UNIVERSITÀ DI PARMA

Epipolar Geometry

Summary

UNIVERSITÀ DI PARMA

- Why stereo?
- Epipolar constraints
- Essential and Fundamental matrices
- Stereo Images rectification

Credits

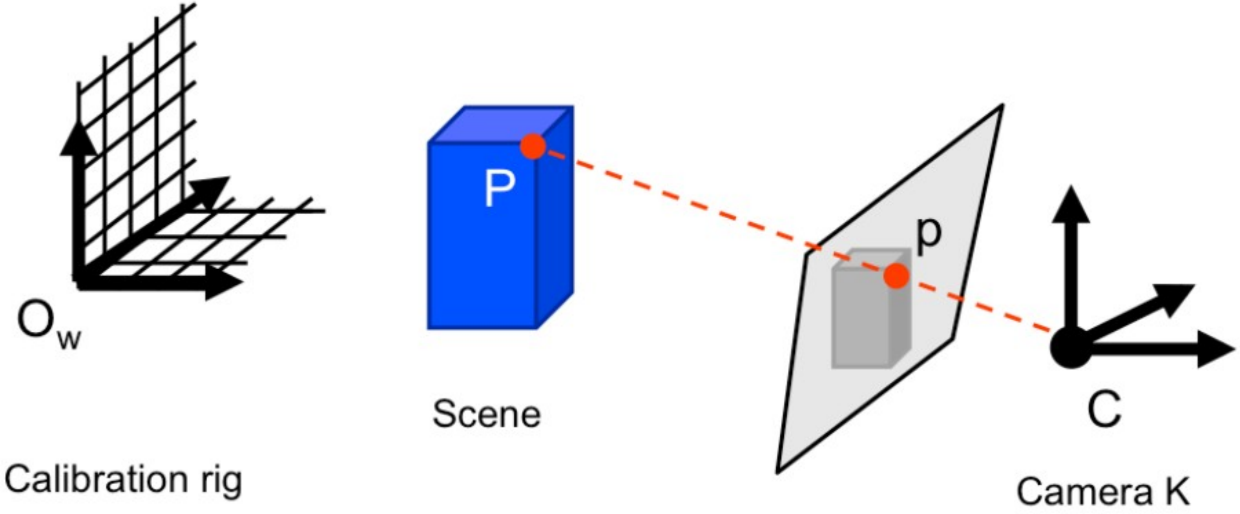
- [FP] D. A. Forsyth and J. Ponce. Computer Vision: A Modern Approach (2nd Edition). Prentice Hall, 2011.

- [HZ] R. Hartley and A. Zisserman. Multiple View Geometry in Computer Vision. Cambridge University Press, 2003.

- **CS231A · Computer Vision: from 3D reconstruction to recognition, Prof. Silvio Savarese – Stanford University**

Credits for many images/equations: Silvio Savarese

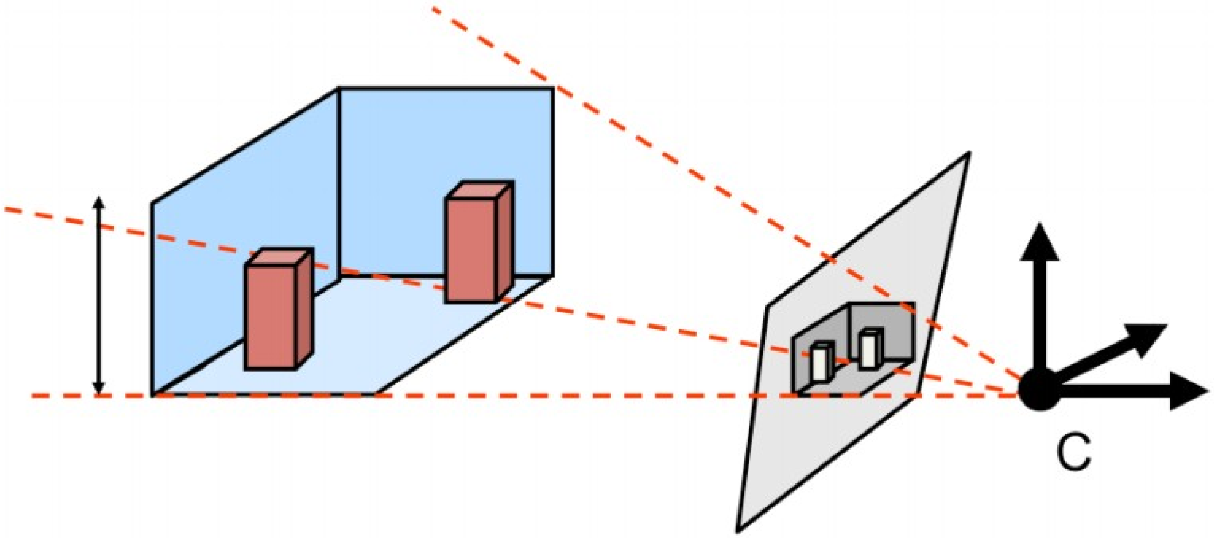
Without stereo...



Calibration rig + rig position

We can compute K and RT

Without stereo...



Vanishing lines and points + orthogonal planes

We can compute K

Without stereo...

Why it is **soooo** difficult?

Line of Sight: multiple potential correspondances for p in the 3D world..

Without stereo...

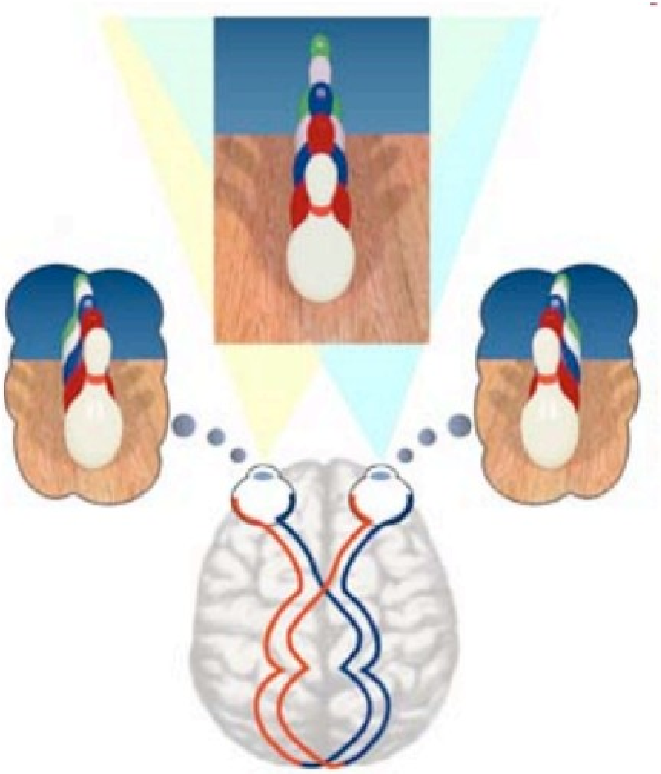
Without stereo...

Without stereo…

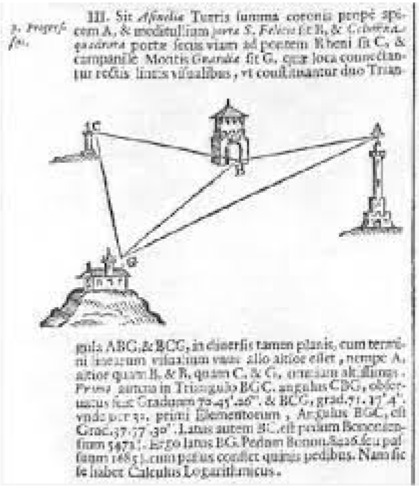
Without stereo...

Nature and monocular vision

Stereo vision wins



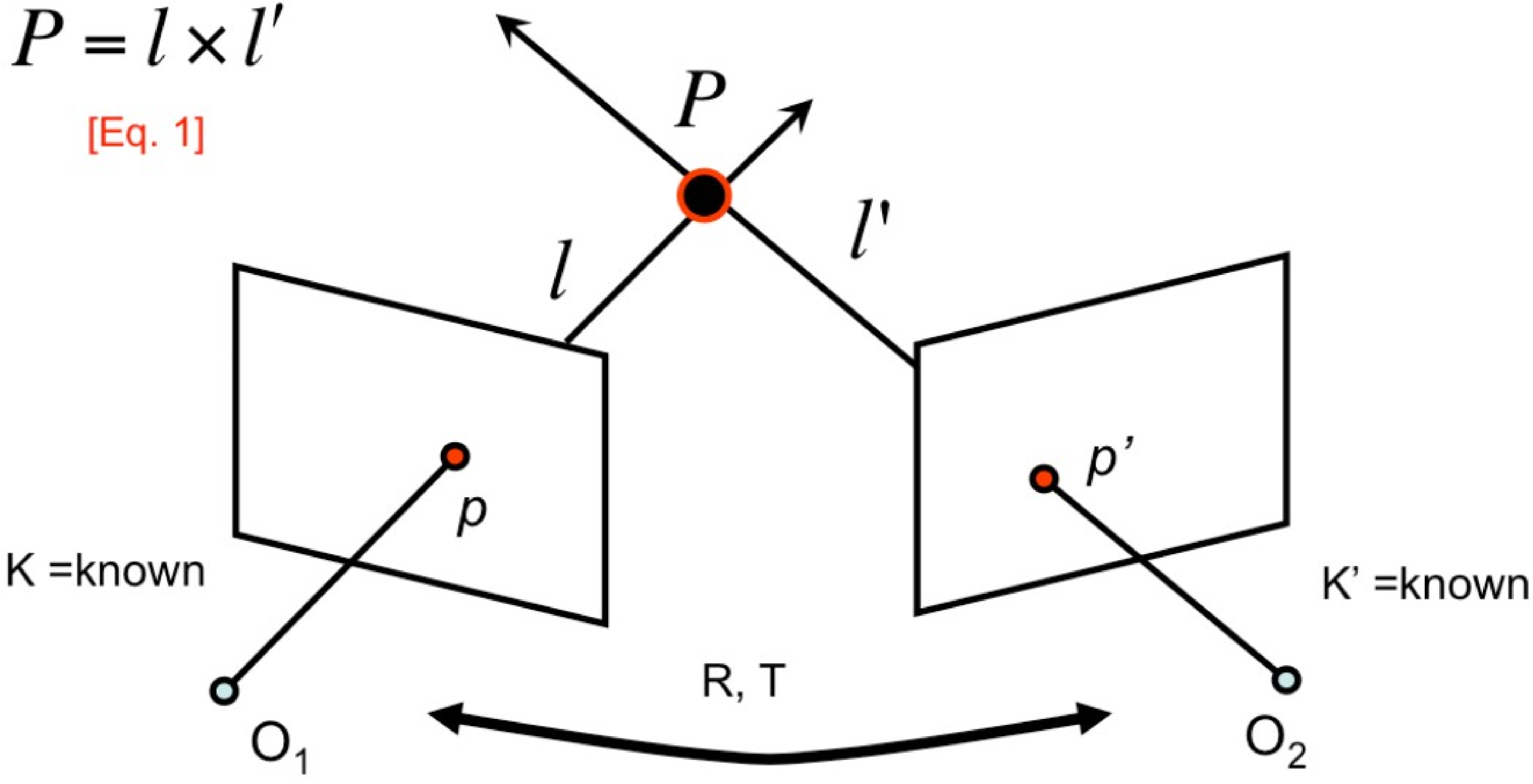
Triangulation


III. Sit *Asinelia* Turris summa coronis propè apicem A, & meditullium *porta S. Felicis* sit B, & *Columna quadrata* portæ secus viam ad pontem Rheni sit C, & campanile Montis *Guardia* sit G, quæ loca connectantur rectis lineis visualibus, vt constituantur duo Triangula ABG, & BCG, in diuersis tamen planis, cum termini linearum visualium vnus alio altior esset, nempe A altior quam B, & B, quam C, & G, omnium altissimus. *Primo* autem in Triangulo BGC, angulus CBG, obseruatus fuit Graduum 70.45'.26". & BCG, grad. 71.17'.4". vnde per 32. primi Elementorum, Angulus BGC, est Grad. 37.57'.30". Latus autem BC, est pedum Bononiensium 5471. Ergo latus BG, Pedum Bonon. 8426. seu passuum 1685 ⅕, cum passus constet quinis pedibus. Nam sic se habet Calculus Logarithmicus.

