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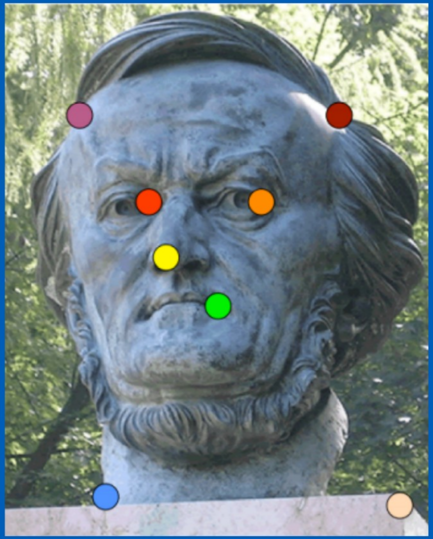
8 Points Algorithm

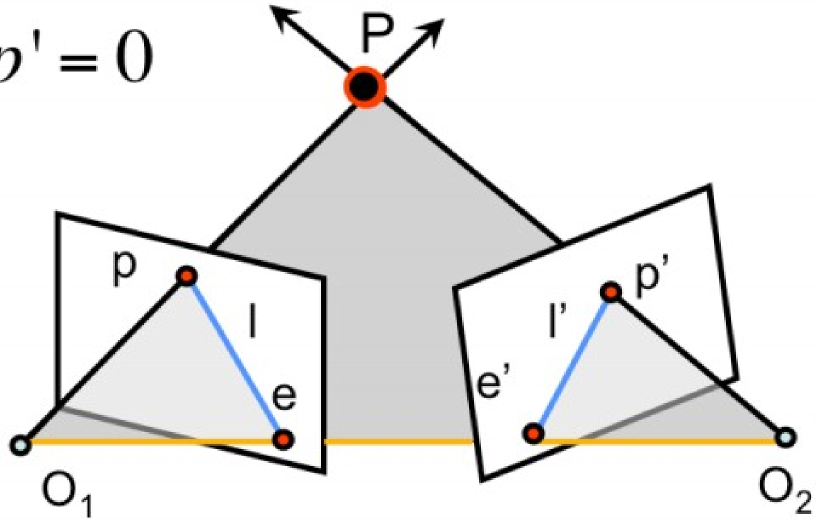


Summary

- Why stereo?
- Epipolar constraints
- Essential and Fundamental matrices
- Stereo Images rectification

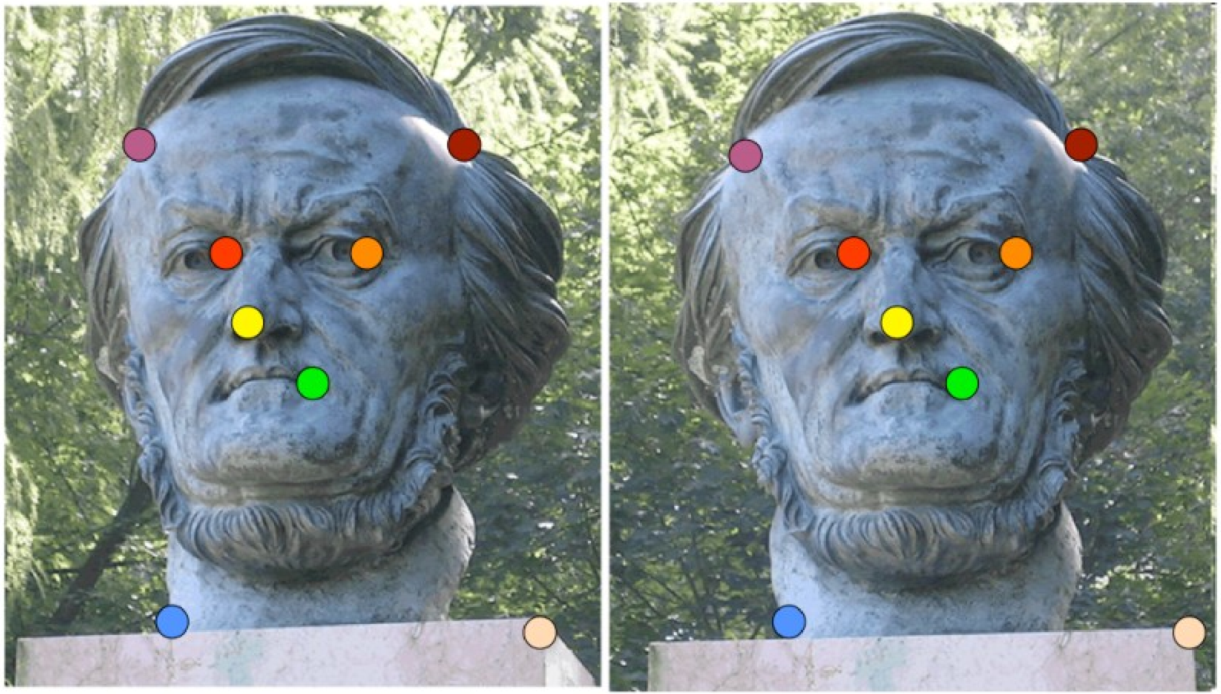
Fundamental Matrix

$$p^T \cdot F\ p' = 0$$



- l = Fp’ $\longrightarrow$ epipolar line for p
- l’ = F^Tp $\longrightarrow$ epipolar line for p’
- Fe’ = 0 F^Te = 0 $\longrightarrow$ namely epipoles are solution for homogeneous equation
- F is a 3x3 matrix with 7 DOF
- F is singular and its rank is 2

