Randomized Block Designs

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Review

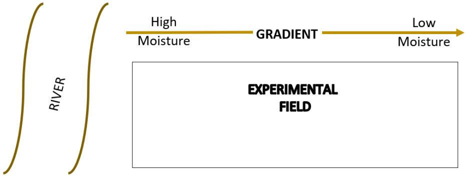
- Experimental units are generally not homogeneous, in other words experimental units are markedly heterogeneous with respect to some criteria of classification
  - Plots differ in fertility, trees differ in age or height, analysts differ in efficiency
- Differences among experimental units are contributes to experimental error and the design should account for that heterogeneity
- **Nuisance factor**- factor that may have an effect on the response variable but of not an interest to the researcher; alo known as **extraneous** factor
  - *Unknown and uncontrollable*- we hope that randomization balances out its impact across the experiment

Review

- **Nuisance factor**
  - *Known but uncontrollable*- measure the value of this factor and include them in the analysis using **Analysis of Covariance**
  - *Known and controllable*- group the experimental units with respect to the nuisance factor
- Blocking is used to come with homogeneous experimental units
- The extraneous variable(s) that may affect the response but not of interest to the researcher is considered as the blocking factor
- A *block* is a group of homogeneous experimental units
- *Blocking* is used to minimize variability in experimental units so as to increase precision of the experiment

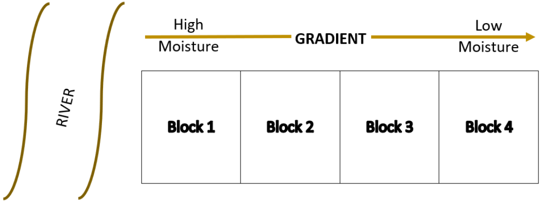
Review

- Blocking is one way of controlling the effect of extraneous factors
- Blocking is most effective when the experimental area has a predictable pattern of variability



Review

- Blocks should be perpendicular to the direction of the gradient



Things to consider in blocking experimental units

- Selection of the blocking variable
  - sex, initial weight, batch, age, elevation, degree program
- Selection of the block shape and orientation
  - *Gradient is unidirectional*: orient the blocks such that their lengths is perpendicular to the direction of the gradient; use long and narrow blocks
  - *Gradient occurs in 2 directions, equally strong and perpendicular to each other*: use long and narrow blocks with their length perpendicular to the direction of one gradient and use covariance technique to take care of the other gradient; use two way blocking (Latin square design)

Things to consider in blocking experimental units

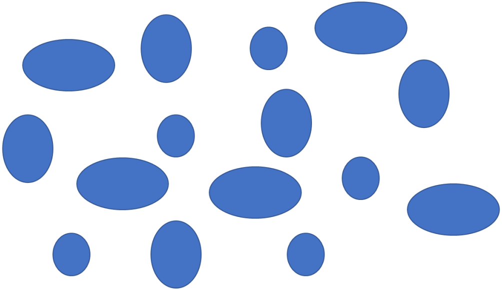
- Selection of the block shape and orientation
  - *Gradient occurs in 2 directions with one gradient much stronger that the other*: ignore the weaker gradient; follow the case of the unidirectional gradient
  - *Pattern of variability is not predictable*: blocks should be as square as possible

Randomized block designs

- Randomized complete block design (RCBD)
- Latin square design (LSD)
- Graeco-Latin square design (GLSD)

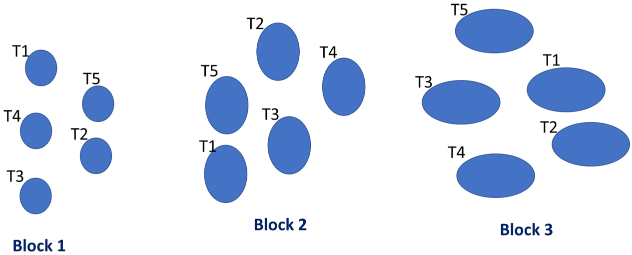
The randomized complete block design (RCBD)

- RCBD is one of the most widely used experimental designs
- It is suited for field experiments where there exists **one extraneous factor** which can be used as *stratifying* variable to form homogeneous experimental units
- Consider an experiment with 5 treatments (t=5): T1, T2, T3, T4, T5 and 3 replicates (r=3)



The randomized complete block design (RCBD)

- A block forms a complete set of replicate for the treatments
- Treatments are randomly assigned to the experimental units independently in each block



The randomized complete block design (RCBD)

Objective: To compare t treatments (means or effetcs)

Replication: Prepare $r(r > 1)$ blocks so that one block contains t experimental units

Local Control: Group t homogeneous experimental units together into one block. This gives us one complete set of t treatments in one block.

