Two-dimensional L-systems

L-systems (also called *Lindenmayer systems* or *parallel string-rewrite systems*) are a compact way to describe iterative graphics using a turtle analogy, similar to that used by the LOGO programming language (about which I know nothing). An L-system is created by starting with an *axiom*, such as a line segment, and one or more *production rules*, which are statements such as "replace every line segment with a left turn, a line segment, a right turn, another segment...". When this system is iterated several times, the result is often a complicated fractal curve.

When programming L-systems, one typically represents the axiom as a sequence of characters, such as `F`, and the production rules as replacement rules of the form `F -> F+F--F+F`. We then carry out the string replacements (in parallel) as many times as desired. For example, we iterate the axiom and production rule just described in the following way:

We start with the axiom `F`, and replace every occurrence of `F` with `F+F--F+F`, resulting in the string

```
F+F--F+F.
```

We then replace every occurrence of `F` in the result with `F+F--F+F`, resulting in the string

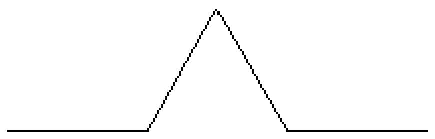
```
F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F.
```

Carrying out the replacement one more time results in the following:

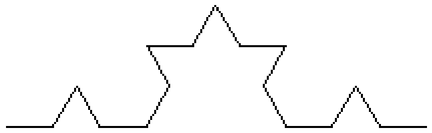
```
F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F-- F+F--F+F+F+F--F+F--F+F--F+F+F+F--
F+F+F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F.
```

After the desired number of iterations has been carried out, we render the L-system string using the turtle analogy. A typical interpretation of our string is that an `F` is an instruction to draw a line segment one unit in the current direction, a `+` is an instruction to rotate the current direction one angular unit clockwise, and a `-` is an instruction to rotate the current direction one angular unit counterclockwise. (Other characters and interpretations are of course up to us.) If we assume for this example that one (angular) unit is Pi/3 radians, the following are the partial results (omitting the zeroth iteration, the axiom).

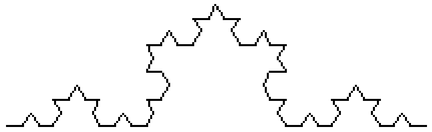
First iteration (`F+F--F+F`):



Second iteration (`F+F--F+F+F+F--F+F--F+F--F+F+F+F--F+F`):



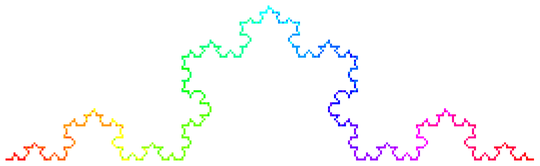
Third (`F+F--F+F+F+F--F+F...(148 characters in all)...F+F+F+F--F+F`):



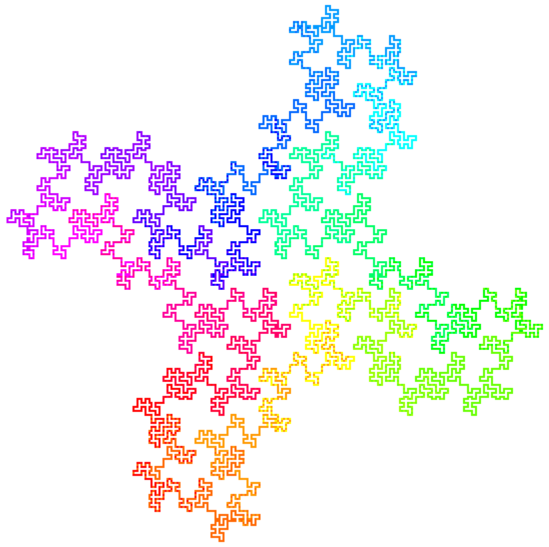
In the following pictures, the rendered L-systems are colored using the progression red-orange-yellow-green-blue-purple-red from beginning to end, to better illustrate the "path" of the curves.

