

# Introduction to Piezoelectric Transducers

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# Introduction to Piezoelectricity

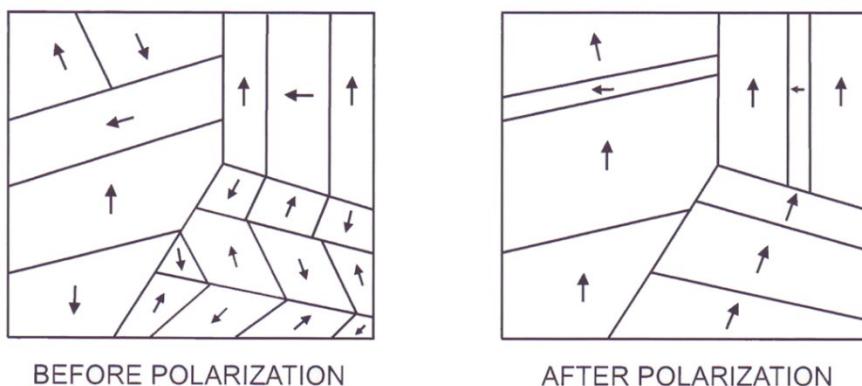
## 1. Introduction to Piezoelectricity

### 1.1 Piezoelectric Phenomenon

Piezoelectricity is a property of certain dielectric materials to physically deform in the presence of an electric field, or conversely, to produce an electrical charge when mechanically deformed. There are a wide variety of materials which exhibit this phenomenon to some degree, including natural quartz crystals, semi-crystalline polyvinylidene polymer, polycrystalline piezoceramic, bone and even wood.

Piezoelectricity is due to the spontaneous separation of charge with certain crystal structures under the right conditions. This phenomenon, referred to as **spontaneous polarization**, is caused by a displacement of the electron clouds relative to their individual atomic centers, i.e., a displacement of the positive ions relative to the negative ions within their crystal cells. Such a situation produces an electric dipole.

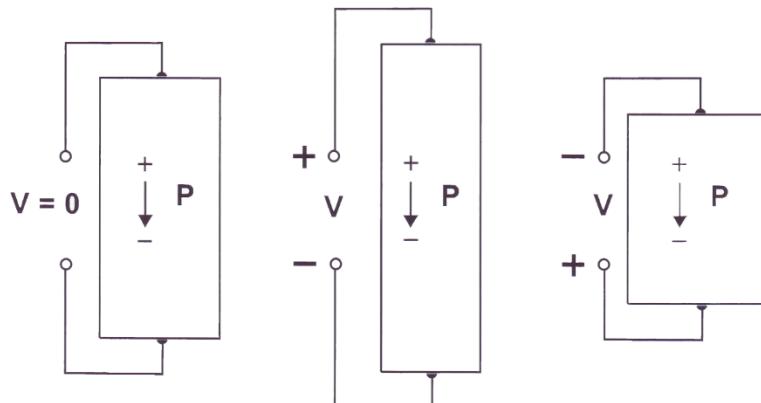
Polycrystalline ceramic, one of the most active piezoelectric materials known, is composed of randomly oriented minute crystallites. Each crystallite is further divided into tiny "domains," or regions having similar dipole arrangements. The overall effect of randomly oriented polar domains is an initial lack of piezoelectric behavior. However, the material may be induced to exhibit **macroscopic polarization** in any given direction by subjecting it to a strong electric field, as shown in *Figure 1*. Such inducible materials are termed **ferroelectric**. Polarization is accomplished by applying a field of ~2350 volts/mm (60 V/mil) across electrodes deposited on outer surfaces. Once polarized, the ferroelectric material will remain polarized until it is depoled by an opposite field or elevated temperature.



**Figure 1.** Inducing macroscopic polarization in a polycrystalline piezoceramic by applying a strong electric field across randomly oriented microscopic polar domains

During electrical polarization, the material elongates permanently in the direction of the poling field (polar axis) and contracts in the transverse direction. When voltage is subsequently applied to the poled material in the same direction as the poling voltage, the piece experiences further elongation along the polar axis and transverse contraction as stipulated by Poisson's ratio. When the voltage is removed, the piece reverts to its original pole dimensions. When voltage is applied opposite to the poled direction (**depoling direction**), the piece contracts along the polar axis and expands in the transverse direction. Again, it reverts to its original poled dimensions after removing the voltage. These distortions are illustrated in *Figure 2* for a rectangularly shaped piece. If too large a voltage is applied in the depoling direction, the original polarization will be degraded (partially or fully **depolarized**). Or, the electric dipoles may be partially or completely flipped around  $180^\circ$ , causing the piece to be repoled in the opposite direction. The maximum depoling field a piece can withstand without experiencing depolarization is its **coercive field**,  $E_c$ .

When stress is applied along the poling axis, an electric field arises within the body which tends to oppose the force acting upon it. Compressive stress generates an electric field with the same orientation as the original poling field, trying to induce the piece to elongate in opposition to the compressive forces. The piece reverts to its original poled dimensions after removing the stress. Tensile stress generates an electric field with an orientation opposite to that of the original poling field. These electric fields are illustrated in *Figure 2* for a rectangularly shaped piece.



**Figure 2.** Physical deformation of a rectangular piezoelectric body under the influence of an applied electric field

## 1.2 Piezoelectric and Material Properties of Piezoceramic

### 1.2.1 TERMINOLOGY AND RELATIONS

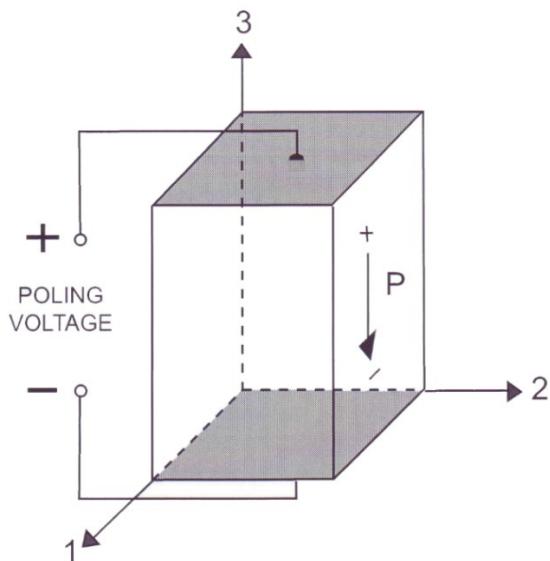
This section describes the terminology commonly used in the discussion of piezoceramics and notes the fundamental relationships useful in motor applications. It also defines commonly used notations and sign conventions.

Relationships between applied electric fields and the resultant responses depend upon the piezoelectric properties of the ceramic, the geometry of the piece, and the direction of electrical excitation. The properties of piezoceramic vary as a function of both strain and temperature. It should be recognized that the data commonly presented represents values measured at very low levels at  $\sim 20^{\circ}\text{C}$ .

Directions are identified using the three axes, labeled 1, 2 and 3, shown in *Figure 3*. The “polar” or 3-axis is chosen parallel to the direction of polarization.

In order to conveniently express all the relevant terms, 2<sup>nd</sup> order tensors such as stress and strain are expressed as 6x1 matrices with only the unique terms listed, as demonstrated by the stress terms in Eq. (1). This allows 3<sup>rd</sup> and 4<sup>th</sup> order tensors to be expressed as 6x3 and 6x6 matrices.

$$T_1 = \sigma_{11}, T_2 = \sigma_{22}, T_3 = \sigma_{33}, T_4 = \sigma_{23}, T_5 = \sigma_{13}, T_6 = \sigma_{12} \quad (1)$$



**Figure 3.** Definition of axes for a rectangular piezoelectric body showing the polar or 3-axis, and the transverse or 1 and 2-axis

The **polarization vector**, established during manufacture by a high DC voltage applied to the electrodes, is represented by an arrow pointing from the positive to the negative poling electrode. This information is conveyed by a dot or stripe on the electrode surface held at high voltage during the poling process.

It is helpful to remember that the polarization arrow represents the force exerted on a positive charge by a positive potential field. Thus, it signifies the direction a positively charged particle would displace when influenced by a nearby positive charge. "Conventional" (positive) current flow moves in this direction. It should be kept in mind that electron flow moves in the opposite direction.

**Piezoelectric coefficients**, relating input parameters to output parameters, use double subscripts. The first subscript denotes the direction of the electric field E or dielectric displacement D, and the second subscript refers to the direction of mechanical stress T or strain S.

The **piezoelectric charge coefficient** is a 3<sup>rd</sup> order tensor that can be expressed as a 3x6 matrix that correlates the charge displaced unit area (with electrodes short circuited), associated with an applied stress, according to the relation:

$$D_i = d_{ij}T_j \quad (2)$$

where  $d_{ij}$  is the piezoelectric charge coefficient [Coulombs/Newton], D is the dielectric displacement [C/m<sup>2</sup>], and T is the applied stress [N/m<sup>2</sup>]. The first subscript gives the direction of the charge motion associated with the applied stress. The second subscript gives the direction of mechanical stress.  $D_{33}$  relates the ratio of charge motion along the 3-axis to the stress applied along the 3-axis, assuming the electrodes are shorted, and no other stresses are present.  $D_{31}$  relates the charge flow along the 3-axis to the stress along the 1-axis (or 2-axis) under similar conditions.

The **piezoelectric voltage coefficient**, g, is a 3 x 6 matrix and correlate the electric field, E, developed (with electrodes open circuited), associated with an applied stress, T, according to the relation:

$$E_i = -g_{ij}T_j \quad (3)$$

Tensile stress is positive, compressive stress is negative, and  $g_{ij}$  is expressed in units of volts / meter per Newton / meter<sup>2</sup>. The first subscript gives the direction of the electric field associated with the applied stress. The second subscript gives the direction of mechanical stress.  $g_{33}$  relates the ratio of the electric field developed along the 3-axis to the stress applied along the 3-axis, assuming the electrodes are open circuited, and no other stresses are present.  $g_{31}$  relates the ratio of the electric field along the 3-axis to the stress along the 1-axis (or 2-axis) under similar conditions.

Piezoelectric coefficients, used to relate input parameters to output parameters, use double subscripts. The **piezoelectric strain coefficients**,  $d_{ij}$ , correlate the strain produced by an applied electric field according to the relation:

$$S = d_{ij}E \quad (4)$$

Where  $d_{ij}$  is expressed in meters/volt. The first subscript gives the direction of the electric field associated with the applied voltage. The second subscript gives the direction of mechanical strain.  $d_{33}$  relates the ratio of the strain along the 3-axis to the electric field applied along the 3-axis, assuming the piece is free to distort in any direction.  $d_{31}$  relates the strain along the 1-axis (or 2-axis) to the excitation along the 3-axis under similar conditions.

The **coupling coefficient**,  $k$  (lower case), is an indication of the materials ability to convert electrical energy to mechanical energy. Specifically, the square of the coupling coefficient equals the ratio of mechanical energy output to the electrical energy input. It is more relevant to the maximum work output of solid ceramic devices than to bending elements because a practical bending element stores a portion of its mechanical energy in the mount and metal shim center layer. For a bending element  $k_{\text{effective}} \sim \frac{3}{4} k_{31}$ .

