Multiple Comparison Procedures

Demo Using R/RStudio

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Import data

```
chap4demo <- read_excel("chap4demo.xlsx")

chap4demo$Trt <- factor(chap4demo$fertilizer)

head(chap4demo)
```

```
## # A tibble: 6 x 4
##   trt   fertilizer yield Trt
##   <chr>      <dbl> <dbl> <fct>
## 1 T1             0  4.89 0
## 2 T1             0  4.79 0
## 3 T1             0  4.65 0
## 4 T1             0  4.47 0
## 5 T2            50  5.08 50
## 6 T2            50  5.19 50
```

Run ANOVA

```
aov1 <- with(chap4demo, aov(yield ~ Trt))

anova(aov1)
```

```
## Analysis of Variance Table
##
## Response: yield
##           Df Sum Sq  Mean Sq F value   Pr(>F)
## Trt        5 1.3555 0.271107  19.567 1.04e-06 ***
## Residuals 18 0.2494 0.013856
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Pairwise comparison using agricolae package

LSD test

```
LSD.test(y = aov1,
         trt = "Trt",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: aov1 ~ "Trt"
##
## LSD t Test for yield
##
## Mean Square Error:  0.01385556
##
## Trt,  means and individual ( 95 %) CI
##
##     yield        std r         se      LCL      UCL  Min  Max    Q25   Q50
## 0    4.70 0.18220867 4 0.05885481 4.576351 4.823649 4.47 4.89 4.6050 4.720
## 100  5.23 0.03559026 4 0.05885481 5.106351 5.353649 5.18 5.26 5.2175 5.240
## 150  5.38 0.06164414 4 0.05885481 5.256351 5.503649 5.31 5.46 5.3550 5.375
## 200  5.36 0.13190906 4 0.05885481 5.236351 5.483649 5.25 5.55 5.2875 5.320
## 250  5.28 0.08755950 4 0.05885481 5.156351 5.403649 5.21 5.40 5.2175 5.255
## 50   5.02 0.14071247 4 0.05885481 4.896351 5.143649 4.89 5.19 4.9125 5.000
##        Q75
## 0   4.8150
## 100 5.2525
## 150 5.4000
## 200 5.3925
## 250 5.3175
## 50  5.1075
##
## Alpha: 0.05 ; DF Error: 18
## Critical Value of t: 2.100922
##
## least Significant Difference: 0.1748666
##
## Treatments with the same letter are not significantly different.
##
##     yield groups
## 150  5.38      a
## 200  5.36      a
## 250  5.28      a
## 100  5.23      a
## 50   5.02      b
## 0    4.70      c
```

Scheffe test

```r
scheffe.test(y = aov1,
         trt = "Trt",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: aov1 ~ "Trt"
##
## Scheffe Test for yield
##
## Mean Square Error  : 0.01385556
##
## Trt,  means
##
##     yield        std r         se  Min  Max    Q25   Q50    Q75
## 0    4.70 0.18220867 4 0.05885481 4.47 4.89 4.6050 4.720 4.8150
## 100  5.23 0.03559026 4 0.05885481 5.18 5.26 5.2175 5.240 5.2525
## 150  5.38 0.06164414 4 0.05885481 5.31 5.46 5.3550 5.375 5.4000
## 200  5.36 0.13190906 4 0.05885481 5.25 5.55 5.2875 5.320 5.3925
## 250  5.28 0.08755950 4 0.05885481 5.21 5.40 5.2175 5.255 5.3175
## 50   5.02 0.14071247 4 0.05885481 4.89 5.19 4.9125 5.000 5.1075
##
## Alpha: 0.05 ; DF Error: 18
## Critical Value of F: 2.772853
##
## Minimum Significant Difference: 0.309917
##
## Means with the same letter are not significantly different.
##
##     yield groups
## 150  5.38      a
## 200  5.36      a
## 250  5.28     ab
## 100  5.23     ab
## 50   5.02      b
## 0    4.70      c
```

Tukey test

```r
HSD.test(y = aov1,
         trt = "Trt",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: aov1 ~ "Trt"
##
## HSD Test for yield
##
## Mean Square Error:  0.01385556
##
```

