Weekly Assignment 2

RC

9/8/2021

```
import sympy as sy
import sympy.vector as sv

N = sv.CoordSys3D('N')
t = sy.Symbol('t')
```

Question 1

Part A

```
P = -2*N.i + N.j - 3*N.k
Q = -1*N.i + 3*N.j + 4*N.k
R = 4*N.i + 5*N.j - 3*N.k
```

Line through R and Q

$$l_{QR}(t) = Q + (R - Q)t$$

$$l_{QR}(0) = Q, \quad l_{QR}(1) = R$$

```
print(f"Line through Q and R = {Q + (R-Q)*t}")
```

```
## Line through Q and R = (5*t - 1)*N.i + (2*t + 3)*N.j + (4 - 7*t)*N.k
```

Line through P and parallel to QR

$$l_P(t') = P + (R - Q)t'$$

```
print(f"Line through P parallel to QR = {P + (R-Q)*t}")
```

```
## Line through P parallel to QR = (5*t - 2)*N.i + (2*t + 1)*N.j + (-7*t - 3)*N.k
```

Check that they are parallel

$$l_P(1) - l_P(0) = \lambda(l_{QR}(1) - l_{QR}(0))$$

$$R - Q = \lambda(R - Q)$$

Part B

```
print(f"Another point on the line in A = {P + (R-Q)*2}")
```

```
## Another point on the line in A = 8*N.i + 5*N.j + (-17)*N.k
```

Part C

Solve 3 equations with 3 unknowns:

$$Ax + By + Cz = 0$$

or

$$z = Ax + By + D$$

$$-3 = D - 2A + B4 = D - A + 3B - 3 = D + 4A + 5B$$

row 1 - row 3

$$0 = 0 - 6A - 4B, \quad A = \frac{-2}{3}B$$

3 * row 2 + 4 * row 3

$$0 = 7D + 13A + 29B0 = 7D - \frac{26}{3}B + 29B0 = 21D + 61BD = \frac{-61}{21}B$$

plug back in

$$-3 = \frac{-61}{21}B - 2\frac{-2}{3}B + B$$

$$-63 = B(-61 + 28 + 21)B = \frac{-63}{-12} = \frac{21}{4}$$

$$A = \frac{-7}{2}$$

$$D = \frac{-61}{4}$$

$$z = \frac{-61}{4} - \frac{7}{2}x + \frac{21}{4}y$$

Using Linear Algebra …

```
X <- matrix(c(1,-2,1,1,-1,3,1,4,5), nrow = 3, byrow = TRUE)
Z <- matrix(c(-3, 4, -3), ncol = 1)
#solve(t(X)%*%X)%*%t(X)%*%Z
print(paste("D,A,B = ", paste(c(solve(X)%*%Z), collapse = ",")))
```

```
## [1] "D,A,B =  -15.25,-3.5,5.25"
```

Part D

Another point on the plane = <0, 0, -61/4>

Question 2

Parametric equation of a line through $A =< 2, -3, 6 >$ and perpendicular to $3x + 2y - 4z = 7$

Picking off three points on the plane, forming 2 vectors in the plane, and finding their cross product yields a vector perpendicular to the plane

```
A = 2*N.i - 3*N.j + 6*N.k

# y=0, z=0
p1 = 7/3*N.i
# x=0, z=0
p2 = 7/2*N.j
# x=0, y=0
p3 = -7/4*N.k

# two vectors in the plane
v1 = p2 - p1
v2 = p3 - p1

print(f"Line through A, perpendicular to the plane = {A + v1.cross(v2)*24/49*t}")
```

```
## Line through A, perpendicular to the plane = (2 - 3.0*t)*N.i + (-2.0*t - 3)*N.j + (4.0*t + 6)*N.k
```

Question 3

Line of intersection of $5x + 2y - z = 4$ and $x - 2y + 3z = 5$

Rewriting...

$$z = f(x, y) = 5x + 2y - 4z = g(x, y) = \frac{-1}{3}x + \frac{2}{3}y + \frac{5}{3}$$

Setting equal

$$5x + 2y - 4 = \frac{-1}{3}x + \frac{2}{3}y + \frac{5}{3}$$

$$16x + 4y - 17 = 0$$

allow $x(t) = t$

$$y(t) = -4t + \frac{17}{4}$$

Plugging back into $f(x, y) = 5x + 2y - 4$

$$z(t) = 5t + 2(-4t + 17/4) - 4$$

$$z(t) = -3t + \frac{9}{2}$$

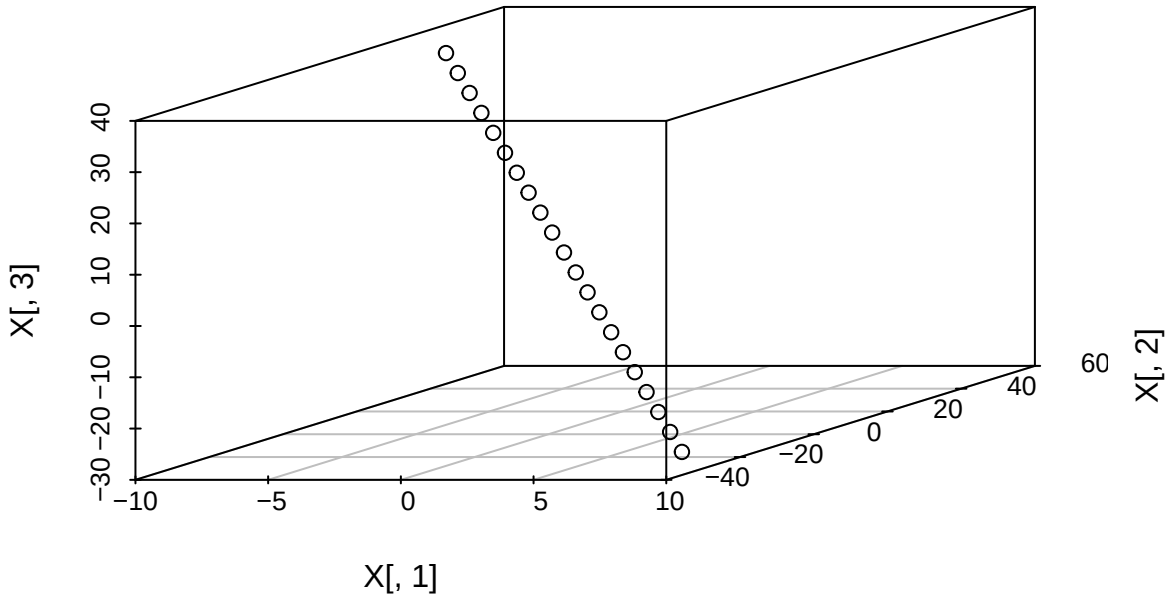
Checking against $g(x, y)$ shows these parameterizations are correct

