Moving Averages

Lesson 2.1

Introduction

- Smoothing is the process of removing random variations that appear as coarseness in a plot of raw time series data.
- Smoothing is usually done to help us better see patterns, trends for example, in time series.
- Generally, we smooth out the irregular roughness to see a clearer signal.
- Analysts also refer to the smoothing process as filtering the data
- For seasonal data, we might smooth out the seasonality so that we can identify the trend.
- Smoothing doesn't provide us with a model, but it can be a good first step in describing various components of the series.

Moving Averages

- The traditional use of the term moving average is that at each point in time we determine (possibly weighted) averages of observed values that surround a particular time.
- This might be done by looking at a "one-sided" moving average in which you average all values for the previous year's worth of data or a centered moving average in which you use values both before and after the current time.

One-sided moving averages

- One-sided moving averages include the current and previous observations for each average. For example, the formula for a moving average (MA) of X at time t with a length of 7 is the following:

$$\frac{x_{t-6} + x_{t-5} + x_{t-4} + +x_{t-3} + x_{t-2} + x_{t-1} + x_t}{7}$$

Centered moving averages

- For instance, at time t, a "centered moving average of length 3" with equal weights would be the average of values at times $t-1$, t, $t+1$.
- To take away seasonality from a series so we can better see trend, we would use a moving average with a length = seasonal span.
- Thus in the smoothed series, each smoothed value has been averaged across all seasons.
- For quarterly data, for example, we could define a smoothed value for time t as

$$\frac{x_t + x_{t-1} + x_{t-2} + x_{t-3}}{4}$$

the average of this time and the previous 3 quarters. In R code this will be a one-sided filter.

- To smooth away seasonality in quarterly data, in order to identify trend, the usual convention is to use the moving average smoothed at time t is

$$\frac{1}{8}x_{t-2} + \frac{1}{4}x_{t-1} + \frac{1}{4}x_t + \frac{1}{4}x_{t+1} + \frac{1}{8}x_{t+2}$$

- To smooth away seasonality in monthly data, in order to identify trend, the usual convention is to use the moving average smoothed at time t which is given by

$$\frac{1}{24}x_{t-6} + \frac{1}{12}x_{t-5} + \frac{1}{12}x_{t-4} + \cdots + \frac{1}{12}x_{t+4} + \frac{1}{12}x_{t+5} + \frac{1}{24}x_{t+6}$$

- That is, we apply weight $\frac{1}{24}$ to values at times $t-6$ and $t+6$ and weight $\frac{1}{12}$ to all values at all times between $t-5$ and $t+5$.
- A centered moving average creates a bit of a difficulty when we have an even number of time periods in the seasonal span (as we usually do).
- For example, the formula for a centered moving average with a length of 8 is as follows:

$$\frac{1}{2}x_{t-4} + \frac{1}{8}x_{t-3} + \frac{1}{8}x_{t-2} + \frac{1}{8}x_{t-1} + \frac{1}{8}x_t + \frac{1}{8}x_{t+1} + \frac{1}{8}x_{t+2} + \frac{1}{8}x_{t+3} + \frac{1}{2}x_{t+4}$$

Some examples

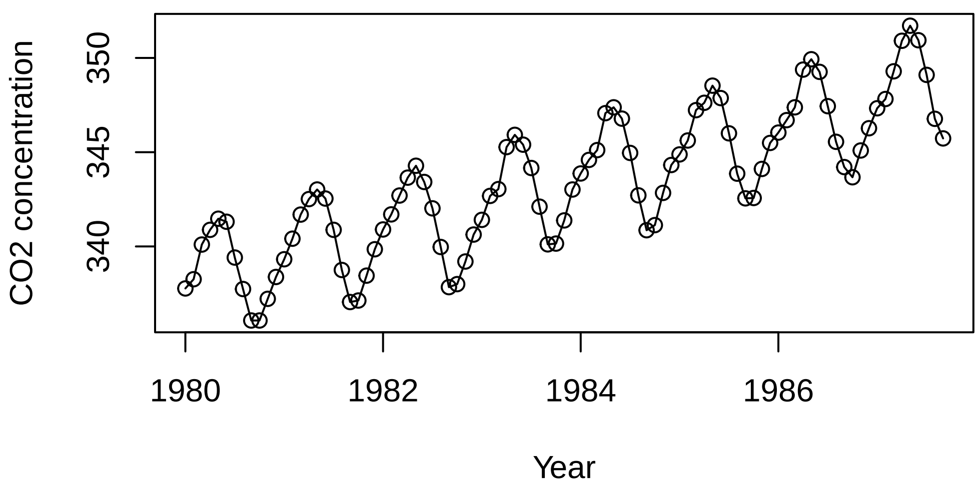
Let us load some packages and the CO_2 data.

```r
library(readxl)
library(ggplot2)
library(tidyverse)
library(forecast)
library(fpp2)

co2 <- read_excel("CO2 1980-1987.xlsx")
```

```r
co2_ts<-ts(co2$CO2,start=c(1980,1),end=c(1987,9),frequency=12)

plot(co2_ts,type="o",xlab="Year",
     ylab="CO2 concentration")
```