Randomization: Assign completely at random the t treatments to the experimental units within a block. Randomization is independent from one block to another.

RCBD: Linear model

$$Y_{ij} = \mu + \tau_i + \beta_j + \epsilon_{ij}, \quad i = 1, 2, \cdots, t; \; j = 1, 2, \cdots, r$$

where:

- Y_{ij} is the response of the experimental unit in the j^{th} block given the i^{th} treatment
- μ is the overall mean response
- τ_i is the effect of i^{th} treatment
- β_j is the effect of the j^{th} block
- ϵ_{ij} is random variation

RCBD: Estimators of the parameters

- $\hat{\mu} = \overline{Y}_{..}$
- $\hat{\tau}_i = \overline{Y}_{i.} - \overline{Y}_{..}$
- $\hat{\beta}_j = \overline{Y}_{.j} - \overline{Y}_{..}$
- $\hat{\epsilon}_{ij} = Y_{ij} - \overline{Y}_{i.} - \overline{Y}_{.j} + \overline{Y}_{..}$

RCBD: Assumptions of the linear model

- For fixed model (Model I)
  - The effects τ_i and β_j are fixed under the restriction that $\sum_{i=1}^{t} \tau_i = 0$ and $\sum_{j=1}^{r} \beta_j = 0$, respectively
- For random model (Model II)
  - The effects τ_i and β_j are random with the following distributions:

$$\tau_i \sim N(0, \sigma_\tau^2)$$
$$\beta_j \sim N(0, \sigma_\beta^2)$$

- $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$
- The effects τ_i and β_j are additive.

RCBD: Analysis of variance

- There are three sources of variation in the response in RCBD experiments: *treatments*, *blocks*, *experimental error*

Source of variation	df	SS	MS	F
Treatments	$t-1$	SSTr	MSTr	$\frac{MSTr}{MSE}$
Blocks	$r-1$	SSR	MSR	$\frac{MSR}{MSE}$
Experimental Error	$(t-1)(r-1)$	SSE	MSE	
TOTAL	$n-1$	SST		

Remarks:

- In Model II, $\sigma_Y^2 = \sigma_\tau^2 + \sigma_\beta^2 + \sigma_\epsilon^2$
- We expect that σ_τ^2 contributes most to σ_Y^2
- By design we also wanted that σ_β^2 contributes some amount of variability on Y_{ij}

RCBD: Computing formulas of the sums of squares

$$SST = \sum_{i=1}^{t}\sum_{j=1}^{r} Y_{ij}^2 - \frac{Y_{..}^2}{rt}$$

$$SSTr = \frac{1}{r}\sum_{i=1}^{t} Y_{i.}^2 - \frac{Y_{..}^2}{rt}$$

$$SSR = \frac{1}{t}\sum_{j=1}^{r} Y_{.j}^2 - \frac{Y_{..}^2}{rt}$$

$$SSE = SST - SSTr - SSR$$

RCBD: Expected mean squares

- Model I:
  - Treatment: $\sigma_\epsilon^2 + \frac{r\sum_{i=1}^{t}\tau_i^2}{t-1}$
  - Block: $\sigma_\epsilon^2 + \frac{t\sum_{j=1}^{r}\beta_j^2}{r-1}$
  - Exptl. Error: σ_ϵ^2
- Model II
  - Treatment: $\sigma_\epsilon^2 + r\sigma_\tau^2$
  - Block: $\sigma_\epsilon^2 + t\sigma_\beta^2$
  - Exptl. Error: σ_ϵ^2

RCBD: Test of hypotheses

Test of hypothesis for treatment means or treatment effects

- Means model
  - $H_0 : \mu_1 = \mu_2 = \cdots = \mu_t$
  - $H_1 : \mu_i \neq \mu_{i'}$, for every pair of $i \neq i'$
- Effects model (Model I)
  - $H_0 : \tau_i = 0, \ \forall i$
  - $H_1 : \tau_i \neq 0, \ \exists i$
- Effects model (Model II)
  - $H_0 : \sigma^2_\tau = 0$
  - $H_1 : \sigma^2_\tau > 0$

