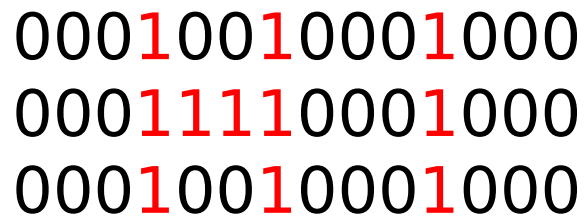


- Binary images
- Thresholding
- Mathematic Morphology
 - (also for non binary images)
- Connected components extraction

- In some cases it is necessary/useful to acquire/produce and/or process images whose pixels can only have 0/1 logical values
 - PBM or XBM image formats

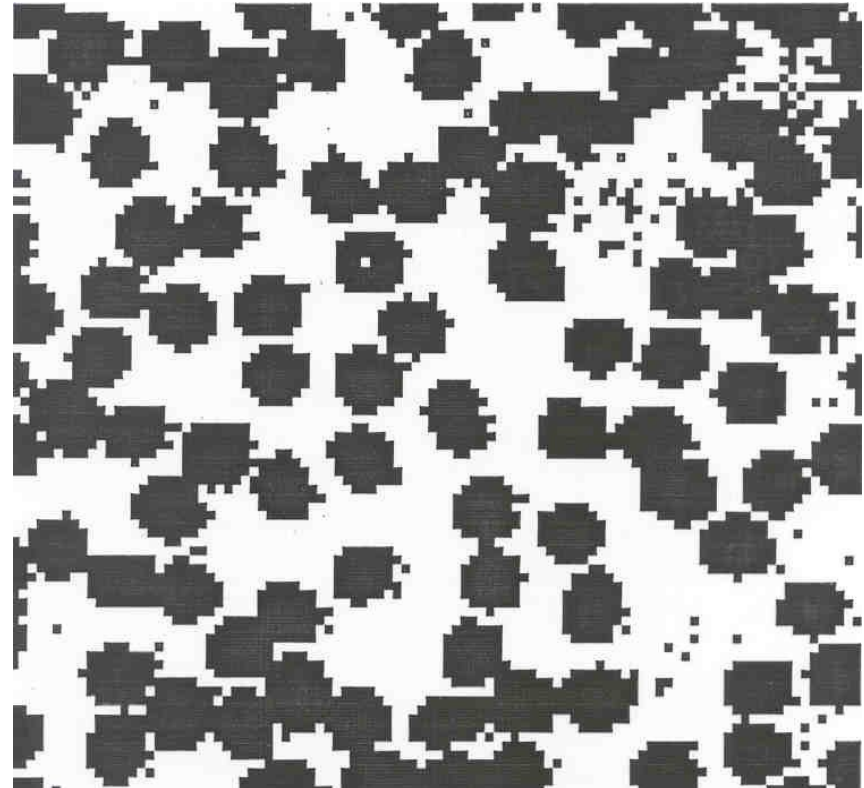


```
00010010001000  
00011110001000  
00010010001000
```

- 1 actually can be considered as !=0
- 255 is often used for simplicity

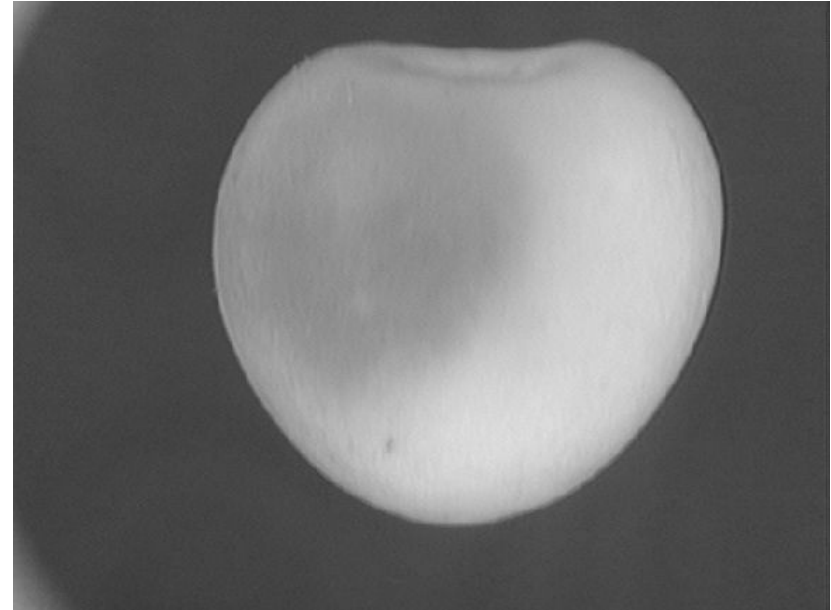
0	0	0	255	0	0	255	0	0	0	255	0	0	0
0	0	0	255	255	255	255	0	0	0	255	0	0	0
0	0	0	255	0	0	255	0	0	0	255	0	0	0

- Real image of blood cells
- We would like to estimate red blood cells
- Anyway they are not isolated ($<50\%$)
- How we can separate them?