Triangulation

$$P = l \times l'$$

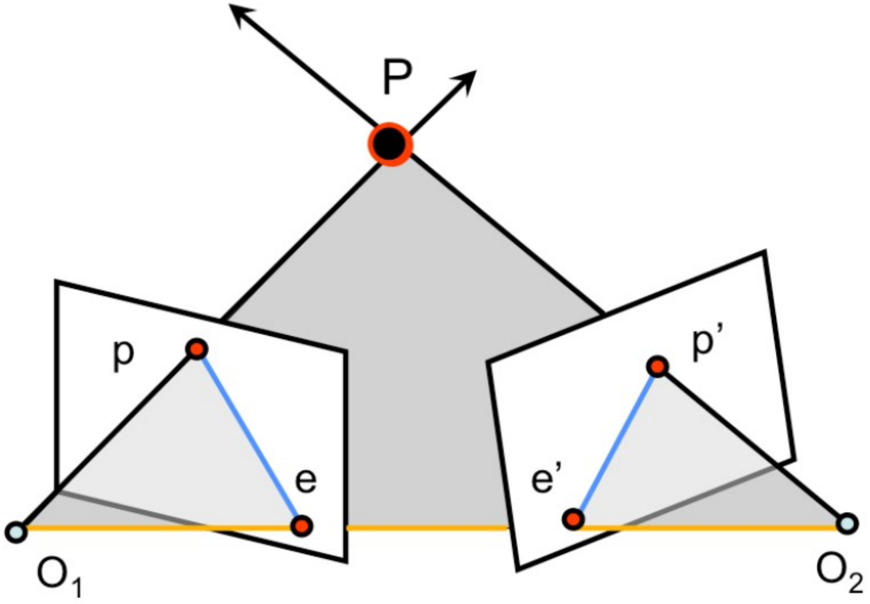
[Eq. 1]



Key problems

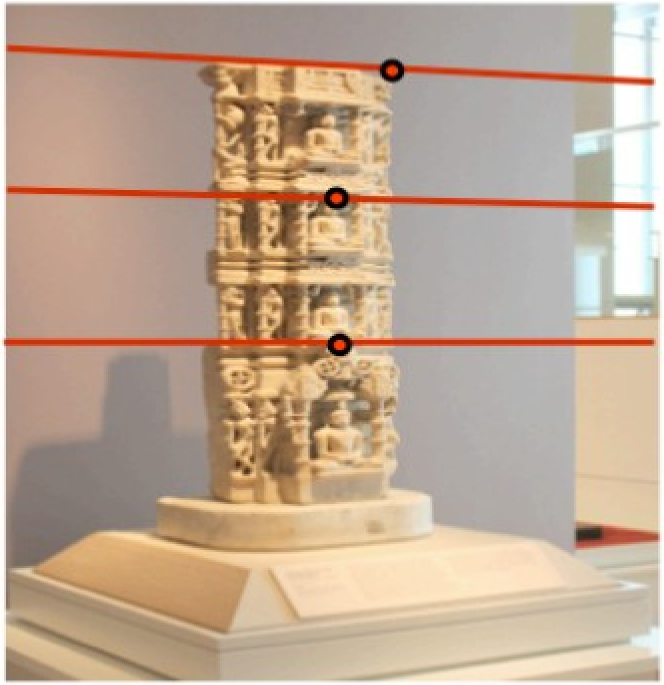
- ~~**Correspondances:** given a p in one image find p' in the other image~~
- **Camera geometry:** given a set of correspondences find camera parameters
- **Scene Geometry:** given a set of correspondences and parameters of both cameras reconstruct the 3D scene

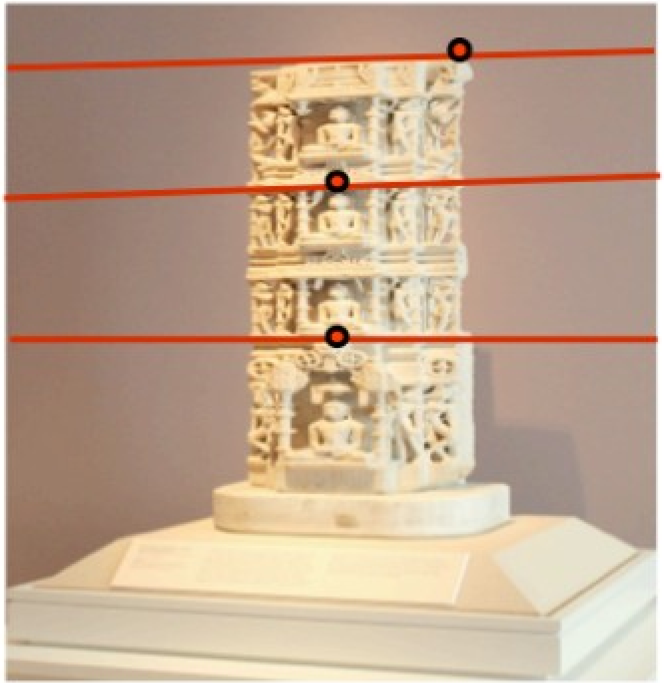
Epipolar geometry



- O1-O2-P: epipolar plane
- O1-O2: baseline
- pe & p'e': epipolar lines (i.e. they meet!)
- e & e': epipoles
  - Intersection of baseline with image planes
  - Projection of O1&O2

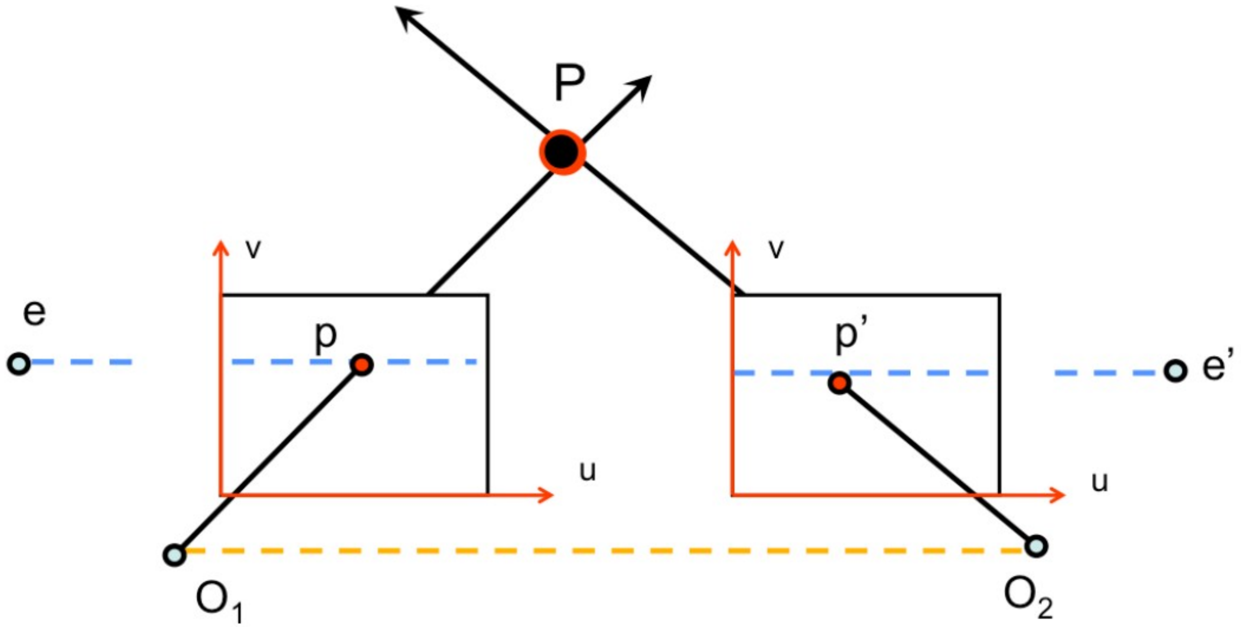
Epipolar lines





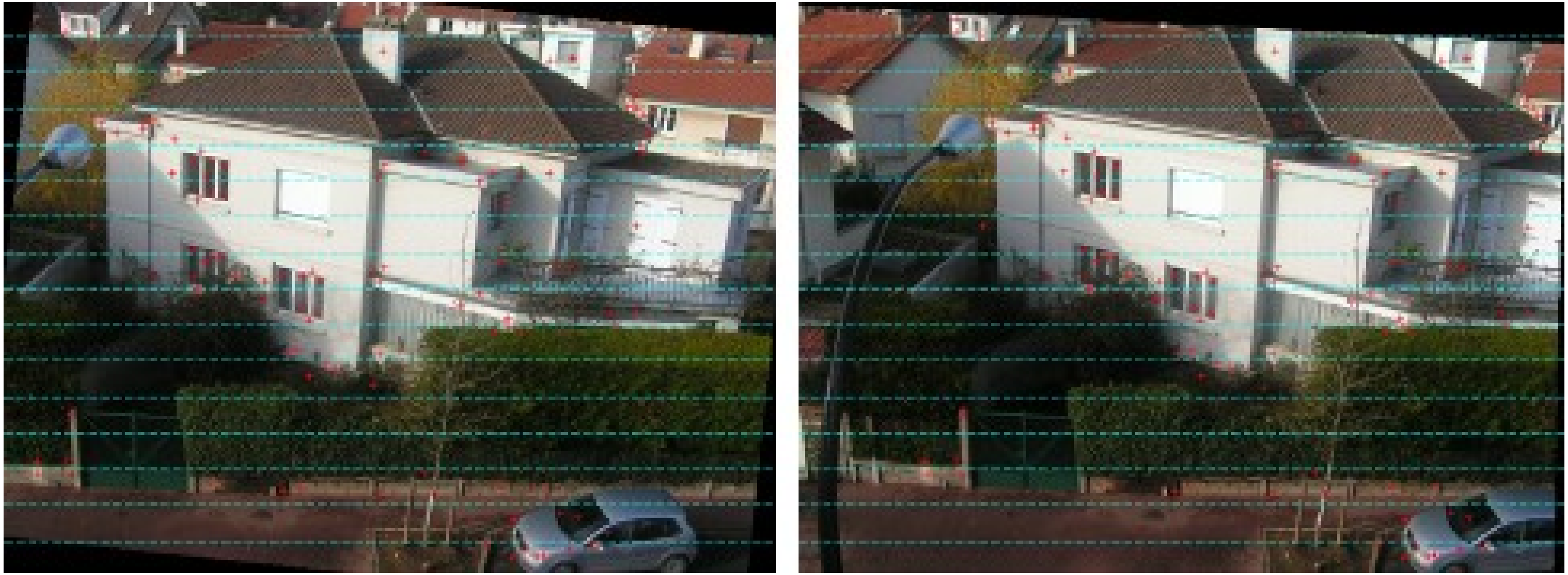
Epipolar lines

Special case: parallel image planes



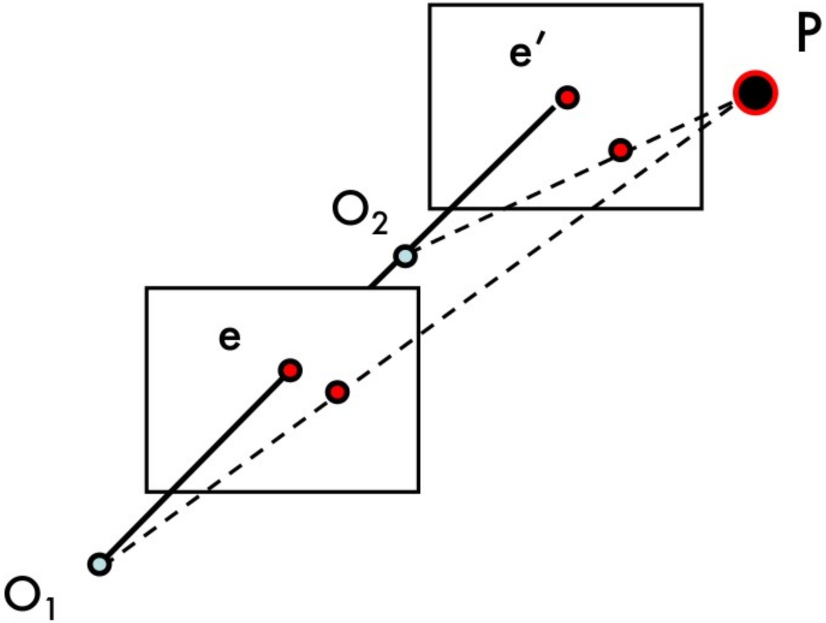
- Epipoles are at infinity
- Also intersection between baseline and image plane are at infinity
- Epipolar lines are parallel to u axis

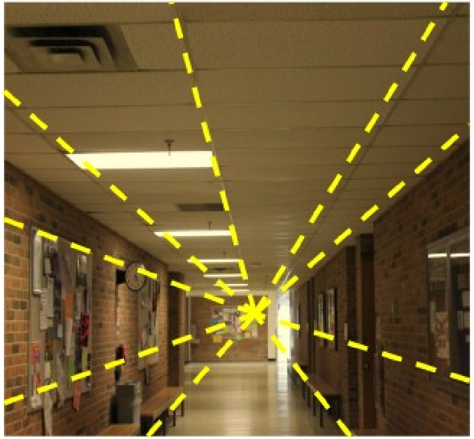
Special case: parallel image planes

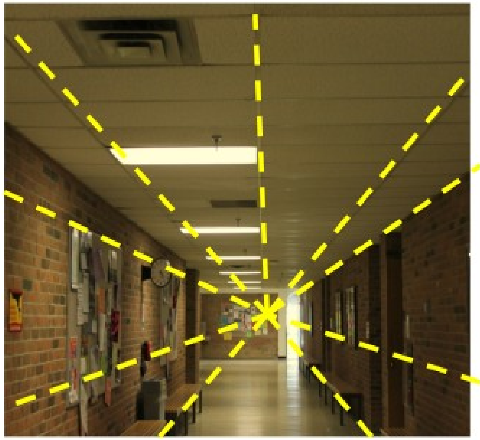


- Example of two images whose planes are parallel to each other
- Epipolar lines are parallel to each other