([Axioms, production rules, and *Mathematica* code]() are below the pictures. These ideas can also be extended to [three dimensions]().)

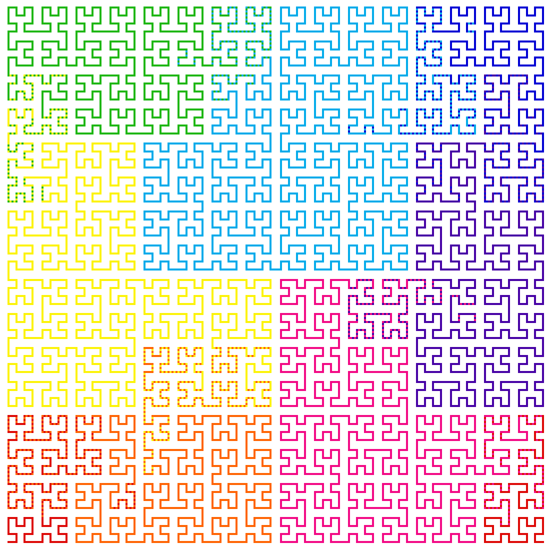
Koch curve (`F -> F+F--F+F`, 60°):



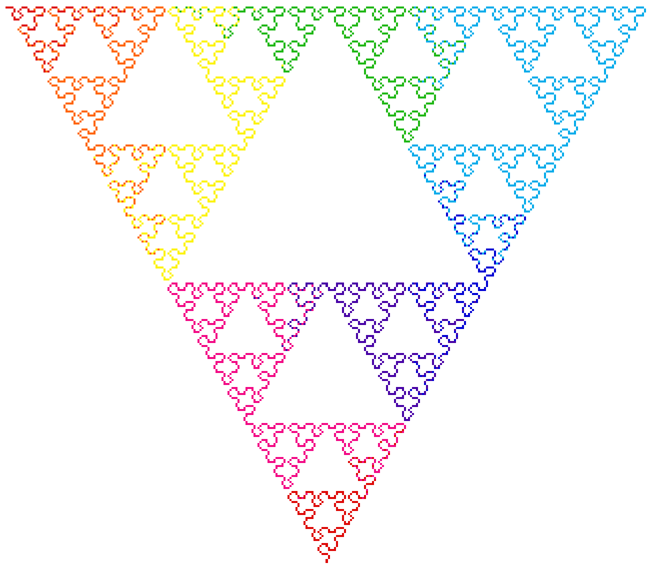
32-segment curve
(`F -> -F+F-F-F+F+FF-F+F+FF+F-F-FF+FF-FF+F+F-FF-F-F+FF-F-F+F+F-F+`):



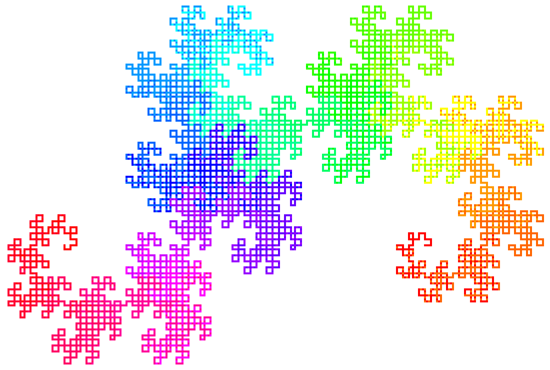
Hilbert curve (`L -> +RF-LFL-FR+`, `R -> -LF+RFR+FL-`):



