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Features Extraction

Summary

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- What is a feature?
- Keypoints ideal characteristics
- Harris Corners
- Scale Invariant Detectors

References

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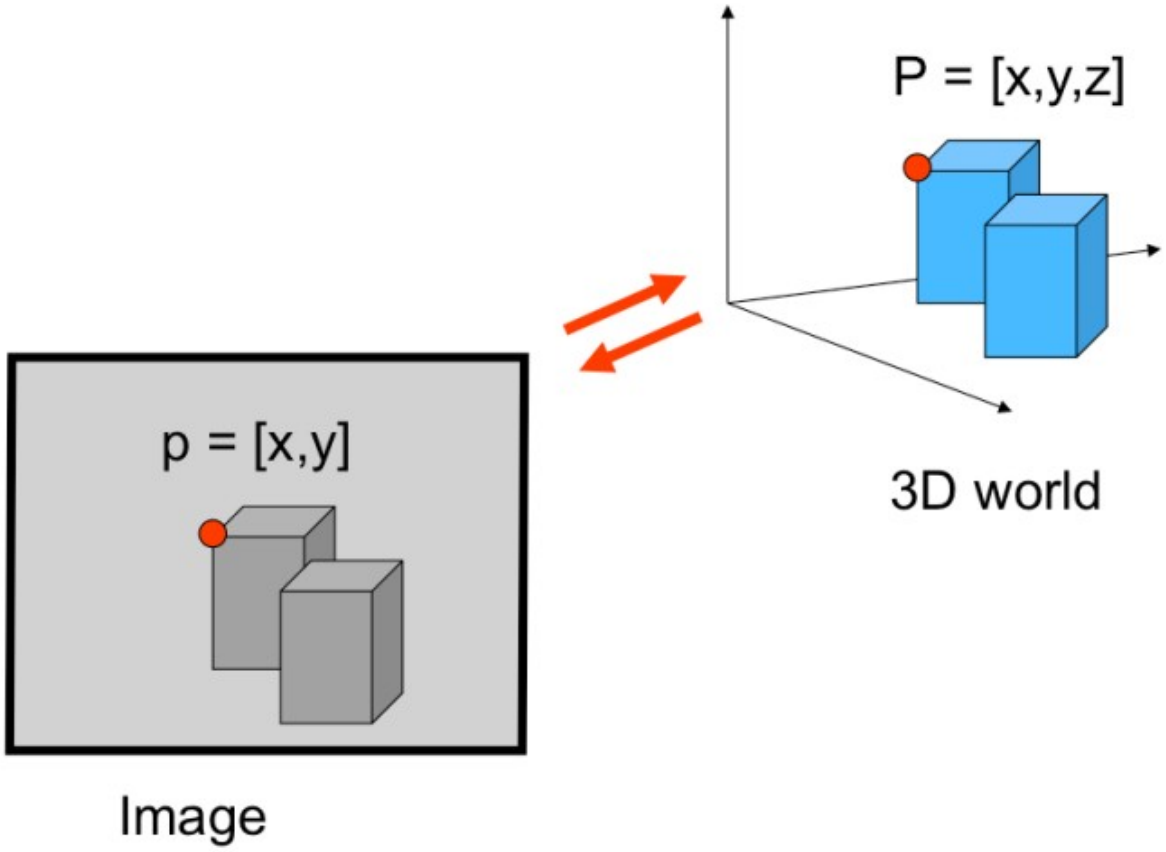
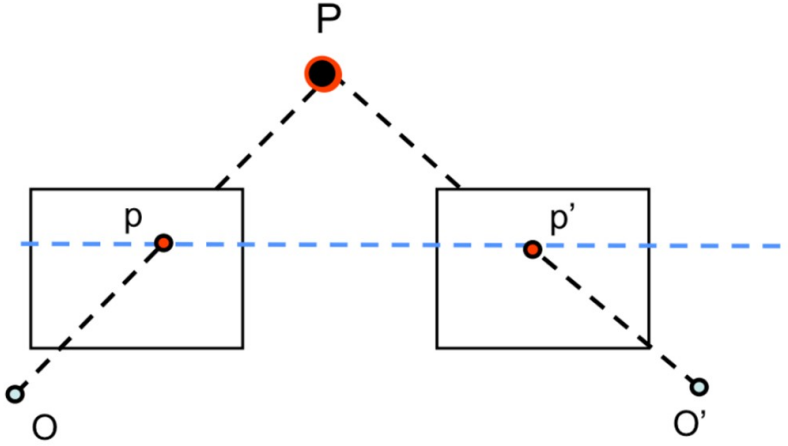
- [FP] D. A. Forsyth and J. Ponce. **Computer Vision: A Modern Approach** (2nd Edition). Prentice Hall, 2011.
- **CS231A · Computer Vision: from 3D reconstruction to recognition**, Prof. Silvio Savarese – Stanford University
- **CS131 · Computer Vision: Foundations and Applications,** Prof. Fei-Fei Li – Stanford University
- **Elementi di Analisi per Visione Artificiale**
  - Paolo Medici http://www.ce.unipr.it/people/medici

Small Recap



- We found a geometrical relation between world and image

Small Recap



- We found a geometrical relation between world and image

Beyond Stereo Vision

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Beyond Stereo Vision

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by Diva Sian

Beyond Stereo Vision

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Beyond Stereo Vision

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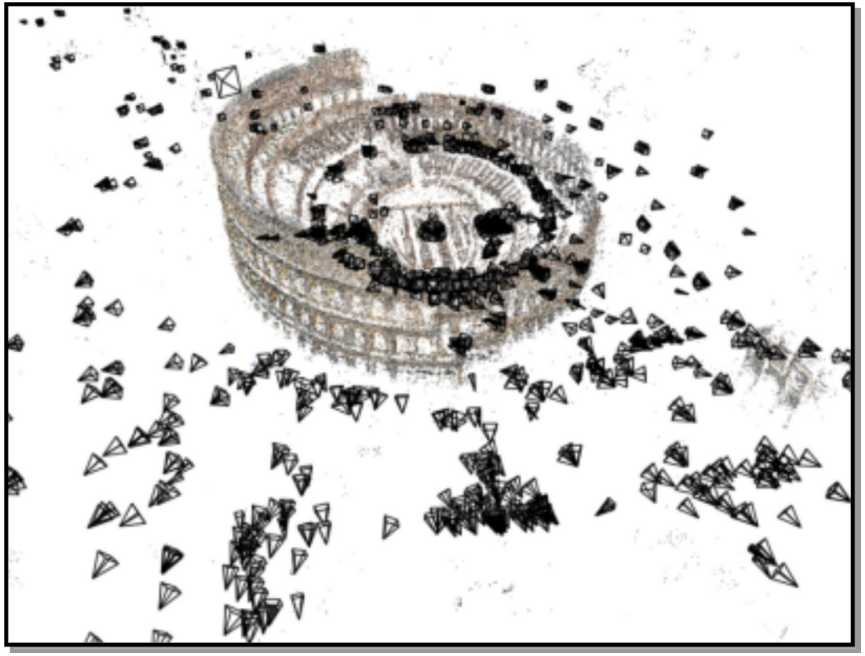

Beyond Stereo Vision

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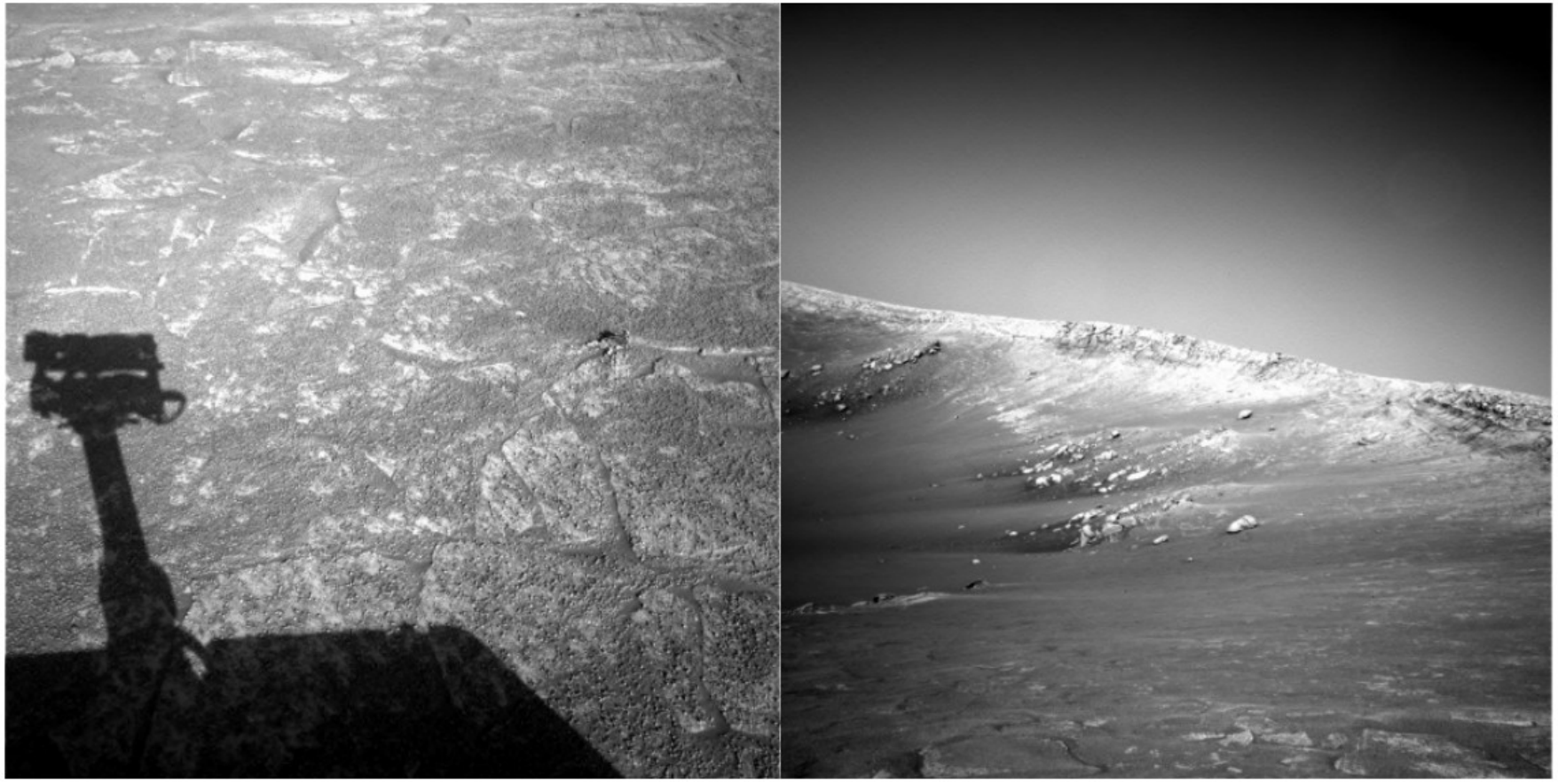

NASA Mars Rover images

Beyond Stereo Vision

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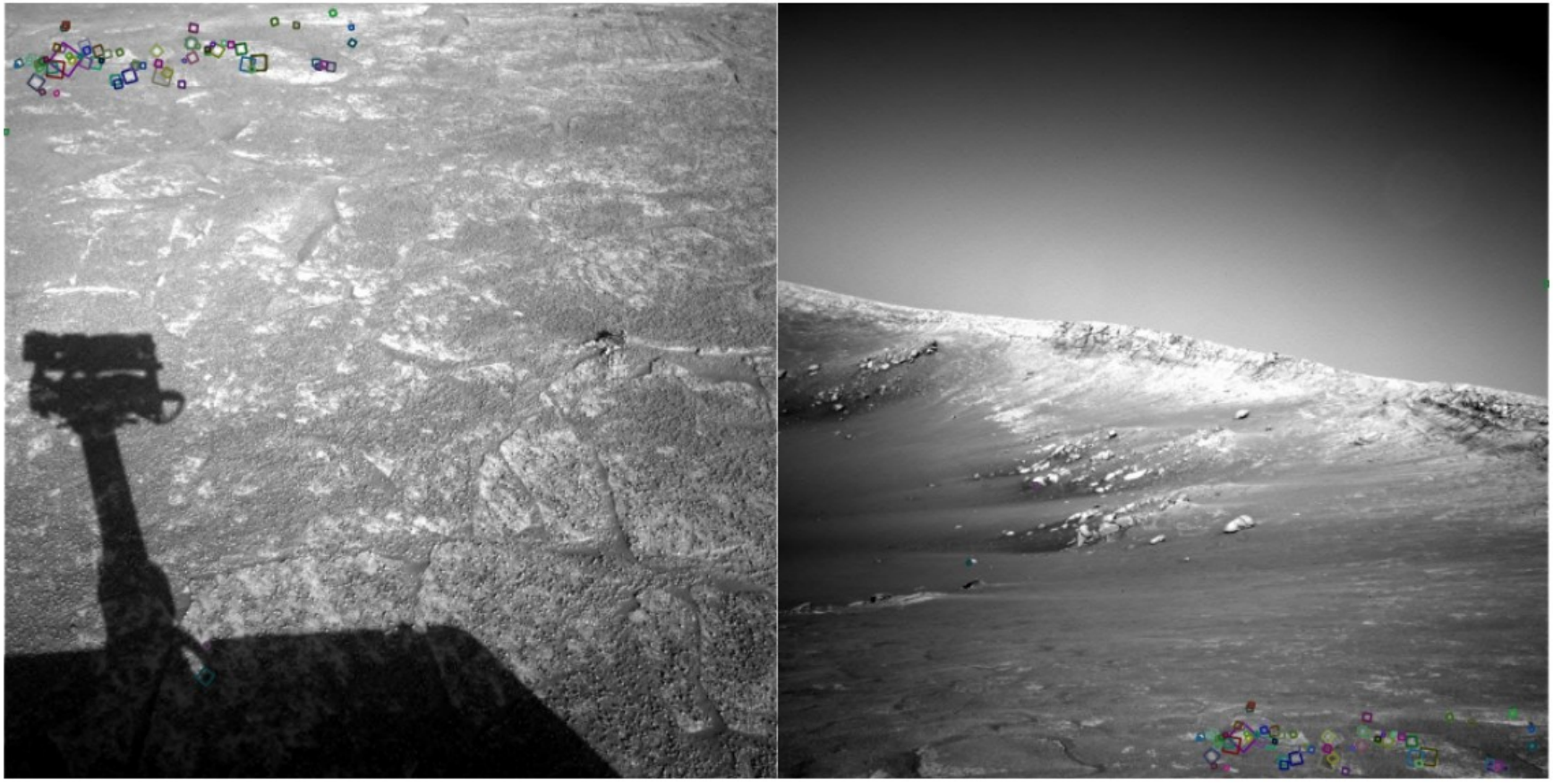

NASA Mars Rover images with SIFT feature matches

Applications

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- During this lesson and the following one we are going to learn:
  - What are “features” and their properties
  - How to extract “features” from images
  - How to match them among all images