```
## Trt,  means
##
##     yield        std r          se  Min  Max    Q25   Q50    Q75
## 0    4.70 0.18220867 4 0.05885481 4.47 4.89 4.6050 4.720 4.8150
## 100  5.23 0.03559026 4 0.05885481 5.18 5.26 5.2175 5.240 5.2525
## 150  5.38 0.06164414 4 0.05885481 5.31 5.46 5.3550 5.375 5.4000
## 200  5.36 0.13190906 4 0.05885481 5.25 5.55 5.2875 5.320 5.3925
## 250  5.28 0.08755950 4 0.05885481 5.21 5.40 5.2175 5.255 5.3175
## 50   5.02 0.14071247 4 0.05885481 4.89 5.19 4.9125 5.000 5.1075
##
## Alpha: 0.05 ; DF Error: 18
## Critical Value of Studentized Range: 4.49442
##
## Minimun Significant Difference: 0.2645182
##
## Treatments with the same letter are not significantly different.
##
##     yield groups
## 150  5.38      a
## 200  5.36      a
## 250  5.28     ab
## 100  5.23     ab
## 50   5.02      b
## 0    4.70      c
```

SNK test

```
SNK.test(y = aov1,
         trt = "Trt",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: aov1 ~ "Trt"
##
## Student Newman Keuls Test
## for yield
##
## Mean Square Error:  0.01385556
##
## Trt,  means
##
##     yield        std r          se  Min  Max    Q25   Q50    Q75
## 0    4.70 0.18220867 4 0.05885481 4.47 4.89 4.6050 4.720 4.8150
## 100  5.23 0.03559026 4 0.05885481 5.18 5.26 5.2175 5.240 5.2525
## 150  5.38 0.06164414 4 0.05885481 5.31 5.46 5.3550 5.375 5.4000
## 200  5.36 0.13190906 4 0.05885481 5.25 5.55 5.2875 5.320 5.3925
## 250  5.28 0.08755950 4 0.05885481 5.21 5.40 5.2175 5.255 5.3175
## 50   5.02 0.14071247 4 0.05885481 4.89 5.19 4.9125 5.000 5.1075
##
## Alpha: 0.05 ; DF Error: 18
```

```
##
## Critical Range
##         2         3         4         5         6
## 0.1748666 0.2124249 0.2352414 0.2516804 0.2645182
##
## Means with the same letter are not significantly different.
##
##     yield groups
## 150  5.38      a
## 200  5.36      a
## 250  5.28      a
## 100  5.23      a
## 50   5.02      b
## 0    4.70      c
```

DMRT

```
duncan.test(y = aov1,
         trt = "Trt",
         group = TRUE,
         console = TRUE)
```

```
##
## Study: aov1 ~ "Trt"
##
## Duncan's new multiple range test
## for yield
##
## Mean Square Error:  0.01385556
##
## Trt,  means
##
##     yield        std r         se  Min  Max    Q25   Q50    Q75
## 0    4.70 0.18220867 4 0.05885481 4.47 4.89 4.6050 4.720 4.8150
## 100  5.23 0.03559026 4 0.05885481 5.18 5.26 5.2175 5.240 5.2525
## 150  5.38 0.06164414 4 0.05885481 5.31 5.46 5.3550 5.375 5.4000
## 200  5.36 0.13190906 4 0.05885481 5.25 5.55 5.2875 5.320 5.3925
## 250  5.28 0.08755950 4 0.05885481 5.21 5.40 5.2175 5.255 5.3175
## 50   5.02 0.14071247 4 0.05885481 4.89 5.19 4.9125 5.000 5.1075
##
## Alpha: 0.05 ; DF Error: 18
##
## Critical Range
##         2         3         4         5         6
## 0.1748666 0.1834731 0.1889036 0.1926667 0.1954172
##
## Means with the same letter are not significantly different.
##
##     yield groups
## 150  5.38      a
## 200  5.36      a
```

```
## 250  5.28      a
## 100  5.23      a
## 50   5.02      b
## 0    4.70      c
```

Pairwise comparison using ExpDes package

LSD test

```
with(chap4demo, crd(treat = Trt,
                    resp = yield,
                    quali = TRUE,
                    mcomp = "lsd"))
```

```
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
##            DF     SS       MS     Fc      Pr>Fc
## Treatament  5 1.3555 0.271107 19.567 1.0399e-06
## Residuals  18 0.2494 0.013856
## Total      23 1.6049
## ------------------------------------------------------------------------
## CV = 2.28 %
##
## ------------------------------------------------------------------------
## Shapiro-Wilk normality test
## p-value:  0.6315903
## According to Shapiro-Wilk normality test at 5% of significance, residuals can be considered normal.
## ------------------------------------------------------------------------
##
## ------------------------------------------------------------------------
## Homogeneity of variances test
## p-value:  0.1868725
## According to the test of bartlett at 5% of significance, residuals can be considered homocedastic.
## ------------------------------------------------------------------------
##
## T test (LSD)
## ------------------------------------------------------------------------
## Groups  Treatments  Means
## a     150     5.38
## a     200     5.36
## a     250     5.28
## a     100     5.23
##  b    50      5.02
##   c   0   4.7
## ------------------------------------------------------------------------
```

Tukey test

