Weekly Assignment 6

RC

2021-10-07

Question 1

$$w = 4x^3yz^2 + 3x^2y$$
$$dw = (12x^2yz^2 + 6xy)dx + (4x^3z^2 + 3x^2)dy + (8x^3yz)dz$$

Question 2

Part A

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

Part B

$$\frac{\partial w}{\partial r} = 6rx^2y^2 + 6yz + \frac{2x^3y + z^2 + \frac{5}{y^2}}{4s^5}$$

Part C

$$\left.\frac{\partial w}{\partial r}\right|_{r=2,s=-1} = -35$$

Question 3

Part A

A vector in the direction of the greatest rate of increase

$$\nabla f = (3x^2y^2)\hat{\mathbf{i}}_{\mathbf{N}} + (2x^3y - 12y^2)\hat{\mathbf{j}}_{\mathbf{N}}$$
$$\left.\nabla f\right|_{2,-1} = (12)\hat{\mathbf{i}}_{\mathbf{N}} + (-28)\hat{\mathbf{j}}_{\mathbf{N}}$$

Part B

The greatest rate of increase

$$||\nabla f||_{2,-1} = 4\sqrt{58}$$

Part C

Vector in the direction of the greatest rate of decrease

$$-\left.\nabla f\right|_{2,-1} = (-12)\hat{\mathbf{i}}_{\mathbf{N}} + (28)\hat{\mathbf{j}}_{\mathbf{N}}$$

Part D

$$D_v f = \frac{\nabla f \cdot v}{||v||} = -\frac{88\sqrt{34}}{17}$$

Question 4

$$f = 2\sqrt{x^2 + y^2}$$

$$v = (7)\hat{\mathbf{i}}_{\mathbf{N}} + (-5)\hat{\mathbf{j}}_{\mathbf{N}}$$

$$\nabla f = (\frac{2x}{\sqrt{x^2 + y^2}})\hat{\mathbf{i}}_{\mathbf{N}} + (\frac{2y}{\sqrt{x^2 + y^2}})\hat{\mathbf{j}}_{\mathbf{N}}$$

$$D_v f = \frac{\nabla f \cdot v}{||v||} = \frac{7\sqrt{74}x}{37\sqrt{x^2 + y^2}} - \frac{5\sqrt{74}y}{37\sqrt{x^2 + y^2}}$$

$$D_v f = \frac{\nabla f \cdot v}{||v||}\Big|_{4,-3} = \frac{43\sqrt{74}}{185}$$

Question 5

$$g = 3xz^2 + 2y^2z^3$$

$$a = (4)\hat{\mathbf{i}}_{\mathbf{N}} + (2)\hat{\mathbf{j}}_{\mathbf{N}} + (-3)\hat{\mathbf{k}}_{\mathbf{N}}$$

$$\nabla g = (3z^2)\hat{\mathbf{i}}_{\mathbf{N}} + (4yz^3)\hat{\mathbf{j}}_{\mathbf{N}} + (6xz + 6y^2z^2)\hat{\mathbf{k}}_{\mathbf{N}}$$

$$D_a g = \frac{\nabla g \cdot a}{||a||} = \frac{\sqrt{29}\left(-18xz - 18y^2z^2 + 8yz^3 + 12z^2\right)}{29}$$

$$D_a g = \frac{\nabla g \cdot a}{||a||}\Big|_{4,-2,-1} = \frac{28\sqrt{29}}{29}$$

