Weekly Assignment 10

RC

2021-11-11

Question 1

First Quadrant

$$z = 12 - x - 2y$$

$$x^2 + y^2 = 4$$

$$V = \int_{x=0}^{2} \int_{y=0}^{\sqrt{4-x^2}} \int_{z=0}^{12-x-2y} dzdydx$$

In cylindrical coordinates:

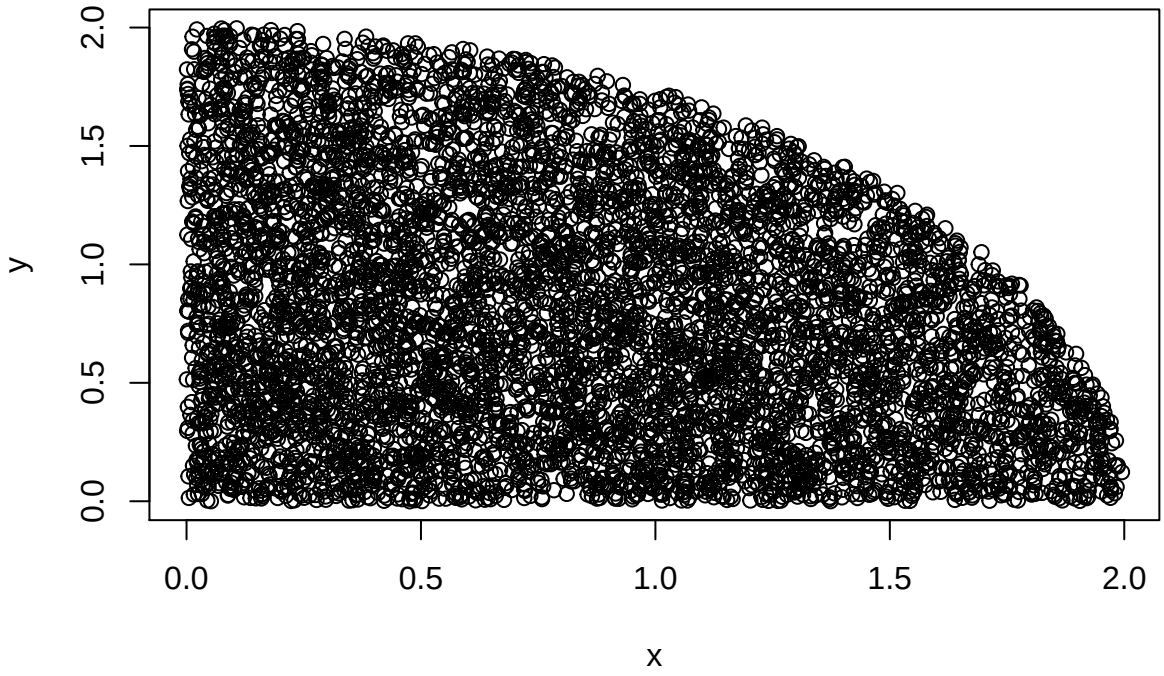
$$z = 12 - rcos\theta - 2rsin\theta$$

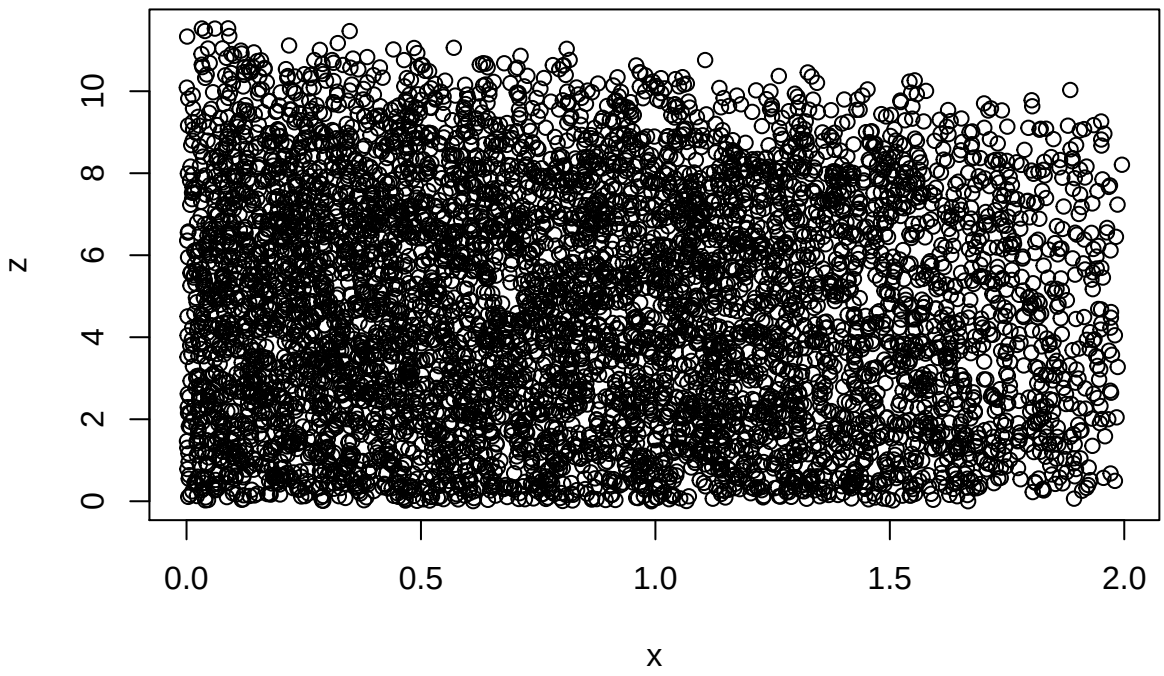
$$r^2 = 4$$

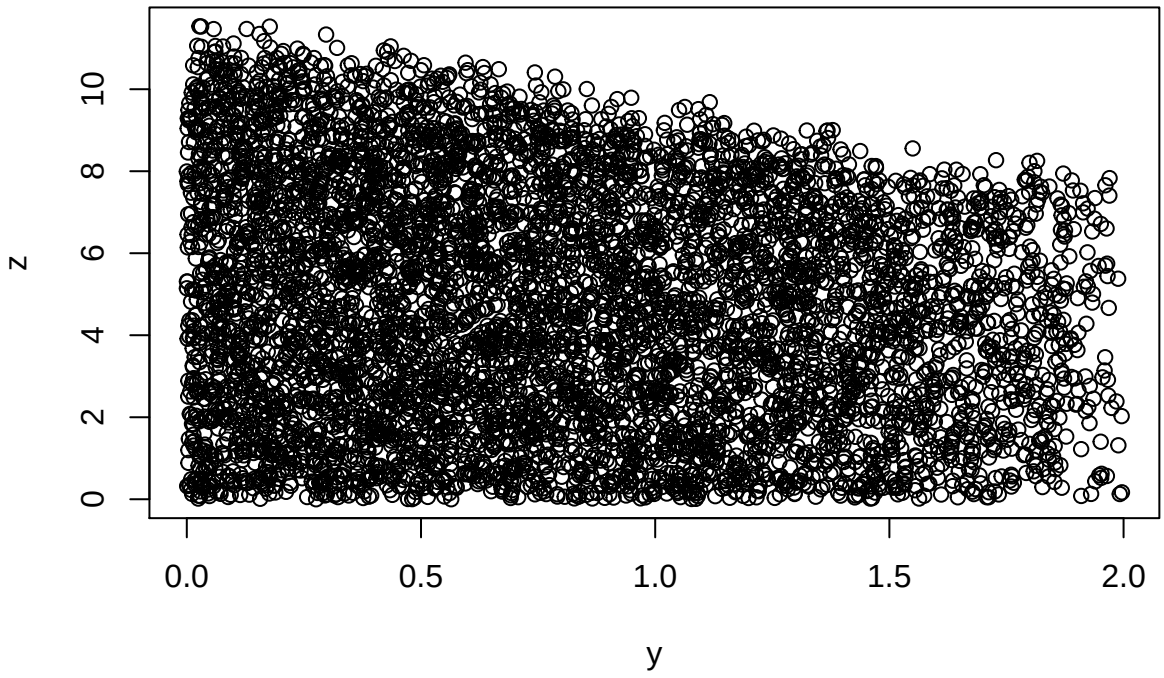
at the cap

$$r = \frac{12 - z}{cos\theta + 2sin\theta}$$

$$V = \int_{\theta=0}^{\pi/2} \int_{r=0}^{2} \int_{z=0}^{12-2cos\theta-4sin\theta} rdzdrd\theta + \int_{\theta=0}^{\pi/2} \int_{z=12-2cos\theta-4sin\theta}^{12} \int_{r=0}^{\frac{12-z}{cos\theta+2sin\theta}} rdrdzd\theta$$


y
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5
2.0
x


z
0
2
4
6
8
10
0.0
0.5
1.0
1.5
2.0
x




```
## [1] "Monte Carlo Volume: 29.8224"
## [1] "Analytic Volume: 29.6991118430775"
```

Question 2

$$I = \int\int\int_Q xydV$$

$$x^2 + y^2 = 9$$

$$z = 2x^2 + 2y^2$$

$$\rho cos(\phi) = 2\rho^2 cos^2(\theta) sin^2(\phi) + 2\rho^2 sin^2(\theta) sin^2(\phi)$$

