

Stat 175 (Probability)

Problem Set No. 3

INSTRUCTIONS: Answer the following as indicated. Show detailed solutions.

1. Prove that if $Y \sim b(n, p)$, then its moment generating function is given by

$$m_Y(t) = (q + pe^t)^n, \text{ where } q = 1 - p$$

2. Using the MGF of a binomial distribution, show that the mean and variance of the distribution are, respectively,

$$E(Y) = np$$

and

$$V(Y) = np(1 - p)$$

3. Show that if Y has a geometric distribution with success probability p , then

$$m_Y(t) = \frac{pe^t}{1 - qe^t}.$$

4. Using the definition of expected value, show that if $Y \sim Pois(\lambda)$, then

$$E(Y) = V(Y) = \lambda$$

5. The telephone lines serving an airline reservation office are all busy about 60% of the time.

- a. If you are calling this office, what is the probability that you will complete your call on the first try?

- b. If you are calling this office, what is the probability that you will complete your call on the third try?
 - c. If you and a friend must both complete calls to this office, what is the probability that a total of four tries will be necessary for both of you to get through?
- 6. A salesperson has found that the probability of a sale on a single contact is approximately 0.03. The salesperson contacts 100 prospects.
 - a. What is the exact probability of making at least one sale?
 - b. What is the approximate probability of making at least one sale?
- 7. A small voting district has 1000 female and 4000 male voters. A random sample of 10 voters is selected.
 - a. Find the exact probability that there at most 7 males in the sample.
 - b. Find the approximate probability that there at most 7 males in the sample.