$$-3t + \frac{9}{2} = \frac{-1}{3}t + \frac{2}{3}(-4t + \frac{17}{4}) + \frac{5}{3}$$

```
g <- function(t)
{
   c(t, -4*t+17/4, -3*t+9/2)
}

X <- as.data.frame(t(sapply(-10:10, g)))

scatterplot3d::scatterplot3d(x = X[,1], y=X[,2], z=X[,3])
```



Question 4

Find the exact coordinates of the point of intersection of the plane $x - y + 2z = 9$ and the line $x = 3 + 2t$, $y = 2 - t$, $z = 5t$

$$(3 + 2t) - (2 - t) + 2(5t) = 9$$

$$t = \frac{8}{13}$$

coordinates $= < \frac{55}{13}, \frac{18}{13}, \frac{40}{13}$

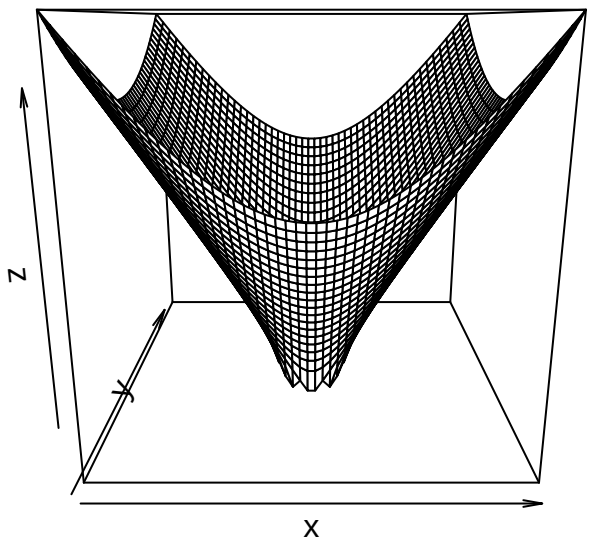
check = TRUE

Question 5

A - Hyperboloid of one sheet

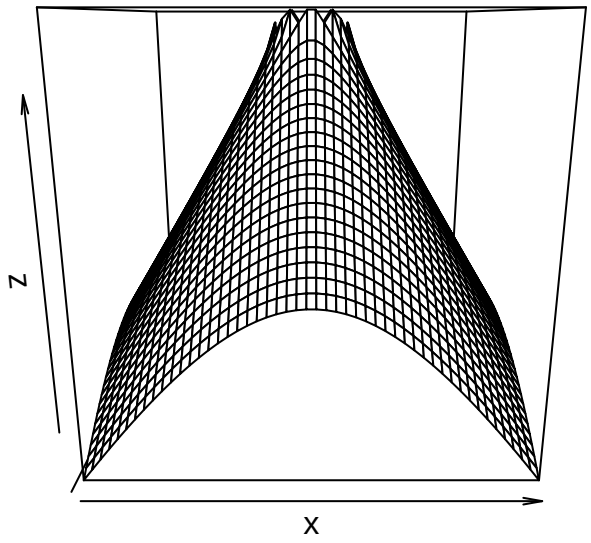
```
f <- function(x,y) ifelse((x^2/5 + y^2/3 - 1) > 0, sqrt(16*(x^2/5 + y^2/3 - 1)), NA)
g <- function(x,y) ifelse((x^2/5 + y^2/3 - 1) > 0, -sqrt(16*(x^2/5 + y^2/3 - 1)), NA)
xseq <- seq(-10, 10, length = 50)
yseq <- seq(-10, 10, length = 50)
persp(x = xseq, y = yseq,
      z = matrix(f(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```




```
persp(x = xseq, y = yseq,
      z = matrix(g(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```

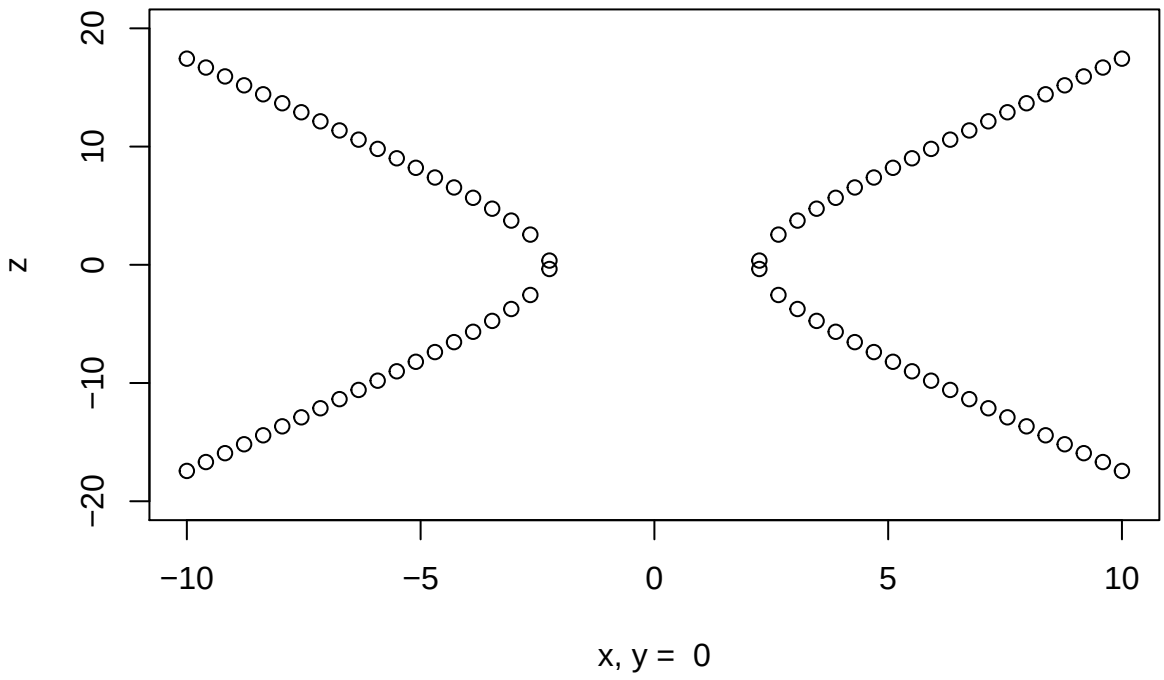



```
plot(x = xseq, y = f(xseq, rep(0, 50)), xlab = "x, y =  0", ylab = "z", ylim=c(-20, 20))
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```