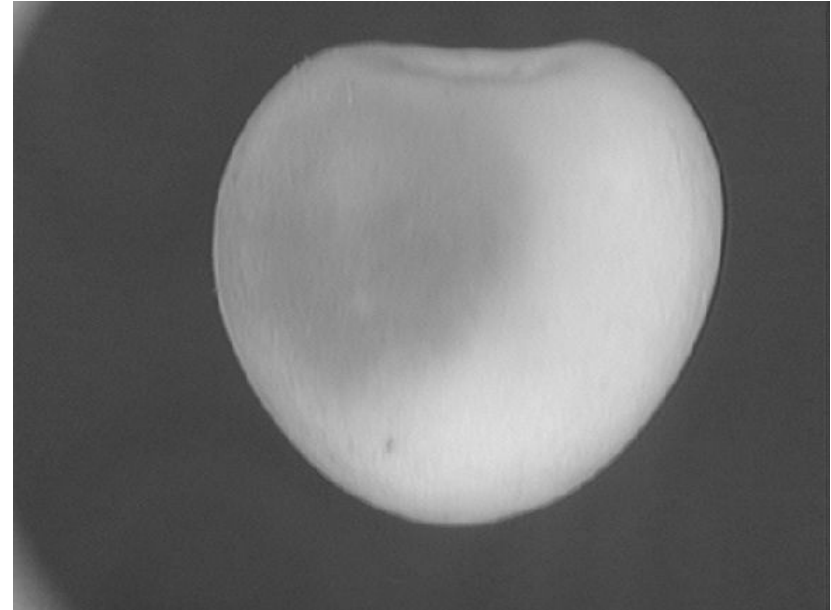


- We would like to discard a bruised apple
- How we can select the bruised part?
 - Extract “dark” pixels
- We can only use pixel brightness
- Thresholding:

$$out(r,c) = \begin{cases} 0 & \text{where } in(r,c) < th \\ 1 & \text{where } in(r,c) \geq th \end{cases}$$

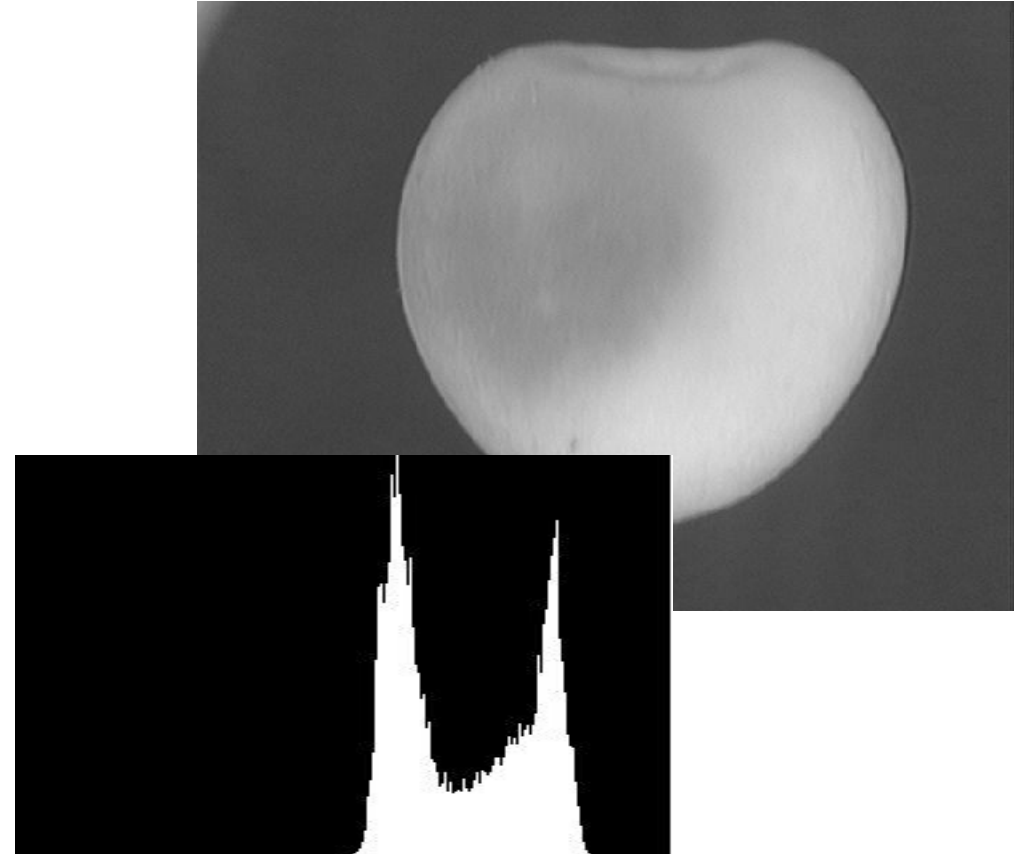


- Anyway we have:
 - “Good” apple part
 - Bruised apple part
 - Background
- How we decide about the threshold?



