Weekly Assignment 5

RC

'r Sys.Date()

```
import sympy as sy
import sympy.vector as sv

N = sv.CoordSys3D('N')
t = sy.Symbol('t', real = True)
```

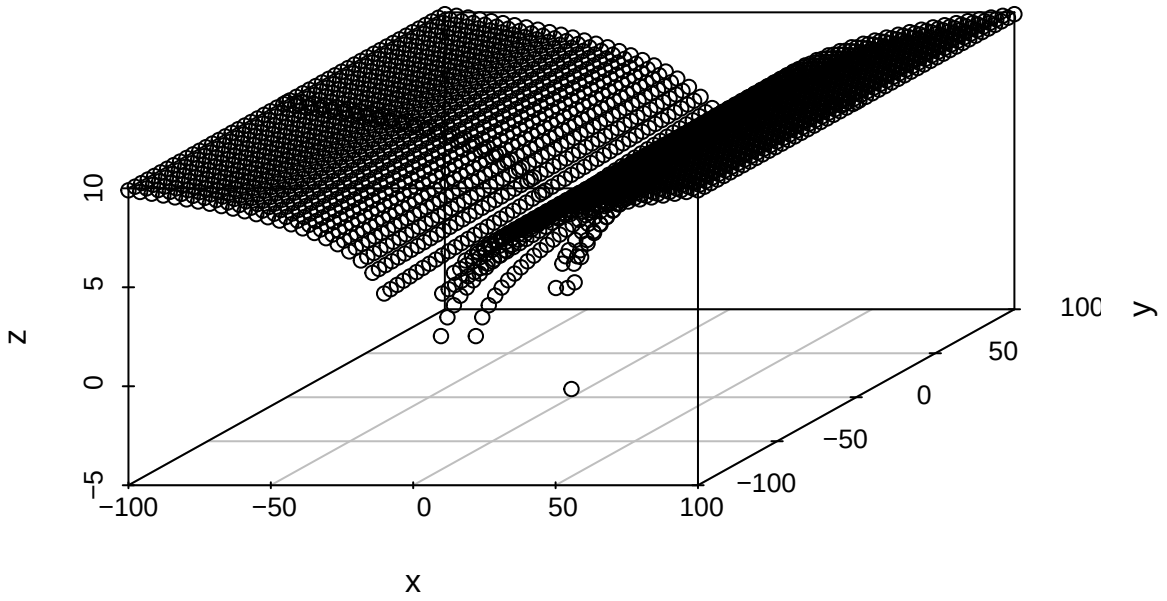
Question 1

$$f(x,y) = ln(2x^2 + y)$$

$$x \in (-\infty, \infty), \quad 2x^2 + y > 0, \quad y \in (-2x^2, \infty)$$

$$f(x,y) \in (-\infty, \infty)$$

```
f <- function(x, y) log(2*x^2 + y)
x <- c(seq(-100, 100, length = 50), 0.01)
y <- c(seq(-100, 100, length = 50), 0.01)
X <- expand.grid(x,y)
names(X) <- c("x", "y")
suppressWarnings(scatterplot3d::scatterplot3d(x = X$x, y = X$y, z = f(X$x, X$y),
                                              xlab = "x", ylab = "y", zlab = "z"))
```



Question 2

Part A

$$g(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$$

$$x, y, z \in [-5, 5], \quad where \;\; 25 - x^2 - y^2 - z^2 >= 0$$

$$g(x,y,z) \in [0, 5]$$

Question 3

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$$

Translating to polar coordinates, $x = rcos\theta$ and $y = rsin\theta$

Note that $x \to 0$ and $y \to 0$ as $r \to 0$ for any $\theta \in [0, 2\pi)$

$$= \lim_{r\to 0} \frac{r^2 cos^2\theta}{r^2} = \lim_{r\to 0} cos^2\theta$$

the limit is undefined.

