Split Plot Experiments

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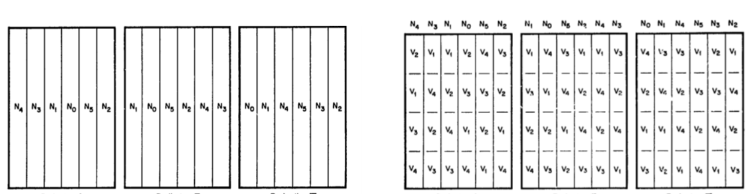
Basic ideas

- a two-factor experiment wherein levels of one of the factors require large plot size for execution and also show large differences in their effects
- the experiment will consist of a set of large plots called *main plots* in which levels for the main plot factor are assigned
- each main plot is divided into *subplots* to which the second factor, called the subplot factor, are assigned
- the precision for measuring the effects of the main-plot factor is sacrificed to improve that of the subplot factor
- the more important factor of which a higher precision is desired must be used as the sub-plot factor
- involves two-stage randomization process
- in the first stage the levels of factor A are randomized over the main plots and in the second stage the levels of factor B are randomized over the subplots within each main plot
- this randomization scheme results in two distinct error terms, one appropriate for the main plots and another for the subplots
- the error term for the main plots is expected to be larger than the subplot error which implies that a higher precision is expected in the subplots than in the main plots

Uses of split-plot experiments

- When the level of one factor requires larger experimental units than the other factor
- When greater precision is desired for comparisons among levels of one factor than that desired for another factor
- When it is known that larger variation can be expected among the levels of one factor than those expected on levels of the other factor
- When an additional factor is added to the experiment to increase the scope of the experiment

Example layout: Split Plot in RCBD



Linear model (CRD)

$$Y_{ijk} = \mu + \alpha_i + \delta_{ij} + \beta_k + (\alpha\beta)_{ik} + \epsilon_{ijk}$$

where:

- Y_{ijk} = observation from subplot given the k^{th} level of B in the j^{th} whole plot given the i^{th} level of A

- μ = overall mean
- α_i = effect of the i^{th} level of A
- δ_{ij} = random error associated with the j^{th} whole plot given the i^{th} level of A
- β_k = effect of the k^{th} level of B
- $(\alpha\beta)_{ik}$ = interaction effect between the i^{th} level of A and the k^{th} level of B
- ϵ_{ijk} = random error associated with the subplot given the k^{th} level of B in the j^{th} whole plot given the i^{th} level of A

Sources of variation	Sums of Squares	df	Mean Squares	F	p-value
Main plot factor (A)					
Error (A)					
Sub-plot factor (B)					
AxB					
Error (B)					
TOTAL					

Test of hypothesis

- The test procedure for split plot experiments is similar to factorial experiments

  1. Test of interaction effect
  2. Test of main effects, if interaction effect is not significant; to be followed with post analysis if main effects are significant
  3. Post hoc analysis for simple effects (effect of A at each level of B, or vice versa), if interaction effect is significant

- Test statistic:

  - F ratio for the effect of the main plot factor (A) is $F_A = \frac{MS_A}{MS_{A:Rep}}$

Split plot experiment in CRD: an example

An experiment is conducted to evaluate effects of grass species (G1 and G2) and stocking density (20 and 24) on the daily gain of sheeps kept on a pasture. Six 1-hactare pastures were planted with the two species of grass. Each grass species is randomly assigned to three pastures (replicates). Then each pasture is split into two where different numbers of sheeps (20 and 24) is randomly assigned. At the end of the experiment the daily gains in weight were determined.