```r
with(chap4demo, crd(treat = Trt,
                    resp = yield,
                    quali = TRUE,
                    mcomp = "tukey"))
```

```
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
##            DF     SS       MS     Fc      Pr>Fc
## Treatament  5 1.3555 0.271107 19.567 1.0399e-06
## Residuals  18 0.2494 0.013856
## Total      23 1.6049
## ------------------------------------------------------------------------
## CV = 2.28 %
##
## ------------------------------------------------------------------------
## Shapiro-Wilk normality test
## p-value:  0.6315903
## According to Shapiro-Wilk normality test at 5% of significance, residuals can be considered normal.
## ------------------------------------------------------------------------
##
## ------------------------------------------------------------------------
## Homogeneity of variances test
## p-value:  0.1868725
## According to the test of bartlett at 5% of significance, residuals can be considered homocedastic.
## ------------------------------------------------------------------------
##
## Tukey's test
## ------------------------------------------------------------------------
## Groups Treatments Means
## a     150     5.38
## a     200     5.36
## ab    250     5.28
## ab    100     5.23
##  b    50      5.02
##   c   0   4.7
## ------------------------------------------------------------------------
```

SNK test

```r
with(chap4demo, crd(treat = Trt,
                    resp = yield,
                    quali = TRUE,
                    mcomp = "snk"))
```

```
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
```

```
##            DF     SS       MS     Fc      Pr>Fc
## Treatament  5 1.3555 0.271107 19.567 1.0399e-06
## Residuals  18 0.2494 0.013856
## Total      23 1.6049
## ------------------------------------------------------------------------
## CV = 2.28 %
##
## ------------------------------------------------------------------------
## Shapiro-Wilk normality test
## p-value:  0.6315903
## According to Shapiro-Wilk normality test at 5% of significance, residuals can be considered normal.
## ------------------------------------------------------------------------
##
## ------------------------------------------------------------------------
## Homogeneity of variances test
## p-value:  0.1868725
## According to the test of bartlett at 5% of significance, residuals can be considered homocedastic.
## ------------------------------------------------------------------------
##
## Student-Newman-Keuls's test (SNK)
## ------------------------------------------------------------------------
## Groups  Treatments  Means
## a     150         5.38
## a     200         5.36
## a     250         5.28
## a     100         5.23
##  b    50          5.02
##   c   0       4.7
## ------------------------------------------------------------------------
```

DMRT

```
with(chap4demo, crd(treat = Trt,
                    resp = yield,
                    quali = TRUE,
                    mcomp = "duncan"))
```

```
## ------------------------------------------------------------------------
## Analysis of Variance Table
## ------------------------------------------------------------------------
##            DF     SS       MS     Fc      Pr>Fc
## Treatament  5 1.3555 0.271107 19.567 1.0399e-06
## Residuals  18 0.2494 0.013856
## Total      23 1.6049
## ------------------------------------------------------------------------
## CV = 2.28 %
##
## ------------------------------------------------------------------------
## Shapiro-Wilk normality test
## p-value:  0.6315903
## According to Shapiro-Wilk normality test at 5% of significance, residuals can be considered normal.
```

```
## ---------------------------------------------------------------------------
##
## ---------------------------------------------------------------------------
## Homogeneity of variances test
## p-value:  0.1868725
## According to the test of bartlett at 5% of significance, residuals can be considered homocedastic.
## ---------------------------------------------------------------------------
##
## Duncan's test
## ---------------------------------------------------------------------------
## Groups  Treatments  Means
## a      150         5.38
## a      200         5.36
## a      250         5.28
## a      100         5.23
##  b     50          5.02
##   c    0       4.7
## ---------------------------------------------------------------------------
```

Group and trend comparisons

Suppose we are interested in testing the following comparisons:

- Control (T1) vs Treated (T2 thru T6)
- (T2, T3) versus (T4, T5, T6)
- T2 versus T3
- T4 versus (T5, T6)
- T5 versus T6

QUESTIONS????

1. What are the coefficients of each comparison?
2. Are the comparisons linear contrasts?
3. Is the above set of comparison orthogonal?

Test of significance of the first comparison

```
c1 <- rbind("Control (T1) vs Treated (T2 thru T6)" = c(5, -1, -1, -1, -1, -1))
summary(glht(aov1,linfct=mcp(Trt = c1)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
```

