Stat 136 (Bayesian Statistics)

Second Semester AY 2024-2025

Laboratory Exercise No. 2

INSTRUCTIONS: Answer the following as indicated.

1. A city is considering building a new museum. The local paper wishes to determine the level of support for this project, and is going to conduct a poll of city residents. Out of the sample of 120 people, 74 support the city building the museum. Use a uniform prior for p, the proportion of the target audience that support the museum.

a. What is the posterior distribution of p?

b. Describe the posterior distribution based on the mean and statndard deviation.

2. Suppose Sophie, the editor of a student newspaper, is going to conduct a survey of students to determine the level of support for the current president of the students' organization. She needs to determine her prior distribution for p, the proportion of students who support the president. She decides her prior mean is 0.5, and her prior standard deviation is 0.15.

a. Show algebraically that $a = b = 5.06$.

b. Out of the 68 students that she polls, $y = 21$ support the current president. Determine posterior distribution using the $Beta(5.06, 5.06)$ prior.

c. Plot both the prior and posterior distributions.

d. Construct and interpret a 90% credible interval for p.

3. In a research program on human health risk from recreational contact with water contaminated with pathogenic microbiological material, the Department of Environment and Natural Resources (DENR) instituted a study to determine the quality of stream water at a variety of catchment types. One-liter water samples (n=116) werre obtained from sites identifed as having a heavy environmental impact from birds (seagulls) and waterfowl. Out of these samples, $y = 17$ samples contained *Giardia* cysts.

a. Suppose it is known that the 25^{th} and 75^{th} percentiles of a $Beta(a, b)$ prior are 0.393 and 0.607, respectively. Find a and b.

b. Find and plot the posterior distribution.

c. Determine the mean, median, and standard deviation of the posterior distribution.

d. If additional $m = 100$ water samples are going to be collected, how many of these will likely contain *Giardia* cysts?