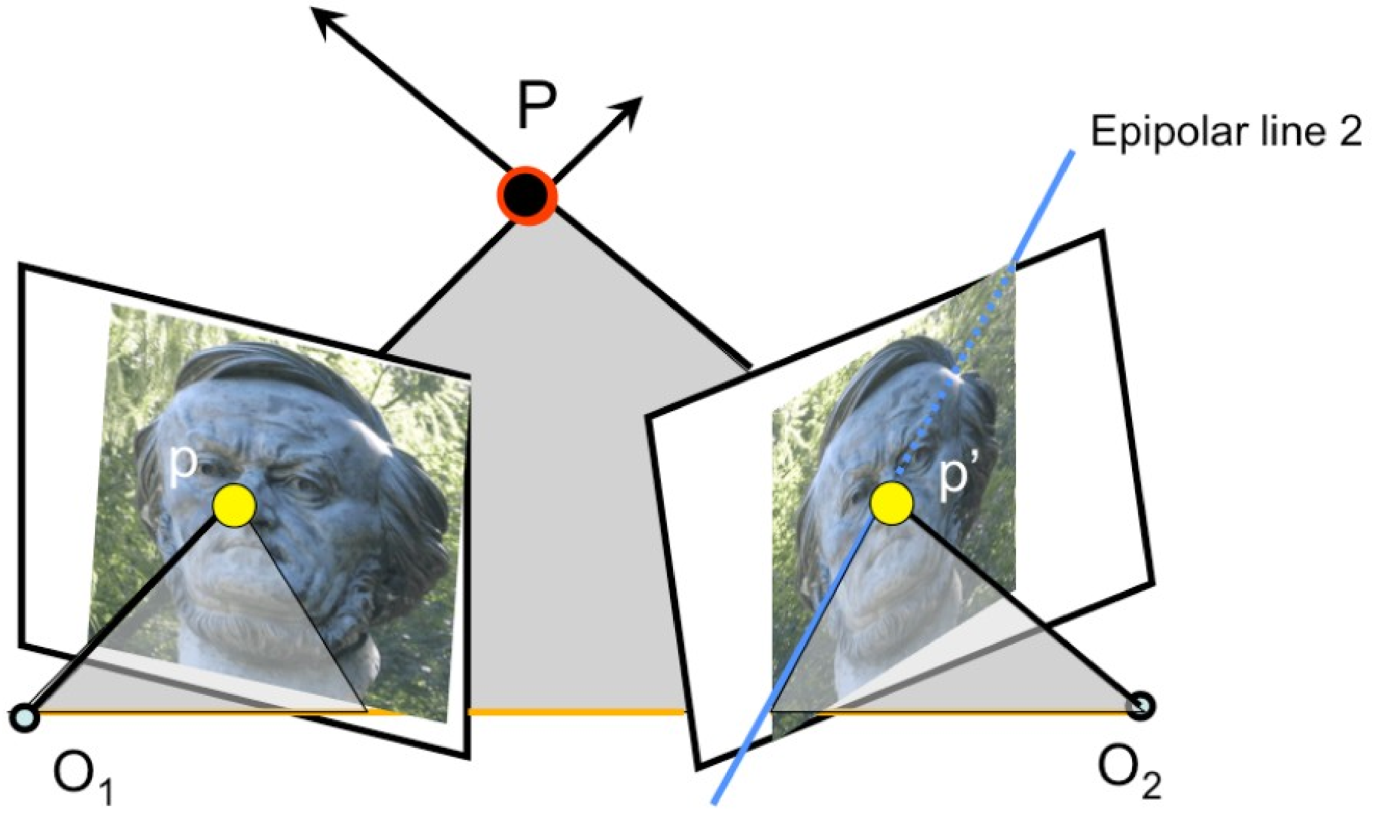
Special case: forward translation



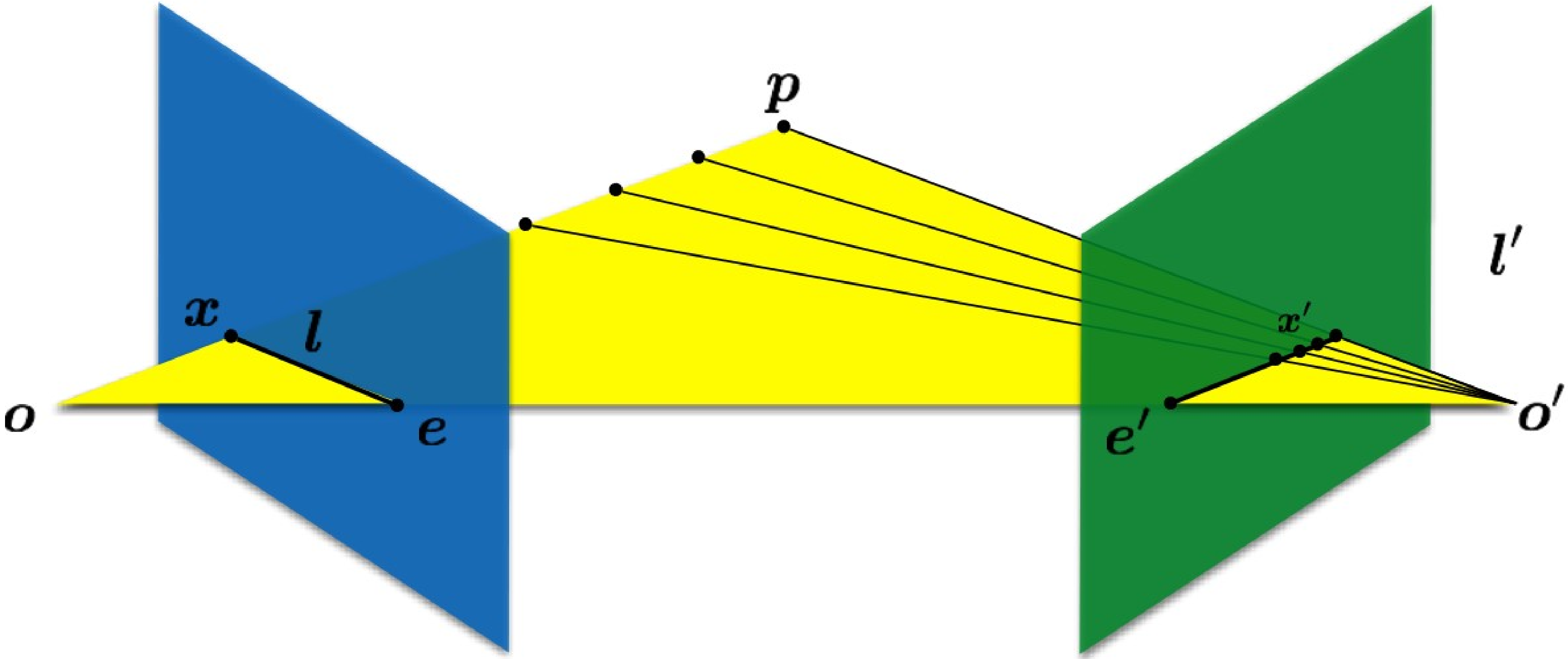




Epipolar constraint

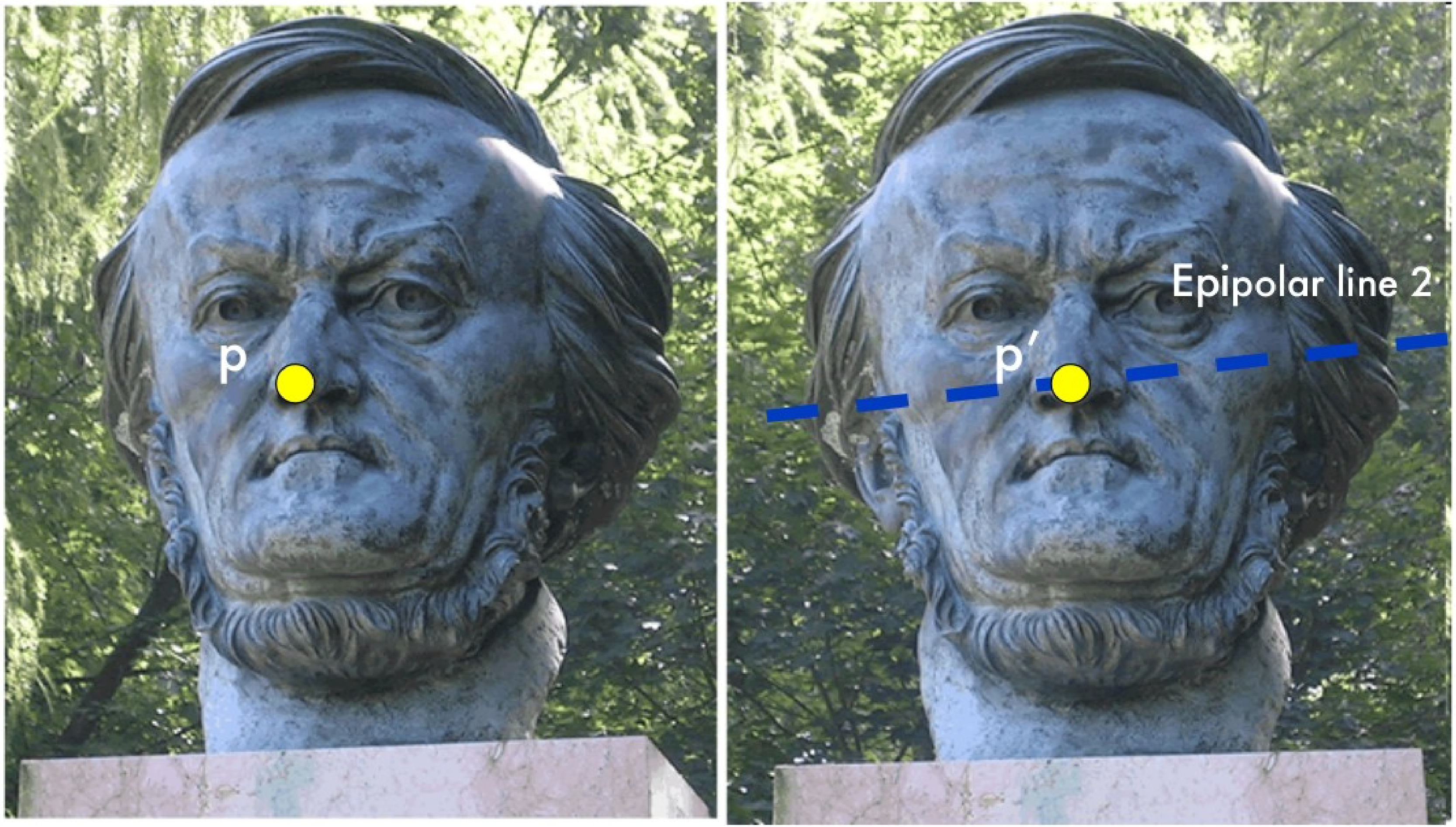


Epipolar constraint

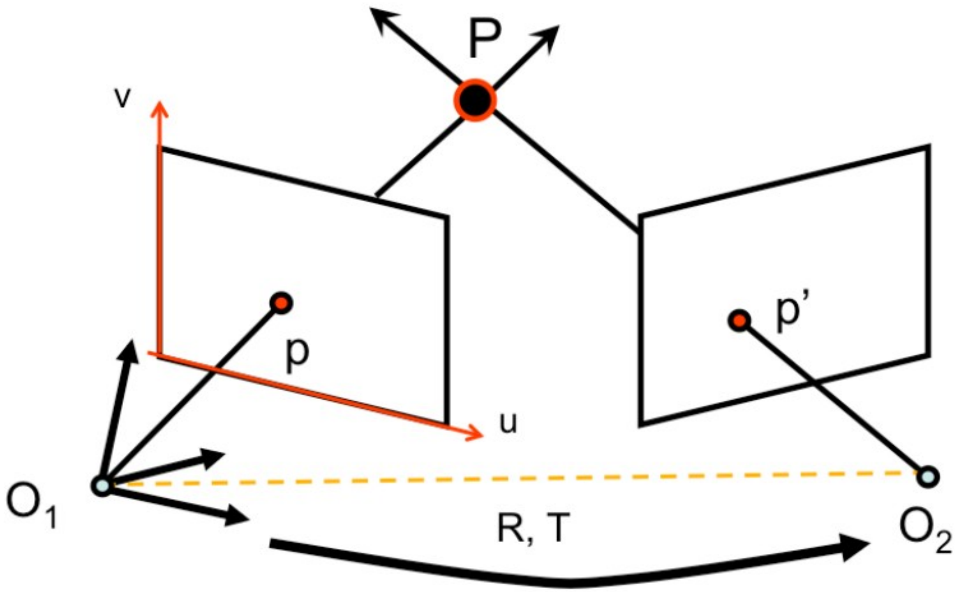


Credits: Ioannis Gkioulekas

Epipolar constraint



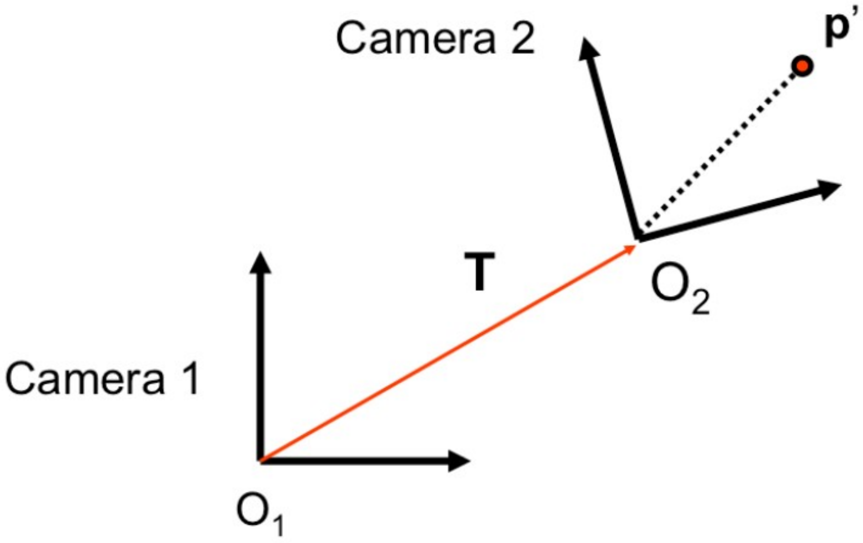
Epipolar constraint



$$M = K\begin{bmatrix} I & 0 \end{bmatrix} \qquad\qquad M' = K'\begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

$$M\ P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = p \quad \text{[Eq. 3]} \qquad\qquad M'\ P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = p' \quad \text{[Eq. 4]}$$

Relative position for cameras

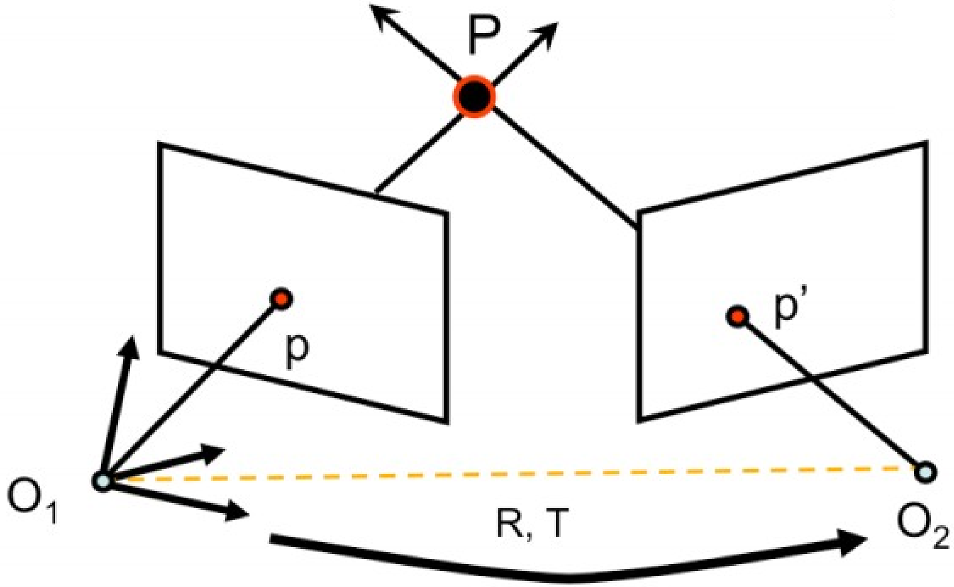


$$p'_{O_1} = Rp'_{O_2} + T$$

$$p'_{O_2} = R^T (p'_{O_1} - T)$$

- T is O_2 in the O_1 reference system
  - Actual translation
- R is the rotation matrix such as **free vector** p' for O_2 is Rp' for O_1
  - Rotation limited to epipolar plane

Epipolar constraint



$$K_{canonical} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

$$\downarrow$$

$$M = \begin{bmatrix} I & 0 \end{bmatrix} \quad \text{[Eq. 5]}$$

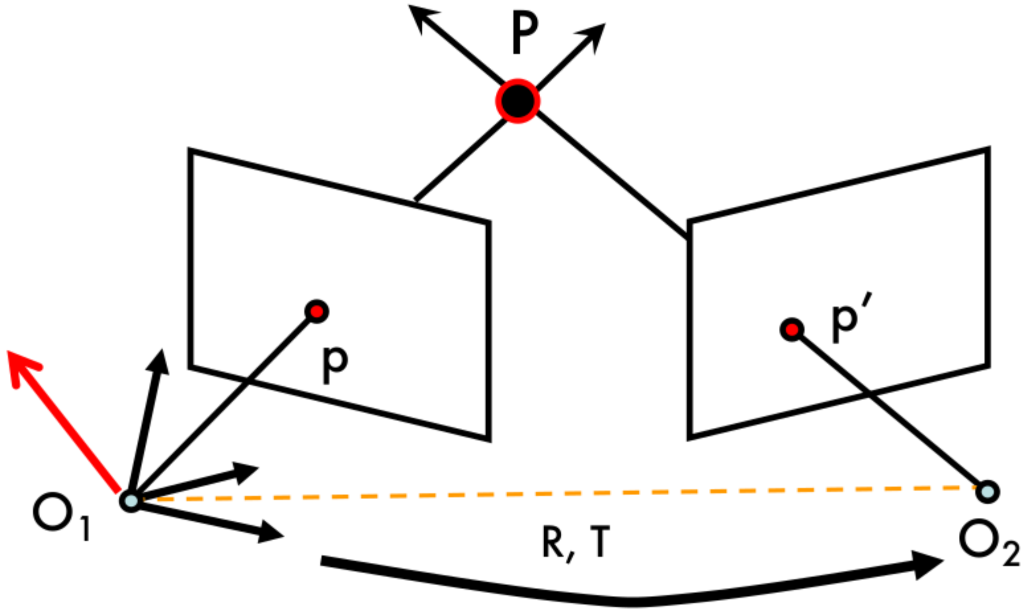
K = K' are known
(canonical cameras)

$$M' = K'\begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

$$\downarrow$$

$$M' = \begin{bmatrix} R^T & -R^T T \end{bmatrix} \quad \text{[Eq. 6]}$$