Also, taking limits consecutively

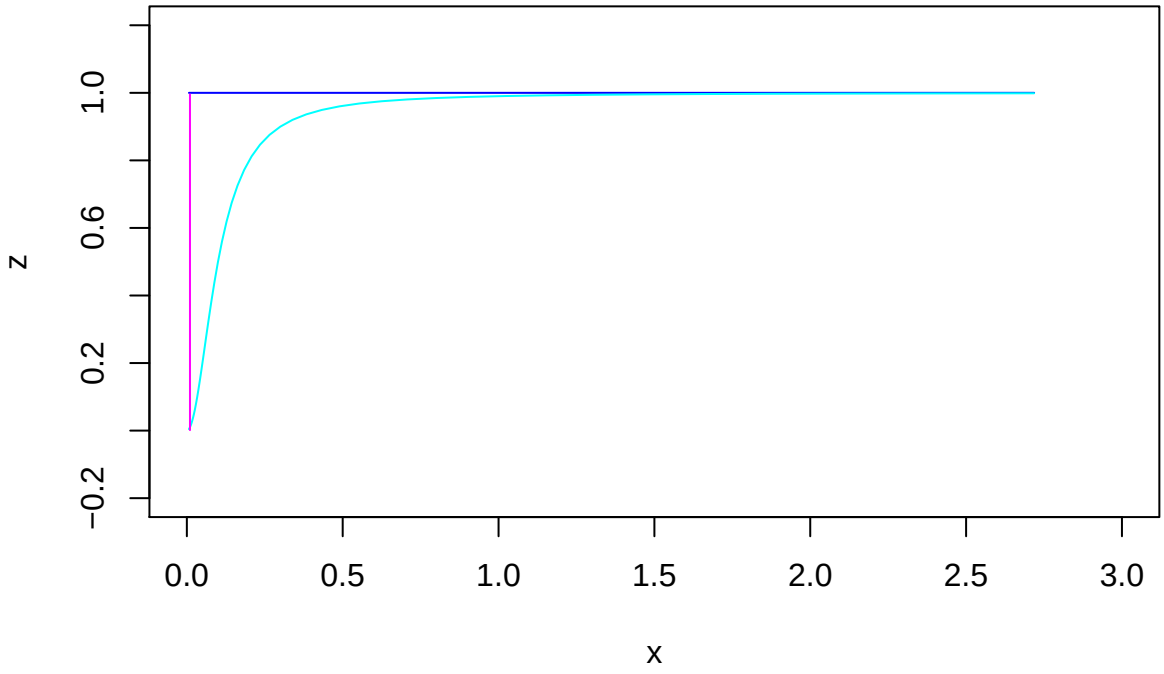
$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} = \lim_{x\to 0}\lim_{y\to 0} \frac{x^2}{x^2+y^2}$$

$$\lim_{x\to 0} \frac{x^2}{x^2} = 1$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2} = \lim_{y\to 0}\lim_{x\to 0} \frac{x^2}{x^2+y^2}$$

$$\lim_{y\to 0} \frac{0}{y^2} = 0$$

```
f <- function(x, y) x^2 / (x^2 + y^2)
x <- exp(seq(1, -5, length = 50))
plot(x, f(x, 0), type="l", col = "blue", xlab = "x", ylab = "z", xlim = c(0,3), ylim = c(-0.2, 1.2))
lines(x, f(x, 0.1), col = "cyan")
lines(rep(0, 50), f(0, x), col = "red")
lines(rep(0.01, 50), f(0.1, x), col = "magenta")
```