```
points(x = xseq, y = g(xseq, rep(0, 50)))
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```

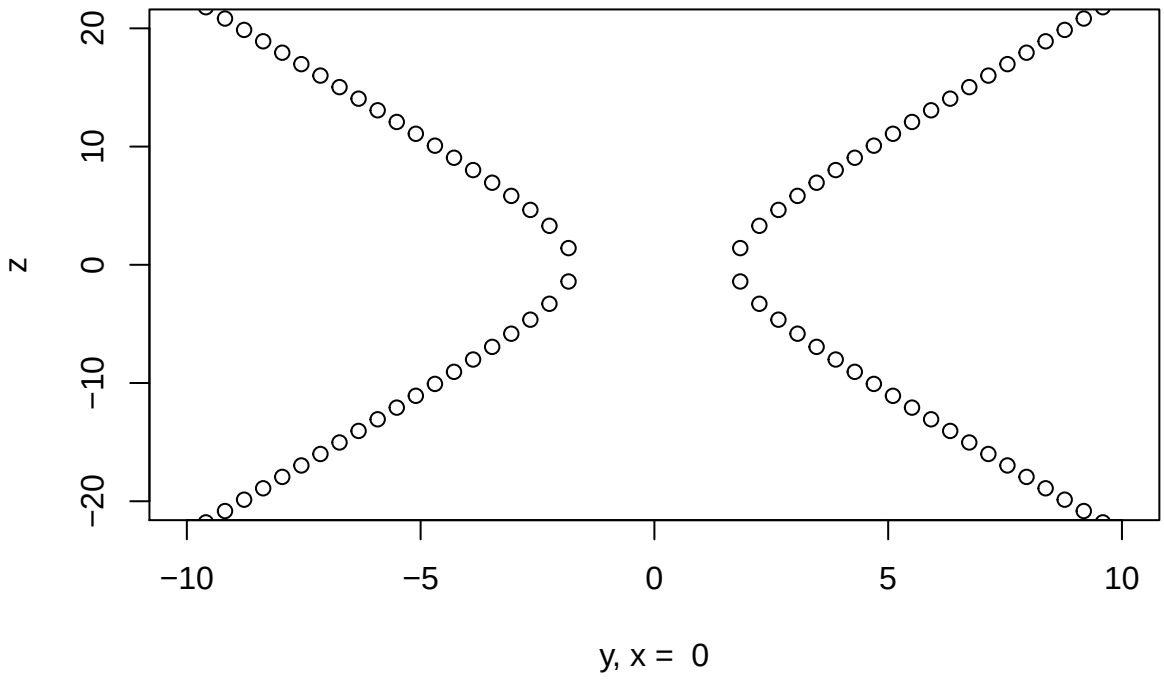



```
plot(x = yseq, y = f(rep(0, 50), yseq), xlab = "y, x =  0", ylab = "z", ylim=c(-20, 20))
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```

```
points(x = yseq, y = g(rep(0, 50), yseq))
```

```
## Warning in sqrt(16 * (x^2/5 + y^2/3 - 1)): NaNs produced
```

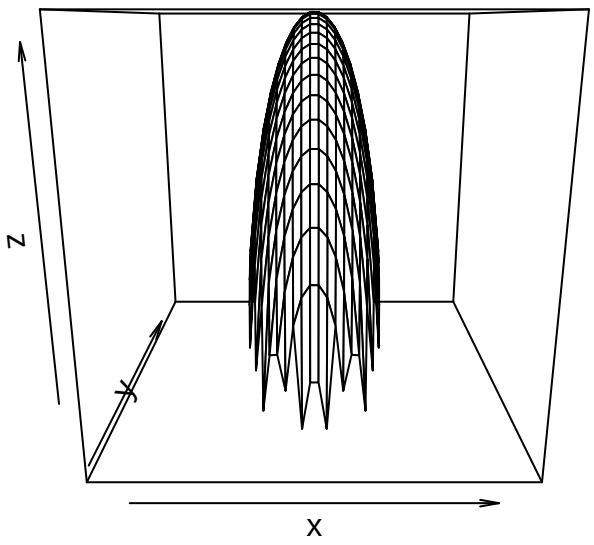


B - Ellipsoid

$2x^2 + 5y^2 + z^2 = 18$

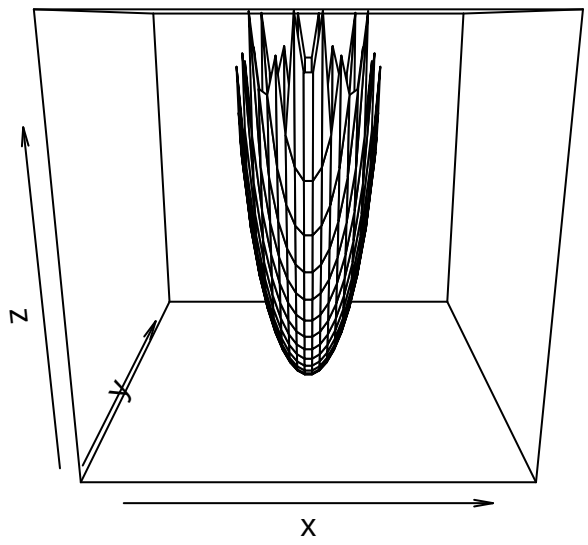
```
f <- function(x,y) ifelse((18 - 2*x^2 - 5*y^2) > 0, sqrt(18 - 2*x^2 - 5*y^2), NA)
g <- function(x,y) ifelse((18 - 2*x^2 - 5*y^2) > 0, -sqrt(18 - 2*x^2 - 5*y^2), NA)
xseq <- seq(-5, 5, length = 50)
yseq <- seq(-5, 5, length = 50)
persp(x = xseq, y = yseq,
      z = matrix(f(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```

```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```