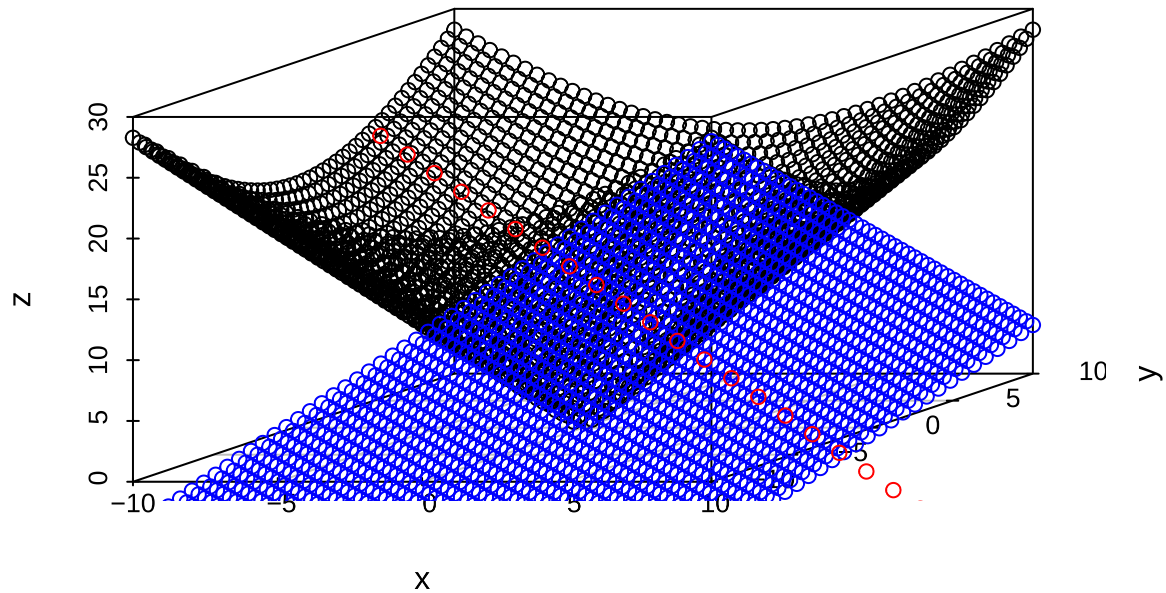
Question 6

$$f = 2\sqrt{x^2 + y^2}$$

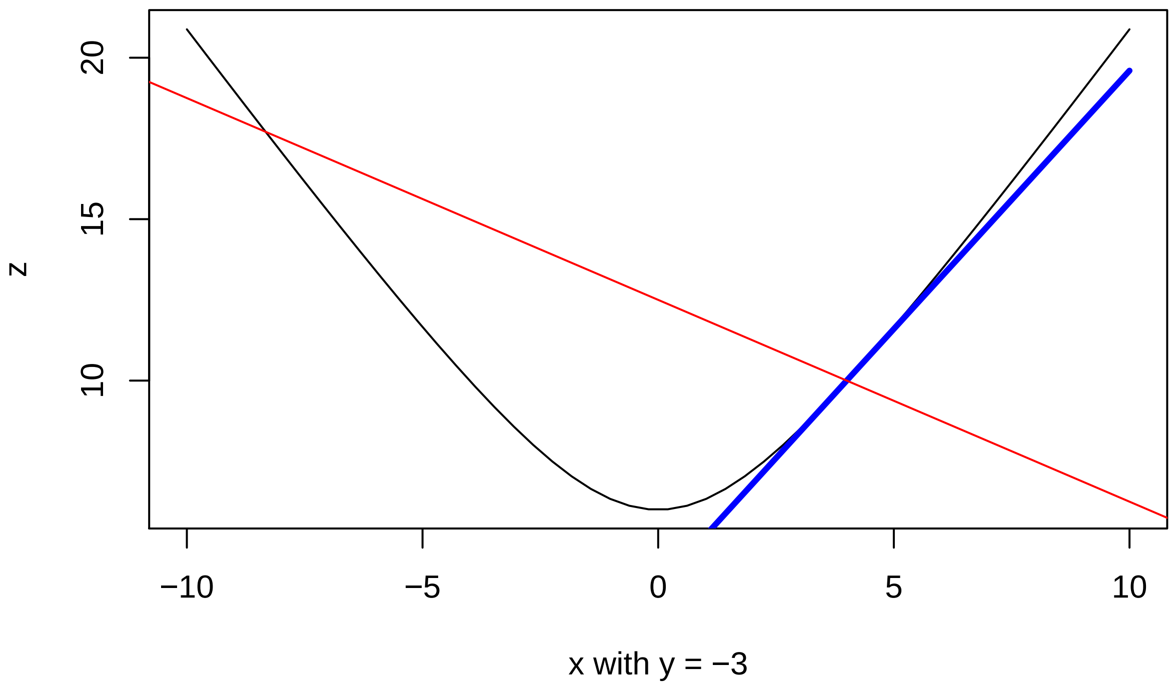
$$z - f(x_0, y_0) = \frac{2x_0\left(x - x_0\right)}{\sqrt{x_0^2 + y_0^2}} + \frac{2y_0\left(y - y_0\right)}{\sqrt{x_0^2 + y_0^2}}$$