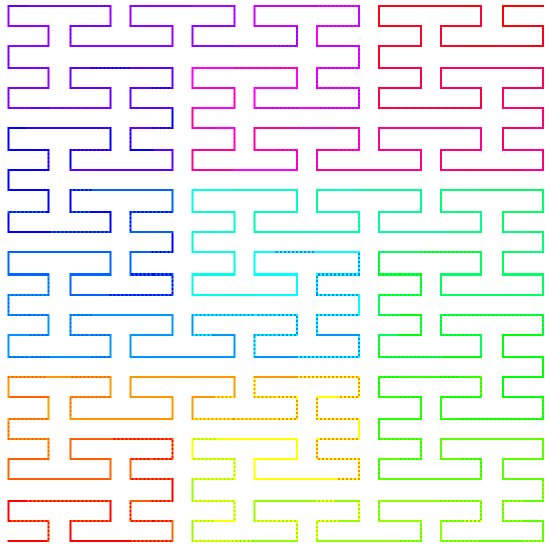
Arrowhead curve (`X -> YF+XF+Y`, `Y -> XF-YF-X`, 60°):



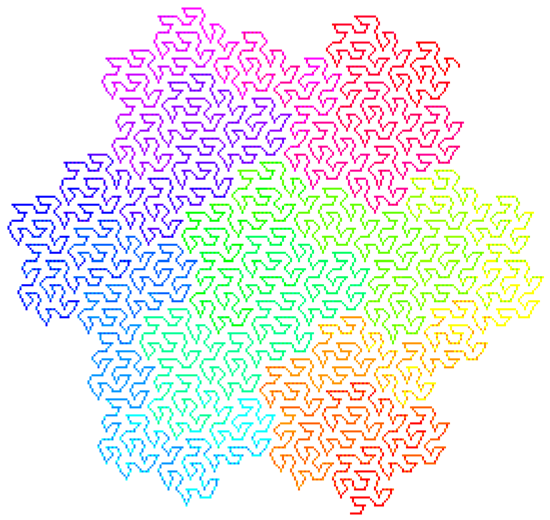
Dragon curve (`X -> X+YF+, Y -> -FX-Y`):



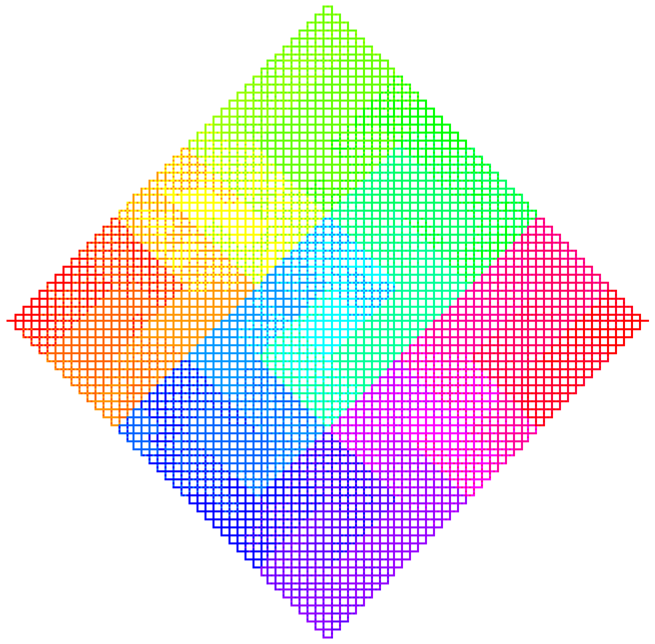
Hilbert curve II (`X -> XFYFX+F+YFXFY-F-XFYFX, Y -> YFXFY-F-XFYFX+F+YFXFY`):



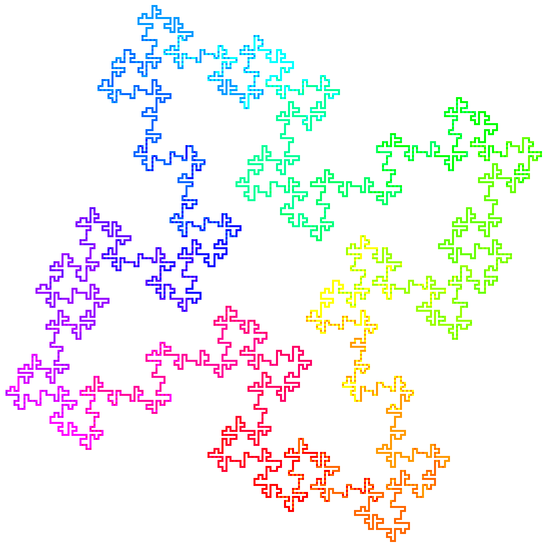
Peano-Gosper curve (`X -> X+YF++YF-FX--FXFX-YF+`, `Y -> -FX+YFYF++YF+FX--FX-Y`, 60°):