$$\rho cos(\phi) = 2\rho^2 sin^2(\phi)$$

$$\rho^2 cos^2(\theta) sin^2(\phi) + \rho^2 sin^2(\theta) sin^2(\phi) = 9$$

$$\rho^2 sin^2(\phi) = 9$$

$$\rho sin(\phi) = 3$$

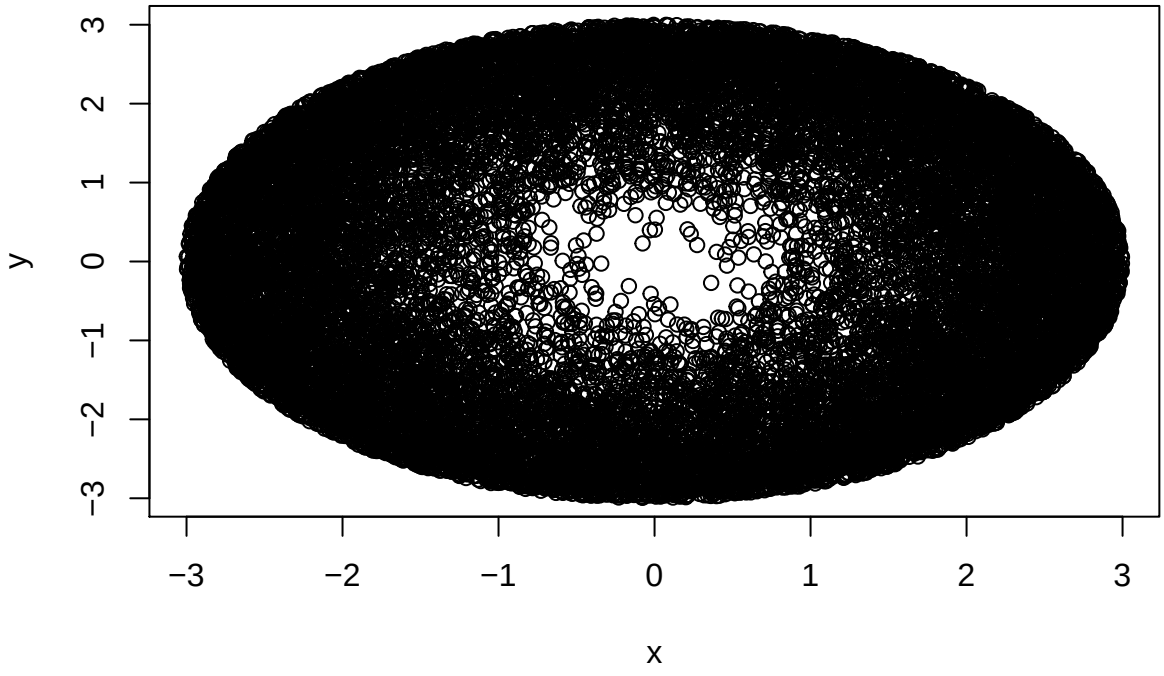
At the intersection

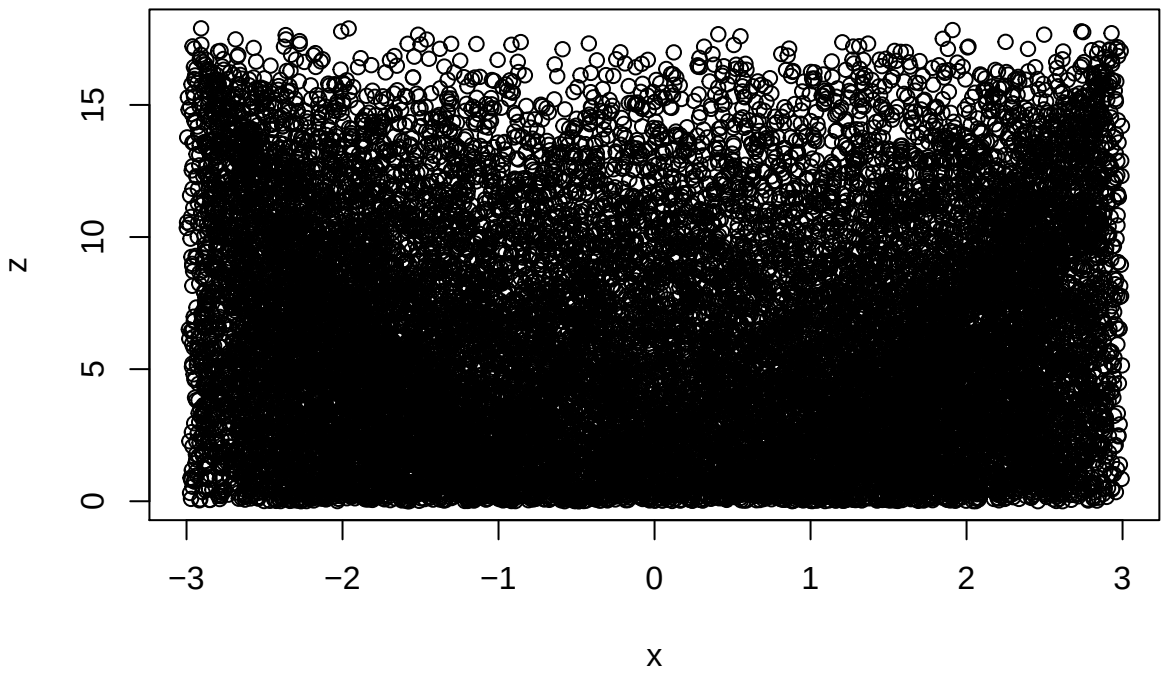
$$cos(\phi) = 2\frac{3}{sin(\phi)}sin^2(\phi)$$

