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Single View Reconstruction

- Calibration math recap
- Vanishing points and lines
- Calibration using a single view
- Estimation of geometry using a single view

- [FP] D. A. Forsyth and J. Ponce. **Computer Vision: A Modern Approach (2nd Edition)**. Prentice Hall, 2011.
- [HZ] R. Hartley and A. Zisserman. **Multiple View Geometry in Computer Vision**. Cambridge University Press, 2003.
- CS231A · **Computer Vision: from 3D reconstruction to recognition**, Prof. Silvio Savarese – Stanford University

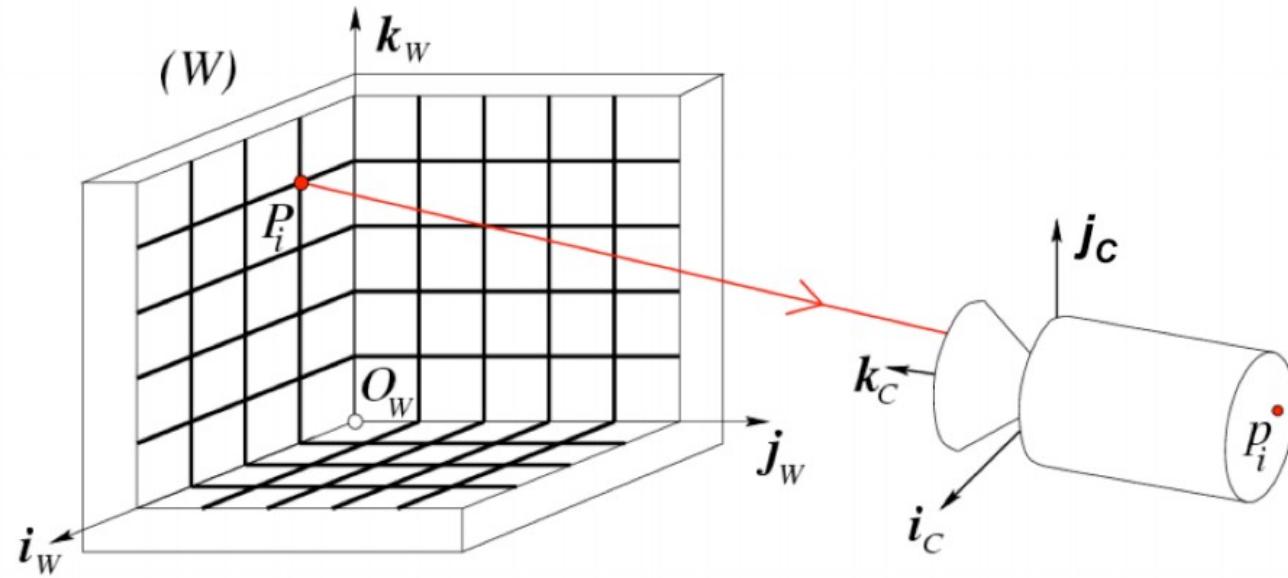


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From pixels to world...

- We now should know how the 3D points are mapped inside the image plane
- We computed a M matrix ($K + RT$) that helps the mapping
- Let's try to do the opposite
 - Given a single image can we compute the 3D geometry?

Camera model



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = M P_i$$

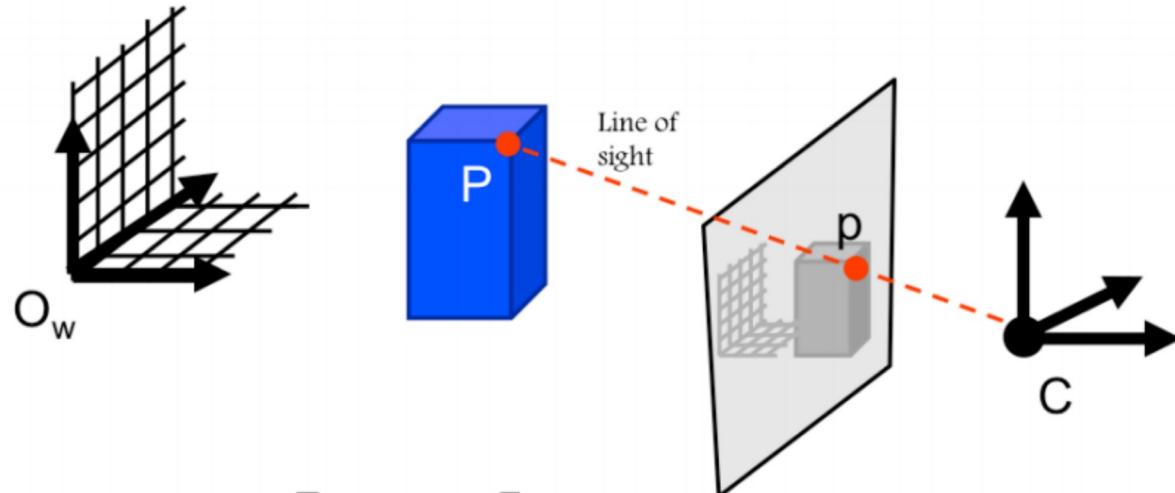
In pixels

World ref. system

$$\mathbf{M} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

11 unknowns
Need at least 6 correspondences

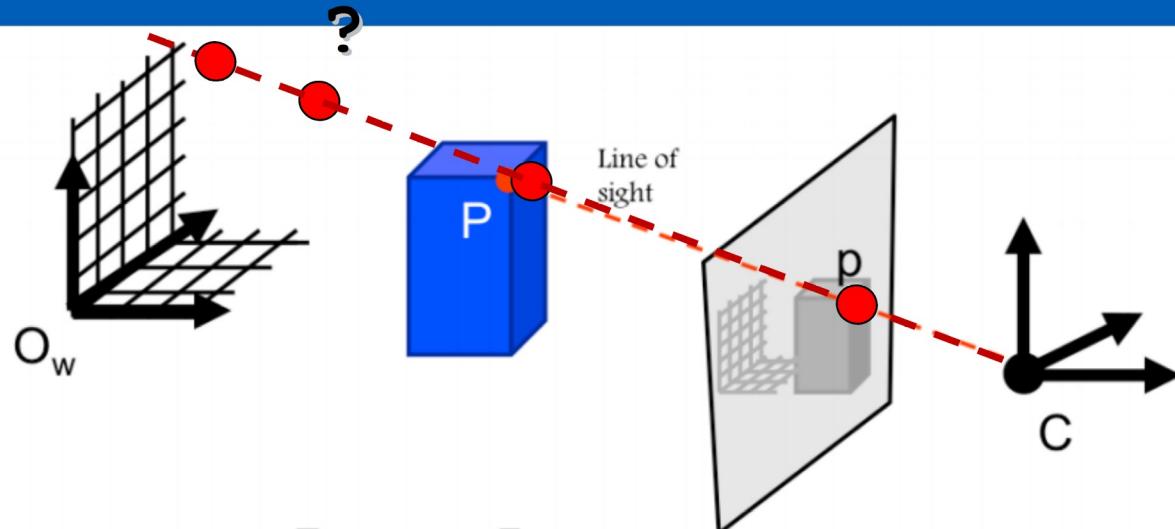
Camera model



$$M = K[R \ T]$$

- After calibration we know the transformation world \rightarrow camera
 - We can discover p if we know P

