

Stat 122 (Mathematical Statistics 2)

Problem Set No. 5

INSTRUCTIONS: Answer the following as indicated. Show detailed solutions.

1. Suppose that Y_1, Y_2, Y_3 denote a random sample from an exponential distribution with density function

$$f_Y(y) = \begin{cases} \frac{1}{\theta} e^{-\frac{y}{\theta}}, & y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Consider the following five estimators of θ :

$$\begin{aligned} \hat{\theta}_1 &= Y_1 \\ \hat{\theta}_2 &= \frac{Y_1 + Y_2}{2} \\ \hat{\theta}_3 &= \frac{Y_1 + 2Y_2}{3} \\ \hat{\theta}_4 &= \min(Y_1, Y_2, Y_3) \\ \hat{\theta}_5 &= \bar{Y} \end{aligned}$$

- a. Which of these estimators are unbiased for θ ?
 - b. Among the unbiased estimators, which has the smallest variance?
2. The reading on a voltage meter connected to a test circuit is uniformly distributed over the interval $(\theta, \theta + 1)$, where θ is the true but unknown voltage of the circuit. Suppose Y_1, Y_2, \dots, Y_n denote a random sample of such readings.
 - a. Show that \bar{Y} is a biased estimator of θ and compute its bias.
 - b. Find the MSE of \bar{Y} .
 - c. Find a function of \bar{Y} that is unbiased estimator for θ .