```
persp(x = xseq, y = yseq,
      z = matrix(g(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```

```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```

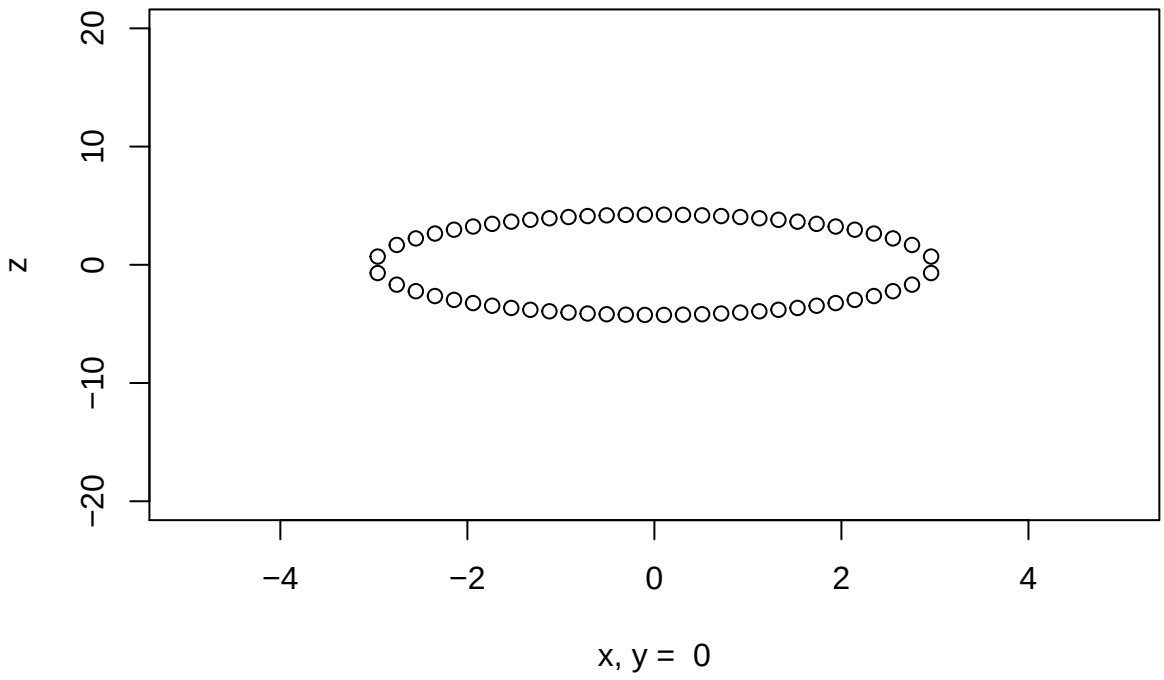



```
plot(x = xseq, y = f(xseq, rep(0, 50)), xlab = "x, y =  0", ylab = "z", ylim=c(-20, 20))
```

```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```

```
points(x = xseq, y = g(xseq, rep(0, 50)))
```

```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```

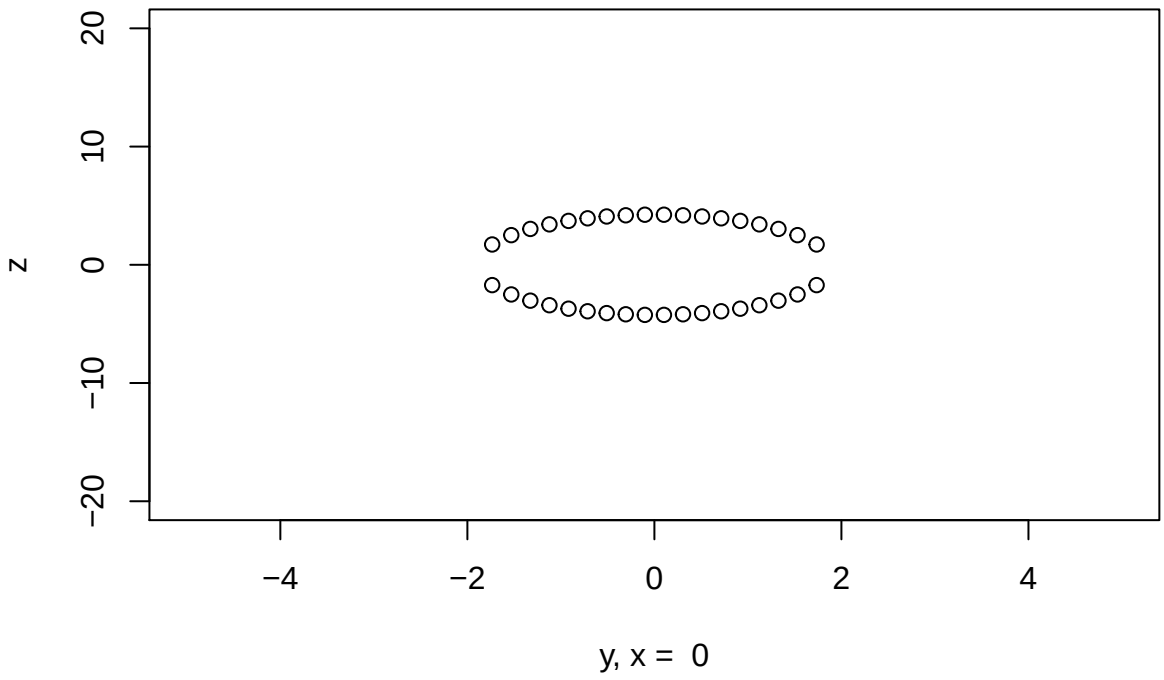



```
plot(x = yseq, y = f(rep(0, 50), yseq), xlab = "y, x =  0", ylab = "z", ylim=c(-20, 20))
```

```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```

```
points(x = yseq, y = g(rep(0, 50), yseq))
```