Epipolar constraint



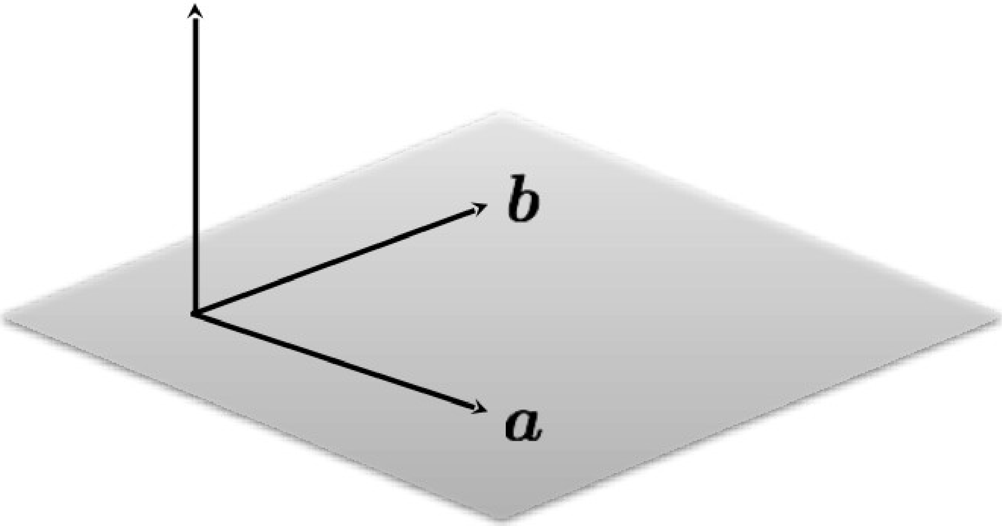
p′ in first camera reference system is $= R\ p' + T$

$T \times ((R\ p') + T) = T \times (R\ p')$ is perpendicular to epipolar plane

$$\rightarrow p^T \cdot \left[T \times (R\ p') \right] = 0$$ [Eq. 7]

Cross product recall

$$\boldsymbol{c} = \boldsymbol{a} \times \boldsymbol{b}$$



$$\boldsymbol{c} \cdot \boldsymbol{a} = 0 \qquad \boldsymbol{c} \cdot \boldsymbol{b} = 0$$

$$\boldsymbol{a} \times \boldsymbol{b} = \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

cross product of two vectors in the same direction is zero

$$\boldsymbol{a} \times \boldsymbol{a} = 0$$

remember this!!!

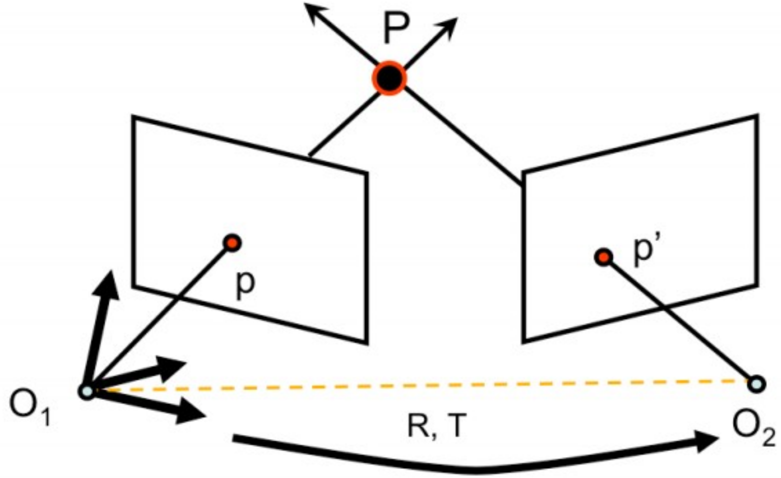
Credits: Ioannis Gkioulekas

Cross product as matrix multiplication

UNIVERSITÀ DI PARMA

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Essential Matrix



$$p^T \cdot \left[T \times (R\, p') \right] = 0 \rightarrow p^T \cdot \boxed{\left[T_\times \right] \cdot R}\, p' = 0$$

[Eq. 8] [Eq. 9]

E = Essential matrix

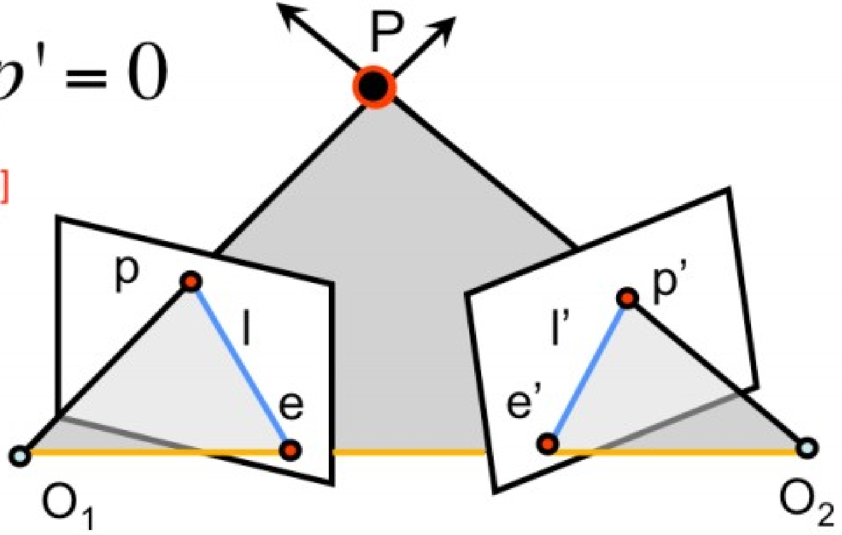
(Longuet-Higgins, 1981)

- E is the **Essential Matrix**

Essential Matrix

$$p^T \cdot E\, p' = 0$$

[Eq. 10]



- $l = Ep'$ → epipolar line for p
- $l' = E^T p$ → epipolar line for p'
- $Ee' = 0$ $E^T e = 0$ → namely epipoles are solution for homogeneous equation
- E is a 3×3 matrix with 5 DOF
- E is singular and its rank is 2

