



UNIVERSITÀ DI PARMA

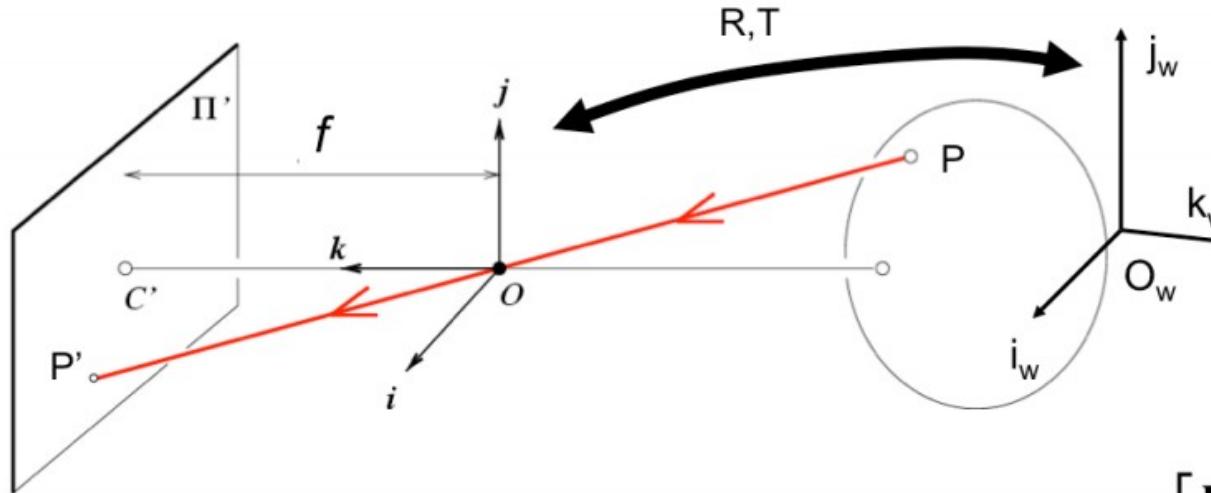
Camera Models (2)

Summary



- Pin Hole Camera recap
- Calibration

Perspective Transformation

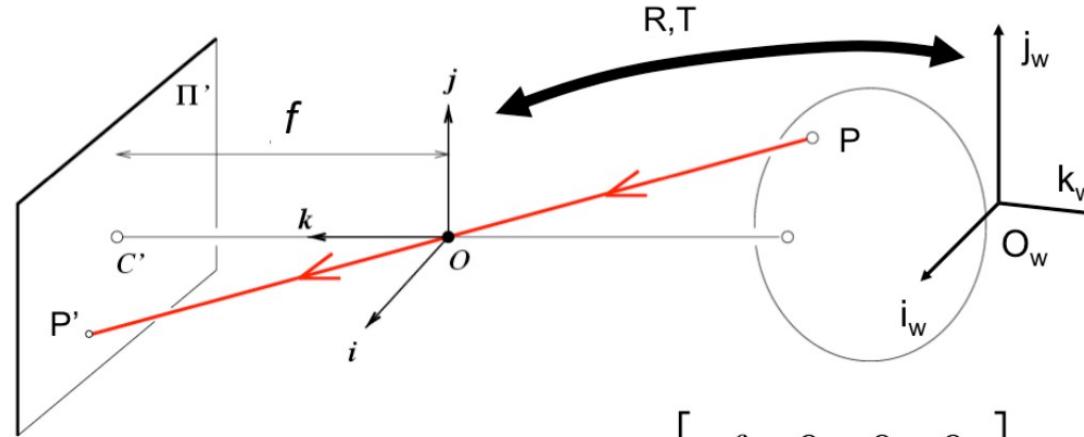


$$\begin{aligned}
 P'_{3 \times 1} &= M P_w = K_{3 \times 3} \begin{bmatrix} R & T \end{bmatrix}_{3 \times 4} P_w_{4 \times 1} & M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} \\
 &= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} P_w = \begin{bmatrix} \mathbf{m}_1 P_w \\ \mathbf{m}_2 P_w \\ \mathbf{m}_3 P_w \end{bmatrix} & \mathbf{E} \rightarrow \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right) \quad [\text{Eq.12}]
 \end{aligned}$$

Perspective Transformation (ideal case)



- Simplified situation: no rotation, no translation, no skew, no offset, squared pixels



$$M = K \begin{bmatrix} R & T \end{bmatrix} = K \begin{bmatrix} I & 0 \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\rightarrow P'_E = \left(\frac{\mathbf{m}_1 P_w}{\mathbf{m}_3 P_w}, \frac{\mathbf{m}_2 P_w}{\mathbf{m}_3 P_w} \right) = \left(f \frac{x_w}{z_w}, f \frac{y_w}{z_w} \right)$$

$$P_w = \begin{bmatrix} x_w \\ y_w \\ z_w \\ 1 \end{bmatrix}$$

Perspective Transformation



- 11 degrees of freedom

$$P' = M P_w = \boxed{K} \boxed{\begin{bmatrix} R & T \end{bmatrix}} P_w$$

Internal parameters External parameters

$$\mathcal{M} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix}_{3 \times 4}$$

$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_o \\ 0 & \frac{\beta}{\sin \theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \quad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

