Weekly Assignment 8

RC

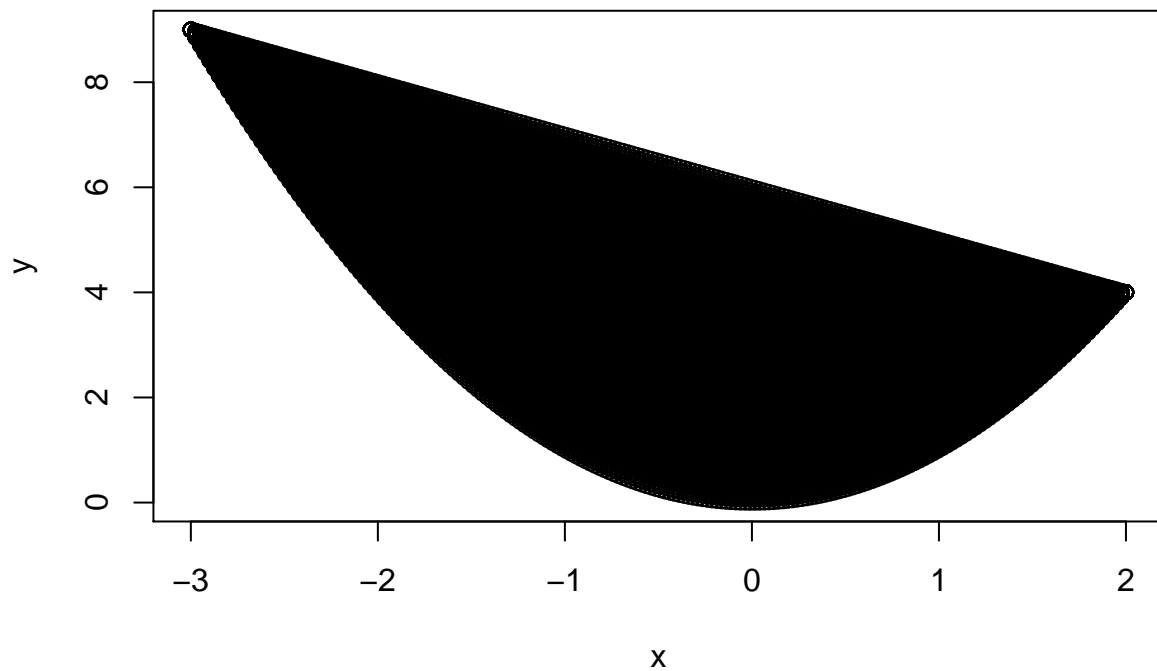### 2021-10-25

Question 1A

$$\int_{x=0}^{2}\int_{y=x}^{x^2} 2xy - 3x\,dy\,dx = \int_{x=0}^{2} \frac{x^6}{2} - \frac{x^4}{2} - 3x^3 + 3x^2\,dx = 8/3$$

Question 1B

$$\int_{\theta=0}^{\pi}\int_{0}^{1-sin(\theta)} r^2 cos(\theta)\,dr\,d\theta = \int_{\theta=0}^{\pi} \frac{(1 - \sin(\theta))^3 \cos(\theta)}{3}\,d\theta = 0$$

Question 2

$$\int_{-3}^{2}\int_{x^2}^{6-x} f(x,y)\,dy\,dx$$

$$= \int_{y=0}^{4} \int_{x=-\sqrt{y}}^{\sqrt{y}} f(x,y)dxdy + \int_{y=4}^{9} \int_{x=-\sqrt{y}}^{6-y} f(x,y)dxdy$$

check

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## -1625/24
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## -1625/24
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Question 3