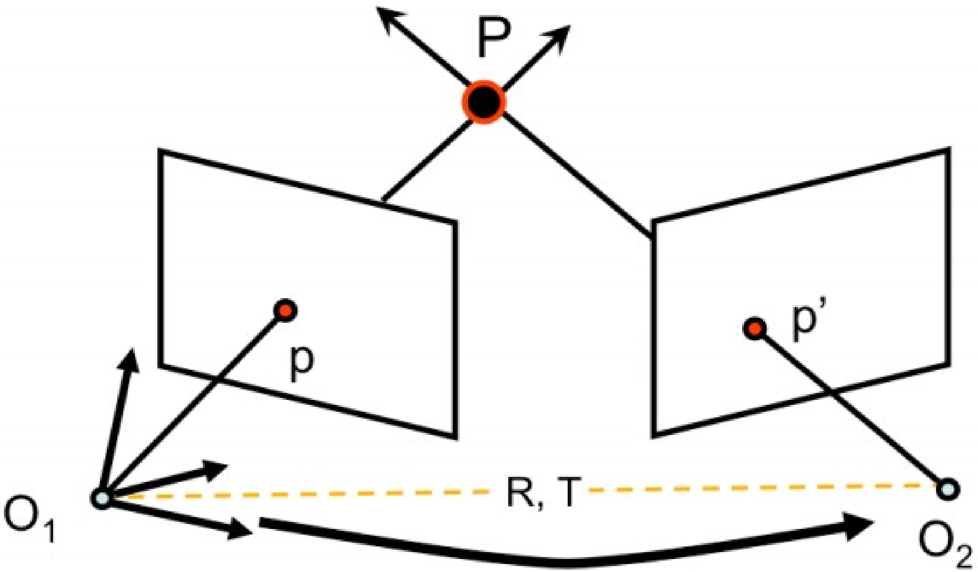
Essential matrix

$$\boldsymbol{p}^T \boldsymbol{E}\boldsymbol{p}'=\boldsymbol{0} \qquad Longuet-Higgins\,Equation$$

- Usually we do not use ideal cameras
- Is then E so useful?
- Yes if we think about normalized camera coordinates and not image coordinates
  - For image coordinates we need to know K

$$p_c = K^{-1} p$$

Epipolar constraint (non ideal camera)



$$M = K\begin{bmatrix} I & 0 \end{bmatrix} \qquad M' = K'\begin{bmatrix} R^T & -R^T T \end{bmatrix}$$

$$p_c = K^{-1}\, p \quad \text{[Eq. 11]} \qquad p'_c = K'^{-1}\, p' \quad \text{[Eq. 12]}$$

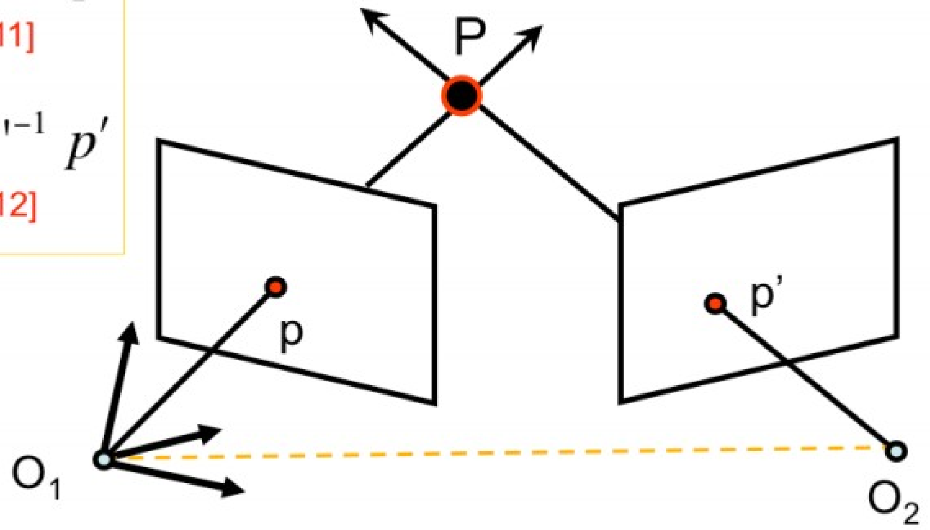
Epipolar constraint (non ideal camera)

$$p_c = K^{-1}\, p$$

[Eq. 11]

$$p'_c = K'^{-1}\, p'$$

[Eq. 12]



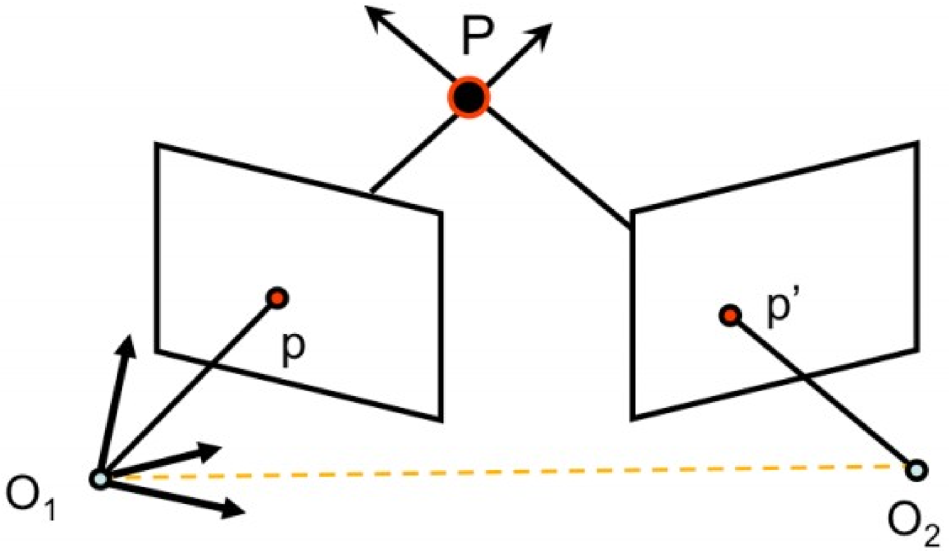
[Eq.9]

$$p_c^T \cdot [T_\times] \cdot R\, p'_c = 0 \rightarrow (\mathrm{K}^{-1}\,\mathrm{p})^\mathrm{T} \cdot [\mathrm{T}_\times] \cdot \mathrm{R}\,\mathrm{K}'^{-1}\,\mathrm{p}' = 0$$

$$\mathrm{p}^\mathrm{T}\, \boxed{\mathrm{K}^{-\mathrm{T}} \cdot [\mathrm{T}_\times] \cdot \mathrm{R}\,\mathrm{K}'^{-1}}\, \mathrm{p}' = 0 \rightarrow \mathrm{p}^\mathrm{T}\, \boxed{\mathrm{F}}\, \mathrm{p}' = 0$$

[Eq. 13]

Fundamental Matrix



[Eq. 13] $$\boxed{p^T F p' = 0}$$

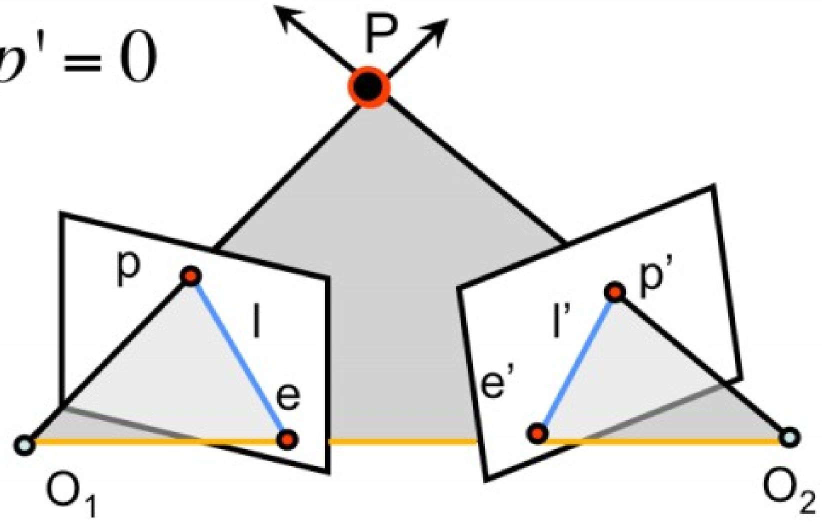
$$F = K^{-T} \cdot [T_{\times}] \cdot R\, K'^{-1}$$

[Eq. 14]

- F ⟶ Fundamental Matrix (Faugeras and Luong 1992)

Fundamental Matrix

$$p^T \cdot F\ p' = 0$$



- l = Fp' $\longrightarrow$ epipolar line for p
- l' = F^Tp $\longrightarrow$ epipolar line for p'
- Fe' = 0 F^Te = 0 $\longrightarrow$ namely epipoles are solution for homogeneous equation
- F is a 3x3 matrix with 7 DOF
- F is singular and its rank is 2

Essential & Fundamental Matrices

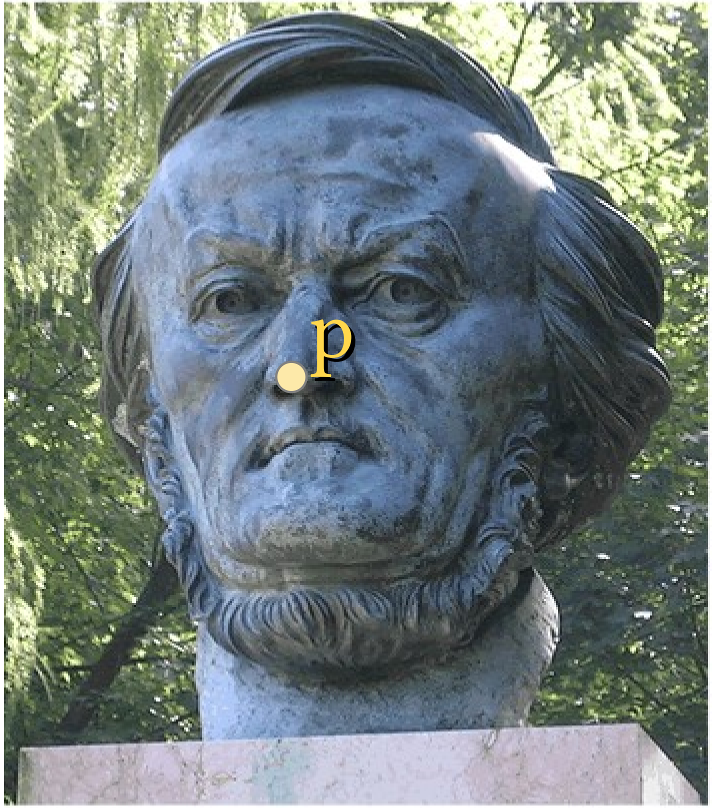
$$p_c^T E \, p'_c = 0 \qquad p^T F \, p' = 0$$

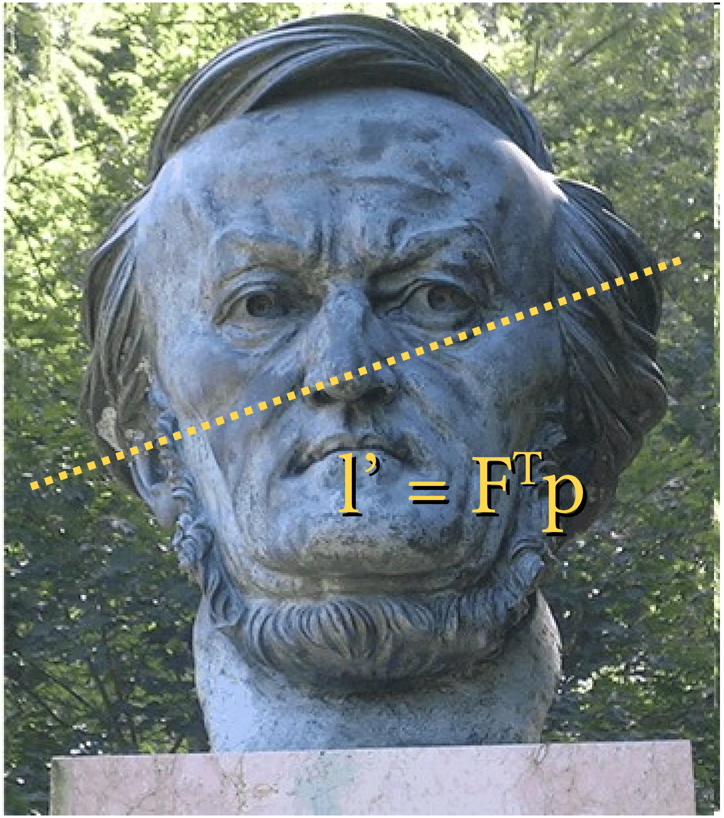
$$p_c = K^{-1} p \qquad p'_c = K'^{-1} p'$$

