Stat 121 (Mathematical Statistics I)

Problem Set No. 5

Instruction: Answer the following as indicated.

1. The length of time required by students to complete a one-hour exam is a random variable (Y) with a density function given by

$$f_Y(y) = \begin{cases} ky^2 + y, \ 0 \leq y \leq 1 \\ 0, \ \text{elsewhere} \end{cases}$$

   a. Find k that would make $f_Y(y)$ a valid pdf.

   b. Find the cumulative distribution function $F_Y(y)$.

   c. Find the probability that a randomly selected student will finish in less than half an hour.

   d. Given that a particular student needs at least 15 minutes to complete the exam, find the probability that she will require at least 30 minutes to finish.

   e. Calculate the mean and variance of Y.

2. Consider the function

$$f_Y(y) = \begin{cases} kye^{-2y}, \ y \geq 0 \\ 0, \ \text{otherwise} \end{cases}$$

   a. Find the value of k that makes a $f_Y(y)$ a valid probability density function.

   b. Derive the moment-generating function for Y.

   c. Compute the mean and variance for Y using the MGF.