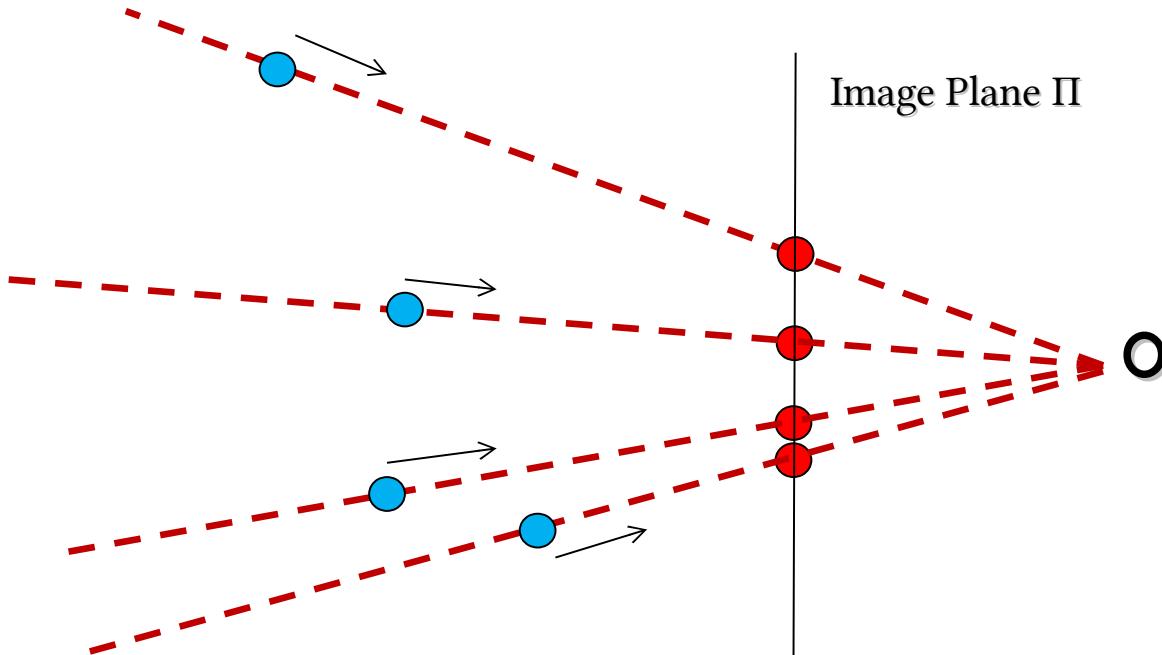
Camera model



$$M = K[R \ T]$$

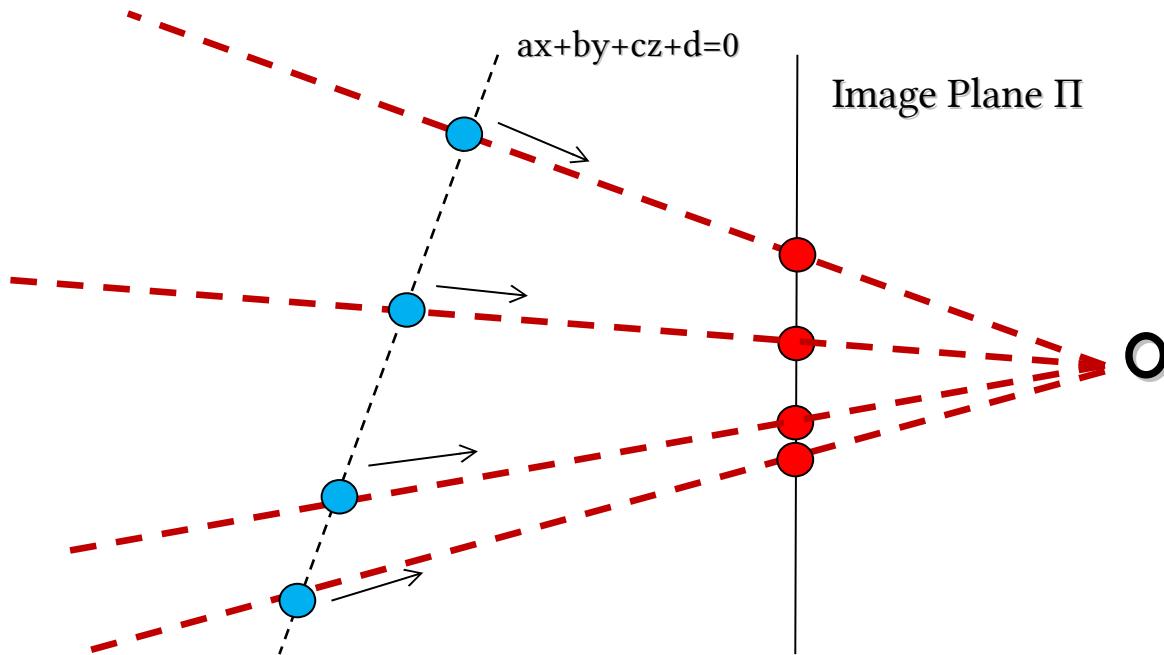
- Can we say where is P when we know only p?
 - NO: any P on the line of sight can generate p...
 - M can not be inverted (3×4)

Ill posed problem



- Projective space \rightarrow we lost one dimension
- 3D $[x,y,z]$ are projected in $[x/z,y/z,1]$

Add a constraint



- Just imagine that the (red) image points are projection of points that lie on the same plane
- In such a case for each image point there is only one world point (cyan)

Add a constraint

- Given a plane $aX + bY + cZ + d = 0$

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = M \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} u \\ v \\ w \\ 0 \end{bmatrix} = \begin{bmatrix} 4 \times 4 \\ M \\ abc \\ d \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ Z \\ W \end{bmatrix} = \begin{bmatrix} M \\ abc \\ d \end{bmatrix}^{-1} \begin{bmatrix} u \\ v \\ 1 \\ 0 \end{bmatrix} \quad \xrightarrow{\text{euclidean}} \quad \begin{bmatrix} X/W \\ Y/W \\ Z/W \end{bmatrix}$$

- We can obtain euclidean world coordinates of pixel (u, v)

Specific case, plane Z=0

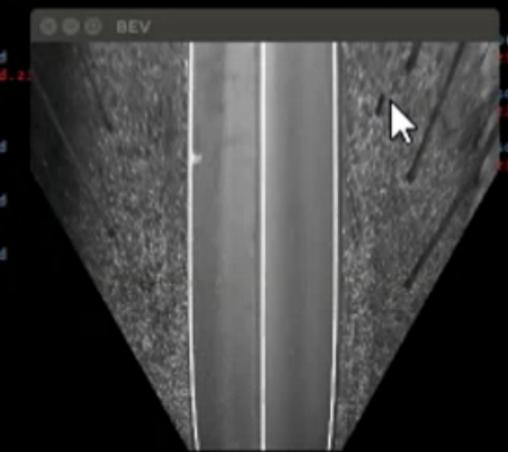
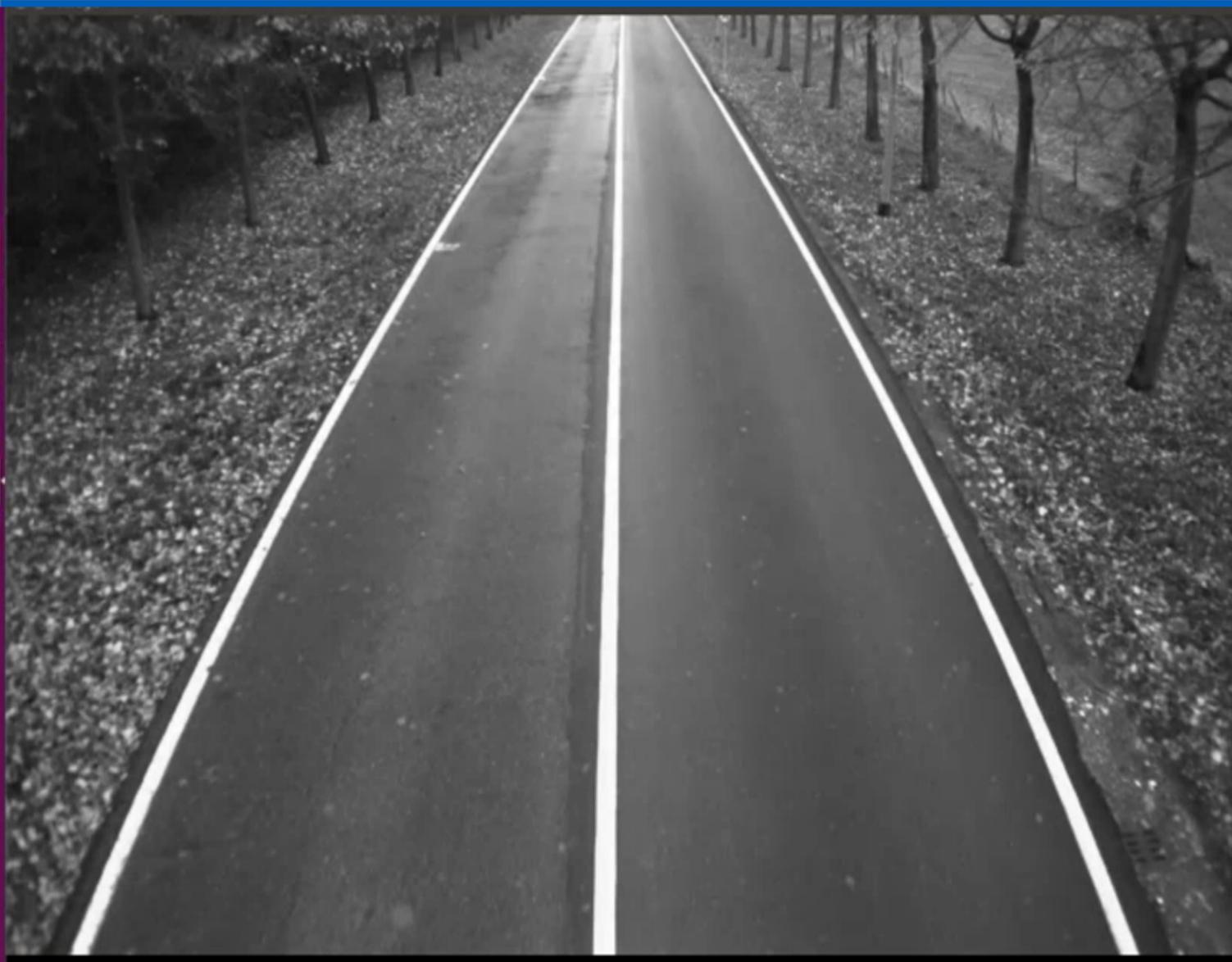


- Consider a very specific plane $\rightarrow Z=0$

$$\begin{bmatrix} X \\ Y \\ 0 \\ W \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & r_{14} \\ m_{21} & m_{22} & m_{23} & r_{24} \\ 0 & 0 & 0 & r_{34} \\ m_{41} & m_{42} & m_{43} & r_{44} \end{bmatrix} \begin{bmatrix} u \\ v \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} X \\ Y \\ W \end{bmatrix} = H \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

- H is 3×3
- This actually works for every possible plane!

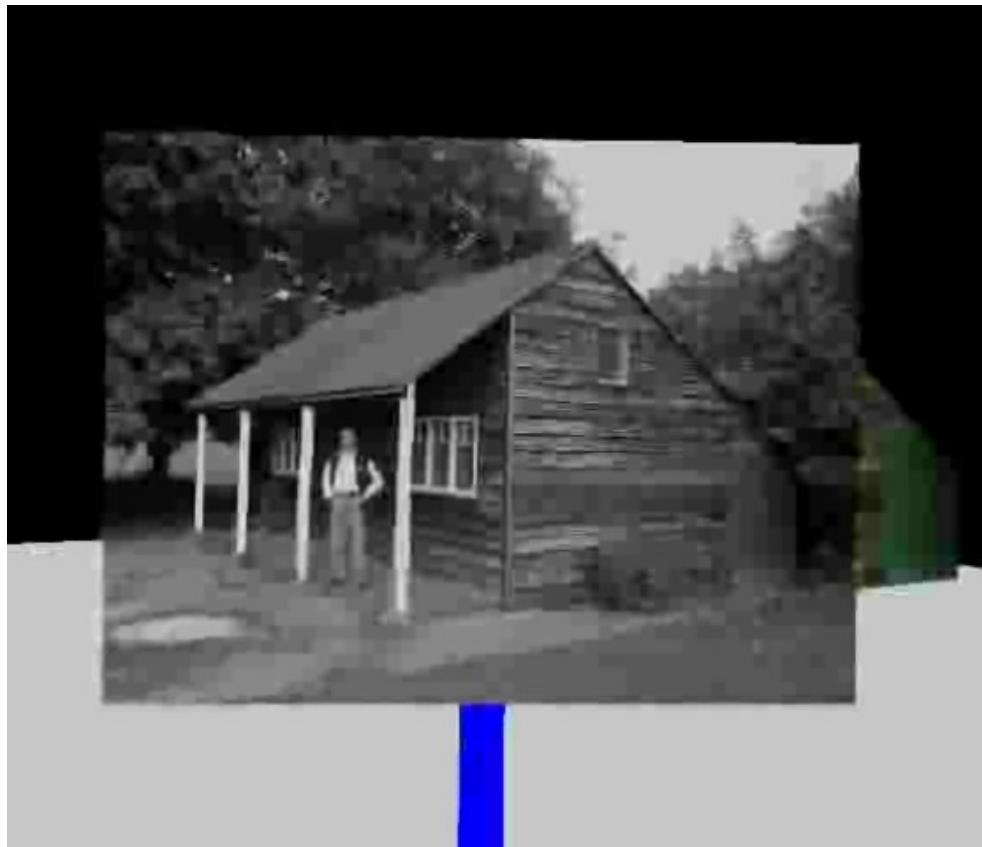


Single View Reconstruction



<http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl>

Single View Reconstruction



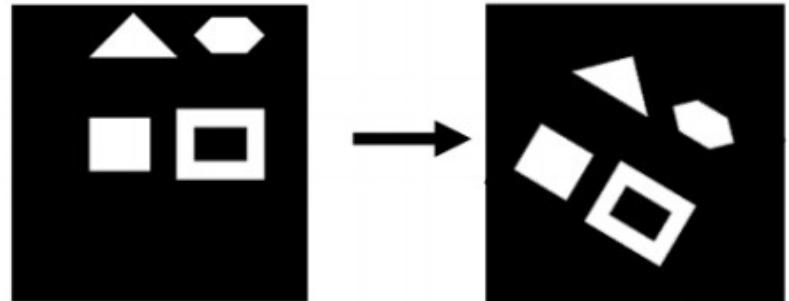
<http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/hut/hutme.wrl>