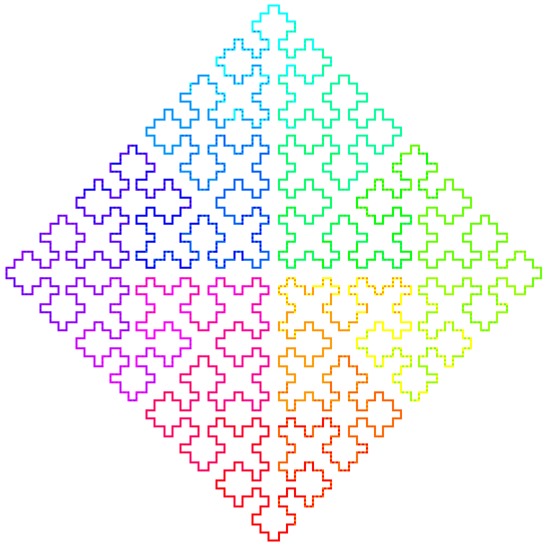
Peano curve (`F -> F+F-F-F-F+F+F+F-F`):



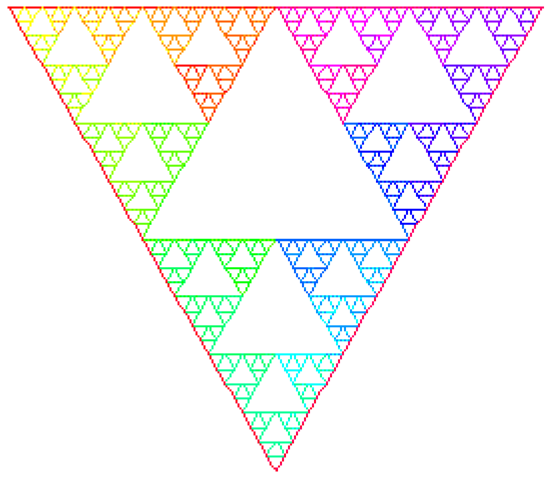
Quadratic Koch island (`F -> F-F+F+FFF-F-F+F`):



Square curve (`X -> XF-F+F-XF+F+XF-F+F-X`):



Sierpinski triangle (`F -> FF`, `X -> --FXF++FXF++FXF--`, 60°):