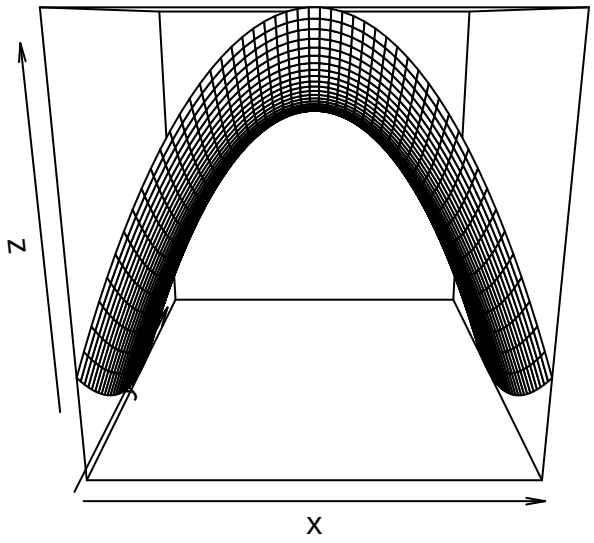
```
## Warning in sqrt(18 - 2 * x^2 - 5 * y^2): NaNs produced
```



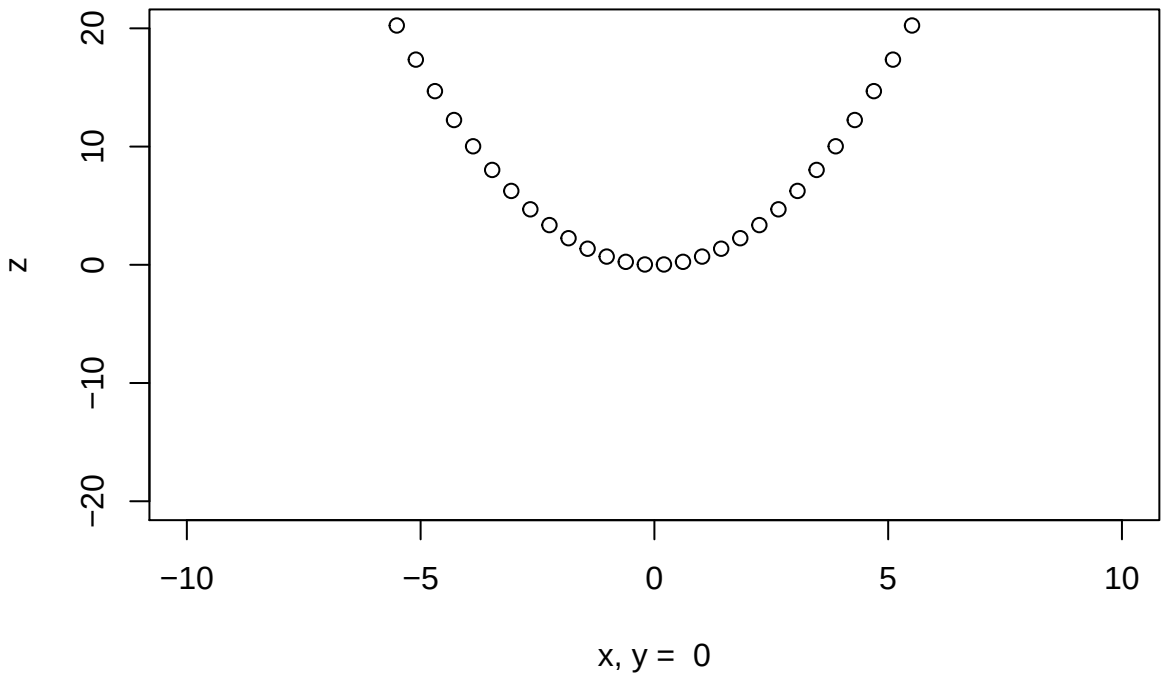
C - Hyperbolic Paraboloid

$2x^2 - 6y^2 - 3z = 0$

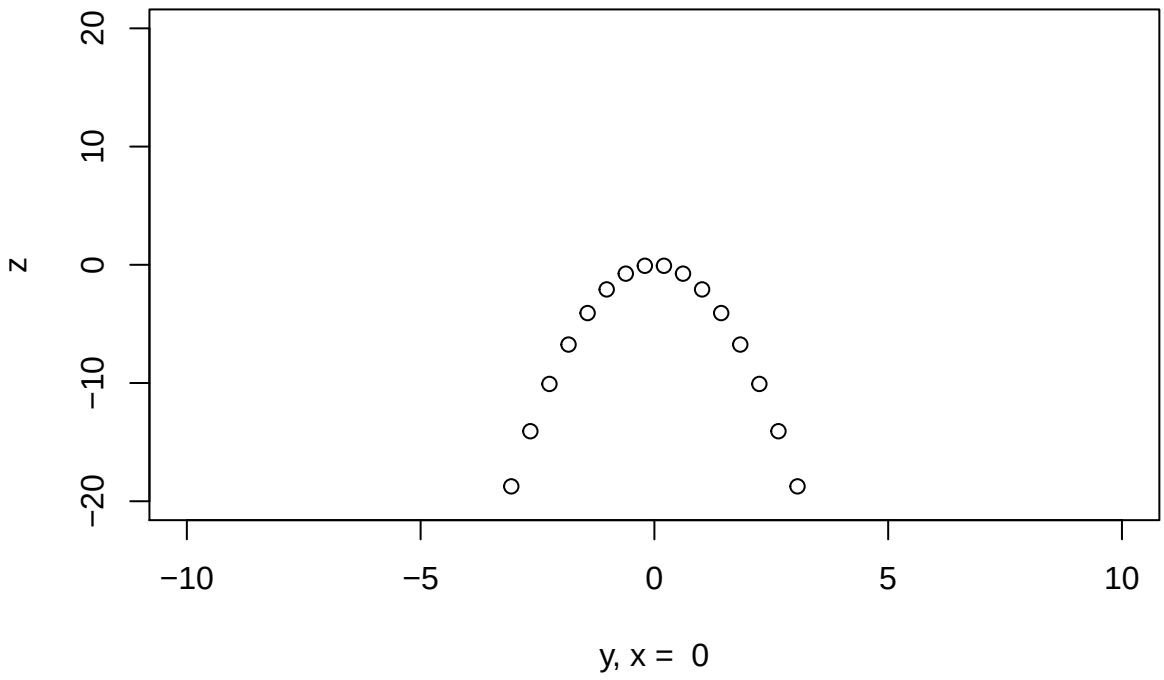
```
f <- function(x,y) 1/3*(2*x^2-6*y^2)
xseq <- seq(-10, 10, length = 50)
yseq <- seq(-10, 10, length = 50)
persp(x = xseq, y = yseq,
      z = matrix(f(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```