Applications

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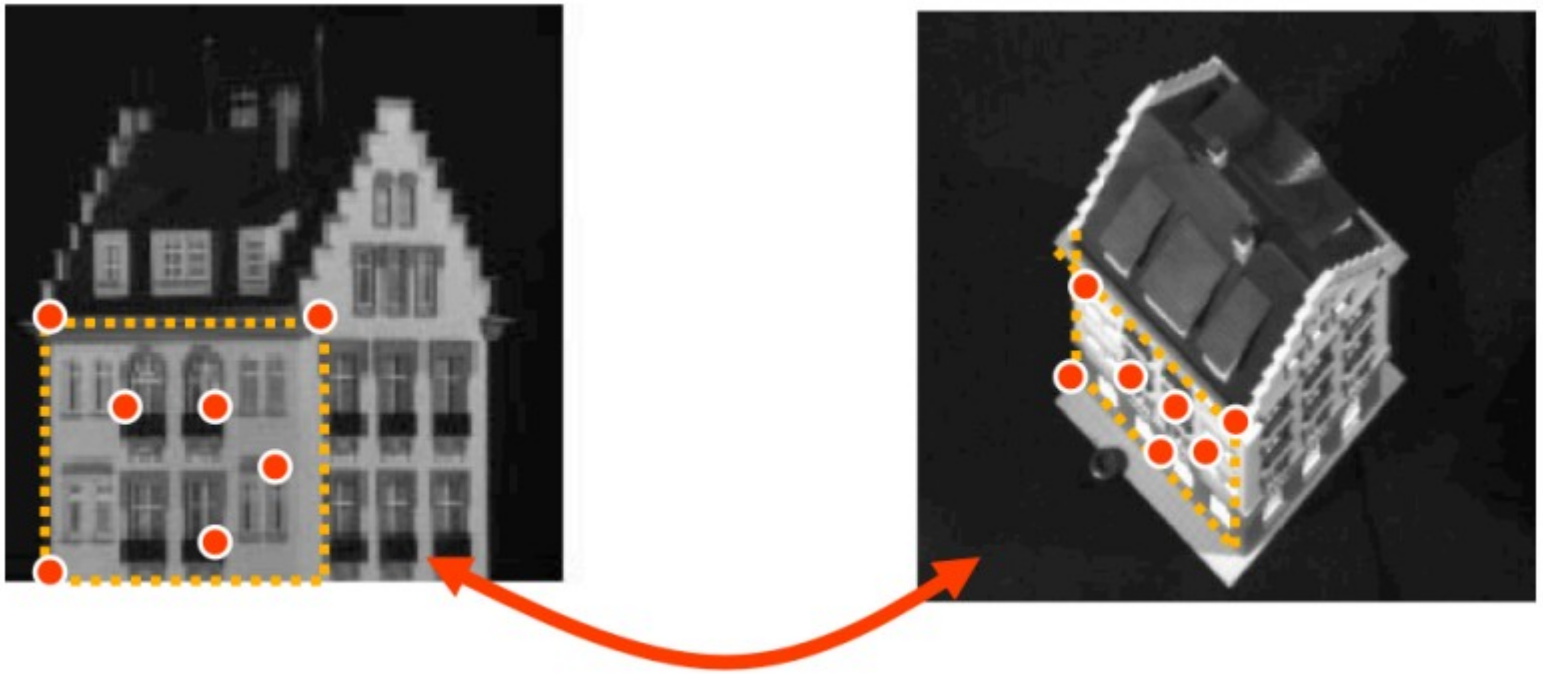
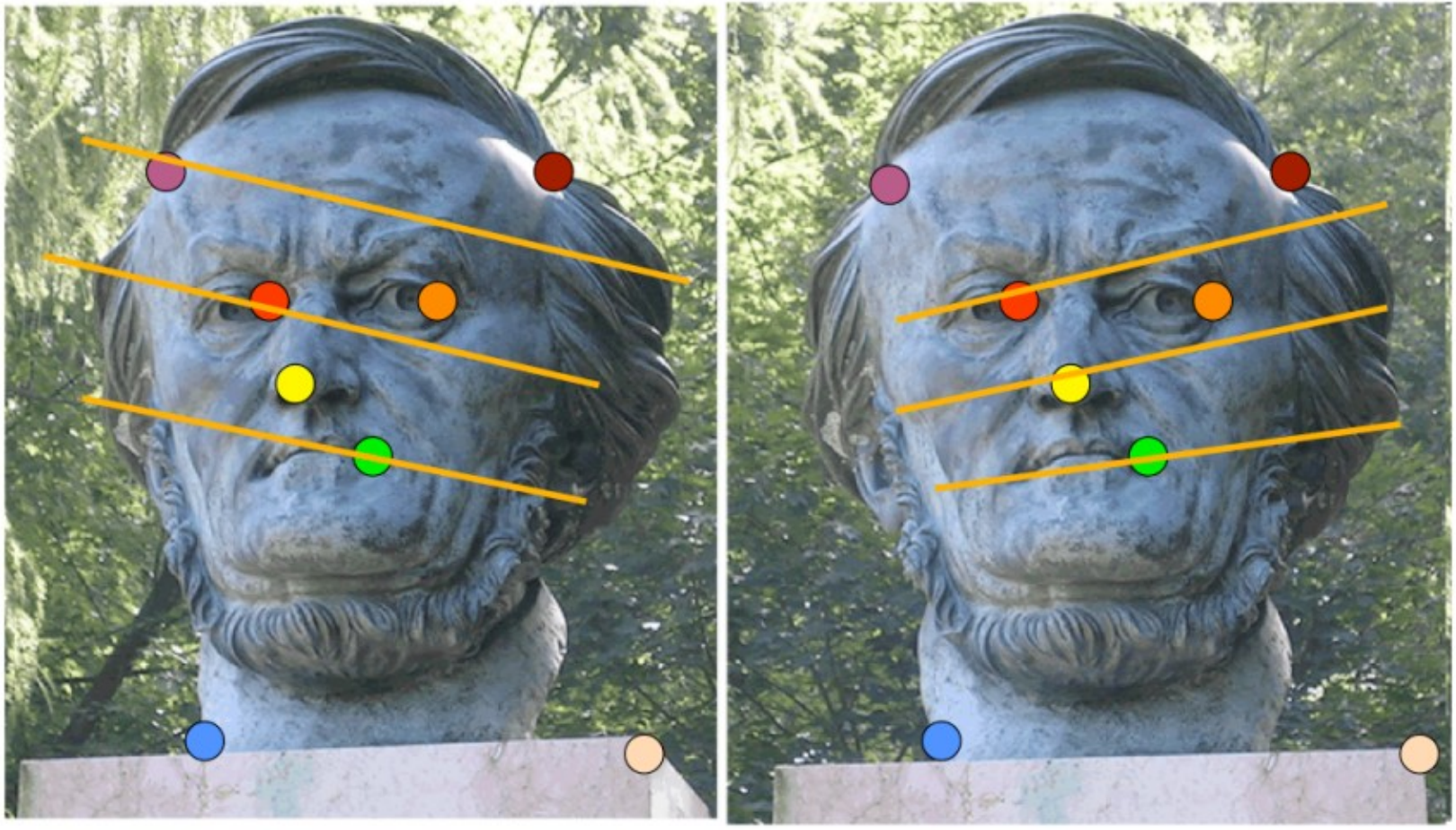
- Potential outcomes are:
  - Fit or estimate models that describe an image or a portion of it
  - Match or index images
  - Detect objects or actions from images
  - “Combine” different images

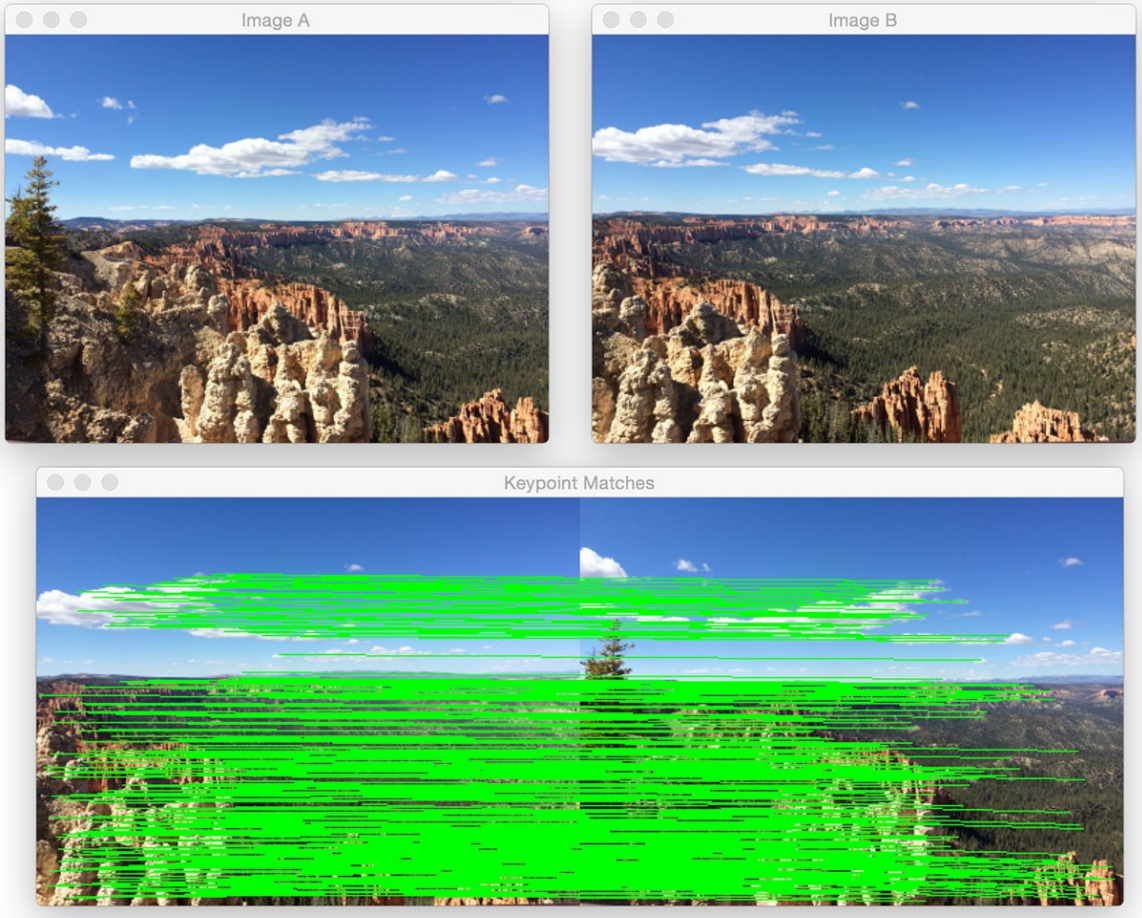
Applications



Applications



Applications



Applications

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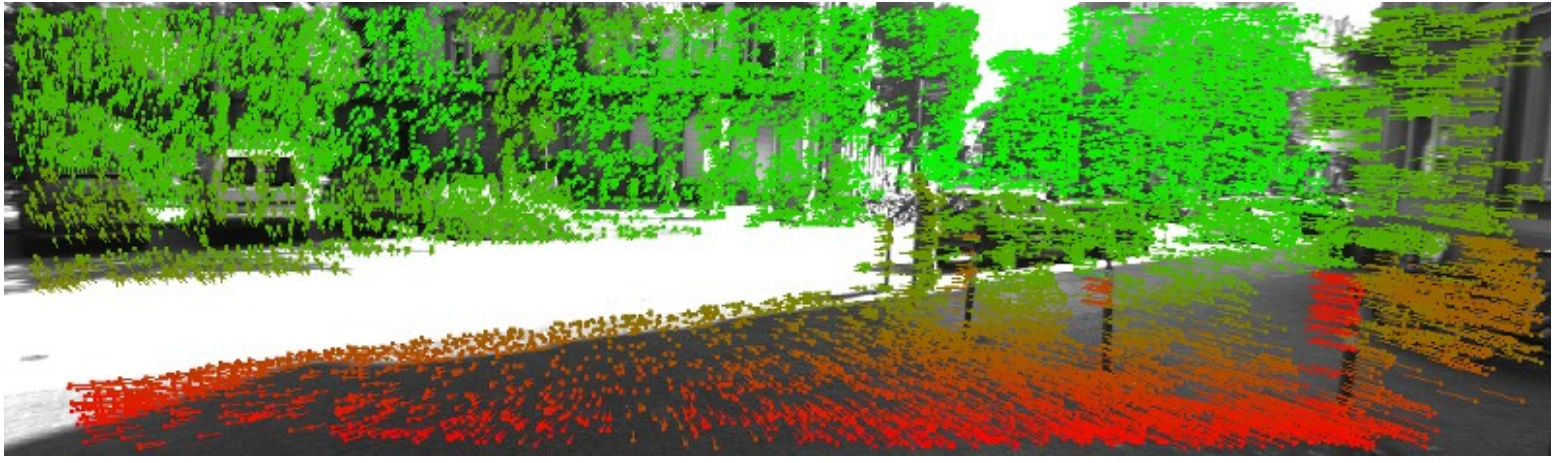

Applications

Applications

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How to do this?

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- Theoretically we could use stereo vision techniques
  - Too much expensive!
  - Moreover, we do not have to obtain a full 3D reconstruction
- Just look at the following case
  - Joining of different images…

Example

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- How to combine following images?