RCBD: Test of hypotheses

Test of hypothesis for block means or block effects

- Test if blocking strategy is effective
  - H_0: There are no differences in the mean response among blocks. (blocking strategy is not effective)
  - H_1: There are differences in the mean response among blocks. (blocking strategy is effective)
- Effects model (Model I)
  - $H_0 : \beta_j = 0, \ \forall j$
  - $H_1 : \beta_j \neq 0, \ \exists j$
- Effects model (Model II)
  - $H_0 : \sigma^2_\beta = 0$
  - $H_1 : \sigma^2_\beta > 0$

RCBD: Measures of precision

- Standard error of a treatment mean

$$\hat{\sigma}_{\overline{Y}_{i.}} = \sqrt{\frac{MSE}{r}}$$

- Standard error of the difference between two treatment means

$$\hat{\sigma}_{\overline{Y}_{i.}-\overline{Y}_{i'.}} = \sqrt{\frac{2MSE}{r}}$$

- Coefficient of variation

$$CV = \frac{\sqrt{MSE}}{\overline{Y}_{..}} \times 100$$

RCBD: Relative efficiency

- To compare an experiment in RCBD with t treatment levels and r blocks to a CRD experiment with the same number of treatment levels and the same number of replications

$$RE = \frac{(r-1)MSR + r(t-1)MSE}{(rt-1)MSE} \times 100$$

- If $RE > 100\%$, then RCBD is better (more efficient) than CRD with the same number of replications
- f $RE < 100\%$, then CRD is better (more efficient) than RCBD with the same number of replications

RCBD: Relative efficiency

Question:

How do we design a CRD experiment with the same efficiency as an RCBD experiment? (When $RE > 100\%$)

Answer:

The CRD experiment must have

$$r_{CRD} = RE \times r_{RCBD}$$

replicates per treatment.

RCBD: An example

A researcher conducted an experiment to compare the effects of three different insecticides on a variety of string beans. To obtain a sufficient amount of data, it was necessary to use four different plots of land. Since the plots had somewhat different soil fertility, drainage characteristics, and sheltering from winds, the researcher decided to conduct a randomized complete block design with the plots serving as the blocks. Each plot was subdivided into three rows. A suitable distance was maintained between rows within a plot so that the insecticides could be confined to a particular row. Each row was planted with 100 seeds and then maintained under the insecticide assigned to the row. The insecticides were randomly assigned to the rows within a plot so that each insecticide appeared in one row within all four plots. The response Y_{ij} of interest was the number of seedlings that emerged per row.

RCBD: An example

Insecticide	Plot				Insecticide Total
	1	2	3	4	
1	56	48	66	62	232
2	83	78	94	93	348
3	80	72	83	85	320
Plot Total	219	198	243	240	900

Sums of squares:

$$SST = \sum_{i=1}^{t}\sum_{j=1}^{r} Y_{ij}^2 - \frac{\overline{Y}_{..}^2}{rt} = (56^2 + 48^2 + \cdots + 85^2) - \frac{900^2}{12} = 2296$$

RCBD: An example

Sums of squares:

$$SSTr = \frac{1}{r}\sum_{i=1}^{t}\overline{Y}_{i.}^{2} - \frac{\overline{Y}_{..}^{2}}{rt} = \frac{1}{4}(232^2 + 348^{+}320^2) - \frac{900^2}{12} = 1832$$

$$SSR = \frac{1}{t}\sum_{j=1}^{r}\overline{Y}_{.j}^{2} - \frac{\overline{Y}_{..}^{2}}{rt} = \frac{1}{3}(219^2 + 198^2 + 243^2 + 240^2) - \frac{900^2}{12} = 438$$

$$SSE = SST - SSTr - SSR = 2296 - 1832 - 438 = 26$$

RCBD: An example

ANOVA Table

Source of variation	df	SS	MS	F
Treatments	2	1832	916	211.55
Blocks	3	438	146	33.72
Experimental Error	6	26	4.33	
TOTAL	11	2296		

RCBD: An example

Test of hypotheses

- Treatment effect (Assuming Model I)
  - $H_0 : \tau_i = 0, \ \forall i$
  - $H_1 : \tau_i \neq 0, \ \exists i$
  - $\alpha = 0.05$
  - Test statistic: $F = 211.55$ with $p - value = 2.7338722 \times 10^{-6}$
  - Decision: Reject H_0.
  - Conclusion: At the 5% level of significance, treatment effect is significant. In other words, the number of seedlings that emerged per row is significantly affected by the type of insecticides applied.