The **relative dielectric constant**,  $K$  (upper case), is the ratio of the permittivity of piezoceramic to that of empty space,  $\epsilon_0$  ( $\epsilon_0 = 8.854 \times 10^{-12}$  farads/meter).  $K_3$ , the relative dielectric constant between the poling electrodes, determines the capacitance of the piece according to the relationship,

$$C = \frac{K_3 \epsilon_0 A}{T} \quad (5)$$

Where  $A$  is the area of the surface electrode, and  $T$  is the thickness of the ceramic layer or layers between electrodes.

Certain piezoceramic material constants are written with superscripts to specify the experimental context in which the parameter is measured.

$T$  = Constant Stress (mechanically free)

$S$  = Constant Strain (mechanically clamped)

$E$  = Constant Electric Field (electrodes short-circuited)

$D$  = Constant Electric Displacement (electrodes open-circuited)

For example,  $K^T_3$ , is the dielectric constant measured across the poling electrodes of a mechanically free piece.

**Young's Modulus**, Y, the ratio of stress required to produce a unit of strain, describes the material stiffness of piezoceramic in units of newtons/square meter. Because mechanical stressing of the ceramic produces an electrical response opposing the resultant strain, the effective Young's Modulus with electrodes short circuited is lower than with the electrodes open circuited. Furthermore, the stiffness differs in the 3 direction from that of the 1 or 2 direction. This,  $Y^E_{11}$ , is the ratio of stress in the 1 direction to stain in the 1 direction with the electrodes shorted.

### **1.2.2 PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5A PIEZOCERAMIC**

The piezoelectric and material properties of PZT-5A piezoceramic are listed in *Table 1*.

**TABLE 1. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5A CERAMIC**

<b>PIEZOELECTRIC</b>			
Composition			Lead Zirconate Titanate, Navy Type-II
Material Designation			PZT-5A
Relative Dielectric Constant (@1KHz)	K <sub>T<sub>3</sub></sub>	1800	
	K <sub>T<sub>1</sub></sub>	1800	
Piezoelectric Strain Coefficient	d <sub>33</sub>	390 x 10 <sup>-12</sup>	Meters / Volt
	d <sub>31</sub>	-190 x 10 <sup>-12</sup>	Meters / Volt
	d <sub>15</sub>	~550 x 10 <sup>-12</sup>	Meters / Volt
Piezoelectric Voltage Coefficient	g <sub>33</sub>	24.0 x 10 <sup>-3</sup>	Volt Meters / Newton
	g <sub>31</sub>	-11.8 x 10 <sup>-3</sup>	Volt Meters / Newton
	g <sub>15</sub>	~26.0 x 10 <sup>-3</sup>	Volt Meters / Newton
Coupling Coefficient	k <sub>33</sub>	0.72	
	k <sub>31</sub>	0.32	
	k <sub>15</sub>	0.59	
Polarization Field	E <sub>p</sub>	2 x 10 <sup>6</sup>	Volts / Meter
Coercive Field (DC) (@ 60 Hz)	E <sub>c</sub>	5 x 10 <sup>5</sup>	Volts / Meter
		6 x 10 <sup>5</sup>	Volts / Meter
<b>MECHANICAL</b>			
Density	ρ	7750	Kg / Meter <sup>3</sup>
Elastic Modulus	Y <sub>E<sub>33</sub></sub>	4.9 x 10 <sup>10</sup>	Newtons / Meter <sup>2</sup>
	Y <sub>E<sub>11</sub></sub>	6.2 x 10 <sup>10</sup>	Newtons / Meter <sup>2</sup>
Poisson' Ratio	ν	0.31	
Compressive Strength		5.2 x 10 <sup>8</sup>	Newtons / Meter <sup>2</sup>
Tensile Strength (Static) (Dynamic)		7.5 x 10 <sup>7</sup>	Newtons / Meter <sup>2</sup>
		2.0 x 10 <sup>7</sup>	Newtons / Meter <sup>2</sup>
Mechanical Q		80	
<b>THERMAL</b>			
Curie Temperature		350	°C
Pyroelectric Coefficient		~420 x 10 <sup>-6</sup>	Coulombs / Meter <sup>2</sup> °C
Thermal Expansion Coefficient		~4 x 10 <sup>-6</sup>	Meters / Meter °C
Specific Heat	C <sub>p</sub>	440	Joules / Kg °C

### 1.2.3 MATERIAL PROPERTIES FOR PIEZOCERAMICS

The piezoelectric and other material properties of PZT-5A, PZT-5H and PZT-5J piezoceramic are listed in *Table 2*.

**TABLE 2. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5H, PZT-5A, AND PZT-5J PIEZOCERAMIC**

Property	Symbol	Units	Material Type		
			PZT-5H 3203HD	PZT-5A 3195HD	PZT-5J 3222HD
Dielectric Constant (1kHz)	K <sup>T</sup> <sub>3</sub>		3200	1900	2650
Dielectric Loss Factor (1kHz)	Tanδ <sub>e</sub>	%	2.0	0.02	0.02
Dielectric Constant (1kHz)	K <sup>T</sup> <sub>1</sub>			1600	2948
Clamped Dielectric Constant	K <sup>S</sup> <sub>3</sub>		1200	900	800
Density	ρ	g/cm <sup>3</sup>	7.87	7.95	7.90
Curie Point	T <sub>c</sub>	°C	225	350	270
Mechanical Quality Factor	Q <sub>m</sub>		30	80	80
Coercive Field (Measured < 1Hz)	E <sub>c</sub>	kV/cm	8.0	12.0	
Remanent Polarization	P <sub>r</sub>	μCoul/cm <sup>2</sup>	39.0	39.0	
	k <sub>p</sub>		0.75	0.68	0.72
	k <sub>33</sub>		0.75	0.72	0.74
Coupling Coefficients	k <sub>31</sub>		0.43	0.40	0.45
	k <sub>t</sub>		0.55	0.49	0.53
	k <sub>15</sub>		0.78	0.61	0.77
	d <sub>31</sub>	Coul/N x 10 <sup>-12</sup>	-320	-190	-270
Piezoelectric Charge (Displacement Coefficient)	d <sub>33</sub>		650	390	485
	d <sub>15</sub>	or m/V x 10 <sup>-12</sup>	1000	460	850
	g <sub>31</sub>		-9.5	-11.3	-11.5
Piezoelectric Voltage Coefficient (Voltage Coefficient)	g <sub>33</sub>	V · m/N x 10 <sup>-3</sup>	19.0	23.2	21.3
	g <sub>15</sub>		35.3	32.4	32.6
Frequency Constants Radial	N <sub>r</sub>	kHz · cm			191
Resonant Thickness	N <sub>tr</sub>	kHz · cm	202	211	205
Anti-Resonant Thickness	N <sub>ta</sub>	kHz · cm	236	236	235

**TABLE 2. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5H, PZT-5A, and PZT-5J PIEZOCERAMIC, CONTINUED**

Property	Symbol	Units	Material Type		
			PZT-5H 3203HD	PZT-5A 3195HD	PZT-5J 3222HD
Thermal Expansion (Perpendicular to Poling)	$\alpha$	ppm/ $^{\circ}$ C	3.5	3.0	
Specific Heat	$C_p$	J/kg $\cdot$ $^{\circ}$ C J/mol $\cdot$ $^{\circ}$ C	420 138	440 145	
Thermal Conductivity with Au Electrodes	$K_d$	W/cm $\cdot$ $^{\circ}$ C W/m $\cdot$ $^{\circ}$ K W/m $\cdot$ $^{\circ}$ K	1.9-2.3 1.2 1.45	1.9-2.3 1.2 1.45	
Poisson's Ratio	$\nu$		0.31	0.34	0.31
Elastic Constants Short Circuit	$S_{11}^E$ $S_{33}^E$ $S_{12}^E$ $S_{13}^E$ $S_{55}^E$		16.6 21.0 $\times 10^{-12}$ m $^2$ /N -4.8 52.4	15.1 18.6 -7.6 40.0 47.0	15.8 18.8 -5.0 -7.7 12.6
Elastic Constants Open Circuit	$S_{11}^D$ $S_{33}^D$ $S_{55}^D$		13.9 $\times 10^{-12}$ m $^2$ /N 8.8 20.5	12.7 9.0 25.1	12.6 8.5 19.1
Elastic Constants Short Circuit	$Y_{11}^E$ $Y_{33}^E$		6.2 $\times 10^{10}$ N/m $^2$ 4.9	6.6 5.4	6.4 5.3
Elastic Constants Open Circuit	$Y_{11}^D$ $Y_{33}^D$		7.0 $\times 10^{10}$ N/m $^2$ 11.0	7.9 11.1	7.9 11.7

# Piezoelectric Actuators

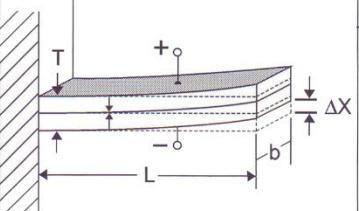
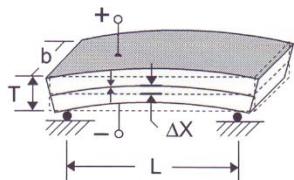
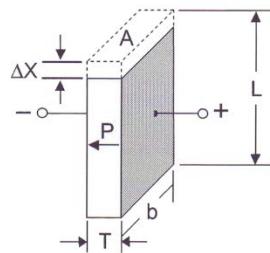
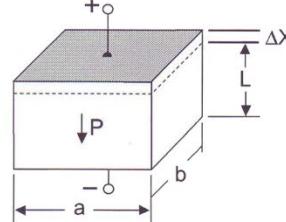
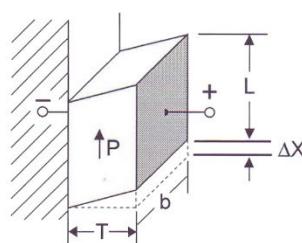
## 2. Designing Piezoelectric Actuators

### 2.1 The Spectrum of Piezoelectric Motor Transducers

Transducers which convert electrical energy to mechanical energy (i.e., motors) come in a wide range of shapes and sizes, each having their own characteristic force-displacement capabilities. Stiff (low compliance) transducers provide tremendous force but tiny motion. On the other hand, highly compliant transducers provide substantial motion but small force.

As a general-purpose reference guide, *Table 3* shows the spectrum of motor transducers commonly considered in piezoelectric applications. The equations for displacement, force, and resonant frequency, are based on linear relationships and low signal values of their piezoelectric strain coefficients.