Fundamental Matrix estimation



- Can we estimate F without knowing <u>nothing</u> about K, R, T?
- Yes → 8-point algorithm

Expand equation

$$p^T F p' = 0$$

$$p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad p' = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} u & v & 1 \end{bmatrix} \begin{bmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{bmatrix} \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} uu' & uv' & u & vu' & vv' & v & u' & v' & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Correspondences

- Given the i^{th} correspondence

$$\begin{bmatrix} u_i u'_i & u_i v'_i & u_i & v_i u'_i & v_i v'_i & v_i & u'_i & v'_i & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = 0$$

Correspondences

- Therefore 1 correspondence $\longrightarrow$ 1 equation
- 9 unknowns $\longrightarrow$ 9 equations?
  - No, fundamental matrix has 7 DOF
- Therefore 7 correspondences should be enough
  - Yes, if we would like to explore complex non linear methods
  - We definitely would opt for a fast linear solution
  - Then use 8 points!

Correspondences

- Take then, at least, 8 correspondences

$$\begin{bmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{bmatrix} = Af = 0$$

Fundamental Matrix Estimation

- Homogeneous system
- F_{ij}s are the unknowns
  - We can find them up to a scale $\longrightarrow$ impose norm as 1
- A not 0 solution is possible
  - For 8 points when rank of A is 8
  - Noise and quantization can lead to have 9 as rank (9 columns)

Fundamental Matrix Estimation

- Again, total least squares approach
  - $Af = 0 \longrightarrow$ minimize $||Af||^2$
- SVD can be used
  - In order to avoid the trivial zero solution and bypass scale issue impose the norm to be 1
  - $||F||^2 = 1$

Fundamental Matrix Estimation

- Find the SVD of $A^T A$
- Entries of f (or F) are the column of V corresponding to the least singular value

Fundamental Matrix Estimation

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- Solving previous system gives us a $\hat{F}$ solution that satisfies
  - $p^T\hat{F}p'=0$
- Anyway $\hat{F}$ is not necessarily a proper fundamental matrix
  - i.e. it can not be a rank 2 matrix
- Let's look for a best rank 2 approximation
  - We seek for an F that minimizes the Froebenius norm of $\hat{F}$
  - Subject to the constraint that det(F) = 0

Fundamental Matrix Estimation

- Therefore we have to solve
  - $||F - \hat{F}|| = 0$ (Froebenius norm)
  - $\det(F) = 0$
- Froebenius norm of a matrix:
  - Sq. root of sum of squares of all entries of the matrix
- How to solve? $\longrightarrow$ SVD again

Fundamental Matrix Estimation

$$\|F-\hat{F}\|=0 \qquad det(F)=0$$

$$F=U\begin{bmatrix} s_1 & 0 & 0 \\ 0 & s_2 & 0 \\ 0 & 0 & 0 \end{bmatrix}V^T \qquad UDV^T=SVD(\hat{F})$$

[HZ] pag 281, chapter 11, “Computation of F”

Fundamental Matrix Estimation

- The 8-point algorithm may be then formulated as consisting of two steps
  1) Linear solution. A solution $\hat{F}$ is obtained from the vector f corresponding to the smallest singular value of A
  2) Constraint enforcement. Replace $\hat{F}$ by F , the closest singular matrix to $\hat{F}$ under a Frobenius norm. This correction is done using the SVD
- The algorithm thus stated is extremely simple, and readily implemented

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Normalized 8 Points Algorithm

Normalized 8 points algorithm

- No invariance wrt similarity
- The 8-point algorithm features a numerical issue
- A can be a not well conditioned (unbalanced) matrix
  - A values are not necessarily in the same order of magnitude
  - This can badly affect the SVD decomposition

See HZ page 108 for details

Normalized 8 points algorithm

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- Solution $\longrightarrow$ normalize A
- Apply a transformation T (translation + scale) to image coordinates
- We want to have:
  - Centroid of points as origin
  - Average distance from the origin is $\sqrt{2}$