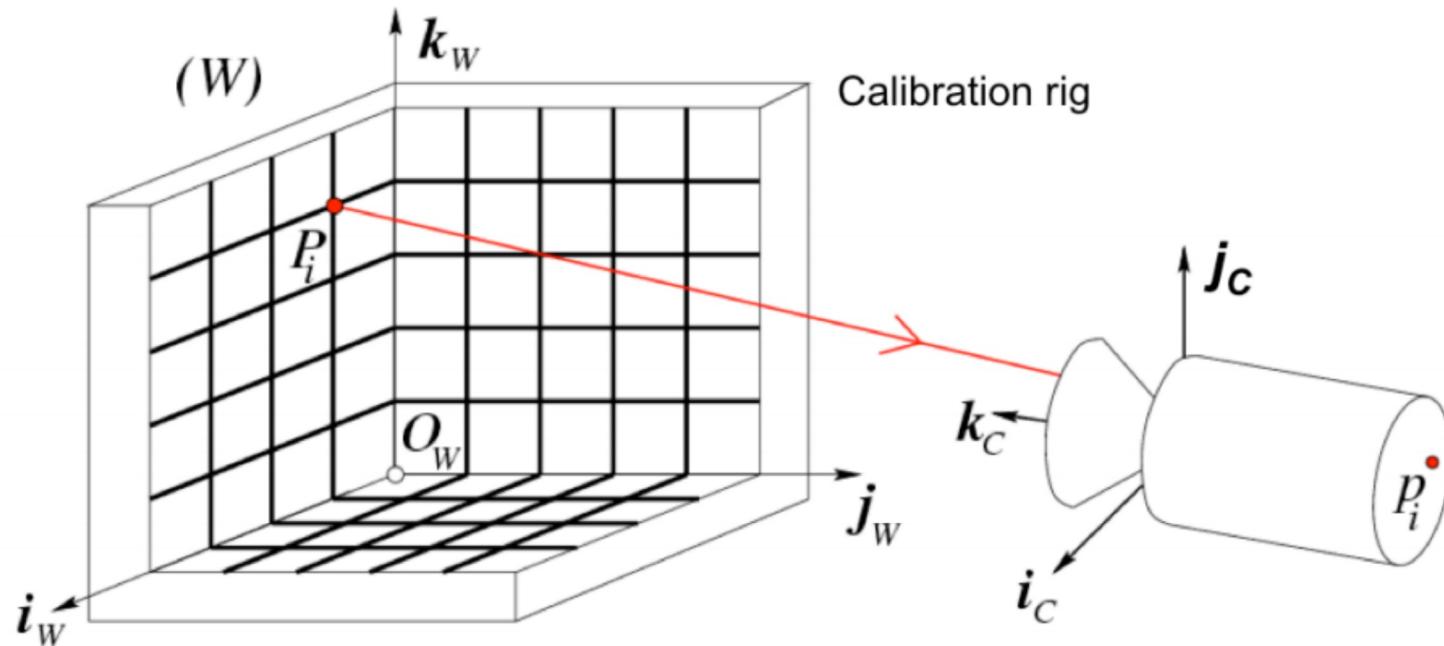
$$P' = M P_w = K \begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters External parameters

- Calibration → **estimation of intrinsic/extrinsic parameters**
- We need 1 or, rather, more images