**TABLE 3. SPECTRUM OF COMMON PIEZOELECTRIC TRANSDUCERS**

PIEZOELECTRIC CONFIGURATION	FREE DEFLECTION	BLOCKED FORCE	RESONANT FREQUENCY	GENERAL FEATURES
CANTILEVER BENDING MOTOR				
	$\frac{3 d_{31} L^2 E}{2 T}$	$\frac{3 d_{31} Y b T^2 E}{8 L}$	$\frac{.16 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$	\$1 - \$100 10 - 500 Hz 10 - 500 grams 5 mm
SIMPLE BENDING MOTOR				
	$\frac{3 d_{31} L^2 E}{8 T}$	$\frac{3 d_{31} Y b T^2 E}{2 L}$	$\frac{.48 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$	
TRANSVERSE ( D31 ) CONTRACTION MOTOR				
	$d_{31} L E$	$d_{31} Y A E$ where $A = b T$	$\frac{1}{2 L} \sqrt{\frac{Y_{11}}{\rho}}$	INCREASING COST INCREASING RESONANT FREQUENCY INCREASING FORCE INCREASING DISPLACEMENT
LONGITUDINAL ( D33 ) EXTENSION MOTOR				
	$d_{33} L E$	$d_{33} Y A E$ where $A = a b$	$\frac{1}{2 L} \sqrt{\frac{Y_{33}}{\rho}}$	$\mu M$ \$100 1 MHz $10^3$ Kg
SHEAR MODE MOTOR				
	$d_{15} T E$	$d_{15} G A E$ where $A = b L$	$\frac{1}{2 T} \sqrt{\frac{Y_{55}}{\rho}}$	

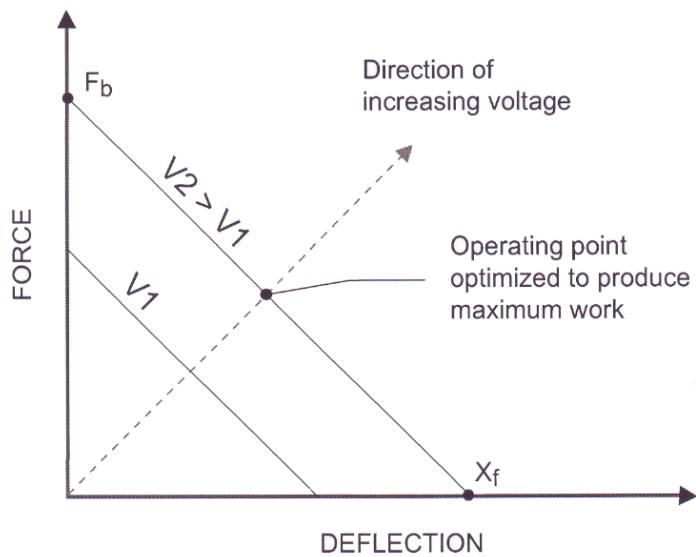
## 2.2 Basic Engineering Considerations

## 2.2.1 PERFORMANCE CONSIDERATIONS

### DEFLECTION AND FORCE

Piezoelectric actuators are usually specified in terms of their free deflection and blocked force. Free deflection,  $X_f$ , refers to displacement attained at the maximum recommended voltage level when the actuator is completely free to move and is not asked to exert any force. Blocked force,  $F_b$ , refers to the force exerted at the maximum recommended voltage level when the actuator is totally blocked and not allowed to move. Deflection is at a maximum when the force is zero, and force is at a maximum when the deflection is zero. All other values of simultaneous displacement and force are determined by a line drawn between these points on a force versus deflection line, as shown in *Figure 4*.

Generally, a piezo motor must move a specified amount and exert a specified force, which determines its operating point on the force vs. deflection line. Work is maximized when the deflection performed permits one half the blocked force to be developed. This occurs when the deflection equals one half the free deflection.



*Figure 4. The force versus displacement diagram for a piezoelectric motor element*

For cantilevered bending motors,  $X_f$  is determined by measuring the tip deflection after energizing, and  $F_b$  is measured by holding a force gauge in position against the tip during energization.

## RESPONSE TIME

For “on demand” actuator applications, it is useful to know how fast an actuator will operate. The fundamental resonate frequency,  $F_r$ , is a guidepost towards answering this question. A piezo actuator can follow a sinusoidal signal up to its resonant frequency. Beyond this point, its inertia inhibits it from keeping up with electrical excitation. From a practical standpoint, it's a good idea to limit operation to  $\sim 3/4 F_r$ . The response time,  $t_r$ , for an actuator to travel through its full range, for bipolar operation (from 0 to positive voltage, then to negative voltage, and back to 0) is  $\frac{1}{4}$  cycle. Thus,

$$t_r(\text{bipolar operation}) = \frac{1}{4 \times 0.75F_r} \quad (6)$$

For example, an actuator driven by a bipolar power supply, and having a resonant frequency of 500 Hertz, has a response time of 0.67 milliseconds. Any mass added to the end of the actuator will lengthen the response time.

## STABILITY

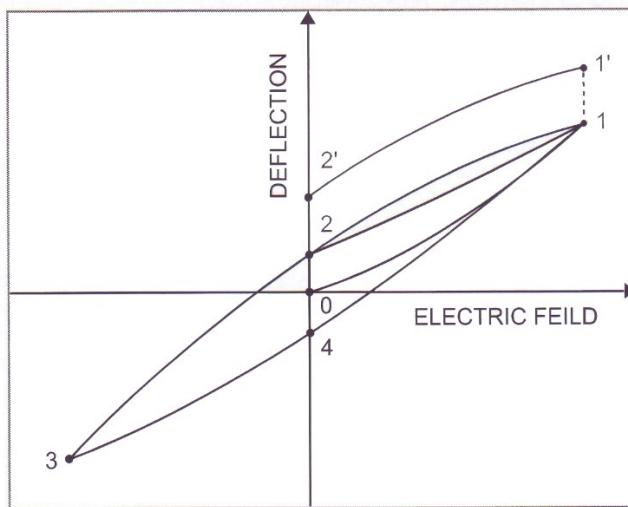
For those applications which require the piezo actuator to hold its position accurately for a long time, an understanding of hysteresis and creep is important. For dynamic applications, where position is changing continuously, these issues are less critical.

**Hysteresis:** When a polycrystalline piezoelectric body is deformed, part of the mechanical energy is stored as elastic strain energy, and part is dissipated as heat during small internal sliding events. Hysteresis appears as an offset between the position path traveled during the application and removal of the excitation field. The size of the offset depends on the field level, the cycle time, and the materials used. It is often specified as a percentage of the total deflection achieved, and ranges from .1% to 10%. Hysteresis is a consideration wherever high frequencies ( $> 1 \text{ kHz}$ ) are concerned because of heat accumulation within the piece after each cycle of operation. This is especially the case for low voltage piezo stacks made of high strain material. This can lead to excessive temperatures if care is not taken in the design. Other portions of the piezoelectric system also contribute to losses, such as adhesive bonds, mounts, and attachments. These show up in ultrasonic designs.

**Creep (or Drift):** Creep is the result of time dependent plastic deformation. Usually it is not a concern under oscillating drive conditions, however any high-level DC voltage application should pay close attention to creep. After voltage has been applied to a piezoelectric actuator, the deflection increases with time. For moderate drive levels ( $> 10 \text{ volts/mil}$ ), the creep rate decreases with time. However, as the drive level increases ( $> 20 \text{ volts/mil}$ ), the creep rate accelerates. Upon removal of the drive voltage, the piece retains a set, a portion of which is irreversible even after a long relaxation period. At high drive levels ( $> 30 \text{ volts/mil}$ ), creep may proceed to the point where the piece finally cracks. Increased temperature exacerbates creep.

When excessive creep is encountered, certain strategies may be required, such as stops to limit excessive travel, or closed loop feedback control to lock in a desired position.

*Figure 5* demonstrates both typical hysteresis and creep behavior of a blending element. When a piece has been at rest for some length of time (~1 day), it will reside at its equilibrium position, 0. Upon initial energization, it will move to position 1. After de-energization it will go to position 2. If it is allowed to rest for a sufficient length of time (~1 day) it will revert back to position 0 again. However, if it is reenergized immediately, it will follow path 2-1. When the piece is energized and left on, it will creep along the path 1-1', and come to rest at position 2' after de-energization. A piece operating in AC mode will follow path 1-2-3-4-1.



*Figure 5. Typical hysteresis and creep behavior of a single blending element*

## 2.2.2 MECHANICAL CONSIDERATIONS

### MOUNTING

An ideal mount permits the normal distortion of the entire active portion of the motor element, while at the same time preventing motion in certain directions at the mounting point or points. Generally piezo motors are either bonded, clamped, or spring loaded to their mounting points. Mounts introduce some mechanical damping into the system since some of the energy from the motor distorts the mount itself. This may or may not be desirable.

### RESONANCE

Mechanical resonance is a manifestation of the trading back and forth of kinetic energy (moving mass) and potential energy (elasticity) in an oscillating body. At certain frequencies, known as resonances, the amount of stored energy becomes very large compared to the excitation energy.

This phenomenon can be useful for achieving large deflections at low voltages, and for obtaining high efficiency. Piezo fans and ultrasonic devices utilize this property. Because of the high amplitudes exhibited at resonance, care must be taken to overstrain and crack the actuator.

The resonant frequencies of a piezo motor depend on its dimensions, material properties, and the manner of mounting. The cantilever beam element has the lowest fundamental (first) resonant frequency per unit length of all configurations and mounting schemes. Equations for determining the fundamental resonant frequencies for several motor configurations are shown on *Table 3* on page 16. These frequencies apply to unloaded elements only. Attachments to the element will add to the resonating mass and lower the resonant frequency.

### **OPERATING FREQUENCY BAND**

Up to resonant frequency, the deflection of a piezoelectric bending element is nearly independent of frequency and proportional to the operating voltage. Around the resonant frequency, deflection rises rapidly to a multiple of its normal value. The amplitude and narrowness of the resonance peak depend on the internal and external losses acting on the actuator. Above resonance, deflection decreases steadily with the square of the frequency. First resonance marks the limit of the usable frequency band for quasi-static actuators. For resonant applications, the useable frequency range is limited to a small band around the useful resonant modes.

### **STRENGTH LIMITATIONS**

Piezoceramic is very strong in compression but weak in tension. Bending elements always have one side in compression and the other side in tension, where the magnitude of stress increases linearly from the midplane to the outside surface. Therefore, the element is always limited by the maximum recommended tensile strength, generally considered to be in the range of  $20\text{-}35 \times 10^6$  Newtons/meters<sup>2</sup>. From a strain point of view, the ceramic should not be allowed to strain more than  $500 \times 10^{-6}$  meter/meter.

### **2.2.3 DRIVE CIRCUIT CONSIDERATIONS**

#### **QUASI-STATIC OPERATION**

A piezoelectric actuator operating below its fundamental resonance can be treated as a capacitive load. The circuit must supply charge to cause a motion and must withdraw charge to cause a retraction (i.e., charge applied to the device does not bleed off internally). When held motionless in any position, piezoelectric actuators draw negligible current, typically much less than a microamp.

#### **NEAR RESONANCE OPERATION**

A piezoelectric actuator operating near resonance can be modeled as a capacitor (having a value equal to the transducer capacitance) with a resistor in parallel (typically 10 to 100 ohms). The power dissipated by this resistance represents the work which the actuator does on its

environment. The drive circuit must have sufficient current capacity to maintain the desired voltage on the resistor.

### CHARGE/DISCHARGE PROTECTION

Instantaneous charging or discharging of piezoelectric actuators causes acoustic shockwaves within the piezoceramic which can lead to localized stress concentrations and fractures. Therefore, the peak current to any actuator must be limited. One simple method places a protection resistor in series with the actuator, the value of which can be estimated using the following relation:

$$5R_p C \geq \frac{4}{F_r} \quad (7)$$

For a series operated cantilevered bending element, substituting for  $C$  and  $F_r$ :

$$R_p = \left( \frac{5L^2}{\varepsilon_0 K_3 b} \right) \sqrt{\frac{\rho}{Y_{11}}} \quad (8)$$

This essentially limits operation to a frequency region below the fundamental resonance.

### OUTPUT STAGE PROTECTION

Piezoelectric bending elements can generate high voltages (>100 volts) under external vibration, shock, or temperature shifts. If these conditions are expected, the drive circuitry of the output stage must be protected against transient voltages of all polarities.