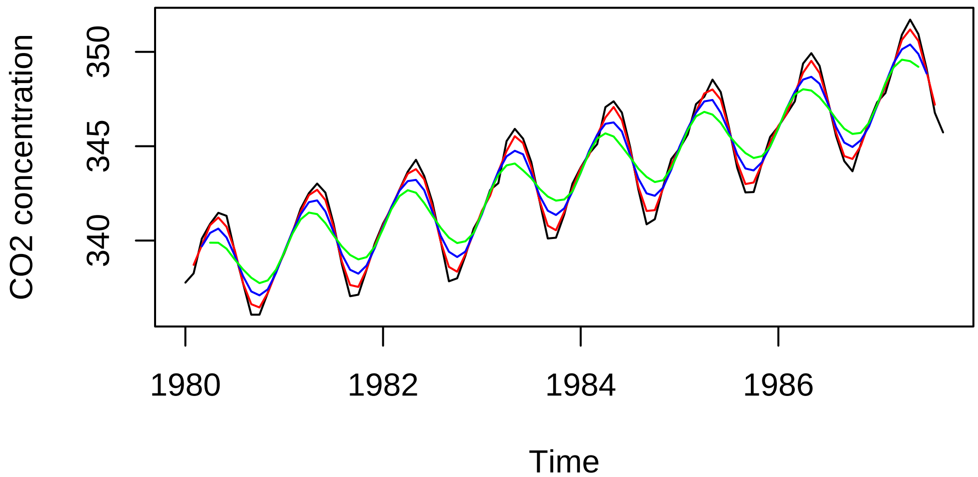


Again, this time series shows both seasonality and trend.The peaks are observed in the month of May every year.

Let us try computing moving averages as outlined above and observe which period or length best captures the patterns in the data.

```r
cma.3<-ma(co2_ts,order=3)
cma.5<-ma(co2_ts,order=5)
cma.7<-ma(co2_ts,order=7)
plot.ts(cbind(co2_ts,cma.3,cma.5,cma.7),
```

```
    plot.type = 'single',
    col = c("black","red","blue","green"),
    ylab = "CO2 concentration")
```



Moving averages of moving averages

It is possible to apply a moving average to a moving average. One reason for doing this is to make an even-order moving average symmetric.

For example, we might take a moving average of order 4, and then apply another moving average of order 2 to the results. In the following table, this has been done for the first few years of the Australian quarterly beer production data.

```
beer <- window(ausbeer,start=1992)
ma4 <- ma(beer, order=4, centre=FALSE)
ma2x4 <- ma(beer, order=4, centre=TRUE)

as.data.frame(cbind(beer, ma4, ma2x4))
```

```
   beer    ma4   ma2x4
1   443     NA      NA
```

2	410	451.25	NA
3	420	448.75	450.000
4	532	451.50	450.125
5	433	449.00	450.250
6	421	444.00	446.500
7	410	448.00	446.000
8	512	438.00	443.000
9	449	441.25	439.625
10	381	446.00	443.625
11	423	440.25	443.125
12	531	447.00	443.625
13	426	445.25	446.125
14	408	442.50	443.875
15	416	438.25	440.375
16	520	435.75	437.000
17	409	431.25	433.500
18	398	428.00	429.625
19	398	433.75	430.875
20	507	433.75	433.750
21	432	435.75	434.750
22	398	440.50	438.125
23	406	439.50	440.000
24	526	439.25	439.375
25	428	438.50	438.875
26	397	436.25	437.375
27	403	438.00	437.125
28	517	434.50	436.250
29	435	439.75	437.125
30	383	440.75	440.250
31	424	437.25	439.000
32	521	442.00	439.625
33	421	439.50	440.750
34	402	434.25	436.875
35	414	441.75	438.000
36	500	436.25	439.000
37	451	436.75	436.500
38	380	434.75	435.750
39	416	429.00	431.875
40	492	436.00	432.500
41	428	433.50	434.750
42	408	437.00	435.250
43	406	438.75	437.875
44	506	431.75	435.250