Part B

$$\lim_{(x,y)\to(1,1)} \frac{xy-y^3}{x-y^2} = \lim_{(x,y)\to(1,1)} y = 1$$

Question 4

Part A

$$f(x,y) = cos(3xy^2) + 4x^3 sin(5y) + e^{y^3}$$

$$\frac{\partial f}{\partial x} = -sin(3xy^2)(3y^2) + 12sin(5y)x^2$$

$$\frac{\partial f}{\partial y} = -sin(3xy^2)(6xy) + 20x^3 cos(5y) + e^{y^3}(3y^2)$$

```
x = sy.Symbol('x')
y = sy.Symbol('y')
f = sy.cos(3*x*y**2) + 4*x**3*sy.sin(5*y) + sy.exp(y**3)
print(f"df/dx = {sy.diff(f, x)}")
```

```
## df/dx = 12*x**2*sin(5*y) - 3*y**2*sin(3*x*y**2)
```

```
print(f"df/dy = {sy.diff(f, y)}")
```

```
## df/dy = 20*x**3*cos(5*y) - 6*x*y*sin(3*x*y**2) + 3*y**2*exp(y**3)
```

Part B

$$z = \sqrt{16x^2 + 4y^2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}(16x^2 + 4y^2)^{-1/2}(32x) = \frac{16x}{\sqrt{16x^2 + 4y^2}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(16x^2 + 4y^2)^{-1/2}(8y) = \frac{4y}{\sqrt{16x^2 + 4y^2}}$$

```
z = sy.sqrt(16*x**2 + 4*y**2)
print(f"dz/dx = {sy.diff(z, x)}")
```

```
## dz/dx = 16*x/sqrt(16*x**2 + 4*y**2)
```

```
print(f"dz/dy = {sy.diff(z, y)}")
```

```
## dz/dy = 4*y/sqrt(16*x**2 + 4*y**2)
```

Part C

$$g(x,y) = \frac{3x^2}{6y+2}$$

$$\frac{\partial g}{\partial x} = \frac{6x}{6y+2}$$

$$\frac{\partial g}{\partial y} = \frac{(-1)(3x^2)(6)}{(6y+2)^2} = \frac{-18x^2}{(6y+2)^2}$$

```
g = 3*x**2 / (6*y+2)
print(f"dg/dx = {sy.diff(g, x)}")
```

```
## dg/dx = 6*x/(6*y + 2)
print(f"dg/dy = {sy.diff(g, y)}")
```

```
## dg/dy = -18*x**2/(6*y + 2)**2
```

Question 5

$$f(x,y) = \frac{2x^3}{y^2} + sin(x^3y)$$

$$f_x(x,y) = \frac{6x^2}{y^2} + 3x^2y\ cos(x^3y)$$

$$f_y(x,y) = \frac{-4x^3}{y^3} + x^3cos(x^3y)$$

$$f_{xx}(x,y) = \frac{12x}{y^2} + 3x^2y(-1)sin(x^3y)(3x^2y) + cos(x^3y)(6xy) = \frac{12x}{y^2} - 9x^4y^2sin(x^3y) + 6xycos(x^3y)$$

$$f_{yy}(x,y) = \frac{12x^3}{y^4} - x^6sin(x^3y)$$

$$f_{xy}(x,y) = \frac{-12x^2}{y^3} - 3x^5ysin(x^3y) + 3x^2cos(x^3y)$$

```
f = 2*x**3 / y**2 + sy.sin(x**3*y)
print(f"df/dx = {sy.diff(f, x)}")
```

```
## df/dx = 3*x**2*y*cos(x**3*y) + 6*x**2/y**2
print(f"df/dy = {sy.diff(f, y)}")
```

```
## df/dy = x**3*cos(x**3*y) - 4*x**3/y**3
print(f"d**2 f / dx**2 = {sy.diff(sy.diff(f, x), x)}")
```

```
## d**2 f / dx**2 = -9*x**4*y**2*sin(x**3*y) + 6*x*y*cos(x**3*y) + 12*x/y**2
print(f"d**2 f / dy**2 = {sy.diff(sy.diff(f, y), y)}")
```

```
## d**2 f / dy**2 = -x**6*sin(x**3*y) + 12*x**3/y**4
print(f"d**2 f / dxdy = {sy.diff(sy.diff(f, x), y)}")
```

```
## d**2 f / dxdy = -3*x**5*y*sin(x**3*y) + 3*x**2*cos(x**3*y) - 12*x**2/y**3
```