$$p^T K^{-T} E K^{-1} p' = 0$$

$$K^{-T} E K^{-1} = F$$

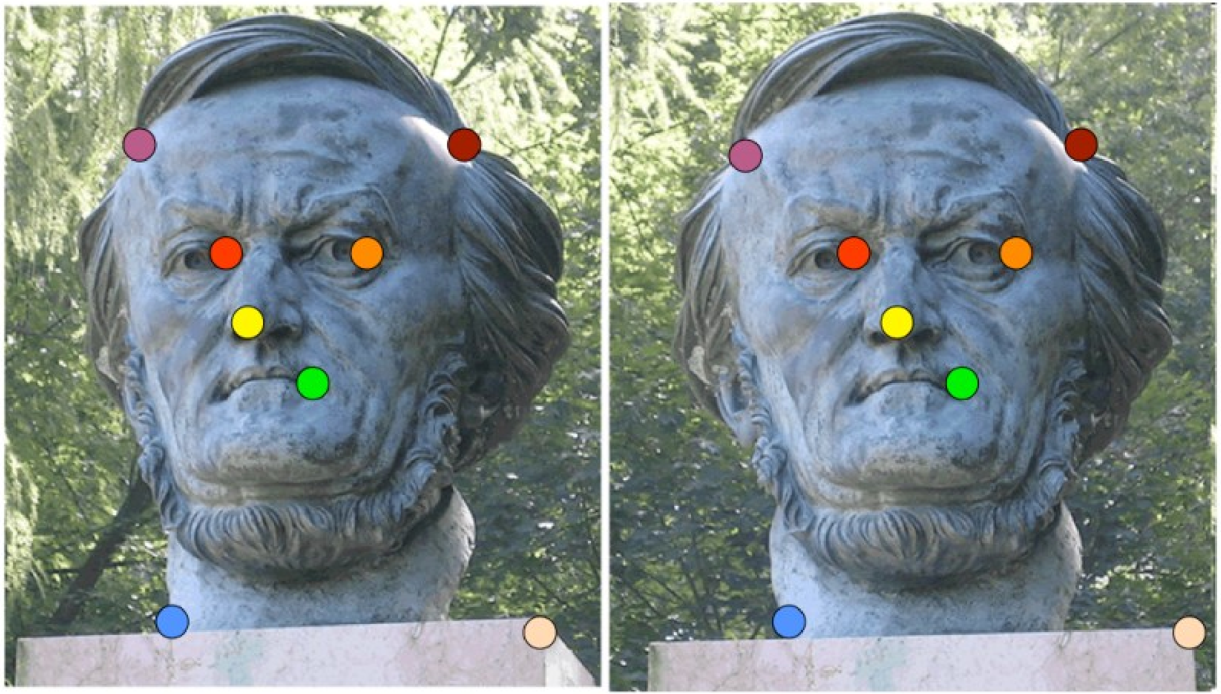
Why we want F?





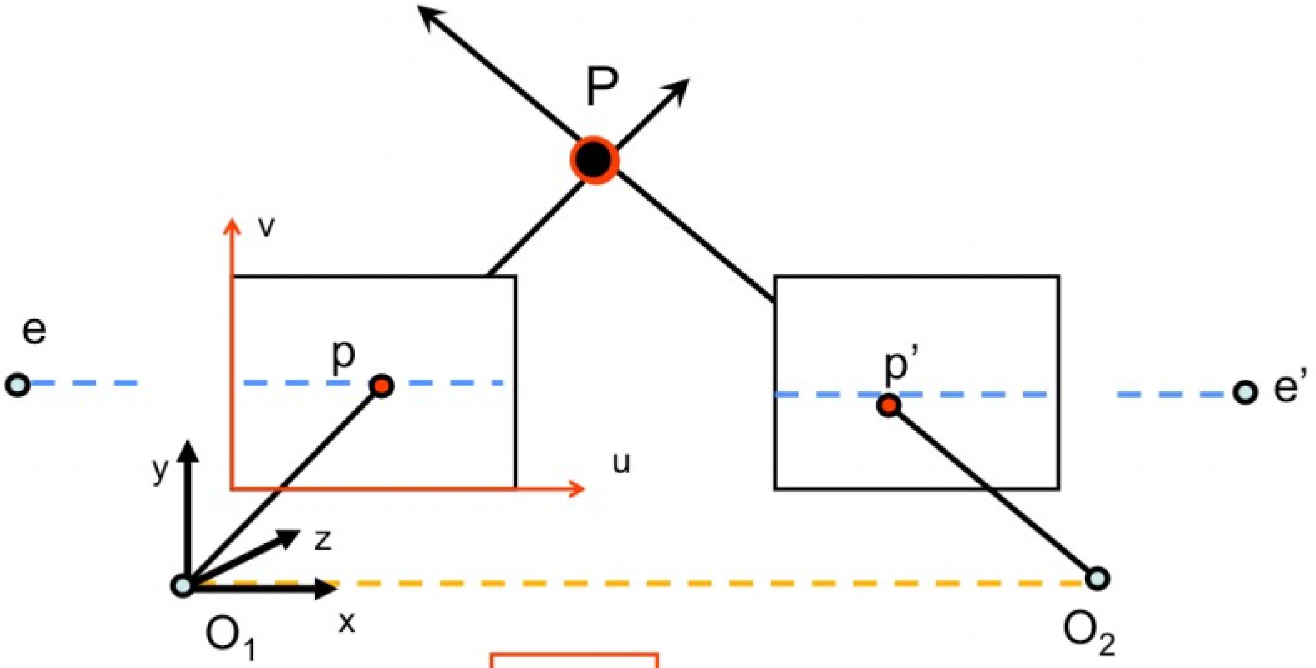
Fundamental Matrix estimation

UNIVERSITÀ DI PARMA



- Can we estimate F without knowing <u>nothing</u> about K, R, T?
- Yes → 8-point algorithm

Special case: parallel image planes



$K_1 = K_2$ = known

x parallel to O_1O_2

$\boxed{E=?}$

Hint :

$\boldsymbol{R} = I \qquad \boldsymbol{T} = (T, 0, 0)$

Special case: parallel image planes

$$\mathbf{E} = \left[\mathbf{T}_\times\right] \cdot \mathbf{R}$$

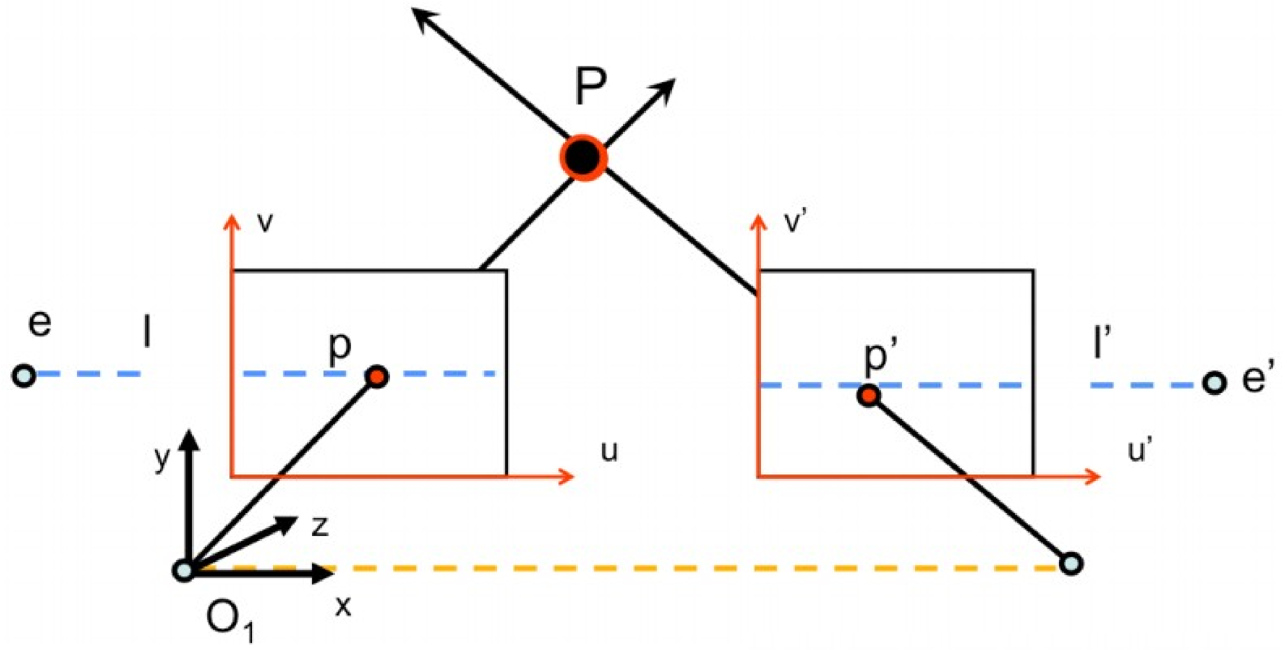
$$\mathbf{E} = \begin{bmatrix} 0 & -T_z & T_y \\ T_z & 0 & -T_x \\ -T_y & T_x & 0 \end{bmatrix} \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

[Eq. 20]

$\mathbf{T} = [\ \mathrm{T}\ \ 0\ \ 0\ ]$

$\mathbf{R} = \mathbf{I}$

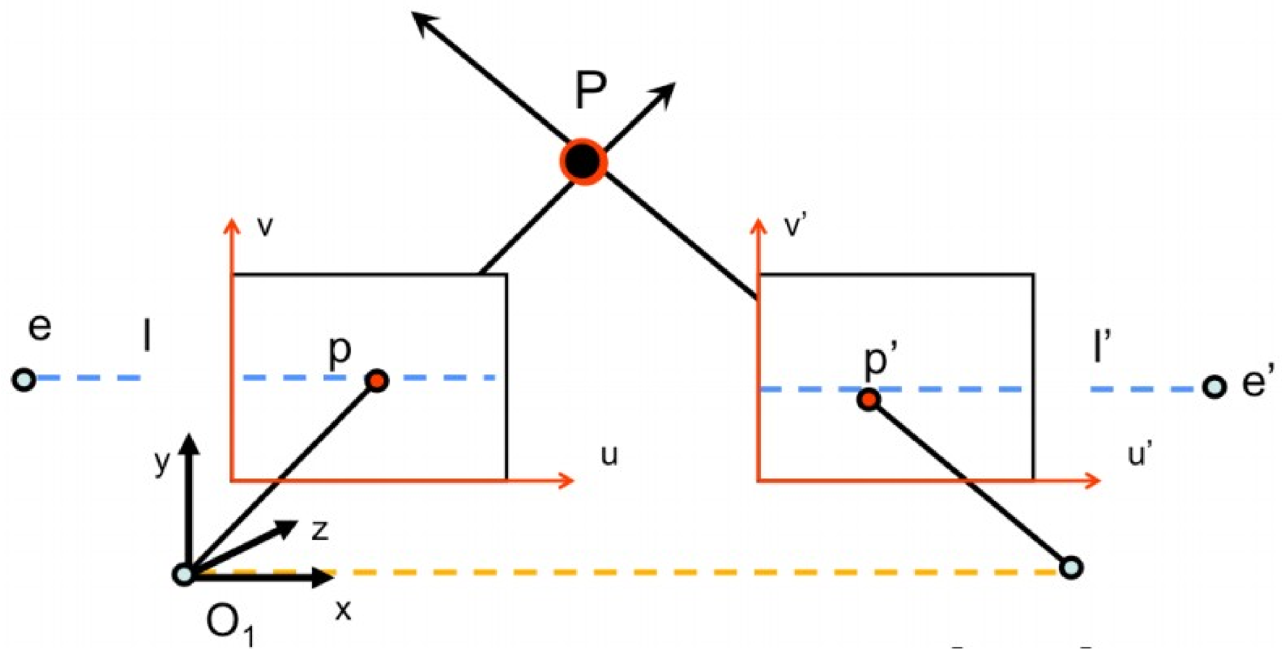
Special case: parallel image planes



- Epipolar lines are horizontal

$$l = E\,p' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -T \\ T v' \end{bmatrix}$$ horizontal!

