# Stat 122 (Mathematical Statistics III)

Problem Set No. 1

2025-08-19

*INSTRUCTION*: Present neat and detailed solutions.

1. Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample from the uniform distribution on the interval $(\theta, \theta+1)$. Let

$$\hat{\theta}_1 = \overline{Y} - \frac{1}{2}$$

and

$$\hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}$$

a. Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators for $\theta$. [10 points]

b. Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$. [10 points]

2. Let $Y_1, Y_2, \cdots, Y_n$ denote a random sample from the uniform distribution over the interval $(0, \theta)$. Show using the Factorization Theorem that $Y_{(n)} = max(Y_1, Y_2, \cdots, Y_n)$ is sufficient for $\theta$. [10 points]