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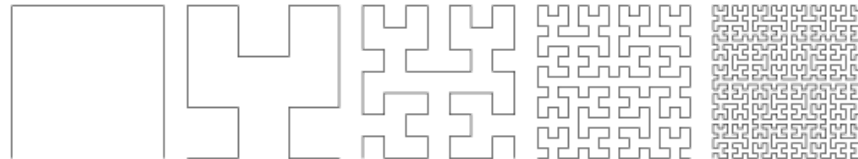
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*Created, developed, and  
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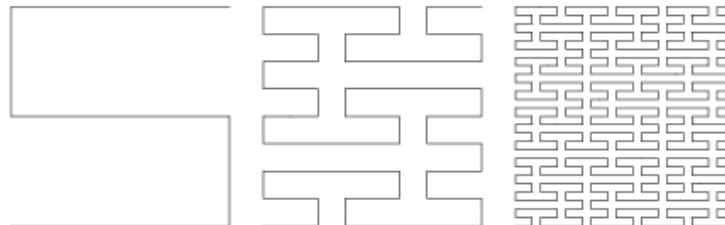
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# Hilbert Curve

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The Hilbert curve is a [Lindenmayer system](#) invented by Hilbert (1891) whose limit is a [plane-filling function](#) which fills a square. Traversing the [polyhedron vertices](#) of an  $n$ -dimensional [hypercube](#) in [Gray code](#) order produces a generator for the  $n$ -dimensional Hilbert curve. The Hilbert curve can be simply encoded with initial string "L", [string rewriting](#) rules "L"  $\rightarrow$  "+RF-LFL-FR+", "R"  $\rightarrow$  "-LF+RFR+FL-", and angle  $90^\circ$  (Peitgen and Saupe 1988, p. 278). The  $n$ th iteration of this Hilbert curve is implemented in the [Wolfram Language](#) as [HilbertCurve\[n\]](#).



A related curve is the Hilbert II curve, shown above (Peitgen and Saupe 1988, p. 284). It is also a [Lindenmayer system](#) and the curve can be encoded with initial string "X", [string rewriting](#) rules "X"  $\rightarrow$  "XFYFX+F+YFXFY-F-XFYFX", "Y"  $\rightarrow$  "YFXFY-F-XFYFX+F+YFXFY", and angle  $90^\circ$ . The  $n$ th iteration of this curve is implemented in the [Wolfram Language](#) as [PeanoCurve\[n\]](#).

**WolframAlpha**  
computational knowledge engine.

hilbert curve

THINGS TO TRY:

- = hilbert curve
- =  $((3+4i)/5)^{10}$
- = factor  $x^{12} - y^{12}$

Interactive knowledge apps from  
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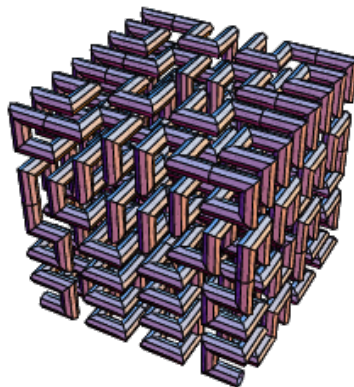
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**Interpolating the  
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A three-dimensional analog of the Hilbert curve can also be generated (Trott 2004, pp. 93-97).

#### SEE ALSO:

[Lindenmayer System](#), [Peano Curve](#), [Plane-Filling Function](#), [Sierpiński Curve](#), [Space-Filling Function](#)

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**UPDATE  
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Referenced on Wolfram|Alpha: [Hilbert Curve](#)

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