RCBD: An example

Test of hypotheses

- Block effect (Assuming Model I)
  - $H_0 : \beta_j = 0, \ \forall j$
  - $H_1 : \beta_j \neq 0, \ \exists j$
  - $\alpha = 0.05$
  - Test statistic: $F = 33.72$ with $p - value = 3.7582101 \times 10^{-4}$
  - Decision: Reject H_0
  - Conclusion: At the 5% level of significance, block effect is significant. In other words, blocking strategy is effective.

RCBD: An example

Relative efficiency

$$RE = \frac{(4-1)146 + 4(3-1)4.33}{(4 \times 3 - 1)4.33} \times 100\% \approx 992.32\%$$

RCBD with subsampling

- Experiment is laid out in RCBD but more than one observation or measurement is taken from every experimental unit

$$Y_{ijk} = \mu + \tau_i + \beta_j + \epsilon_{ij} + \delta_{ijk}, \ \ i = 1, 2, \cdots, t; \ j = 1, 2, \cdots, r; \ k = 1, 2, \cdots,$$

where:

$Y_{ijk} = k^{th}$ observation from the j^{th} experimental unit applied with treatment i

$\mu =$ the overall mean

$\tau_i =$ effect of treatment i

$\beta_j =$ effect of block j

$\epsilon_{ij} =$ random error associated with the experimental units in the j^{th} block given treatment i

$\delta_{ijk} =$ random error associated with the observations of the k^{th} sampling unit in the j^{th} block given treatment i

RCBD with subsampling: Estimators of the parameters

- $\hat{\mu} = \overline{Y}_{...}$
- $\hat{\tau}_i = \overline{Y}_{i..} - \overline{Y}_{...}$
- $\hat{\beta}_j = \overline{Y}_{.j.} - \overline{Y}_{...}$
- $\hat{\epsilon}_{ij} = \overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{Y}_{...}$
- $\hat{\delta}_{ijk} = Y_{ijk} - \overline{Y}_{ij.}$

RCBD with subsampling: Assumptions of the linear model

- For fixed model (Model I)
  - The effects τ_i and β_j are fixed under the restriction that $\sum_{i=1}^{t} \tau_i = 0$ and $\sum_{j=1}^{r} \beta_j = 0$, respectively
- For random model (Model II)
  - The effects τ_i and β_j are random with the following distributions:

$$\tau_i \sim N(0, \sigma_\tau^2)$$
$$\beta_j \sim N(0, \sigma_\beta^2)$$

- $\epsilon_{ij} \sim N(0, \sigma_\epsilon^2)$
- $\delta_{ijk} \sim N(0, \sigma_\delta^2)$
- The effects τ_i and β_j are additive.

RCBD with subsampling: Sums of squares

$$SST = \sum_{i=1}^{t}\sum_{j=1}^{r}\sum_{k=1}^{s} Y_{ijk}^2 - \frac{Y_{...}^2}{rts}$$

$$SSTr = \frac{1}{rs}\sum_{i=1}^{t} Y_{i..}^2 - \frac{Y_{...}^2}{rts}$$

$$SSR = \frac{1}{ts}\sum_{j=1}^{r} Y_{.j.}^2 - \frac{Y_{...}^2}{rts}$$

$$SSU = \frac{1}{s}\sum_{i=1}^{t}\sum_{j=1}^{r} Y_{ij.}^2 - \frac{Y_{...}^2}{rts}$$

$$SSE = SSU - SSTr - SSR$$

$$SS(SE) = SST - SSU$$

RCBD with subsampling: Analysis of Variance

- There are four sources of variation in the response in RCBD with subsamples: *treatments*, *blocks*, *experimental error*, *sampling error*