### ELECTRICAL ISOLATION

The outer electrode surfaces of certain motor elements are electrically "live" in many configurations. For product or experimental safety, consideration should be given to insulating or shielding the electrodes, mount, and power take-off sections of the motor element.

### ELECTRICAL BREAKDOWN

The highest value of applied electric field is determined by electrical breakdown occurring either through the body of the piezoceramic sheet or over its edges. Piece of dust and debris adhering to edges can initialize edge discharge at fields as low as 400-800 volts/mm. However, the discharge arc vaporizes the debris, thereby cleaning itself. A number of these edge-debris arcs may occur during the initial energization or the bending motor, but they will not occur again. Continuous breakdown occurs around 3,000-4,000 volts/mm, usually at impurity or defect regions within the bulk of the material. This can lead to a short circuit across the sheet due to vapor

deposition of electrode or shim material near the site of arcing. A current limiting resistor or in-line fuse is recommended when excessive electric fields are used.

## ELECTRICAL LOSSES

The bulk resistivity of piezoceramic is  $\sim 10^{12} \Omega\text{-cm}$ . Therefore, electrical losses are minimal under static or low frequency operation. However, dielectric losses are significant under cycled operation and can lead to heating under high frequency/high power operations. The loss tangent, the ratio of series resistance to series reactance, for PZT-5A is  $\sim 0.015$ .

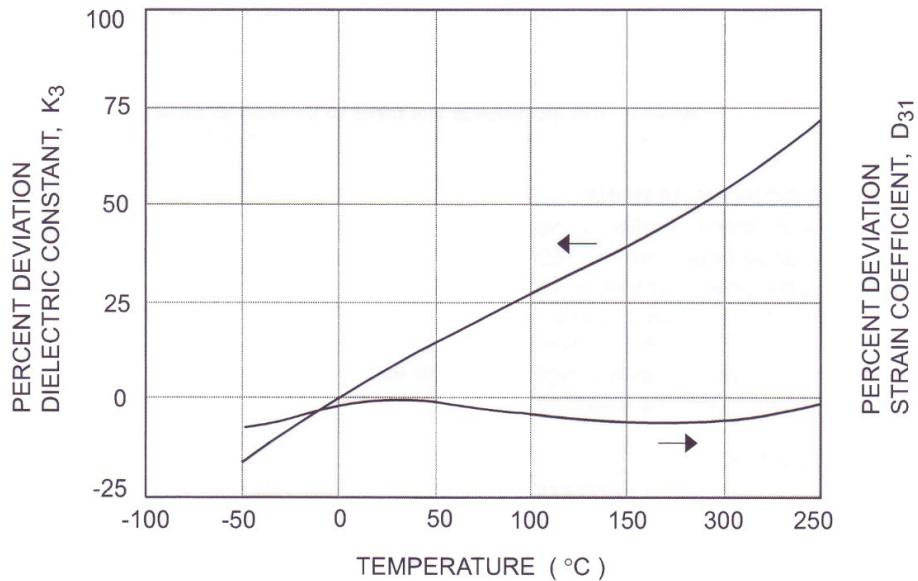
### 2.2.4 THERMAL CONSIDERATIONS

#### CURIE TEMPERATURE

For each piezoceramic material there is a critical temperature, known as its Curie point, which represents its maximum operating temperature before suffering a permanent and complete loss of piezoelectric activity. In practice, the operating temperature must be limited to some value substantially below the Curie point because at elevated temperatures depoling is greatly facilitated, the aging process is accelerated, electrical and mechanical losses increase, and the maximum safe stress is reduced. As a rule of thumb, a temperature equal to one half the Curie temperature is considered the maximum safe operating temperature.

#### PIEZOELECTRIC AND MATERIAL PROPERTIES AS A FUNCTION OF TEMPERATURE

Piezoelectric properties are strongly temperature dependent, and thermal dependence varies markedly from one material to the next. *Figure 6* demonstrates the temperature dependence of  $K_3$  and  $d_{31}$  for PZT-5A.



**Figure 6.** The relative dielectric constant and transverse strain coefficient as a function of temperature

## PYROELECTRIC EFFECTS

An electric field is induced on the electrodes of a piezo motor when it is exposed to a thermal change. The induced field is

$$E = \frac{\alpha(\Delta T)}{\epsilon_0 K_3} \quad (9)$$

Where  $\alpha$  is the pyroelectric coefficient in units of coulombs /m<sup>2</sup> °C,  $\Delta T$  is the temperature change,  $K_3$  is the relative dielectric constant in the poling direction, and  $\epsilon_0$  is the permittivity of free space. It should be noted that a depoling voltage will be developed across a layer of piezoceramic when the temperature drops. This can happen during processing, testing, and normal usage. If a temperature drop is sufficient, over a time interval which is too short to allow charge to leak away, a voltage greater than the coercive field can result. This can degrade the original polarization causing reduced performance. It is always good practice to short circuit the electrodes of any piezo device during a cool down procedure.

## THERMAL EXPANSION

One must account for thermal displacements over the temperature range anticipated. *The actuator must be capable of compensating for thermal displacements and still have a useful motion range.*

In addition, differential thermal expansions of adjacent assembly parts will cause moments and warping. The standard bending motor element has a symmetrical construction. Distortion due to thermal excursion should be negligible. However, care should be taken in the design of the mount or any other attachments not to introduce thermal distortion. This is facilitated by properly matching the thermal expansion coefficients of adjacent members to that of the ceramic element. The coefficient of thermal expansion of PZT-5A is  $\sim 4 \text{ } \mu\text{m/m } ^\circ\text{C}$ .

### **CRYOGENIC OPERATION**

The low signal values of the strain coefficients for operation at 4.2 K are *reduced by a factor of 5-7 times*. The value of the coercive field increases substantially, however. Cycling the transducer between these temperature extremes does not seem to affect them adversely.

### **2.2.5 VACUUM CONSIDERATIONS**

Because piezo actuators are solid state devices, they lend themselves to high vacuum operation. However, several issues should be understood. First, voltage should not be applied to the electrodes during the vacuum pump-down process because of the low insulation resistance of air and nitrogen between the range of 10 to 0.1 torr. Arcing between electrodes is possible within this pressure range. Secondly, outgassing of the part is possible depending on construction materials. Motors to be used in high vacuum environments should have small cross sections of outgassing materials (primarily the adhesive). If bake-out is necessary, the transducer will have to be built to withstand solvents and bake-out temperatures.

### **2.2.6 ELECTRIC FIELD CONSIDERATIONS**

As mentioned earlier, under adverse conditions piezoelectric properties may degrade, vanish completely, or be flipped around  $180^\circ$ . A strong electric field applied to a piezoceramic in a sense opposite to the original poling voltage will tend to cause depoling. The field strength necessary to initiate depoling depends on the material, duration of application, and temperature, but is typically in the range of 475 volts/mm at  $20^\circ\text{C}$  for PZT-5A under static conditions. Alternating fields may also degrade the piezoceramic, but the peak field level is higher because the duration is shorter before the field is reversed. A peak field of 600 volts/mm may be tolerated for 60 Hz operation at  $20^\circ\text{C}$ .

### **2.2.7 STRESS CONSIDERATIONS**

When the mechanical stress on a piezoceramic element becomes too high, again there is a danger of degrading the piezoelectric properties. Generally, compressive or hydrostatic stress levels of  $-50 \times 10^6 \text{ N/m}^2$  are required to degrade PZT-5A if no other degrading influences are present.

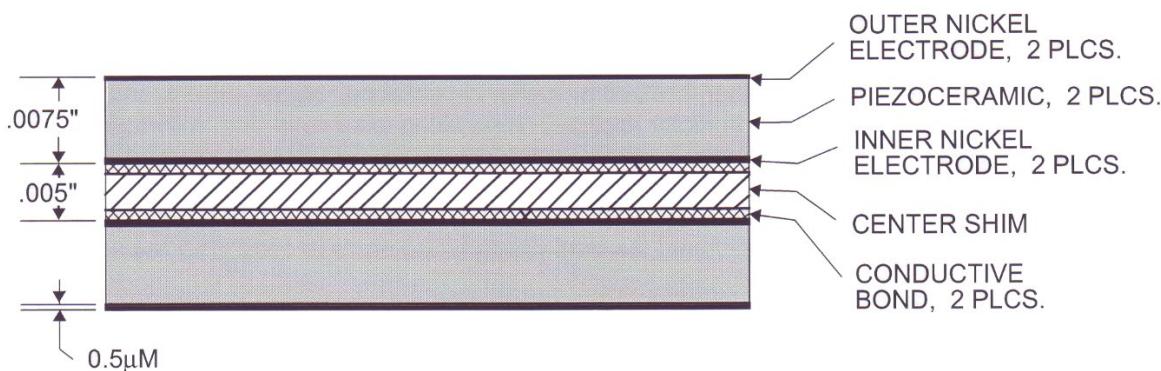
## 2.2.8 AGING

Piezoelectric properties change gradually with time. The changes tend to be logarithmic with time after the original polarization. Therefore, the rate reduces rapidly with time. Aging depends on the ceramic material, manufacturing process, and ambient conditions such as temperature, vibration or shock. Pieces may be heated for a specific time to accelerate the aging process.

### 3. Piezoelectric Bending Motors

#### 3.1 Principles of Operation

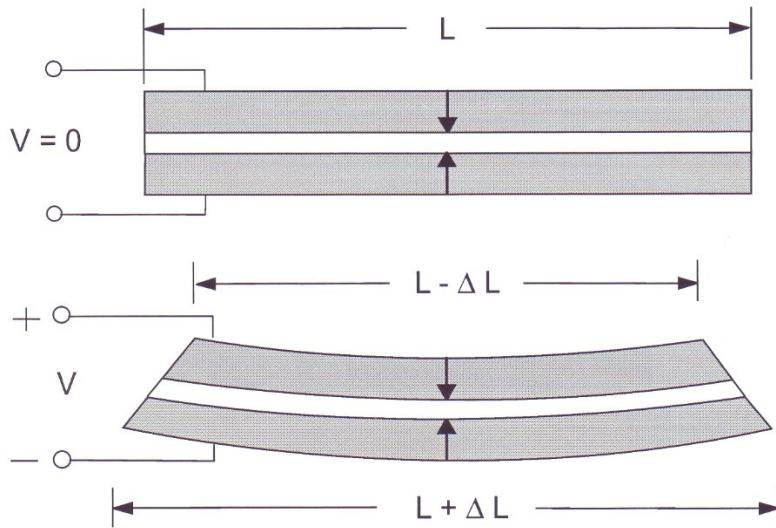
The most common type of piezoelectric bending motor is composed of two layers of piezoceramic bonded to a thin metal shim sandwiched in the middle. The construction and typical dimensions of the 2-layer elements in this kit are shown in *Figure 7*.



*Figure 7. Construction of laminated 2-layer motor element*

The application of voltage to the element is analogous to the application of heat to a bimetallic strip. The voltage across the bender element forces one layer to expand, while the other contracts, as depicted in *Figure 8*. The result of these physical changes is a strong curvature and large deflection at the tip when the other end is clamped. The tip deflection is much greater than the change in length of either ceramic layer.

