Stat 113 (Introduction to Mathematical Statistics)

Problem Set No. 4

INSTRUCTION: Answer the following as indicated. Show detailed solutions. If you express your answer in decimal form, retain at most 3 decimal places in your final answer.

1. Consider the function below.

   a. Verify that the function is a probability density function.

   b. Derive the moment generating function of Y.

   c. Use the MGF to compute the mean and variance of Y.

$$f_Y(y) = \begin{cases} 2e^{-2x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

2. The length of time to failure (in hundreds of hours) for a transistor is a random variable Y with cumulative distribution function given by

$$F_Y(y) = \begin{cases} 1 - e^{-y^2}, & y \geq 0 \\ 0, & y < 0 \end{cases}$$

   a. Find the pdf of Y.

   b. Find the probability that the transistor operates for at least 200 hours.

   c. Find $P(Y > 100 | Y \leq 200)$.