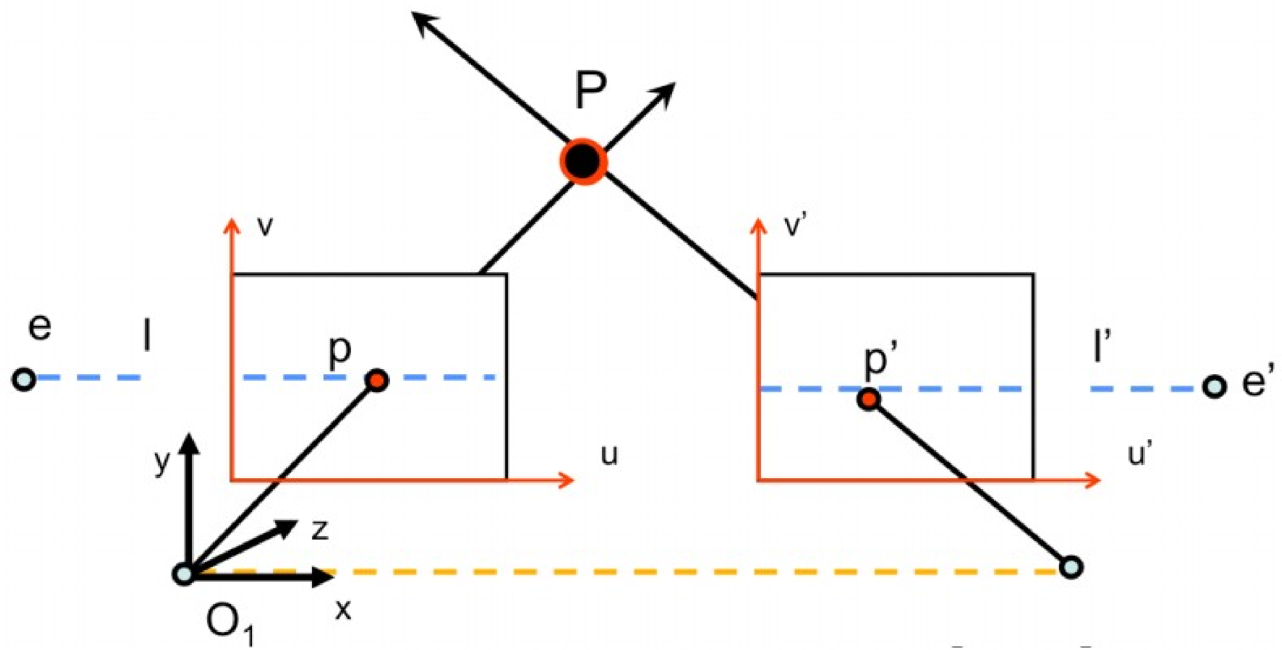
Special case: parallel image planes



- How p & p' are related?

$$p^T \cdot E\, p' = 0$$

Special case: parallel image planes

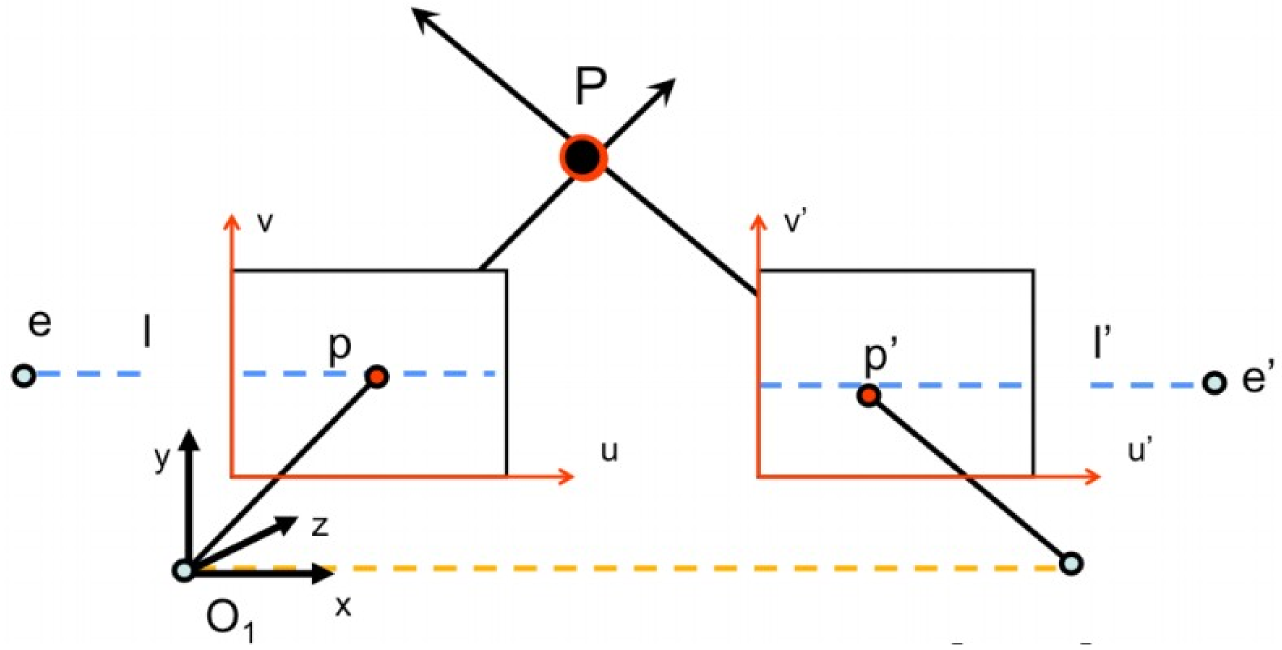


- How p & p' are related?

$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \Rightarrow \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \Rightarrow Tv = Tv'$$

$$\Rightarrow v = v'$$

Special case: parallel image planes



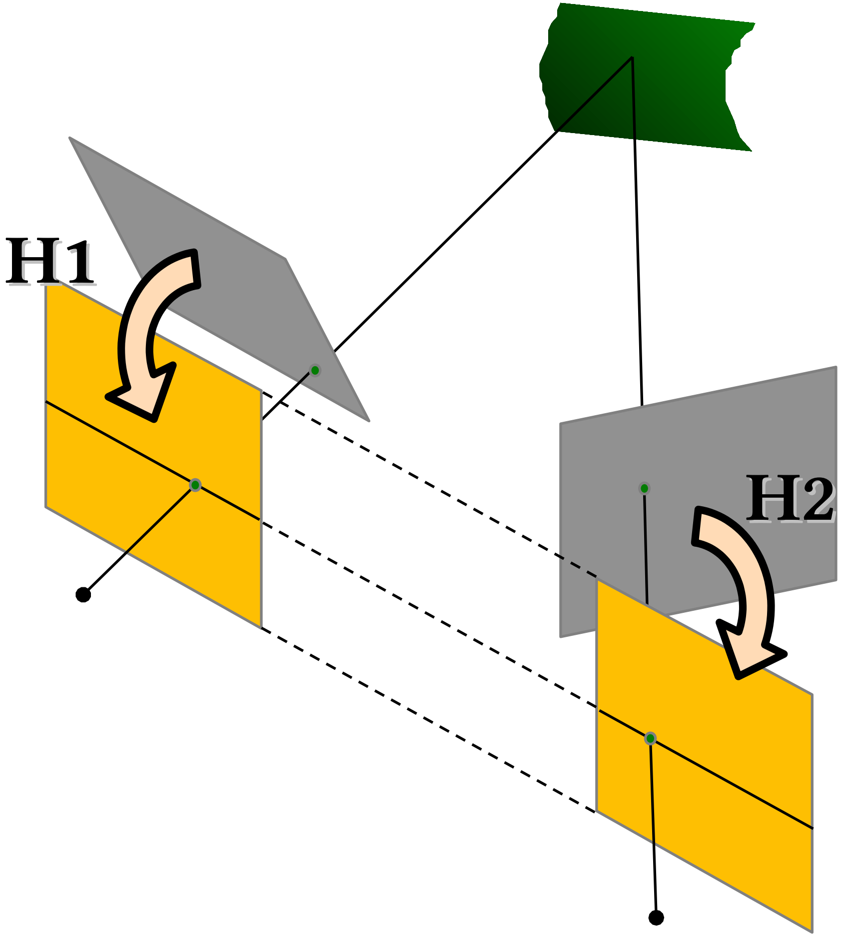
- v=v' means the same line!
- The search for p' is simpler…

True for canonical cameras!

Also when K=K'!

Image Rectification

- Parallel image planes are better to handle for stereovision
- Simply accurately align cameras
  - Good idea, but it does not work!
  - Precision, noise, thermal effects…
- Reproject camera planes onto a common plane parallel to the line between camera centers



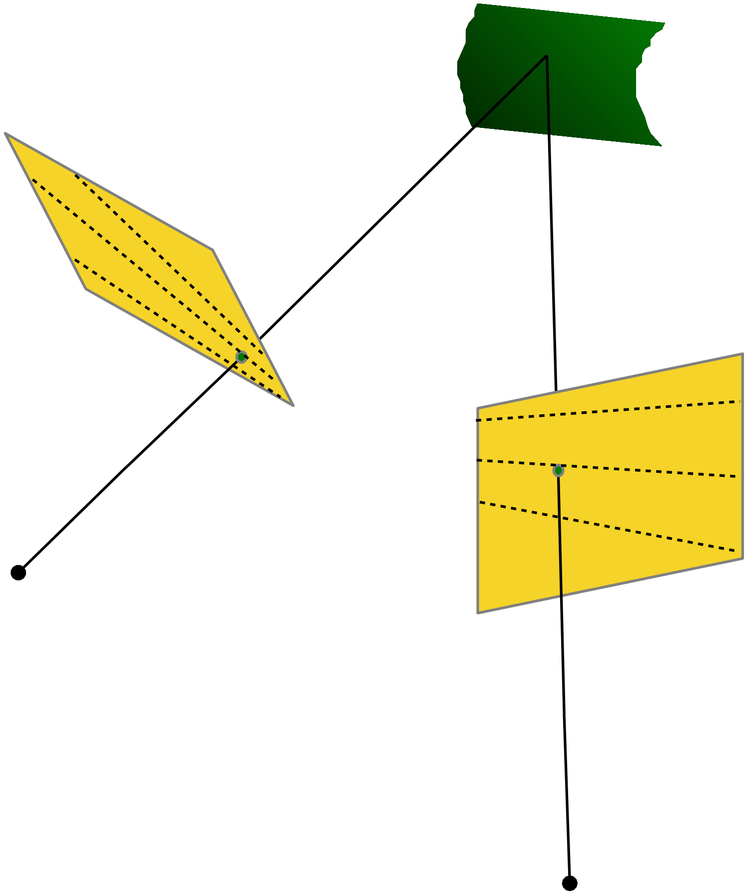
Stereo Image Rectification

- Assuming K=K' as known

1) Compute E $\longrightarrow$ we get R
2) Rotate right image by R
3) Rotate both images to move e and e' to ∞
   - We need to compute a R_{rect}
4) Adjuste the scale

