Weekly Assignment 13

RC

2021-12-12

Question 1

$$\oint_C y^2 dx + 3x^2 dy$$

$$C : (0,0) \rightarrow (3,0) \rightarrow (3,4) \rightarrow (0,4)$$

$y = 0$, $y = \sqrt{x}$, $x = 4$

Greens Theorem:

$$\oint_C F \cdot dr = \oint_C P dx + Q dy = \int\int_D (Q_x - P_y) dA$$

$$= \int_{y=0}^{4} \int_{x=0}^{3} 6x - 2y \ dx dy = \int_{y=0}^{4} (27 - 6y) dy = 27(4) - 3(16) = 60$$

Question 2

$$\oint (1 - 2xy) dx + (x^2 - y^2) dy$$

$$C : \quad r = 1 + sin\theta$$

$$Q_x - P_y = 2x + 2x = 4x \rightarrow 4r cos\theta$$

$$= \int_{\theta=0}^{2\pi} \int_{r=0}^{1+sin\theta} 4r^2 cos\theta dr d\theta = \int_0^{2\pi} \frac{4}{3}(1 + sin\theta)^3 cos\theta d\theta = \frac{1}{3}1^4 - \frac{1}{3}1^4 = 0$$

Question 3

$$r(u,v) = x(u,v)\hat{i} + y(u,v)\hat{j} + z(u,v)\hat{k}$$

$$z = x^2 + y^2 + 5$$

$$x = \sqrt{v}cos(u)$$

1

$$y = \sqrt{v}sin(u)$$

$$z = v + 5$$

Check

$$v + 5 = (\sqrt{v}cos(u))^2 + (\sqrt{v}sin(u))^2 + 5$$

Question 4

$$\frac{x^2}{4} + \frac{z^2}{16} = 1$$

$$x = 2cos(u)$$

$$z = 4sin(u)$$

$$y = v$$

check

$$\frac{(2cos(u))^2}{4} + \frac{(4sin(u))^2}{16} = 1$$

Question 5

$$r(u,v) = 4ucos(v)\hat{i} + 4usin(v)\hat{j} + \frac{1}{8}u^2\hat{k}$$

$$x^2 + y^2 = 16u^2$$

$$x^2 + y^2 = 128z$$

Paraboloid

Question 6

$$r(u,v) = u\hat{i} + 3v\hat{j} + u^2v\hat{k}$$

at $< x, y, z >=< 2, 9, 12 >$ which is $< u, v >=< 2, 3 >$

$$r_u =< 1, 0, 2uv >, \quad r_u(a,b) =< 1, 0, 12 >$$

$$r_v =< 0, 3, u^2 >, \quad r_v(a,b) =< 0, 3, 4 >$$

$$r_u(a,b) \times r_v(a,b) = -36\hat{i} - 4\hat{j} + 3\hat{k}$$

Surface

$$-36x - 4y + 3z = -36(2) + (-4)(9) + 3(12) = -72$$