$$out(r,c) = \begin{cases} 0 & \text{where } in(r,c) < th \\ 1 & \text{where } in(r,c) \geq th \end{cases}$$

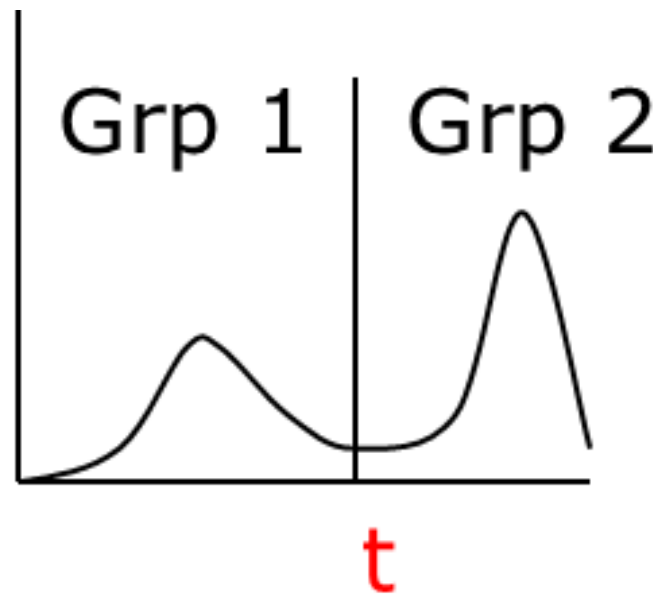
- Anyway we have:
 - “Good” apple part
 - Bruised apple part
 - Background
- How we decide about the threshold?
- Histogram!



- Best threshold computation
- Problem: find a th that minimize weighted sum of each group variance

$$\sigma^2(th) = \sigma_1^2(th) \cdot w_1(th) + \sigma_2^2(th) \cdot w_2(th)$$

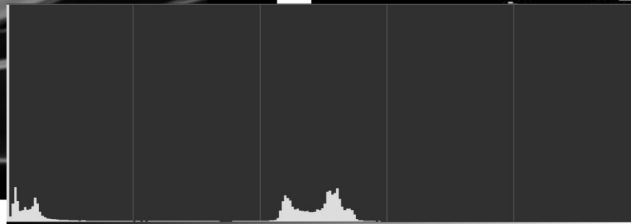
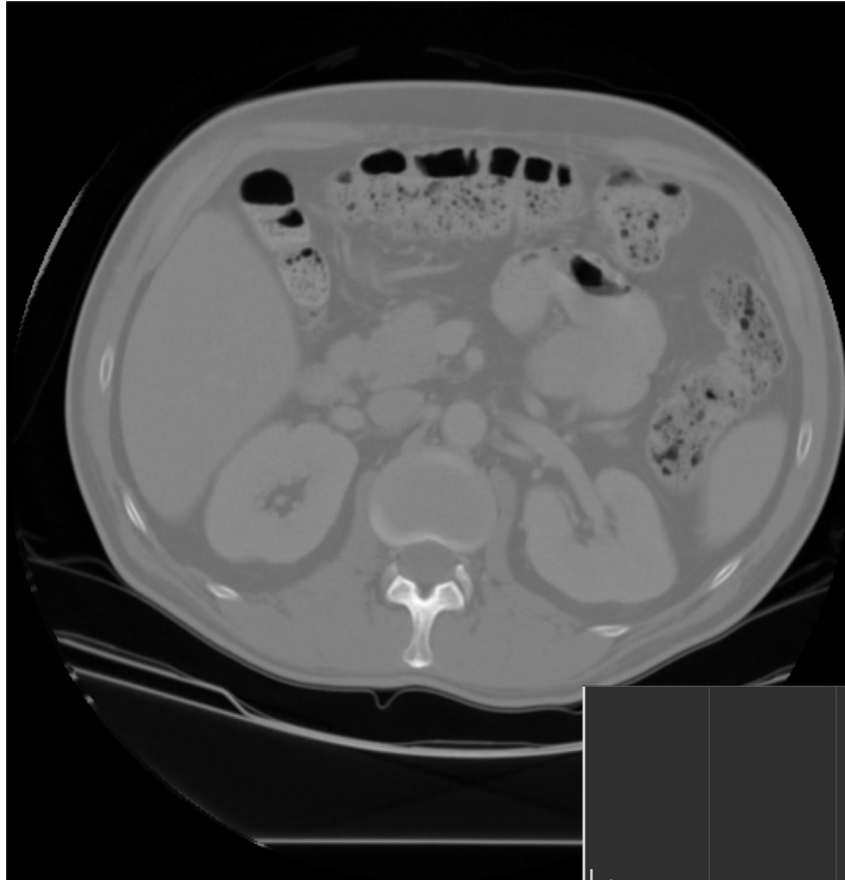
- $w_1(th)$ & $w_2(th)$ are the probability of belonging to group 1 or 2



$$w_1(th) = \sum_{i=0}^{th-1} h(i)$$

$$w_2(th) = \sum_{i=th}^L h(i)$$

Otsu's Method



Otsu's Method



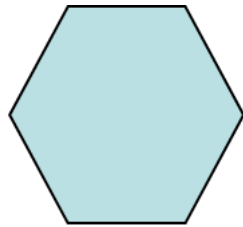
- Used to extract image components that are useful in the representation and description of region shape, such as
 - boundaries extraction
 - skeletons
 - convex hull
 - morphological filtering
 - thinning
 - pruning