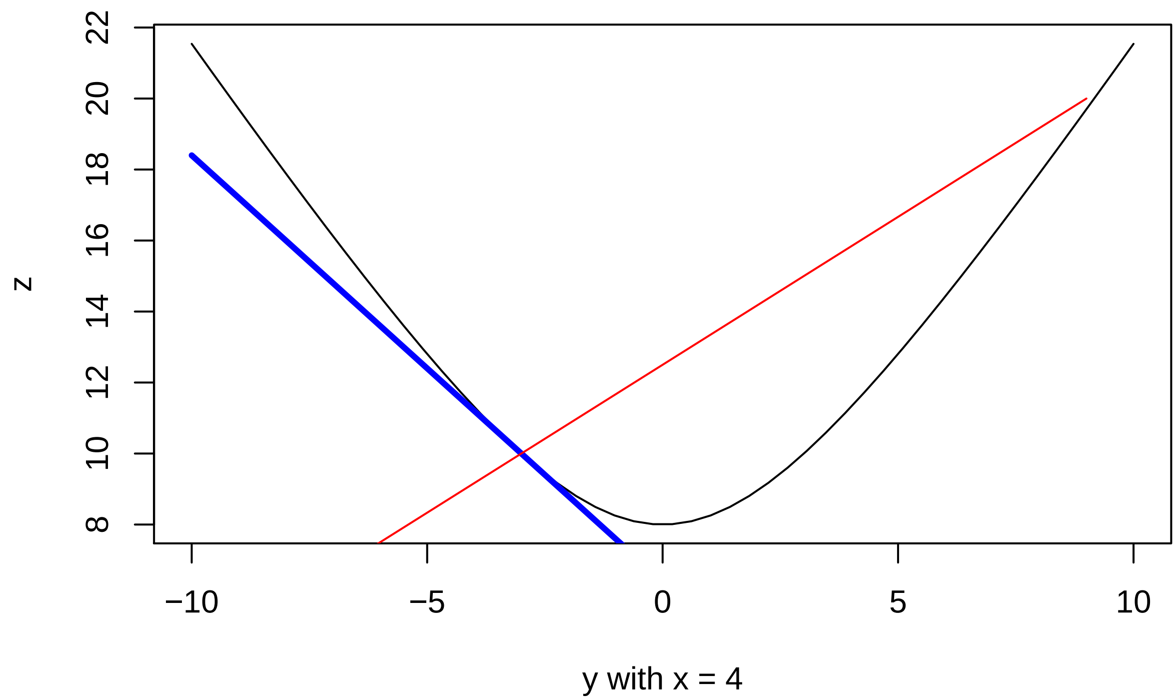
$$z = \frac{8x}{5} - \frac{6y}{5}$$

$$l(t) = (\frac{8t}{5} + 4)\hat{\mathbf{i}}_{\mathbf{N}} + (-\frac{6t}{5} - 3)\hat{\mathbf{j}}_{\mathbf{N}} + (10 - t)\hat{\mathbf{k}}_{\mathbf{N}}$$