Stereo Image Rectification

1) Compute E $\longrightarrow$ we get R

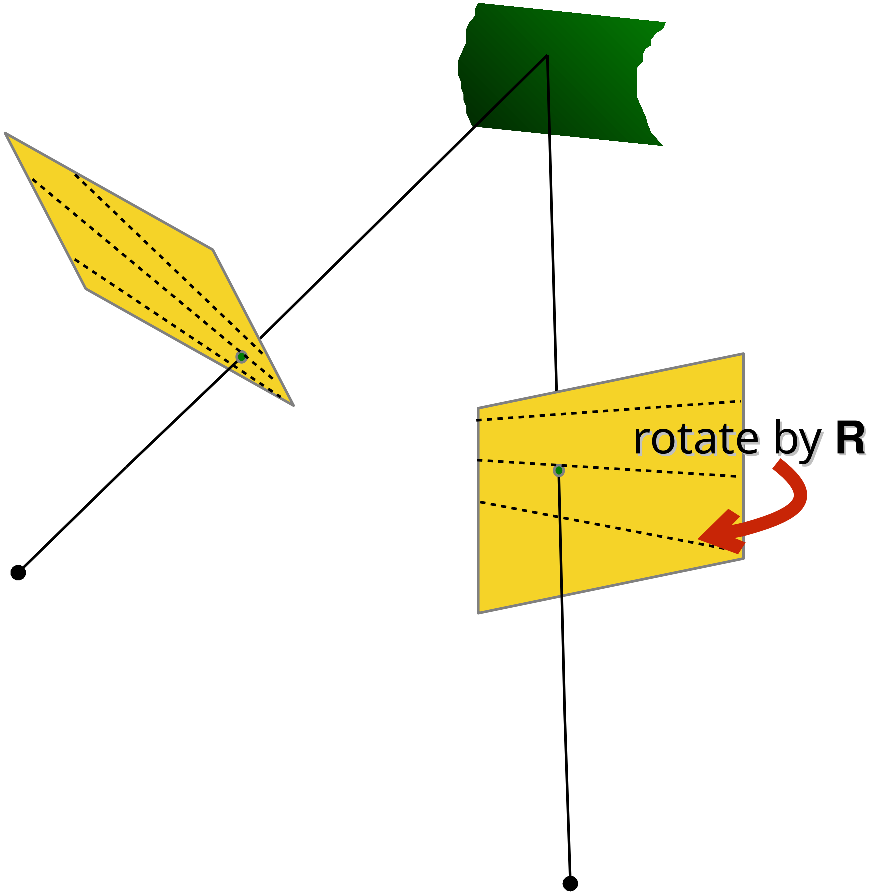


Credits: Ioannis Gkioulekas

Stereo Image Rectification

1) Compute E $\longrightarrow$ we get R

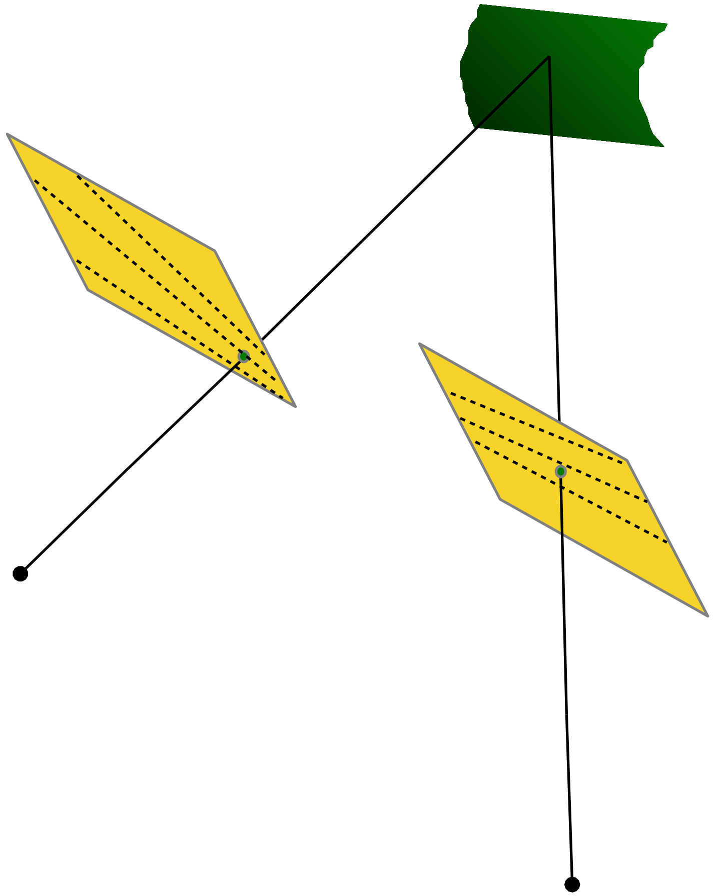
2) Rotate left image by R



Credits: Ioannis Gkioulekas

Stereo Image Rectification

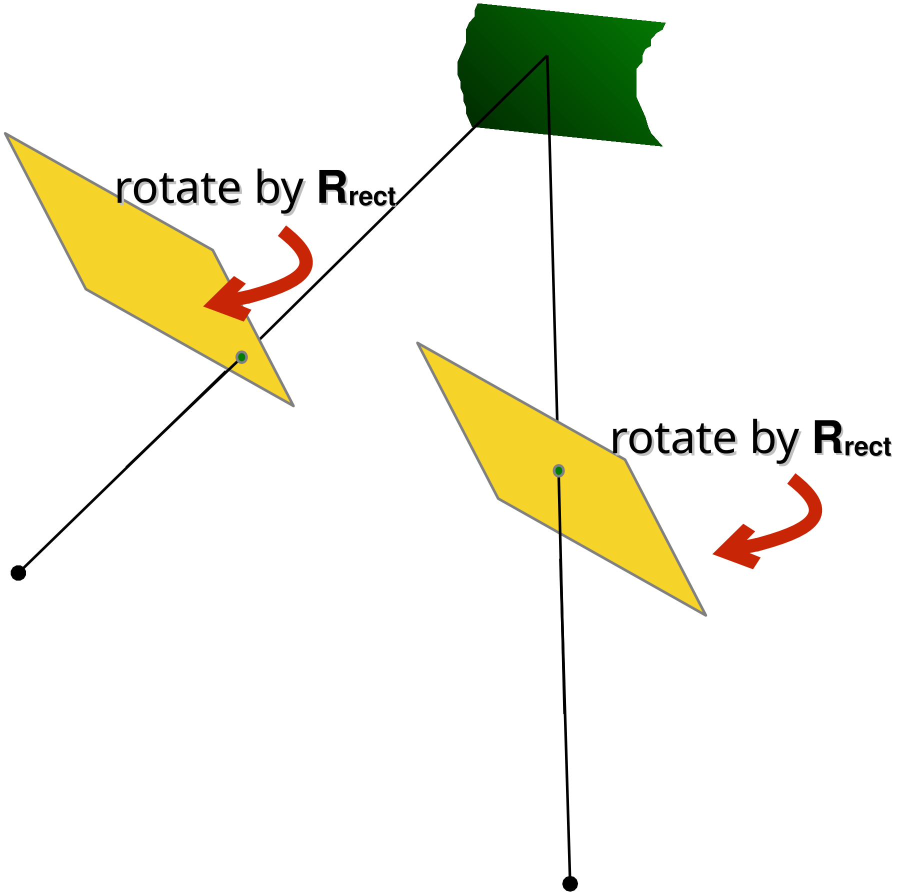
1) Compute E $\longrightarrow$ we get R
2) Rotate right image by R



Credits: Ioannis Gkioulekas

Stereo Image Rectification

1) Compute E $\longrightarrow$ we get R
2) Rotate right image by R
3) Rotate both images by R_{rect}



Credits: Ioannis Gkioulekas

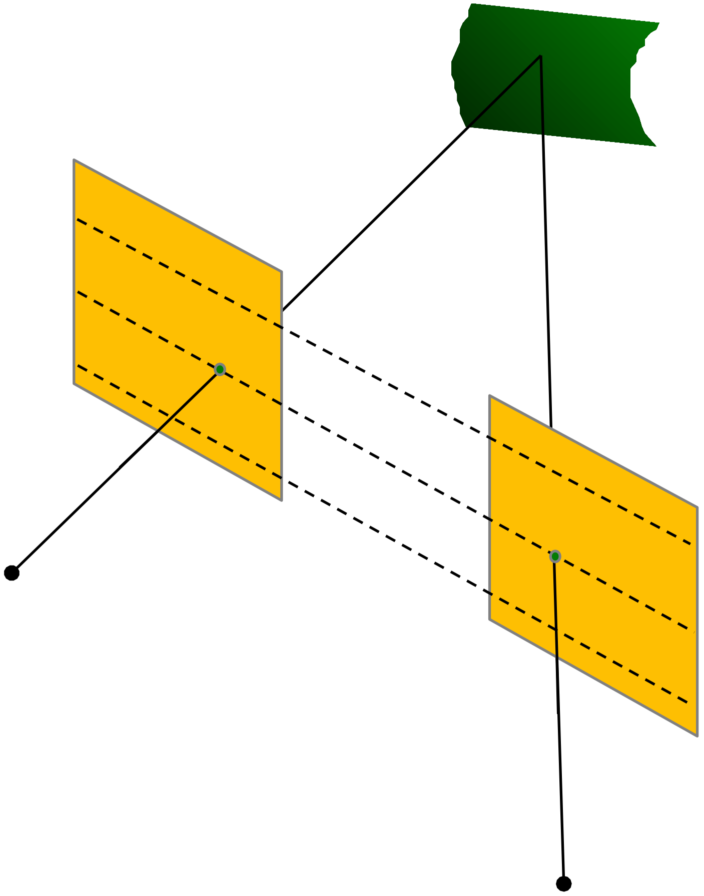
Stereo Image Rectification

UNIVERSITÀ DI PARMA

1) Compute E $\longrightarrow$ we get R

2) Rotate right image by R

3) Rotate both images by R_{rect}

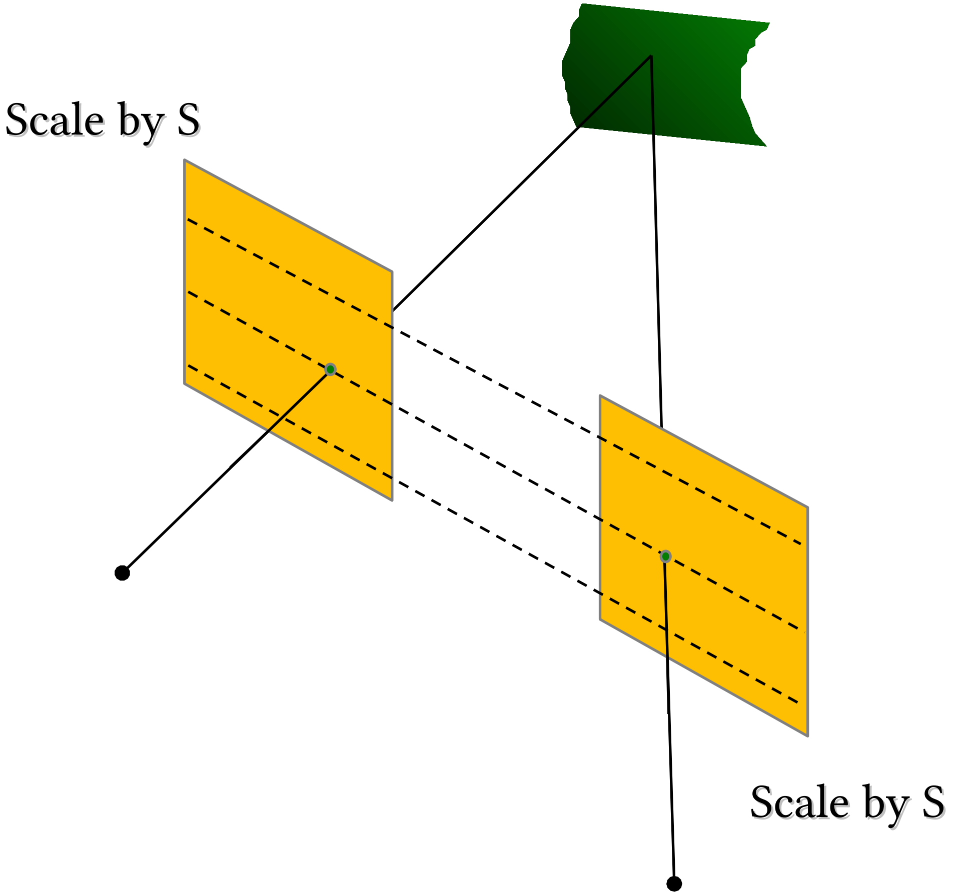


Credits: Ioannis Gkioulekas

Stereo Image Rectification

UNIVERSITÀ DI PARMA

1) Compute E $\longrightarrow$ we get R
2) Rotate right image by R
3) Rotate both images by R_{rect}
4) Scale images

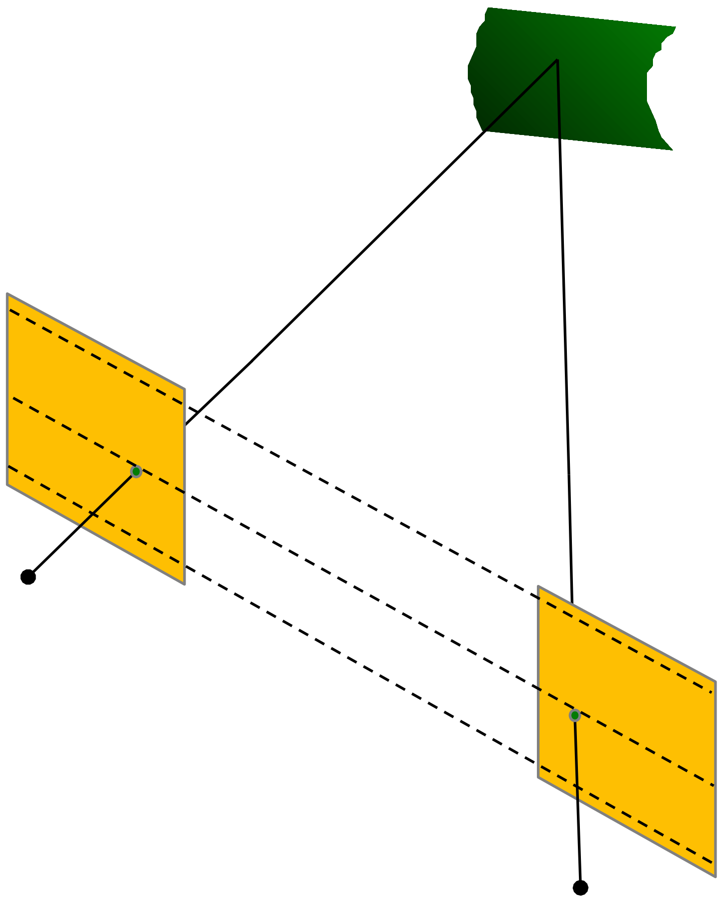


Credits: Ioannis Gkioulekas

Stereo Image Rectification

UNIVERSITÀ DI PARMA

1) Compute E $\longrightarrow$ we get R
2) Rotate right image by R
3) Rotate both images by R_{rect}
4) Scale images



Credits: Ioannis Gkioulekas

Stereo Image Rectification

Actually not so simple…

1) Estimate E using the 8 point algorithm
2) Estimate the epipole e (solve Ee=0)
3) Build R_{rect} from e
4) Decompose E into R and T
5) Set $R1=R_{rect}$ and $R2 = RR_{rect}$
6) Rotate each left camera point x'~ Hx where H = KR1
7) Alter the focal length (inside K) to keep as much points within the original image size
8) Repeat 6 and 7 for right camera points using R2

Credits: Ioannis Gkioulekas

UNIVERSITÀ DI PARMA

Epipolar Geometry

Question time!