- Main operations
 - Erosion $\rightarrow \ominus$
 - Dilation $\rightarrow \oplus$
 - Opening $\rightarrow \circ$
 - Closing $\rightarrow \bullet$
 - Hit or Miss $\rightarrow \otimes$

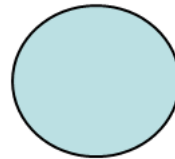
- Small “mask” to probe the image under study
- Structuring Element features an origin
- Shape and size must be adapted to geometric properties we would like to mask/obtain



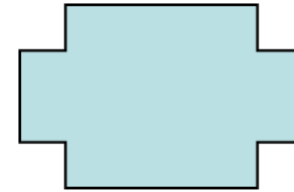
box



hexagon



disk

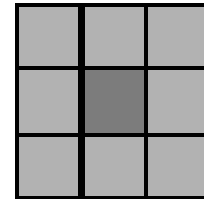
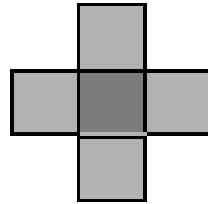
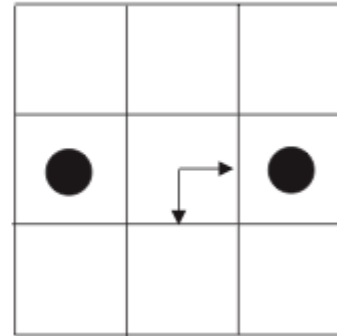
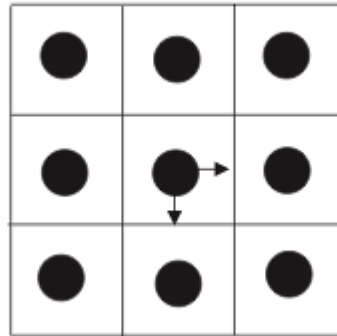
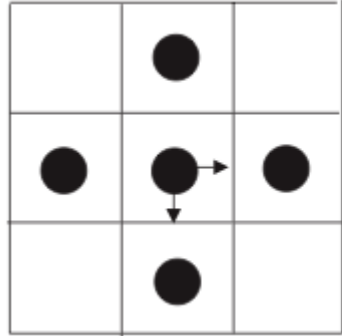


something

box(length,width)

disk(diameter)

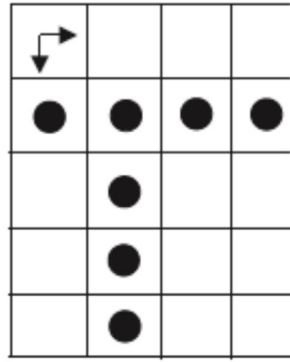
Structuring Element



- The structuring element B is “moved” (B_z) on the image A for a set of values z belonging to an Euclidean Space E
- The result is the set of z values for which B_z is in A

$$A \ominus B = \left\{ z \in E^2 : B_z \subseteq A \right\}$$

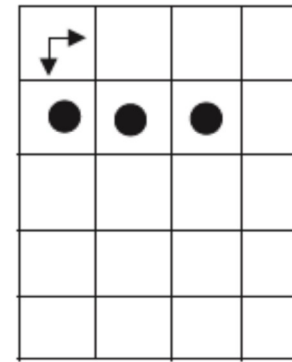
- Practically the structuring element is superimposed on all the pixel of the image.
 - When it completely fits the binary image the result is 1, otherwise 0
 - Logical AND
 - It shrinks image elements