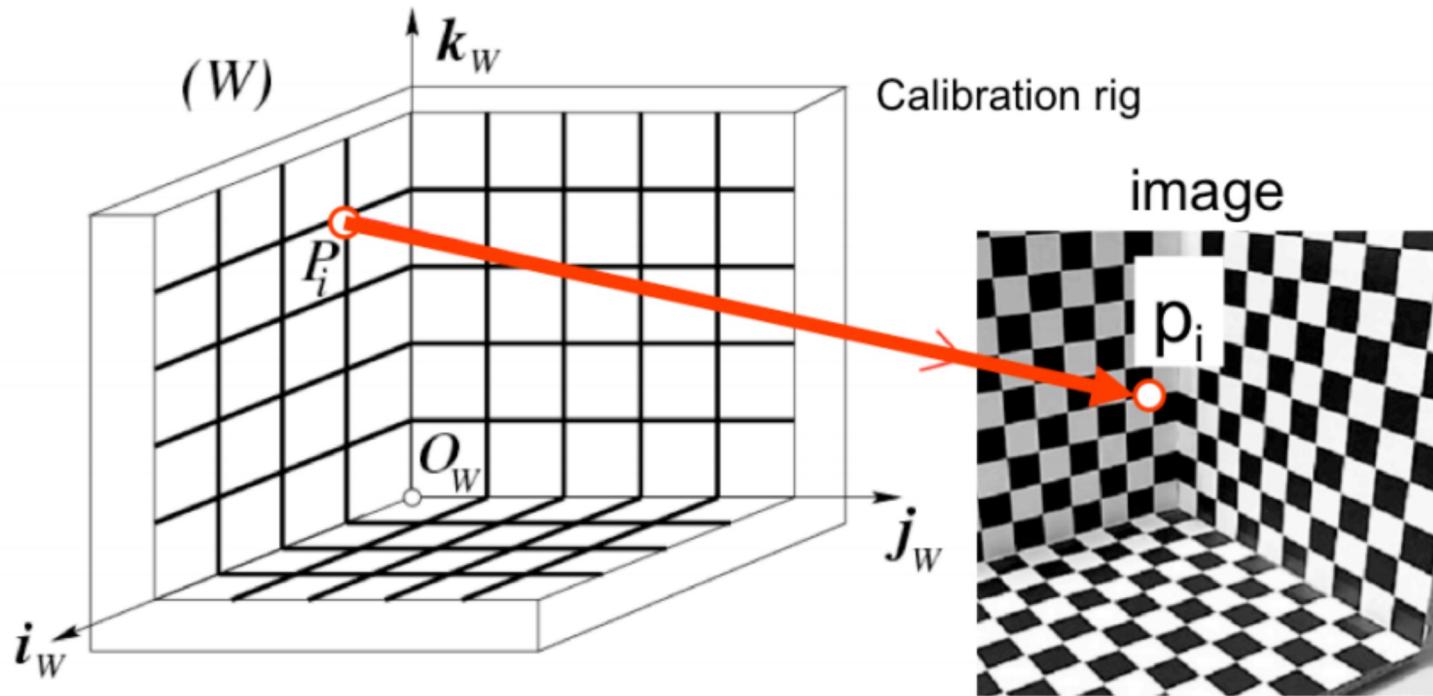
Change notation:
 $P = P_w$
 $p = P'$

Calibration



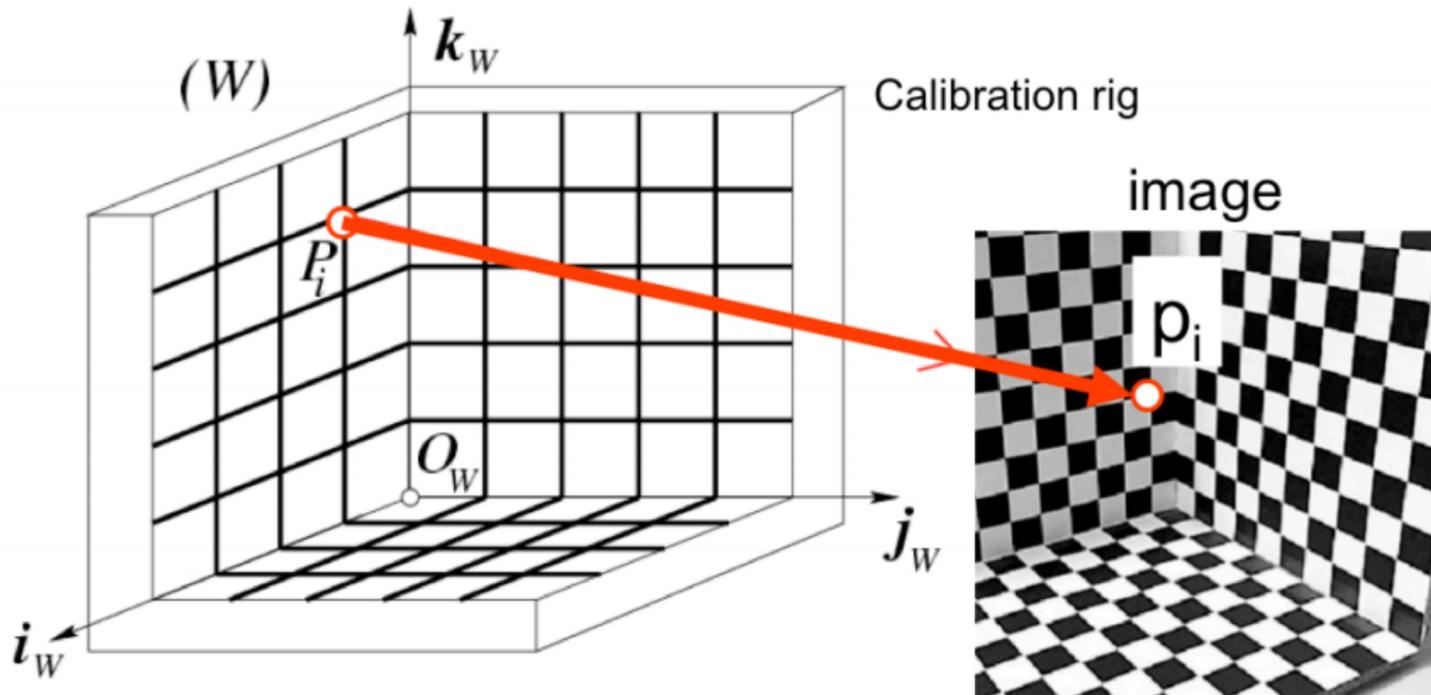
- We need: n points (P_1, P_2, \dots, P_n) in known positions [O_w, i_w, j_w, k_w] in the world reference system

Calibration



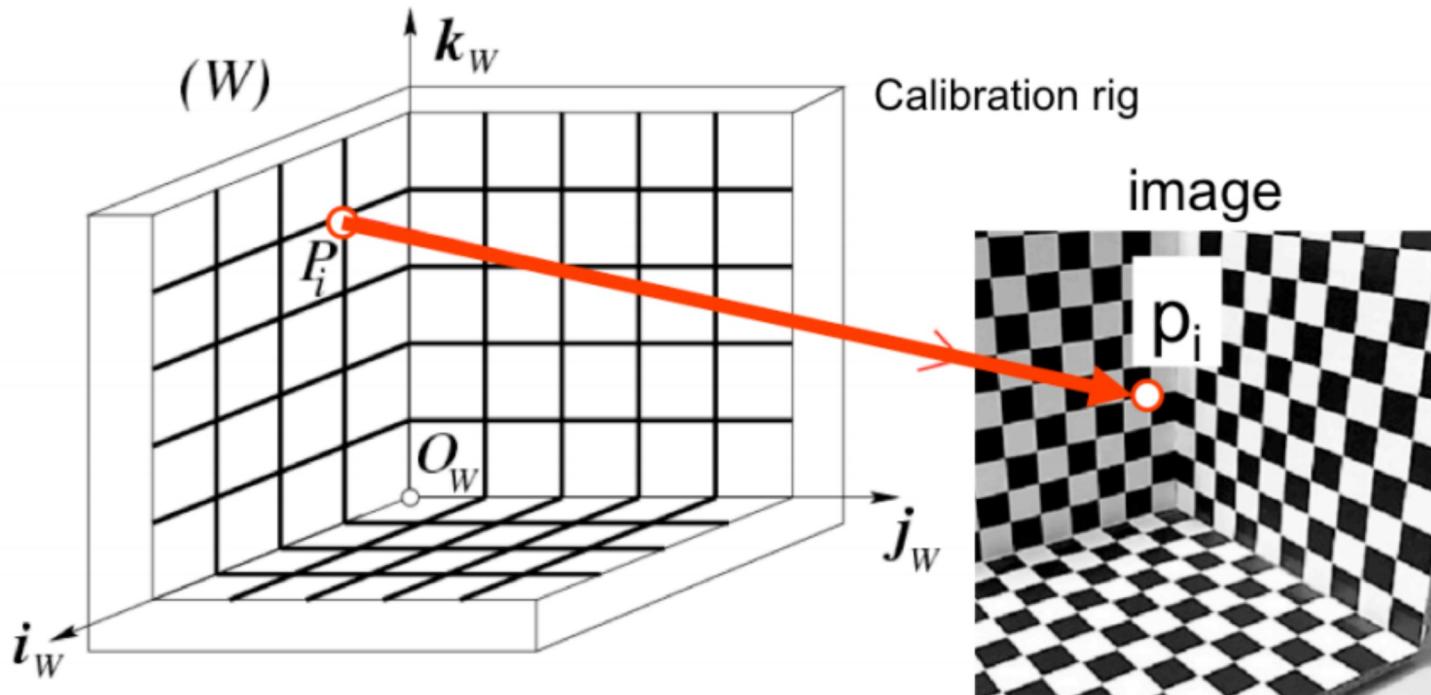
- We also need: camera coordinates for the n points projections (p_1, p_2, \dots, p_n) as (i_c, j_c)