$$x = y^2 - 2$$

$$y - x = 4$$

$$y = -1, \quad y = 4$$

$$\int_{y=-1}^{4} \int_{x=y-4}^{y^2-2} dxdy$$

$$\int_{y=-1}^{4} \int_{x=y-4}^{y^2-2} dxdy = 145/6$$
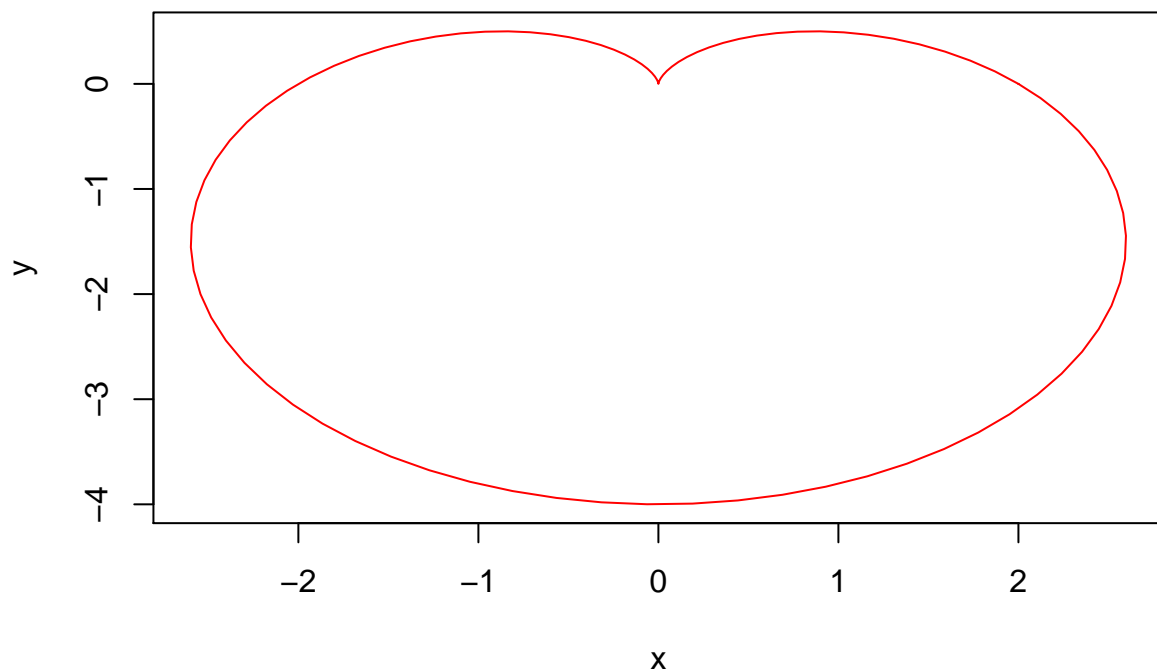
Question 4

$$2x + y + z = 4$$

Above the xy plane

$$f(x,y) = 4 - 2x - y$$

$$\int_{x=0}^{2} \int_{y=0}^{4-2x} 4 - 2x - ydydx = 16/3$$

Question 5

$$r = 2 - 2sin(\theta)$$

$$\int_{\theta=0}^{2*\pi} \int_{r=0}^{2-2sin(\theta)} r\, dr\, d\theta = \int_{\theta=0}^{2*\pi} \frac{(2-2\sin(\theta))^2}{2} d\theta = 6\pi$$

Question 6

$$r = 1$$

$$r = 2sin(\theta)$$

Find the intersections:

$$1 = 2sin(\theta)$$

$$\theta = \frac{\pi}{6}, \quad \frac{5\pi}{6}$$

$$\int_{\theta=\pi/6}^{5\pi/6} \int_{r=1}^{2sin(\theta)} r\, dr\, d\theta = \int_{\theta=\pi/6}^{5\pi/6} 2\sin^2(\theta) - \frac{1}{2} d\theta = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$$

Question 7

$$\int\int_{x^2+y^2=1} \frac{1}{\sqrt{4-x^2-y^2}}\,dA$$

$$= \int_{r=0}^{1}\int_{\theta=0}^{\pi/2} \frac{r}{\sqrt{4-r^2}}\,drd\theta$$

check with Monte Carlo integration (not exactly equal)

0.420893607238466

[1] 0.420015

Question 8

$$\int_{x=-3}^{3}\int_{y=0}^{\sqrt{9-x^2}} \frac{x+2y}{x^2+y^2}\,dydx$$

$$\int_{r=0}^{3}\int_{\theta=0}^{\pi} \left(cos(\theta)+2sin(\theta)\right)\,drd\theta$$

check

12

12

Question 9

$$z = x^2 + 2y^2$$

$$z = 12 - 2x^2 - y^2$$

Intersection

$$x^2 + 2y^2 = 12 - 2x^2 - y^2$$

$$0 = 4 - x^2 - y^2$$

$$f(x,y) = 12 - 3x^2 - 3y^2$$

$$f(r,\theta) = 12 - 3r^2$$

$$\int_{0}^{2\pi}\int_{0}^{2} r(12-3r^2)drd\theta = 24\pi$$

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