z
30
25
20
15
10
5
0
−10
−5
0
5
10
x
y
10
5
0
−5


z
20
15
10
−10
−5
0
5
10
x with y = −3

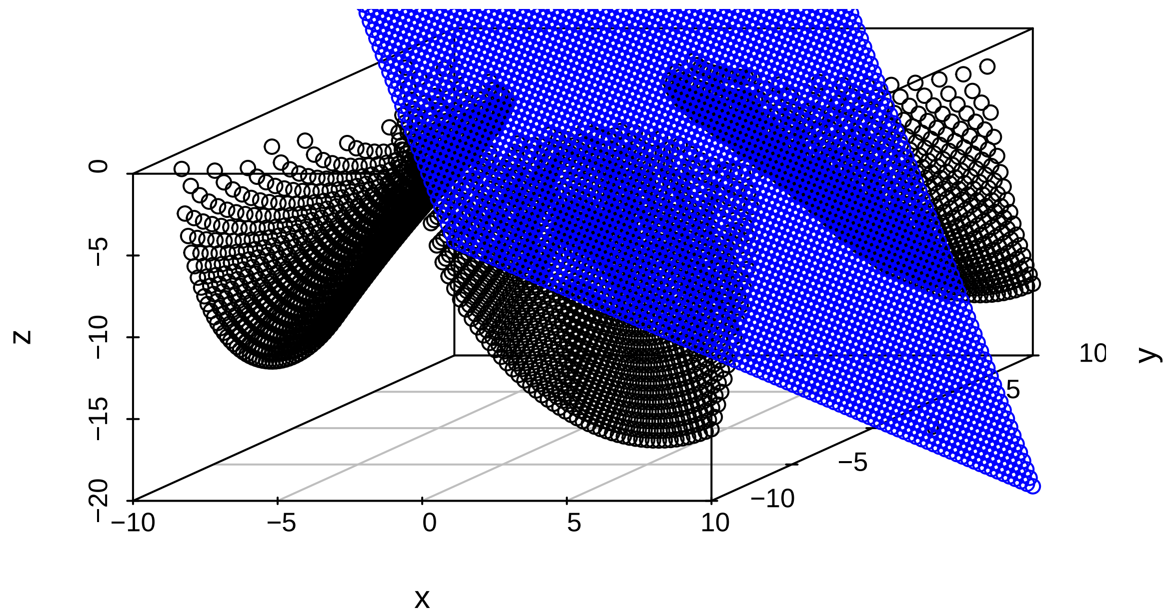


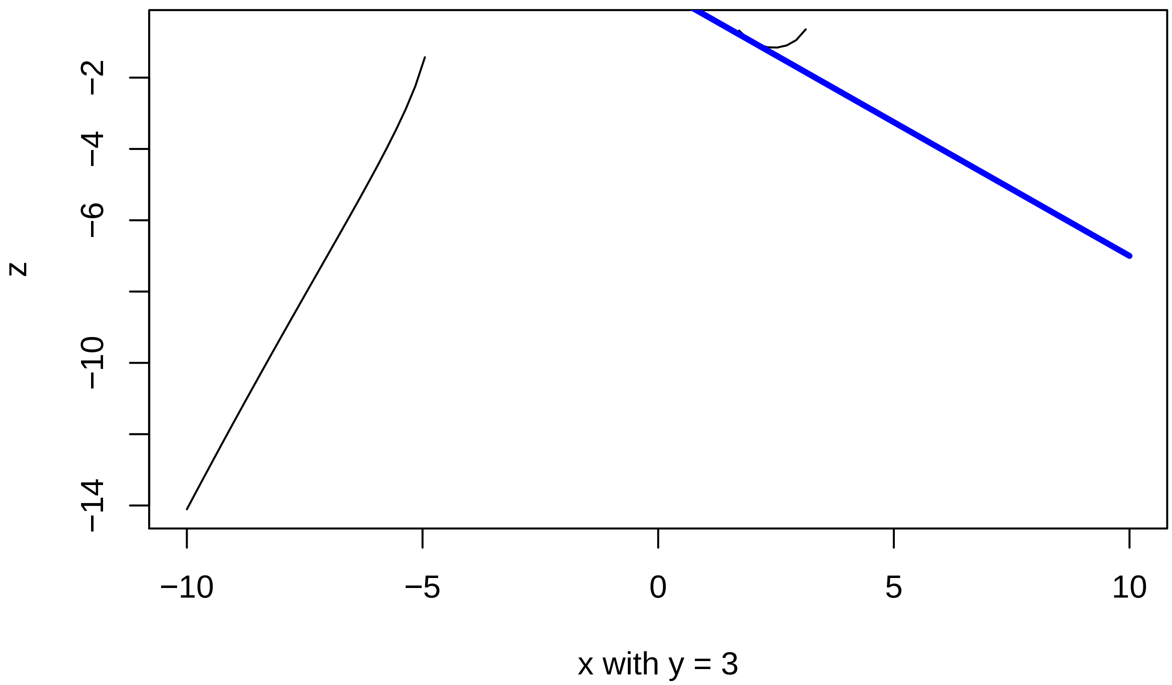
Question 7

$$f = -0.5\sqrt{-x^3 + 2xy^2 - 24}$$

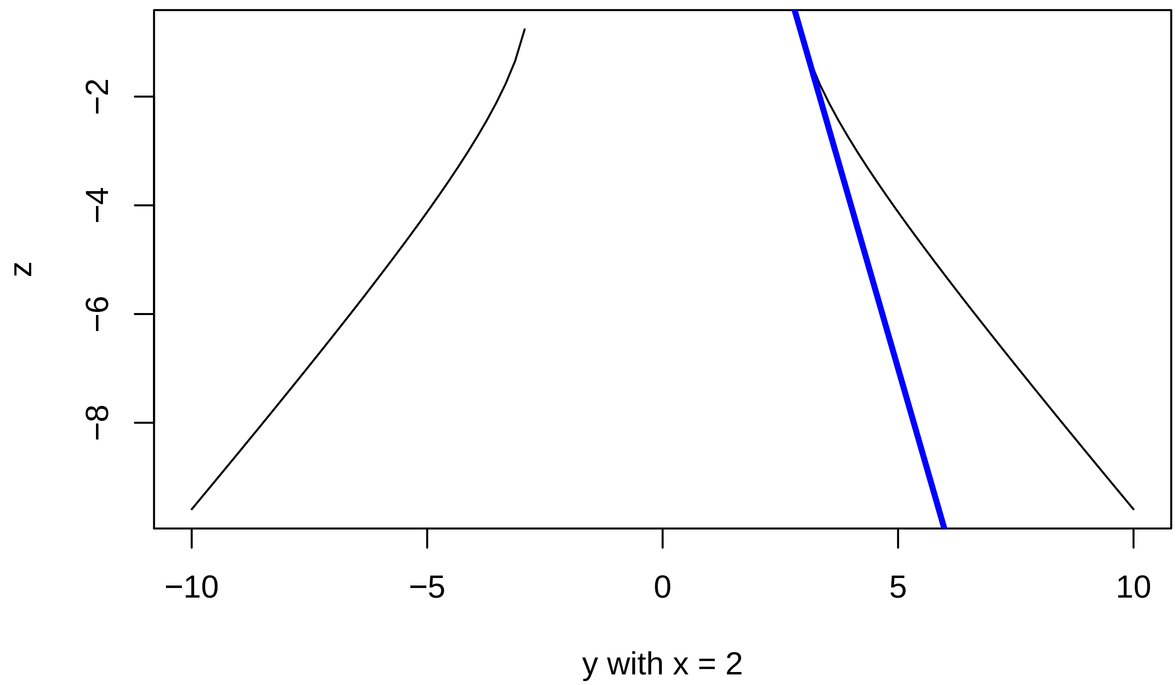
$$z - f(x_0, y_0) = -\frac{1.0x_0y_0\,(y - y_0)}{\sqrt{-x_0^3 + 2x_0y_0^2 - 24}} - \frac{0.5\,(x - x_0)\left(-\frac{3x_0^2}{2} + y_0^2\right)}{\sqrt{-x_0^3 + 2x_0y_0^2 - 24}}$$

$$z = -0.75x - 3.0y + 9.5$$


z
0
−5
−10
−15
−20
x
−10
−5
0
5
10
y
−10
−5
5
10



z
−2
−4
−6
−10
−14
−10
−5
0
5
10
x with y = 3



Question 8

$$x^2 + 4y^2 - z^2 = 28$$

$$z = \pm\sqrt{x^2 + 4y^2 - 28}$$

$$\frac{\partial z}{\partial x} = \pm\frac{x}{\sqrt{x^2 + 4y^2 - 28}}$$

$$\frac{\partial z}{\partial y} = \pm\frac{4y}{\sqrt{x^2 + 4y^2 - 28}}$$

Tangent plane at (a, b, c)

$$z = c + \frac{\partial z}{\partial x}|_{a,b,c}(x - a) + \frac{\partial z}{\partial y}|_{a,b,c}(y - b)$$

For positive z,

$$z_+ = c + \frac{a(x - a)}{\sqrt{a^2 + 4b^2 - 28}} + \frac{4b(y - b)}{\sqrt{a^2 + 4b^2 - 28}}$$

For negative z,

$$z_- = c - \frac{a(x-a)}{\sqrt{a^2+4b^2-28}} - \frac{4b(y-b)}{\sqrt{a^2+4b^2-28}}$$

In order for this plane to be tangent to the target plane, they must have the same coefficients on x and y, but can be at a different level (c)