Bending motors exhibit unique properties. They may be energized proportionately and be held in the energized position with negligible consumption of energy and generation of heat. They may be operated over billions of cycles without wear or deterioration. Their low profile allows their use in very restricted locations.



**Figure 8.** The curvature of a bending motor is due to the expansion of one layer and contraction of the other

### 3.2 Standard Polarization Configurations

There are three standard polarization configurations for the two-layer bender construction: series, parallel, and single plate (monomorph). These are illustrated in *Figure 9*. Both the series and parallel elements are limited to electric fields below the coercive field ( $\sim 475$  V/mm for PZT-5A).

#### 3.2.1 SERIES OPERATION

The bender poled for series operation is the simplest and most economical. It requires two connections to the outside surfaces of the piezoceramic layers which are electrically in series. It is characterized by a lower capacitance, lower current, and higher voltage.

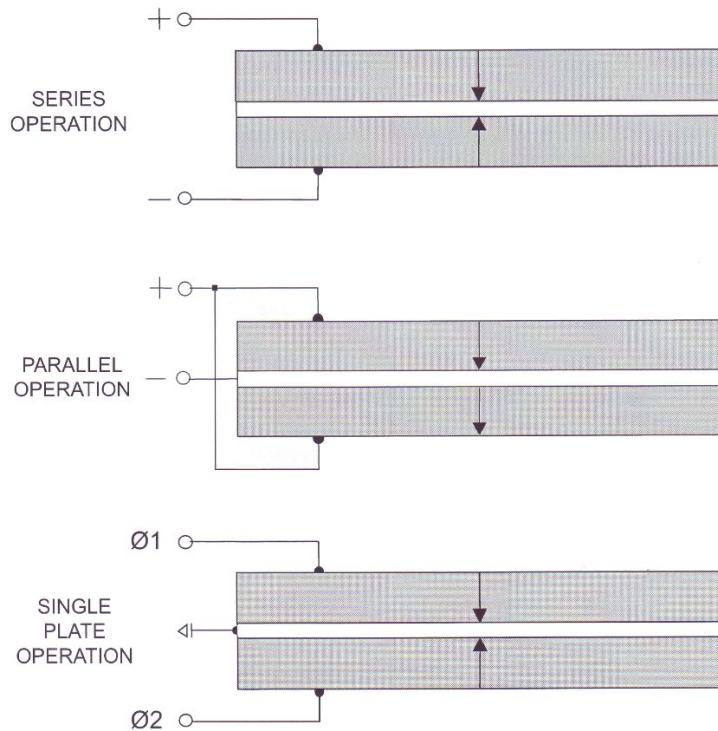
#### 3.2.2 PARALLEL OPERATION

The bender poled for parallel operation requires three electrical connections. The third connection accesses the center shim of the bender, requiring an extra manufacturing step and therefore a higher cost. Voltage is applied across the individual layers. The advantage of the parallel bender is that its deflection per volt is twice that of a series bender. However, the maximum deflection is the same for both. The parallel bender is characterized by higher capacitance, higher current, and lower voltage.

#### 3.2.3 SINGLE PLATE OPERATION

In motor applications requiring extra deflection, a third alternative is available. In this case a bender poled for series operation is used with three electrical connections. Voltage is applied to either layer of piezoceramic in the direction of polarization. Usually only one side is energized at

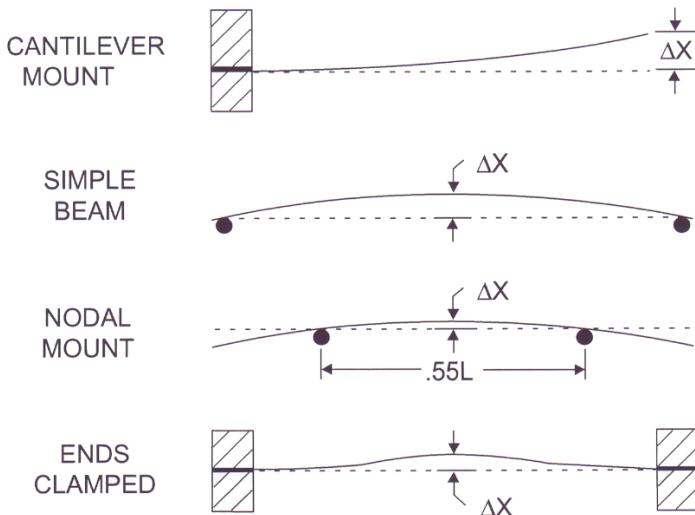
a time. The excitation field may exceed the coercive field since there is no concern for depolarization. Fields as high as 1,500-2,000 volts/mm (the level where arcing starts to occur near the edges) may be applied.



**Figure 9.** The three standard polarization configurations for 2-layer bending motors

### 3.3 Standard Mounts for Bending Motors

Standard mounts for bending motors are illustrated in *Figure 10* and fall into two general categories. The first category has power taken off at one end and is mounted at the other. Known as the cantilever mount, it provides maximum compliance and deflection. The second category has power taken off at the center and is mounted at the ends. The simple beam mount allows the ends to move in and out as well as rotate but fixes their vertical position. Compared to the cantilever mount, the simple beam mount produces reduced deflections, increased forces, and increased frequency. For high frequency-resonant applications, power dissipation at the mounts can be minimized by using nodal mounts. The nodes are evenly spaced, .55L apart, where L is the length of the beam. The beam may also be rigidly clamped at both ends, although this results in a significant proportion of the beam being constrained.



*Figure 10. Standard mounting techniques for bending motors*

### 3.4 Bending Motor Equations

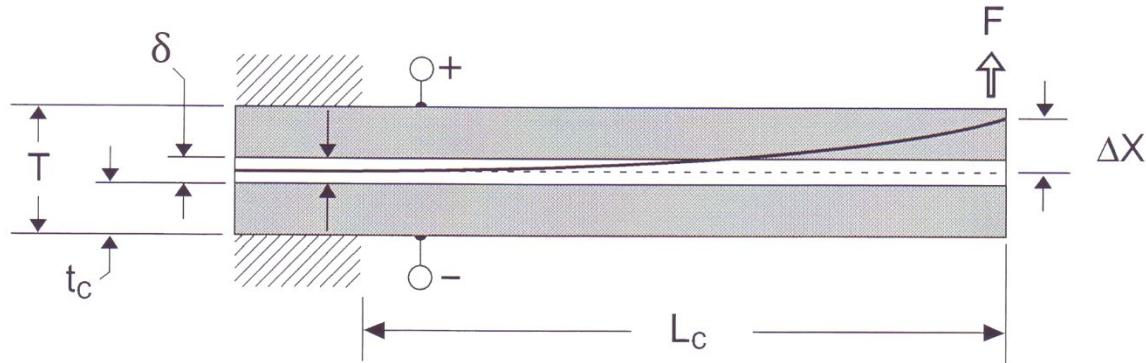
The “Bending Motor Equations” describe non-linear behavior and take into account the center shim. They can be used to estimate the motion and blocking force of 2-layer bending elements based on their dimensions and material property values. The relations below were developed by C.P. Germano of Vernitron Piezoelectric, with minor modifications by PIEZO.COM.

#### Definition of terms:

- 
- L = Total Length of the bending motor (m)
  - $L_c$  = Cantilever length (m)
  - T = Total thickness of bending motor (m)
  - $\delta$  = Thickness of center shim and adhesive layers (m)
  - $t_c$  = Thickness of a single layer of piezoceramic (m)
  - b = Width of bending motor (m)
  - $d_{31}$  = Piezoelectric transverse strain coefficient (m/V)
  - E = Electric field strength (V/m)
  - $E \text{ (series operation)} = \text{Voltage} / 2t_c$
  - $E \text{ (parallel operations)} = \text{Voltage} / t_c$
  - $Y$  = Young's modulus of elasticity (N/m<sup>2</sup>)

- $K$  = Relative dielectric constant of piezoceramic  
 $\epsilon_0$  = Permittivity of free space ( $8.85 \times 10^{-12}$  Farads/Meter)  
 $\rho$  = Average density of bending motor  
 $\beta$  = A non-linearity constant related to the electric field strength  
 $\gamma$  = A non-linearity constant related to the electric field strength

### 3.4.1 BENDING MOTOR MOUNTED AS A CANTILEVER



**Figure 11.** Terminology used for cantilevered bending motor equations

### FREE DEFLECTION

The free deflection,  $X_f$ , is usually twice the required operating deflection.

$$X_f = 3\beta d_{31} \left( \frac{L_c^2}{T^2} \right) \left( 1 + \frac{\delta}{T} \right) t_c E \quad (10)$$

where  $0 < \delta < t_c$

### BLOCKED FORCE

The blocked force,  $F_b$ , is usually twice the required operating force.

$$F_b = \left(\frac{3}{4}\right) \gamma Y_{11} d_{31} \left(\frac{bT}{L_c}\right) \left(1 + \frac{\delta}{T}\right) t_c E \quad (11)$$

where  $0 < \delta \leq t_c$

## RESONANT FREQUENCY

$$F_r = \left(\frac{16T}{L_c^2}\right) \sqrt{\frac{Y_{11}}{\rho}} \quad (12)$$

## MAXIMUM SURFACE STRAIN

$\sim 500 \times 10^{-6}$  is the maximum recommended strain limit in tension.

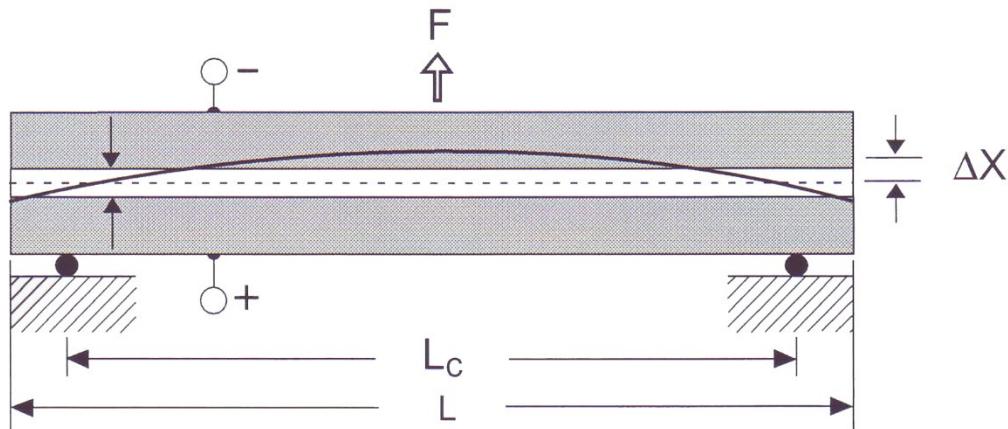
$$S = \frac{X_f T}{L_c^2} \quad (13)$$

## CAPACITANCE

$$C (\text{series operation}) = K_3 \varepsilon_0 L b / 2t_c \quad (14)$$

$$C (\text{parallel operation}) = 4 \times C (\text{series operation}) \quad (15)$$

### 3.4.2 BENDING MOTOR MOUNTED AS A SIMPLE BEAM



**Figure 12.** Terminology used for simple beam bending motor equations

Free Deflection:                    Multiply the cantilever  $X_f$  by  $\frac{1}{4}$

Blocked Force:                    Multiply the cantilever  $F_b$  by 4

Resonant Frequency:            Multiply the cantilever  $F_r$  by 8

Surface Strain:                    Multiply the cantilever  $S$  by 4

Capacitance:                    Same as cantilever for both series and parallel operation