See HZ page 108 for details

Normalized 8 points algorithm

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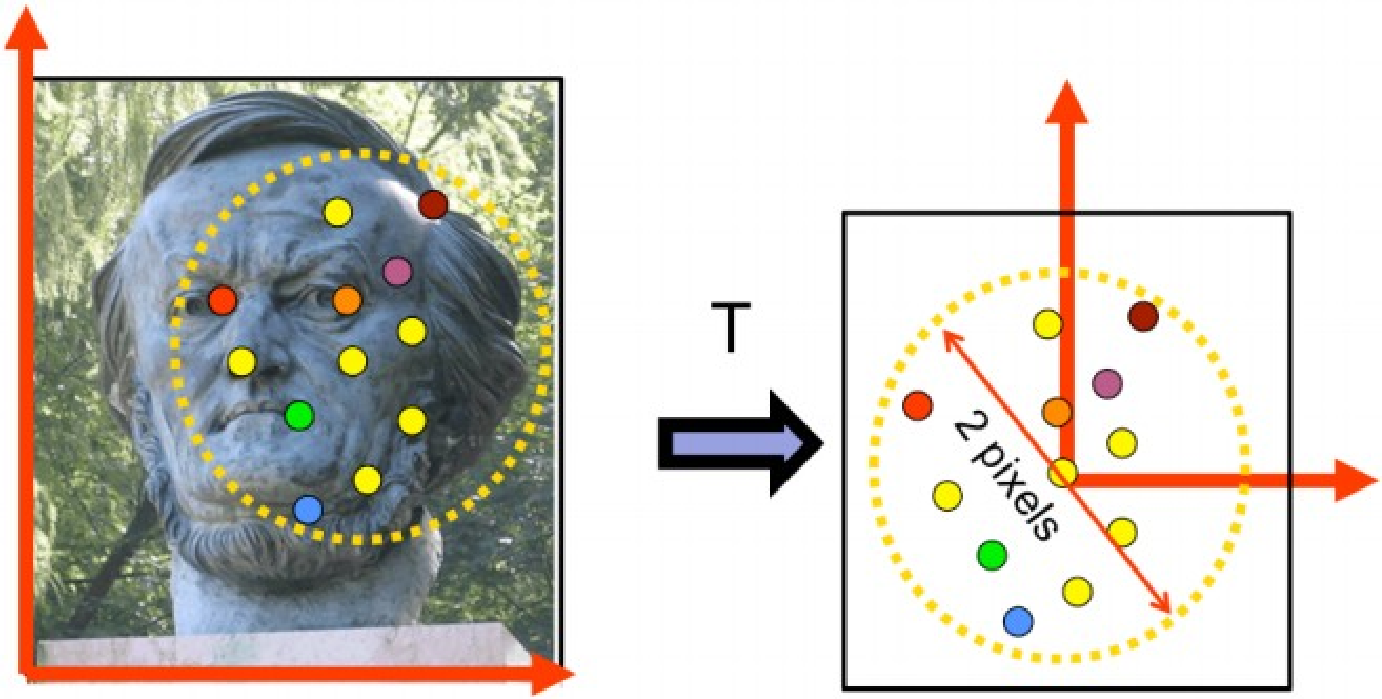


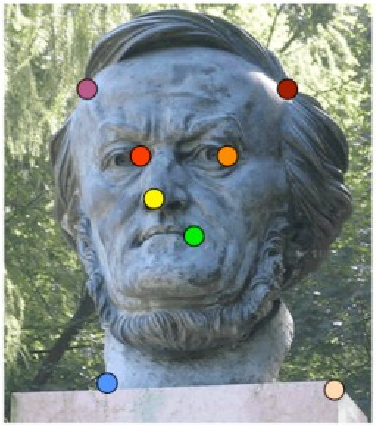
Coordinate system of the image before applying T

Coordinate system of the image after applying T

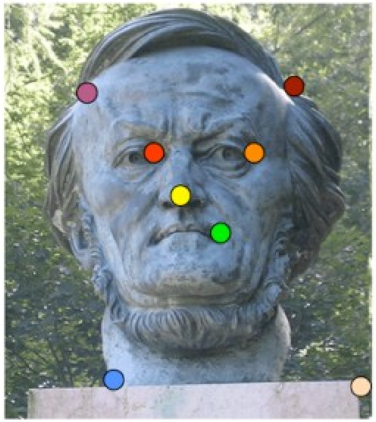
See HZ page 108 for details

Normalized 8 points algorithm

- Apply a T and T’ to left and right images
- Obtain a “new” set of, at least, 8 points and proceed as before



$$q_i = T\ p_i$$



$$q_i' = T'\ p_i'$$

See HZ page 108 for details

Normalized 8 points algorithm

- Once we obtain a fundamental matrix F_q we need to de-normalize it
- Simply invert the transformation

$$\begin{cases} q^T F_q q' = 0 \\ \det(F_q) = 0 \end{cases}$$

$$F = T^T F_q T'$$

See HZ page 108 for details

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Essential Matrix Estimation

Essential Matrix Estimation

- In such a case 5 points should be enough
- Same approach as F
- Essential matrix gives us relative pose estimation between two views:

$$E = [T_x] \cdot R$$

Essential Matrix Estimation

- How we can recover R and T from previous equation?

$$E=[T_x]\cdot R=UDV^T$$

- Consider a rotation matrix W
- It can be demonstrated that:

$$W=\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$T=UDWU^T$$

$$R=UW^{-1}V^T$$

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8 Points Algorithm

Question time!