Calibration



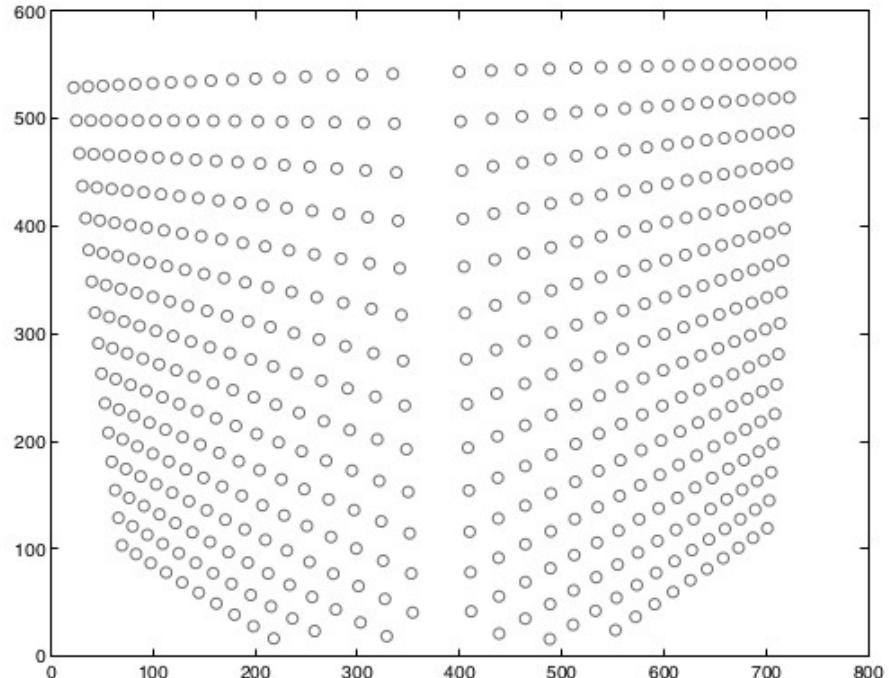
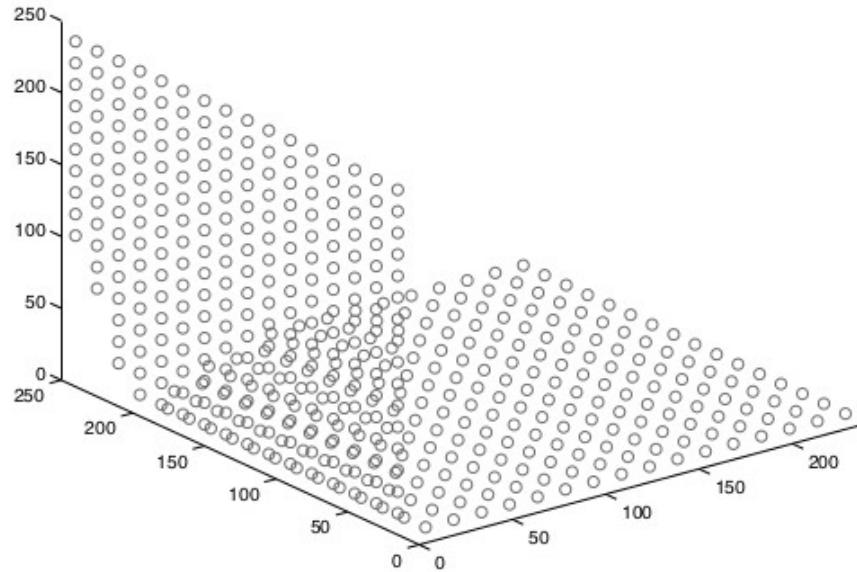
- 11 degrees of freedom → how many matches we need?
- Each correspondence → 2 equations (world coordinates → row/column)
- We need, at least, 6 correspondences

Calibration



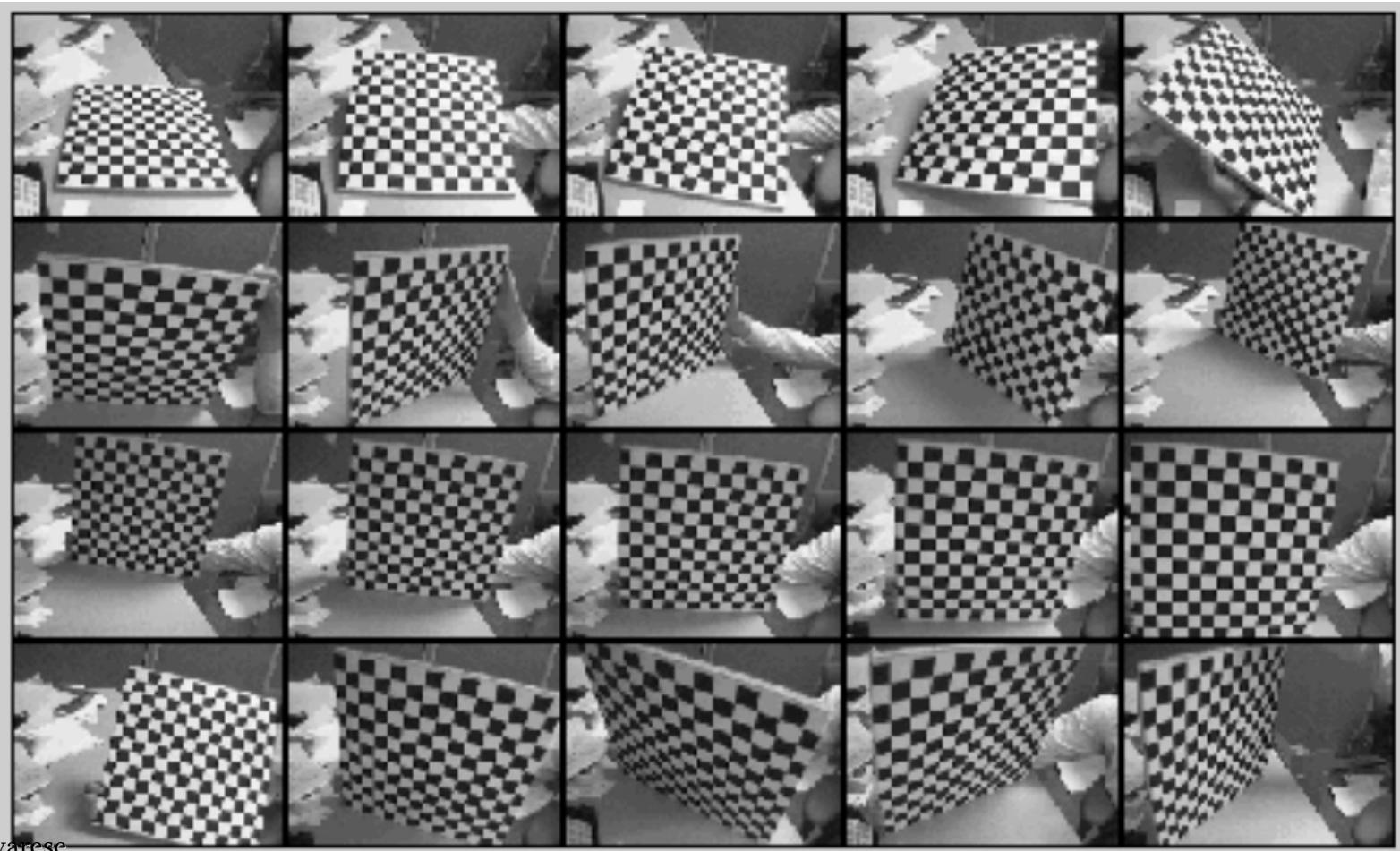
- Practically we need more than 6 correspondences!
- Consider error in measurements + digitalization error!

