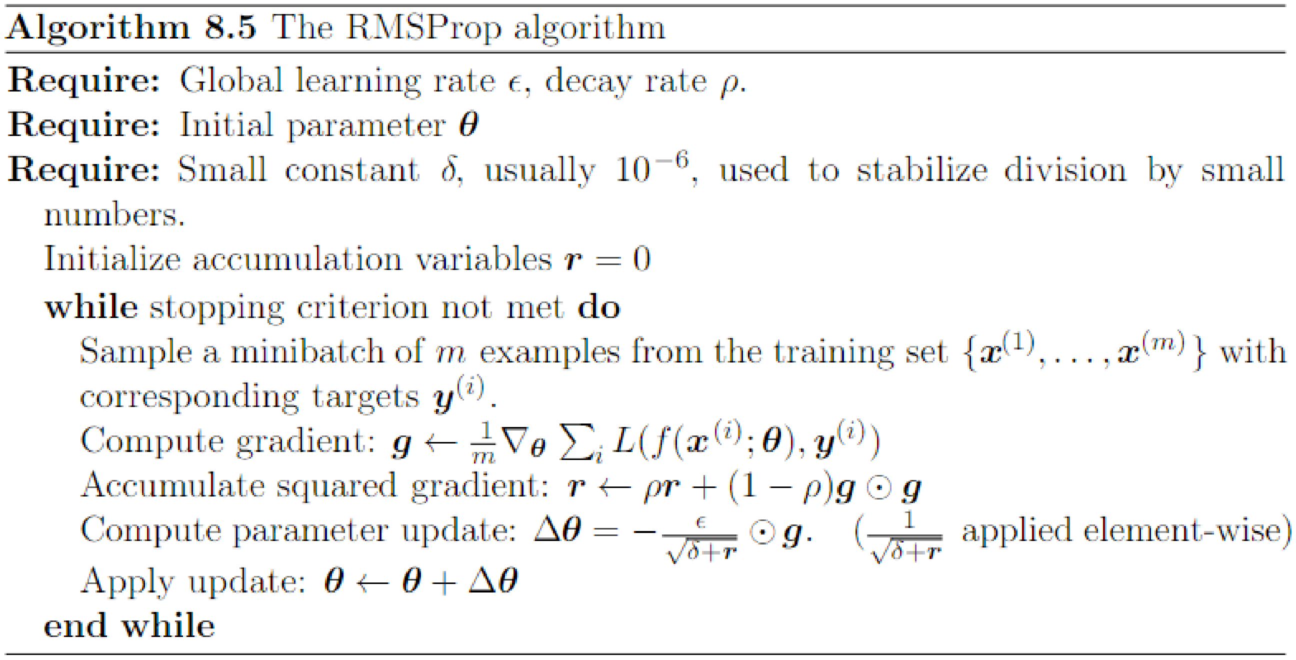
```
Require: Global learning rate $\epsilon$
Require: Initial parameter $\boldsymbol{\theta}$
Require: Small constant $\delta$, perhaps $10^{-7}$, for numerical stability
  Initialize gradient accumulation variable $\boldsymbol{r} = \mathbf{0}$
  while stopping criterion not met do
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ with corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    Accumulate squared gradient: $\boldsymbol{r} \leftarrow \boldsymbol{r} + \boldsymbol{g} \odot \boldsymbol{g}$
    Compute update: $\Delta\boldsymbol{\theta} \leftarrow -\frac{\epsilon}{\delta + \sqrt{\boldsymbol{r}}} \odot \boldsymbol{g}$.   (Division and square root applied element-wise)
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \Delta\boldsymbol{\theta}$
  end while
```