```
plot(x = xseq, y = f(xseq, rep(0, 50)), xlab = "x, y =  0", ylab = "z", ylim=c(-20, 20))
```



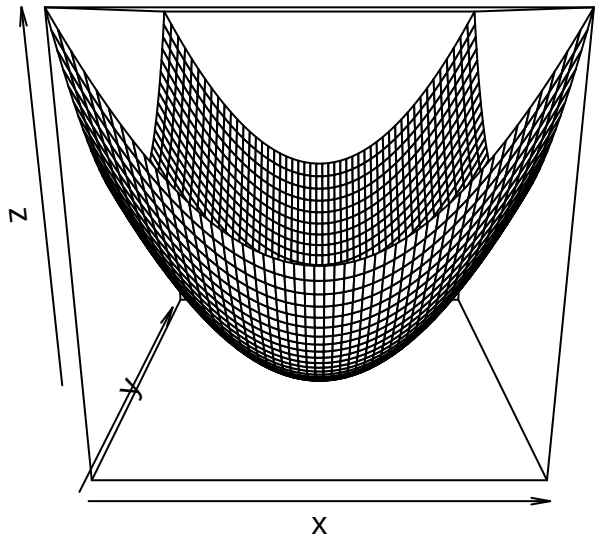

```
plot(x = yseq, y = f(rep(0, 50), yseq), xlab = "y, x =  0", ylab = "z", ylim=c(-20, 20))
```



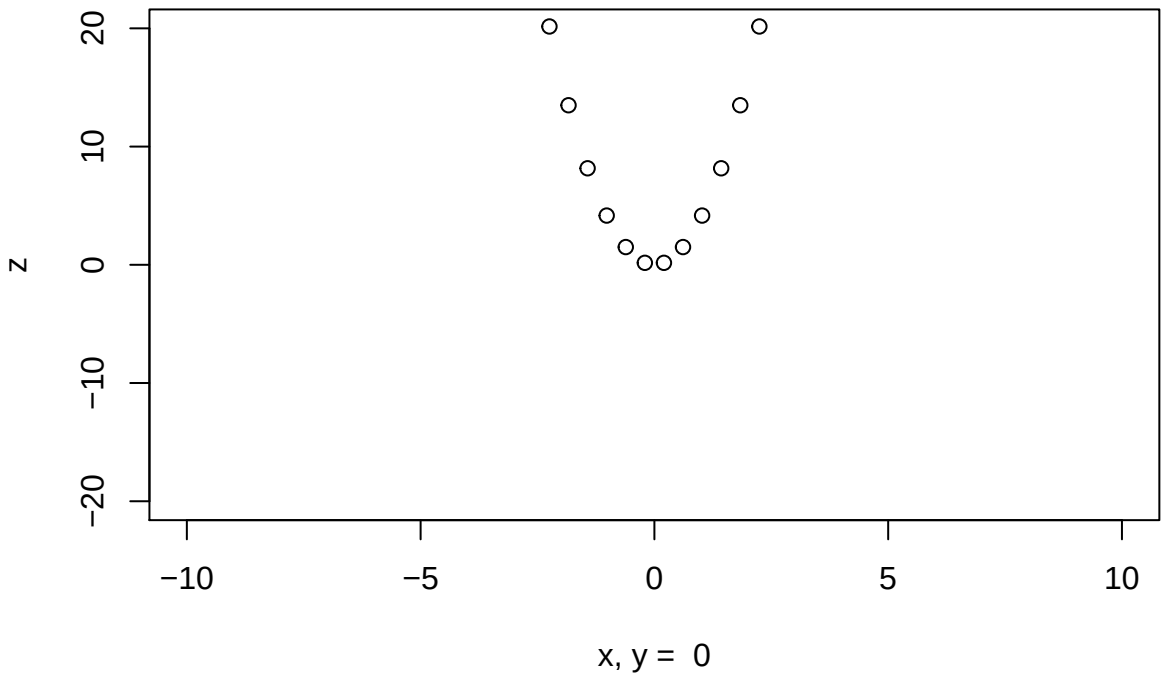
D - Paraboloid

$4x^2 + 4y^2 = z$

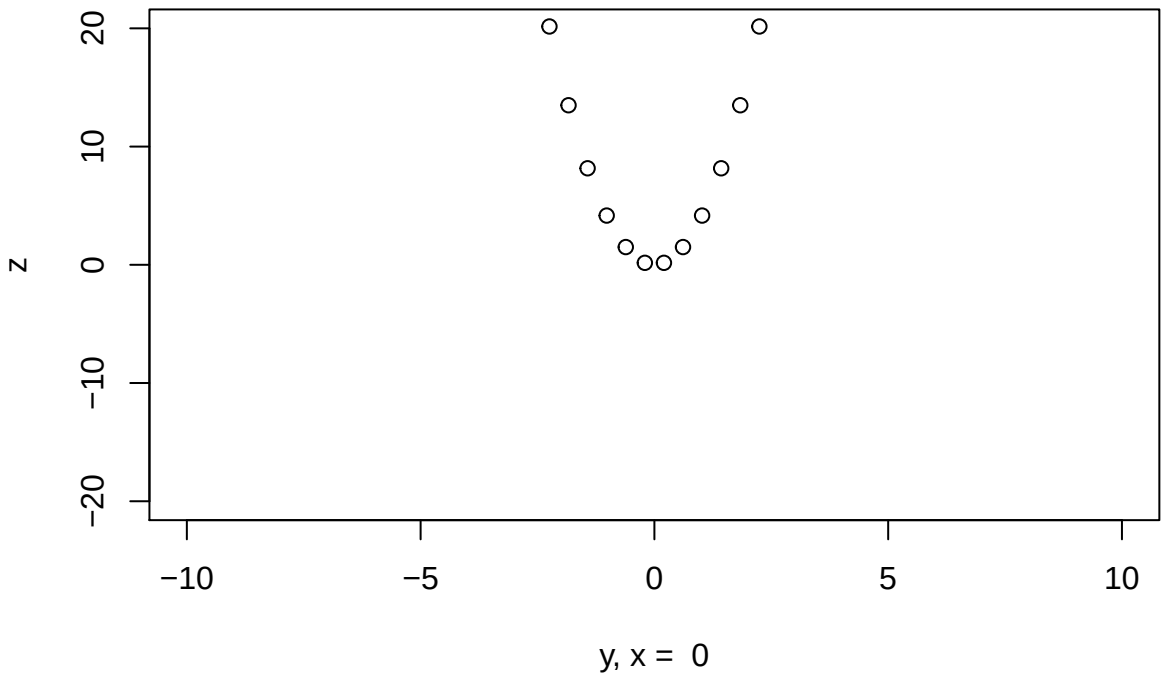
```
f <- function(x,y) 4*x^2 + 4*y^2
xseq <- seq(-10, 10, length = 50)
yseq <- seq(-10, 10, length = 50)
persp(x = xseq, y = yseq,
      z = matrix(f(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```




```
plot(x = xseq, y = f(xseq, rep(0, 50)), xlab = "x, y =  0", ylab = "z", ylim=c(-20, 20))
```



