Stat 136 (Bayesian Statistics)

Laboratory Exercise 3

1. A medical researcher collected the systolic blood pressure reading for a random sample of 30 female students under the age of 21 who visited the Student's Health Service. The blood pressures are:120, 122, 121, 108, 133, 119, 136, 108, 106, 105, 122, 139, 133, 115, 104, 94, 118, 93, 102, 114, 123, 125, 124, 108, 111, 134, 107, 112, 109, 125.

Assume that systolic blood pressure comes from a $N(\mu, \sigma^2)$ distribution, where the σ^2 is unknown. Use a $N(120, 152)$ prior for μ.

a. Calculate the posterior distribution of μ.

b. Find and interpret a 95% Bayesian credible interval for μ.

c. Based on the answer in (b), can we reject at the 5% level of significance $H_0 : \mu = 135$ in favor of $H_1 : \mu \neq 135$? Why or why not?

2. A School District Supervisor is interested to verify the claim that female school heads/principals are better at managerial skills than their male counterparts. He obtained data on management skills of random samples of 13 male and 12 female school heads from four school districts using an adapted management skills inventory. The data is presented below.

Female	75	65	55	80	67	65	67	71	79	59	63	69	
Male	66	61	60	56	78	63	59	68	77	62	50	49	57

a. Suppose these samples were drawn randomly from independent populations of female and male school heads and that these populations are $N(\mu_f, \sigma_f^2)$ and $N(\mu_m, \sigma_m^2)$, where $\sigma_f^2 \neq \sigma_m^2$ and both are unknown. Use the following as prior distributions of μ_f and μ_m, respectively, $N(70, 49)$ and $N(60, 64)$. Find the posterior distributions of μ_f and μ_m.

b. Find the posterior distribution of $\mu_d = \mu_f - \mu_m$

c. Construct a 95% credible interval for μ_d.

d. Test the hypothesis that females have superior managerial skills than males. Use a 5% level of significance.

3. With the COVID-19 pandemic, public school teachers in the country were busy preparing and printing course modules for distribution to their pupils. One of the important parts of the module is a pretest and a posttest. The pretest and posttest scores of a class of 15 pupils in Mathematics are obtained from the class record of a teacher. The data are shown below.

Student	Pre test	Post test
1	18	22
2	21	25
3	16	17
4	22	24
5	19	16
6	24	29
7	17	20
8	21	23
9	23	19
10	18	20
11	14	15
12	16	15
13	16	18
14	19	26
15	18	18

a. Assume that the pairwise differences are distributed as $N(\mu_d, \sigma_d^2)$, with σ_d^2 unknown. Using a $N(-3, 4)$ prior for μ_d, find the posterior distribution for μ_d.

b. At the 5% level of significance, test the hypothesis that the module is effective, that is, the mean post test score is greater than the mean pre test score.