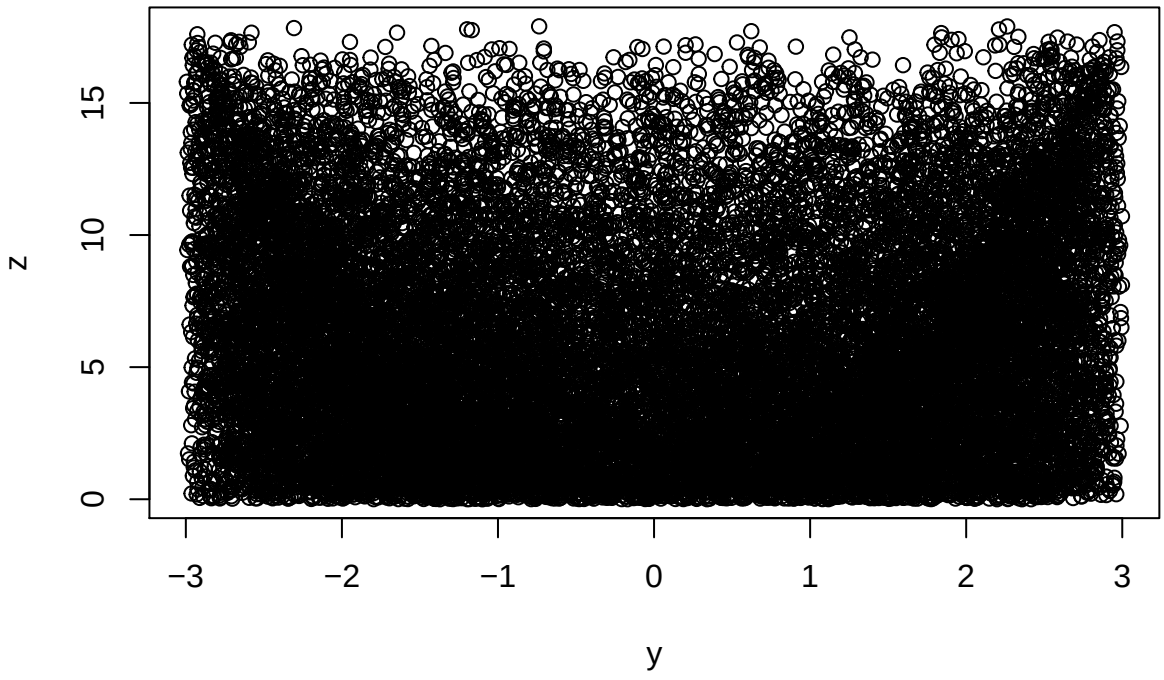
$$\phi = arctan(\frac{1}{6})$$

$$I = \int_{\theta=0}^{2\pi}\int_{\phi=arctan(1/6)}^{\pi/2}\int_{\rho=\frac{cos\phi}{2sin^2\phi}}^{\frac{3}{sin\phi}} (\rho cos(\theta)sin(\phi))(\rho sin(\theta)sin(\phi))\rho^2 sin(\phi)drd\phi d\theta$$

$$= \int_{\theta=0}^{2\pi}\int_{\phi=arctan(1/6)}^{\pi/2}\int_{\rho=\frac{cos\phi}{2sin^2\phi}}^{\frac{3}{sin\phi}} \rho^4 cos(\theta)sin(\theta)sin^3(\phi))d\rho d\phi d\theta$$




z
0
5
10
15
−3
−2
−1
0
1
2
3
x




```
## [1] "Monte Carlo Integral: -0.121714188949609"
## [1] "Analytic Integral: 0"
## [1] "Monte Carlo Volume: 254.78064"
## [1] "Analytic Volume: 254.469004940773"
```

Question 3

$$I = \int_{-1}^{1} \int_{0}^{\sqrt{1-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{x^2+y^2}} 4xy^2 dz dy dx$$

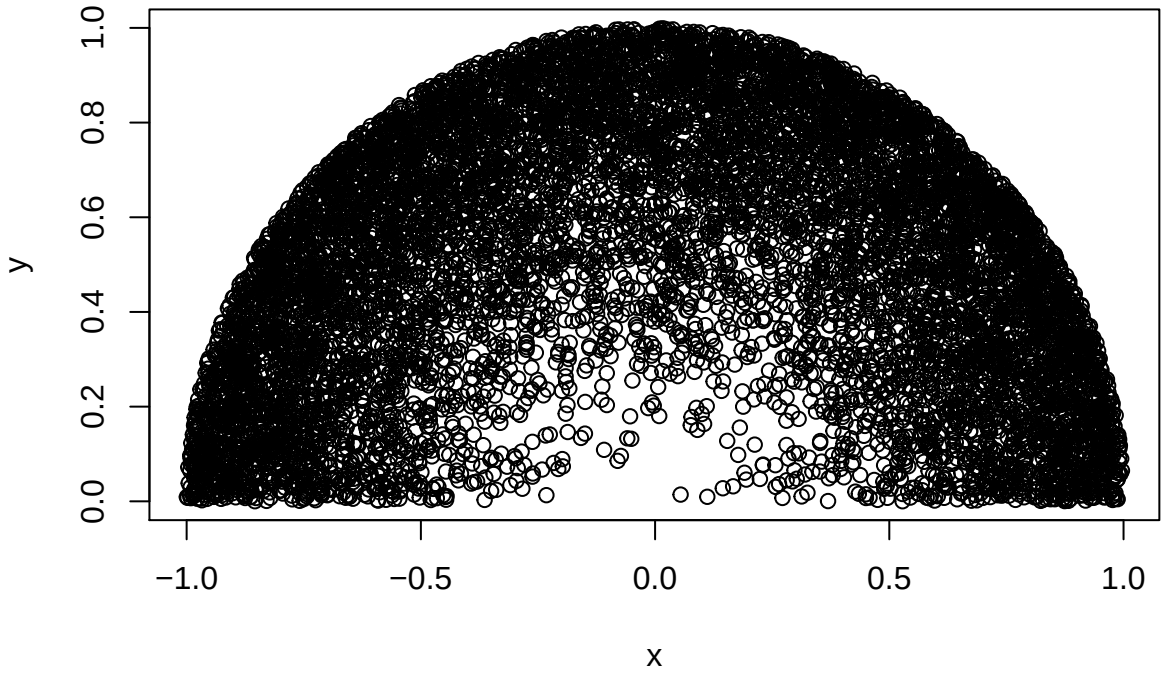
$$I = \int_{\theta=0}^{\pi} \int_{r=0}^{1} \int_{z=-r^2}^{r^2} 4(rcos\theta)(rsin\theta)^2 r dz dr d\theta$$