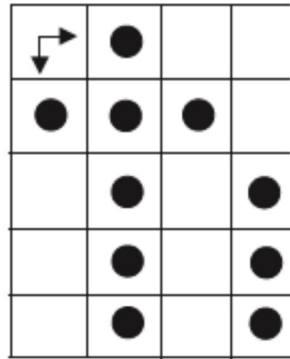
A



B



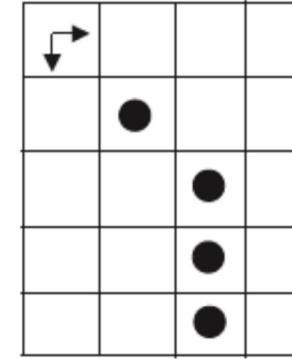
$A \ominus B$



A



B

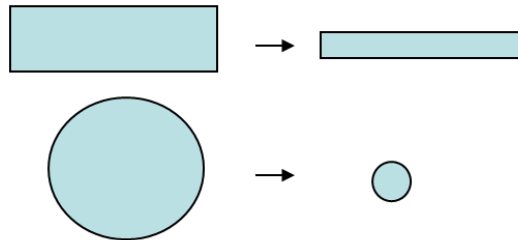


$A \ominus B$

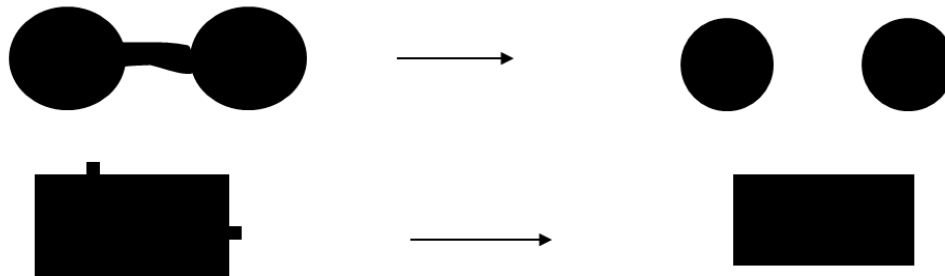
Erosion **shrinks** the connected sets of 1s of a binary image.

It can be used for

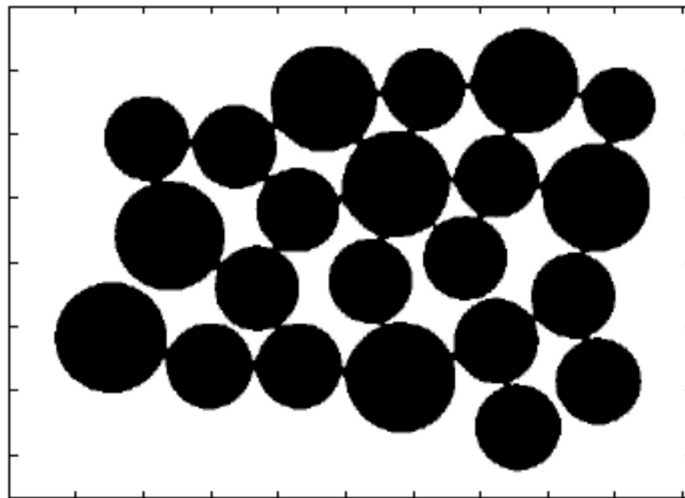
1. shrinking features



2. Removing bridges, branches and small protrusions



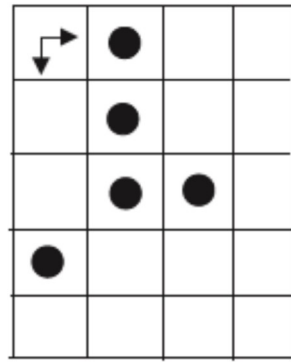
- Consider the following image



- We can use erosion to separate elements

- Again the structuring element is superimposed on all the pixel of the image.
 - When it hits the binary image the result is 1, otherwise 0
 - Logical OR
- It “dilates” image elements

