Stat 121 (Mathematical Statistics I)

Problem Set No. 2

2025-09-05

INSTRUCTION: Present neat and detailed solutions.

1. A student is allowed to pass a test on three tries. The probability that the student passes the test on the first attempt is 0.5, that he passes on the second attempt (of course, given that he failed on the first attempt) is 0.7, and the probability that he passes on the third attempt is 0.8 (given he failed on the first two attempts). What is the probability that he will pass the test?

2. Let A be the event that a family has children of both sexes and B be the event that a family has at most one boy.

   a) Show that A and B are independent if the family has three children.
   b) Show that A and B are not independent if the family has two children.

3. It is estimated that 50% of emails are spam emails. Some software has been applied to filter these spam emails before they reach your inbox. A certain brand of software claims that it can detect 99% of spam emails, and the probability for a false positive (a non-spam email detected as spam) is 5%. Now if an email is detected as spam, then what is the probability that it is in fact a non-spam email?

4. At the Visayas State University 35%, 25% and 40% of the students in STAT 21 are taught by Prof. X, Prof. Y and Prof. Z, respectively. The respective probabilities of failing in STAT 21 for the three professors are 6%, 7% and 5%. A student in STAT 21 in the previous semester was randomly selected and was found out to have failed in the subject. Under which class most likely did he belong? Explain your answer.

5. Prove that if $P(A|B) = P(A|B^c)$, then A and B are independent events.