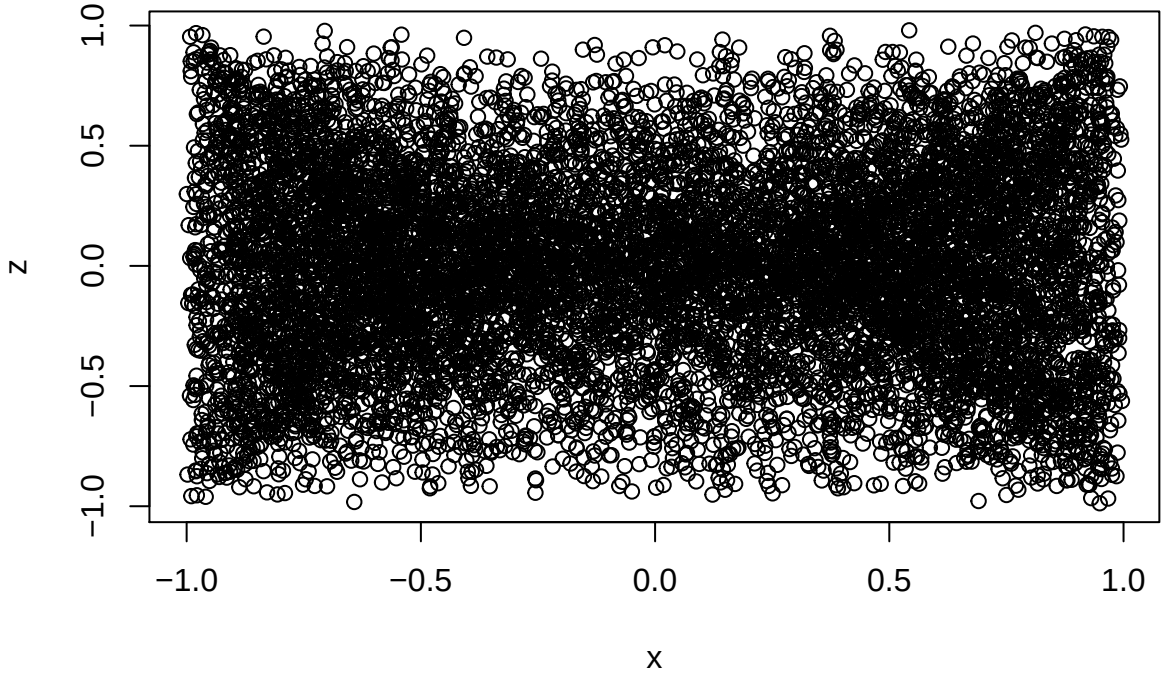
$$= \int_{\theta=0}^{\pi} \int_{r=0}^{1} \int_{z=-r^2}^{r^2} 4r^4 cos\theta sin^2\theta dz dr d\theta$$

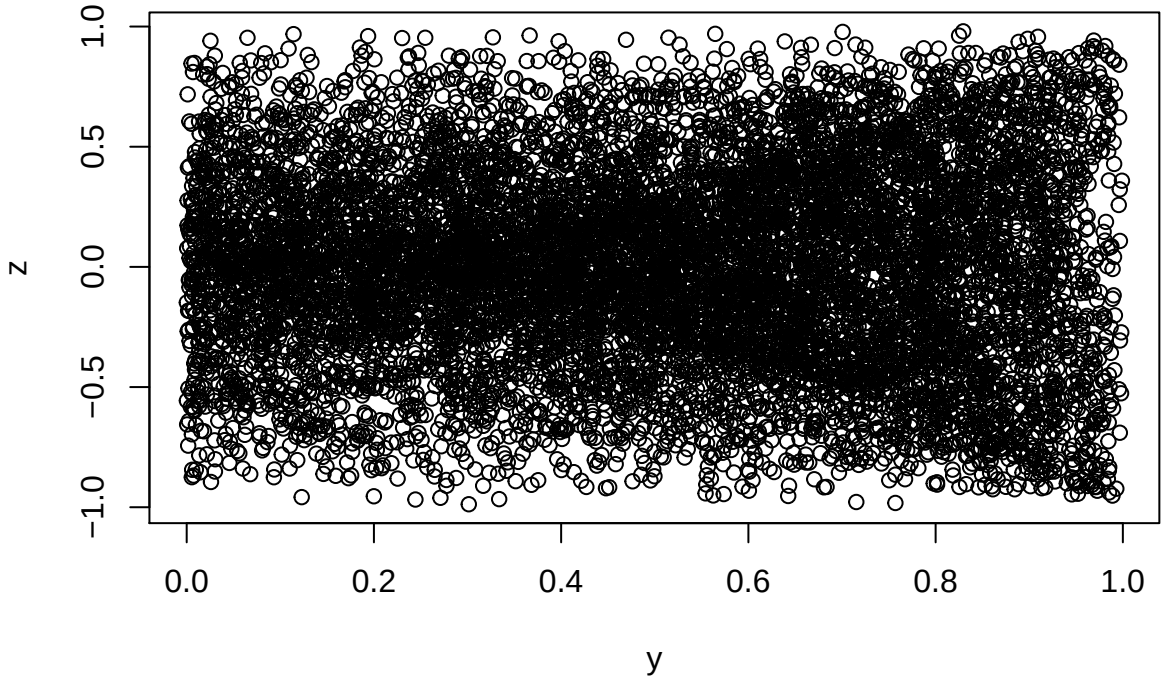
$$= \int_{\theta=0}^{\pi} \int_{r=0}^{1} 8r^6 \sin^2(\theta)\cos(\theta) dr d\theta$$

$$= \int_{\theta=0}^{\pi} \frac{8\sin^2(\theta)\cos(\theta)}{7} d\theta$$

$$= 0$$


y
0.0
0.2
0.4
0.6
0.8
1.0
−1.0
−0.5
0.0
0.5
1.0
x


z
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
x




```
## [1] "Monte Carlo Integral: 1.19105024226613"
## [1] "Analytic Integral: 0"
## [1] "Monte Carlo Volume: 1.56288"
## [1] "Analytic Volume: 1.5707963267949"
```