- Let us introduce some transformations
- For sake of simplicity in 2D
 - Isometric transformations
 - Similarity transformations
 - Affine transformations
 - Projective transformations or **homographies**

Isometric Transformation



- Concatenation of rotation and translation
- 3 degrees of freedom
 - 2 for translation and 1 for rotation
- **It preserves distance**
- Motion of rigid object



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_i \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$H_i = \begin{bmatrix} \cos \theta & -\sin \theta & t_x \\ \sin \theta & \cos \theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

R is a 2×2 matrix

Similarity Transformation

- Concatenation of rotation, translation, and scale
- 4 degrees of freedom
 - 2 for translation, 1 for rotation, 1 for scale
- **It preserves shape**
- Angles among lines & Lengths ratio



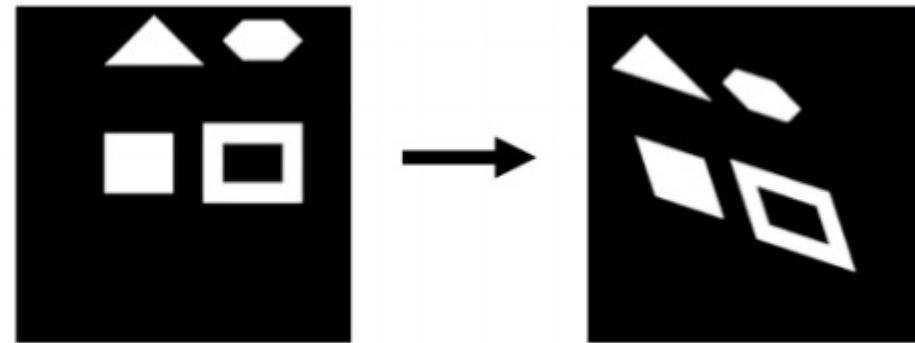
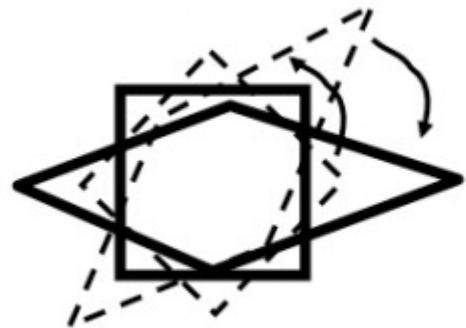
$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S & R & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_s \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
$$S = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix}$$

SR is still a 2×2 matrix

Affine Transformation



- It preserves ??



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \quad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

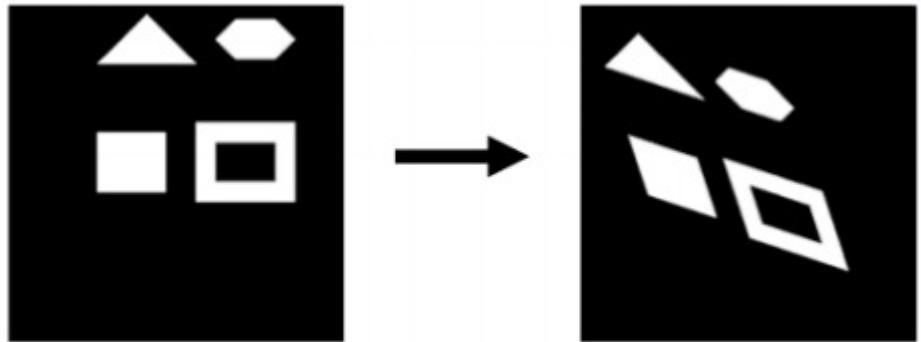
- Matrix A can be seen as a Single Value Decomposition of 2 orthogonal matrices and a diagonal one

$$A = UDV^T = (UV^T)(VDV^T) = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi)$$

$$UV^T = R(\theta), \quad V^T = R(\phi), \quad D = D$$

Affine Transformation

- It preserves
 - Parallel lines
 - Areas ratio
 - Collinear segment ratio
- 6 degrees of freedom



$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = R(\theta) \cdot R(-\phi) \cdot D \cdot R(\phi) \quad D = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

Projective Transformation

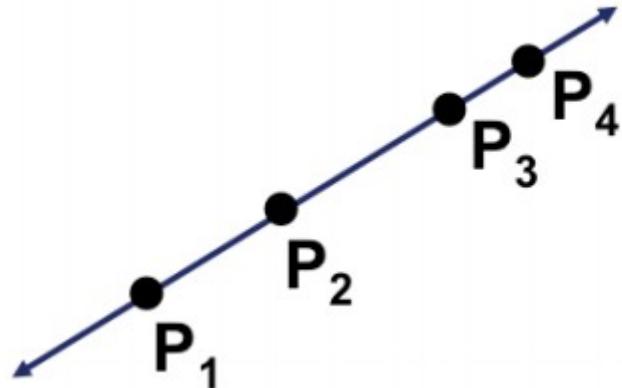


- It preserves
 - Collinearity
 - Cross-ratio for 4 collinear points
- 8 degrees of freedom



$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} A & t \\ v & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_p \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Cross-ratio for 4 collinear points



[Eq. 9]

$$\frac{\|\mathbf{P}_3 - \mathbf{P}_1\| \|\mathbf{P}_4 - \mathbf{P}_2\|}{\|\mathbf{P}_3 - \mathbf{P}_2\| \|\mathbf{P}_4 - \mathbf{P}_1\|}$$

$$\mathbf{P}_i = \begin{bmatrix} X_i \\ Y_i \\ Z_i \\ 1 \end{bmatrix}$$

Projective Transformation

- 8 degrees of freedom
 - Consider a scale factor
- The previous equation can be written as

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = k \begin{bmatrix} A & t \\ v & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = k' \begin{bmatrix} \frac{A}{c} & \frac{t}{c} \\ \frac{v}{c} & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine transformation vs Homography

- It can be noticed that “up” to the affine transformation z is left unchanged

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_a \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Homographies also affect z!

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} A & t \\ v & c \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = H_p \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

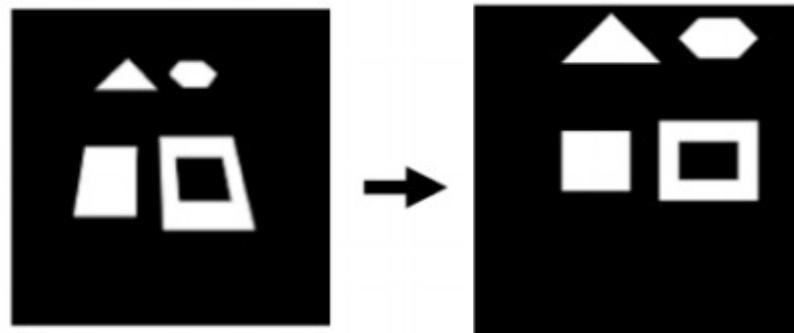
Affine transformation vs Homography

- Affine transformations:
 - move the (image) point
 - originally the projection of a (world) point
 - in another point
 - that it is the projection of another (world) point at the same distance as the previous one
 - $(x,y,z) \rightarrow (x',y',z)$
- Omographies do not have that constraint!
 - Let's consider the last H_P line as $[a\ b\ c]$
 - $(x,y,z) \rightarrow (x',y',ax+by+cz)$

Planar Homography



- A plane in the \mathbb{R}^3 space is
 - $ax+by+cz+d=0 \rightarrow ax+by+cz=-d$
- Projection of points that lies on such plane are $(x', y', -d)$
- Homographies transform points that lie on a plane on another plane: plane \rightarrow another plane with a different point of view

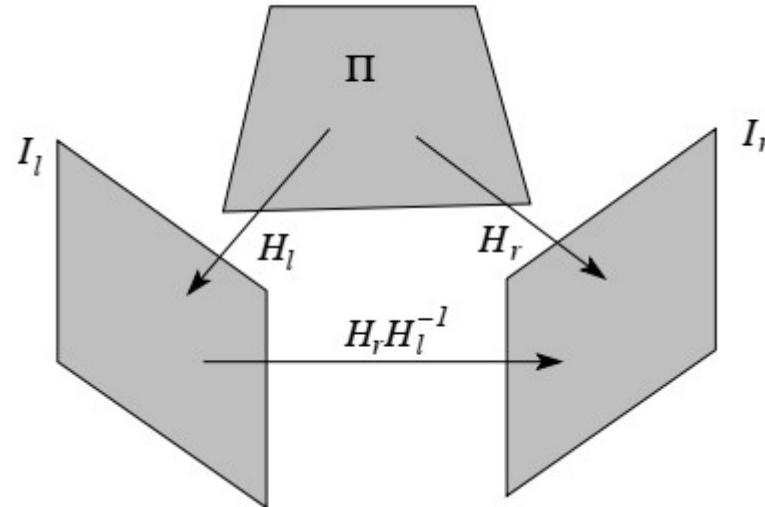


- Consider 2 planes
 - $ax+by+cz+d=0$ and $a'x+b'y+c'z+d'=0$
- It can be easily demonstrated that we can compute an homography that bring from one plane to the other
- We can compute H and H' homographies
- We can then combine them to obtain another projection
 - $H_{\Pi} = (H) \cdot (H')^{-1} (d/d')$

Planar Homography



- We can see this in a different fashion
- The projection of a plane on another 2 planes can be computed using a homographic transformation



- Assuming the 2 planes are the sensor planes of 2 cameras
- The induced homography can be computed using intrinsic camera parameters and their relative position

$$H_{\Pi} = K' \left(R + \frac{tn^T}{d} \right) K^{-1}$$

- K and K' are intrinsic camera matrices
- R and t are relative rototranslation matrices
- $n \cdot p = d$ is plane equation



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Points and Lines at Infinity