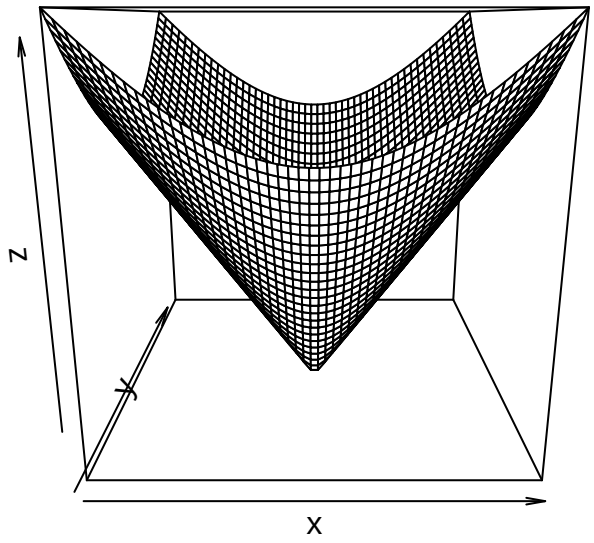

```
plot(x = yseq, y = f(rep(0, 50), yseq), xlab = "y, x =  0", ylab = "z", ylim=c(-20, 20))
```



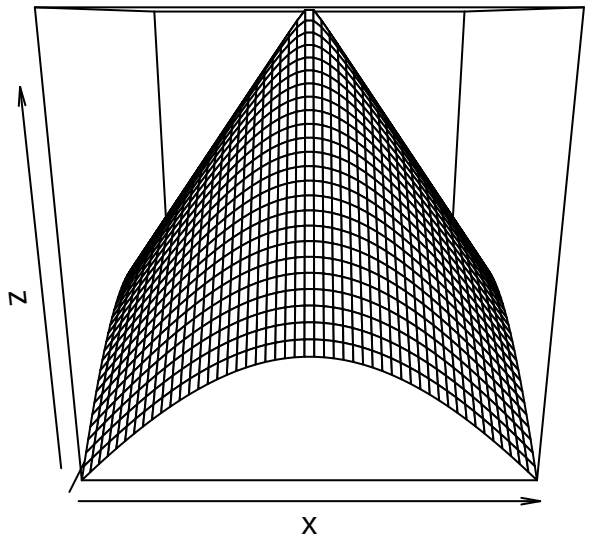
E - Hyperboloid of two sheets??

