



UNIVERSITÀ DI PARMA

Epipolar Geometry

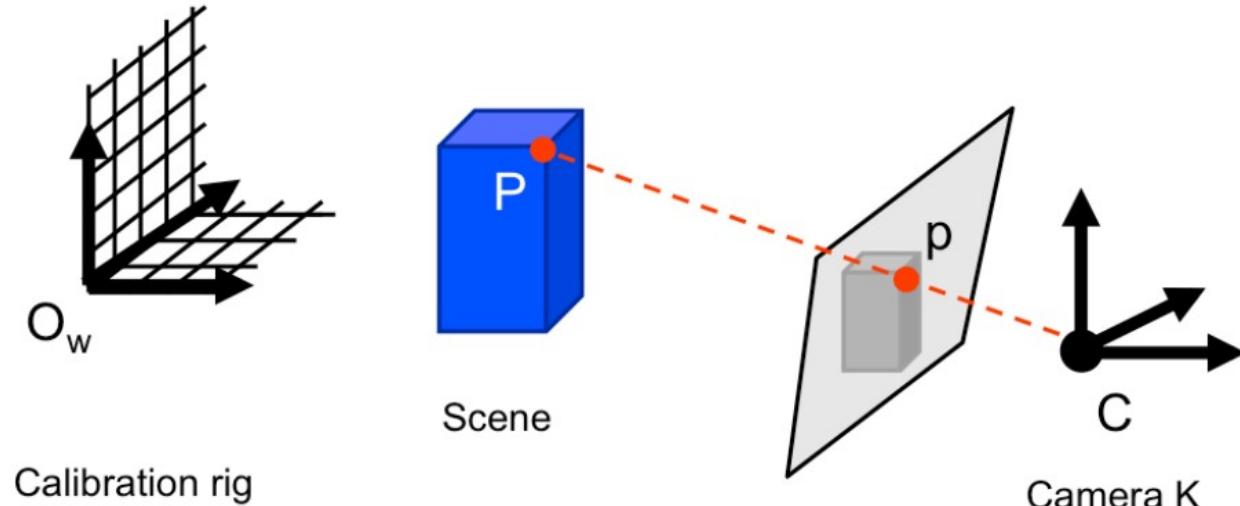


- Why stereo?
- Epipolar constraints
- Essential and Fundamental matrices
- Stereo Images rectification

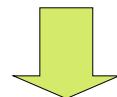
- [FP] D. A. Forsyth and J. Ponce. Computer Vision: A Modern Approach (2nd Edition). Prentice Hall, 2011.
- [HZ] R. Hartley and A. Zisserman. Multiple View Geometry in Computer Vision. Cambridge University Press, 2003.
- **CS231A · Computer Vision: from 3D reconstruction to recognition, Prof. Silvio Savarese – Stanford University**

Credits for many images/equations: Silvio Savarese

Without stereo...

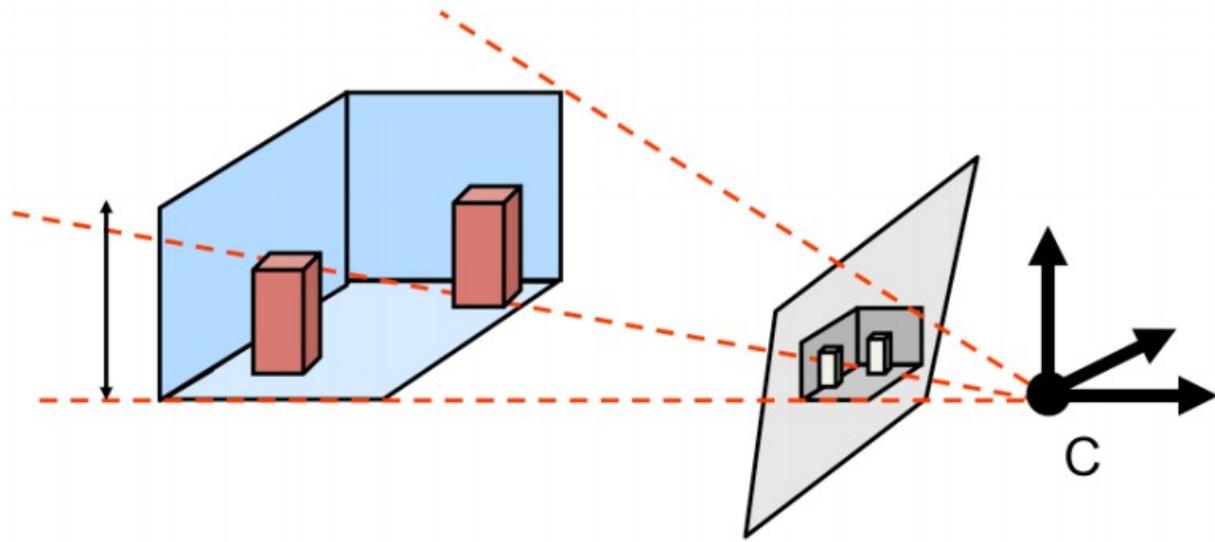


Calibration rig + rig position

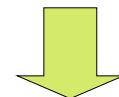


We can compute K and RT

Without stereo...



Vanishing lines and points + orthogonal planes

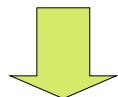


We can compute K

Without stereo...



Why it is **soooo** difficult?



Line of Sight: multiple potential correspondances for p in the 3D world..

Without stereo...



Without stereo...



Without stereo...



Without stereo...

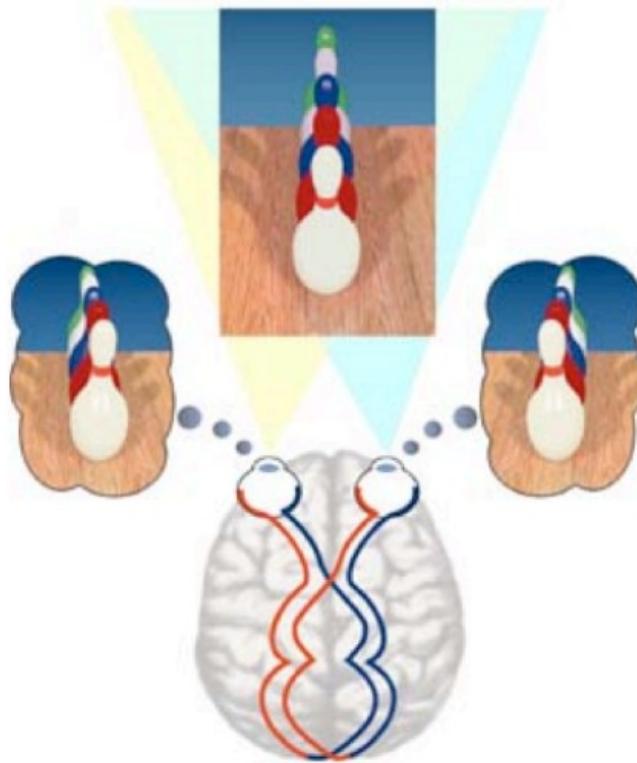


Nature and monocular vision

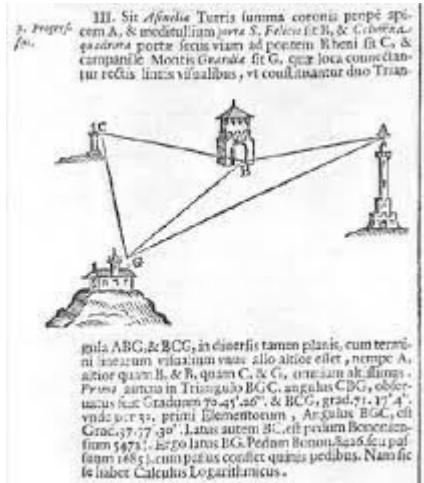


Stereo vision wins

UNIVERSITÀ
DI PARMA



Triangulation

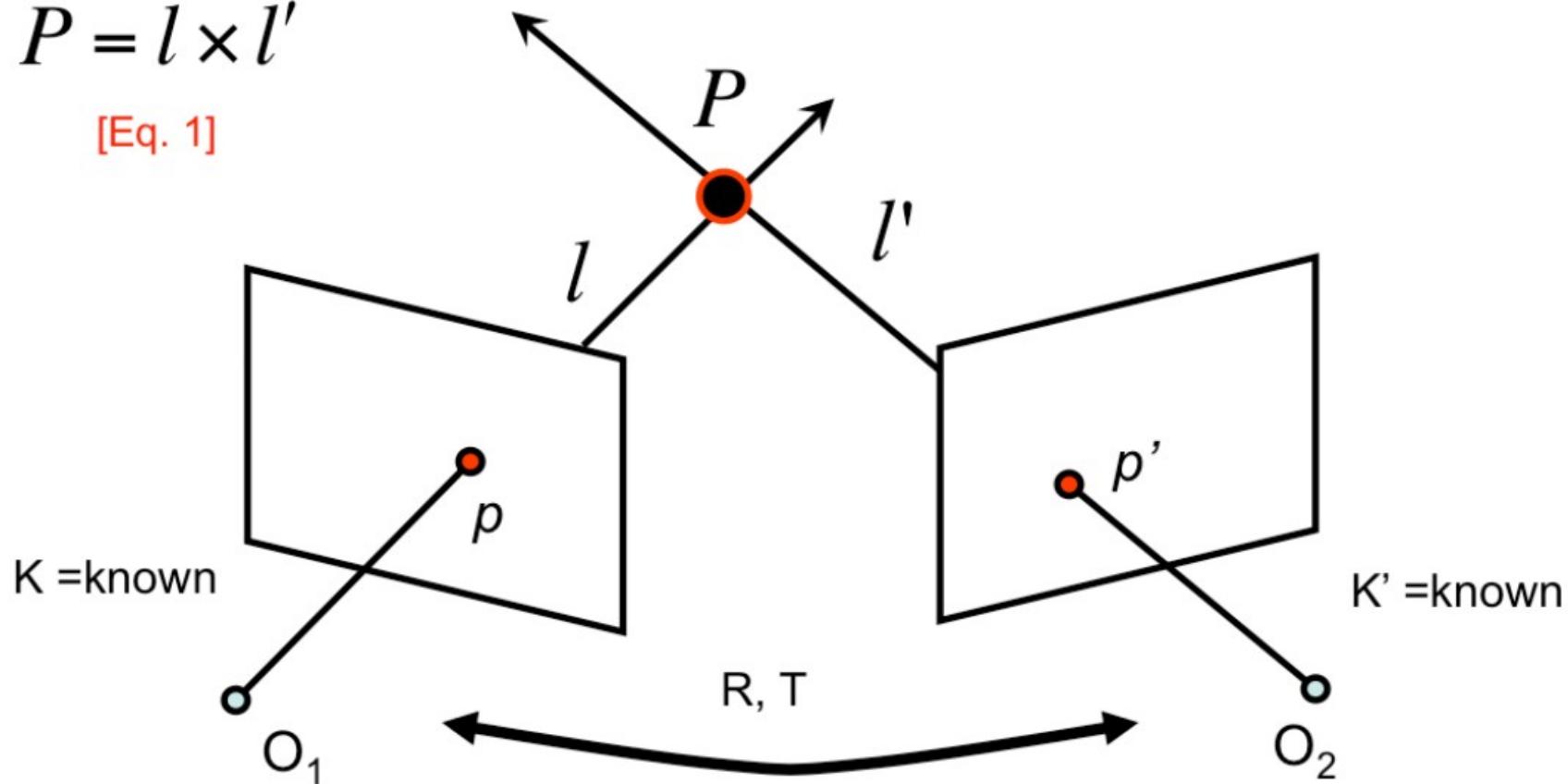


Triangulation



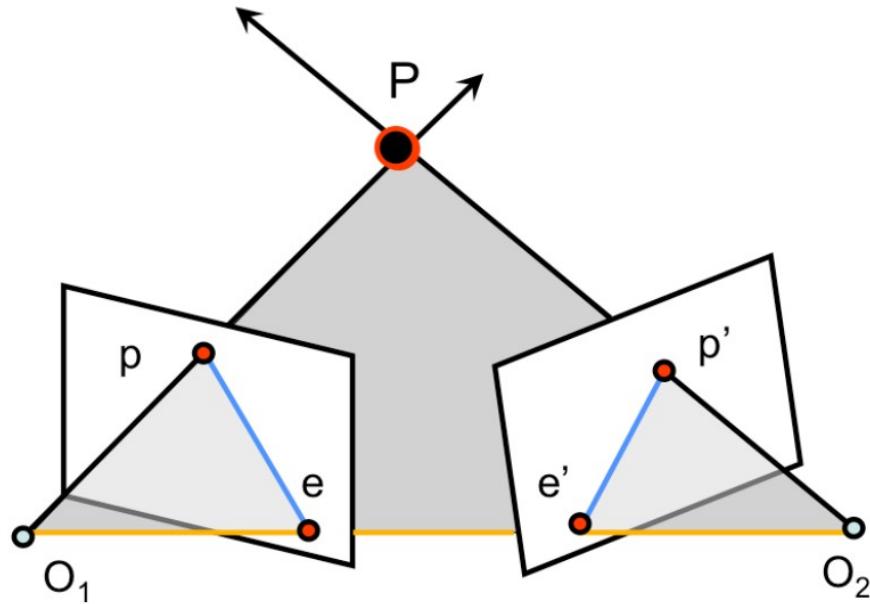
$$P = l \times l'$$

[Eq. 1]