Question 4

$$z = \sqrt{36 - x^2 - y^2}$$

$$x^2 + y^2 = 9$$

$$z = \sqrt{36 - r^2}$$

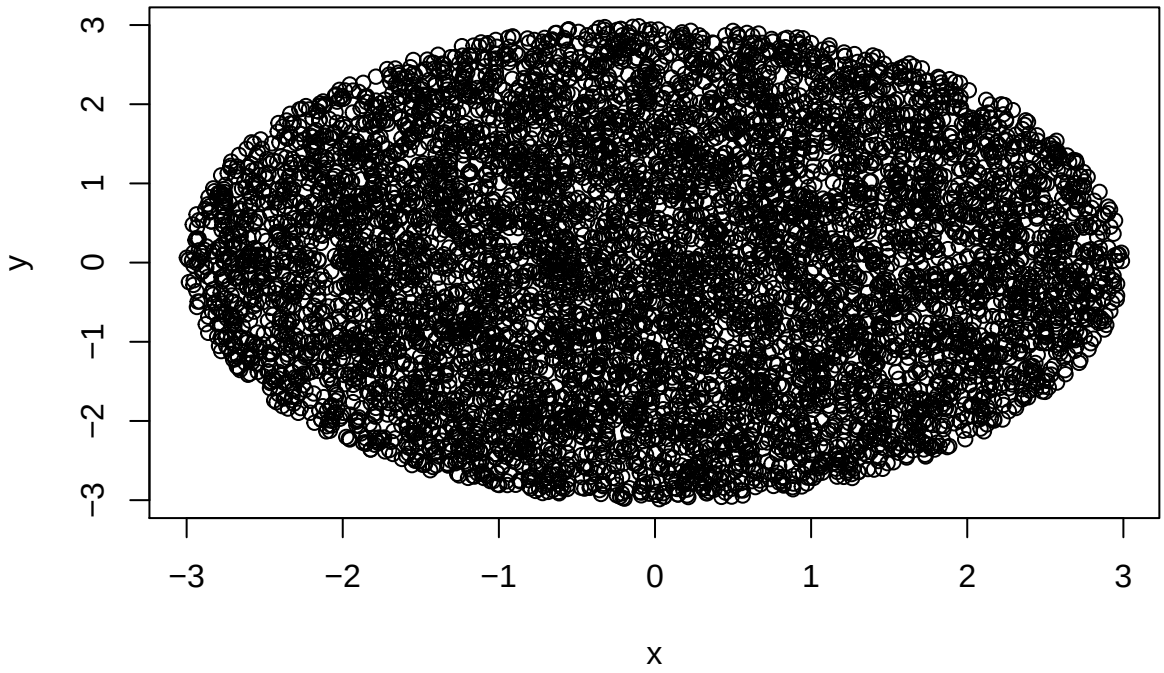
$$r = 3$$

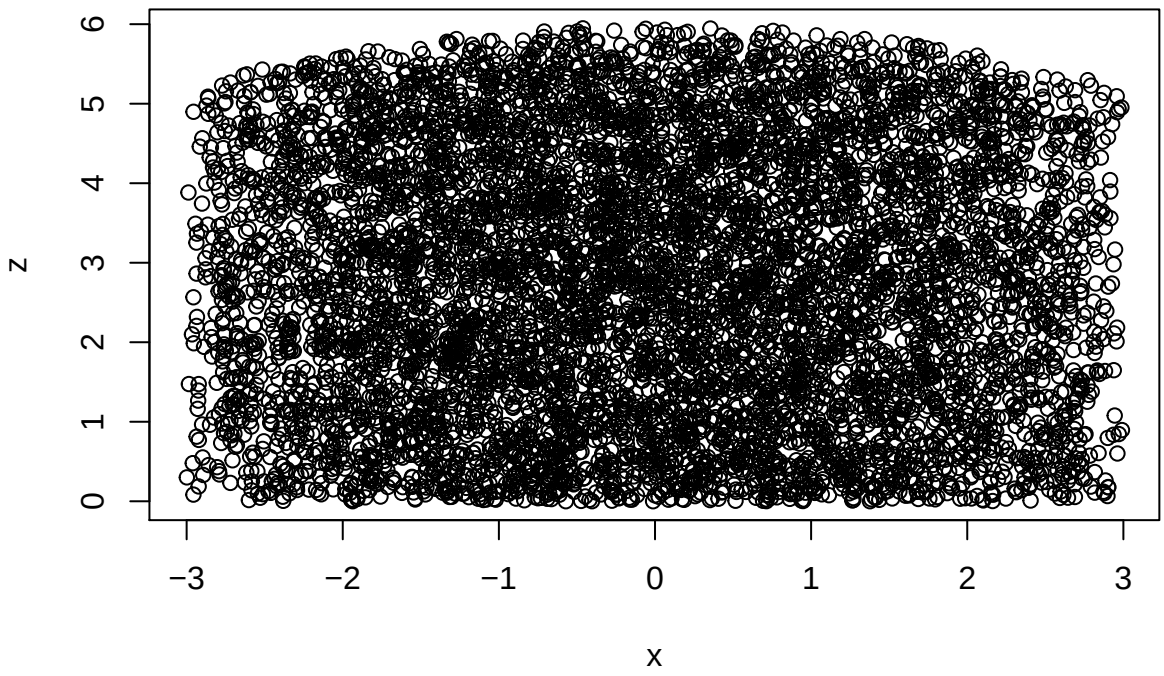
Intersection

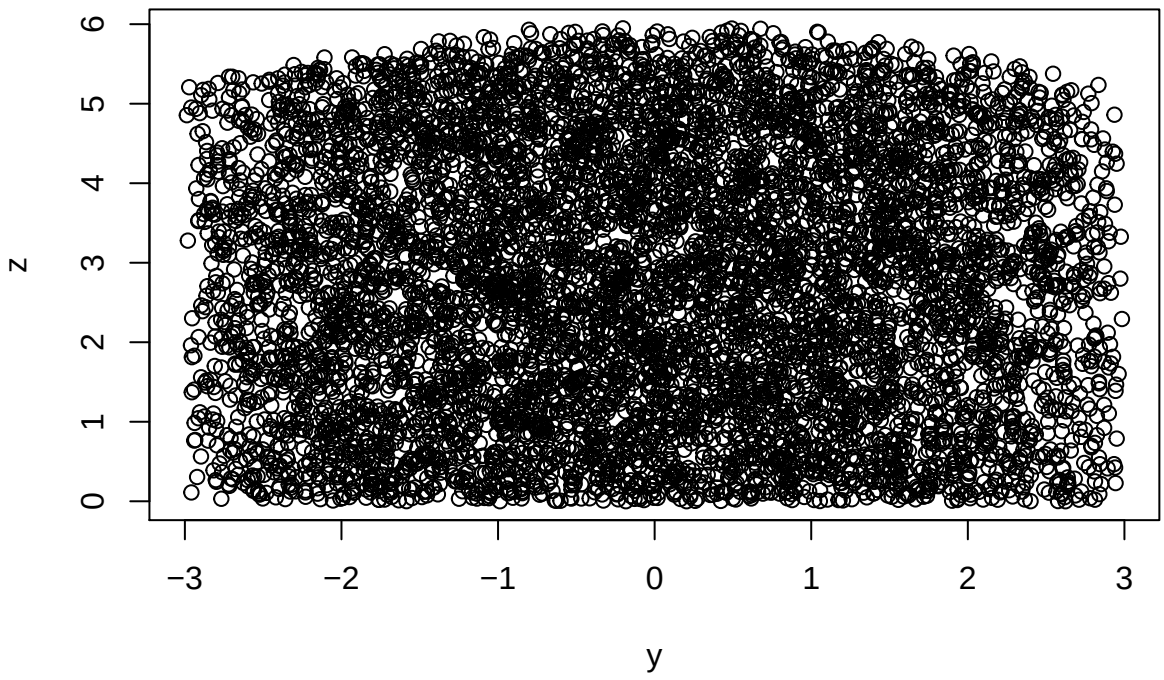
$$z = \sqrt{27}$$

$$V = \int_{\theta=0}^{2\pi} \int_{r=0}^{3} \int_{z=0}^{\sqrt{27}} r dz dr d\theta + \int_{\theta=0}^{2\pi} \int_{z=\sqrt{27}}^{6} \int_{r=0}^{\sqrt{36-z^2}} r dr dz d\theta$$

$$V = 2\pi \left(72 - \frac{81\sqrt{3}}{2} \right) + 27\sqrt{3}\pi$$




z
0
1
2
3
4
5
6
−3
−2
−1
0
1
2
3
x




```
## [1] "Monte Carlo Volume: 157.9392"
## [1] "Analytic Volume: 158.553845110987"
```

