Weekly Assignment 11

RC

2021-12-11

Question 1

$$F = 2xy^2\hat{\imath} - 5x^2z^3\hat{\jmath} + 3y^2z\hat{k}$$

A

$$5\mathbf{y_N}^2$$

B

$$(15\mathbf{x_N}^2\mathbf{z_N}^2 + 6\mathbf{y_N}\mathbf{z_N})\hat{\mathbf{i}}_\mathbf{N} + (-4\mathbf{x_N}\mathbf{y_N} - 10\mathbf{x_N}\mathbf{z_N}^3)\hat{\mathbf{k}}_\mathbf{N}$$

Question 2

$$F = (9x^2 + 3y^2 + 5y)\hat{\imath} + (6xy - 6y^2 + 5x + 2)\hat{\jmath}$$

F is a conservative field if and only if

$$\nabla f = F$$

F is defined everywhere in $\mathbb{R}^3$

If F is conservative, then the $curl(F) = 0$

$$\hat{\mathbf{0}}$$

Find the potential f

$$\frac{\partial f}{\partial x} = F \cdot \hat{\imath} = 9x^2 + 3y^2 + 5y$$

$$f = 3x^3 + 3xy^2 + 5xy + g(y)$$

$$\frac{\partial f}{\partial y} = F \cdot \hat{\jmath} = 6xy - 6y^2 + 5x + 2$$

$$f = 3xy^2 - 2y^3 + 5xy + 2y + h(x)$$

Therefore

$$f = 3x^3 + 3xy^2 + 5xy - 2y^3 + 2y + C$$

Since f can be found, F is conservative

Using python...

$$3\mathbf{x_N}^3 + \mathbf{x_N}\left(3\mathbf{y_N}^2 + 5\mathbf{y_N}\right) - 2\mathbf{y_N}^3 + 2\mathbf{y_N}$$

Is F conservative? True

Question 3

$$\int_C (x^3 + y)ds$$

$$r(t) = 3t\hat{\imath} + t^3\hat{\jmath} \qquad 0 \le t \le 2$$

$$\int_C f(r(t))ds = \int_a^b f(r(t))||r'(t)||dt$$

$$\int_C (x^3 + y)ds = \int_0^2 \left((3t)^3 + t^3\right)\sqrt{9 + 9t^4}dt = \int_0^2 28t^3\sqrt{9 + 9t^4}dt$$

$$= \frac{28}{36} * \frac{2}{3}(9 + 9t^4)^{3/2}\Big|_0^2 = \frac{14}{27}\left(153^{3/2} - 27\right) = 14 * 17^{3/2} - 14 = 238\sqrt{17} - 14$$

Using python...

$$-14 + 238\sqrt{17}$$
$$-14 + 238\sqrt{17}$$

Question 4

$$\int_C xy^2dx + x^3dy$$

$$r(t) = 2t\hat{\imath} + (t^2 - 3)\hat{\jmath} \qquad -1 \le t \le 1$$

$$\int_C xy^2dx + x^3dy = \int_{-1}^1 (2t)\left(t^2 - 3\right)^2(2)dt + \int_{-1}^1 (2t)^3(2t)dt$$

$$= \int_{-1}^1 4t\left(t^2 - 3\right)^2dt + \int_{-1}^1 16t^4dt = \frac{2}{3}(t^2 - 3)^3\Big|_{-1}^1 + \frac{16}{5}t^5\Big|_{-1}^1$$

$$= \frac{2}{3}\left[(-2)^3 - (-2)^3\right] + \frac{16}{5}\left[1 - (-1)\right] = \frac{32}{5}$$

Using python

$$\frac{32}{5}$$

Question 5

$$\int_C F \cdot dr$$

$$F = (4x^2 + 5y)\hat{i} + (2x - 6y)\hat{j} + (x^3 - 6z)\hat{k}$$

$$C : (0,0,0) \to (1,0,0) \to (1,-3,0) \to (1,3,2)$$

$$r_1(t) = t\hat{i} \qquad 0 \le t \le 1$$

$$r_2(t) = \hat{i} - 3t\hat{j} \qquad 0 \le t \le 1$$

$$r_3(t) = \hat{i} + (-3+6t)\hat{j} + 2t\hat{k} \qquad 0 \le t \le 1$$

$$\int_{C1} F \cdot dr_1 + \int_{C2} F \cdot dr_2 + \int_{C3} F \cdot dr_3$$

$$= \int_0^1 4t^2 dt + \int_0^1 (2(1) - 6(-3t))\,(-3)dt + \int_0^1 (2(1) - 6(-3+6t))\,(6) + (1 - 6(2t))\,(2)dt$$

$$= \int_0^1 4t^2 dt + \int_0^1 (-6 - 54t)dt + \int_0^1 [6(20 - 36t) + 2(1 - 12t)]dt$$

$$= \frac{4}{3} - 6 - 27 + 122 - 120 = \frac{4}{3} - \frac{93}{3} = -\frac{89}{3}$$

Using python...

$$-\frac{89}{3}$$

Question 6

$$\int_C F \cdot dr$$

$$F = (2y - z)\hat{i} + (2z - x)\hat{j} + (3x - y)\hat{k}$$

$$C : (-1,2,1) \to (3,-2,5)$$

$$r(t) = (-1+4t)\hat{i} + (2-4t)\hat{j} + (1+4t)\hat{k} \qquad 0 \le t \le 1$$

$$\int_C F \cdot dr = \int_0^1 [2(2-4t) - (1+4t)](4) + [2(1+4t) - (-1+4t)](-4) + [3(-1+4t) - (2-4t)](4)dt$$

$$= \int_0^1 [12 - 48t] + [-12 - 16t] + [-20 + 64t]dt$$

$$= \int_0^1 -20dt = -20$$

Using python...

-20

Question 7

C is a smooth curve given by $r(t)$ $a \leq t \leq b$

f is a function whose gradient vector ∇f is continuous on C then

$$\int_C \nabla f \cdot dr = f(r(b)) - f(r(a))$$

$$F = 6x^2\hat{i} + (3y^2 - 4z)\hat{j} - (4y - 8z)\hat{k}$$

F is a conservative vector field

C is a smooth curve from $(1, 3, 2)$ to $(3, 4, 5)$

Since F is conservative, then f must exist where $F = \nabla f$

$$f = 2x^3 + g(y, z)$$

$$f = y^3 - 4yz + h(x, z)$$

$$f = -4yz + 4z^2 + m(x, y)$$

Therefore

$$f = 2x^3 + y^3 - 4yz + 4z^2 + C$$

$$\int_C F \cdot dr = \int_C \nabla f \cdot dr = 2(3^3) + 4^3 - 4(4)(5) + 4(5^2) - [2(1) + 3^3 - 4(3)(2) + 4(2^2)] = 117$$

Using python...

$$f = 2\mathbf{x_N}^3 + \mathbf{y_N}^3 - 4\mathbf{y_N}\mathbf{z_N} + 4\mathbf{z_N}^2$$

Is F conservative? True

$$f(r(b)) - f(r(a)) = 117$$