Source of variation	df	SS	MS	F
Treatments	$t-1$	SSTr	MSTr	
Blocks	$r-1$	SSR	MSR	
Experimental Error	$(t-1)(r-1)$	SSE	MSE	
Sampling error	$rt(s-1)$	SS(SE)	MS(SE)	
TOTAL	$n-1$	SST		

RCBD with subsampling: Sequential test of hypothesis

A. Test on the variability among experimental units given the same treatment

- $H_0 : \sigma_\epsilon^2 = 0$ versus $H_1 : \sigma_\epsilon^2 > 0$
- Test statistic: $F = \frac{MSE}{MS(SE)}$

B. Test on the differences among treatment means

- $H_0 : \mu_1 = \mu_2 = \cdots = \mu_t$ vs $H_1 : \mu_i \neq \mu_{i'}$ for at least one pair $i \neq i'$
- Test statistic (CASE 1, $H_0 : \sigma_\epsilon^2 = 0$ is rejected): $F = \frac{MSTr}{MSE}$
- Test statistic (CASE 2, $H_0 : \sigma_\epsilon^2 = 0$ is not rejected): $F = \frac{MSTr}{MSE_{pooled}}$

$$MSE_{pooled} = \frac{SSE + SS(SE)}{(r-1)(t-1) + rt(s-1)}$$

NOTE: Block effect is tested similarly!

RCBD with subsampling: An example

An agronomist wishes to determine the efficacy of two fumigants C and S for controlling wireworms. A field is divided into 5 blocks, each containing 3 plots. Each of the three treatments fumigant C, fumigant S, and the control O were randomly assigned to one plot within each block. In each experimental plot, the number of wireworms was counted in each of four subsamples. The data is gven below.

Fumigant	*Block 1*	*Block 2*	*Block 3*	*Block 4*	*Block 5*
Control	12	7	9	12	7
	20	4	6	22	8
	8	4	7	13	5
	8	5	11	17	9
C	5	0	4	7	4
	4	9	4	3	9
	5	3	3	5	8
	2	3	9	12	6
S	5	6	2	6	2
	5	4	9	4	9
	1	5	3	8	7
	2	4	7	4	3

RCBD with subsampling: An example

Table of totals:

Fumigant	Plot1	Plot2	Plot3	Plot4	Plot5	TOTAL
Control	48	20	33	64	29	194
C	16	15	20	27	27	105
S	13	19	21	22	21	96
TOTAL	77	54	74	113	77	395

ANOVA Table

Source of variation	df	SS	MS	F
Fumigant	2	293.43	146.72	
Blocks	4	151.17	37.792	
Experimental Error	8	196.23	24.53	
Sampling error	45	409.75	9.11	
TOTAL	59	1050.58		

RCBD with subsampling: An example

A. Test on the variability among experimental units given the same treatment

- $H_0 : \sigma_\epsilon^2 = 0$ versus $H_0 : \sigma_\epsilon^2 > 0$
- Test statistic: $F = \frac{MSE}{MS(SE)} = \frac{24.53}{9.11} = 2.69$ with $p - value = 0.0165382$

B. Test of treatment effect

- $H_0 : \mu_1 = \mu_2 = \mu_3$ vs $H_1 : \mu_i \neq \mu_{i'}$ for at least one pair $i \neq i'$
- Test statistic: $F = \frac{MSTr}{MSE} = \frac{146.72}{24.53} = 5.98$ with $p - value = 0.0258058$

C. Test of block effect

- $H_0 : \beta_1 = \beta_2 = \cdots = \beta_5$ vs $H_1 : \beta_j \neq \beta_{j'}$ for at least one pair $j \neq j'$
- Test statistic: $F = \frac{MSR}{MSE} = \frac{37.792}{24.53} = 1.54$ with $p - value = 0.2791743$

Latin Square Design

- Takes care of two nuisance factors by using two-dimensional blocking of experimental materials according to these nuisance factors
- Complete block design- complete set of treatments are present in a row and in a column
- An experimental unit should belong to one of the row classifications, and to one of the column classifications
- Each treatment must be applied once to each row and once to each column

Latin Square Design

- The number of treatments (t), should be the same as the number of rows (r) and number of columns (c)
- It is practical to use only for a small number of treatments due to the restrictions imposed by the number of required rows and columns
- With very few treatments, there is a very small degrees of freedom left for the experimental error, hence, it usually advised that the Latin squares be repeated to increase the precision of the estimates

Latin Square Design

A food chemist wants to investigate the chemical residue levels in 4 kinds of dried fish (F_1, F_2, F_3, and F_4). She suspects that the source of the dried fish (S_1, S_2, S_3, and S_4) and the chemists (C_1, C_2, C_3, and C_4) doing the analysis contribute to the variation in the results. To eliminate the variations contributed by sources and chemists, a 4x4 Latin square design was used. It is assumed that the sources, the chemists and the kinds of dried fish do not have interaction. The experimental layout is given below.

