Weekly Assignment 9

RC

2021-11-01

Question 1

First Quadrant

$$y = x^2$$

$$y = 9$$

$$\rho(x, y) = x^2 y$$

Mass

$$\int_{x=0}^{3} \int_{y=x^2}^{9} x^2 y dy dx = \int_{x=0}^{3} -\frac{x^6}{2} + \frac{81x^2}{2} dx = 1458/7$$

Moments

$$M_y = \int_{x=0}^{3} \int_{y=x^2}^{9} x^3 y dy dx = 6561/16$$

$$M_x = \int_{x=0}^{3} \int_{y=x^2}^{9} x^2 y^2 dy dx = 1458$$

Center of Mass

$$\bar{x} = 63/32$$

$$\bar{y} = 7$$

Question 2

$$f(x, y) = 16 - x^2 - y^2$$

Intersects with the xy plane at

$$f(x, y) = 0 = 16 - x^2 - y^2$$

$$f(r,\theta) = 16 - r^2$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{4)} r(16-r^2)d\theta dr = \int_{\theta=0}^{2\pi} 64 d\theta = 128 * pi$$

Question 3

$$\int_0^1\int_0^x\int_0^{x-2y} x^2 dz dy dx$$

$$\int_0^1\int_0^x\int_0^{x-2y} x^2 dz dy dx = \int_0^1\int_0^x x^2\,(x-2y)\,dy dx = \int_0^1 0 dx = 0$$

Question 4

$$3x + y + 4z = 12$$

$$x = 0 \quad y = 0 \quad z = 0$$

Intersection on boundary:

$$\int_{x=0}^{4}\int_{y=0}^{12}\int_{z=0}^{3}(12-3x-y-4z)dzdydx$$

$$\int_{x=0}^{4}\int_{y=0}^{12}\int_{z=0}^{3}(12-3x-y-4z)dzdydx = \int_{x=0}^{4}\int_{y=0}^{12} 9x+3y-18 dydx = \int_{x=0}^{4} 108x dx = 486$$

Question 5

$$x + 2y + 3z = 6$$

$$x^2 + y^2 = 4$$

First quadrant

$$r^2 = 4$$

$$rcos(\theta) + 2rsin(\theta) + 3z = 6$$

$$\int_{r=0}^{2}\int_{\theta=0}^{\pi/2}\int_{z=0}^{2} r(rcos(\theta)+2rsin(\theta)+3z-6)dzd\theta dr$$