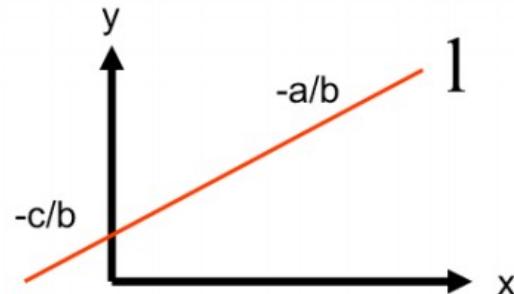
- Let us consider a straight line
 - How we can write it
 - When a point lies on that line



$$ax + by + c = 0$$

$$\mathbf{l} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

if $x = [x, y] \in l$



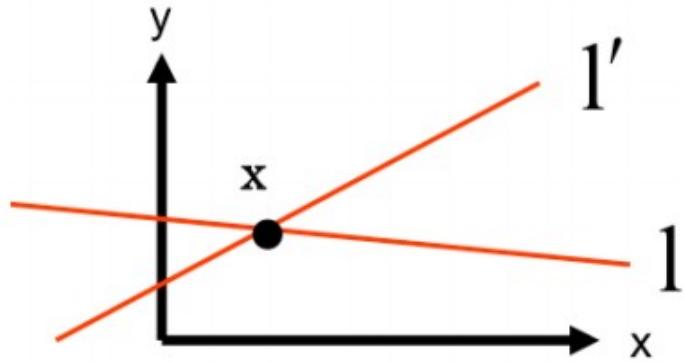
$$\begin{bmatrix} x & y & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = x^T \mathbf{l} = 0$$

- Not parallel lines l and l' intersect at unique point x
- Where is x ?
 - x lies on both lines, therefore: $x^T l = 0$ and $x^T l' = 0$ (1)
 - Let x be $l \times l'$
 - Does this satisfies (1)?
 - Hint: how is the result of a cross product?
 - Solution: it is a vector perpendicular to both l and l'
 - And what is the result of a scalar product between two perpendicular vectors?

2D Lines Intersection



$$x = l \times l' \quad [\text{Eq. 11}]$$



$$l \times l' \perp l \rightarrow (l \times l') \cdot l = 0 \rightarrow x \in l \quad [\text{Eq. 12}]$$

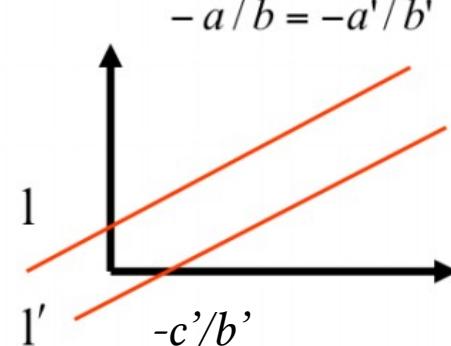
$$\underbrace{l \times l'}_x \perp l' \rightarrow (l \times l') \cdot l' = 0 \rightarrow x \in l' \quad [\text{Eq. 13}]$$

2D Parallel Lines Intersection

$$x = \begin{bmatrix} x \\ y \\ w \end{bmatrix}, w \neq 0$$

$$x_\infty = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

$$\begin{aligned} -a/b &= -a'/b' \\ l &= \begin{bmatrix} a \\ b \\ c \end{bmatrix} \quad l' = \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} = \begin{bmatrix} a \\ b \\ c'' \end{bmatrix} \\ l \times l' &= \begin{bmatrix} bc'' - bc \\ ac - ac'' \\ ab - ab \end{bmatrix} = \begin{bmatrix} b(c'' - c) \\ -a(c'' - c) \\ 0(c'' - c) \end{bmatrix} \propto \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = x_\infty \quad \text{eq. 13} \end{aligned}$$

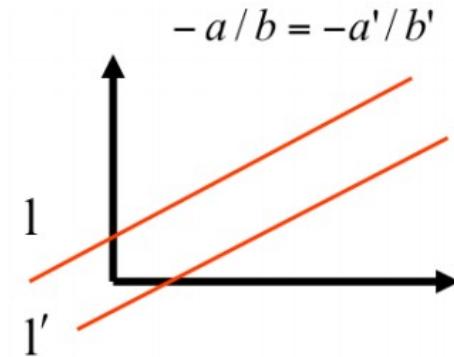


- The result is a point to infinity
 - Ideal point

2D Ideal Points



$$x_{\infty} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$



$$\begin{aligned} l &= \begin{bmatrix} a \\ b \\ c \end{bmatrix} \\ l' &= \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix} \end{aligned}$$

Note: the line $l = [a \ b \ c]^T$ pass trough the ideal point x_{∞}

$$l^T x_{\infty} = \begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} b \\ -a \\ 0 \end{bmatrix} = 0 \quad [\text{Eq. 15}]$$

- This is true for every straight lines with the same slope

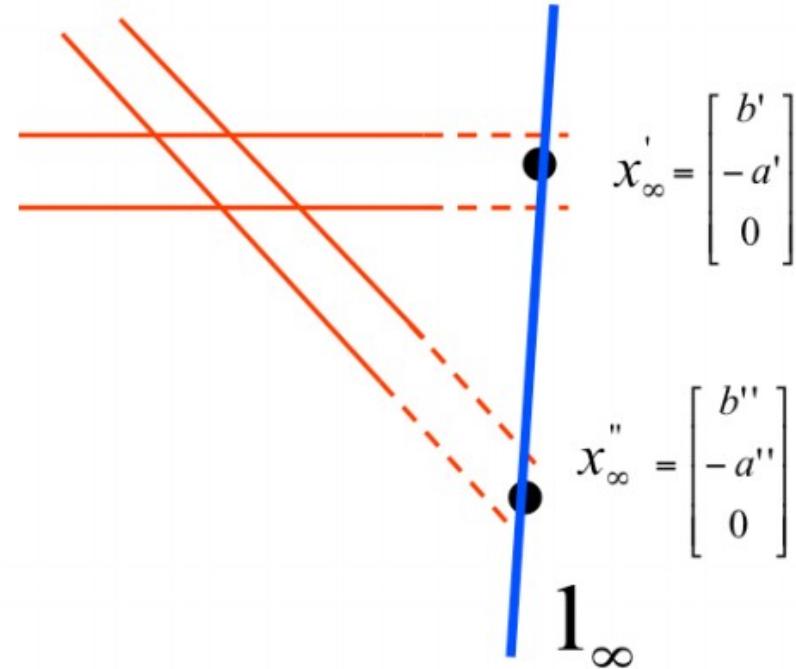
Ideal line



$$l_{\infty} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Indeed:

$$\begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix}^T \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 0$$

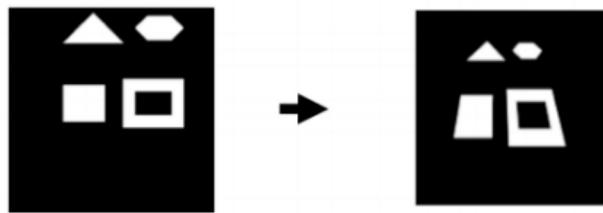


- All ideal points lie on the same line → ideal line or vanishing line
 - set of “directions” of lines in the plane

Ideal points transformations



$$H = \begin{bmatrix} A & t \\ v & b \end{bmatrix}$$



$$p' = H \ p$$

is it a point at infinity?

$$H \ p_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

[Eq. 17]

Ideal line transformation



$$H = \begin{bmatrix} A & t \\ v & b \end{bmatrix}$$



$$l' = H^{-T} l$$

is it a line at infinity?

[Eq. 19]

$$H^{-T} l_\infty = ? = \begin{bmatrix} A & t \\ v & b \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} t_x \\ t_y \\ b \end{bmatrix} \dots \text{no!}$$

[Eq. 20]

$$H_A^{-T} l_\infty = ? = \begin{bmatrix} A & t \\ 0 & 1 \end{bmatrix}^{-T} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A^{-T} & 0 \\ -t^T A^{-T} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

[Eq. 21]



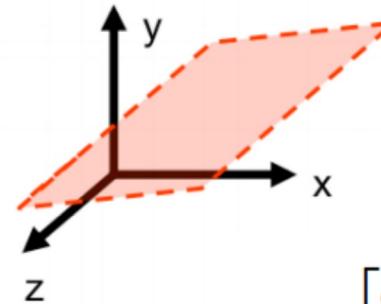
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Back to 3D

3D planes e lines



$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 1 \end{bmatrix} \quad \Pi = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$



[HZ] Ch 3.2.2

$$x^T \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = ax_1 + bx_2 + cx_3 + d = 0$$

$$x \in \Pi \Leftrightarrow x^T \Pi = 0$$

[Eq. 22]

$$ax + by + cz + d = 0$$

[Eq. 23]

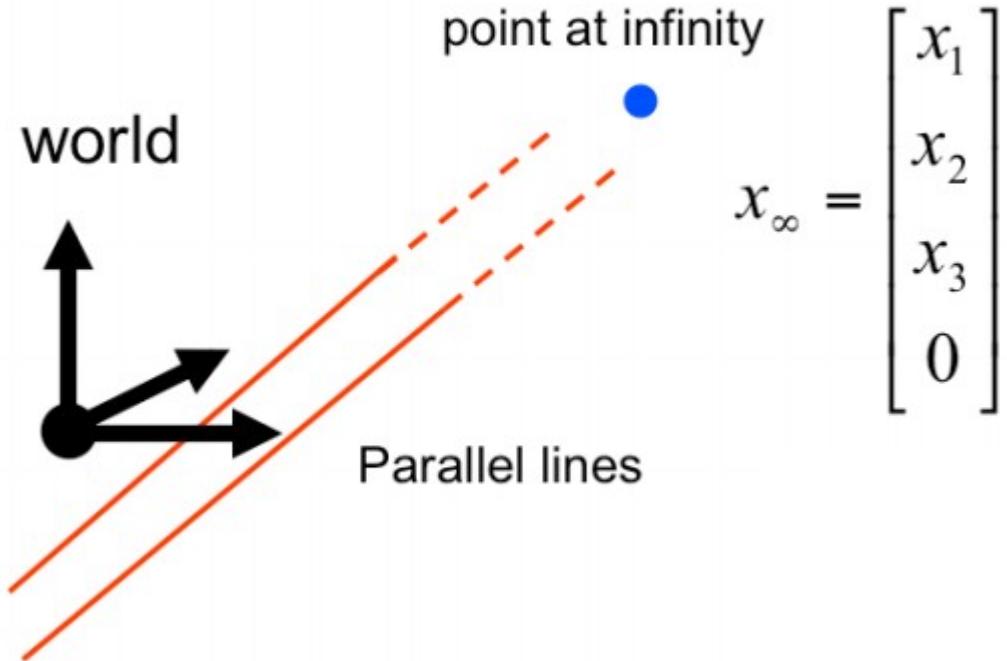
- Lines have 4 degree of freedom
 - They can be seen as intersections of 2 planes