	S_3	S_2	S_1	S_4
C_1	F_3	F_2	F_1	F_4
C_4	F_2	F_1	F_4	F_3
C_3	F_1	F_4	F_3	F_2
C_2	F_4	F_3	F_2	F_1

Latin Square Design

Suppose that we want to test five drugs –A,B,C,D and E, for their effect in alleviating the symptoms of a chronic disease. Five patients (P_1, P_2, P_3, P_4, and P_5) are available for a trial, and each will be available for five weeks (W_1, W_2, W_3, W_4, and W_5). Testing a single drug requires a week. The experimental layout is given below.

	P_5	P_2	P_3	P_4	P_1
W_1	E	B	C	D	A
W_3	B	D	E	A	C
W_2	A	C	D	E	B
W_5	D	A	B	C	E
W_4	C	E	A	B	D

Latin Square Design: Randomization

1. Obtain a basic $t \times t$ Latin Square plan

	C_1	C_2	C_3	C_4
R_1	A	B	C	D
R_2	B	C	D	A
R_3	C	D	A	B
R_4	D	A	B	C

2. Randomize the row assignment

	C_1	C_2	C_3	C_4
R_1	A	B	C	D
R_3	C	D	A	B
R_4	D	A	B	C
R_2	B	C	D	A

Latin Square Design: Randomization

3. Randomize the column assignments

	C_2	C_4	C_3	C_1
R_1	B	D	C	A
R_3	D	B	A	C
R_4	A	C	B	D
R_2	C	A	D	B

Latin Square Design: Linear Model

$$Y_{ijk} = \mu + \tau_i + \beta_j + \gamma_k + \epsilon_{ijk}$$

where:

- Y_{ijk} is the response of the experimental unit in each jk combination given the i^{th} treatment
- μ is the overall mean response
- τ_i is the i^{th} treatment effect; $i = 1, 2, \cdots, t$
- β_j is the j^{th} row effect; $j = 1, 2, \cdots, r$
- γ_k is the k^{th} column effect; $k = 1, 2, \cdots, c$
- ϵ_{ijk} is the random error

Latin Square Design: ANOVA Table

Source of Variation	df	SS	MS	F
Treatment	$t-1$	SSTr	MSTr	$\frac{MSTr}{MSE}$
Row	$r-1$	SSR	MSR	$\frac{MSR}{MSE}$
Column	$c-1$	SSC	MSC	$\frac{MSC}{MSE}$
Experimental Error	$(r-1)(c-2)$	SSE	MSE	
TOTAL	$n-1$	SST		

Latin Square Design: Sums of squares computing formulas

$$SST = \sum_{i=1}^{t}\sum_{j=1}^{r}\sum_{k=1}^{c} Y_{ijk}^2 - \frac{Y_{\dots}^2}{t^2}$$

$$SSTr = \frac{1}{r}\sum_{i=1}^{t} Y_{i..}^2 - \frac{Y_{\dots}^2}{t^2}$$

$$SSR = \frac{1}{c}\sum_{j=1}^{r} Y_{.j.}^2 - \frac{Y_{\dots}^2}{t^2}$$

$$SSC = \frac{1}{r}\sum_{k=1}^{c} Y_{..k}^2 - \frac{Y_{\dots}^2}{t^2}$$

$$SSE = SST - SSTr - SSR - SSC$$

Latin Square Design: Tests of hypotheses

A. Test of treatment effect

- $H_0 : \tau_i = 0, \forall i$
- $H_1 : \tau_i \neq 0, \exists i$
- Test statistic: $F = \frac{MSTr}{MSE}$

B. Test of row effect

- $H_0 : \beta_j = 0, \forall j$
- $H_1 : \beta_j \neq 0, \exists j$
- Test statistic: $F = \frac{MSR}{MSE}$

