Stat 175 (Probability)

Problem Set No. 4

INSTRUCTIONS: Answer the following as indicated. Show detailed solutions.

1. The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with cumulative distribution function given by

$$F_Y(y) = \begin{cases} 1 - e^{-y^2}, \ y \geq 0 \\ 0, \ y < 0 \end{cases}$$

   a. Find the probability that the transistor operates for at least 200 hours.

   b. Find the PDF of Y.

   c. Find $P(Y > 100 | Y \leq 200)$.

2. Consider the function

$$f_Y(y) = \begin{cases} cye^{-2y}, \ y \geq 0 \\ 0, \ y < 0 \end{cases}$$

   a. Find the value of c that makes $f_Y(y)$ a valid probability density function.

   b. Derive the moment-generating function for Y.

   c. Compute the mean and variance for Y using the MGF.

3. Ace Heating and Air Conditioning Service finds that the amount of time a repairman needs to fix an air conditioning unit is uniformly distributed between 1.5 and four hours.

   a. Find the probability that a randomly selected AC unit repair requires more than two hours.

   b. The longest 25% of AC unit repair times take at least how long?

4. In China, four-year-olds average three hours a day unsupervised. Most of the unsupervised children live in rural areas, considered safe. Suppose that the standard deviation is 1.5 hours and the amount of time spent alone is normally distributed. We randomly select one Chinese four-year-old living in a rural area. We are interested in the amount of time the child spends alone per day.

   a. What percent of the children spend over ten hours per day unsupervised?

   b. Seventy percent of the children spend at least how long per day unsupervised?