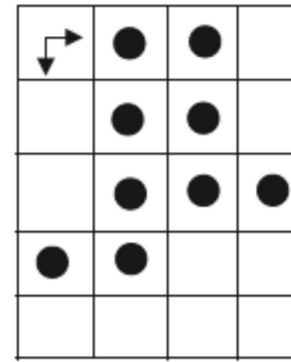
Dilation \oplus



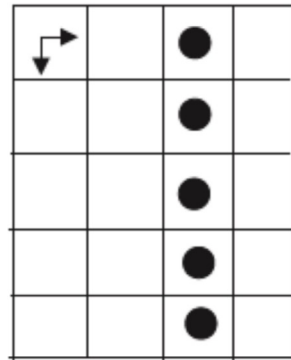
A



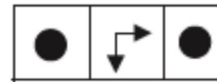
B



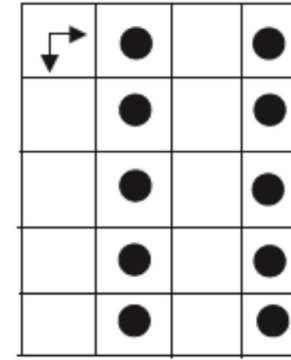
$A \oplus B$



A



B

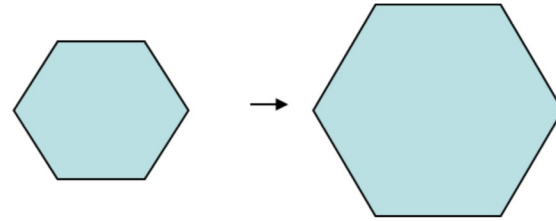


$A \oplus B$

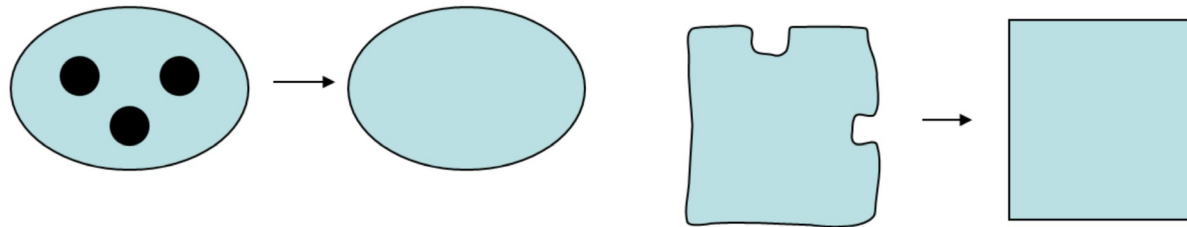
Dilation **expands** the connected sets of 1s of a binary image.

It can be used for

1. growing features



2. filling holes and gaps



- We can use erosion and dilation to remove small details or to fill some gaps
- But they also affect other part of the image...
- Solution
 - Combine them!

- Opening
 - Erosion followed by dilation

$$A \circ B = (A \ominus B) \oplus B$$

- Closing
 - Dilation followed by erosion

$$A \circ B = (A \oplus B) \ominus B$$

- The same structuring element is used

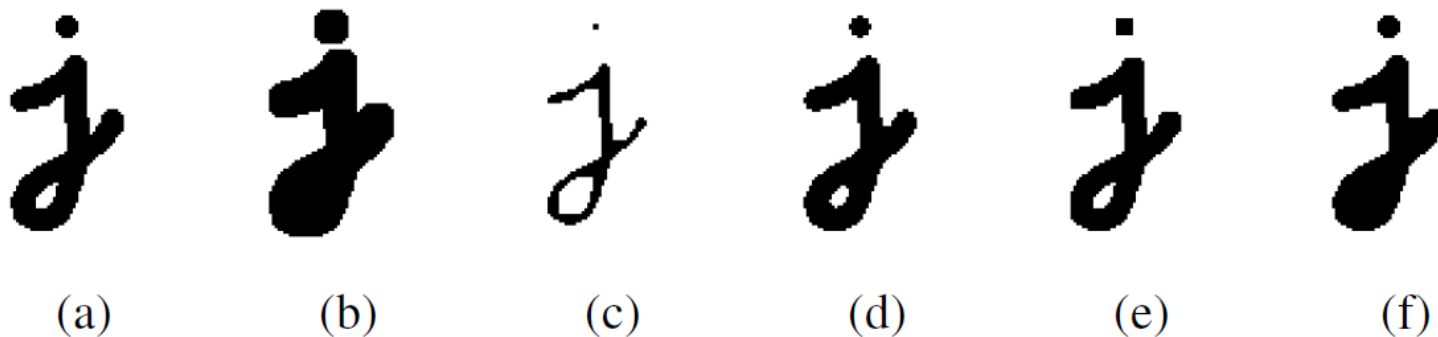


Figure 3.21 Binary image morphology: (a) original image; (b) dilation; (c) erosion; (d) majority; (e) opening; (f) closing. The structuring element for all examples is a 5×5 square. The effects of majority are a subtle rounding of sharp corners. Opening fails to eliminate the dot, since it is not wide enough.

- Two “disjoint” structuring elements are used: A e B
 - $A \cap B = \emptyset$
- Two erosions are used and joined
 - $I \otimes X = (I \ominus A) \cap (I^c \ominus B)$
 - Where I^c is the complement for I and $X = (A, B)$
- The idea is that A should match shapes and B background
 - It can be seen as an “extended” SE