- **Correspondances:** given a p in one image find p' in the other image
- **Camera geometry:** given a set of correspondences find camera parameters
- **Scene Geometry:** given a set of correspondences and parameters of both cameras reconstruct the 3D scene

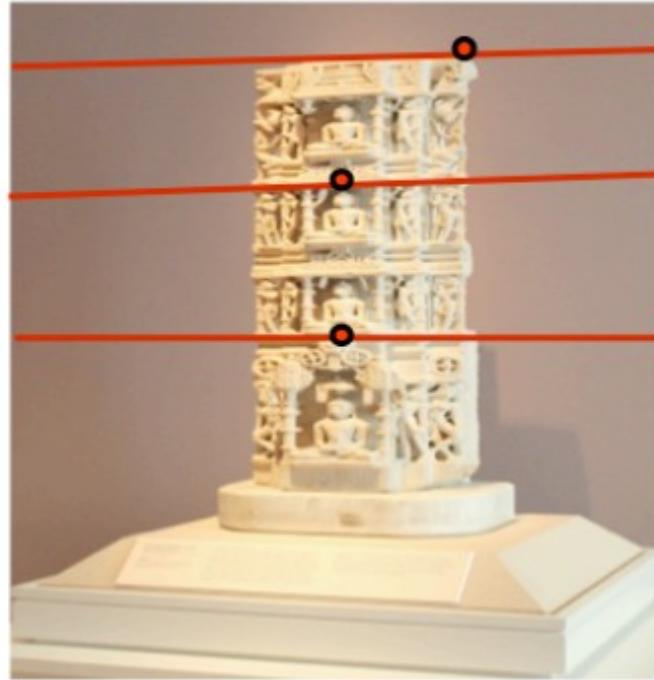
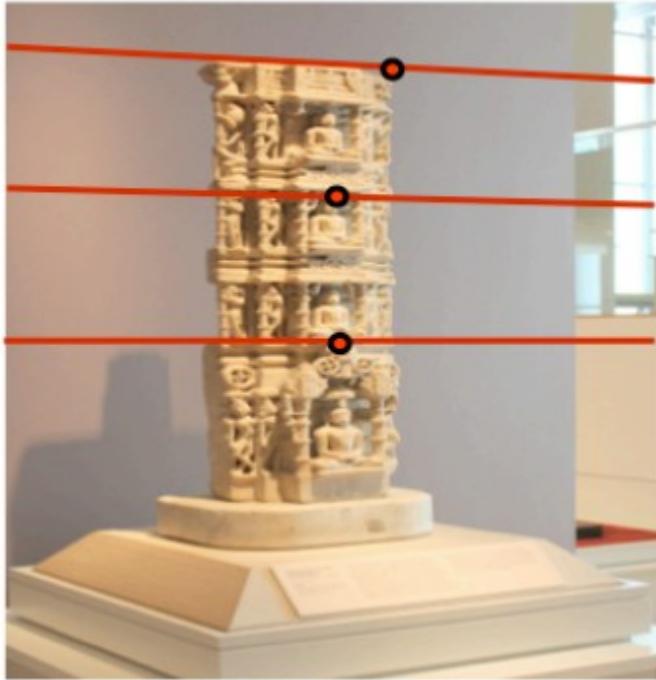
Epipolar geometry



- O_1-O_2-P : epipolar plane
- O_1-O_2 : baseline
- pe & $p'e'$: epipolar lines (i.e. they meet!)
- e & e' : epipoles
 - Intersection of baseline with image planes
 - Projection of O_1 & O_2

Epipolar lines

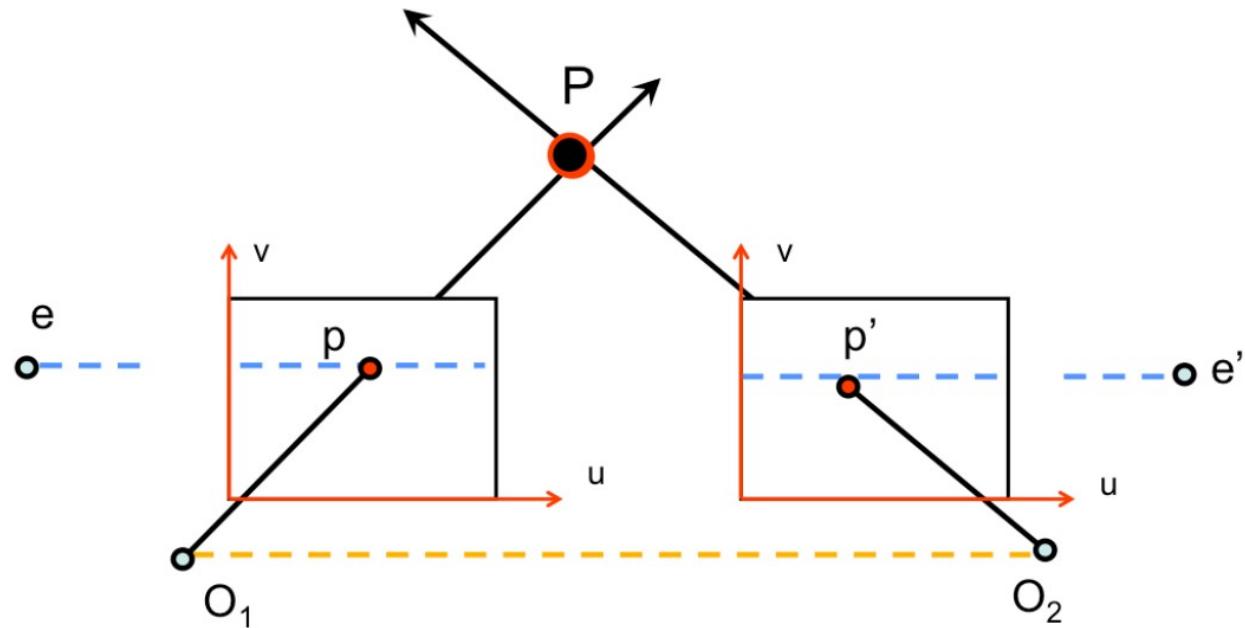
UNIVERSITÀ
DI PARMA



Epipolar lines

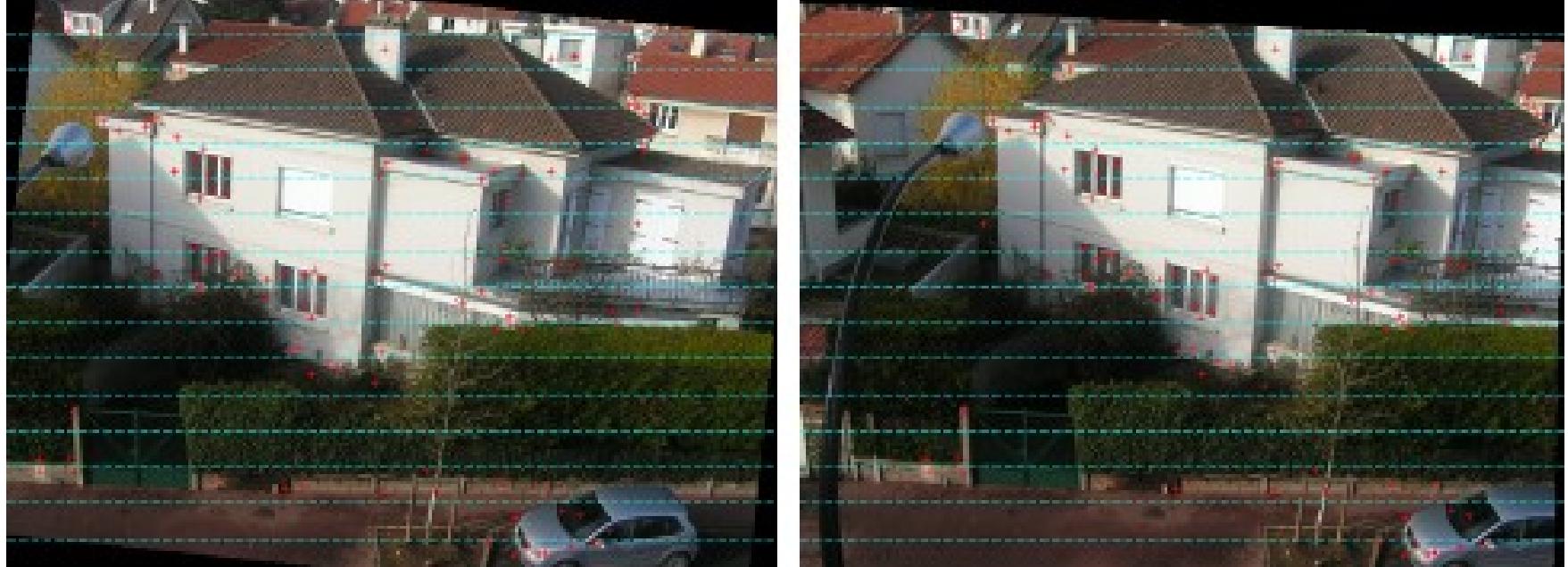


Special case: parallel image planes



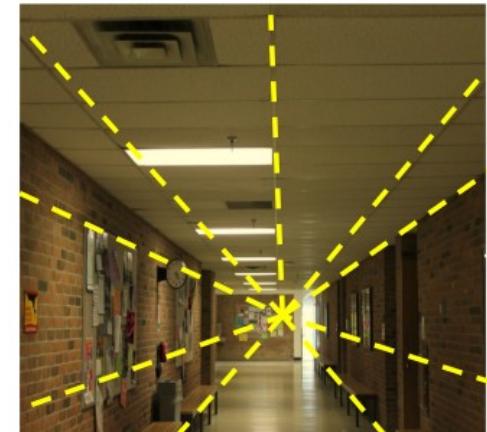
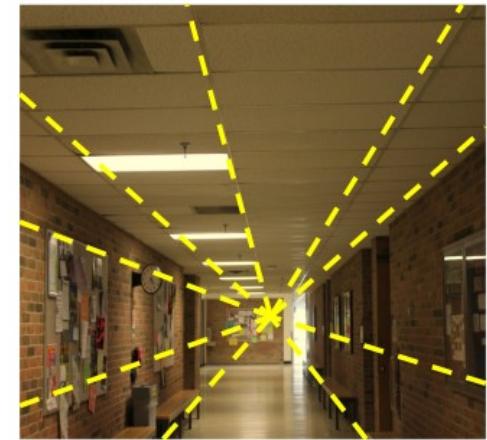
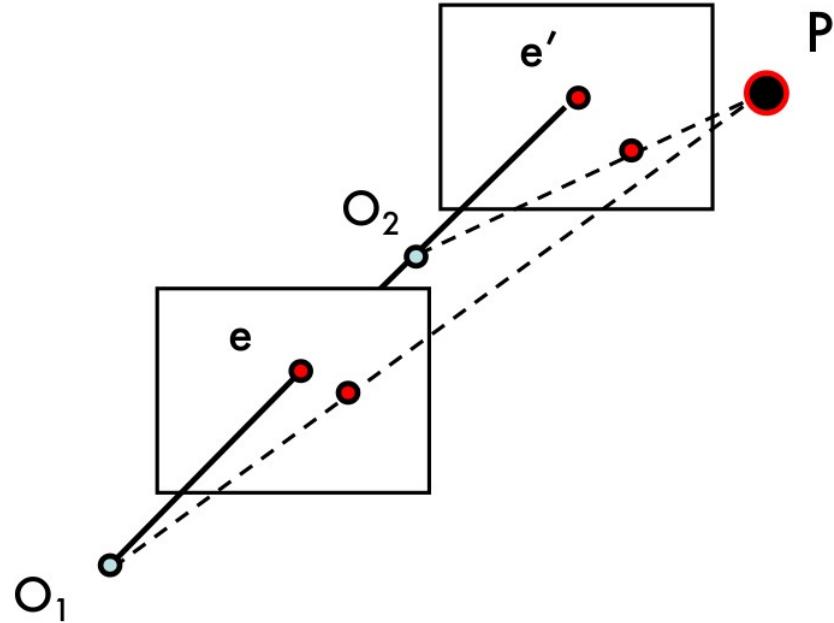
- Epipoles are at infinity
- Also intersection between baseline and image plane are at infinity
- Epipolar lines are parallel to u axis