Here's the embarrassing, non-Wolfram Research-approved code (the functions `LShow` and `LShowColor` are identical except that `LShow` displays the system using only a black line, works a bit faster, and takes less memory):

```
(* carry out the forward and backward moves and the various
    rotations by updating the global location 'Lpos' and
    direction angle 'Ltheta'. *)

Lmove[ z_String, Ldelta_ ] :=
    Switch[ z,
        "+", Ltheta += Ldelta;,
        "-", Ltheta -= Ldelta;,
        "F", Lpos += {Cos[Ltheta], Sin[Ltheta]},
        "B", Lpos -= {Cos[Ltheta], Sin[Ltheta]},
         _ , Lpos += 0. ];

LSystem::usage =
    "LSystem[axiom, {rules}, n, Ldelta:90 Degree]
    creates the L-string for the nth iteration of
    the list 'rules', starting with the string 'axiom'.";

(* make the string: starting with 'axiom', use StringReplace the
    specified number of times *)

LSystem[ axiom_, rules_List,
    n_Integer, Ldelta_:N[90 Degree]] :=

    Nest[ StringReplace[#, rules]&, axiom, n ];
```

```
Off[General::spell1];

(* initialize the position 'Lpos' and the direction angle 'Ltheta';
    create the Line graphics primitive represented by the L-system by
    mapping 'Lmove' over the characters in the L-string, deleting all the
    Nulls; then show the Graphics object *)

LShow[lstring_String, Ldelta_:N[90 Degree]] :=
   (Lpos={0.,0.};
    Ltheta=0.;
    Show[
      Graphics[Line[
        Prepend[
          DeleteCases[
            Map[Lmove[#, Ldelta]&, Characters[lstring]],
           Null ],
        {0,0}]]],
     AspectRatio -> Automatic]);

(* same as above, plus a list of colors for each segment contained in
    'templist' -- unfortunately, 'templist' isn't really 'temp', but
    stays in memory as a global variable; so sue me *)

LShowColor[lstring_String, Ldelta_:N[90 Degree]] :=
   (Lpos = {0., 0.};
    Ltheta = 0.;

    templist =
      Map[ Line,
        Partition[
          Prepend[
            DeleteCases[
              Map[ Lmove[#, Ldelta]&, Characters[lstring] ],
             Null],
           {0,0}],
         2,1]];

    ncol = N[Length[templist]];

    huelist = Table[ Hue[k/ncol], {k, 1., ncol} ];

    Show[
      Graphics[
        N[Flatten[Transpose[{huelist, templist}]]]],
     AspectRatio -> Automatic]);

On[General::spell1];

LShowColor[ (* Koch curve *)
```

```
    LSystem["F", {"F" -> "F+F--F+F"}, 4],
        N[60 Degree]];

LShowColor[ (* Peano curve *)
    LSystem["F", {"F" -> "F+F-F-F-F+F+F+F-F"}, 4]];

LShowColor[ (* Quadratic Koch island *)
    LSystem["F+F+F+F", {"F" -> "F-F+F+FFF-F-F+F"}, 3]];

LShowColor[ (* 32-segment curve *)
    LSystem["F+F+F+F",
    {"F" ->
     "-F+F-F-F+F+FF-F+F+FF+F-F-FF+FF-FF+F+F-FF-F-F+FF-F-F+F+F-F+"},
    2]];

LShowColor[
    LSystem["YF", (* Sierpinski arrowhead *)
     {"X" -> "YF+XF+Y", "Y" -> "XF-YF-X"}, 7],
        N[60 Degree]];

LShowColor[ (* Peano-Gosper curve *)
    LSystem["FX",
      {"X" -> "X+YF++YF-FX--FXFX-YF+",
       "Y" -> "-FX+YFYF++YF+FX--FX-Y"}, 4],
     N[60 Degree]];

LShowColor[ (* Sierpinski triangle *)
    LSystem["FXF--FF--FF",
    {"F" -> "FF",
     "X"->"--FXF++FXF++FXF--"}, 6], N[60 Degree]];

LShowColor@LSystem["F+XF+F+XF", (* Square curve *)
    {"X"->"XF-F+F-XF+F+XF-F+F-X"}, 4];

LShowColor[ (* Dragon curve *)
    LSystem["FX", {"X"->"X+YF+", "Y"->"-FX-Y"}, 12]];

LShowColor@LSystem["L", (* Hilbert curve *)
    {"L" -> "+RF-LFL-FR+", "R" -> "-LF+RFR+FL-"}, 6];

LShowColor@LSystem["X", (* Hilbert curve II *)
    {"X" -> "XFYFX+F+YFXFY-F-XFYFX",
     "Y" -> "YFXFY-F-XFYFX+F+YFXFY"}, 3];
```

Designed and rendered using *Mathematica* versions 2.2 and 3.0 for the Apple Macintosh.

See also Peitgen *et al, Chaos and Fractals*, Springer-Verlag, 1992; Stan Wagon, *Mathematica in Action*, W. H. Freeman and Co., 1991; and Roger T. Stevens, *Fractal Programming in C*, M&T Books, 1989.

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