Pasture	1	2	3	4	5	6
	G1	G2	G2	G1	G1	G2
	D2	D1	D2	D2	D1	D1
	D1	D2	D1	D1	D2	D2
	Rep1		Rep2		Rep3	

```r
grassden <- read.csv("GrassStockingDensity.csv")
head(grassden)
```

```
##   Rep Grass Density weight
## 1  R1    G1      D2    310
## 2  R1    G1      D1    290
## 3  R1    G2      D1    310
## 4  R1    G2      D2    330
## 5  R2    G2      D2    400
## 6  R2    G2      D1    380
```

```r
spcrd.out <- aov(weight ~ Grass + Grass:Rep + Density + Grass:Density,
                 data = grassden)
anova(spcrd.out)
```

```
## Analysis of Variance Table
##
## Response: weight
##               Df Sum Sq Mean Sq F value    Pr(>F)
## Grass          1 7500.0  7500.0   112.5 0.0004472 ***
## Density        1 1200.0  1200.0    18.0 0.0132356 *
## Grass:Rep      4 8266.7  2066.7    31.0 0.0028687 **
## Grass:Density  1   33.3    33.3     0.5 0.5185185
## Residuals      4  266.7    66.7
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- We need to correct the F ratio for *Grass*
- The revised/corrected ANOVA table is given below.

SoV	df	SS	MS	F	p
Grass (A)	1	7500.0	7500.0	3.63	0.1295
Error A	4	8266.7	2066.7		
Density (B)	1	1200.0	1200.0	18.0	0.0132
A*B	1	33.3	33.3	0.5	0.5185
Residual or Error B	4	266.7	66.7		

- There is no interaction effect between grass species and stocking density.
- Gain in weight is not significantly affected by grass species ($p > 0.05$)
- Gain in weight is significantly affected by stocking density ($p < 0.05$)

```r
anova(spcrd.out)["Df"]
```

```
##                Df
## Grass           1
## Density         1
## Grass:Rep       4
## Grass:Density   1
## Residuals       4
```

```r
anova(spcrd.out)["Mean Sq"]
```

```
##                Mean Sq
## Grass           7500.0
## Density         1200.0
## Grass:Rep       2066.7
## Grass:Density     33.3
## Residuals         66.7
```

```r
HSD.test(spcrd.out,
         trt = "Density",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: spcrd.out ~ "Density"
##
## HSD Test for weight
##
## Mean Square Error:  66.66667
##
## Density,  means
##
##      weight      std r       se Min Max   Q25 Q50   Q75
## D1 333.3333 37.77124 6 3.333333 290 380 312.5 320 365.0
## D2 353.3333 42.26898 6 3.333333 310 410 322.5 340 387.5
##
## Alpha: 0.05 ; DF Error: 4
## Critical Value of Studentized Range: 3.926503
##
## Minimun Significant Difference: 13.08834
##
## Treatments with the same letter are not significantly different.
##
##      weight groups
## D2 353.3333      a
## D1 333.3333      b
```

Linear model (RCBD)

$$Y_{ijk} = \mu + \alpha_i + \rho_j + \delta_{ij} + \beta_k + (\alpha\beta)_{ik} + \epsilon_{ijk}$$

where:

- Y_{ijk} = observation from subplot given the k^{th} level of B in the j^{th} whole plot given the i^{th} level of A
- μ = overall mean
- α_i = effect of the i^{th} level of A
- ρ_j = effect of the j^{th} block
- δ_{ij} = random error associated with the i^{th} level of A in the j^{th} block
- β_k = effect of the k^{th} level of B
- $(\alpha\beta)_{ik}$ = interaction effect between the i^{th} level of A and the k^{th} level of B
- ϵ_{ijk} = random error associated with the subplot given the k^{th} level of B in the j^{th} whole plot given the i^{th} level of A

Sources of variation	Sums of Squares	df	Mean Squares	F	p-value
Block					
Main plot factor (A)					
Error (A)					
Sub-plot factor (B)					
AxB					
Error (B)					
TOTAL					

Test of hypothesis

- The test procedure for split plot experiments is similar to factorial experiment
- Test statistic for block effect: $F_{Block} = \frac{MS_{Block}}{MS_{A:Block}}$
- Test statistic for main plot effect: $F_A = \frac{MS_A}{MS_{A:Block}}$

Split plot experiment in RCBD: an example

An experiment was conducted in order to investigate four different treatments of pasture and two mineral supplements on milk yield. The experiment was designed as a split-plot, with pasture treatments (factor A) assigned to the main plots and mineral supplements (factor B) assigned to sub-plots. The experiment was replicated in three blocks.

- Let us load the data and convert the variables with numeric codes to factors.