Example

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- 1. select “specific” points

Example

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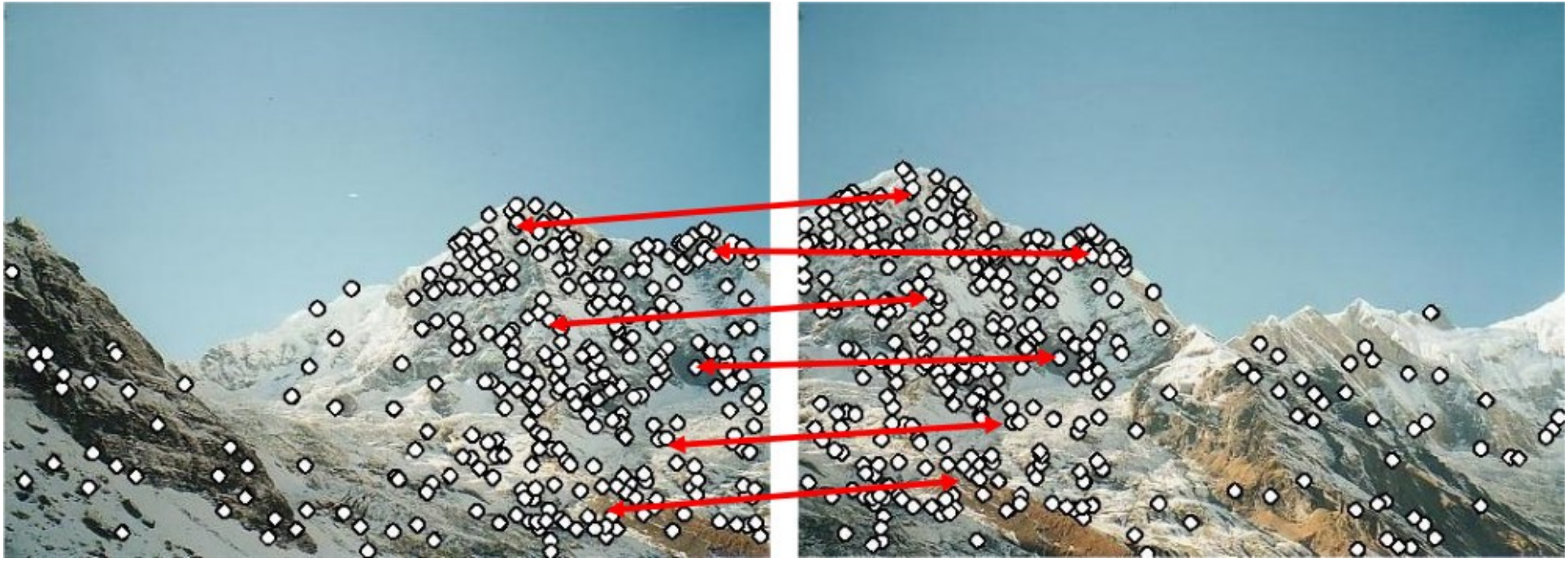
- 2. find potential correspondances



Example

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- 3. compute alignement

Example recap

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- Select specific points → keypoints or features → **features extraction**
- Find potential correspondences → **features matching**
- Application dependent step
  - Matching
  - Indexing
  - Detection
  - Image alignement

Keypoints or Features

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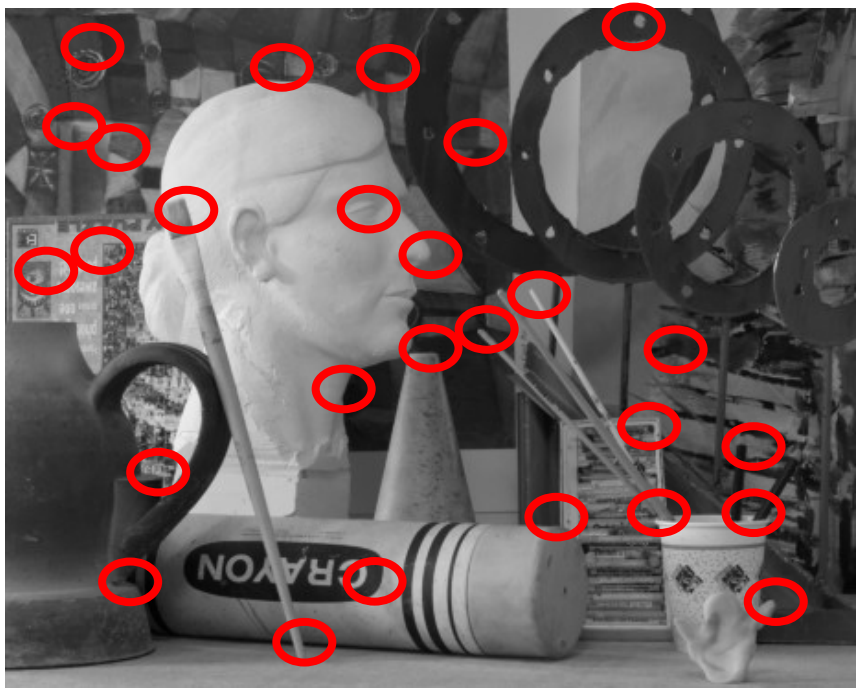
- For Stereo Vision
  - Window match ⟶ SAD
- In a more general situation
  - Different size
  - Different orientation
- A complete match is impraticable
  - Only keypoints ⟶ patches

Keypoints or Features

- What keypoints I have to select?



Characteristics of a good feature

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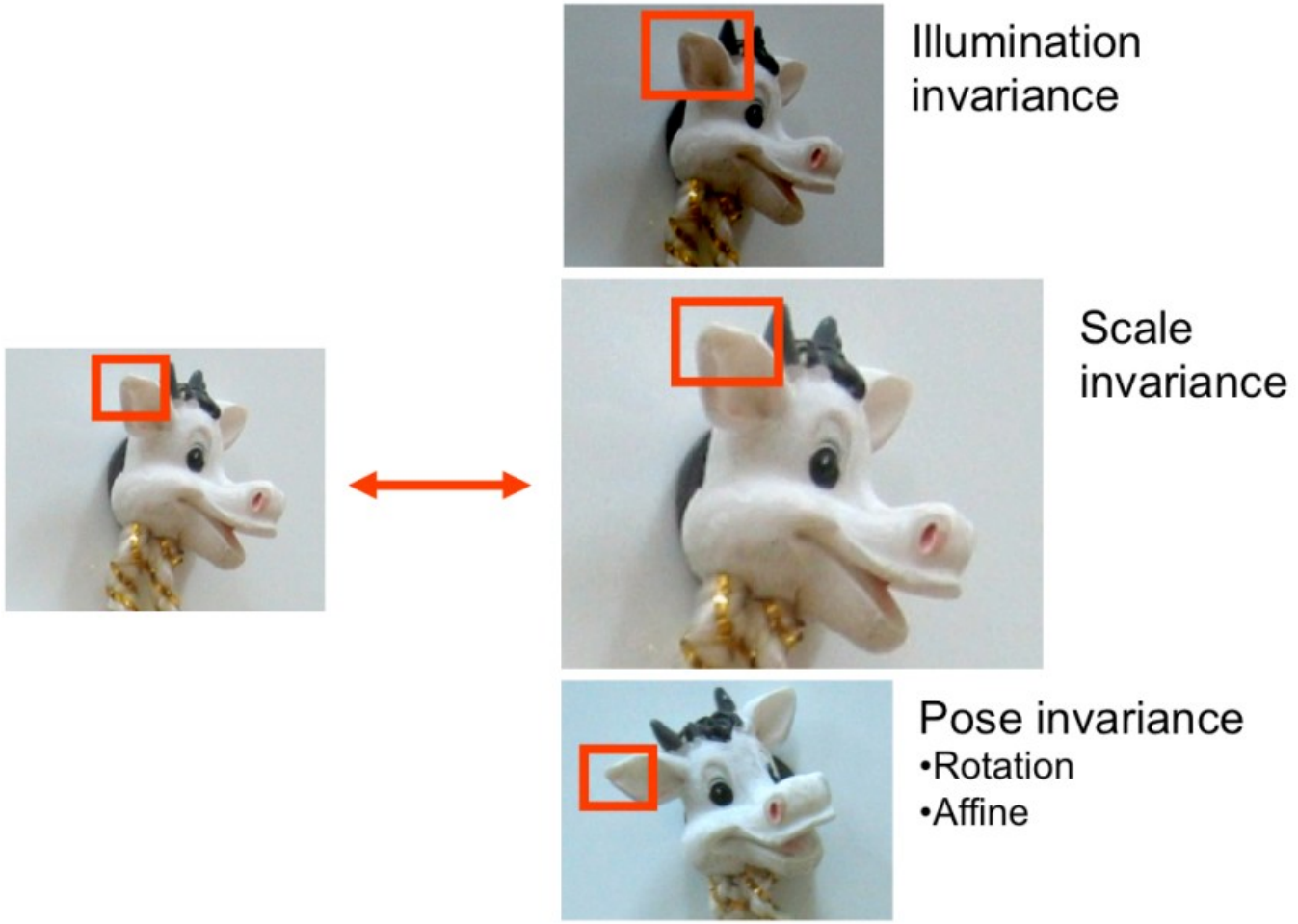
- **Repeatability:** to be able to find the same features on even very different images
- **Saliency:** the feature contains information
- **Locality:** relatively small area of the image, namely more robust wrt occlusions and clutter

Repeatability



Saliency & Locality

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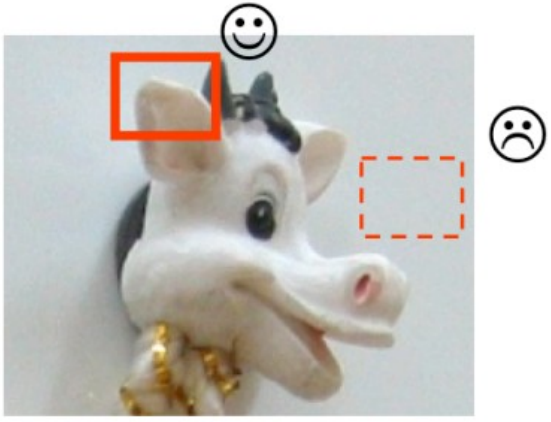
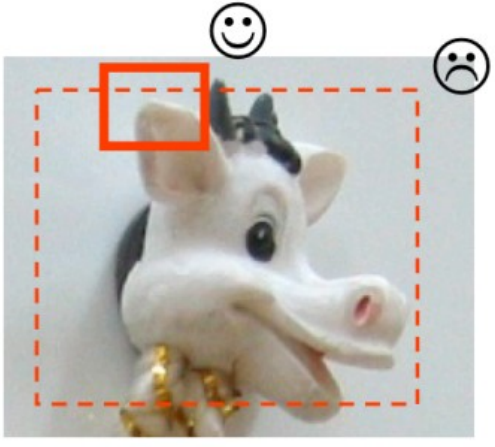
- Saliency



- Locality



Patches

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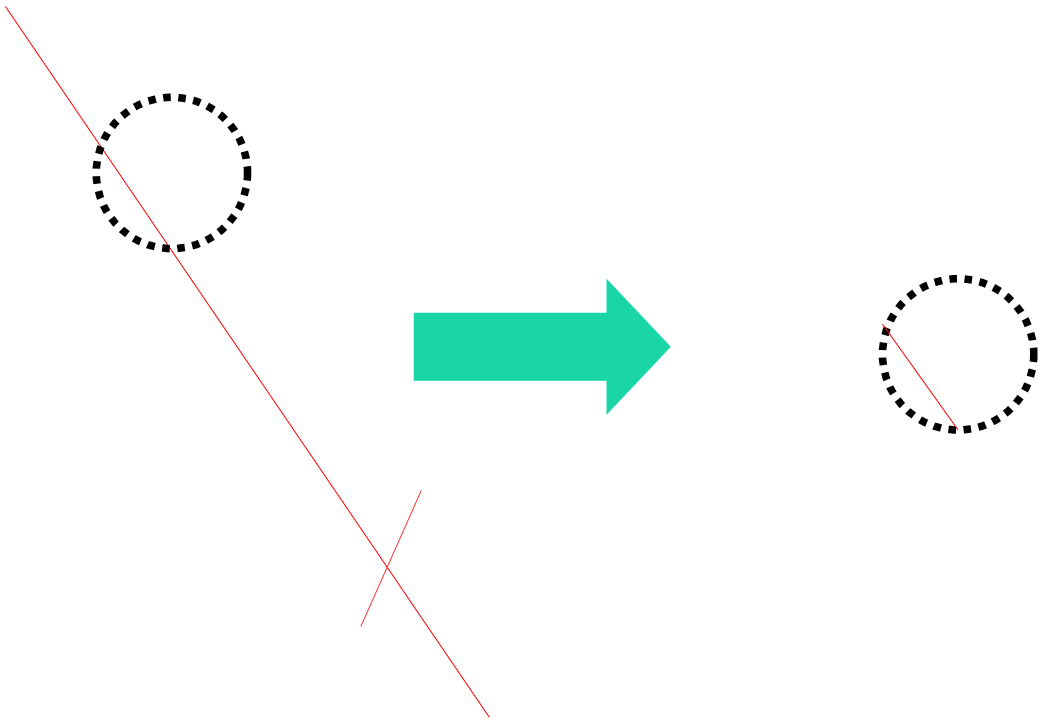
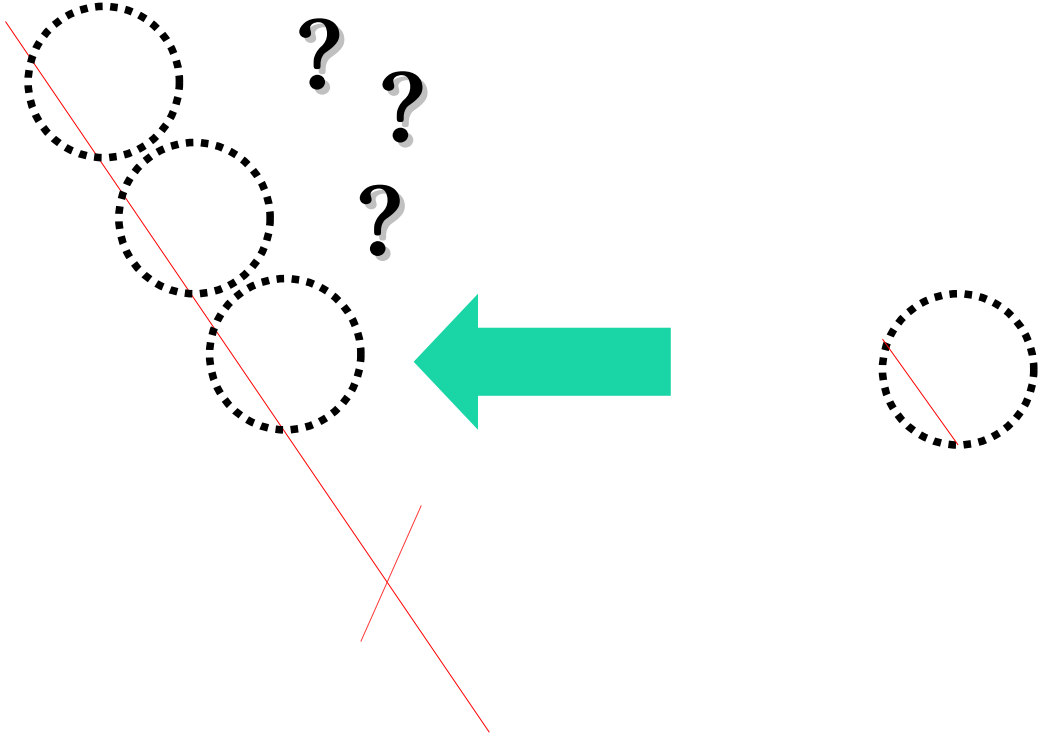
- Textureless patches are nearly impossible to match
- Patches with large contrast changes (gradients) are easier to localize
  - We want edges!
- Are straight edges a good option?
  - Aperture Problem

