Stat 136 (Bayesian Statistics)

Second Semester AY 2024-2025

Laboratory Exercise No. 4

INSTRUCTIONS: Answer the following as indicated.

Let a random sample of 5 observations from a $N(25, \sigma^2)$ distribution be 26.05, 29.39, 23.58, 23.95, and 23.38.

a) What is the equation of the likelihood function of the variance .

b) Out prior belief is that the distribution of the standard deviation has median 4. Find the inverse chi-square prior distribution with 1 degree of freedom that fits our prior belief of the median.

c) Change the variable from the variance to standard deviation and find the prior distribution for the standard deviation.

d) Find the posterior distribution of the variance.

e) Compute Bayesian estimates (mean, median, and mode) of the standard deviation.

f) Test the hypothesis $H_0 : \sigma^2 = 6$ against $H_1 : \sigma^2 \neq 6$.

g) Test the hypothesis $H_0 : \sigma \geq 6$ against $H_1 : \sigma < 6$.