

Stat 131 (Mathematical Statistics III)

Problem Set No. 3

2025-09-30

INSTRUCTION: Present neat and detailed solutions.

1. Consider a test of the simple hypothesis $H_0 : \theta = \theta_0$ versus $H_1 : \theta = \theta_1$. Based on a random sample from a distribution with pmf $P(X = x|\theta)$ for $x = 1, 2, \dots, 7$. The values of the likelihood function at θ_0 and θ_1 are given in the table below.

x	1	2	3	4	5	6	7
$L(\theta_0)$	0.01	0.01	0.01	0.01	0.01	0.01	0.94
$L(\theta_1)$	0.06	0.05	0.04	0.03	0.02	0.01	0.79

- a. Use the Neyman-Pearson Lemma to find the most powerful test for testing H_0 against H_1 with significance level $\alpha = 0.05$.
 - b. Compute the power of the test. Do you think the test has enough power to reject a false null hypothesis? Explain.
2. A single positive random variable X has density function

$$f(x|\theta) = \begin{cases} \frac{\theta}{(\theta+x)^2}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

where $\theta > 0$ is an unknown parameter. Find the uniformly most powerful critical region of size $\alpha = 0.05$ for testing $H_0 : \theta = 2$ versus $H_1 : \theta > 2$.