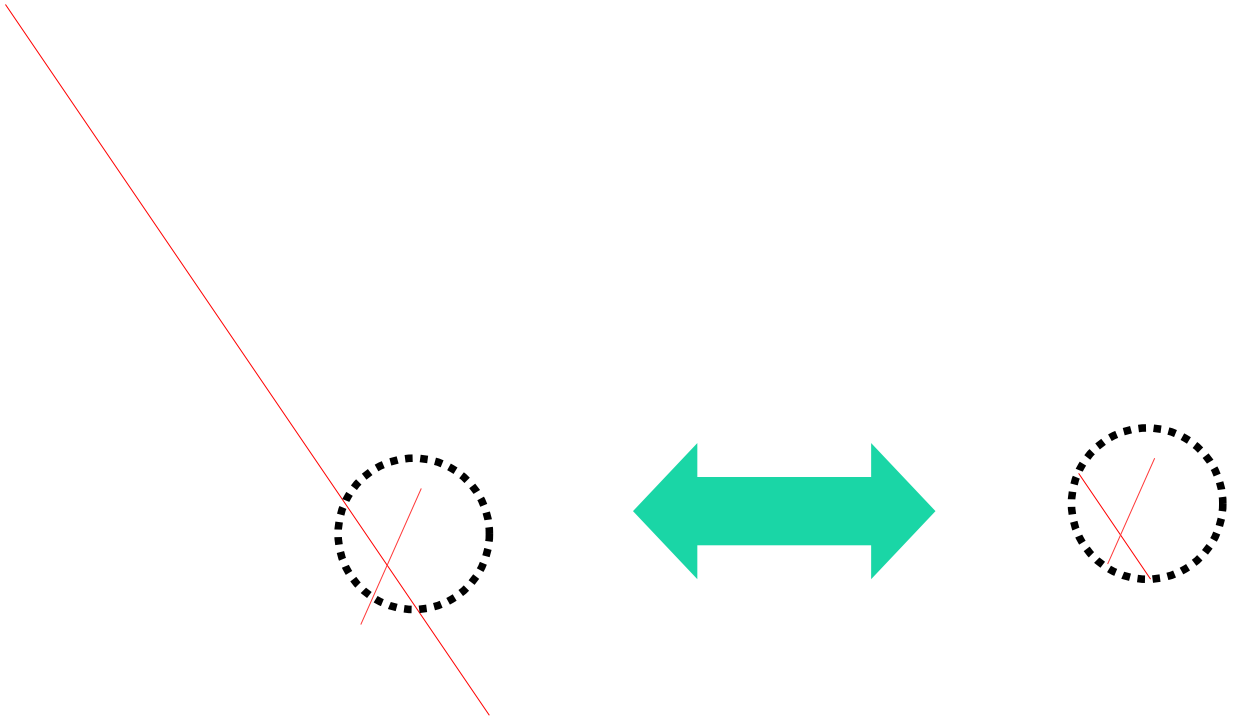
Aperture problem



Aperture problem



Aperture problem



Corner Detection

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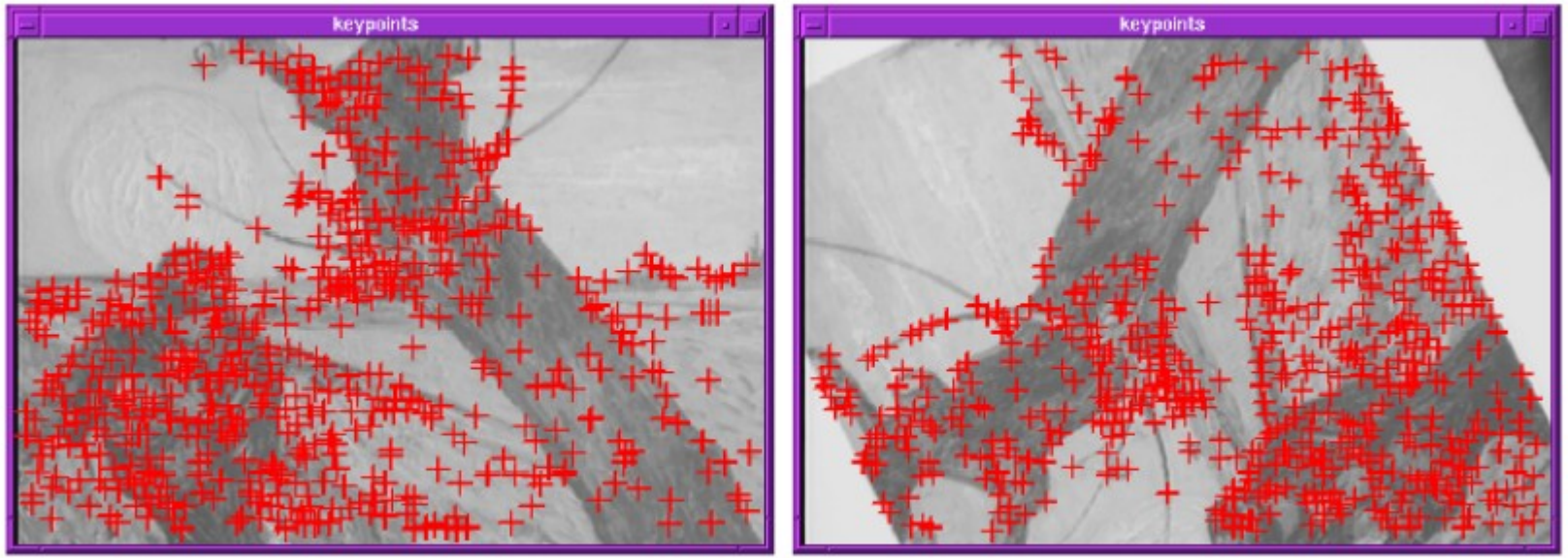
- In “real” images a corner is easier to match
  - 2 “strong” gradients



Repeatibility – Saliency – Locality

Local Measure of Uniqueness

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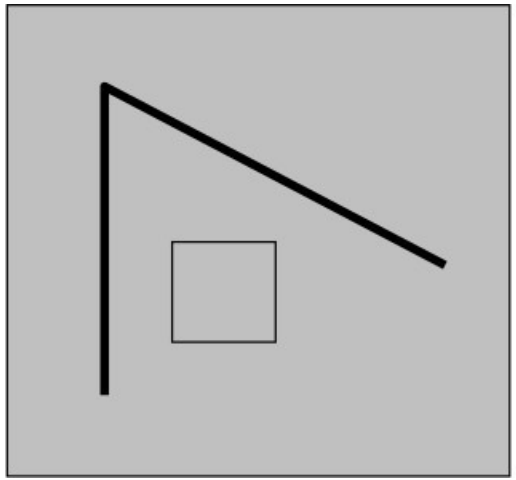
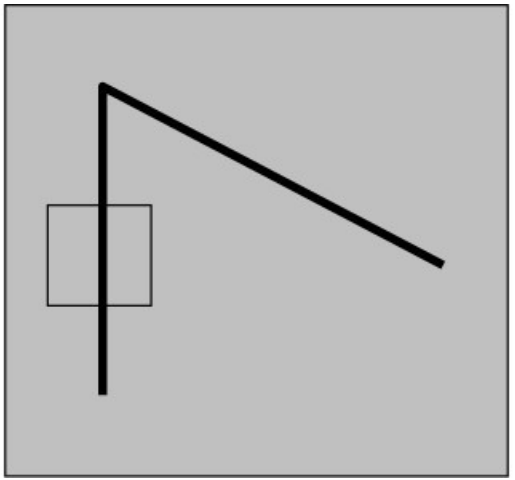
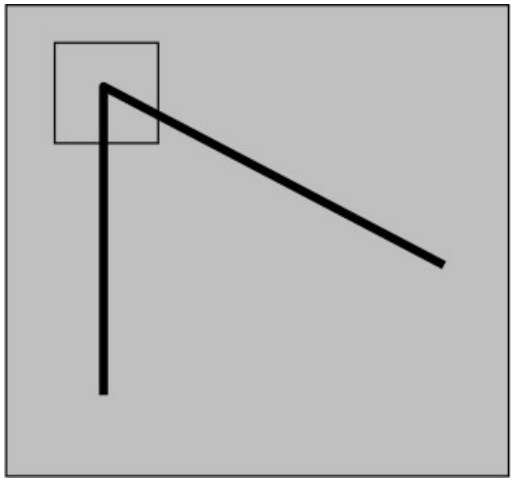
- What defines whether our patch is a good or bad candidate?

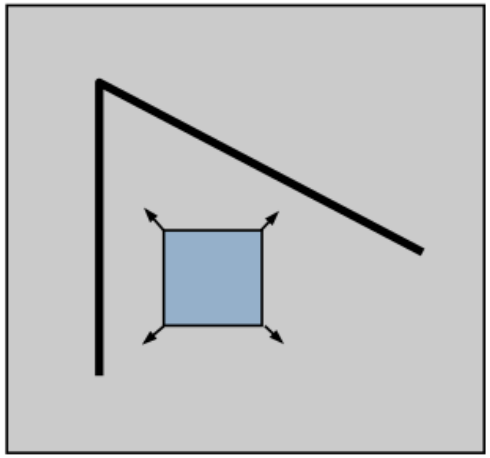




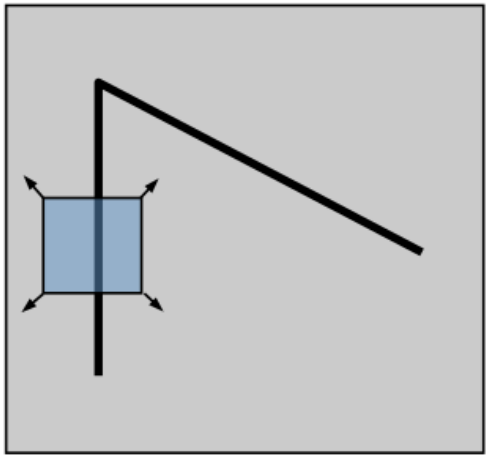


Corner Detection

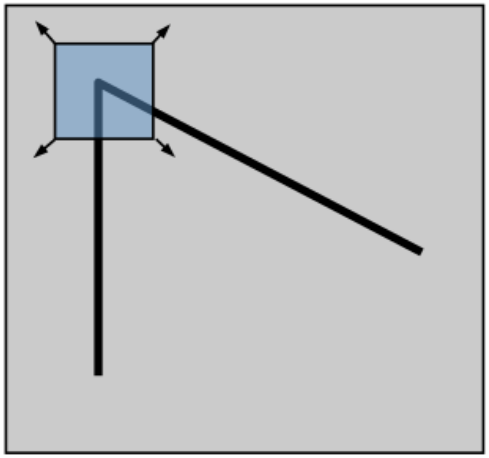
- Let's move a small window (W) on the image
- How the content is changing?



"flat" region:
no change in all directions



"edge":
no change along the edge direction



"corner":
significant change in all directions

Corner Detection

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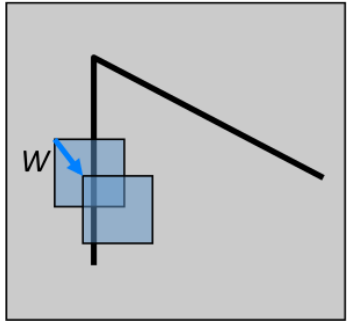
- Let's move window W by (u,v)
  - u and v are small…
- A good estimation can be the Sum of Squared Differences (SSD)
- The SSD error E(u, v) can be defined as

$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y) \right]^2$$



Corner Detector

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- Using a Taylor Expansion

$$I_x = \frac{\partial I(x,y)}{\partial x} \qquad I_y = \frac{\partial I(x,y)}{\partial y}$$

$$I(x+u, y+v) = I(x,y) + \frac{I_x u}{1!} + \frac{I_y v}{1!} + \text{higher order terms}$$

$$\approx I(x,y) + I_x u + I_y v$$

$$\approx I(x,y) + \begin{bmatrix} I_x & I_y \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

Corner Detector

UNIVERSITÀ DI PARMA

- Using a Taylor Expansion

$$I_x = \frac{\partial I(x,y)}{\partial x} \qquad I_y = \frac{\partial I(x,y)}{\partial y}$$

$$E(u,v) = \sum_{(x,y)\in W} \left[I(x+u, y+v) - I(x,y) \right]^2$$

$$\approx \sum_{(x,y)\in W} \left[I(x,y) + I_x u + I_y v - I(x,y) \right]^2$$

$$\approx \sum_{(x,y)\in W} \left[I_x u + I_y v \right]^2$$

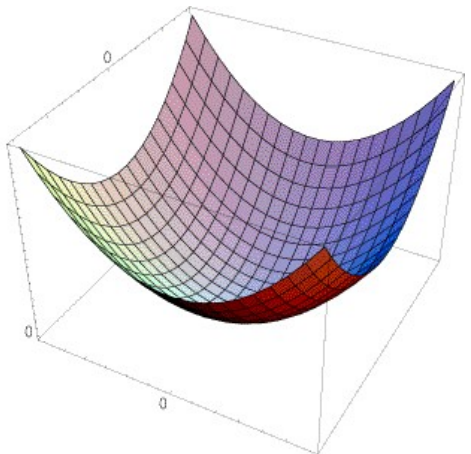
Corner Detector

UNIVERSITÀ DI PARMA

- Using a Taylor Expansion

$$I_x = \frac{\partial I(x,y)}{\partial x} \qquad I_y = \frac{\partial I(x,y)}{\partial y}$$

$$E(u,v) \approx \sum_{(x,y)\in W} \left[I_x u + I_y v \right]^2$$



$$\approx Au^2 + 2Buv + Cv^2$$

$$A = \sum_{(x,y)\in W} I_x^2 \qquad B = \sum_{(x,y)\in W} I_x I_y \qquad C = \sum_{(x,y)\in W} I_y^2$$

Corner Detector

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- Using a Taylor Expansion

$$I_x = \frac{\partial I(x,y)}{\partial x} \qquad I_y = \frac{\partial I(x,y)}{\partial y}$$

$$E(u,v) \approx \sum_{(x,y)\in W} \left[I_x u + I_y v \right]^2$$

$$\approx Au^2 + 2Buv + Cv^2$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u & v \end{bmatrix} H \begin{bmatrix} u \\ v \end{bmatrix}$$

Corner Detector

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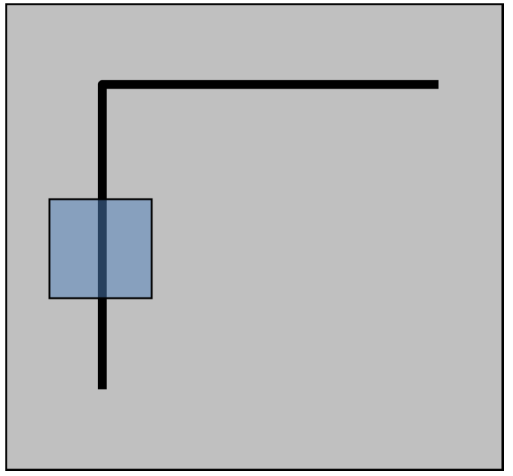
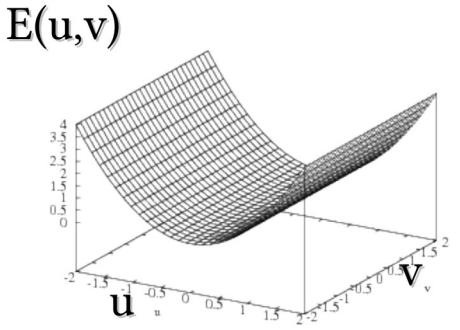
- Vertical Edge $\longrightarrow I_y=0$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$