```
##
## Linear Hypotheses:
##                                            Estimate Std. Error t value Pr(>|t|)
## Control (T1) vs Treated (T2 thru T6) == 0  -2.7700     0.3224  -8.593 8.73e-08
##
## Control (T1) vs Treated (T2 thru T6) == 0 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

Test of significance of the 2nd comparison

```
c2 <- rbind("(T2,T3) vs (T4, T5, T6)" = c(0, 3, 3, -2, -2, -2))
summary(glht(aov1,linfct=mcp(Trt = c2)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
##                               Estimate Std. Error t value Pr(>|t|)
## (T2,T3) vs (T4, T5, T6) == 0  -1.2900     0.3224  -4.002 0.000837 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

Simultaneous test of significance of the set of contrasts

```
contrast <- rbind("Control (T1) vs Treated (T2 thru T6)" = c(5, -1, -1, -1, -1, -1),
                  "(T2,T3) vs (T4, T5, T6)" = c(0, 3, 3, -2, -2, -2),
                  "T2 vs T3" = c(0, 1, -1, 0, 0, 0),
                  "T4 vs (T5, T6)" = c(0, 0, 0, 2, -1, -1),
                  "T5 vs T6" = c(0, 0, 0, 0, 1, -1))

summary(glht(aov1,linfct=mcp(Trt = contrast)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
```

```
##                                            Estimate Std. Error t value Pr(>|t|)
## Control (T1) vs Treated (T2 thru T6) == 0 -2.77000    0.32236  -8.593  < 0.001
## (T2,T3) vs (T4, T5, T6) == 0              -1.29000    0.32236  -4.002  0.00409
## T2 vs T3 == 0                             -0.21000    0.08323  -2.523  0.09678
## T4 vs (T5, T6) == 0                        0.12000    0.14416   0.832  0.92003
## T5 vs T6 == 0                              0.08000    0.08323   0.961  0.86565
##
## Control (T1) vs Treated (T2 thru T6) == 0 ***
## (T2,T3) vs (T4, T5, T6) == 0              **
## T2 vs T3 == 0                             .
## T4 vs (T5, T6) == 0
## T5 vs T6 == 0
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

Test of significance of a linear trend

```
line <- rbind("Linear trend" = c(-5, -3, -1, 1, 3, 5))
summary(glht(aov1,linfct=mcp(Trt = line)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
##                     Estimate Std. Error t value Pr(>|t|)
## Linear trend == 0   4.0700     0.4924   8.265 1.53e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

```
library(tidyverse)
chap4demo %>%
  group_by(Trt) %>%
  summarize(M=mean(yield))
```

```
## # A tibble: 6 x 2
##   Trt       M
##   <fct> <dbl>
## 1 0      4.7
## 2 50     5.02
## 3 100    5.23
## 4 150    5.38
## 5 200    5.36
## 6 250    5.28
```

Test of significance of a quadratic trend

```
quad <- rbind("Quadratic trend" = c(5, -1, -4, -4, -1, 5))
summary(glht(aov1,linfct=mcp(Trt = quad)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
##                      Estimate Std. Error t value Pr(>|t|)
## Quadratic trend == 0  -2.9200     0.5394  -5.413 3.83e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```

Test of significance of a cubic trend

```
cube <- rbind("Cubic trend" = c(-5, 7, 4, -4, -7, 5))
summary(glht(aov1,linfct=mcp(Trt = cube)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
##                  Estimate Std. Error t value Pr(>|t|)
## Cubic trend == 0  -0.0800     0.7896  -0.101     0.92
## (Adjusted p values reported -- single-step method)
```

Simultaneous test of significance of linear, quadratic, and cubic trend

```
trend <- rbind("Linear trend" = c(-5, -3, -1, 1, 3, 5),
                  "Quadratic trend" = c(5, -1, -4, -4, -1, 5),
                  "Cubic trend" = c(-5, 7, 4, -4, -7, 5))
summary(glht(aov1,linfct=mcp(Trt = trend)))
```

```
##
##   Simultaneous Tests for General Linear Hypotheses
```

```
##
## Multiple Comparisons of Means: User-defined Contrasts
##
##
## Fit: aov(formula = yield ~ Trt)
##
## Linear Hypotheses:
##                       Estimate Std. Error t value Pr(>|t|)
## Linear trend == 0      4.0700     0.4924   8.265   <1e-04 ***
## Quadratic trend == 0  -2.9200     0.5394  -5.413   <1e-04 ***
## Cubic trend == 0      -0.0800     0.7896  -0.101    0.999
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## (Adjusted p values reported -- single-step method)
```