$$l_{3D} = \begin{bmatrix} \Pi_1^T \\ \Pi_2^T \end{bmatrix}_{2 \times 4}$$

3D vanishing points



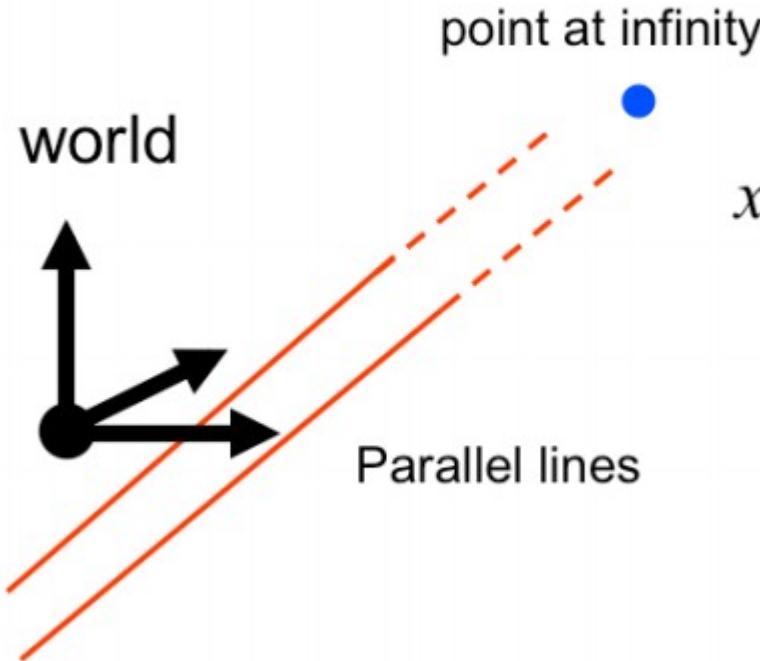
- Similar to 2D case



3D vanishing points

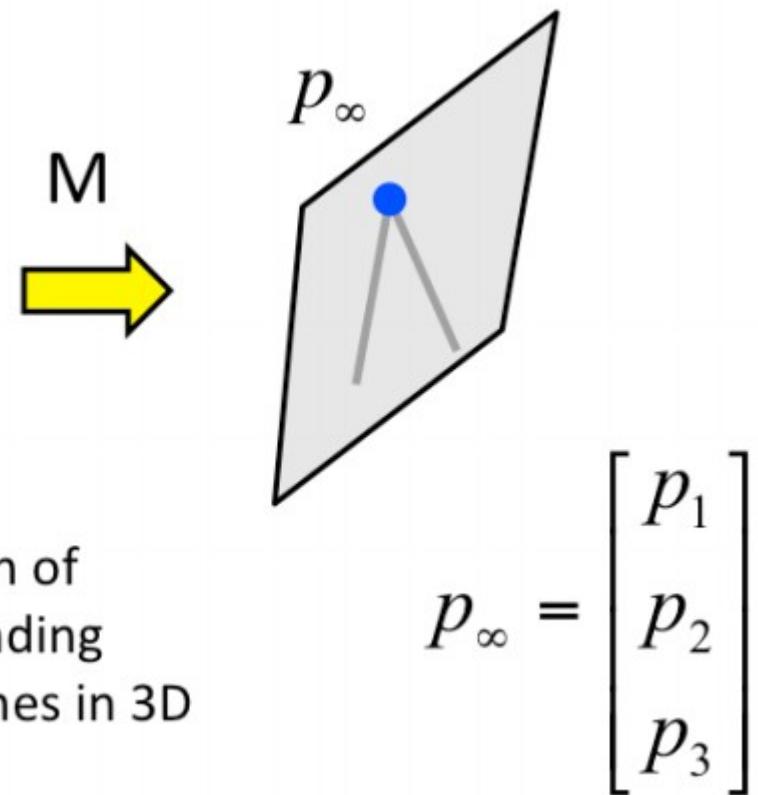


- A projective transformation M can bring point to infinity in a finite place → vanishing point



$$x_{\infty} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ 0 \end{bmatrix}$$

= direction of
corresponding
parallel lines in 3D

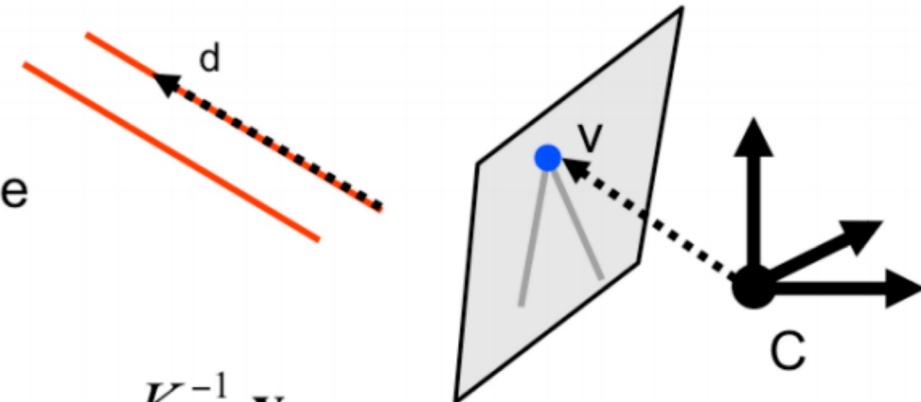


Vanishing points vs Camera parameters



Let's consider a pencil of lines

\mathbf{d} = direction of the line
 $= [a, b, c]^T$



$$\mathbf{v} = K \mathbf{d}$$

[Eq. 24]

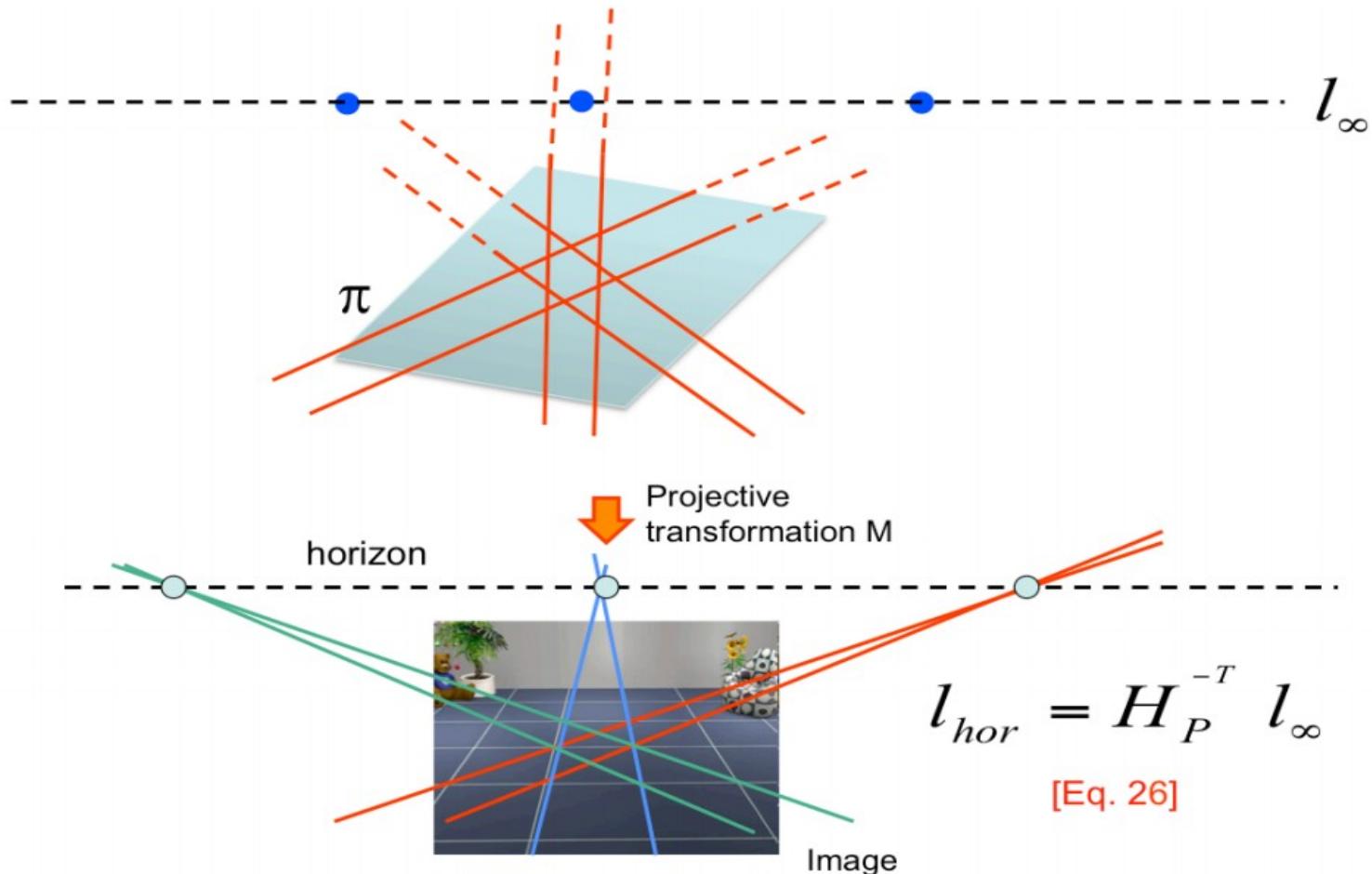
$$\mathbf{d} = \frac{K^{-1} \mathbf{v}}{\|K^{-1} \mathbf{v}\|}$$

[Eq. 25]