Special case: parallel image planes

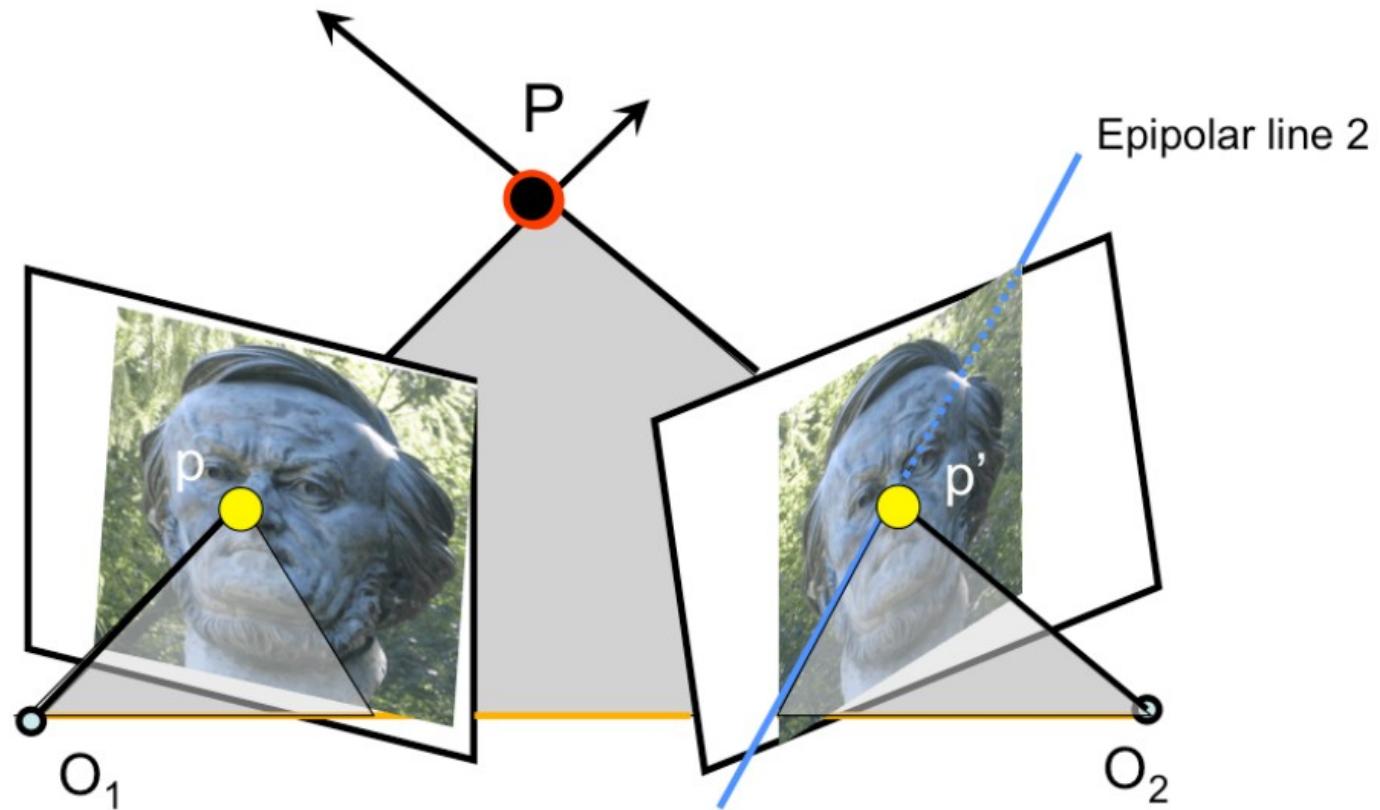


- Example of two images whose planes are parallel to each other
- Epipolar lines are parallel to each other