Example of calibration data

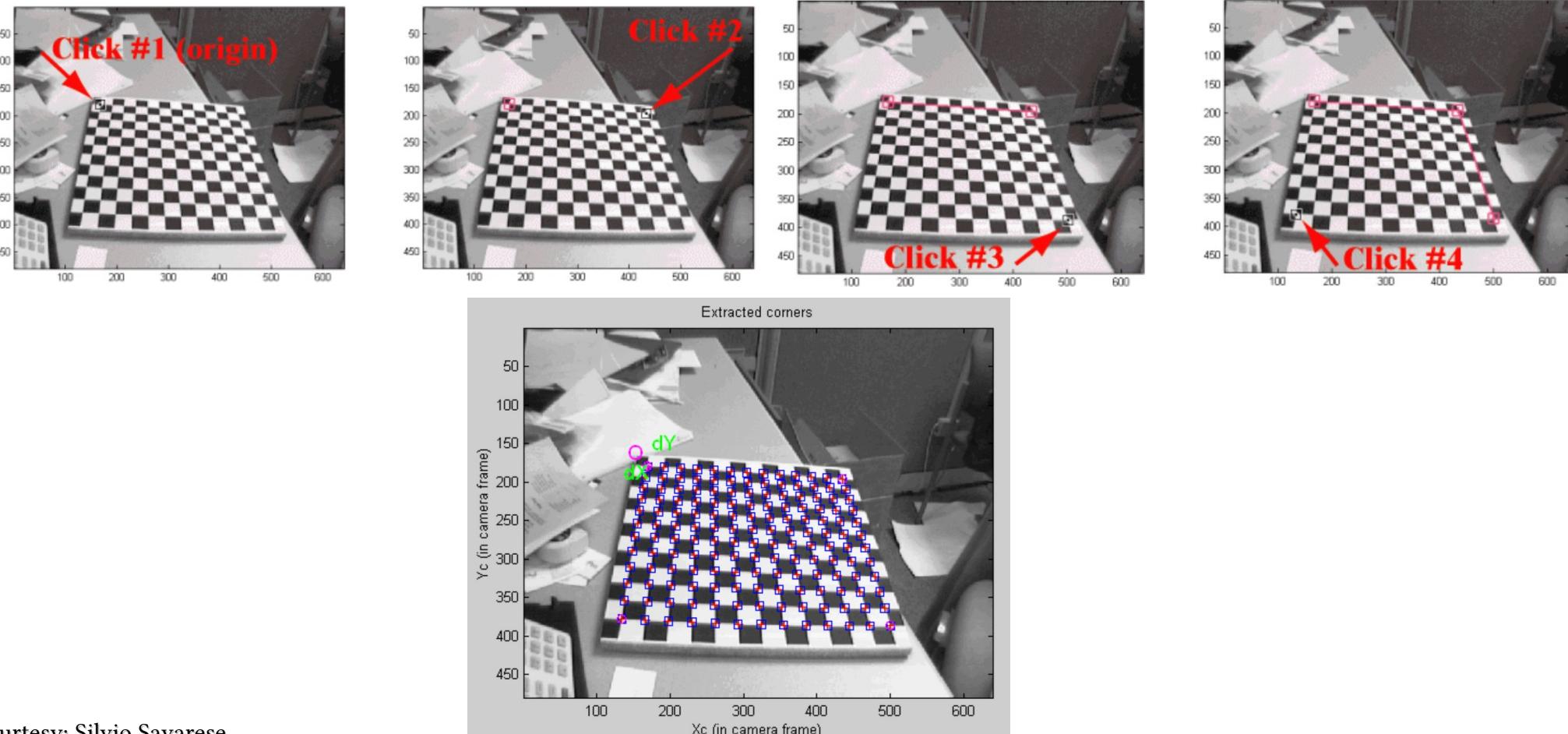


- Example of 491 3D points
 - From: Janne Heikkilä; data copyright 2000, University of Oulu.