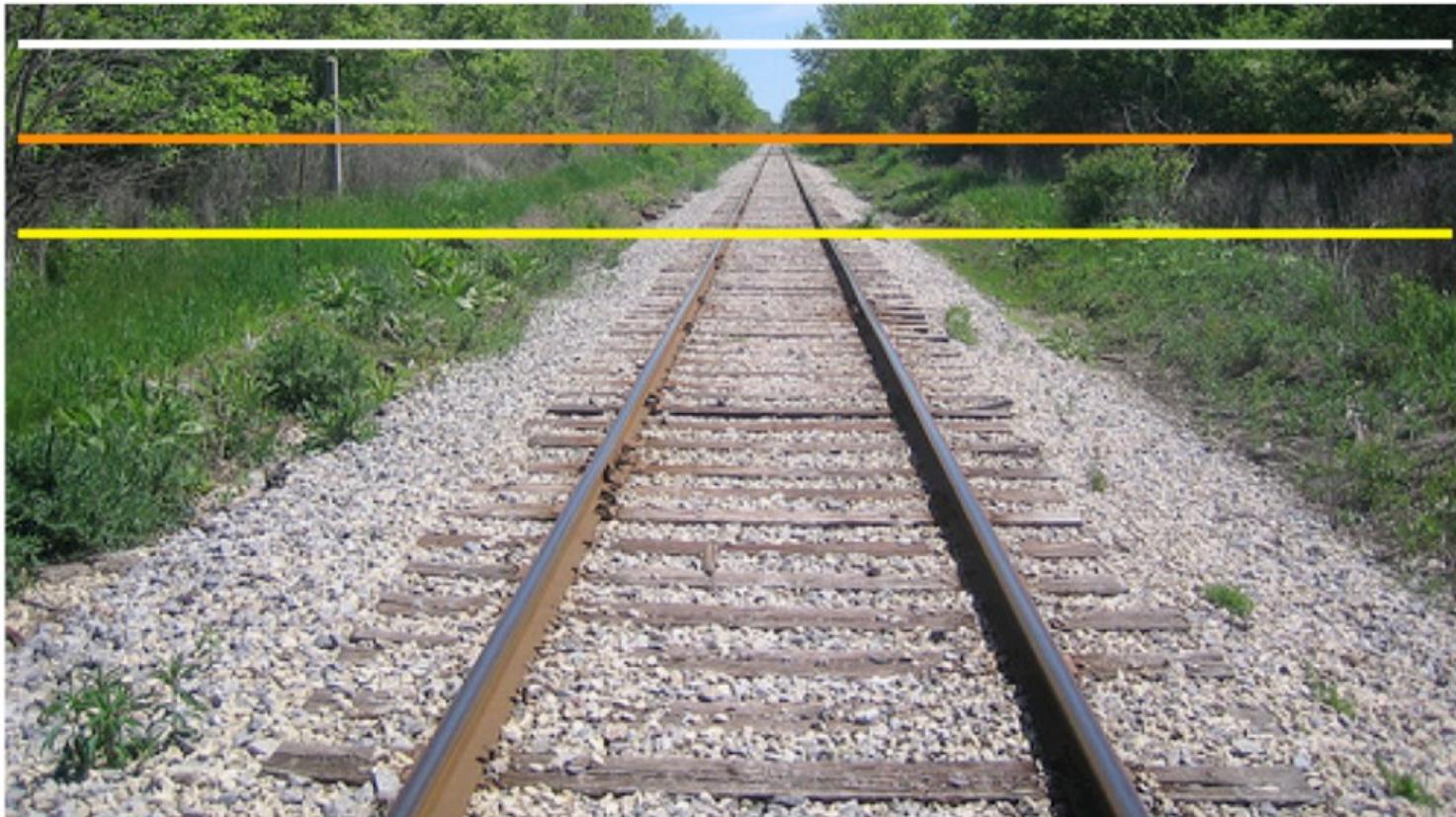
Proof:

$$X_\infty = \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} \xrightarrow{M} \mathbf{v} = M X_\infty = \mathbf{K} [\mathbf{I} \quad \mathbf{0}] \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix} = \mathbf{K} \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}$$

Horizon line

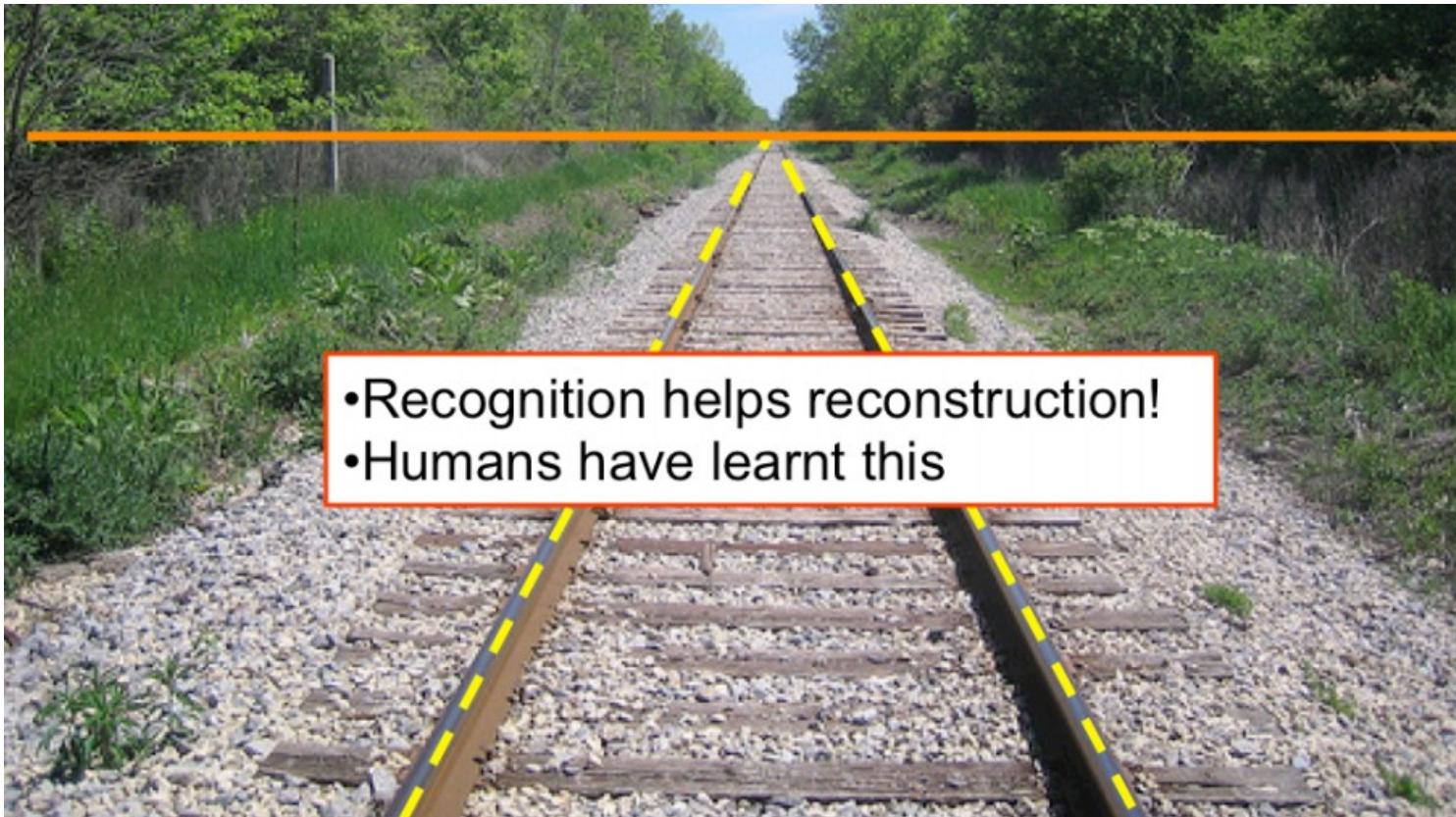


Horizon line



Humans intuitively deduce properties about the image

Horizon line



If lines intersect on horizon line they are parallel

Horizon line



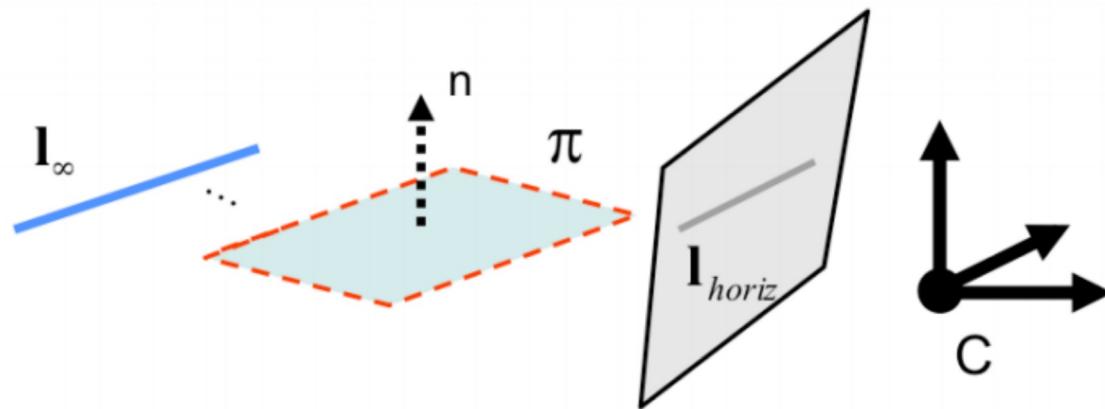
If lines intersect on horizon line they are parallel

Horizon line



If lines intersect on horizon line they are parallel

Horizon line vs Camera parameters



[HZ] Ch 8.6.2

we can estimate the orientation of the ground plane!

$$\mathbf{n} = \mathbf{K}^T \mathbf{l}_{horiz}$$

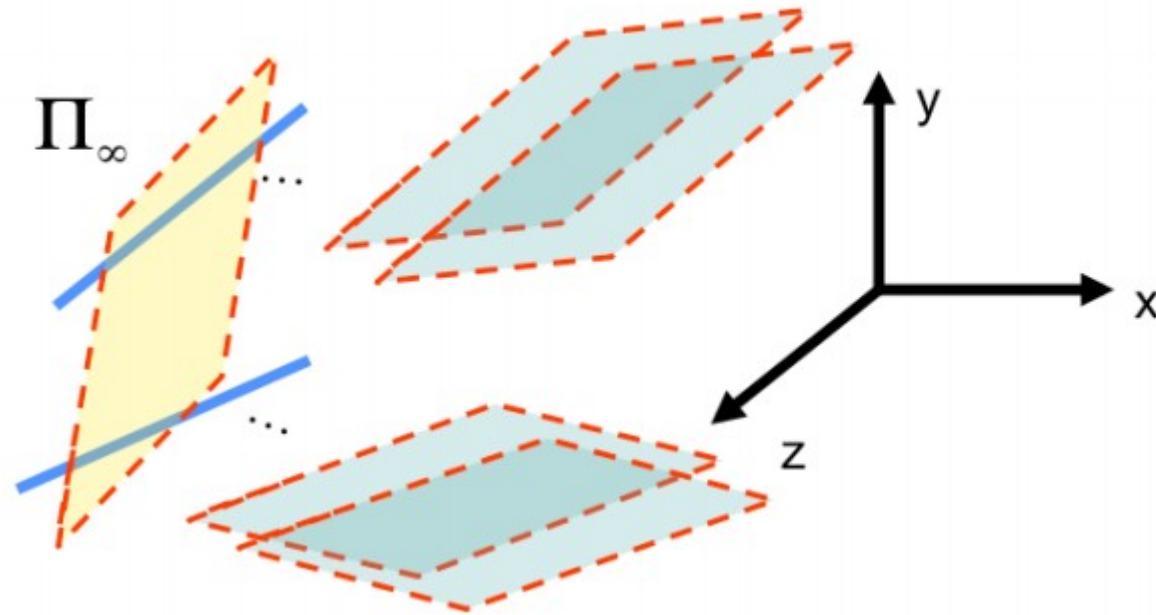
[Eq. 27]