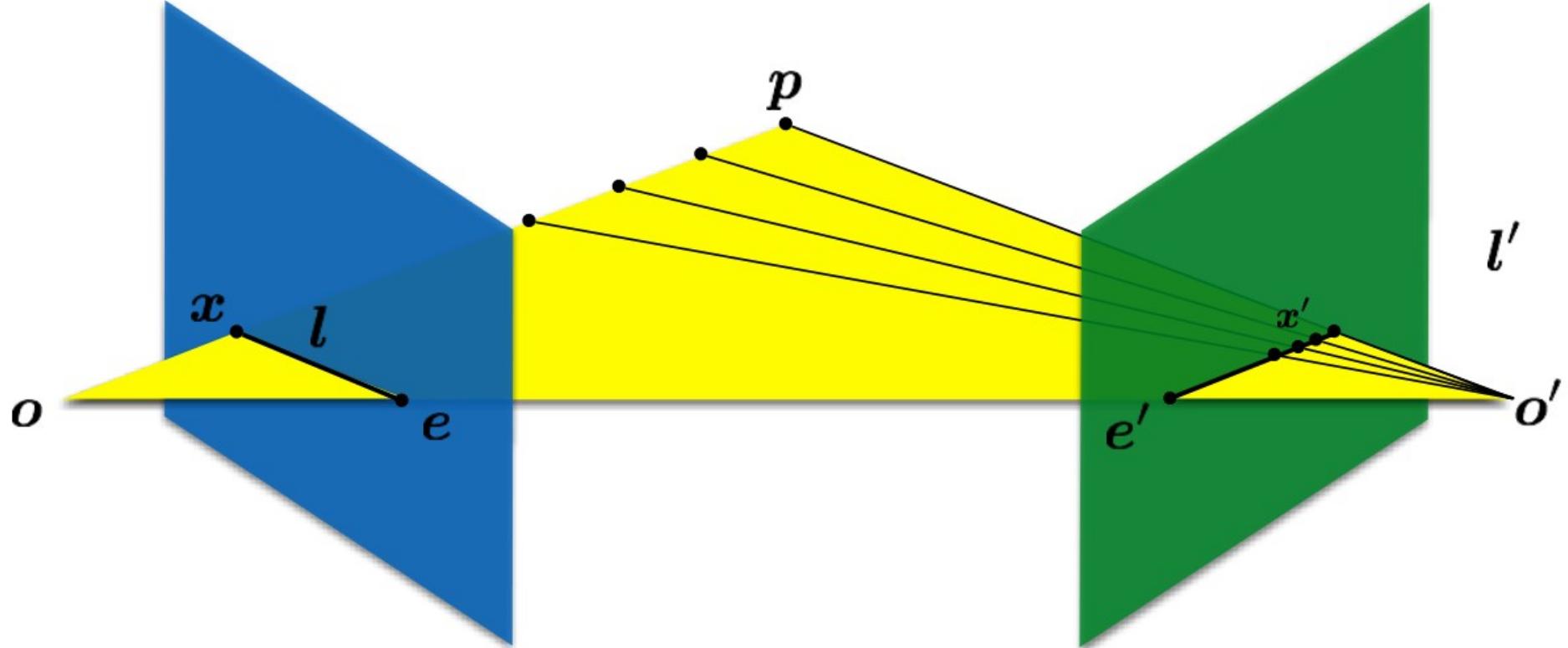
Special case: forward translation



Epipolar constraint

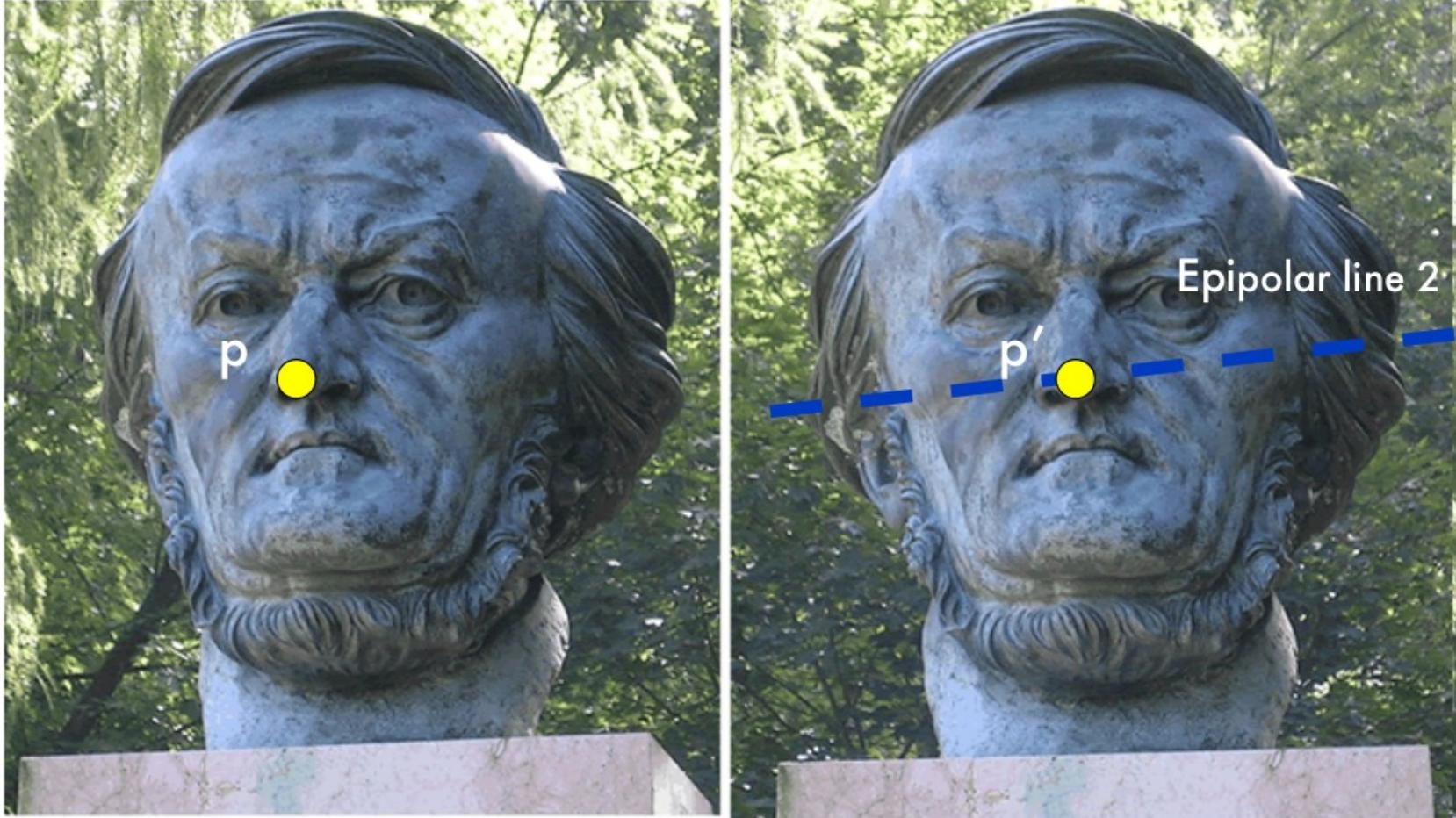


Epipolar constraint

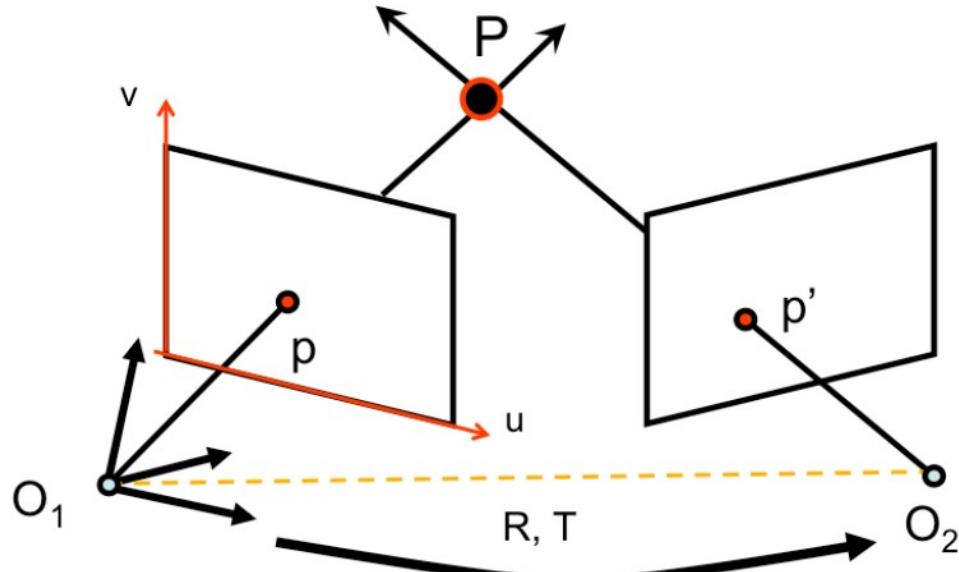


Credits: Ioannis Gkioulekas

Epipolar constraint



Epipolar constraint



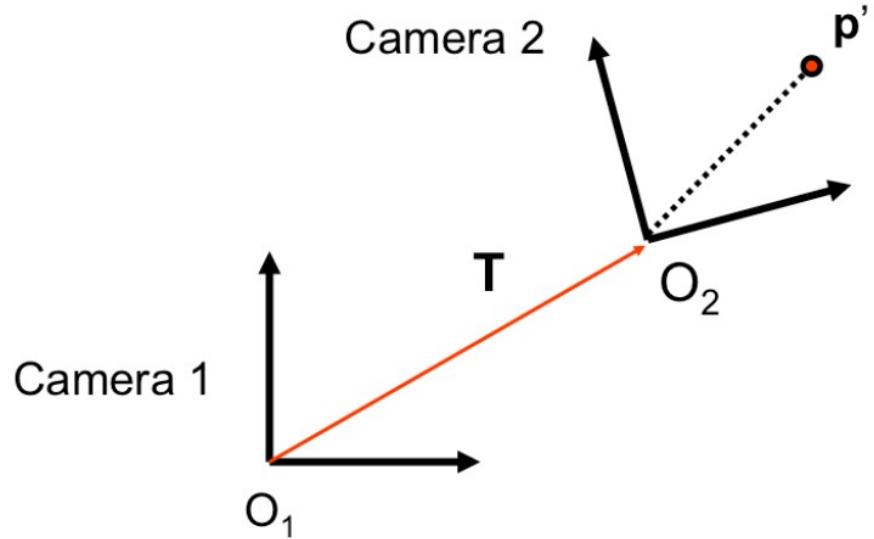
$$M = K \begin{bmatrix} I & 0 \end{bmatrix}$$

$$M P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = p \quad [\text{Eq. 3}]$$

$$M' = K' \begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

$$M' P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = p' \quad [\text{Eq. 4}]$$

Relative position for cameras

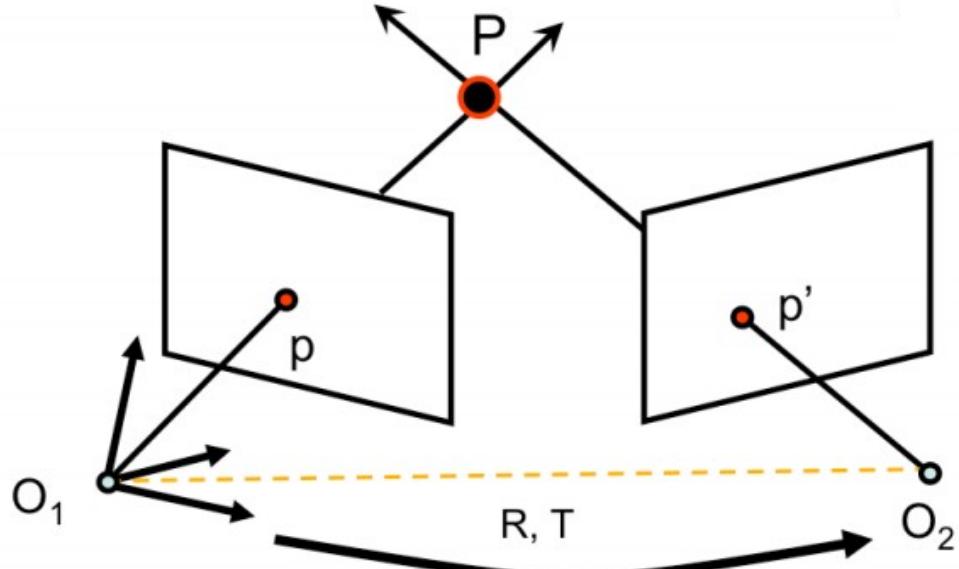


$$p'_{O_1} = Rp'_{O_2} + T$$

$$p'_{O_2} = R^T (p'_{O_1} - T)$$

- T is O_2 in the O_1 reference system
 - Actual translation
- R is the rotation matrix such as **free vector** p' for O_2 is Rp' for O_1
 - Rotation limited to epipolar plane

Epipolar constraint

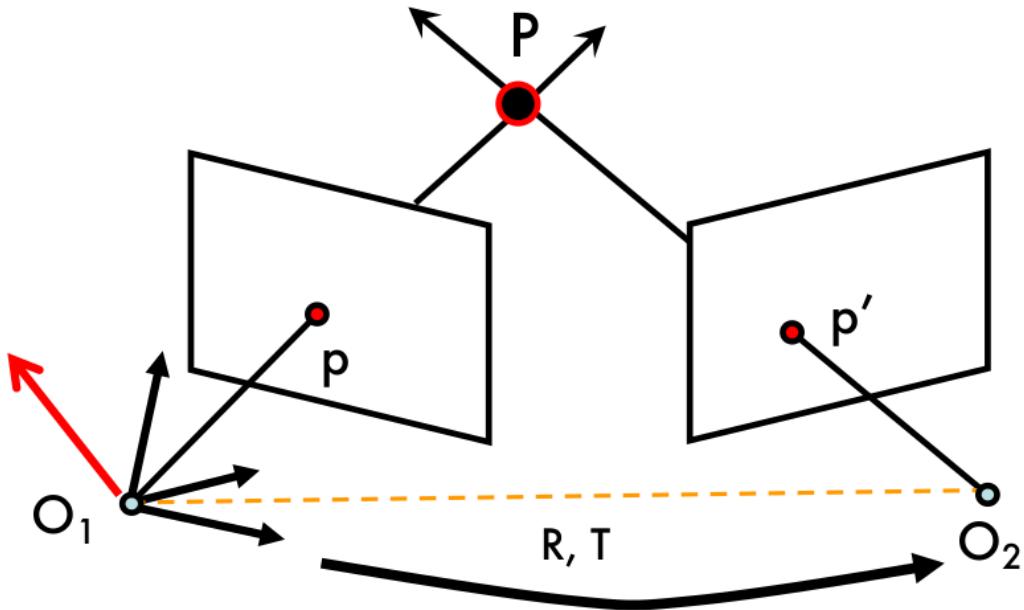


$$K_{\text{canonical}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = K \begin{bmatrix} I & 0 \end{bmatrix} \quad \boxed{K = K' \text{ are known} \\ (\text{canonical cameras})} \quad M' = K' \begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

$$\downarrow \qquad \qquad \downarrow$$
$$M = [I \quad 0] \quad [\text{Eq. 5}] \qquad \qquad M' = [R^T \quad -R^T T] \quad [\text{Eq. 6}]$$

Epipolar constraint



p' in first camera reference system is = $R p' + T$

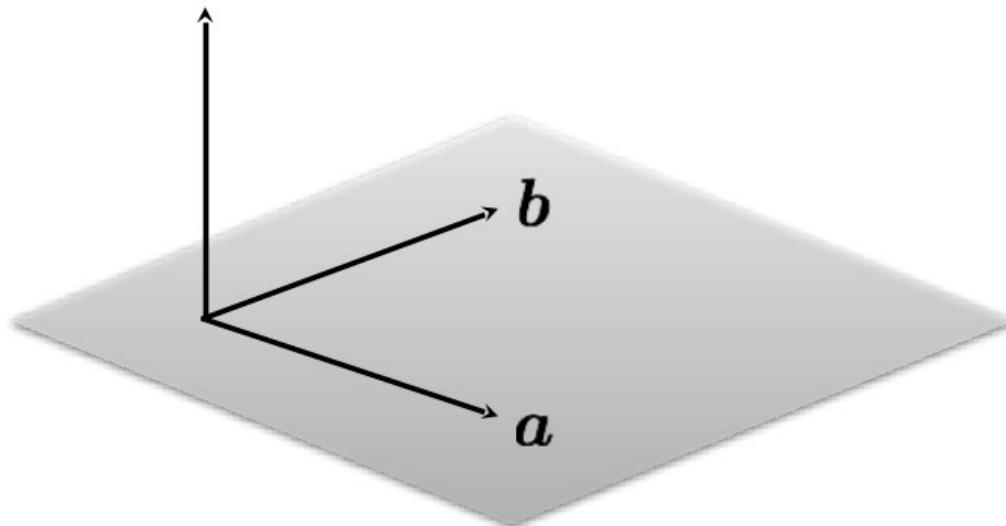
$T \times ((R p') + T) = T \times (R p')$ is perpendicular to epipolar plane

$$\rightarrow p^T \cdot [T \times (R p')] = 0 \quad [\text{Eq. 7}]$$

Cross product recall



$$\mathbf{c} = \mathbf{a} \times \mathbf{b}$$



$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix}$$

cross product of two vectors
in the same direction is zero

$$\mathbf{a} \times \mathbf{a} = 0$$

remember this!!!

$$\mathbf{c} \cdot \mathbf{a} = 0$$

$$\mathbf{c} \cdot \mathbf{b} = 0$$

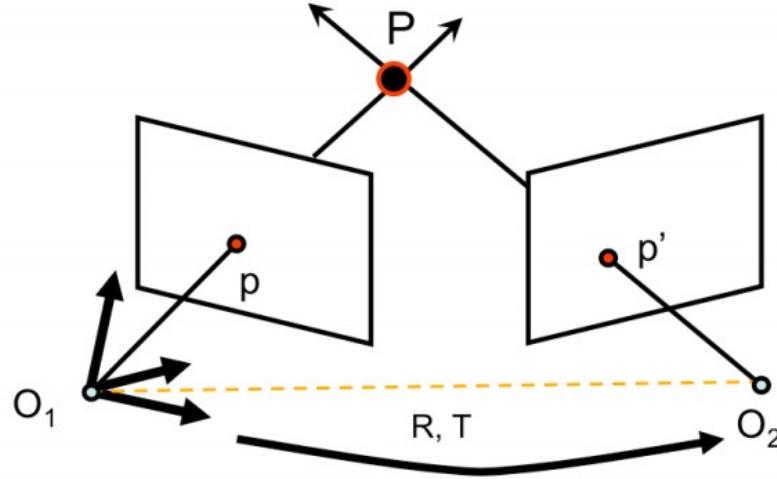
Credits: Ioannis Gkioulekas

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} a_y b_z - a_z b_y \\ a_z b_x - a_x b_z \\ a_x b_y - a_y b_x \end{bmatrix} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & -a_x & a_0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_x] \mathbf{b}$$

$$\mathbf{a} \times \mathbf{b} = [\mathbf{a}_x] \mathbf{b}$$

Essential Matrix



$$p^T \cdot [T \times (R p')] = 0 \rightarrow p^T \cdot [T_x] \cdot R p' = 0 \quad [\text{Eq. 8}]$$

$$E = \text{Essential matrix} \quad [\text{Eq. 9}]$$

(Longuet-Higgins, 1981)

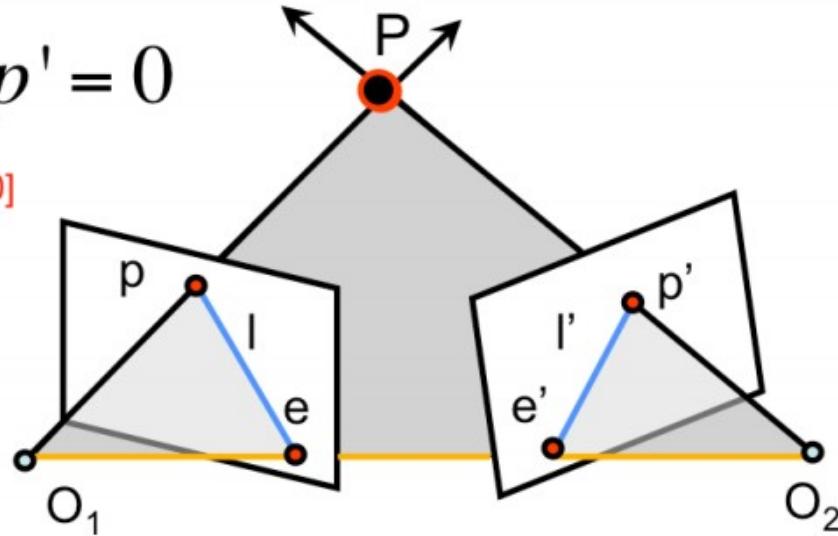
- E is the **Essential Matrix**

Essential Matrix



$$p^T \cdot E \cdot p' = 0$$

[Eq. 10]