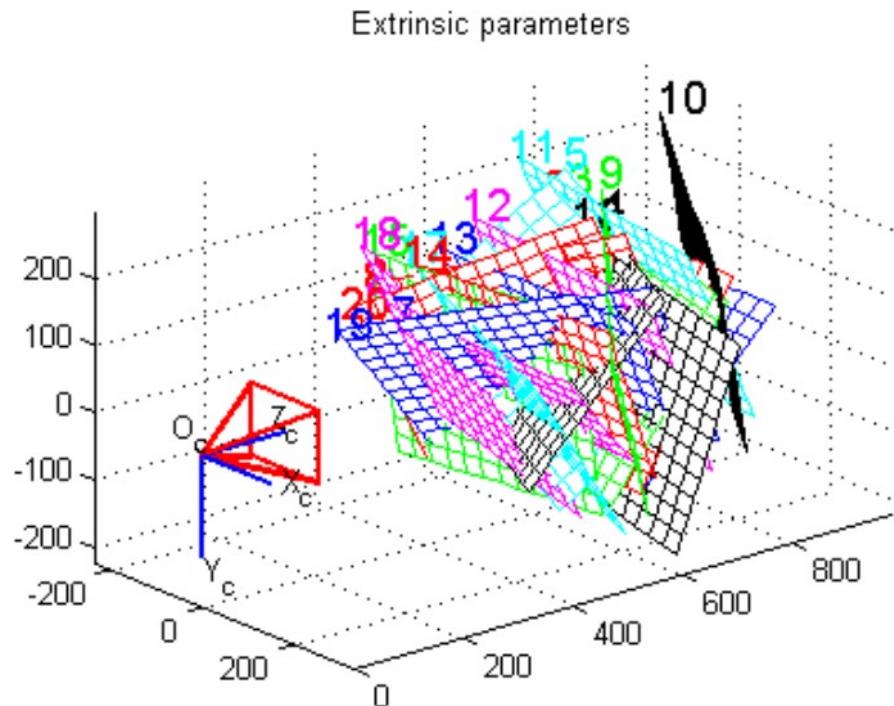
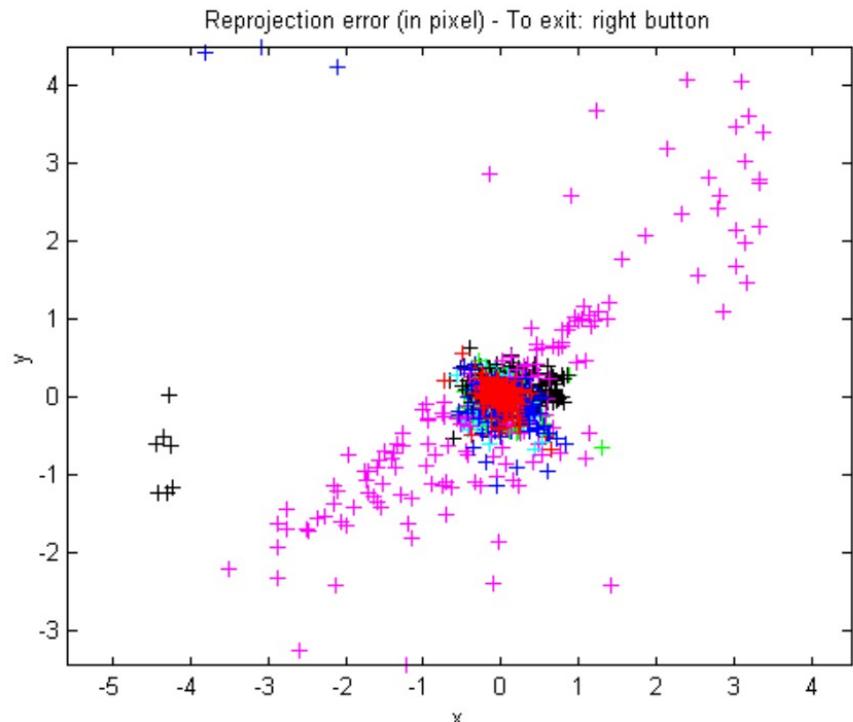
Example of calibration data



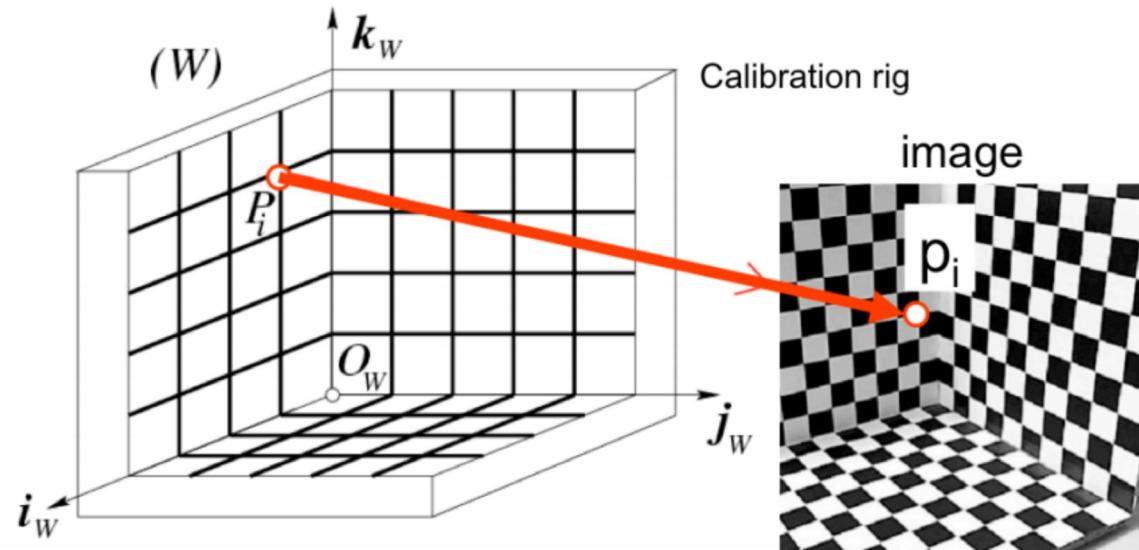
Example of calibration data



Example of calibration data



Calibration



$$p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{\mathbf{m}_1}{\mathbf{m}_3} P_i \\ \frac{\mathbf{m}_2}{\mathbf{m}_3} P_i \end{bmatrix} = M P_i \quad [Eq. 1]$$
$$M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration



[Eq. 1]

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \frac{m_1 P_i}{m_3 P_i} \\ \frac{m_2 P_i}{m_3 P_i} \end{bmatrix}$$

$$u_i = \frac{m_1 P_i}{m_3 P_i} \rightarrow u_i(m_3 P_i) = m_1 P_i \rightarrow u_i(m_3 P_i) - m_1 P_i = 0$$

$$v_i = \frac{m_2 P_i}{m_3 P_i} \rightarrow v_i(m_3 P_i) = m_2 P_i \rightarrow v_i(m_3 P_i) - m_2 P_i = 0$$

[Eqs. 2]

- 2 equations for each correspondence
- m_i are the unknowns

$$\left\{ \begin{array}{l} u_1(\mathbf{m}_3 P_1) - \mathbf{m}_1 P_1 = 0 \\ v_1(\mathbf{m}_3 P_1) - \mathbf{m}_2 P_1 = 0 \\ \vdots \\ u_i(\mathbf{m}_3 P_i) - \mathbf{m}_1 P_i = 0 \\ v_i(\mathbf{m}_3 P_i) - \mathbf{m}_2 P_i = 0 \\ \vdots \\ u_n(\mathbf{m}_3 P_n) - \mathbf{m}_1 P_n = 0 \\ v_n(\mathbf{m}_3 P_n) - \mathbf{m}_2 P_n = 0 \end{array} \right. \quad [\text{Eqs. 3}]$$