Corner Detector

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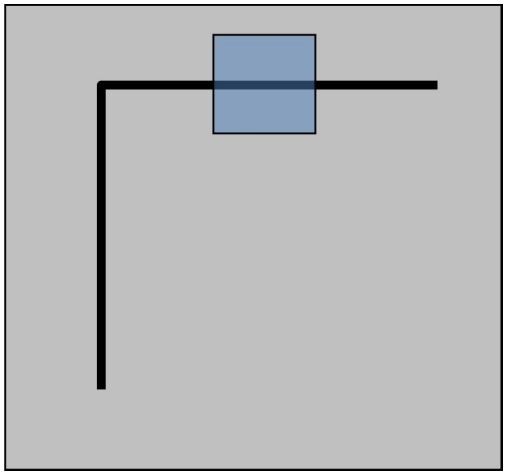
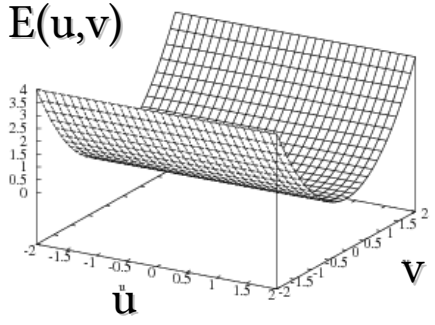
- Horizontal Edge $\longrightarrow I_x=0$

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$

$$\approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}$$





Corner Detector

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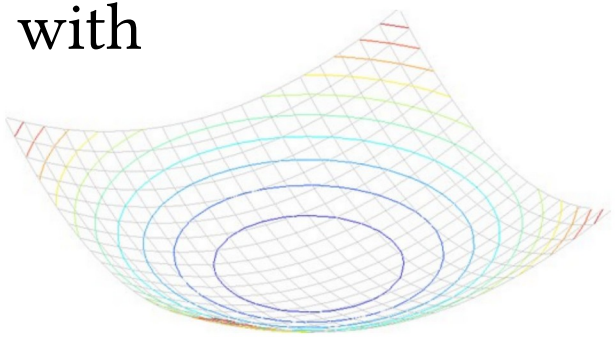
- General Case

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u & v \end{bmatrix} H \begin{bmatrix} u \\ v \end{bmatrix}$$

- H can be considered as describing an ellipse with
  - Axis lenght $\propto$ H eigenvalues
  - Axis orientation $\rightarrow$ H eigenvectors



Eigenvector/Eigenvalues review

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- Given a matrix A eigenvectors x and eigenvalues λ satisfy:

  $Ax=\lambda x$

- Eigenvalues can be find solving

  $det(A-\lambda I)=0$

- For a 2×2 matrix like H we can find $\lambda \rightarrow$

  $$det\begin{bmatrix} h_{11}-\lambda & h_{12} \\ h_{12} & h_{22}-\lambda \end{bmatrix}$$

  - This leads to a **quadratic** equation
  - i.e. λ_1 and λ_2

- Once found λs, x can be computed as $\rightarrow$

  $$\begin{bmatrix} h_{11}-\lambda & h_{12} \\ h_{12} & h_{22}-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$$

Corner Detector

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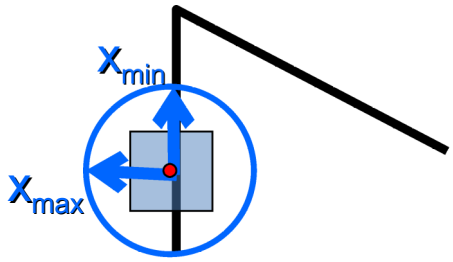
$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} \begin{bmatrix} A & B \\ B & C \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u & v \end{bmatrix} H \begin{bmatrix} u \\ v \end{bmatrix}$$

$$Hx_{max} = \lambda_{max} x_{max}$$

$$Hx_{min} = \lambda_{min} x_{min}$$

- $X_{max} \rightarrow$ direction of largest increase for E(u,v)
- $X_{min} \rightarrow$ direction of smallest increase for E(u,v)
- $\lambda_{max} \rightarrow$ amount of increase along x_{max}
- $\lambda_{min} \rightarrow$ amount of increase along x_{min}



Corner Detector

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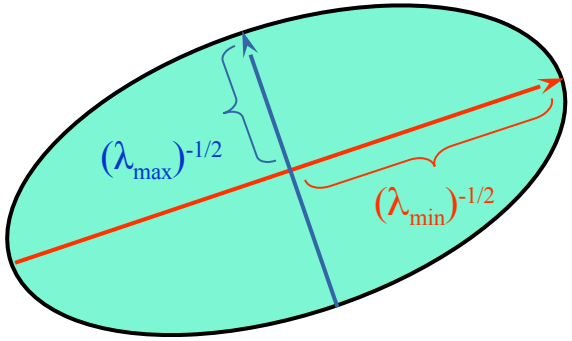
- H can be considered as describing an ellipse with
  - Axis lenght $\propto$ H eigenvalues
  - Axis orientation $\rightarrow$ H eigenvectors

direction of the fastest change

λ_{max}, λ_{min} : eigenvalues of H



direction of the slowest change

$$H = R \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R^{-1}$$

Corner Detector

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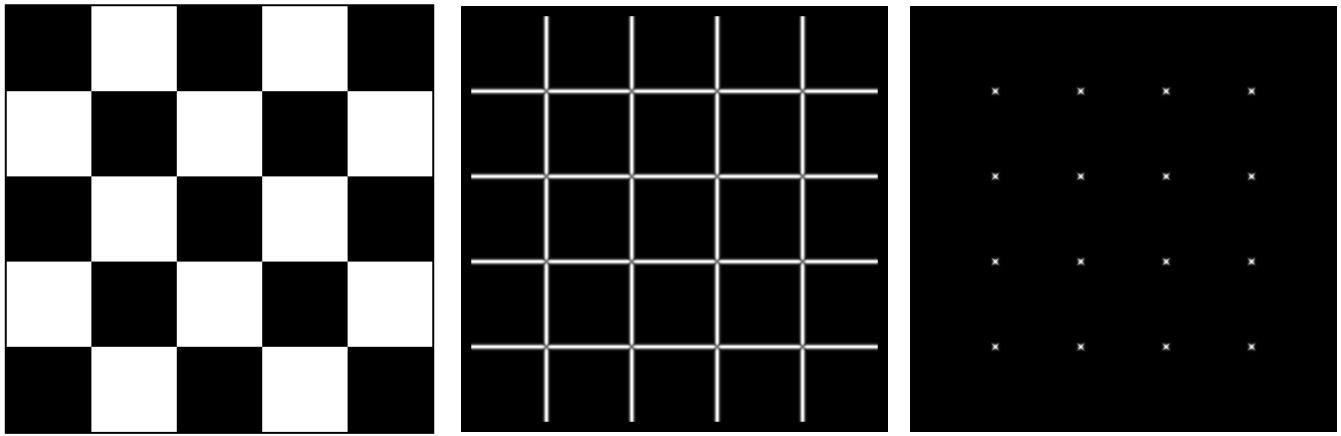
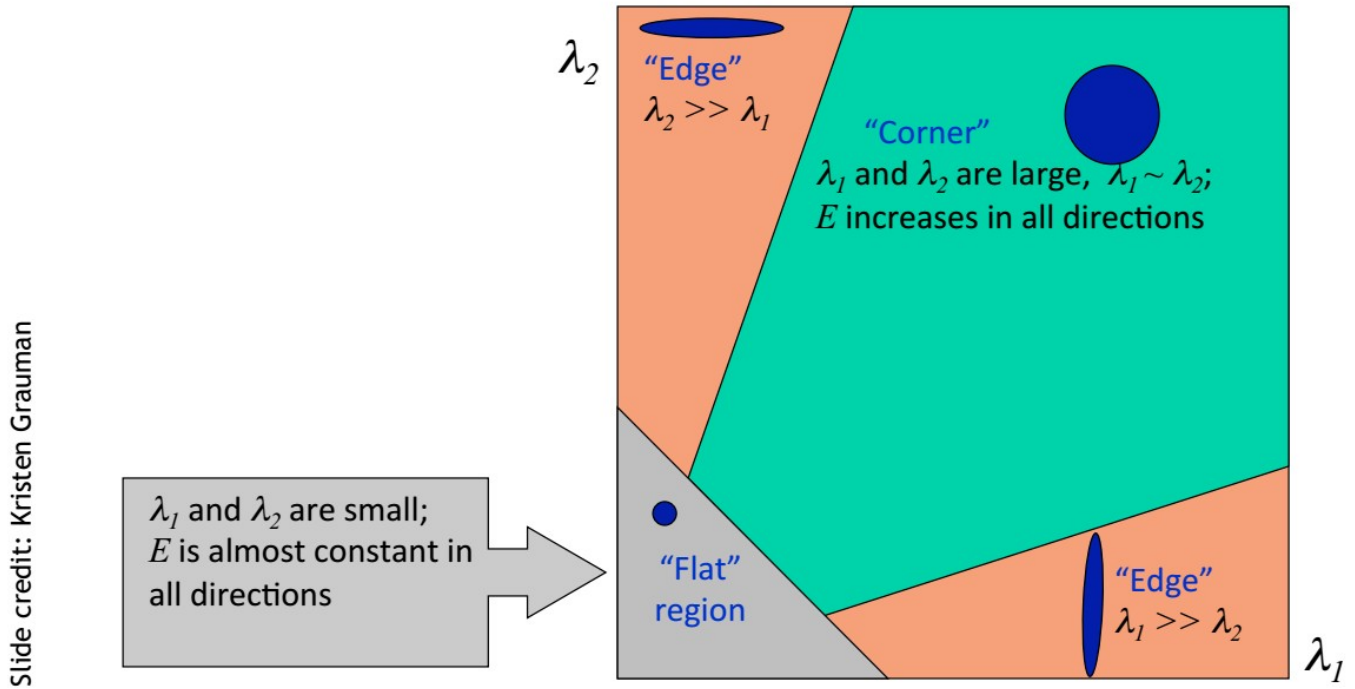
- How λ_{min} and λ_{max} behave on corners?
- λ_{max} is “big” when we have at least one “strong” gradient
- λ_{min} encodes the strenght of the ortogonal gradient



I λ_{max} λ_{min}

Corner Detector

- In a corner both λ_1 and λ_2 are "large"



Corner Detector

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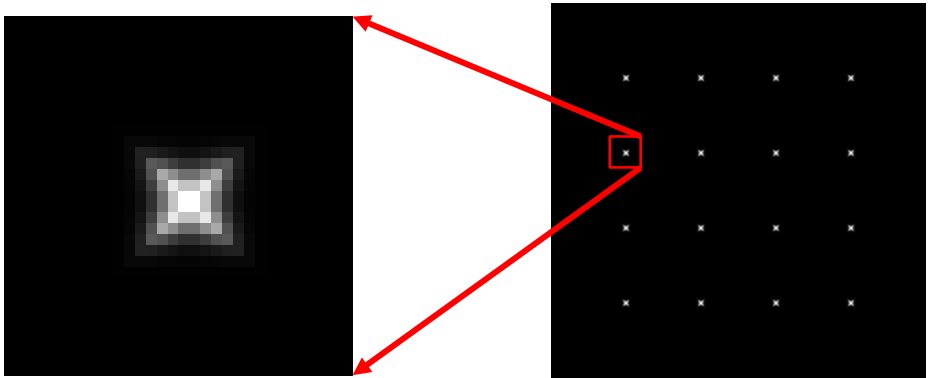
- Summary
  - Compute horizontal and vertical gradients
  - For each pixel
    - Create H matrix from gradient
    - Compute H eigenvalues
    - Discard pixels with $\lambda_{min} << \lambda_{max} <$ threshold
  - Apply a NMS approach to find pixels where λ_{min} is a local maximum

$$H = \begin{bmatrix} \sum_{(x,y)\in W} I_x^2 & \sum_{(x,y)\in W} I_x I_y \\ \sum_{(x,y)\in W} I_x I_y & \sum_{(x,y)\in W} I_y^2 \end{bmatrix}$$



Harris Corner Detector

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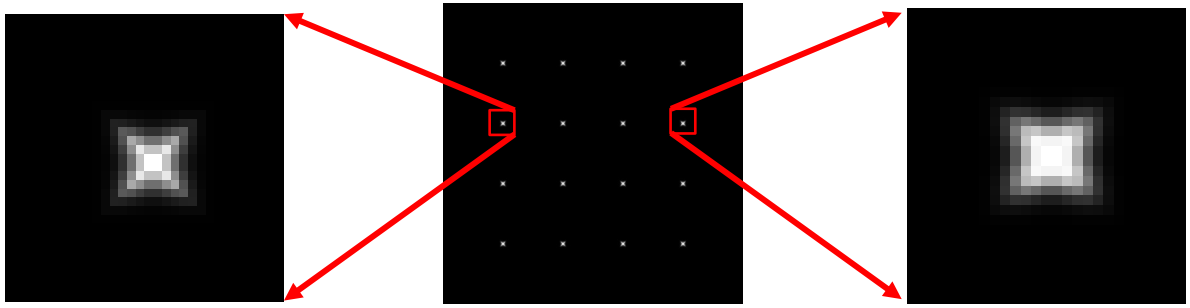
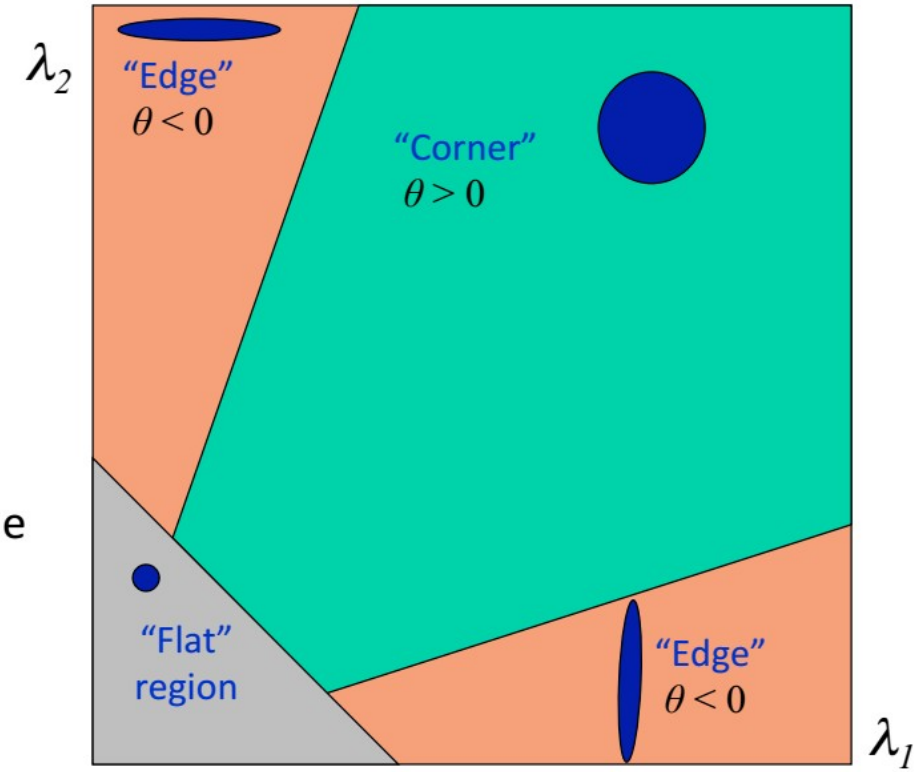
- "Harris" is a little variant of previous detector
  - No need to compute λs ⟶ no square roots
  - Weight function ⟶ based on the distance from the center pixel

$$\theta = det(H) - \alpha\, trace(H)^2$$



Harris Corner Detector

$$\theta = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$



- Fast approximation
  - Avoid computing the eigenvalues
  - α: constant (0.04 to 0.06)

