Hilbert Curve

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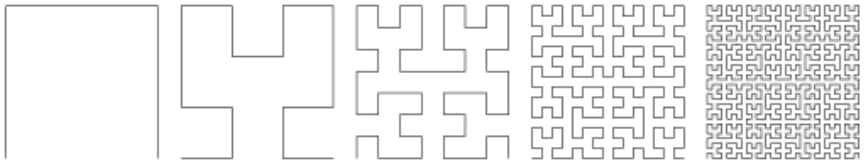
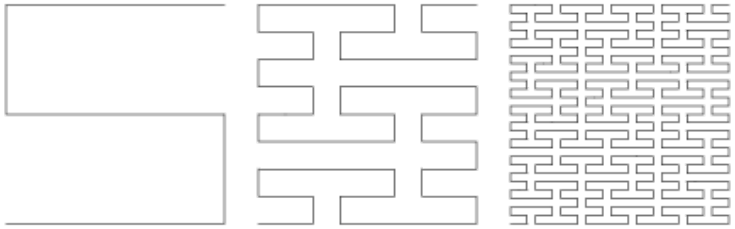
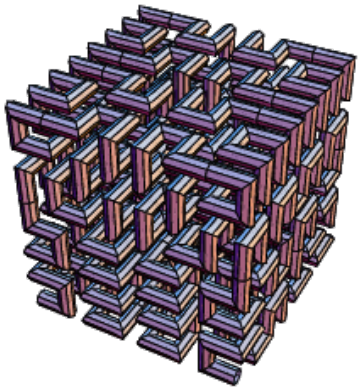


The Hilbert curve is a Lindenmayer system invented by Hilbert (1891) whose limit is a plane-filling function which fills a square. Traversing the polyhedron vertices of an n-dimensional hypercube in Gray code order produces a generator for the n-dimensional Hilbert curve. The Hilbert curve can be simply encoded with initial string `"L"`, string rewriting rules `"L" -> "+RF-LFL-FR+"`, `"R" -> "-LF+RFR+FL-"`, and angle $90°$ (Peitgen and Saupe 1988, p. 278). The nth iteration of this Hilbert curve is implemented in the Wolfram Language as `HilbertCurve`[n].



A related curve is the Hilbert II curve, shown above (Peitgen and Saupe 1988, p. 284). It is also a Lindenmayer system and the curve can be encoded with initial string `"X"`, string rewriting rules `"X" -> "XFYFX+F+YFXFY-F-XFYFX"`, `"Y" -> "YFXFY-F-XFYFX+F+YFXFY"`, and angle $90°$. The nth iteration of this curve is implemented in the Wolfram Language as `PeanoCurve`[n].



A three-dimensional analog of the Hilbert curve can also be generated (Trott 2004, pp. 93-97).

SEE ALSO:
Lindenmayer System, Peano Curve, Plane-Filling Function, Sierpiński Curve, Space-Filling Function

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