	1	
0	1	1
0	0	

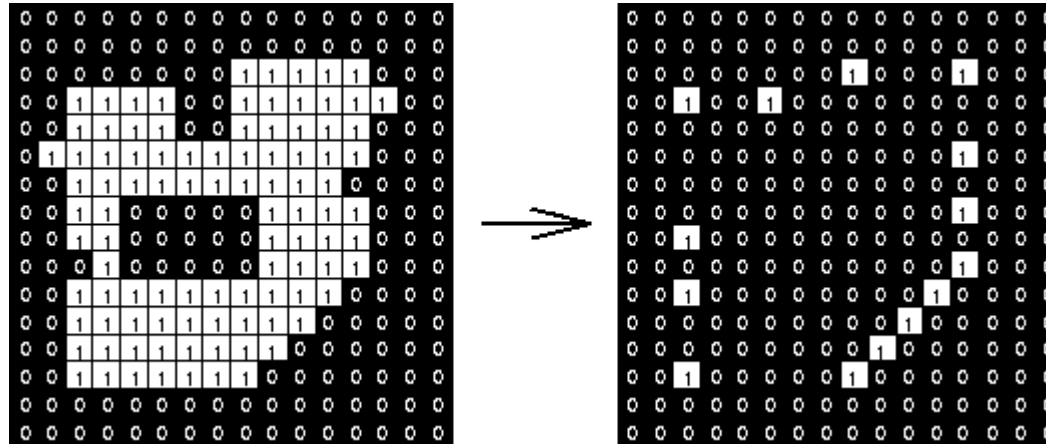
Hit or Miss \otimes

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	

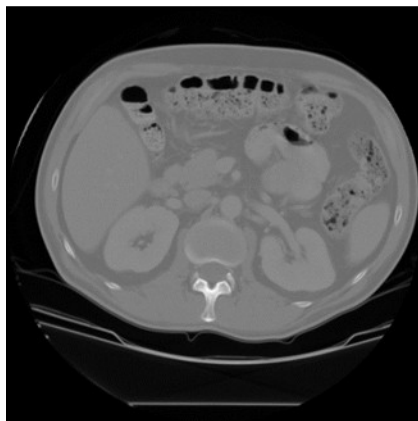


- Other morphological complex operations can be used using union/intersection:
 - Border extraction
 - Skeletonization
 - Thinning
 - ...



- A connected component is a **contiguous** group of pixels that have the **same value**
 - Actually not only binary images

original



thresholded



opening+closing

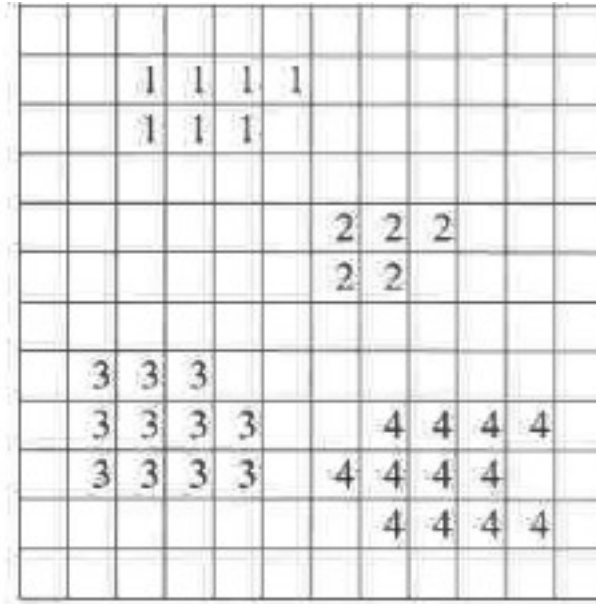
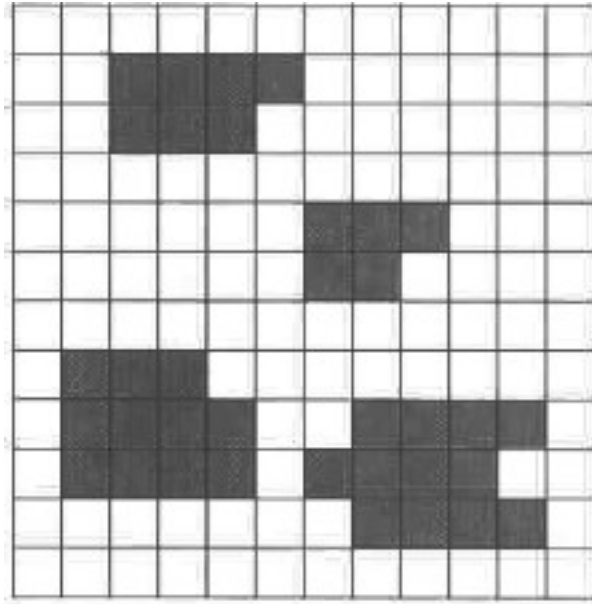