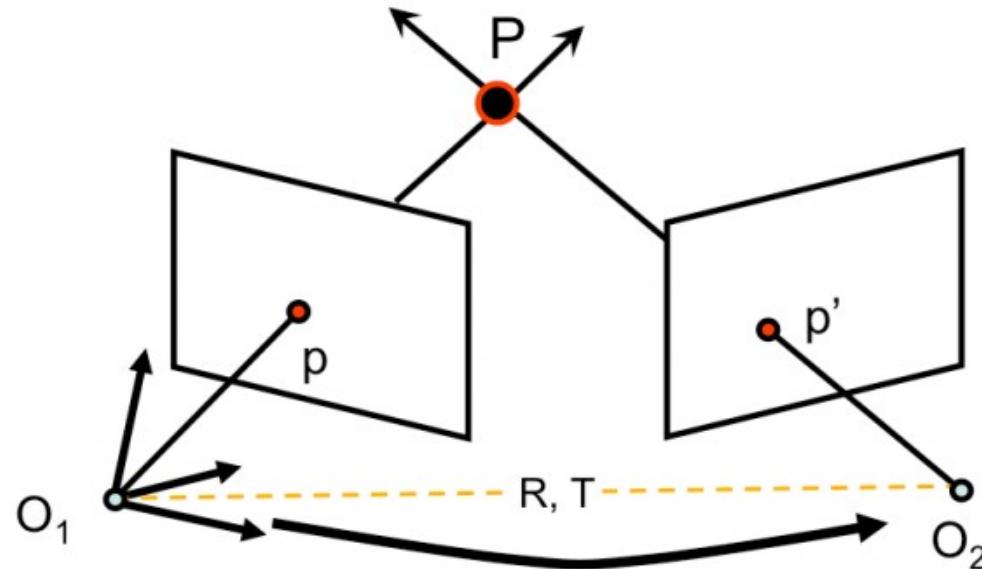
- $l = Ep' \rightarrow$ epipolar line for p
- $l' = E^T p \rightarrow$ epipolar line for p'
- $Ee' = 0 \quad E^T e = 0 \rightarrow$ namely epipoles are solution for homogeneous equation
- E is a 3×3 matrix with 5 DOF
- E is singular and its rank is 2

$$\mathbf{p}^T \mathbf{E} \mathbf{p}' = \mathbf{0} \quad \textit{Longuet-Higgins Equation}$$

- Usually we do not use ideal cameras
- Is then E so useful?
- Yes if we think about normalized camera coordinates and not image coordinates
 - For image coordinates we need to know K

$$\mathbf{p}_c = \mathbf{K}^{-1} \mathbf{p}$$

Epipolar constraint (non ideal camera)



$$M = K \begin{bmatrix} I & 0 \end{bmatrix}$$

$$p_c = K^{-1} p \quad [\text{Eq. 11}]$$

$$M' = K' \begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

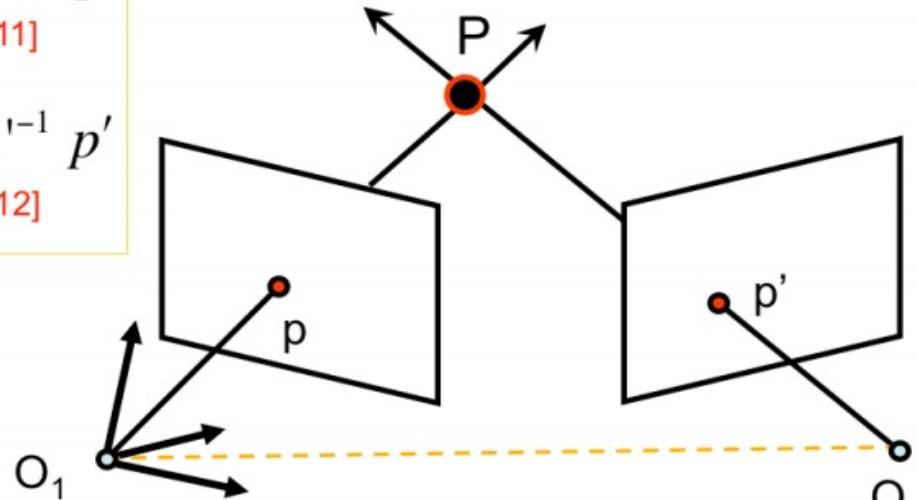
$$p'_c = K'^{-1} p' \quad [\text{Eq. 12}]$$

Epipolar constraint (non ideal camera)



$$p_c = K^{-1} p \quad [\text{Eq. 11}]$$

$$p'_c = K'^{-1} p' \quad [\text{Eq. 12}]$$

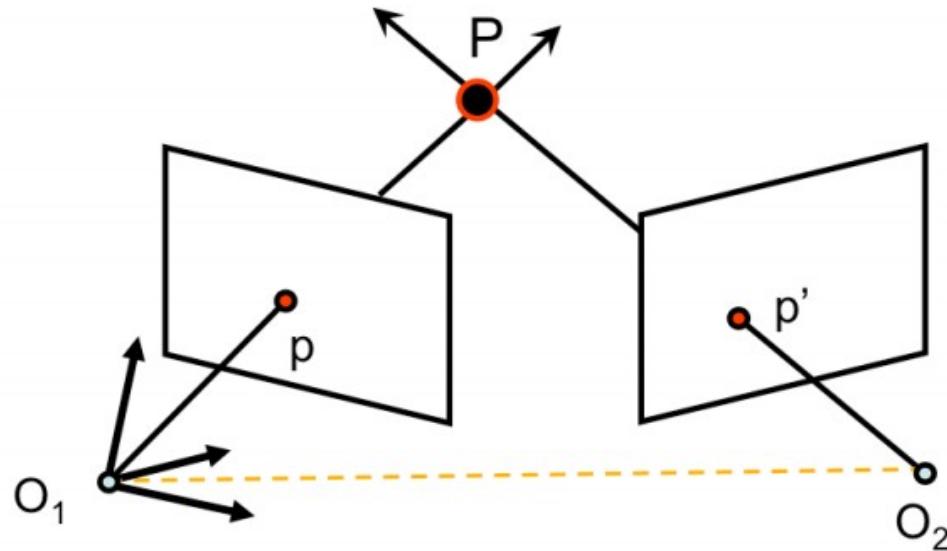


[Eq.9]

$$p_c^T \cdot [T_x] \cdot R p'_c = 0 \rightarrow (K^{-1} p)^T \cdot [T_x] \cdot R K'^{-1} p' = 0$$

$$p^T \boxed{K^{-T} \cdot [T_x] \cdot R K'^{-1}} p' = 0 \rightarrow p^T \boxed{F} p' = 0 \quad [\text{Eq. 13}]$$

Fundamental Matrix



[Eq. 13]

$$p^T F p' = 0$$

$$F = K^{-T} \cdot [T_x] \cdot R \cdot K'^{-1}$$

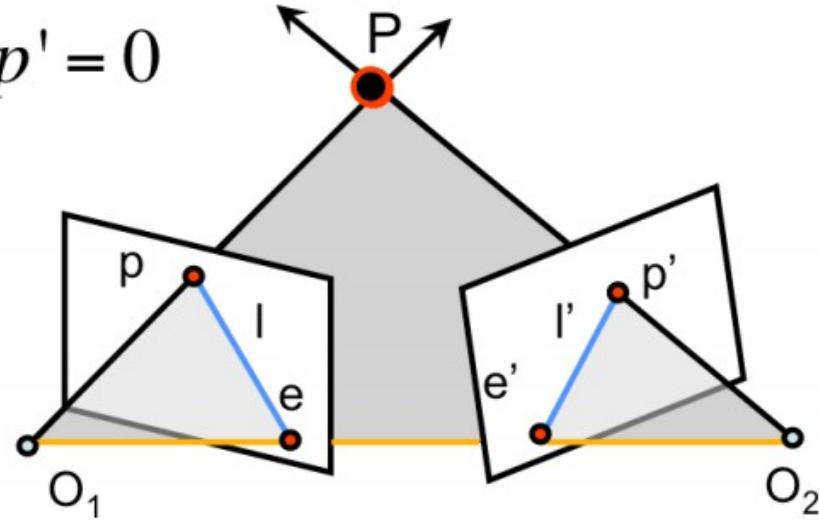
[Eq. 14]

- $F \rightarrow$ Fundamental Matrix (Faugeras and Luong 1992)

Fundamental Matrix



$$p^T \cdot F \cdot p' = 0$$



- $l = Fp' \rightarrow$ epipolar line for p
- $l' = F^Tp \rightarrow$ epipolar line for p'
- $Fe' = 0 \quad F^Te = 0 \rightarrow$ namely epipoles are solution for homogeneous equation
- F is a 3x3 matrix with 7 DOF
- F is singular and its rank is 2