# Algorithms for optimization - adaptive learning rate

## RMSProp(Hinton et al., 2012)

### *Parameter update*

Scales the learning rate with weighted average of square of past gradients



**Algorithm 8.5** The RMSProp algorithm

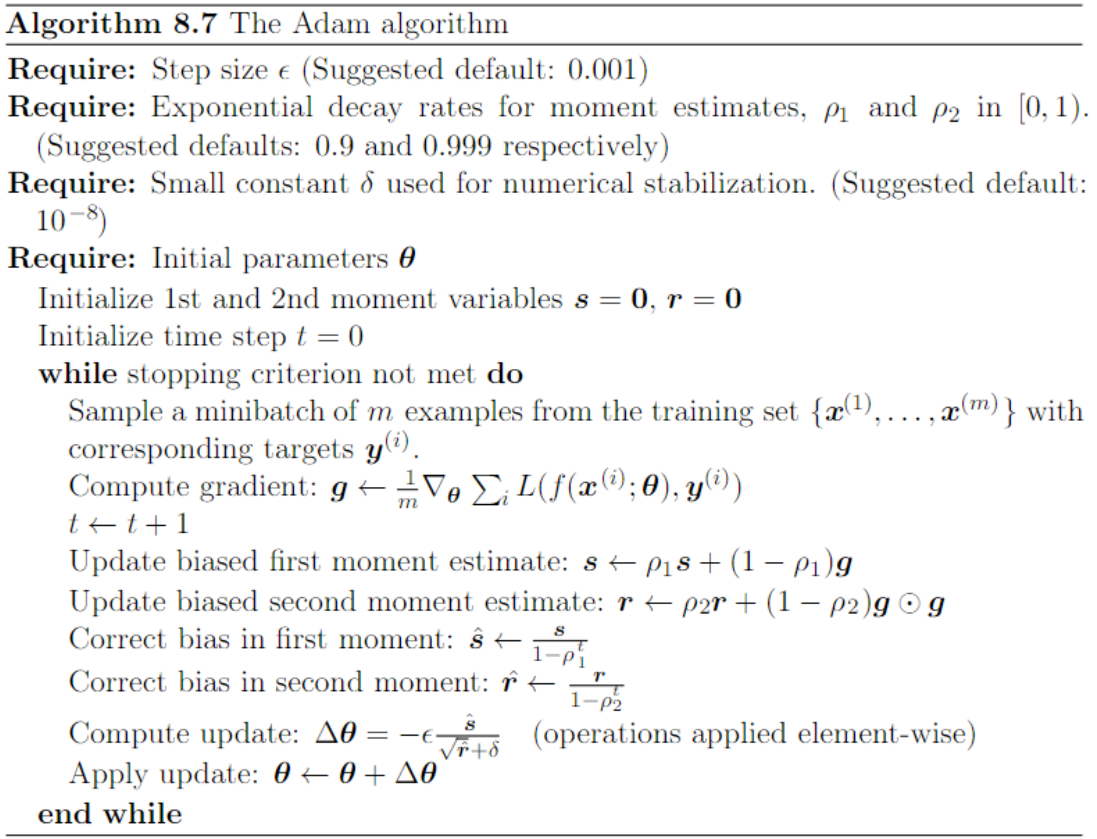
```
Require: Global learning rate $\epsilon$, decay rate $\rho$.
Require: Initial parameter $\boldsymbol{\theta}$
Require: Small constant $\delta$, usually $10^{-6}$, used to stabilize division by small
  numbers.
  Initialize accumulation variables $\boldsymbol{r} = 0$
  while stopping criterion not met do
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ with
    corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    Accumulate squared gradient: $\boldsymbol{r} \leftarrow \rho \boldsymbol{r} + (1 - \rho) \boldsymbol{g} \odot \boldsymbol{g}$
    Compute parameter update: $\Delta \boldsymbol{\theta} = -\frac{\epsilon}{\sqrt{\delta + \boldsymbol{r}}} \odot \boldsymbol{g}$.   ($\frac{1}{\sqrt{\delta + \boldsymbol{r}}}$ applied element-wise)
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \Delta \boldsymbol{\theta}$
  end while
```

# Algorithms for optimization - adaptive learning rate

Adam (Kingma et al., 2014)

*Parameter update*

Combines RMSProp and momentum methods



**Algorithm 8.7** The Adam algorithm

```
Require: Step size $\epsilon$ (Suggested default: 0.001)
Require: Exponential decay rates for moment estimates, $\rho_1$ and $\rho_2$ in $[0, 1)$.
  (Suggested defaults: 0.9 and 0.999 respectively)
Require: Small constant $\delta$ used for numerical stabilization. (Suggested default:
  $10^{-8}$)
Require: Initial parameters $\boldsymbol{\theta}$
  Initialize 1st and 2nd moment variables $\boldsymbol{s} = \boldsymbol{0}$, $\boldsymbol{r} = \boldsymbol{0}$
  Initialize time step $t = 0$
  while stopping criterion not met do
    Sample a minibatch of $m$ examples from the training set $\{\boldsymbol{x}^{(1)}, \dots, \boldsymbol{x}^{(m)}\}$ with
    corresponding targets $\boldsymbol{y}^{(i)}$.
    Compute gradient: $\boldsymbol{g} \leftarrow \frac{1}{m} \nabla_{\boldsymbol{\theta}} \sum_i L(f(\boldsymbol{x}^{(i)}; \boldsymbol{\theta}), \boldsymbol{y}^{(i)})$
    $t \leftarrow t + 1$
    Update biased first moment estimate: $\boldsymbol{s} \leftarrow \rho_1 \boldsymbol{s} + (1 - \rho_1)\boldsymbol{g}$
    Update biased second moment estimate: $\boldsymbol{r} \leftarrow \rho_2 \boldsymbol{r} + (1 - \rho_2)\boldsymbol{g} \odot \boldsymbol{g}$
    Correct bias in first moment: $\hat{\boldsymbol{s}} \leftarrow \frac{\boldsymbol{s}}{1 - \rho_1^t}$
    Correct bias in second moment: $\hat{\boldsymbol{r}} \leftarrow \frac{\boldsymbol{r}}{1 - \rho_2^t}$
    Compute update: $\Delta \boldsymbol{\theta} = -\epsilon \frac{\hat{\boldsymbol{s}}}{\sqrt{\hat{\boldsymbol{r}}} + \delta}$   (operations applied element-wise)
    Apply update: $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \Delta \boldsymbol{\theta}$
  end while
```

*Slide courtesy, Ian Goodfellow et al., deep learning book

## What we covered in Optimization

- Minibatch optimization
- Stochastic Gradient Descent (SGD)
- Momentum method
- Adaptive learning rate algorithms
  - AdaGrad
  - RMSProp
  - Adam

# Summary: Basic Neural Networks

## Introduction

- McCulloch and Pits model
- Rosenblatt’s perceptron

## Perceptrons

- Geometry and linear separability
- XoR problem
- Multi-layer perceptron (MLP)

## Training MLPs

- Error back propagation
- Loss functions

## Regularization and optimization

- Over fitting / Under fitting
- Regularization
- Various algorithm