# Piezoelectric Generators

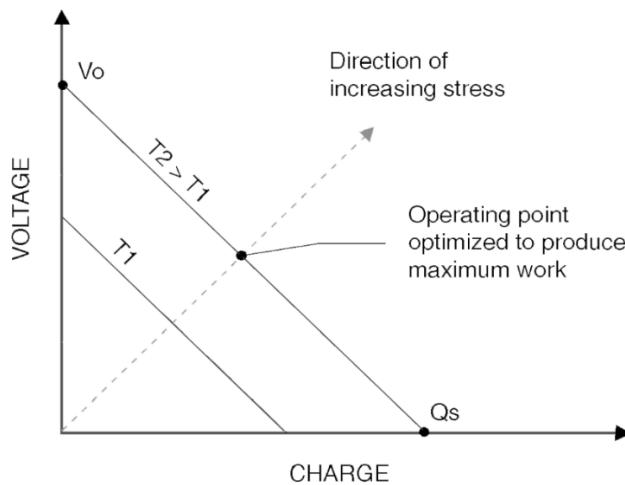
## 4. Designing Piezoelectric Generators

### 4.1 Basic Engineering Considerations

#### 4.1.1 ELECTRICAL OUTPUTS

##### CHARGE AND VOLTAGE

Piezoelectric generators are usually specified in terms of their short-circuit charge and open-circuit voltage. Short-circuit charge,  $Q_s$ , refers to the total charge developed, at the maximum recommended stress level, when the charge is completely free to travel from one electrode to the other, and is not asked to build up any voltage. Open-circuit voltage,  $V_o$ , refers to the voltage developed, at the maximum recommended stress level, when charge is prohibited from traveling from one electrode to the other. Charge is at a maximum when the voltage is zero, and voltage is at a maximum when the charge transfer is zero. All other values of simultaneous charge and voltage levels are determined by a line drawn between these points on a voltage versus charge line, as shown in *Figure 13*. Generally, a piezo generator must move a specified amount of charge and supply a specified voltage, which determines its operating point on the voltage vs. charge line. Work is maximized when the charge moved permits one half the open circuit voltage to be developed. This occurs when the charge equals one half the short-circuit charge.



*Figure 13. The voltage versus charge diagram for a piezoelectric generator element.*

#### 4.1.2 MECHANICAL INPUTS

Mechanical inputs can be described as either forces or displacements acting on a specific point or area of the generator body. These forces may be either static or dynamic, low power (typically

sensing) or high power (typically generating). If the force is oscillating and continuous, then the generator may be driven either at resonance or off-resonance (typically below resonance)

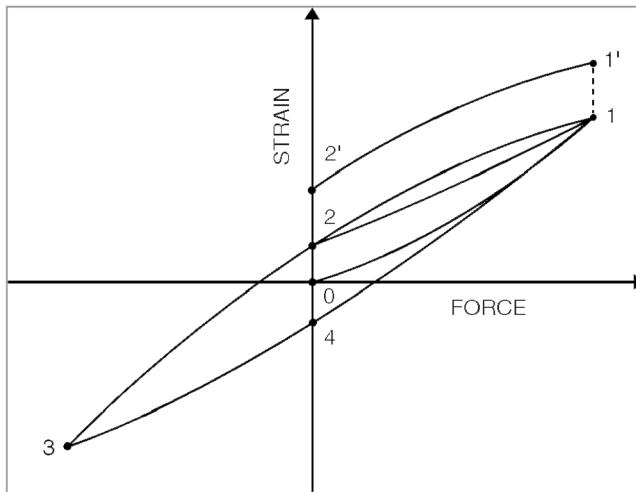
## STATIC INPUT FORCES OR DISPLACEMENTS

For long duration static forces acting on a piezo generator, such as a static pressure measurement, an understanding of creep, hysteresis and electrical leakage is useful.

**Creep & Relaxation:** Creep and relaxation are time dependent plastic phenomena resulting from piezoceramic grains slipping within the polycrystalline material upon application or removal of a load. When a constant load is applied and maintained on a piezo body, it will initially deform and then continue to deform (creep) over a period of time. There are three levels of creep: transient creep occurring immediately after load application; steady state creep characterized by a decreasing creep rate; and accelerating creep. At high drive levels, accelerated creep may proceed to the point where the piece finally cracks. Creep rates vary based on load level, temperature, and time. Upon removal of the load, the piece may slowly relax to its original equilibrium condition or retain a set.

**Mechanical Hysteresis:** Hysteresis is a lagging of strain values during stress cycling. When a polycrystalline piezoelectric body is deformed, part of the input energy is stored as elastic strain energy, and part is dissipated as heat due to small internal slippage mechanisms. Hysteresis appears as an offset in the strain level between the application and removal portions of a stress load. The size of the offset depends on the force level, cycle time, temperature, and materials used. For low stress levels encountered in small signal sensing, hysteresis is inconsequential, but for moderate and high stress levels it may be significant. Hysteresis is described as a percentage of the total strain and ranges around 15% for high stress levels. Voltage or charge production, which is strain dependent, is influenced by the mechanical hysteresis behavior. Hysteresis leads to non-linearity in transducer output.

*Figure 14* demonstrates the typical mechanical hysteresis and creep behavior of a piezoelectric element such as a bender. Imagine a force applied to the tip of a cantilevered bending element. When the element has been at rest for some length of time (~1 day), it will reside at its equilibrium position, 0. Upon initial energization, the tip will move to position 1. After de-energization it will go to position 2. If it is allowed to rest for a sufficient length of time (~1 day) it will revert back to position 0 again. However, if it is forced again immediately, it will follow path 2-1. If the force is left on, it will creep along the path 1-1', and come to rest at position 2' after de-energization. A piece experiencing a full bipolar cycle will follow path 1-2-3-4-1. The size of the loop is time dependent and the area inside the loop represents the energy dissipation occurring during the cycle.



**Figure 14.** Typical mechanical hysteresis and creep behavior of a piezoelectric element

**Electrical Leakage:** After charge has been established on the electrodes of a piezo generator, it will immediately begin to leak away even though the insulation resistance is large. It will leak through the bulk of the piezo slab according to the relation for bulk resistance:

$$R = \frac{\rho t}{A} \quad (16)$$

where  $\rho$  is the bulk resistivity of the piezo material in ohm-meters,  $t$  is the thickness between electrodes, and  $A$  is the electrode area. The value of resistivity for PZT-5A is  $\sim 10^{10}$  ohm-meters. Charge will travel along surfaces and over edges from one electrode to the other. Surface contaminants exacerbate this problem. Charge also drains through the input circuit. Lastly, charge may drain through the bleed resistor often placed across the electrodes to limit voltage swings due to pyroelectric (thermal) sources, triboelectric (surface friction) sources, and transient circuit currents. Eventually, the electrostatic charge will leak back to zero.

Overall, the behavior described above indicates why it is so difficult to design statically driven piezo sensing devices and why only dynamic force is generally measured.

## DYNAMIC INPUT FORCES OR DISPLACEMENTS

Piezo is much more friendly to dynamic applications. Dynamic inputs may either be pulsed or continuous.

**Pulsed Input Forces:** For a short duration transient force, issues of creep and electrical leakage are insignificant since there is insufficient time for their behavior to occur. However, hysteresis still applies.

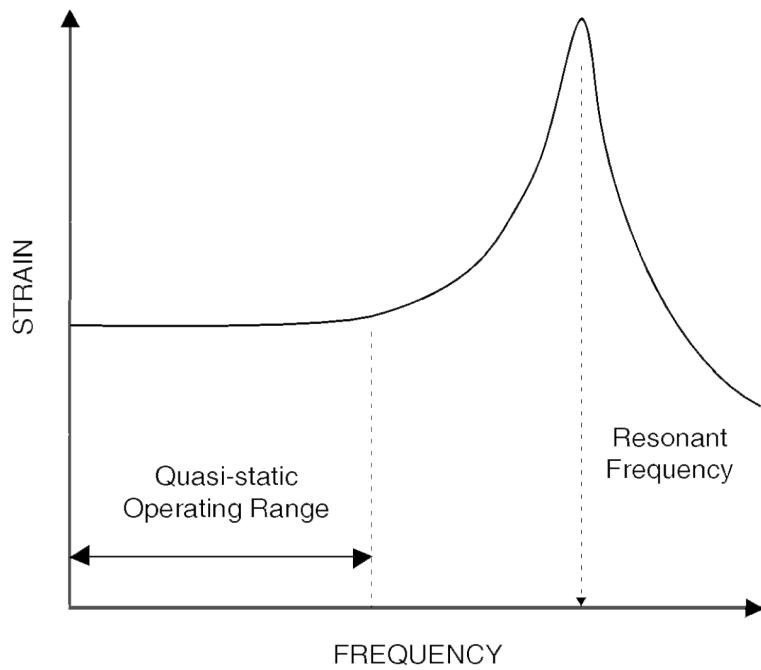
**Continuously Alternating Input Forces:** When the generator will be excited by an oscillating force, it is useful to be familiar with the following concepts:

**Hysteresis:** Hysteresis is a concern with oscillating forces because heat accumulates within the element for each cycle of operation. Heat accumulates from contributions due to mechanical losses described earlier and dielectric losses attributed to the phase lag between charge displacement and electric field. Low voltage piezo stacks are generally limited to < 1kHz operation at full loading due to heat buildup. Care should be taken in the design to account for heating caused by internal piezo losses as well as external system losses, such as strains within adhesive bonds, and friction at the mount or other points of attachment.

**Mechanical Resonance:** When a piezo body is acted upon by a periodic series of impulses, it will be set into relatively large amplitude vibration if the frequency of those impulses corresponds to the natural or resonant frequency of the device. This resonance is a manifestation of the trading back and forth of kinetic energy (moving mass) and potential energy (elasticity) in the oscillating body. At resonance, the amount of stored energy becomes very large compared to the excitation energy. For this reason, it is useful for achieving large voltages at low stress levels, and thus, for obtaining high efficiency. Because of the high amplitudes exhibited, care must be taken not to overstrain and crack the generator.

The resonant frequencies of a piezo generator depend on its dimensions, material properties, and the manner of mounting. The cantilevered piezo bending element, being very compliant, has the lowest fundamental resonant frequency per unit length of all configurations and mounting schemes. The piezo stack, being very stiff, has a high resonant frequency. Equations for determining the fundamental resonant frequencies for several generator configurations are shown in Table 4 on pages 38 and 39. These frequencies apply to unloaded elements only. Attachments to the element will add to the resonating mass and lower the resonant frequency.

**Operating Frequency Band:** Below resonant frequency, the strain of a piezoelectric transducer is nearly independent of frequency and proportional to the applied stress. Around the resonant frequency, strain rises rapidly to a multiple of its normal value. The amplitude and narrowness of the resonance depend on the internal and external losses acting on the generator. Above resonance, strain decreases steadily with the square of the frequency. Generally, for quasi-static transducers, a value of about 2/3 of the fundamental resonance marks the limit of the usable frequency band. For resonant applications, the useable frequency range is limited to a small band around the useful resonant modes. *Figure 15* shows strain as a function of operating frequency.