Essential & Fundamental Matrices

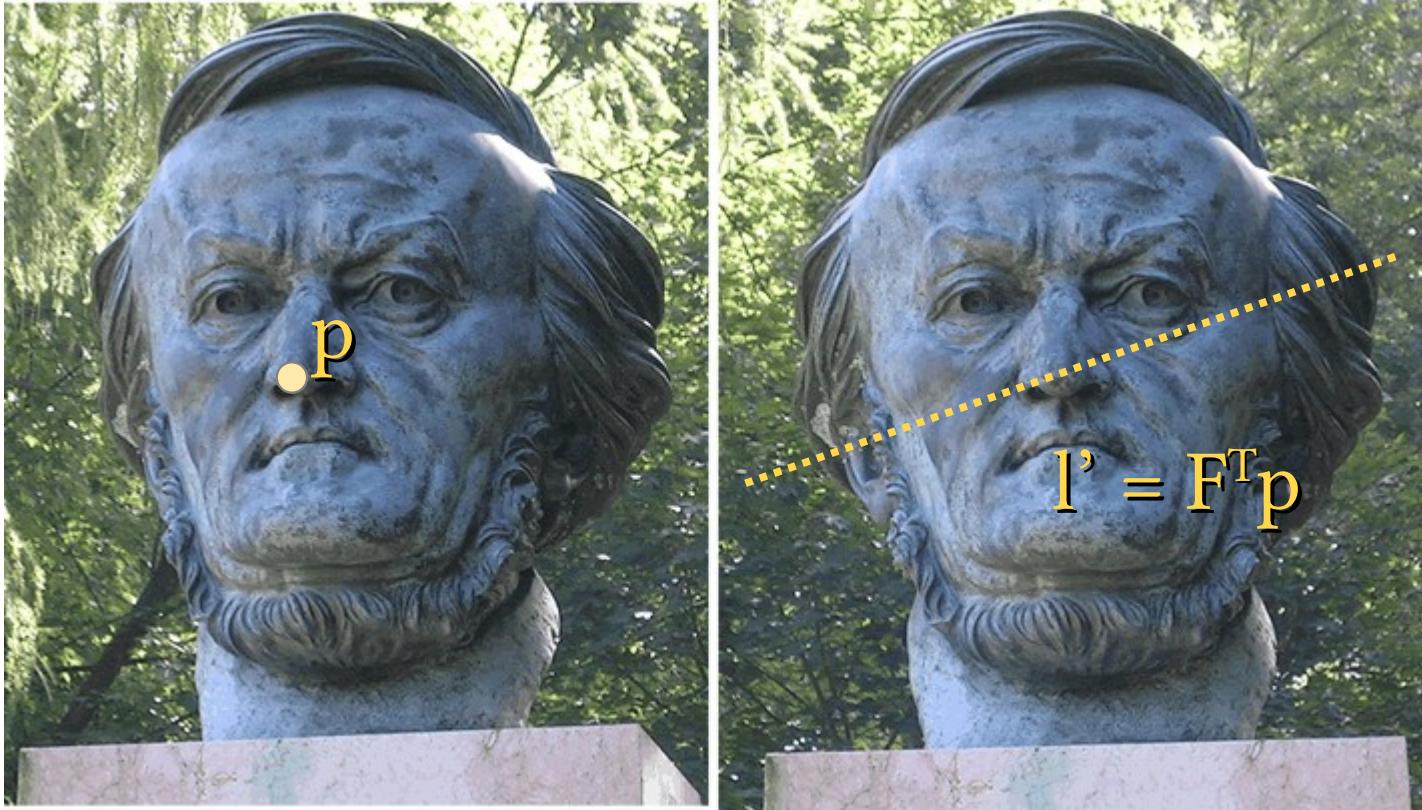
$$p_c^T E p'_c = 0 \quad p^T F p' = 0$$

$$p_c = K^{-1} p \quad p'_c = K'^{-1} p'$$

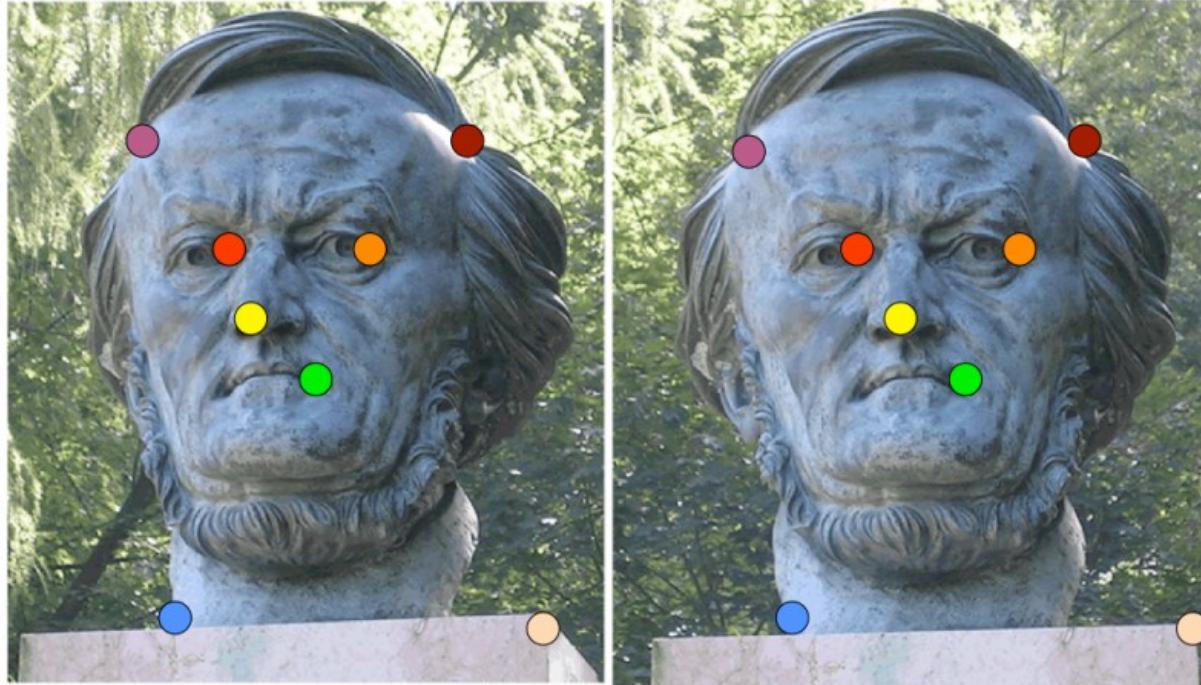
$$p^T K^{-T} E K^{-1} p' = 0$$

$$K^{-T} E K^{-1} = F$$

Why we want F?

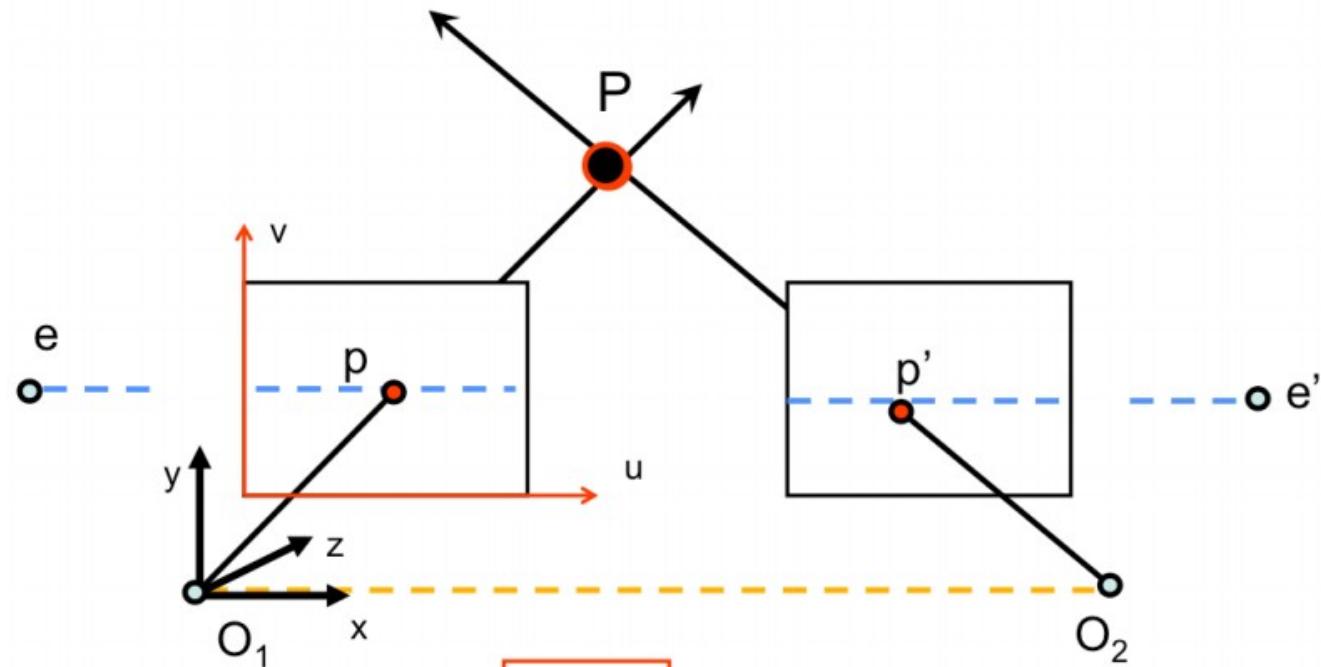


Fundamental Matrix estimation



- Can we estimate F without knowing nothing about K , R , T ?
- Yes → 8-point algorithm

Special case: parallel image planes



$K_1 = K_2 = \text{known}$

x parallel to O_1O_2

$E=?$

Hint :
 $R = I$ $T = (T, 0, 0)$

Special case: parallel image planes

$$\mathbf{E} = [\mathbf{T}_x] \cdot \mathbf{R}$$

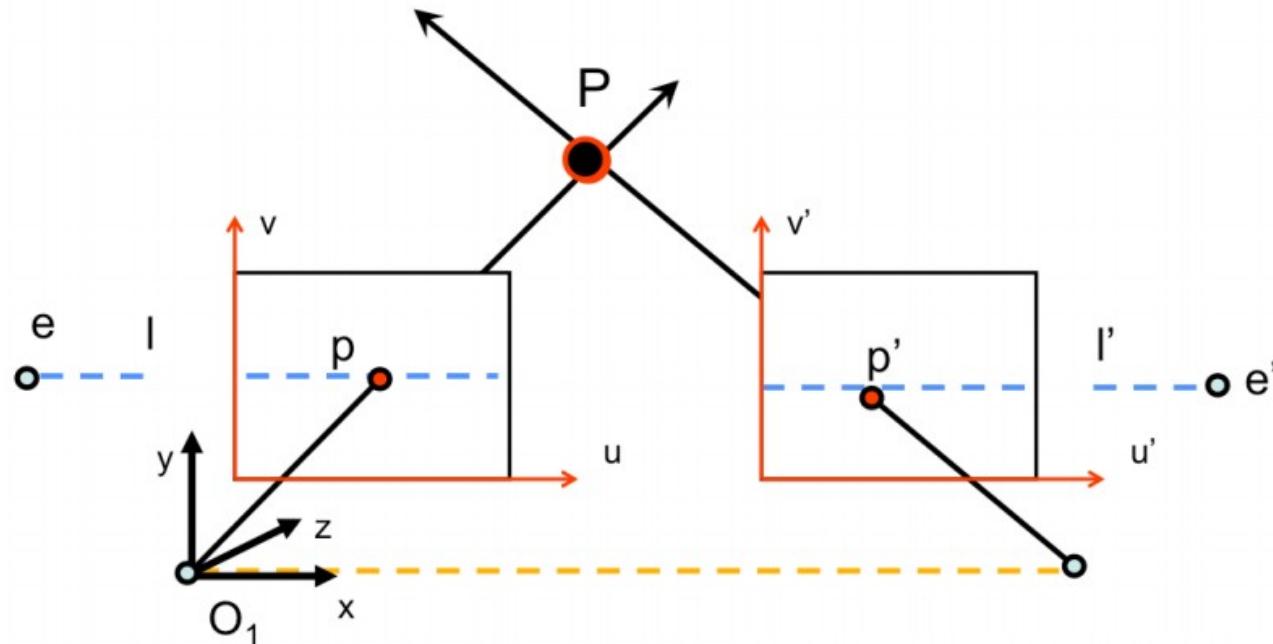
$$\mathbf{E} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

[Eq. 20]

$$\mathbf{T} = [T \ 0 \ 0]$$

$$\mathbf{R} = \mathbf{I}$$

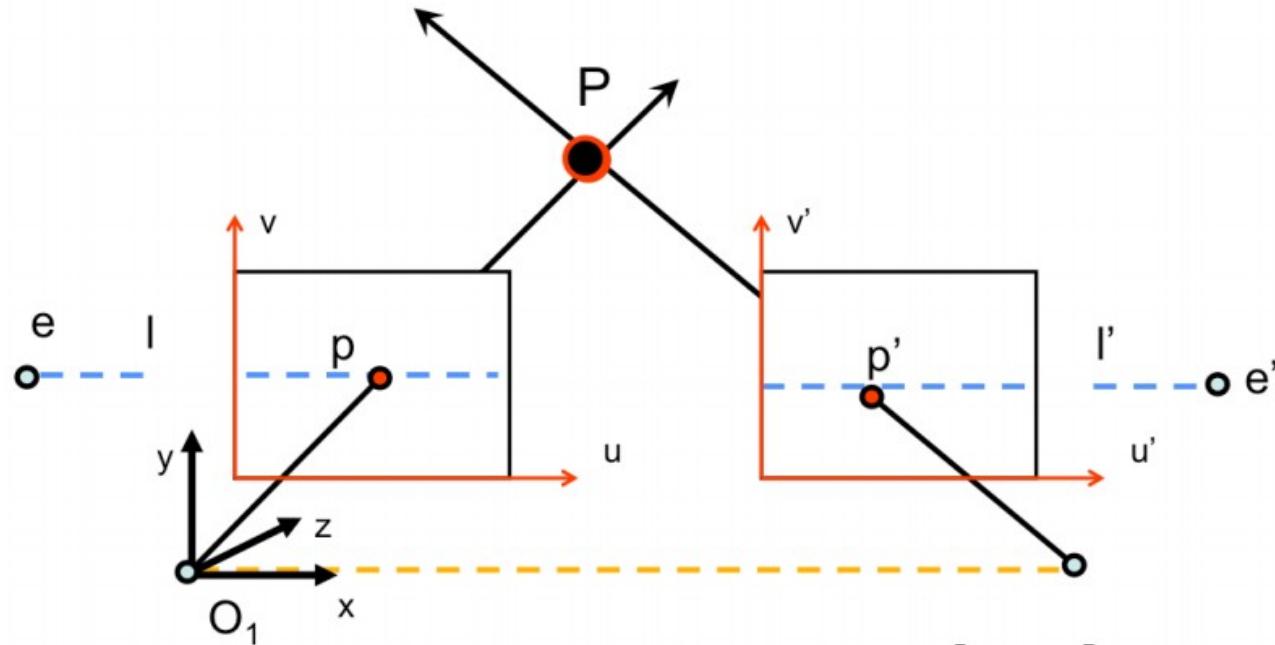
Special case: parallel image planes



- Epipolar lines are horizontal

$$l = E p' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -T \\ T v' \end{bmatrix} \text{ horizontal!}$$