Planes at infinity



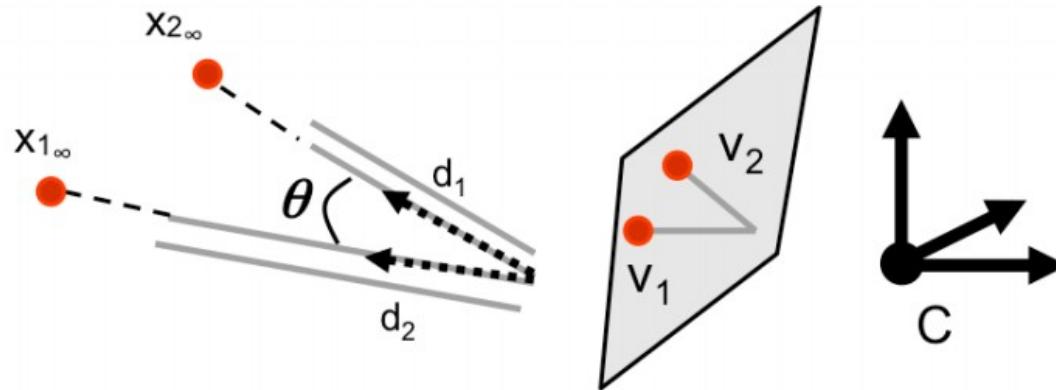
- Parallel planes intersect on a line at infinity
- 2 or more lines at infinity define a plane at infinity



$$\Pi_{\infty} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

plane at infinity

Angle between 2 pencils of straight lines



$$\cos \theta = \frac{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2}{\sqrt{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_1} \sqrt{\mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_2}}$$

$\boldsymbol{\omega} = (K K^T)^{-1}$

[Eq. 28]

If $\theta = 90^\circ \rightarrow \boxed{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0}$ [Eq. 29]

Scalar equation

Small recap



$$\mathbf{v} = K \mathbf{d}$$

[Eq. 24]

$$\mathbf{n} = K^T \mathbf{l}_{\text{horiz}}$$

[Eq. 27]

$$\cos \theta = \frac{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2}{\sqrt{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_1} \sqrt{\mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_2}}$$

[Eq. 28]

$$\theta = 90^\circ$$

$$\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0$$

[Eq. 29]

$$\boldsymbol{\omega} = (K K^T)^{-1}$$

Estimating geometry from a single image



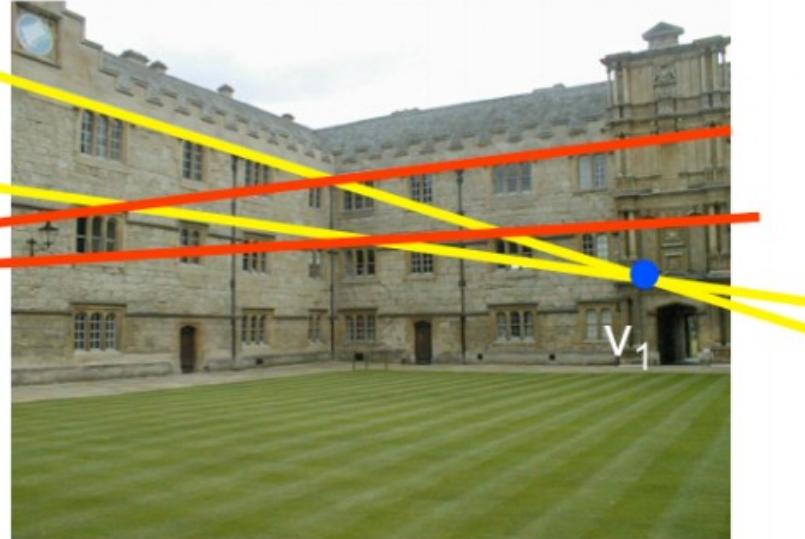
[Eq. 28]

$$\cos \theta = \frac{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2}{\sqrt{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_1} \sqrt{\mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_2}}$$

\mathbf{v}_2

$$\theta = 90^\circ$$

$$\left\{ \begin{array}{l} \mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0 \\ \boldsymbol{\omega} = (\mathbf{K} \mathbf{K}^T)^{-1} \end{array} \right. \quad \xrightarrow{\hspace{1cm}}$$



- Identify two 3D planes and a pair of parallel lines for each plane → $\mathbf{v}_1, \mathbf{v}_2$ and eq. 28
- 3D perpendicular → eq. 29
- Scalar eq. 29 is still not enough (\mathbf{K} has 5 DOF)

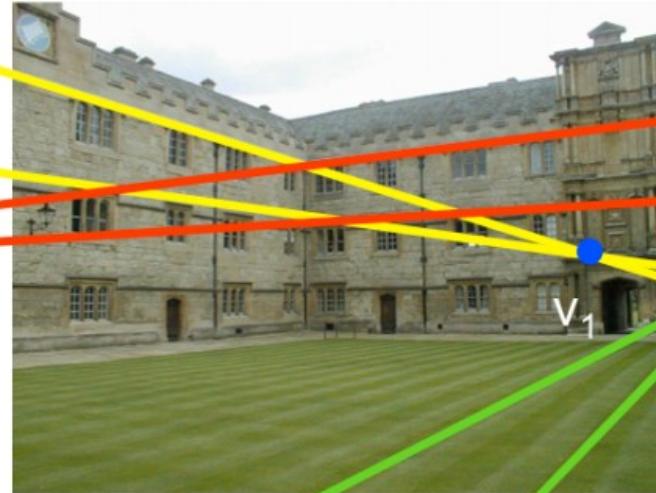
Estimating geometry from a single image



[Eq. 28]

$$\cos \theta = \frac{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2}{\sqrt{\mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_1} \sqrt{\mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_2}}$$

\mathbf{v}_2



[Eqs. 31]

$$\left\{ \begin{array}{l} \mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0 \\ \mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_3 = 0 \\ \mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_3 = 0 \end{array} \right.$$

- If we get a third perpendicular plane...
- Scalar eq. 29 is still not enough (3 vs 5 DOF)

Small recap about K



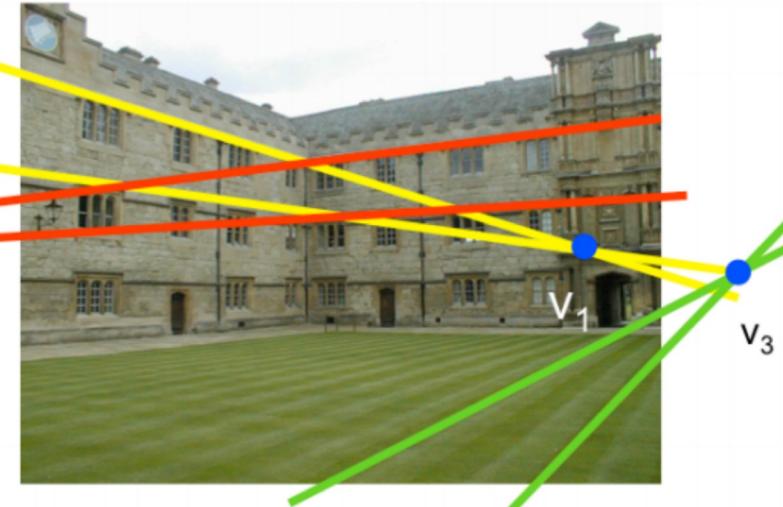
$$K = \begin{bmatrix} \alpha & -\alpha \cot(\theta) & c_x \\ 0 & \frac{\beta}{\sin(\theta)} & c_y \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \begin{array}{l} \text{Square pixels} \\ \text{No Skew} \end{array} \quad \begin{bmatrix} f & 0 & c_x \\ 0 & f & c_y \\ 0 & 0 & 1 \end{bmatrix}$$

Estimating geometry from a single image



$$\omega = \begin{bmatrix} \omega_1 & \omega_2 & \omega_4 \\ \omega_2 & \omega_3 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

- Square pixels
 - No skew
- $\omega_2 = 0$
 $\omega_1 = \omega_3$



[Eqs. 31]

$$\left\{ \begin{array}{l} \mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_2 = 0 \\ \mathbf{v}_1^T \boldsymbol{\omega} \mathbf{v}_3 = 0 \\ \mathbf{v}_2^T \boldsymbol{\omega} \mathbf{v}_3 = 0 \end{array} \right.$$

• Assumptions can reduce $\boldsymbol{\omega}$ DOF

Estimating geometry from a single image



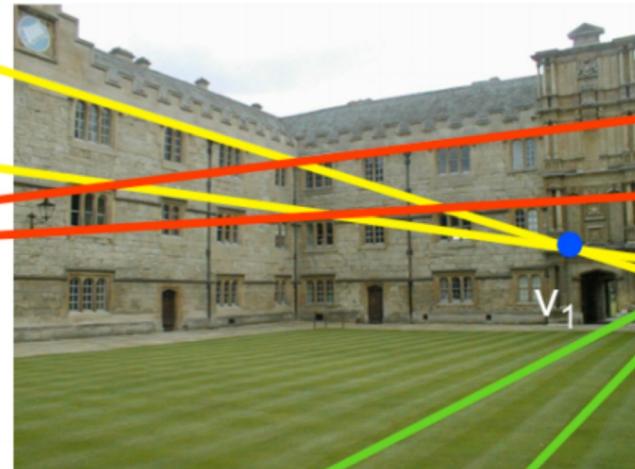
$$\omega = \begin{bmatrix} \omega_1 & 0 & \omega_4 \\ 0 & \omega_1 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

known up to scale

v_2

- Square pixels
- No skew

$$\rightarrow \begin{aligned} \omega_2 &= 0 \\ \omega_1 &= \omega_3 \end{aligned}$$



[Eqs. 31]

$$\left\{ \begin{array}{l} v_1^T \omega v_2 = 0 \\ v_1^T \omega v_3 = 0 \\ v_2^T \omega v_3 = 0 \end{array} \right.$$

→ Compute ω !