$$z' = -2x + 4y$$

For positive z,

$$\frac{a}{\sqrt{a^2+4b^2-28}} = -2, \quad \frac{4b}{\sqrt{a^2+4b^2-28}} = 4$$

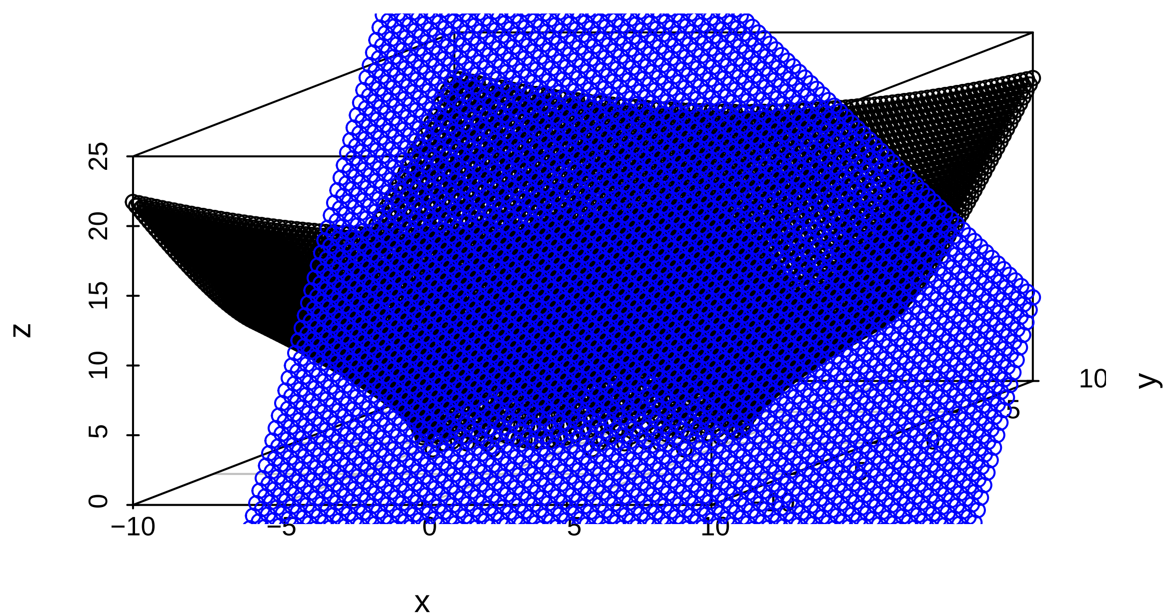
$$\frac{a}{-2} = \frac{4b}{4}, \quad b = \frac{-a}{2}, \quad a = -4, \quad b = 2, \quad c = 2$$