```
supp <- read.csv("mineral_supplement.csv")
supp$Block <- factor(supp$block)
supp$Pasture <- factor(supp$Pasture)
supp$Mineral <- factor(supp$Mineral)
```

```
sprbd.out <- aov(Milk.Yield ~ Block + Pasture + Block:Pasture + Mineral + Pasture:Mineral, data = supp)

anova(sprbd.out)
```

```
## Analysis of Variance Table
##
## Response: Milk.Yield
##                 Df  Sum Sq Mean Sq F value    Pr(>F)
## Block            2 212.583 106.292 47.2407 3.713e-05 ***
## Pasture          3  71.167  23.722 10.5432  0.003742 **
## Mineral          1   8.167   8.167  3.6296  0.093224 .
## Block:Pasture    6  26.083   4.347  1.9321  0.190897
## Pasture:Mineral  3   5.833   1.944  0.8642  0.498130
## Residuals        8  18.000   2.250
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- We need to recompute the F ratio for *Block*:

$$\begin{aligned} F_{Block} &= \frac{MS_{Block}}{MS_{A:Block}} \\ &= \frac{106.292}{4.347} \\ &= 24.45 \\ p - value &= 0.0013 \end{aligned} \tag{1}$$

- Likewise, we need to recompute the F ratio for *Pasture*:

$$\begin{aligned} F_{Pasture} &= \frac{MS_{Pasture}}{MS_{A:Block}} \\ &= \frac{23.722}{4.347} \\ &= 5.46 \\ p - value &= 0.0377 \end{aligned} \tag{2}$$

- The revised/corrected ANOVA table is given below.

SoV	df	SS	MS	F	p
Block	2	212.583	106.292	24.45	0.0013
Pasture (A)	3	71.167	23.722	5.46	0.0377
Error A	6	26.083	4.347		
Mineral (B)	1	8.167	8.167	3.6296	0.0932
A*B	3	5.833	1.944	0.8642	0.4981
Residual or Error B	8	18.000	2.250		

- There is significant block effect ($p < 0.01$) $\Rightarrow$ blocking strategy is successful
- There is no significant interaction effect of Pasture and Mineral ($p > 0.05$)
- Mineral supplement has no significant effect on milk yield ($p > 0.05$)
- Pasture has significant effect on milk yield ($p < 0.05$)

- We perform post hoc analysis on pasture means

```r
#Modified HSD.test()
hsd.test(sprbd.out,
         trt = "Pasture",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: sprbd.out ~ "Pasture"
##
## HSD Test for Milk.Yield
##
## Mean Square Error:  4.347
##
## Pasture,  means
##
##   Milk.Yield      std r Min Max
## 1   29.66667 3.204164 6  25  34
## 2   30.66667 3.777124 6  26  37
## 3   30.00000 3.949684 6  24  33
## 4   34.00000 3.741657 6  29  38
##
## Alpha: 0.05 ; DF Error: 6
## Critical Value of Studentized Range: 4.895599
##
## Minimun Significant Difference: 4.167015
##
## Treatments with the same letter are not significantly different.
##
##   Milk.Yield groups
## 4   34.00000      a
## 2   30.66667     ab
## 3   30.00000     ab
## 1   29.66667      b
```

Analysis of Split-Plot Experiments using the ExpDes package

Split-plot in CRD

```r
library(ExpDes)

with(grassden, split2.crd(factor1 = Grass,
                          factor2 = Density,
                          repet = Rep,
                          resp = weight,
                          quali = c(T, T),
                          fac.names = c("Grass", "Density"),
                          mcomp = "tukey"))
```

```
## ------------------------------------------------------------------------
```

```
## Legend:
## FACTOR 1    (plot):  Grass
## FACTOR 2 (split-plot):  Density
## ------------------------------------------------------------------------
##
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
##                DF      SS     MS     Fc   Pr>Fc
## Grass           1  7500.0 7500.0  3.629 0.12949
## Error a         4  8266.7 2066.7
## Density         1  1200.0 1200.0 18.000 0.01324
## Grass*Density   1    33.3   33.3  0.500 0.51852
## Error b         4   266.7   66.7
## Total          11 17266.7
## ------------------------------------------------------------------------
## CV 1 = 13.24095 %
## CV 2 = 2.378145 %
##
## ------------------------------------------------------------------------
## Shapiro-Wilk normality test (Error b)
## p-value:  0.8805033
## According to Shapiro-Wilk normality test at 5% of significance, residuals can be considered normal.
## ------------------------------------------------------------------------
##
## No significant interaction: analyzing the main effects
## ------------------------------------------------------------------------
## Grass
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels    Means
## 1     G1 318.3333
## 2     G2 368.3333
## ------------------------------------------------------------------------
## Density
## Tukey's test
## ------------------------------------------------------------------------
## Groups Treatments Means
## a     D2      353.3333
##  b    D1      333.3333
## ------------------------------------------------------------------------
```

