Stat 136 (Bayesian Statistics)

Second Semester AY 2024-2025

Laboratory Exercise No. 3

INSTRUCTIONS: Answer the following as indicated.

1. You are the statistician responsible for quality standards at a cheese factory. You want the probability that a randomly chosen block of cheese labelled *1 kg* is actually less than 1 kilogram to be 1% or less. The distribution of the weight (in grams) of blocks of cheese produced by the machine is $N(\mu, 9)$. The weights (in grams) of a random sample of 20 blocks of cheese are: 994, 997, 999, 1003, 994, 998, 1001, 998, 996, 1002, 1004, 995, 994, 995, 998, 1001, 995, 1006, 997, 998.

You decide to use a discrete prior distribution for μ with the following probabilities:

$$f(\mu) = \begin{cases} 0.05, \text{for } \mu \in \{991, 992, \cdots, 1010\} \\ 0, \text{otherwise} \end{cases}$$

a. Calculate your posterior probability distribution.
b. Calculate your posterior probability that ?? < 1000.
c. Should you adjust the machine? Why or why not.

2. A medical researcher collected the systolic blood pressure reading for a random sample of 30 female students under the age of 21 who visited the Student's Health Service. The blood pressures are:120, 122, 121, 108, 133, 119, 136, 108, 106, 105, 122, 139, 133, 115, 104, 94, 118, 93, 102, 114, 123, 125, 124, 108, 111, 134, 107, 112, 109, 125.

Assume that systolic blood pressure comes from a $N(\mu, \sigma^2)$ distribution, where the σ^2 is unknown.

a. Use a $N(120, 152)$ prior for μ. Calculate the posterior distribution of μ.
b. Find a 95% Bayesian credible interval for μ.
c. Based on the answer in (b), can we reject at the 5% level of significance $H_0 : \mu = 135$ in favor of $H_1 : \mu \neq 135$? Why or why not?