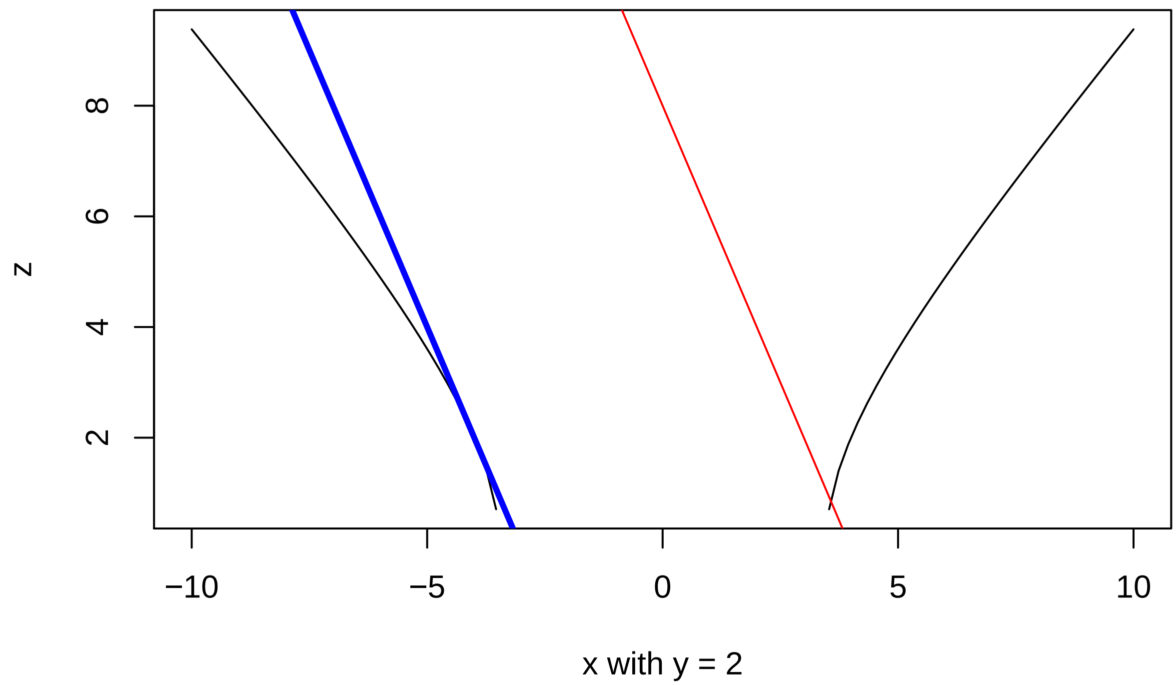
For negative z,

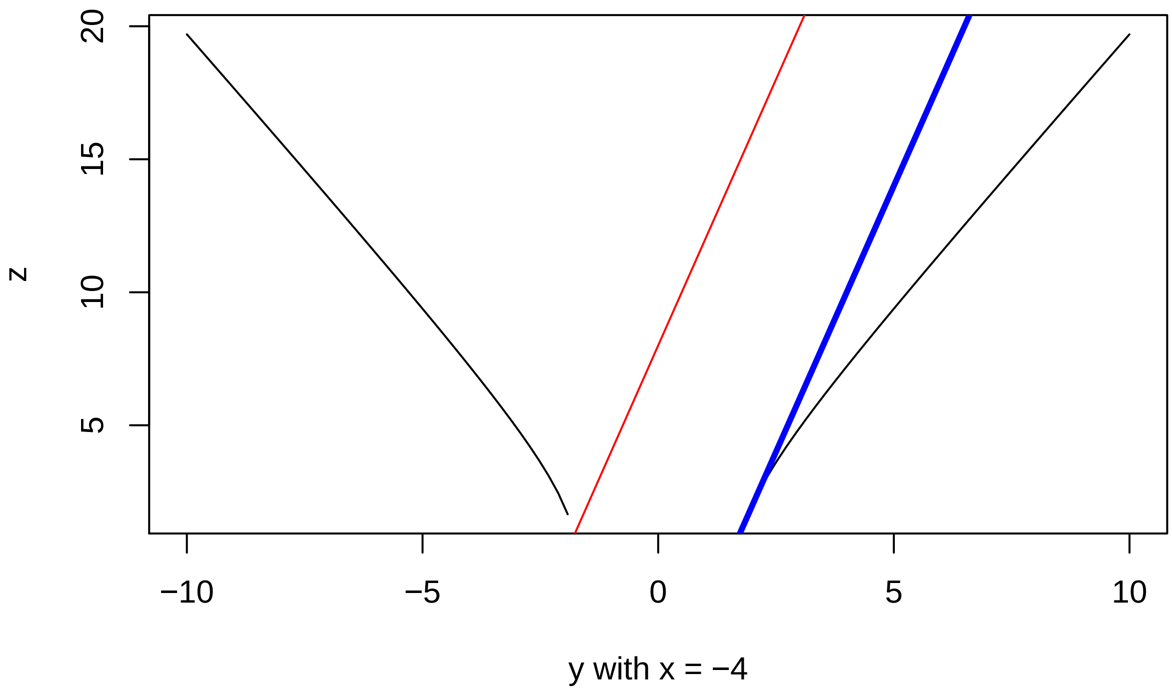
$$\frac{-a}{\sqrt{a^2+4b^2-28}} = -2, \quad \frac{-4b}{\sqrt{a^2+4b^2-28}} = 4$$

$$\frac{-a}{-2} = \frac{-4b}{4}, \quad b = \frac{-a}{2}, \quad a = 4, \quad b = -2, \quad c = -2$$

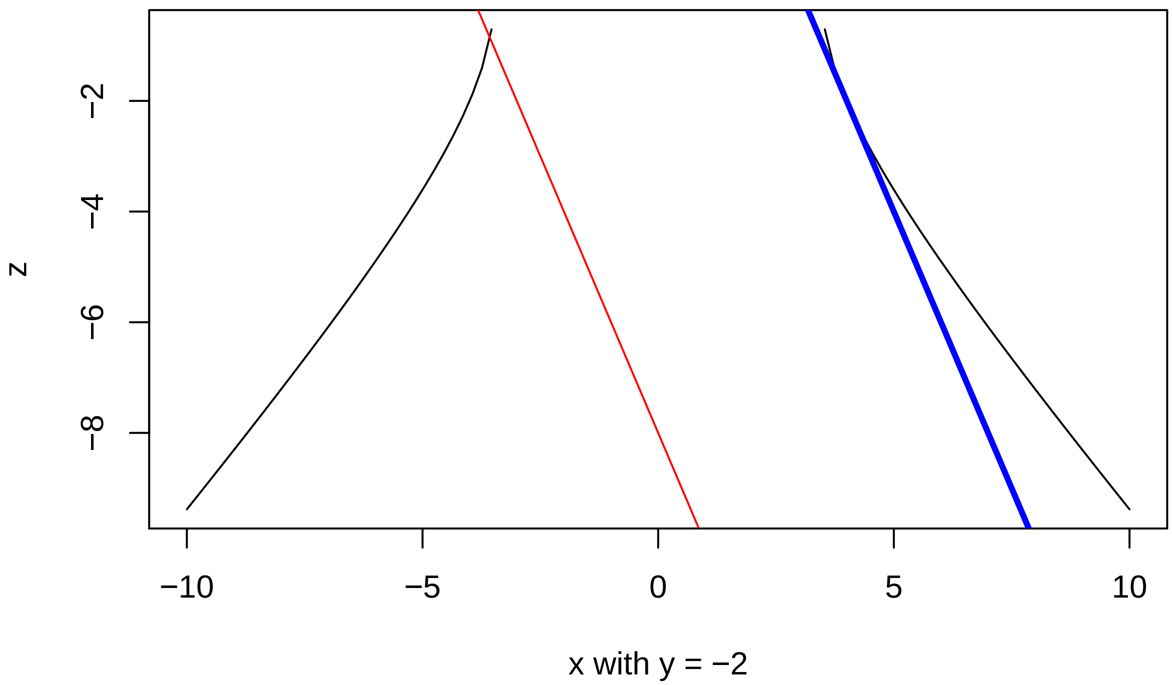
Therefore both $(-4, 2, 2)$ and $(4, -2, -2)$ have tangent planes that are parallel to $z + 2x - 4y = 0$