Slide credit: Kristen Grauman

Corner Detectors

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- Shi et al. (1994) $\longrightarrow$ $\lambda_{min}^{\frac{1}{2}}$
- Harris et al. (1998) $\longrightarrow$ $\theta = det(H) - \alpha\, trace(H)^2 = \lambda_0 \lambda_1 - \alpha(\lambda_0 + \lambda_1)^2$
- Triggs et al. (2004) $\longrightarrow$ $\lambda_{min} - \alpha \lambda_{max}$
- Brown et al. (2005) $\longrightarrow$ $\lambda_{min} \approx \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2} = \frac{det(H)}{trace(H)}$

Window Function

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- Harris also introduced a window function
  - The idea is to weight distance

$$E(u,v)=\sum_{(x,y)\in W} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

Window Function

Shifted Intensity

Intensity

Window function

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

- Option 1: uniform window
  - Sum over square window

$$M = \sum_{x,y} \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

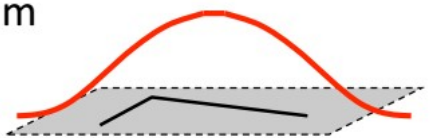
  - Problem: not rotation invariant

1 in window, 0 outside

- Option 2: Smooth with Gaussian
  - Gaussian already performs weighted sum

$$M = g(\sigma) * \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

  - Result is rotation invariant



Gaussian

Slide credit: Bastian Leibe

Harris Corner Detector

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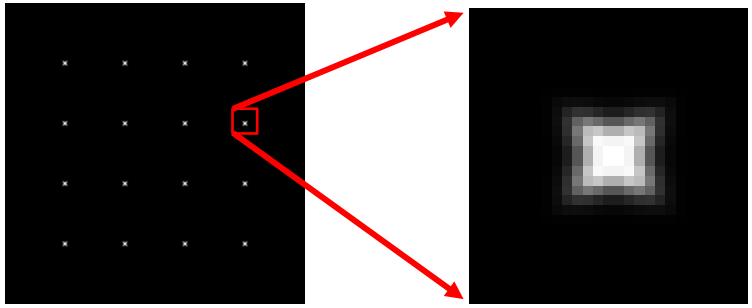
- Summary
  - Compute horizontal and vertical gradients
  - Apply window fuction
  - For each pixel
    - Create H matrix from gradients
    - Compute θ
    - Discard pixels with θ <0 or anyway below a given threshold
  - Apply a NMS approach to find pixels where λ_{min} is a local maximum

$$H(\sigma_I, \sigma_D) = g(\sigma_I) \begin{bmatrix} \sum_{(x,y)\in W} I_x^2(\sigma_D) & \sum_{(x,y)\in W} I_x I_y(\sigma_D) \\ \sum_{(x,y)\in W} I_x I_y(\sigma_D) & \sum_{(x,y)\in W} I_y^2(\sigma_D) \end{bmatrix}$$



Harris Corner Detector steps

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Slide adapted from Darya Frolova, Denis Simakov

Harris Corner Detector steps

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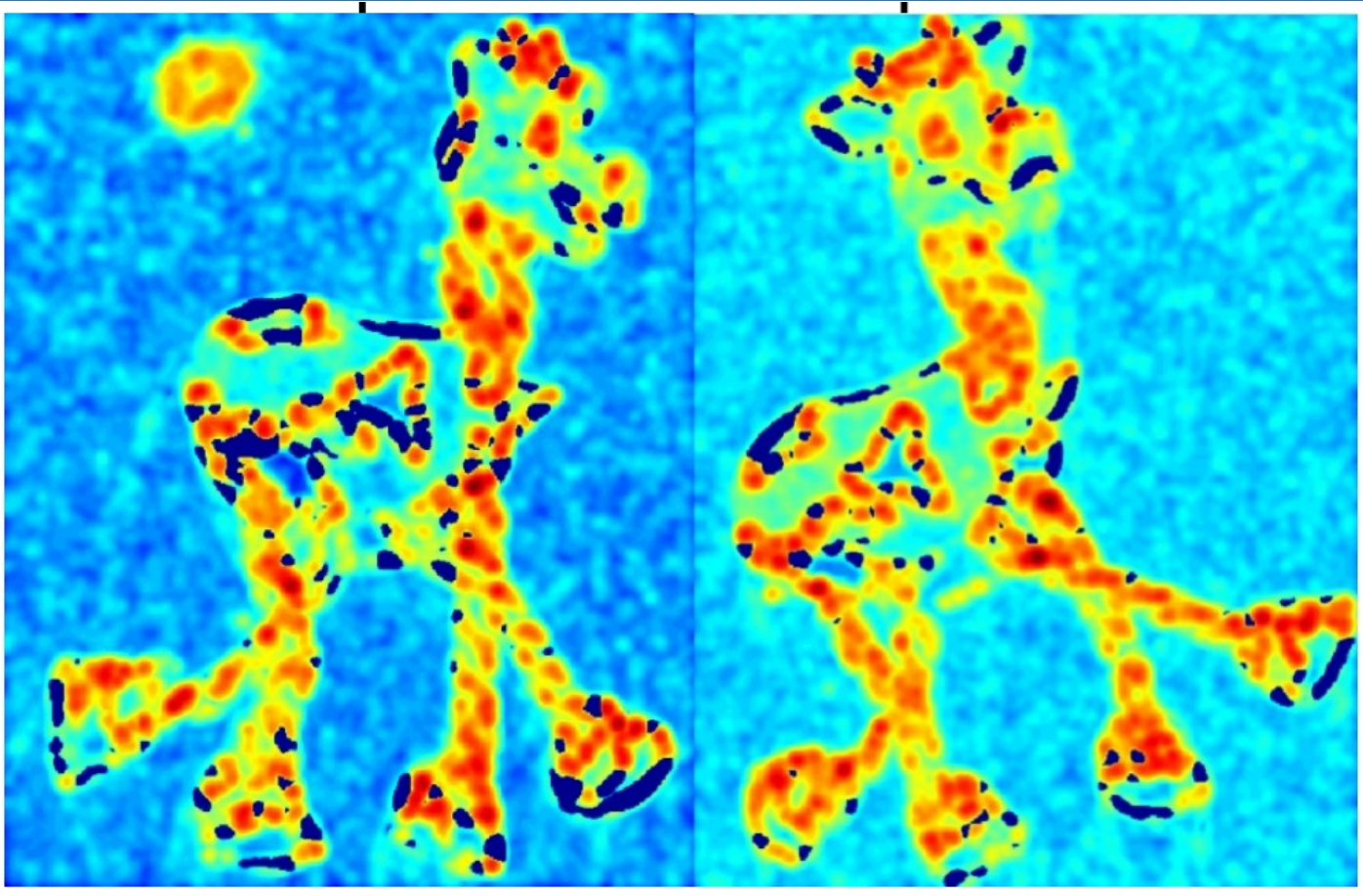

Slide adapted from Darya Frolova, Denis Simakov

Harris Corner Detector steps

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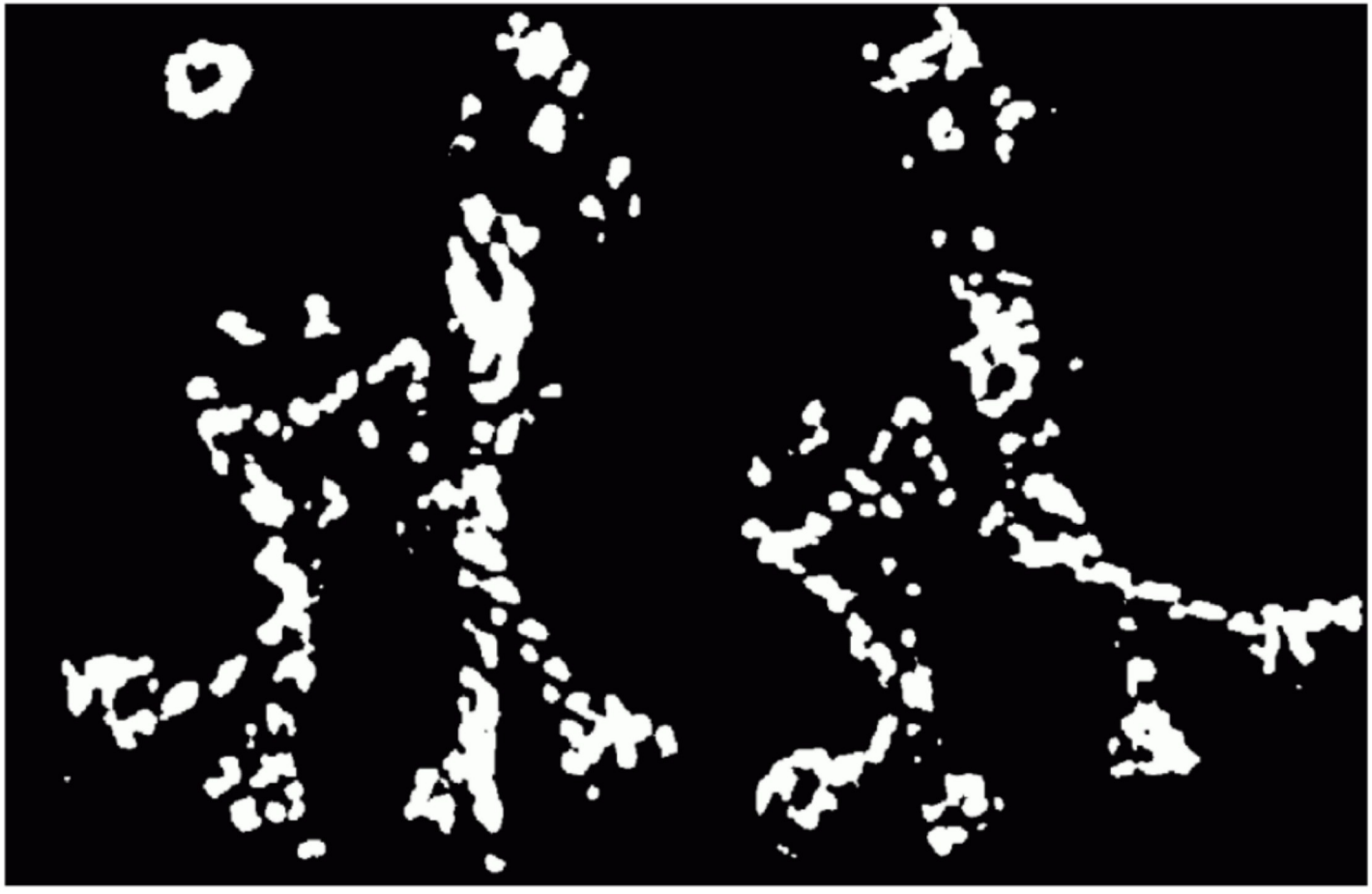

Harris Corner Detector steps

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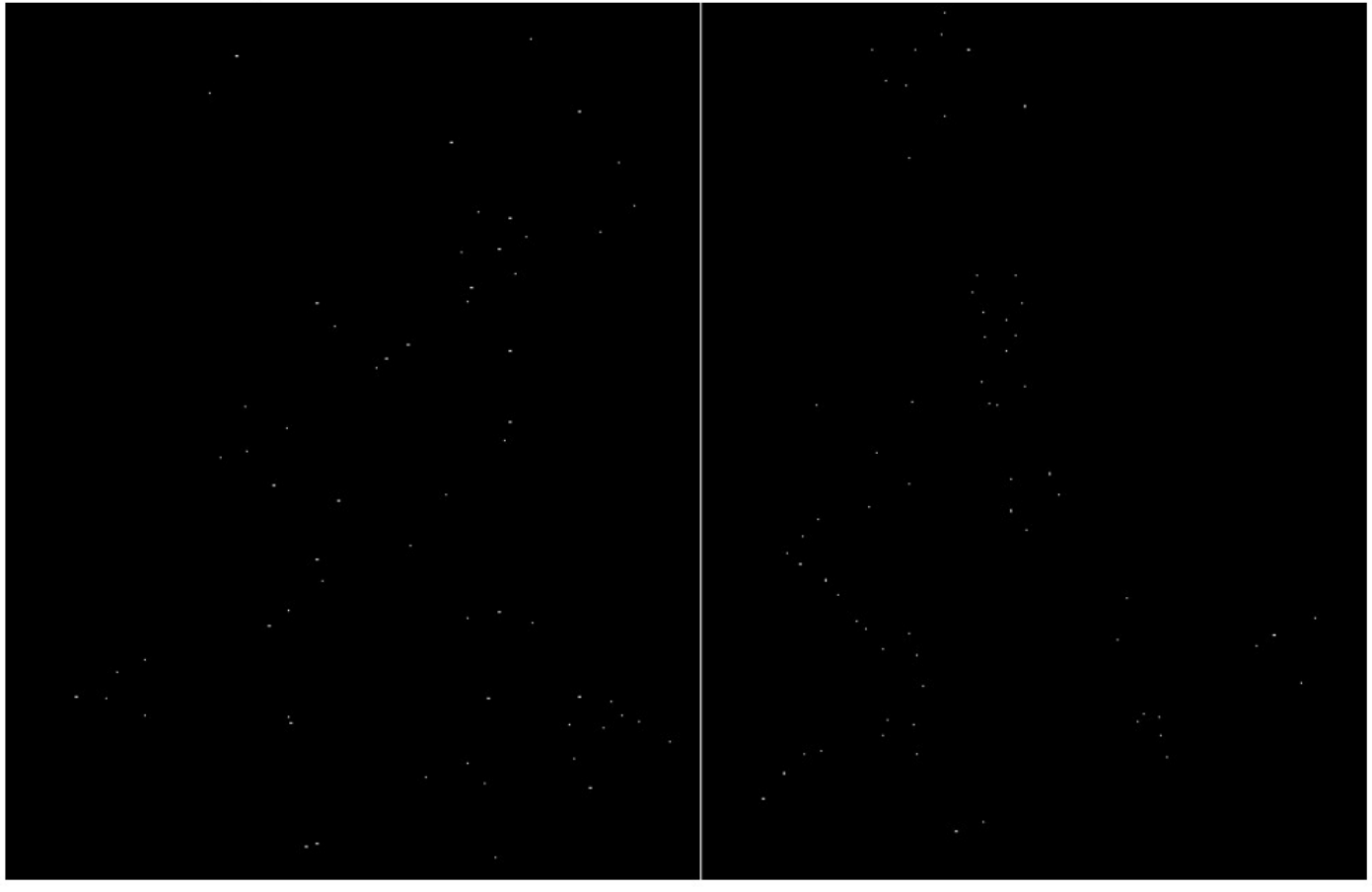
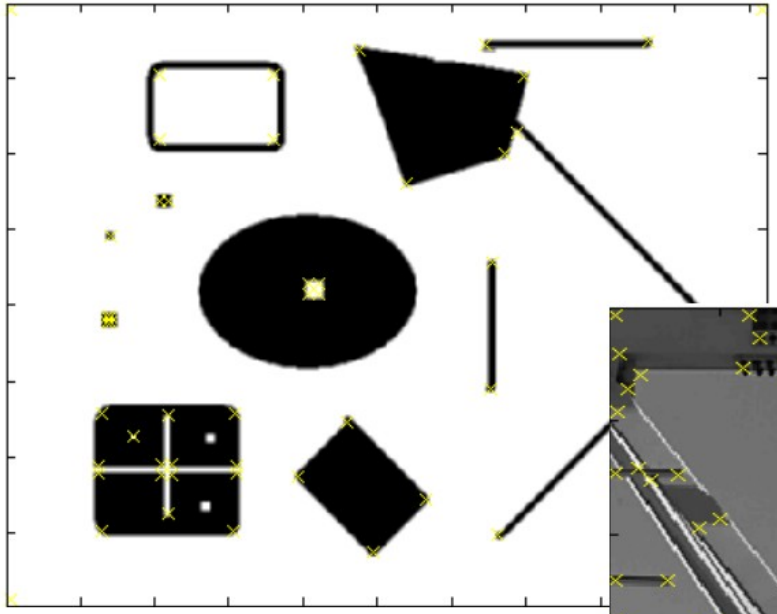