Question 5

$$x^2 + y^2 + z^2 = 16$$

$$x^2 + y^2 + z^2 = 4$$

$$5z = \sqrt{x^2 + y^2}$$

Cylindrical

$$\rho = 4$$

$$\rho = 2$$

$$5\rho cos\phi = \rho sin\phi$$

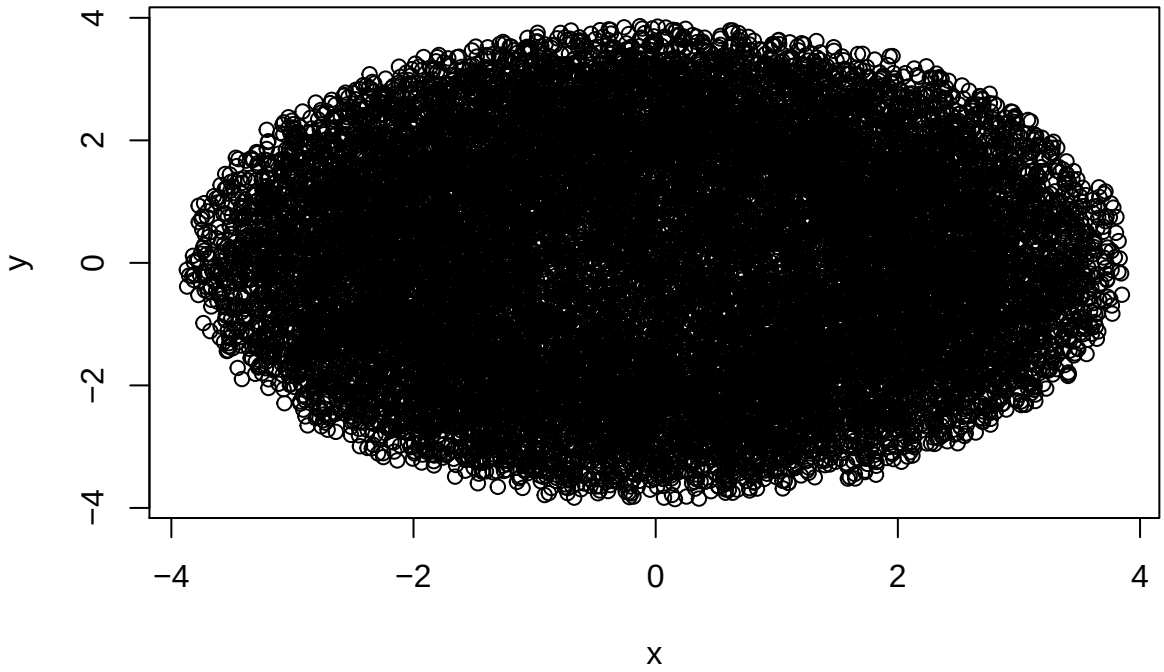
$$\phi = arctan(5)$$

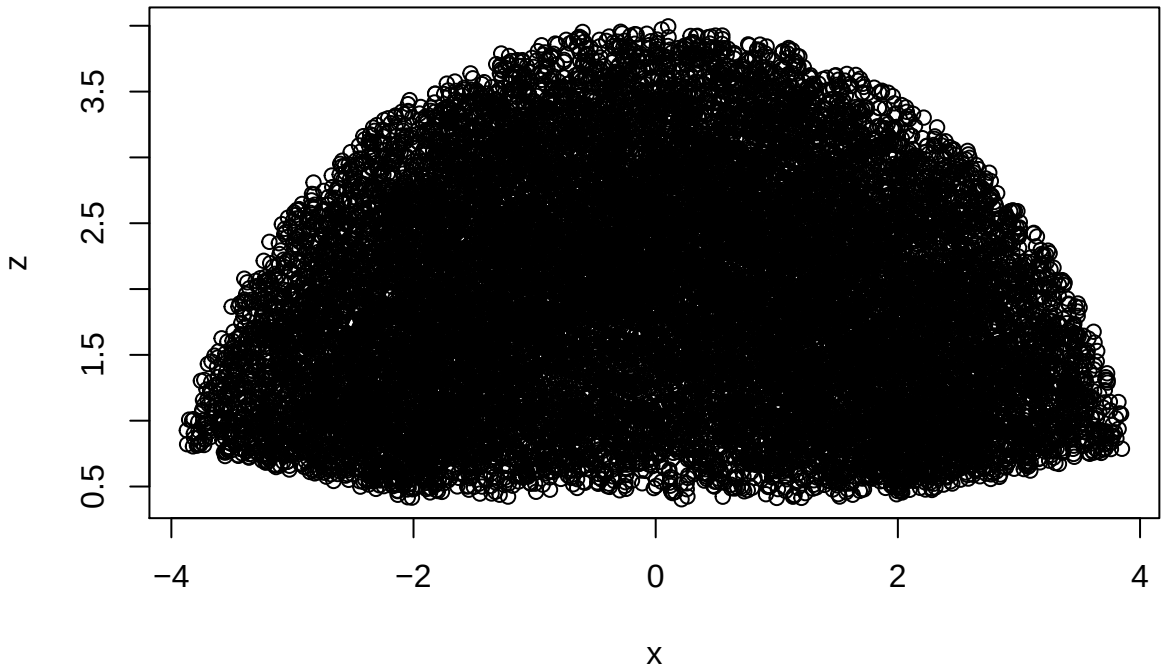
$$V = \int_{\theta=0}^{2\pi} \int_{\phi=0}^{arctan(5)} \int_{\rho=2}^{\rho=4} \rho^2 sin\phi d\rho d\phi d\theta$$