Estimating geometry from a single image



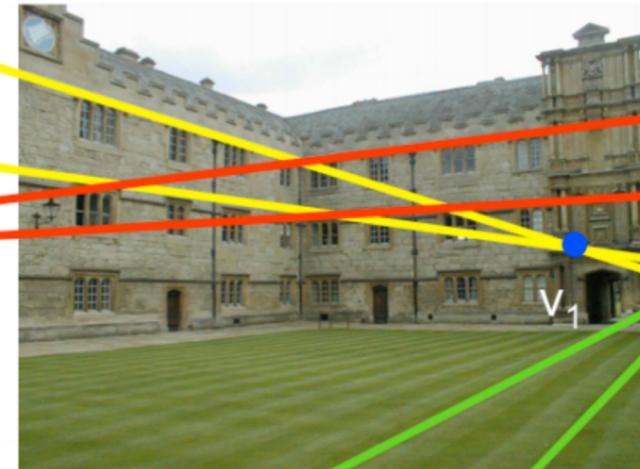
$$\omega = \begin{bmatrix} \omega_1 & 0 & \omega_4 \\ 0 & \omega_1 & \omega_5 \\ \omega_4 & \omega_5 & \omega_6 \end{bmatrix}$$

known up to scale

v_2

- Square pixels
- No skew

$$\rightarrow \begin{aligned} \omega_2 &= 0 \\ \omega_1 &= \omega_3 \end{aligned}$$



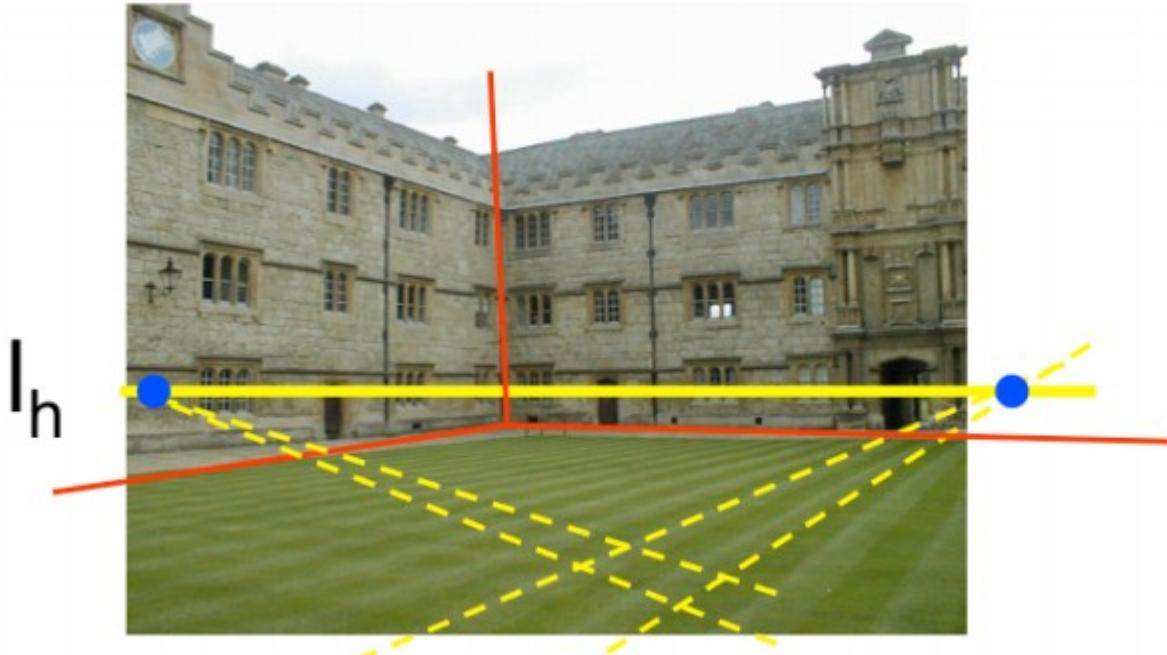
Fact. Cholesky [HZ] Ch pg. 582

[Eqs. 31]

$$\left\{ \begin{array}{l} v_1^T \omega v_2 = 0 \\ v_1^T \omega v_3 = 0 \\ v_2^T \omega v_3 = 0 \end{array} \right.$$

$$\omega = (K \ K^T)^{-1} \longrightarrow K$$

Estimating geometry from a single image



[Eq. 27]

$$K \text{ known} \rightarrow \mathbf{n} = K^T \mathbf{l}_{\text{horiz}}$$

Eq. 27 allows to compute corresponding lines at infinity
We can then select orientation discontinuities (red lines)
And now we can compute the orientation of all planes

Example



Criminisi & Zisserman, 99

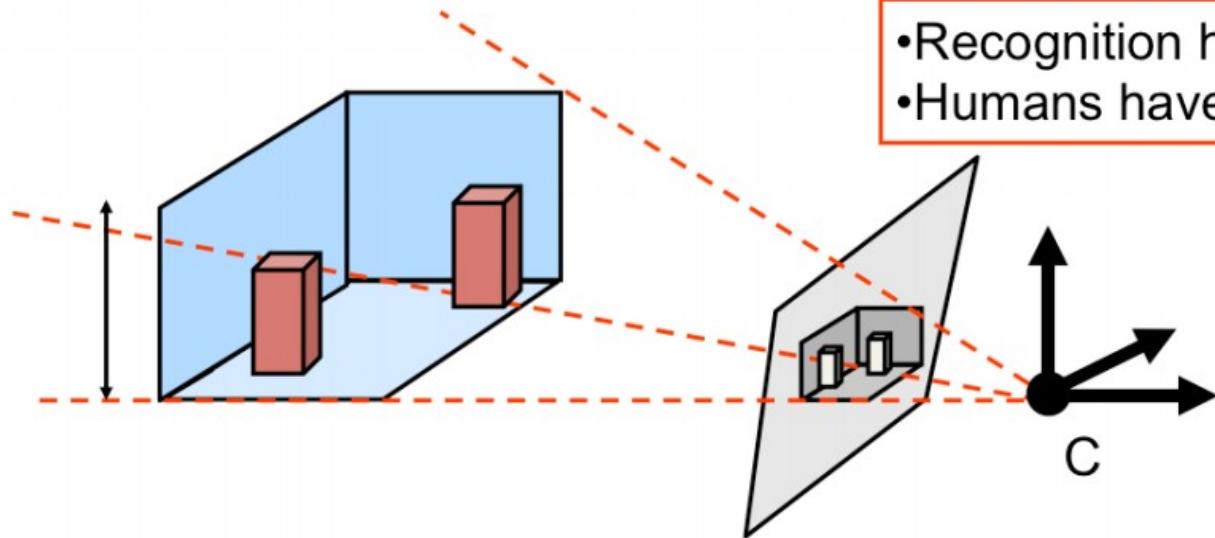


<http://www.robots.ox.ac.uk/~vgg/projects/SingleView/models/merton/merton.wrl>

Example



Small recap

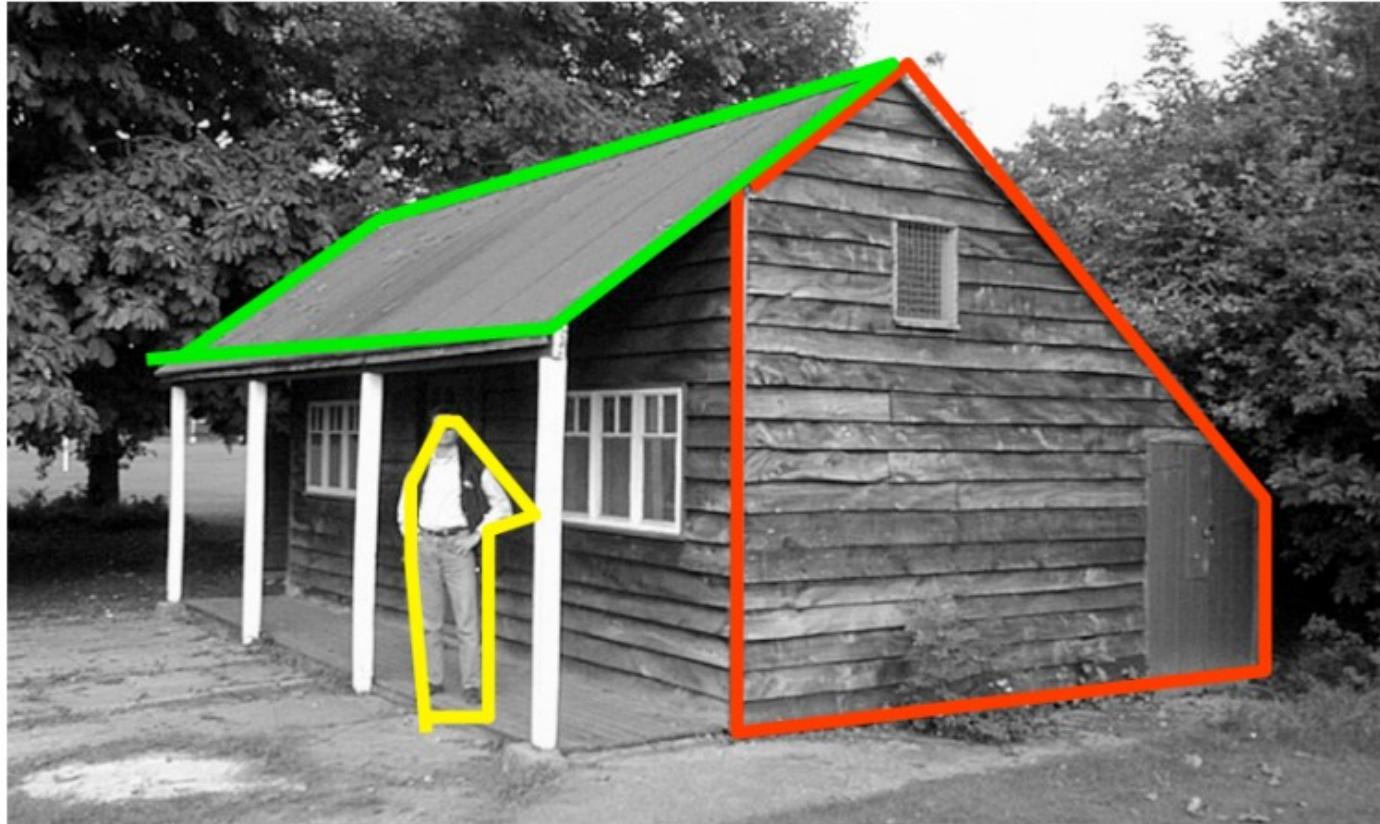


- Now we are able to compute K and some 3D world info using a single camera!
- Scale still unknown
 - We can recover having other info about what we are looking at
- Please this is not a unmanned procedure!

Small recap



Not an automatic procedure...



We are not cheating!



*St. Jerome in
his Study*

H. Steenwick



We are not cheating!



Not an automatic procedure...



- If humans can do this
 - IA also!



(a) input image



(b) superpixels



(c) constellations



(d) labeling



(e) novel view



UNIVERSITÀ DI PARMA

Single View Reconstruction

Question time!