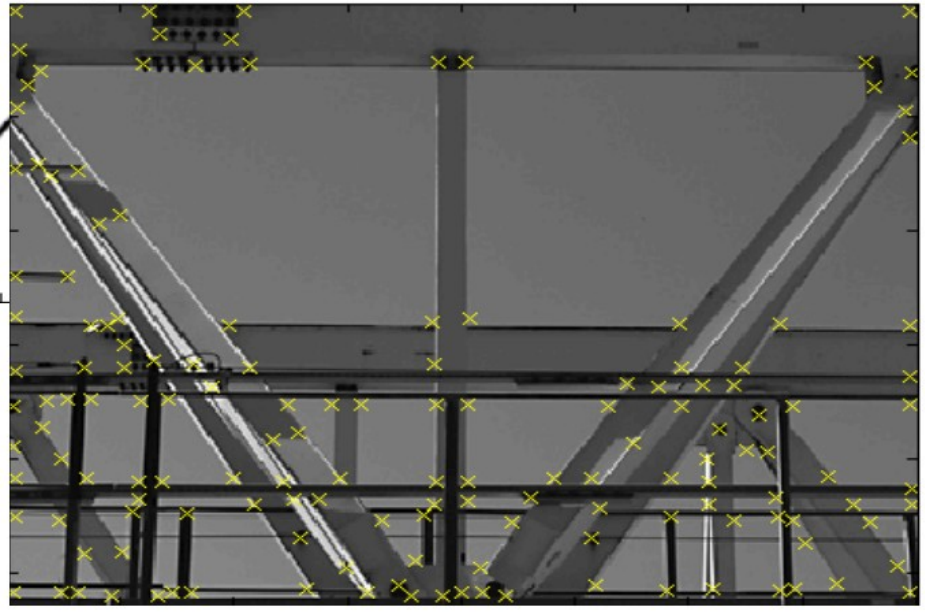
Slide adapted from Darya Frolova, Denis Simakov

Harris Corner Detector steps

Slide adapted from Darya Frolova, Denis Simakov

Examples





Effect: **A very precise corner detector.**

Slide credit: Krystian Mikolajczyk

Examples

Slide credit: Krystian Mikolajczyk

Sparse Stereo

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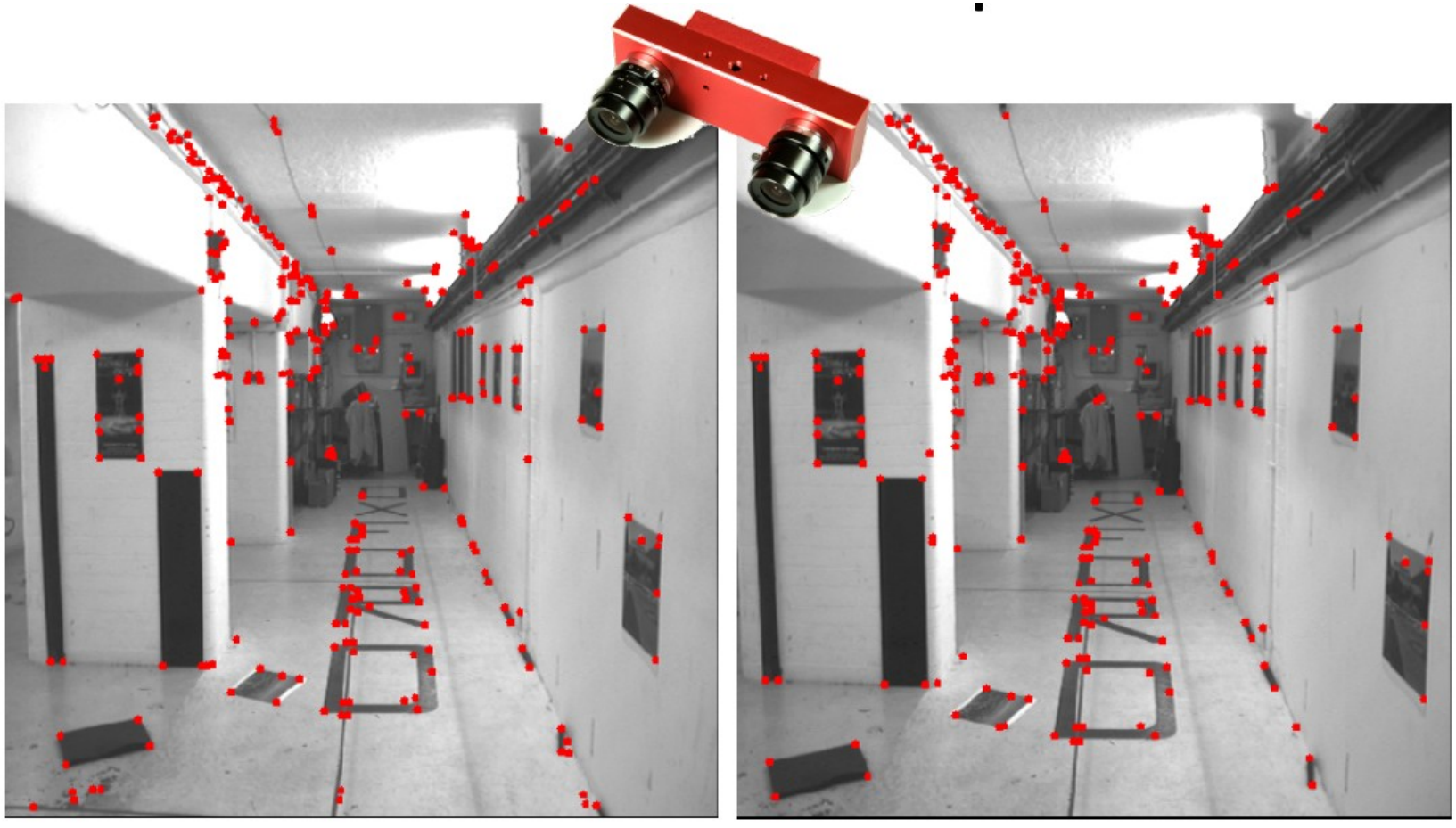

Invariance to transformations

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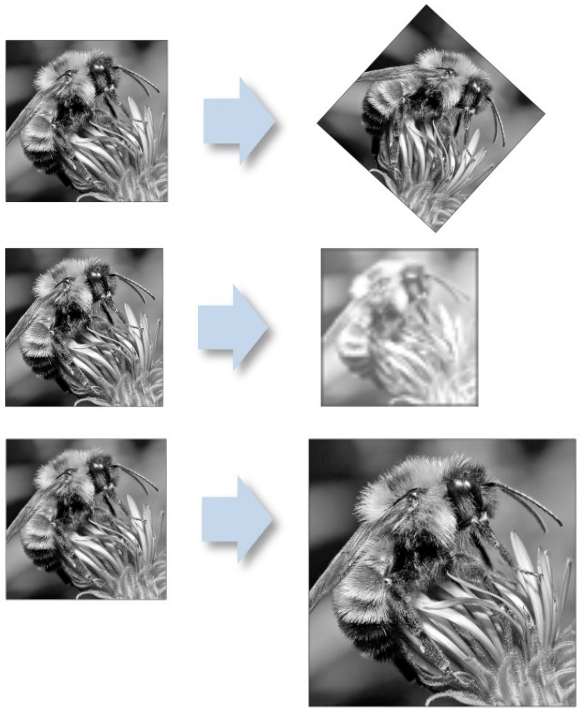
- How much keypoints extraction is robust to image transformations?
  - Rotation (traslation)
  - Intensity change
  - Scale



Rotation

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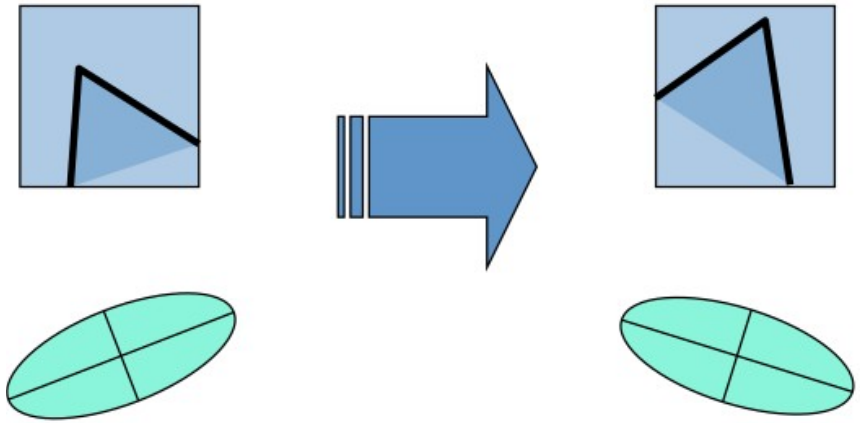
- Ellipse rotates but eigenvalues are the same

$$H = R \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R^{-1}$$



Invariant to Rotation

Intensity Change

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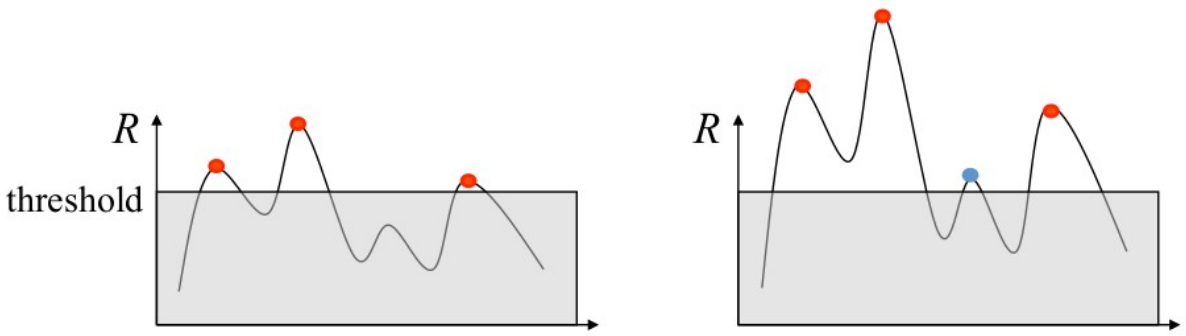
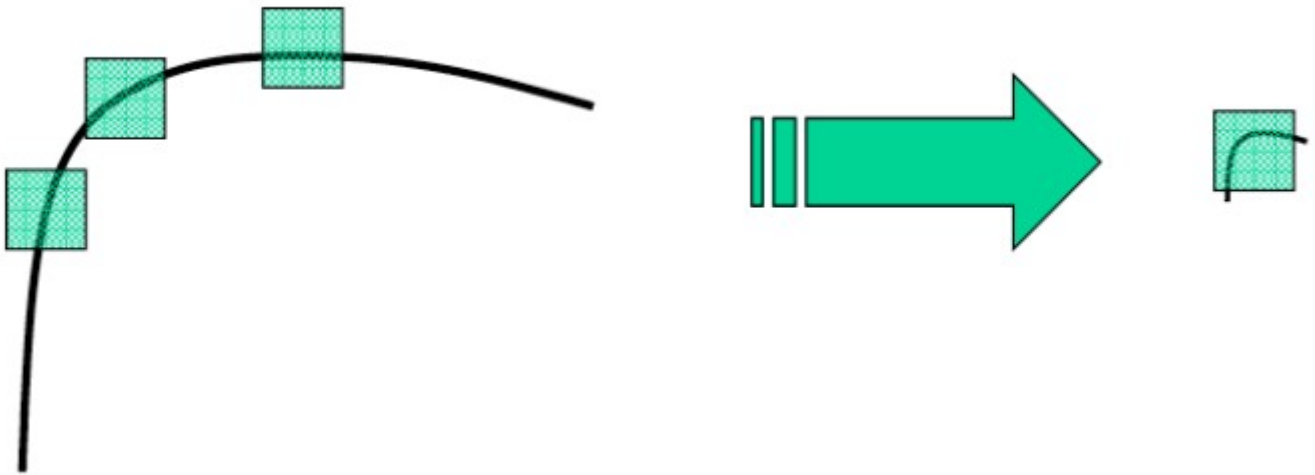
- Intensity change can be modeled as $g(r,c)=a\cdot f(r,c)+b$
- Derivative operations make result invariant wrt b
- Not so true about a …



Partially Invariant to Intensity Changes

Scale

- Right → corner
- Left → corner



Not Invariant to Scale Changes

Scale

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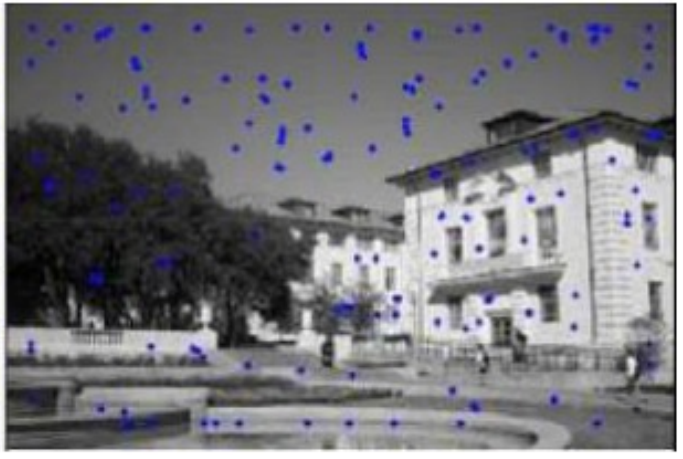
- The keypoints are highly dependent on window size