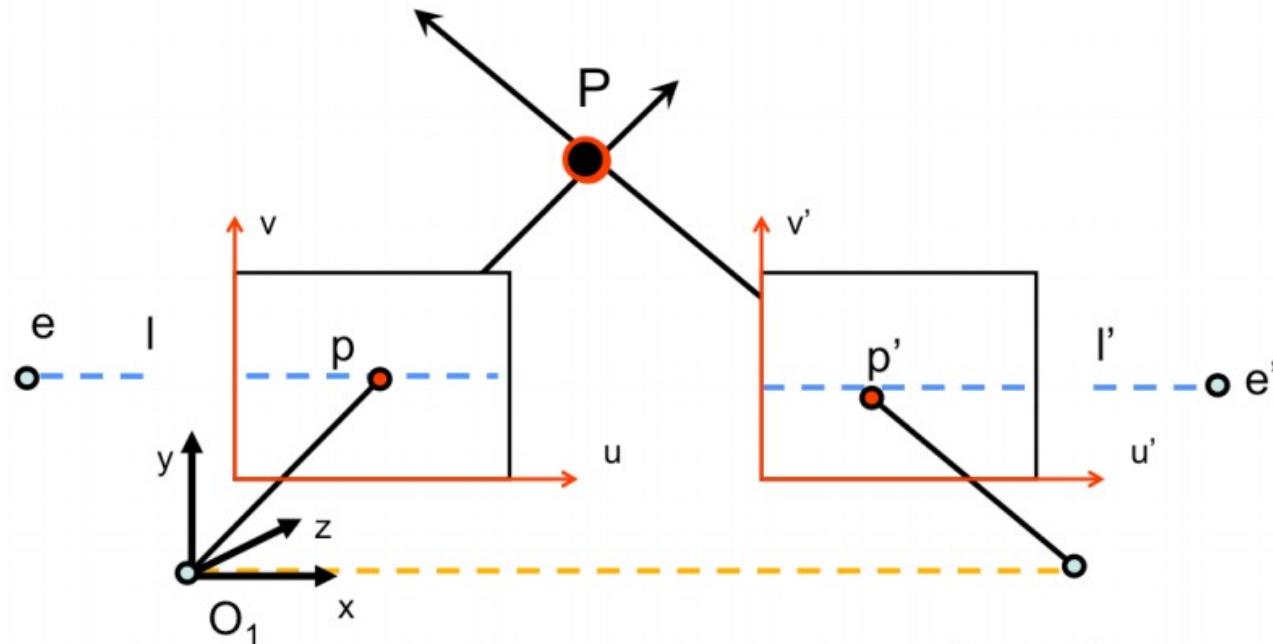
Special case: parallel image planes



- How p & p' are related?

$$p^T \cdot E \cdot p' = 0$$

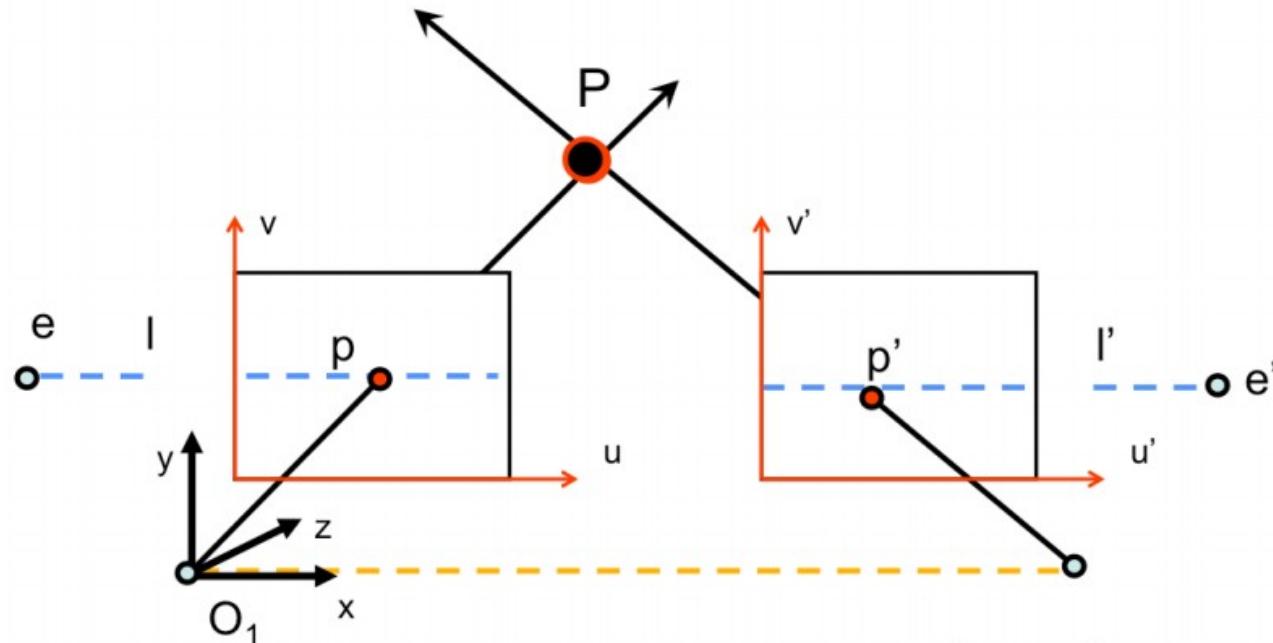
Special case: parallel image planes



- How p & p' are related?

$$(u \quad v \quad 1) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow (u \quad v \quad 1) \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \Rightarrow Tv = Tv' \Rightarrow v = v'$$

Special case: parallel image planes



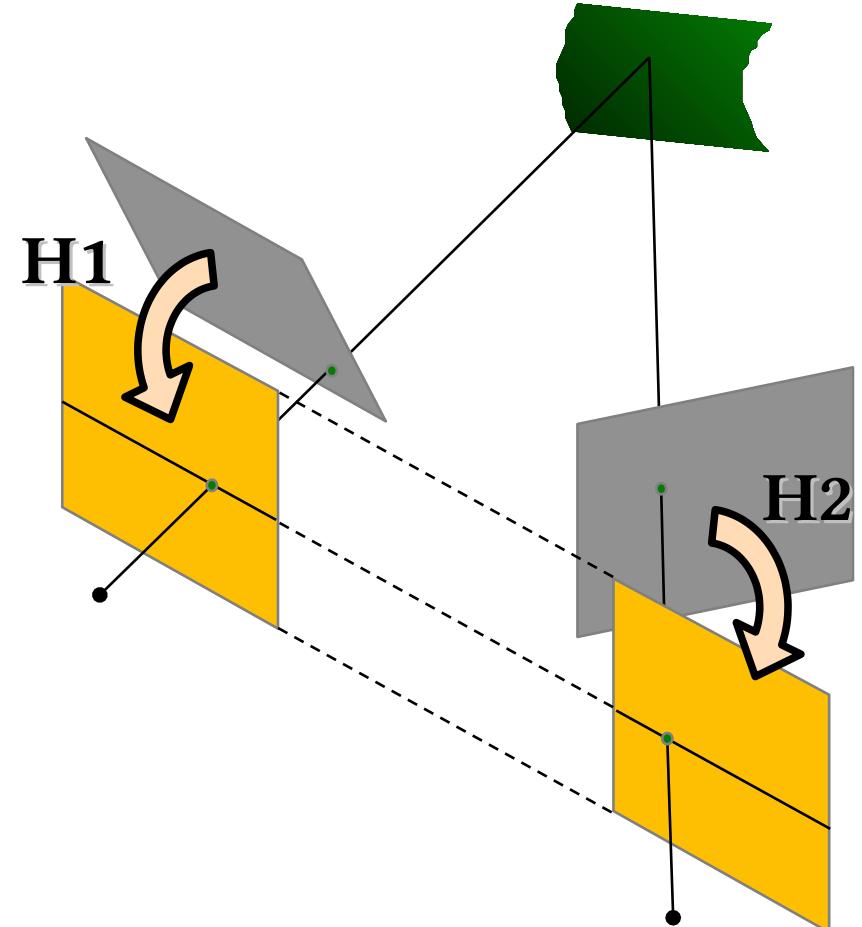
- $v=v'$ means the same line!
- The search for p' is simpler...

True for canonical cameras!

Also when $K=K'$!

Image Rectification

- Parallel image planes are better to handle for stereovision
- Simply accurately align cameras
 - Good idea, but it does not work!
 - Precision, noise, thermal effects...
- Reproject camera planes onto a common plane parallel to the line between camera centers



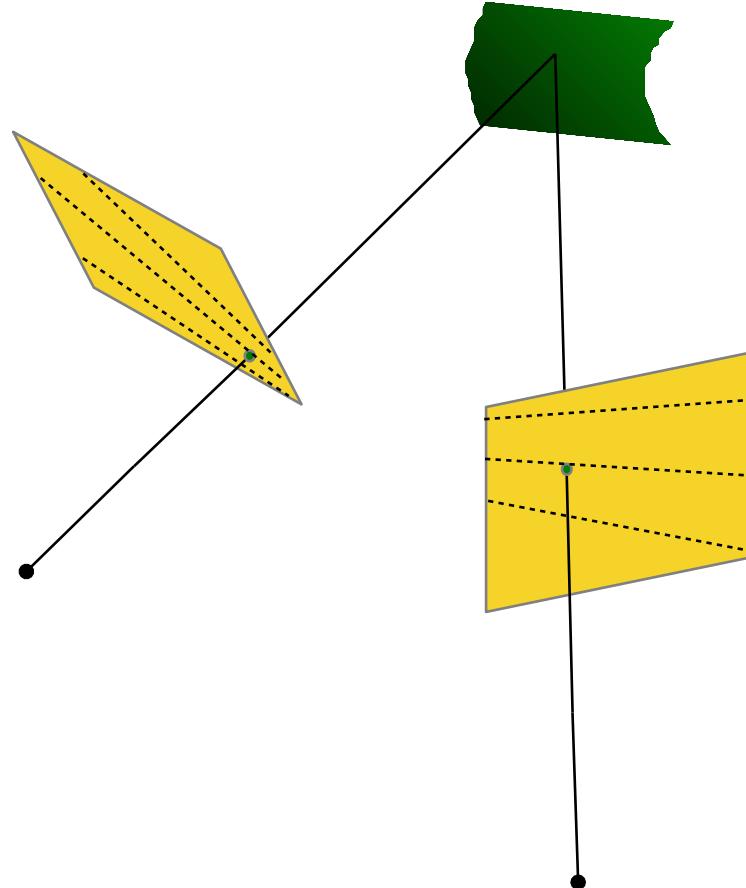
Stereo Image Rectification

- Assuming $K=K'$ as known
 - 1) Compute $E \rightarrow$ we get R
 - 2) Rotate right image by R
 - 3) Rotate both images to move e and e' to ∞
 - We need to compute a R_{rect}
 - 4) Adjuste the scale

Stereo Image Rectification



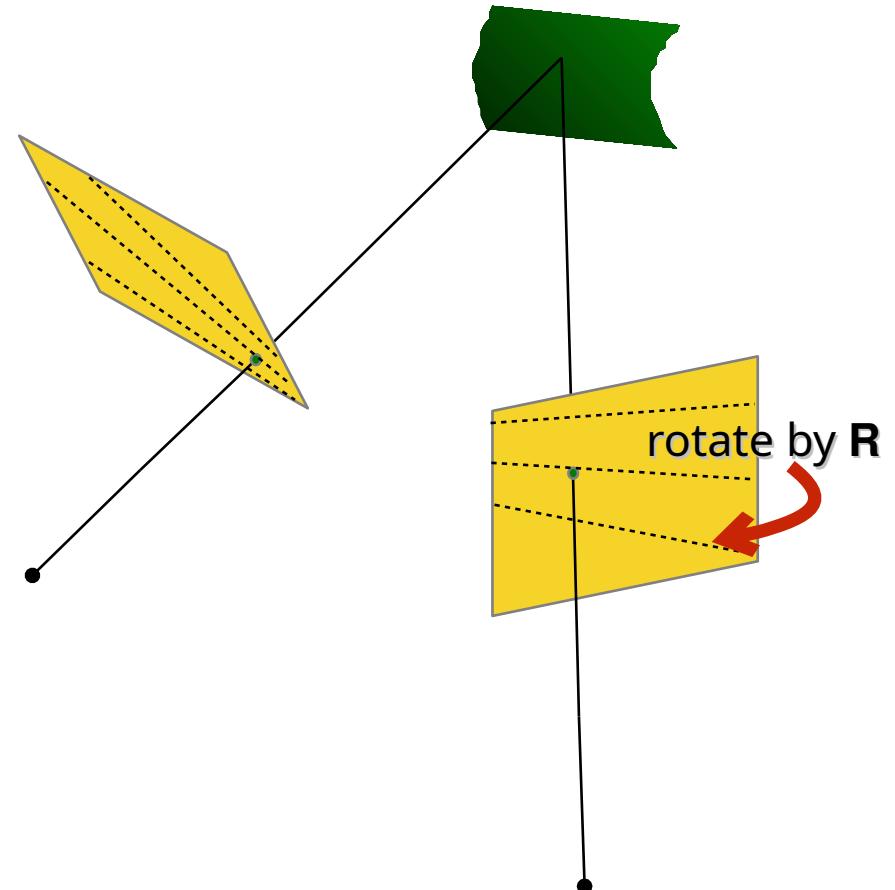
1) Compute $E \rightarrow$ we get R



Stereo Image Rectification



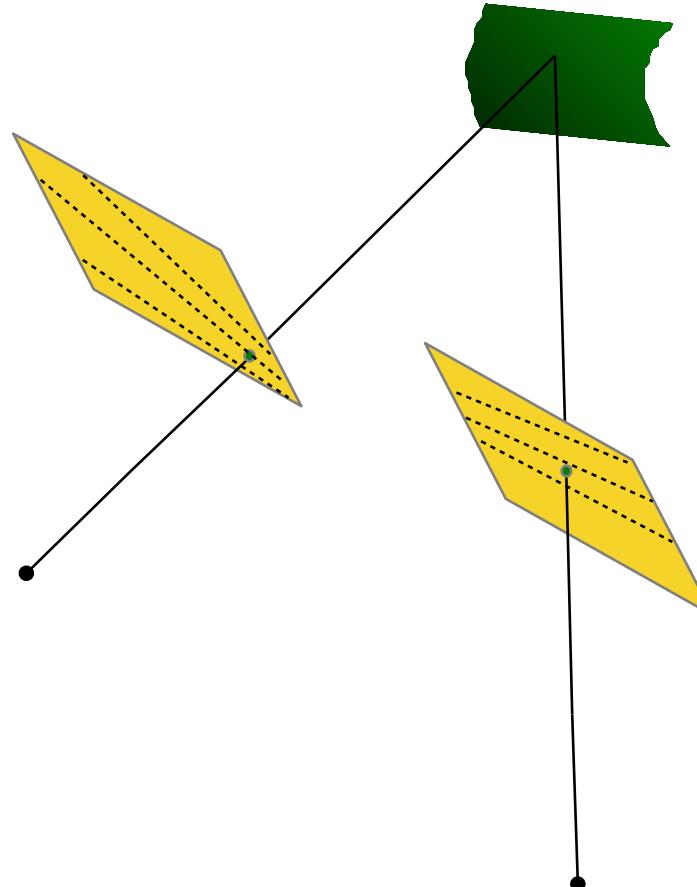
- 1) Compute $E \rightarrow$ we get R
- 2) Rotate left image by R