- \mathbf{n} points $\rightarrow 2\mathbf{n}$ equations

Recap: block matrix multiplication

Consider the 2 block matrices A and B

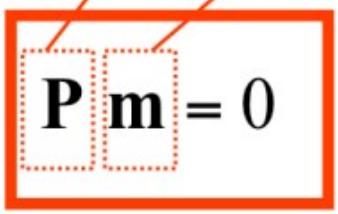
$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \quad B = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix}$$

How can we write the AB product?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration

$$\begin{cases} -u_1(\mathbf{m}_3 P_1) + \mathbf{m}_1 P_1 = 0 \\ -v_1(\mathbf{m}_3 P_1) + \mathbf{m}_2 P_1 = 0 \\ \vdots \\ -u_n(\mathbf{m}_3 P_n) + \mathbf{m}_1 P_n = 0 \\ -v_n(\mathbf{m}_3 P_n) + \mathbf{m}_2 P_n = 0 \end{cases}$$

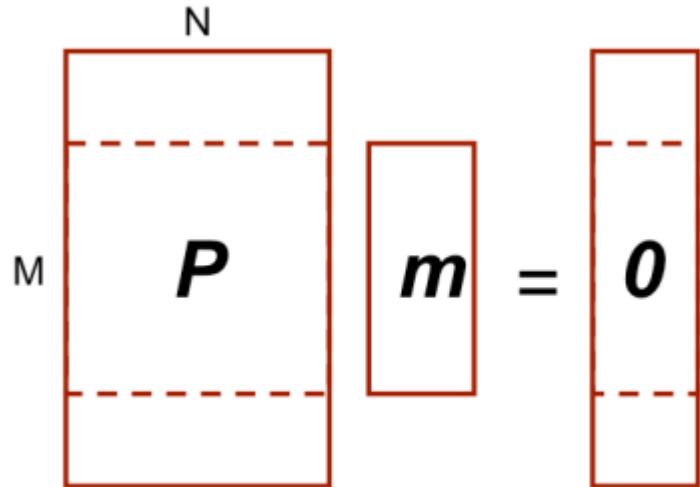
→  [Eq. 4]

Homogenous linear system

$$\mathbf{P} \stackrel{\text{def}}{=} \left(\begin{array}{ccc} \mathbf{P}_1^T & \mathbf{0}^T & -u_1 \mathbf{P}_1^T \\ \mathbf{0}^T & \mathbf{P}_1^T & -v_1 \mathbf{P}_1^T \\ \vdots & & \\ \mathbf{P}_n^T & \mathbf{0}^T & -u_n \mathbf{P}_n^T \\ \mathbf{0}^T & \mathbf{P}_n^T & -v_n \mathbf{P}_n^T \end{array} \right)_{2n \times 12}^{1 \times 4}$$

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix}_{12 \times 1}^{4 \times 1}$$

Calibration



- $M > N$ as we suggested
 - Rectangular system of equations
 - Overdetermined
 - “potentially” no solution other trivial one $m=0$
- The idea is to find the best fitting solution
- Minimize $|Pm|^2$
 - Assuming $m \neq 0 \rightarrow |m|^2 = 1$

- How we can solve such system?

$$Pm=0$$

- Via SVD decomposition!
 - This leads to the optimal solution assuming $|m|^2=1$

Calibration



$$\boxed{P} \mathbf{m} = 0$$

SVD decomposition of P

$$\boxed{U_{2n \times 12} \ D_{12 \times 12} V^T}_{12 \times 12}$$

- The last V column gives us m

$$\mathbf{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^T \\ \mathbf{m}_2^T \\ \mathbf{m}_3^T \end{pmatrix} \quad \xrightarrow{\hspace{1cm}} \quad M$$

Parameters Extraction

Courtesy: Silvio Savarese

$$M = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} \rho$$

- We can recover parameters but with a scale factor ρ
- Actually not an unknown, we can impose $|m|=1$
 - or equivalent Froebius norm $\|M\|=1$
- Extrinsic and Intrinsic parameters are separately extracted

Parameters Extraction

$$\frac{M}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} = \frac{K}{\rho} \begin{bmatrix} R & T \end{bmatrix}$$
$$K = \begin{bmatrix} \alpha & -\alpha \cot \theta & u_0 \\ 0 & \frac{\beta}{\sin \theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

Box 1

$$A = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\rho = \frac{\pm 1}{|\mathbf{a}_3|} \quad u_o = \rho^2 (\mathbf{a}_1 \cdot \mathbf{a}_3)$$
$$v_o = \rho^2 (\mathbf{a}_2 \cdot \mathbf{a}_3)$$
$$\cos \theta = \frac{(\mathbf{a}_1 \times \mathbf{a}_3) \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_1 \times \mathbf{a}_3| \cdot |\mathbf{a}_2 \times \mathbf{a}_3|}$$

Parameters Extraction

$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

A **b**

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

Estimated values

Intrinsic

$$\alpha = \rho^2 |\mathbf{a}_1 \times \mathbf{a}_3| \sin \theta$$

$$\beta = \rho^2 |\mathbf{a}_2 \times \mathbf{a}_3| \sin \theta$$

Parameters Extraction



$$\frac{\mathcal{M}}{\rho} = \begin{pmatrix} \alpha \mathbf{r}_1^T - \alpha \cot \theta \mathbf{r}_2^T + u_0 \mathbf{r}_3^T & \alpha t_x - \alpha \cot \theta t_y + u_0 t_z \\ \frac{\beta}{\sin \theta} \mathbf{r}_2^T + v_0 \mathbf{r}_3^T & \frac{\beta}{\sin \theta} t_y + v_0 t_z \\ \mathbf{r}_3^T & t_z \end{pmatrix} = \mathbf{K} [\mathbf{R} \quad \mathbf{T}]$$