*Figure 15. Strain as a function of operating frequency*

#### 4.1.3 MECHANICAL CONSIDERATIONS

##### MECHANICAL RESPONSE TIME

It is useful to know how fast a generator element can respond to a force input. The fundamental resonant frequency,  $F_r$ , helps answer this question. A piezo transducer can follow a sinusoidal input up to its resonant frequency. Beyond this point, inertia prevents the transducer from keeping up with excitation. The time it takes a generator to travel from its zero position to full positive amplitude at its resonant frequency, is its response time,  $t_r$ . This is 1/4 the time it takes the transducer to travel a full bipolar cycle (from zero to peak positive amplitude to 0 to peak negative amplitude back to zero). Thus,

$$t_r \sim \frac{1}{4F_r} \quad (17)$$

The response time of a generator may be measured by driving it as an actuator with a low-level sinusoidal signal at resonance. For example, a generator having a resonant frequency of 500 Hertz, has a response time of 0.5 milliseconds under ideal conditions. In a practical setting, this response time is rarely achieved due to hysteresis and other losses. A more realistic estimate for  $t_r$  is given by (18). Any mass added to the end of the generator will increase the response time.

$$t_r \sim \frac{1}{3F_r} \quad (18)$$

## MOUNTING

An ideal mount permits the normal distortion of the entire active portion of the generator element, while at the same time preventing motion in certain directions at the mounting point or points. Generally, piezo generators are either bonded, clamped, or spring loaded to their mounting points. Mounts introduce some mechanical damping into the system since some of the energy from the generator distorts the mount itself. This may or may not be desirable.

## STRENGTH LIMITATIONS

Piezoceramic is very strong in compression but weak in tension. Bending elements always have one side in compression and the other side in tension, where the magnitude of stress increases linearly from the midplane to the outside surface. Therefore, the element is always limited by the maximum recommended tensile strength, generally considered to be in the range of  $20-35 \times 10^6$  Newtons/meter<sup>2</sup>. From a strain point of view, the piezoceramic should not be allowed to strain more than  $500 \times 10^{-6}$  meter/meter in tension.

## STRESS DEPOLING

When the mechanical stress on a piezoceramic element becomes too high, again there is a danger of degrading the piezoelectric properties. Generally, compressive or hydrostatic stress levels of  $\sim 50 \times 10^6$  N/m<sup>2</sup> are required to degrade PZT-5A if no other degrading influences are present.

### 4.1.4 ELECTRICAL CONSIDERATIONS

#### QUASI-STATIC OPERATION

A piezoelectric generator operating below its fundamental resonance can be treated simply as a capacitive element. It supplies, withdraws or stores charge. Ideally, this charge does not leak away. However, in practice charge may leak through the bulk material, over its edges, or through external circuitry.

#### NEAR RESONANCE OPERATION

A piezoelectric generator operating at resonance can be treated as a capacitor (having a value equal to the transducer capacitance) with a resistor in parallel. The power dissipated by this resistance represents the work which the transducer does on its environment or the internal loss occurring within the transducer.

## ELECTRICAL ISOLATION

The outer electrode surfaces of certain generator elements are electrically "live" in many configurations. For product or experimental safety, consideration should be given to insulating or shielding the electrodes, mount, and power input sections of the generator element.

## ELECTRICAL BREAKDOWN

The highest value of generated electric field is determined by electrical breakdown occurring either through the body of the piezoceramic sheet or over its edges. Debris adhering to edges can initialize edge discharge at fields as low as 400-800 volts/mm. Continuous breakdown occurs around 3,000-4,000 volts/mm, usually at impurity or defect regions within the bulk of the material. This can lead to a short circuit across the sheet.

## ELECTRICAL LOSSES

The bulk resistivity of piezoceramic is  $\sim 10^{12}$   $\Omega\text{-cm}$ . Therefore, electrical losses are minimal under static or low frequency operation. However, dielectric losses are significant at high frequency, under high load, and can lead to heating under high frequency /high power operation. The loss tangent, the ratio of series resistance to series reactance, for PZT-5A is  $\sim 0.015$ .

## ELECTRICAL DEPOLARIZATION

As mentioned earlier, under adverse conditions piezoelectric polarization may degrade, vanish completely, or be flipped around  $180^\circ$ . A strong electric field applied to a piezoceramic in a sense opposite to the original poling voltage will tend to cause depoling. The field strength necessary to initiate depoling depends on the material, duration of application, and temperature, but is typically in the range of 475 volts/mm at  $20^\circ \text{ C}$  for PZT-5A under static conditions. Alternating fields may also degrade the piezoceramic, but the peak field level is higher because the duration is shorter before the field is reversed. A peak field of 600 volts/mm may be tolerated for 60 Hz operation at  $20^\circ \text{ C}$ .

### 4.1.5 THERMAL CONSIDERATIONS

#### CURIE TEMPERATURE

For each piezoceramic material there is a critical temperature, known as its Curie point, which represents its maximum operating temperature before suffering a permanent and complete loss of piezoelectric activity. In practice, the operating temperature must be limited to some value substantially below the Curie point because at elevated temperatures, depoling is greatly facilitated, the aging process is accelerated, electrical and mechanical losses increase, and the maximum safe stress is reduced. As a rule of thumb, a temperature equal to one half the Curie temperature is considered the maximum safe operating temperature.

## PIEZOELECTRIC AND MATERIAL PROPERTIES AS A FUNCTION OF TEMPERATURE

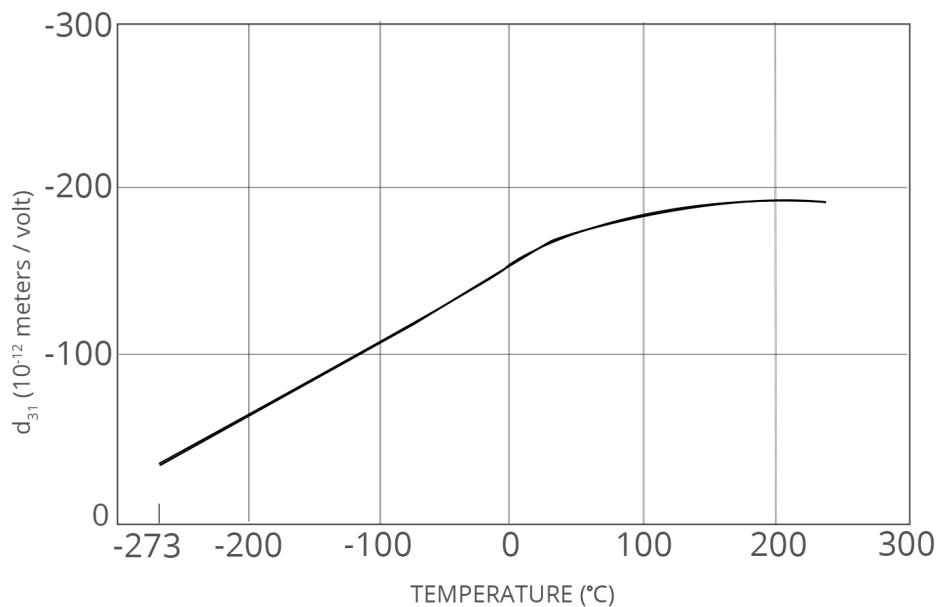
Piezoceramic properties are temperature dependent, and thermal dependence varies markedly from one material to the next. *Figure 16*, *Figure 17* and *Figure 18* demonstrate the temperature dependence of  $d_{31}$ ,  $g_{31}$ , and  $K^T_{33}$  for PZT-5A, respectively.

## PYROELECTRIC EFFECTS

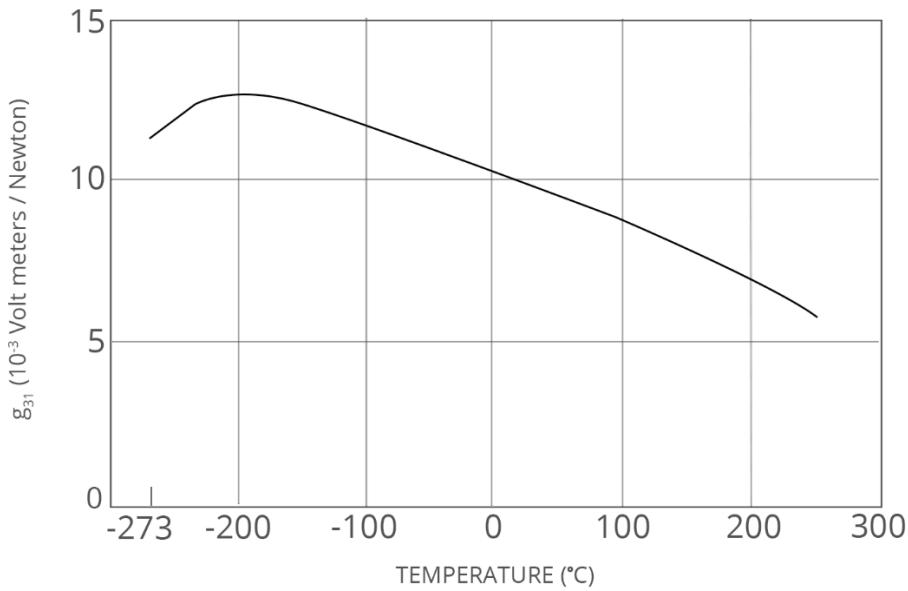
An electric field is induced across the electrodes of a piezo generator when it is exposed to a thermal change. The induced field,  $E$  (volts/meter), is

$$E = \frac{V}{t} = \frac{\alpha(\Delta T)}{\epsilon_0 K^T_{33}} \quad (19)$$

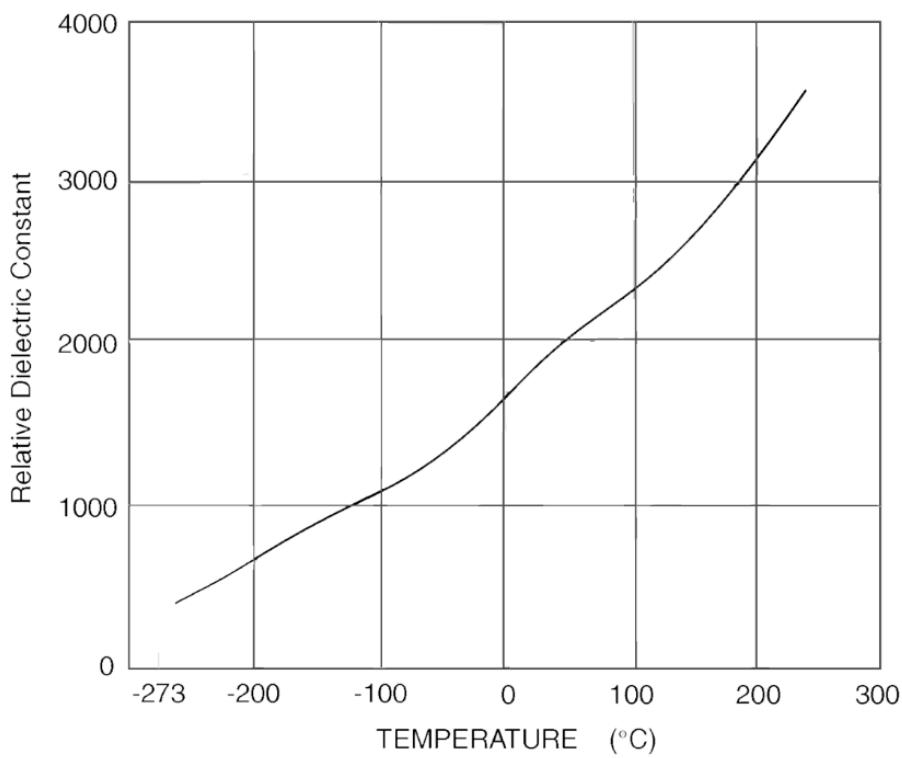
where  $\alpha$  is the pyroelectric coefficient in units of coulombs /  $m^2 \text{ } ^\circ\text{C}$ ,  $\Delta T$  is the temperature change,  $K^T_{33}$  is the relative dielectric constant in the poling direction, and  $\epsilon_0$  is the permittivity of free space. It is important in the design of a sensor to maximize the ratio of mechanical effect to pyroelectric effect.