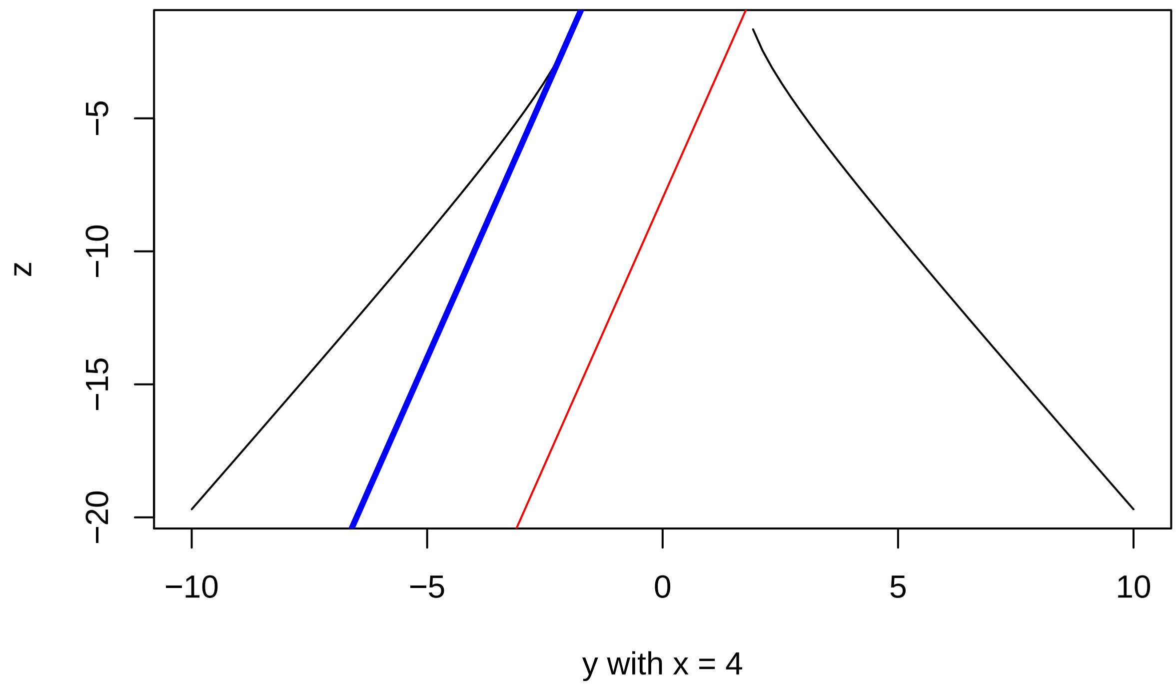

z
2
4
6
8
−10
−5
0
5
10
x with y = 2



20
15
10
5
z
−10
−5
0
5
10
y with x = −4



z
−2
−4
−6
−8
−10
−5
0
5
10
x with y = −2



Question 9

$$y = g(x)$$

is continuous and differentiable

Part A

if $g'(x) > 0$ for all x on (4,7), then $g(x)$ is **increasing** for all x on (4,7)

Part B

if $g'(9) = 0$ and $g''(9) = 4$ then $g(x)$ has a **local minimum** at $x = 9$

Part C

if $g(1) \leq g(x)$ for all x in [-3,3], then $g(1)$ is the **global minimum** or **absolute minimum** of $g(x)$ on $[-3, 3]$