components



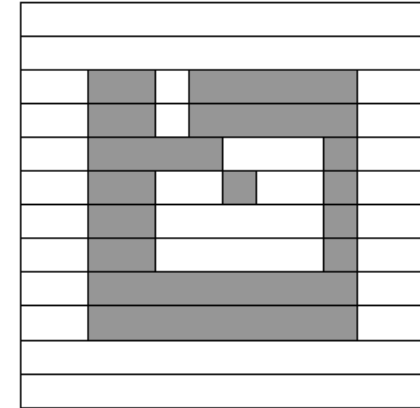
Labelling algorithms for CC

- We assign a unique label to each component



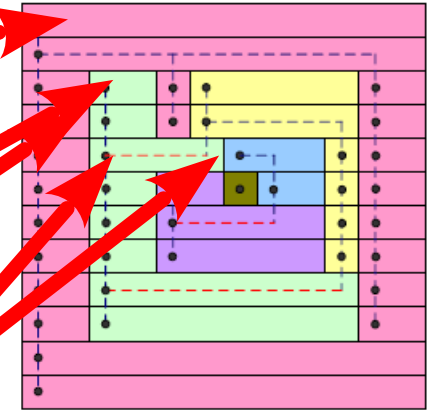
Source: R. Jain

-



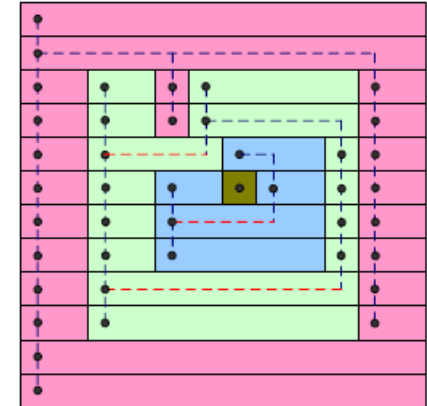
Row by Row Labelling algorithm

- 1st pass:
 - First row:
 - Check left neighbor, do it have the same value?
 - Yes → same label
 - No → new label
 - Other rows
 - Check left and upper neighbor
 - They have same value and same label → same label
 - Current pixel have same value of only one of them → same label of that one
 - They have same value but different label (!) → set lowest label and track the equivalence
 - Otherwise → new label



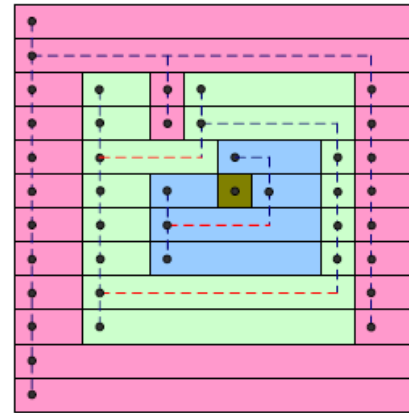
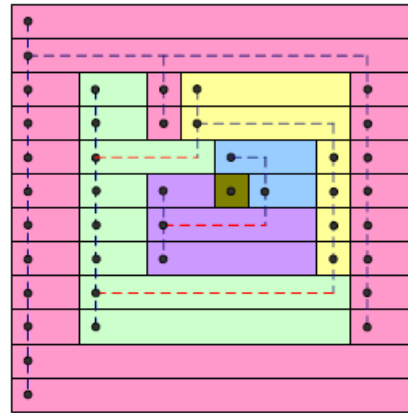
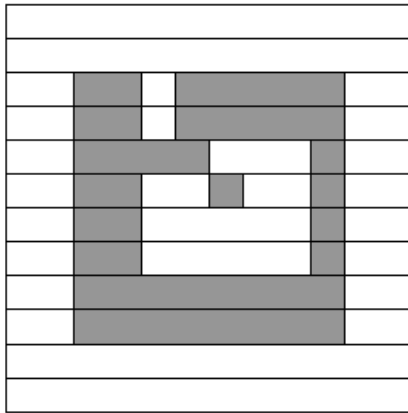
Row by Row Labelling algorithm

- 2nd pass:
 - During 1st pass we have tracked label equivalences
 - Row by row change labels with lowest equivalent

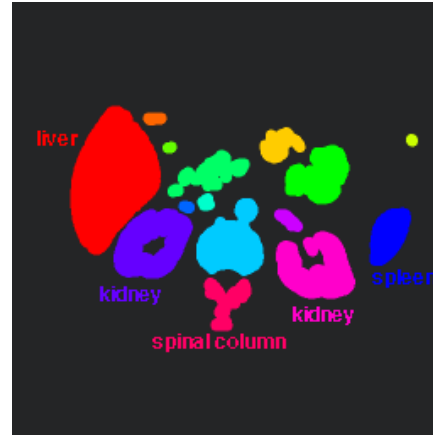


- Row by Row

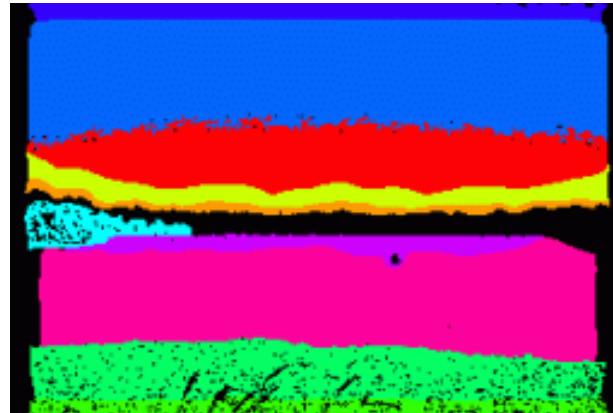
Source: Szeliski



Connected Components



connected
components
of 1's from
cleaned,
thresholded
image



connected
components
of cluster
labels

- Mathematical morphology can be extended to grayscale images
- Applications
 - Contrast enhancement
 - texture description
 - edge detection
 - thresholding

- In a grayscale image there is no “object” and “background”
- Min and Max operators are used
- Structuring element is still a mask
- Dilation \rightarrow max element under the mask wins
- Erosion \rightarrow min element under the mask wins
- Opening&Closing as before

Grayscale Morphology