45	435	435.50	433.625
46	380	431.50	433.500
47	421	431.50	431.500
48	490	434.00	432.750
49	435	431.75	432.875
50	390	422.75	427.250
51	412	418.00	420.375
52	454	421.25	419.625
53	416	420.25	420.750
54	403	427.25	423.750
55	408	432.75	430.000
56	482	428.50	430.625
57	438	427.75	428.125
58	386	430.00	428.875
59	405	427.25	428.625
60	491	426.50	426.875
61	427	423.75	425.125
62	383	419.25	421.500
63	394	417.50	418.375
64	473	419.25	418.375
65	420	423.25	421.250
66	390	427.00	425.125
67	410	425.75	426.375
68	488	427.75	426.750
69	415	430.00	428.875
70	398	430.00	430.000
71	419	429.75	429.875
72	488	423.75	426.750
73	414	NA	NA
74	374	NA	NA

The notation *2 x 4-MA* in the last column means a 4-MA followed by a 2-MA. The values in the last column are obtained by taking a moving average of order 2 of the values in the previous column.

When a 2-MA follows a moving average of an even order (such as 4), it is called a “centred moving average of order 4”. This is because the results are now symmetric. To see that this is the case, we can write the *2 x 4-MA* as follows:

$$\begin{aligned}\hat{T}_t &= \frac{1}{2}[\frac{1}{4}(y_{t-2} + y_{t-1} + y_t + y_{t+1}) + \frac{1}{4}(y_{t-1} + y_t + y_{t+1} + y_{t+2})] \\ &= \frac{1}{8}y_{t-2} + \frac{1}{4}y_{t-1} + \frac{1}{4}y_t + \frac{1}{4}y_{t+1} + \frac{1}{8}y_{t+2}\end{aligned}$$

It is now a weighted average of observations that is symmetric. By default, the ma() function in R will return a centred moving average for even orders (unless *center=FALSE* is specified).

Other combinations of moving averages are also possible. For example, a *3 × 3-MA* is often used, and consists of a moving average of order 3 followed by another moving average of order 3. In general, an even order MA should be followed by an even order MA to make it symmetric. Similarly, an odd order MA should be followed by an odd order MA.

Estimating the trend-cycle with seasonal data

The most common use of centred moving averages is for estimating the trend-cycle from seasonal data. Consider the *2 x 4-MA*:

$$\hat{T}_t = \frac{1}{8}y_{t-2} + \frac{1}{4}y_{t-1} + \frac{1}{4}y_t + \frac{1}{4}y_{t+1} + \frac{1}{8}y_{t+2}$$

When applied to quarterly data, each quarter of the year is given equal weight as the first and last terms apply to the same quarter in consecutive years. Consequently, the seasonal variation will be averaged out and the resulting values of $\hat{T}_t$ will have little or no seasonal variation remaining. A similar effect would be obtained using a *2 × 8-MA* or a *2 × 12-MA* to quarterly data.

In general, a *2 × m-MA* is equivalent to a weighted moving average of order
$m+1$, where all observations take the weight $\frac{1}{m}$, except for the first and last terms which take weights $\frac{1}{2m}$. So, if the seasonal period is even and of order m, we use a *2 × m-MA* to estimate the trend-cycle. If the seasonal period is odd and of order m, we use a *m-MA* to estimate the trend-cycle. For example, a *2 × 12-MA* can be used to estimate the trend-cycle of monthly data and a *7-MA* can be used to estimate the trend-cycle of daily data with a weekly seasonality.

Other choices for the order of the MA will usually result in trend-cycle estimates being contaminated by the seasonality in the data.

Note that combinations of moving averages result in weighted moving averages. For example, the *2 x 4-MA* is equivalent to a weighted 5-MA with weights given by
$[\frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{8}]$