A **b**

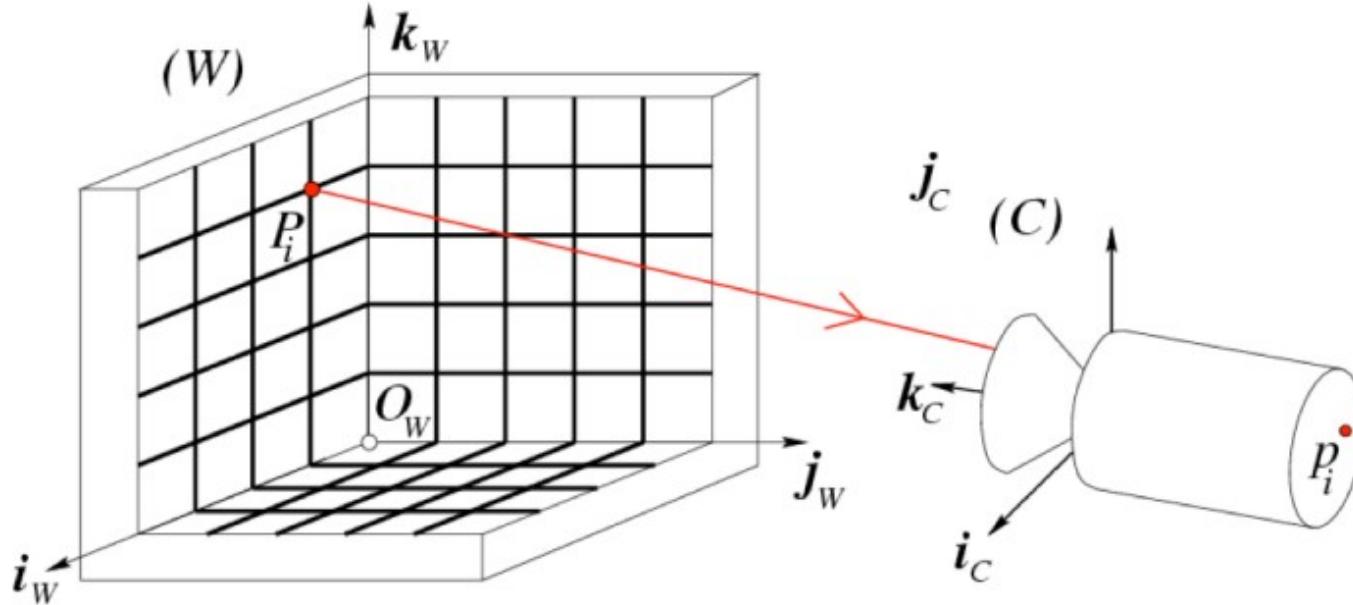
$$\boxed{\mathbf{A} = \begin{bmatrix} \mathbf{a}_1^T \\ \mathbf{a}_2^T \\ \mathbf{a}_3^T \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \\ \mathbf{b}_3 \end{bmatrix}}$$

Estimated values

Extrinsic

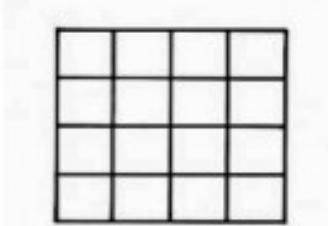
$$\mathbf{r}_1 = \frac{(\mathbf{a}_2 \times \mathbf{a}_3)}{|\mathbf{a}_2 \times \mathbf{a}_3|} \quad \mathbf{r}_3 = \frac{\pm \mathbf{a}_3}{|\mathbf{a}_3|}$$
$$\mathbf{r}_2 = \mathbf{r}_3 \times \mathbf{r}_1 \quad \mathbf{T} = \rho \mathbf{K}^{-1} \mathbf{b}$$

Degenerate Case

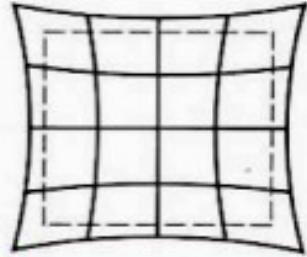


- Not all P_w s lead to a result!
 - P_w points must not lie on the same plane
 - P_w points must not lie on the intersection curve of 2 quadric surfaces

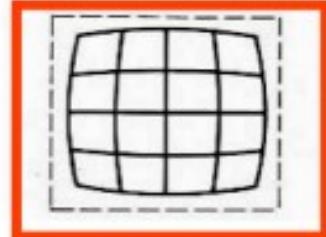
Lens Distortion



No distortion



Pin cushion



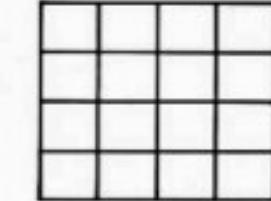
Barrel



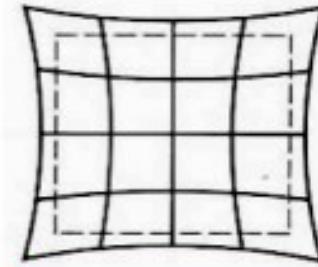
Radial Lens Distortion



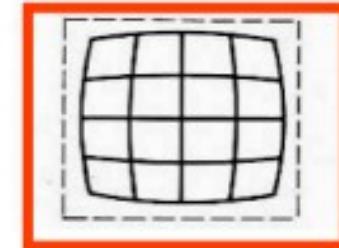
- We assumed a “thin” lens
 - This basically means that lines are lines
 - Unfortunately this is not true!
 - Remember: “never trust Bertozzi”
- Radial Distortion
 - Image magnification/contraction depends on distance from optical axis
 - Deviations are more evident on lateral areas
 - Cheap optics



No distortion

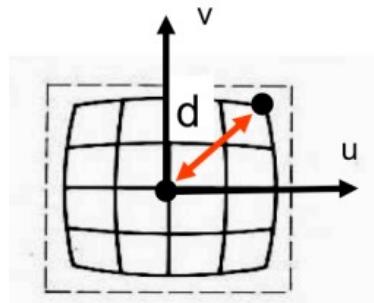


Pin cushion



Barrel

Radial Lens Distortion



$$S_\lambda \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i$$

$$\lambda = 1 \pm \sum_{p=1}^3 \kappa_p d^{2p}$$

[Eq. 5] [Eq. 6]

Distortion coefficient To model radial behavior

Polynomial function

- λ is the distortion factor
 - Actually it is a $\lambda_{(u,v)}$ since it depends on the distance from optical axis

Radial Lens Distortion



$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i \quad Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Q

- λ depends on u, v and this leads to a non linearity in the $P \rightarrow p$ mapping

Radial Lens Distortion



$$\begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & 1 \end{bmatrix} M P_i \rightarrow \begin{bmatrix} u_i \\ v_i \end{bmatrix} = p_i \quad Q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

Q

- Tsai 87, initially estimate m_1 and m_2 by SVD and then $m_3=f(m_1, m_2, \lambda)$



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Camera Models (2)

Question time!