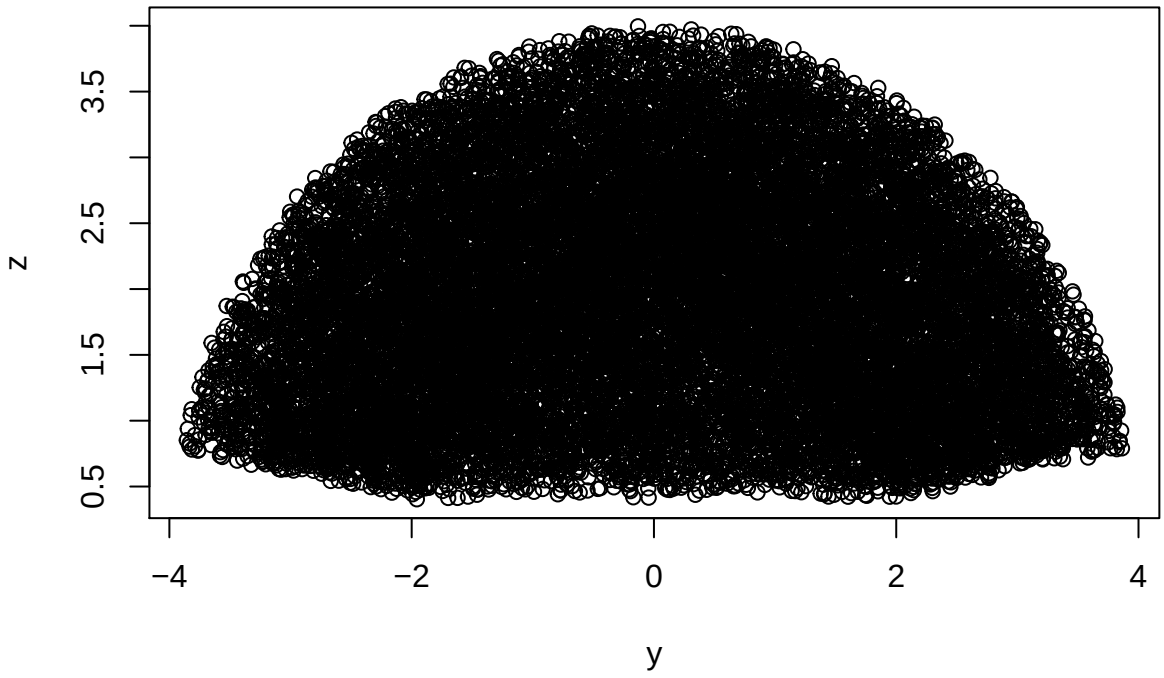
$$= \int_{\theta=0}^{2\pi} \int_{\phi=0}^{arctan(5)} \frac{56\sin(\phi)}{3} d\phi d\theta$$

$$= \int_{\theta=0}^{2\pi} \frac{56}{3} - \frac{28\sqrt{26}}{39} d\theta$$

$$= 2\pi \left(\frac{56}{3} - \frac{28\sqrt{26}}{39} \right)$$




z
0.5
1.5
2.5
3.5
−4
−2
0
2
4
x




```
## [1] "Monte Carlo Volume: 94.44864"
## [1] "Analytic Volume: 94.2844240497321"
```