Split-plot in RCBD

```r
with(supp, split2.rbd(factor1 = Pasture,
                      factor2 = Mineral,
                      block = Block,
                      resp = Milk.Yield,
                      fac.names = c("Pasture","Mineral"),
                      quali = c(T, T),
                      mcomp = "tukey"))
```

```
## ------------------------------------------------------------------------
## Legend:
## FACTOR 1 (plot):  Pasture
## FACTOR 2 (split-plot):  Mineral
## ------------------------------------------------------------------------
##
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
##                  DF     SS      MS      Fc  Pr(>Fc)
## Pasture           3  71.17  23.722  5.4569 0.037703 *
## Block             2 212.58 106.292 24.4505 0.001305 **
## Error a           6  26.08   4.347
## Mineral           1   8.17   8.167  3.6296 0.093224 .
## Pasture*Mineral  3   5.83   1.944  0.8642 0.498130
## Error b           8  18.00   2.250
## Total            23 341.83
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## ------------------------------------------------------------------------
## CV 1 = 6.707773 %
## CV 2 = 4.825737 %
##
## No significant interaction: analyzing the simple effects
## ------------------------------------------------------------------------
## Pasture
## Tukey's test
## ------------------------------------------------------------------------
## Groups Treatments Means
## a     4   34
## ab    2   30.66667
## ab    3   30
##  b    1   29.66667
## ------------------------------------------------------------------------
##
## Mineral
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels    Means
## 1      1 30.50000
## 2      2 31.66667
## ------------------------------------------------------------------------
##
##
##
## Significant interaction: analyzing the interaction
## ------------------------------------------------------------------------
##
## Analyzing  Pasture  inside of each level of  Mineral
## ------------------------------------------------------------------------
##                            DF       SS        MS       Fc  p.value
## Pasture : Mineral 1   3.00000 41.66667 13.888889 4.210526 0.031213
## Pasture : Mineral 2   3.00000 35.33333 11.777778 3.570526 0.048756
## Pooled Error         11.50639 37.95511  3.298611       NA       NA
```

```
## ------------------------------------------------------------------------
##
##
##  Pasture inside of Mineral 1
## ------------------------------------------------------------------------
## Tukey's test
## ------------------------------------------------------------------------
## Groups Treatments Means
## a     4   33.66667
## ab    3   30
## ab    2   29.33333
##  b    1   29
## ------------------------------------------------------------------------
##
##  Pasture inside of Mineral 2
## ------------------------------------------------------------------------
## Tukey's test
## ------------------------------------------------------------------------
## Groups Treatments Means
## a     4   34.33333
## a     2   32
## a     1   30.33333
## a     3   30
## ------------------------------------------------------------------------
##
##
## Analyzing  Mineral  inside of each level of  Pasture
## ------------------------------------------------------------------------
##                      DF        SS        MS       Fc  p.value
## Mineral : Pasture 1   1  2.666667  2.666667 1.185185 0.308008
## Mineral : Pasture 2   1 10.666667 10.666667 4.740741 0.061117
## Mineral : Pasture 3   1  0.000000  0.000000 0.000000 1.000000
## Mineral : Pasture 4   1  0.666667  0.666667 0.296296 0.601052
## Error b               8 18.000000  2.250000       NA       NA
## ------------------------------------------------------------------------
##
##
##  Mineral inside of Pasture 1
## ------------------------------------------------------------------------
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels    Means
## 1      1 29.00000
## 2      2 30.33333
## ------------------------------------------------------------------------
##
##  Mineral inside of Pasture 2
## ------------------------------------------------------------------------
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels    Means
## 1      1 29.33333
## 2      2 32.00000
## ------------------------------------------------------------------------
```

```
##
##  Mineral inside of Pasture 3
## ------------------------------------------------------------------------
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels Means
## 1      1    30
## 2      2    30
## ------------------------------------------------------------------------
##
##  Mineral inside of Pasture 4
## ------------------------------------------------------------------------
## According to F test, the means of this factor are not different.
## ------------------------------------------------------------------------
##   Levels    Means
## 1      1 33.66667
## 2      2 34.33333
## ------------------------------------------------------------------------
```