$x^2 + y^2 = 5z^2$

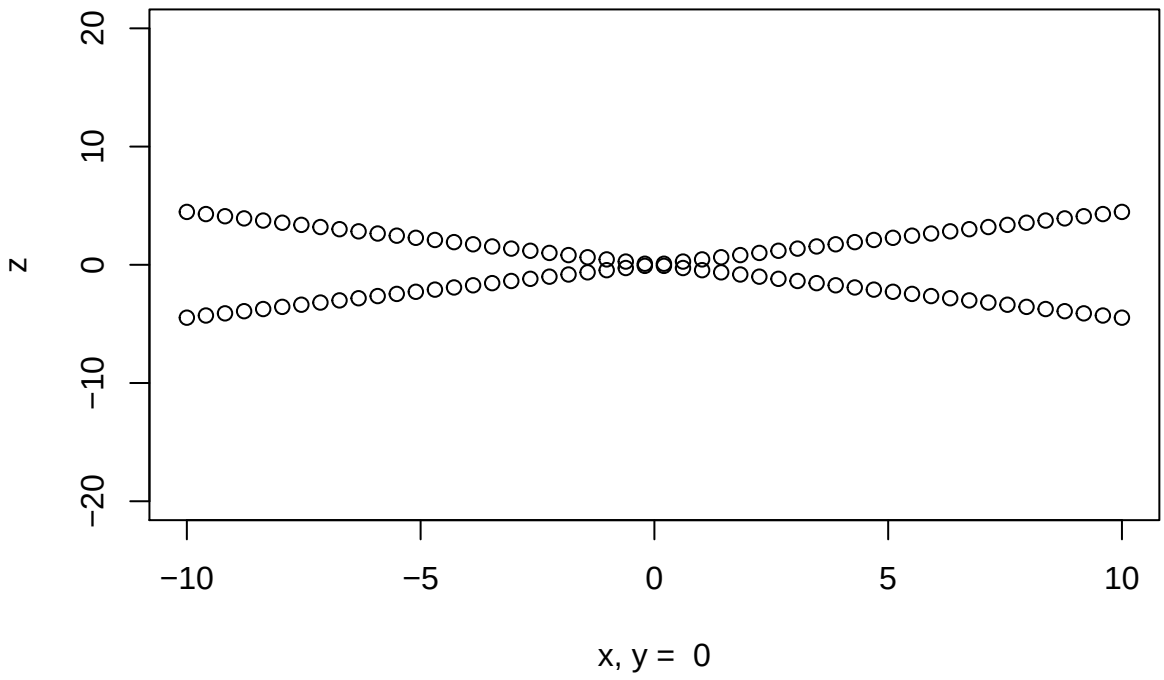
```
f <- function(x,y) sqrt(1/5*(x^2 + y^2))
g <- function(x,y) -sqrt(1/5*(x^2 + y^2))
xseq <- seq(-10, 10, length = 50)
yseq <- seq(-10, 10, length = 50)
persp(x = xseq, y = yseq,
      z = matrix(f(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```



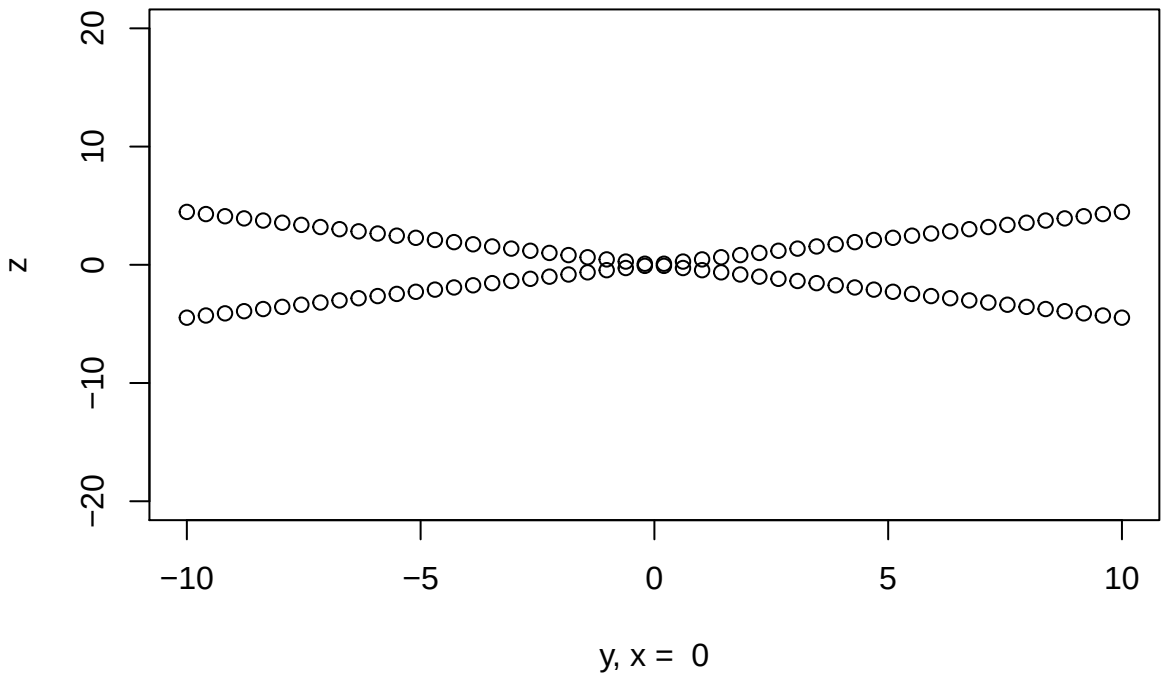

```
persp(x = xseq, y = yseq,
      z = matrix(g(rep(xseq, each=50), rep(yseq, times=50)), nrow=50, ncol=50),
      xlab="x", ylab="y", zlab="z")
```




```
plot(x = xseq, y = f(xseq, rep(0, 50)), xlab = "x, y =  0", ylab = "z", ylim=c(-20, 20))
points(x = xseq, y = g(xseq, rep(0, 50)))
```




```
plot(x = yseq, y = f(rep(0, 50), yseq), xlab = "y, x =  0", ylab = "z", ylim=c(-20, 20))
points(x = yseq, y = g(rep(0, 50), yseq))
```



Question 6

A <-3,3,-2> to cylindrical

$$r = \sqrt{x^2 + y^2} = \sqrt{18}$$

$$theta = tan^{-1}\frac{y}{x} = \frac{3\pi}{4}$$

check = TRUE

$$z = -2$$

B <5, 2/3 , 3/4 > to Rectangular

$$x = \rho sin\phi cos\theta = 5sin\frac{3\pi}{4}cos\frac{2\pi}{3} = 5\frac{1}{\sqrt{2}}\frac{-1}{2} = \frac{-5}{2\sqrt{2}}$$

$$y = \rho sin\phi sin\theta = 5sin\frac{3\pi}{4}sin\frac{2\pi}{3} = 5\frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2} = \frac{5\sqrt{3}}{2\sqrt{2}}$$

$$z = \rho cos\phi = 5cos\frac{3\pi}{4} = \frac{-5}{\sqrt{2}}$$

Question 7

Part A

$$x^2 + y^2 - 3x + z^2 = 25$$

Spherical

$$\rho^2 - 3\rho sin\phi cos\theta = 25$$

Cylindrical

$$r^2 - 3rcos\theta + z^2 = 25$$

B

$$y^2 + z^2 = 4$$

spherical

$$(\rho sin\phi cos\theta)^2 + (\rho cos\phi)^2 = 4$$

$$\rho^2(sin^2\phi sin^2\theta + cos^2\phi) = 4$$

Cylindrical

$$(rsin\theta)^2 + z^2 = 4$$

$$r^2 sin^2\theta + z^2 = 4$$

Question 8

Part A

$$r = 2sin\theta$$

$$r^2 = 2rsin\theta$$

Rectangular

$$x^2 + y^2 = 2y$$

$$x^2 + y^2 - 2y = 0$$

Spherical

$$(\rho sin\phi cos\theta)^2 + (\rho sin\phi sin\theta)^2 - 2\rho sin\phi sin\theta$$

$$\rho^2 sin^2\phi - 2\rho sin\phi sin\theta = 0$$

$$\rho sin\phi - 2sin\theta = 0$$

Part B

$$\rho sin\phi = 5$$

$$\rho^2 sin^2\phi = 25$$

$$\rho^2 sin^2\phi cos^2\theta + \rho^2 sin^2\phi sin^2\theta = 25$$

Rectangular

$$x^2 + y^2 = 25$$

Cylindrical

$$r^2 = 25$$