C. Test of column effect

- $H_0 : \gamma_k = 0, \forall k$
- $H_1 : \gamma_k \neq 0, \exists k$
- Test statistic: $F = \frac{MSC}{MSE}$

Latin Square Design: Measures of precision

- Standard error of a treatment mean

$$\text{s.e.}(\overline{Y}_{i..}) = \sqrt{\frac{MSE}{t}}$$

- Standard error of a row mean

$$\text{s.e.}(\overline{Y}_{.j.}) = \sqrt{\frac{MSE}{r}}$$

- Standard error of a column mean

$$\text{s.e.}(\overline{Y}_{..k}) = \sqrt{\frac{MSE}{c}}$$

Latin Square Design: Measures of precision

- Standard error of the difference between two treatment means (*between two row or column means*)

$$\text{s.e.}(\overline{Y}_{i..} - \overline{Y}_{i'..}) = \sqrt{\frac{MSE}{t}}$$

- Coefficient of variation

$$\text{CV} = \frac{\sqrt{MSE}}{\overline{Y}_{...}} \times 100\%$$

Latin Square Design: Relative efficiency

- Latin Square Design vs CRD

$$\text{RE (LSD/CRD)} = \frac{MSR + MSC + (t-1)MSE}{(t+1)MSE}$$

- Latin Square Design vs RCBD with Rows as blocks

$$\text{RE (LSD/RCDB.r)} = \frac{MSC + (t-1)MSE}{tMSE}$$

- Latin Square Design vs RCBD with Columns as blocks

$$\text{RE (LSD/RCDB.c)} = \frac{MSR + (t-1)MSE}{tMSE}$$

Latin Square Design: An example

An experiment was conducted to evaluate the effect of different additives (A, B, C, D) on reducing pollution. Four different cars (C_1, C_2, C_3, C_4) were used in the experiment which were driven by different drivers (D_1, D_2, D_3, D_4). The response being measured is emission reduction index. The data is given below.

	C_1	C_2	C_3	C_4
D_1	19(A)	24(B)	23(D)	26(C)
D_2	23(D)	24(C)	19(A)	30(B)
D_3	15(B)	14(D)	15(C)	16(A)
D_4	19(C)	18(A)	19(B)	16(D)

Latin Square Design: An example

Solution:

- Additive totals: $Y_{1..} = 72$, $Y_{2..} = 88$, $Y_{3..} = 84$, $Y_{4..} = 76$
- Driver totals: $Y_{.1.} = 92$, $Y_{.2.} = 96$, $Y_{.3.} = 60$, $Y_{.4.} = 72$
- Car totals: $Y_{..1} = 76$, $Y_{..2} = 80$, $Y_{..3} = 76$, $Y_{..4} = 88$
- Overall total: $Y_{...} = 320$

ANOVA Table

Source of Variation	df	SS	MS	F	p-value
Additive	3	40	13.3	2.5	0.1564901
Driver	3	216	72.0	13.5	0.0044658
Car	3	24	8.0	1.5	0.3071741
Experimental Error	6	32	5.3		
TOTAL	15	312			

Latin Square Design: An example

Solution:

- There is no significant additive or car effect.
- There is driver effect
- Relative efficiency of Latin Square Design vs CRD

$$\begin{aligned}
\text{RE (LSD/CRD)} &= \frac{MSR + MSC + (t-1)MSE}{(t+1)MSE} \\
&= \frac{72 + 8 + 3(5.3)}{(4+1)5.3} \\
&\approx 1.40
\end{aligned}$$

Latin Square Design: An example

Solution:

- Relative efficiency of Latin Square Design vs RCBD with Drivers as blocks:

$$
\begin{aligned}
\text{RE (LSD/RCDB.r)} &= \frac{MSC + (t-1)MSE}{tMSE} \\
&= \frac{8 + 3(5.3)}{4(5.3)} \\
&\approx 1.38
\end{aligned}
$$

Latin Square Design: An example

Solution:

- Latin Square Design vs RCBD with Cars as blocks

$$
\begin{aligned}
\text{RE (LSD/RCDB.c)} &= \frac{MSR + (t-1)MSE}{tMSE} \\
&= \frac{72 + 3(5.3)}{4(5.3)} \\
&\approx 1.12
\end{aligned}
$$