Stereo Image Rectification



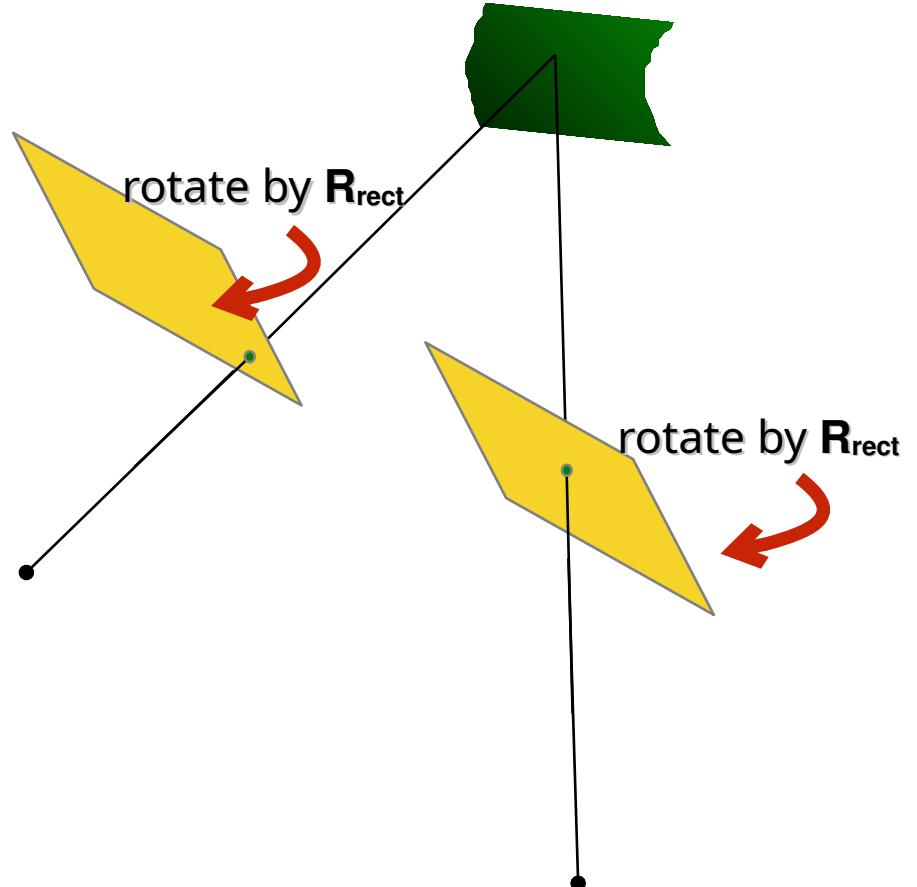
- 1) Compute $E \rightarrow$ we get R
- 2) Rotate right image by R



Stereo Image Rectification



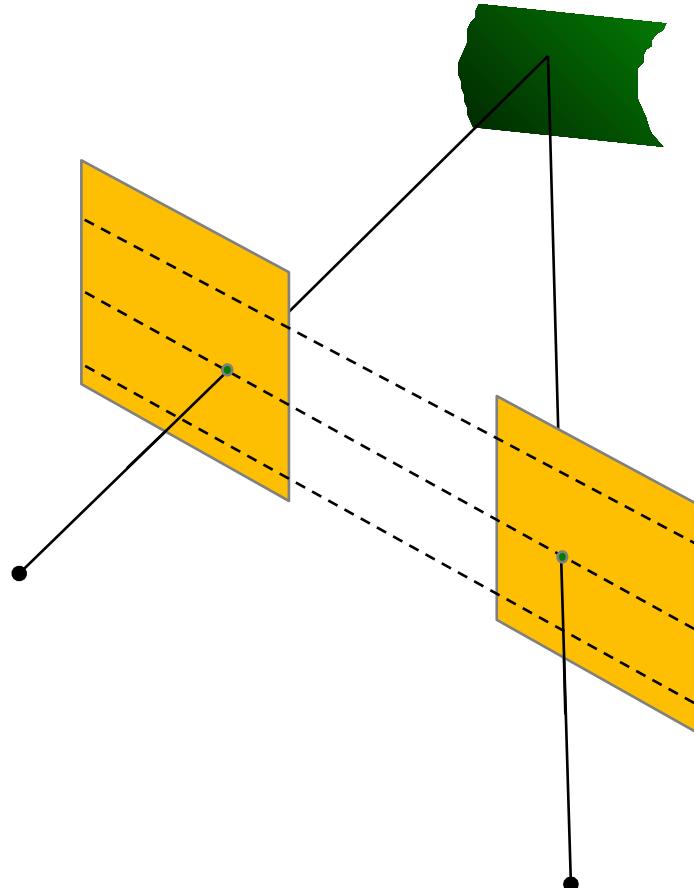
- 1) Compute $E \rightarrow$ we get R
- 2) Rotate right image by R
- 3) Rotate both images by R_{rect}



Stereo Image Rectification



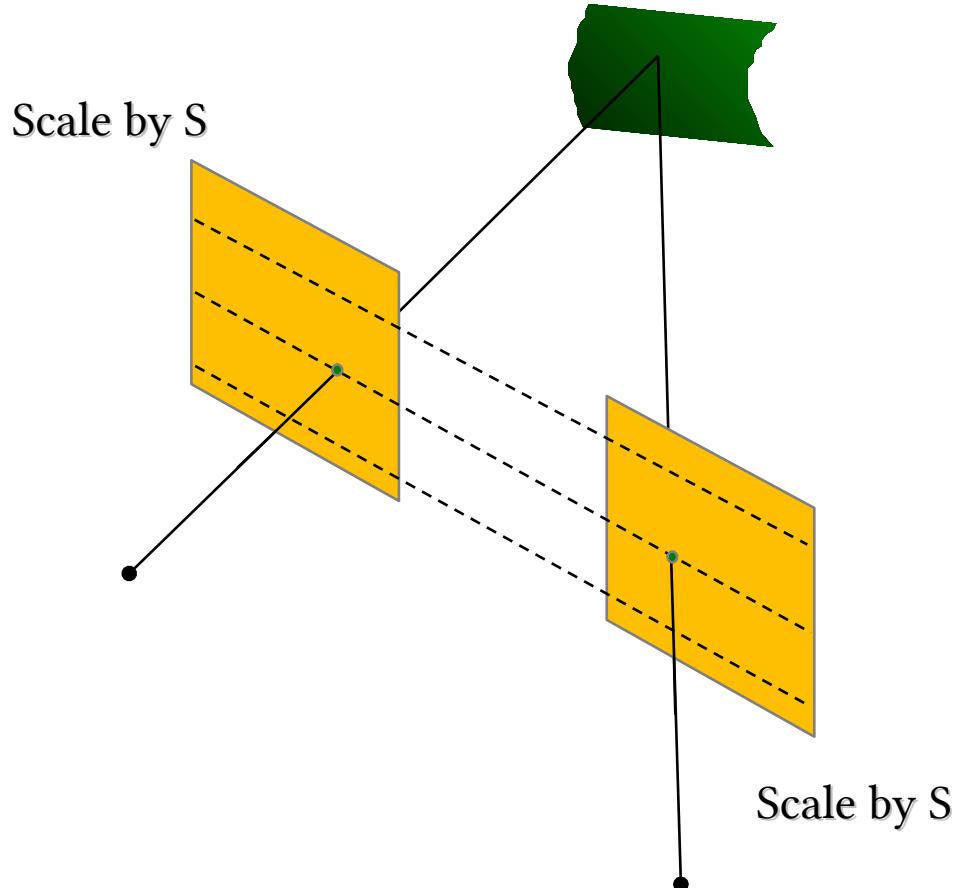
- 1) Compute $E \rightarrow$ we get R
- 2) Rotate right image by R
- 3) Rotate both images by R_{rect}



Stereo Image Rectification



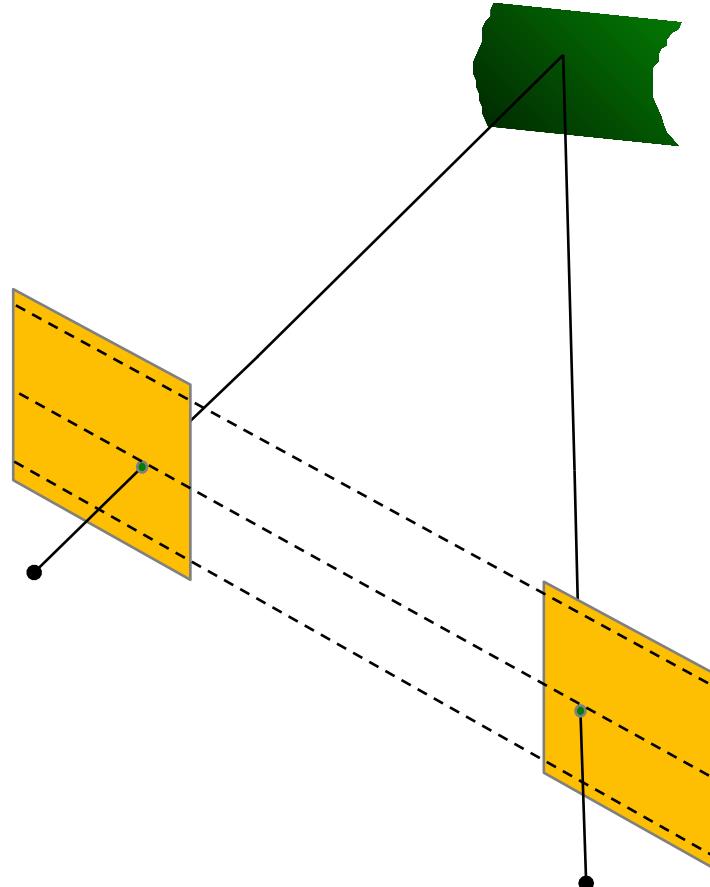
- 1) Compute $E \rightarrow$ we get R
- 2) Rotate right image by R
- 3) Rotate both images by R_{rect}
- 4) Scale images



Stereo Image Rectification



- 1) Compute $E \rightarrow$ we get R
- 2) Rotate right image by R
- 3) Rotate both images by R_{rect}
- 4) Scale images



Stereo Image Rectification

Actually not so simple...

- 1) Estimate E using the 8 point algorithm
- 2) Estimate the epipole e (solve $Ee=0$)
- 3) Build R_{rect} from e
- 4) Decompose E into R and T
- 5) Set $R_1 = R_{\text{rect}}$ and $R_2 = RR_{\text{rect}}$
- 6) Rotate each left camera point $x' \sim Hx$ where $H = KR_1$
- 7) Alter the focal length (inside K) to keep as much points within the original image size
- 8) Repeat 6 and 7 for right camera points using R_2



UNIVERSITÀ DI PARMA

Epipolar Geometry

Question time!