Scale invariant detector

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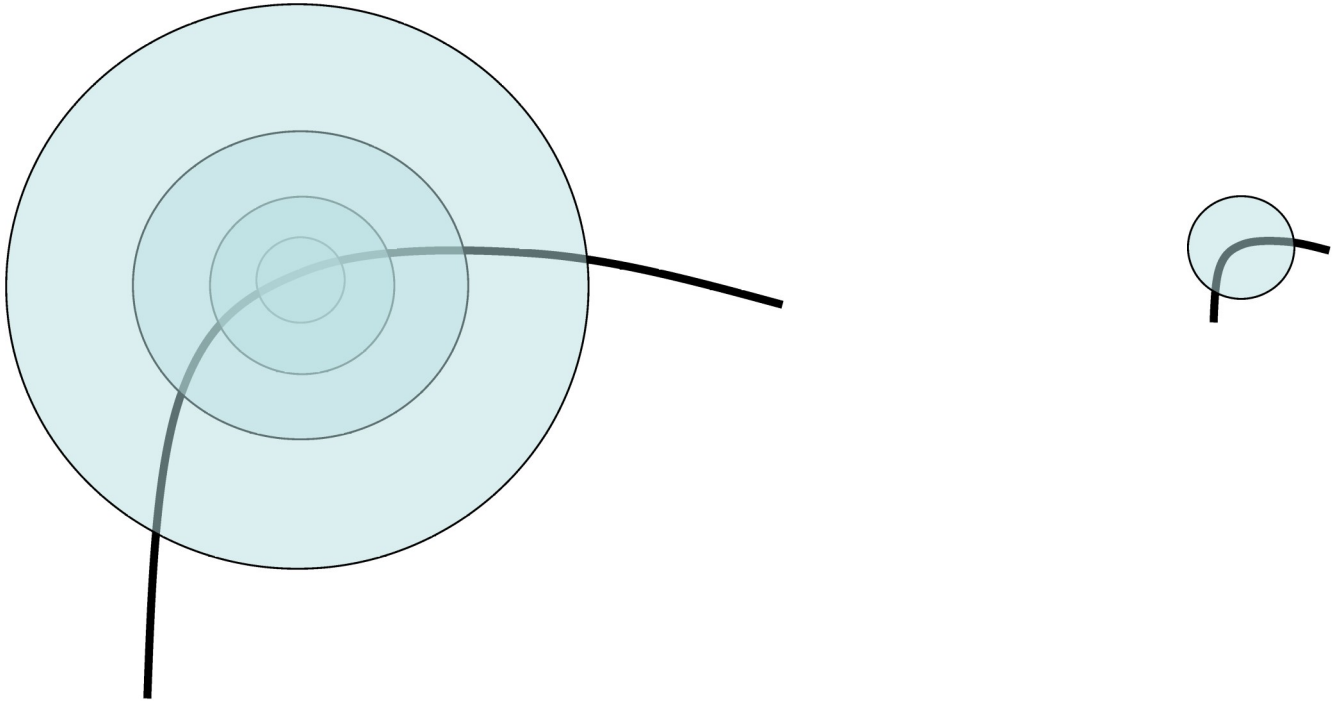
- Consider circles of different size around a point
- We can find a size that look similar in both images



Scale invariant detector

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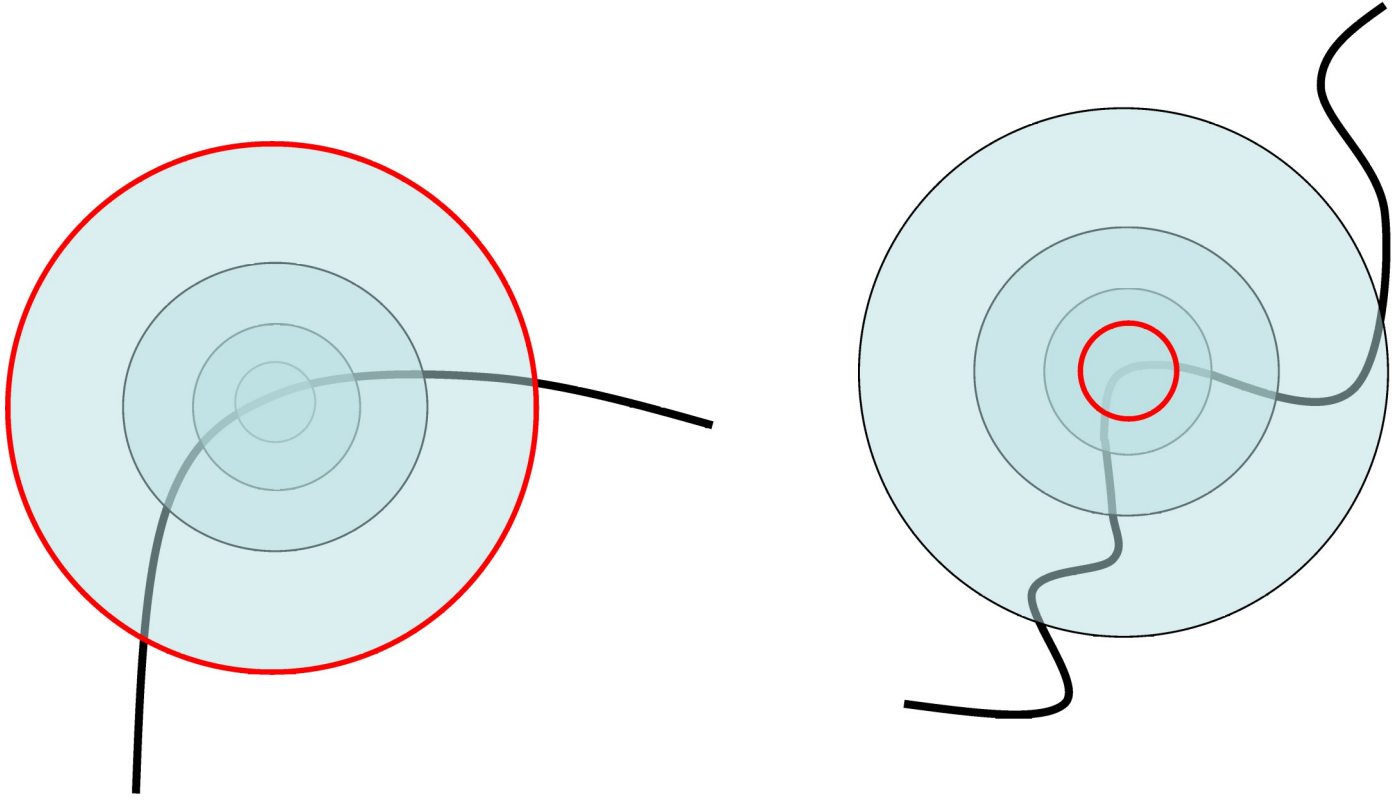
- How do we select appropriate circles size in both images?



Scale invariant detector

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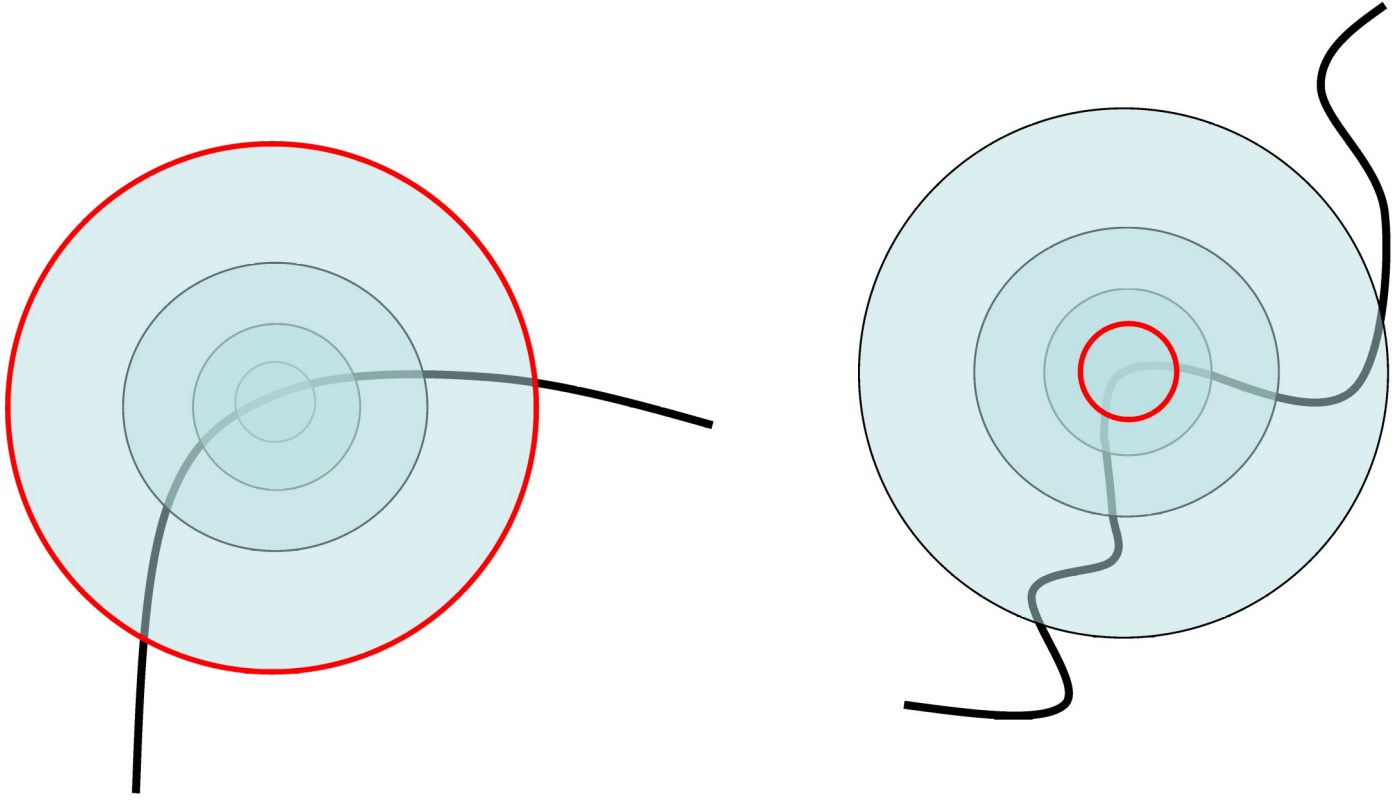
- We can select the window size that **maximize** Harris operator



Scale invariant detector

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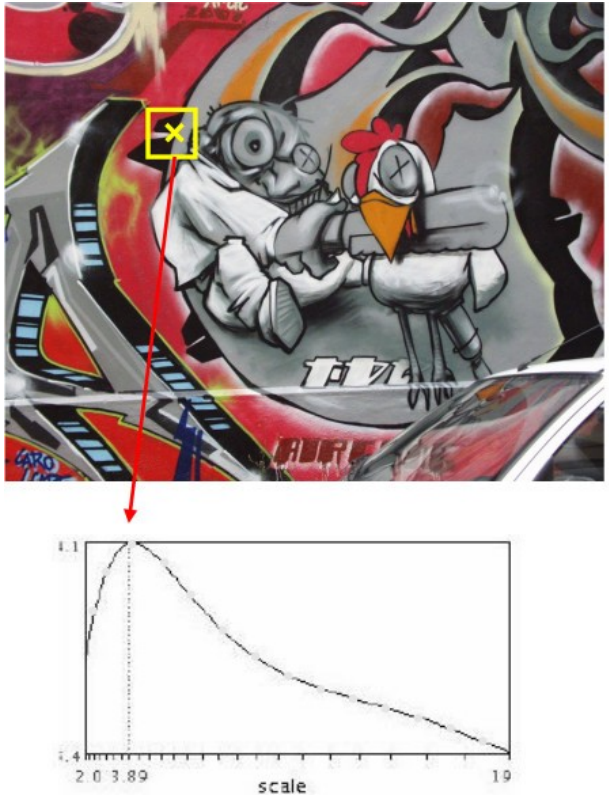
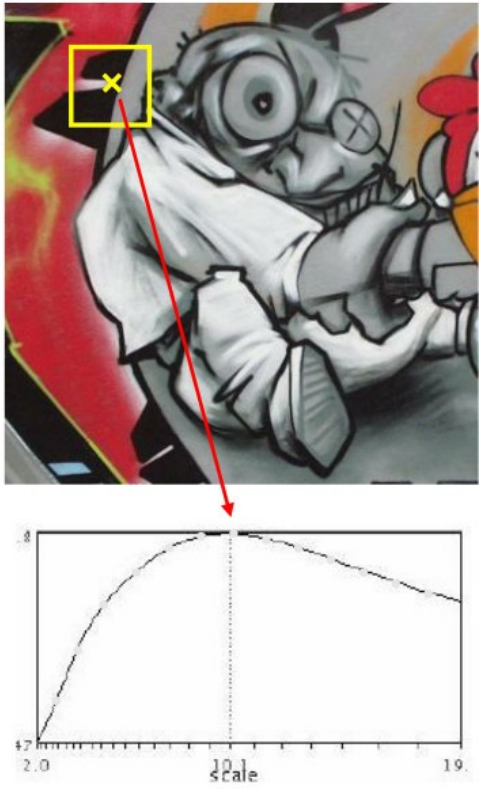
- Example





Scale invariant detector

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- Instead computing θ for larger and larger windows a pyramidal approach can be used

(sometimes need to create in-between levels, e.g. a ¾-size image)

Other scale invariant detectors

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- **HARRIS-LAPLACIAN**
  - K. Mikolajczyk, C. Schmid, Indexing based on scale invariant interest points, ICCV 2001
- **SIFT**: Scale Invariant Feature Transform
  - D. Lowe. “Distinctive Image Features from Scale-Invariant Keypoints“, IJCV 2004
- **FAST**: Features from Accelerated Segment Test
  - Edward Rosten and Tom Drummond, “Machine learning for high speed corner detection” in 9th European Conference on Computer Vision, vol. 1, 2006
- **SURF**: Speeded-Up Robust Features
  - Herbert Bay, Andreas Ess, Tinne Tuytelaars, Luc Van Gool, "SURF: Speeded Up Robust Features", Computer Vision and Image Understanding (CVIU), 2008
- **MSER**: Maximally Stable Extremal Regions
  - J. Matas, O. Chum, M. Urban, and T. Pajdla. "Robust wide baseline stereo from maximally stable extremal regions." Proc. of British Machine Vision Conference, 2002.
- **BRISK**: Binary Robust Invariant Scalable Keypoint
  - Stefan Leutenegger ; Margarita Chli ; Roland Y. Siegwart, 2011 International Conference on Computer Vision

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Features Extraction

Question time!