*Figure 16.* Typical temperature dependence of  $d_{31}$  for PZT-5A



**Figure 17.** Typical temperature dependence of  $g_{31}$  for PZT-5A



**Figure 18.** Typical temperature dependence of  $K_{33}$  for PZT-5A

It should be noted that a depoling voltage will be developed across a layer of piezoceramic when the temperature drops. This can happen during processing, testing, shipping, or normal usage. If a temperature drop is sufficient, over a time interval which is too short to allow charge to leak away, a voltage greater than the coercive field can result. This can degrade the original polarization causing reduced performance. It is always good practice to short circuit the electrodes of any piezo device during a cool down procedure.

### **THERMAL EXPANSION**

Thermal expansion,  $\Delta L$ , results in a dimensional change due to a thermal change,  $\Delta T$ , according to the relation:

$$\frac{\Delta L}{L} = \alpha^E \Delta T \quad (20)$$

The coefficient of thermal expansion (at constant electric field),  $\alpha^E$ , of PZT-5A is  $\sim 4 \mu\text{m} / \text{m } ^\circ\text{C}$ . If positional stability is important, one must account for thermal expansion / contraction displacements over the temperature range anticipated.

Differential thermal expansions of adjacent assembly materials will cause moments, warping, and shifting. The standard 2-layer bending generator element has a symmetrical construction. Distortion due to thermal excursion should be slight. However, care should be taken in the design of the mount or any other attachments not to introduce thermal distortion. This is facilitated by properly matching the thermal expansion coefficients of adjacent members to that of the ceramic element.

### **CRYOGENIC OPERATION**

The low signal values of the charge coefficients for operation at 4.2K are reduced by a factor of 5-7 times. The value of the dielectric constant decreases and the value of the coercive field increases, however. In general, piezo sensors work quite well at cryogenic temperatures where they have been used to monitor magnetic flux motions in superconducting magnets bathed in liquid helium. Cycling the transducer between these temperature extremes does not affect them adversely.

### **THERMAL DEPOLARIZATION**

Thermal agitation can reduce the number of electric dipoles aligned during the original poling process. The higher the temperature, the greater the effect. Eventually, at the Curie temperature, the piezoceramic suffers a complete loss of piezoelectric properties and repoling becomes necessary.

#### 4.1.6 VACUUM CONSIDERATIONS

Because piezo generators are solid state devices, they lend themselves to high vacuum operation. However, several issues should be understood. First, high voltage should not be generated across the electrodes during the vacuum pump-down process because of the low insulation resistance of air and nitrogen between the range of 10 to 0.1 torr. Arcing between electrodes is possible within this pressure range. Secondly, outgassing of the part is possible depending on construction materials. Generators to be used in high vacuum environments should have small cross sections of outgassing materials (primarily the adhesive). If bake-out is necessary, the transducer will have to be built to withstand solvents and bake-out temperatures.

#### 4.1.7 TEMPORAL CONSIDERATIONS

##### AGING

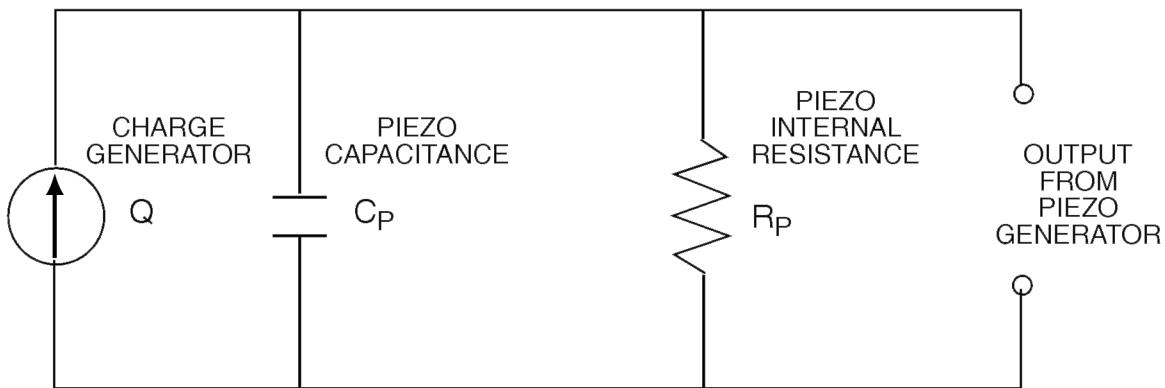
Piezoelectric properties change gradually with time. The changes tend to be logarithmic with time after the original polarization. Therefore, the rate reduces rapidly with time. Aging depends on the ceramic material, manufacturing process, and ambient conditions such as temperature, vibration or shock. Pieces may be heated for a specified time to accelerate the aging process. It is common practice to model piezoelectric devices electrically to describe their piezoelectric, dielectric and electric properties.

#### 4.1.8 CIRCUIT CONSIDERATIONS

It is common practice to use a lumped parameter electronic circuit model to simulate the interaction of piezoelectric devices with electronic circuits attached to them. These circuits are referred to as “equivalent circuits.” Equivalent circuits for piezo generators and vibrational sensors operating quasi-statically are simplest form of useful model. See examples below.

##### PIEZO CHARGE GENERATOR

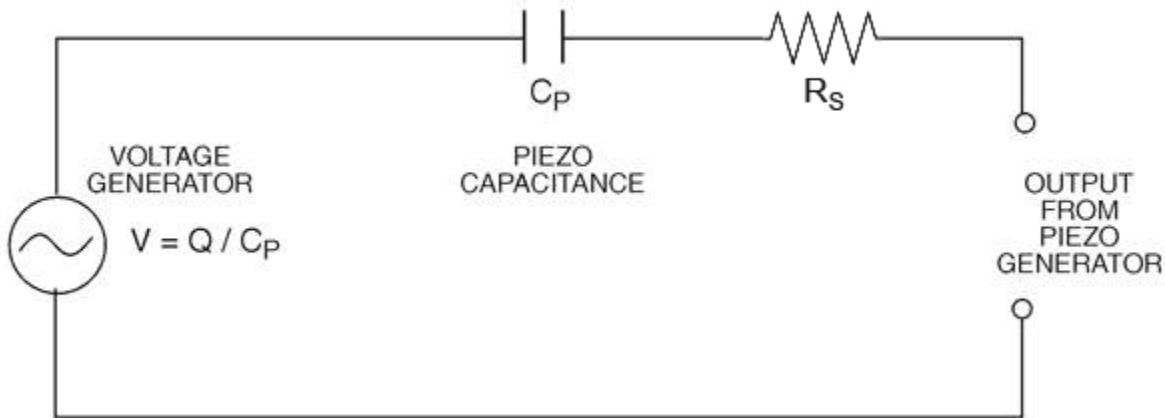
*Figure 19* shows the equivalent circuit for a piezo charge generator. The only thing in this circuit that is accessible to the outside world is the two-terminal output interface. The value of  $C_p$  is the lab measured piezo capacitance and  $R_p$  is the measured internal DC resistance of the piezo device. Since the bulk resistivity of piezoceramic is of the order of  $\sim 10^{12} \Omega\text{-cm}$ ,  $R_c$  routinely runs in the giga Ohms and so is generally neglected.  $Q$  represents the charge generated by time dependent stresses exerted on the piezoceramic and can be calculated based on the design of the part, its mechanical mounting, and applied external forces. See Table 4 for formulas.



*Figure 19. Piezo transducer modeled as a charge generator*

### PIEZO VOLTAGE GENERATOR

Since voltage can be calculated from the charge delivered by a piezo device according to the relationship that  $V = Q / C_p$ , Figure 20 shows a piezo device modeled as a voltage generator. Voltage output  $V$  is the open circuit voltage given in Table 4, and proportional to the stress applied to the piezo device.  $C_p$  is the lab measured capacitance of the piezo sensor.  $R_s$ , the equivalent series resistance of the sensor, is frequently taken as zero. Unlike the  $R_p$  of the Charge Generator model however,  $R_s$  is not entirely internal to the sensor. Both surface resistance of the piezoceramic electrodes (typically sub-Ohm to 10 Ohms), solder connections, cable resistance, and connector contact resistance can contribute  $R_s$ .



*Figure 20. Piezo transducer modeled as a voltage generator*

## ELECTRICAL TIME CONSTANT

The charge on a piezo sensor decays with time at a rate determined by its RC time constant. An RC circuit exhibits the exponential decay. Typically, charge is generated quickly. If the discharge time constant is large, it will not decay faster than the charging rate. Clearly, a long time constant is necessary if low frequencies need to be monitored.

## INPUT SIGNAL CIRCUITS

The signals from piezo sensors can be fed to meters, oscilloscopes, and circuits. The circuits are numerous, ranging from simple to complex, whose purpose is to filter or cancel unwanted portions of the signal, to amplify or switch the signal (transistors and operational amplifiers), or to make decisions based on information from signals (digital logic).

*High Input Impedance for Voltage Sensor Circuits:* Piezo voltage sensor schemes are typically used in strain detection. In practice the applications are largely 'real time', meaning that the output voltage from the sensor is assumed to indicate the instantaneous state of strain that indicates the flex, compression, etc of the structure being measured. Since the piezoceramic itself is a capacitor with near-infinite internal parallel resistance, the limiting factor is the input resistance of the buffer (i.e. first) circuit that is in contact with the output terminals. If it is too low, it causes a significant lag, or phase shift, in the signal right at the start of the signal chain. The job of obtaining input impedances sufficiently high is made relatively easy by integrated circuit operational amplifiers.

*Low Input Impedance for Charge Sensor Circuits:* Piezo charge sensor schemes are typically used in applications in which it is necessary to have a long cable between the sensor itself and the buffer circuit (e.g. an accelerometer). In this case an operational amplifier buffer circuit input is arranged to have a virtual ground potential and 100% of the current issuing from the piezo sensor ends up flowing through that input regardless of the cable length. The primary advantage of this approach is that cable capacitance nor any stray capacitance affects the output calibration. Transient signals can be measured in 'real time', however low frequency signals are prey to noise induced by cable motion, magnetic, and thermal effects, etc.

## INPUT STAGE PROTECTION

Piezoelectric elements can generate high voltages ( $>>100$  volts) under external vibration, shock, or temperature shifts. If these conditions are expected, the circuitry of the input stage must be protected against transient voltages of all polarities. This commonly accomplished with protection diodes that steer excess charge away from the input circuit terminal to ground.

## CABLES

In the case of voltage sensors, the buffer (first) amplifier circuit should be kept as close to the piezo sensor as possible. Shielded coax cable should be used to connect the sensor to the buffer

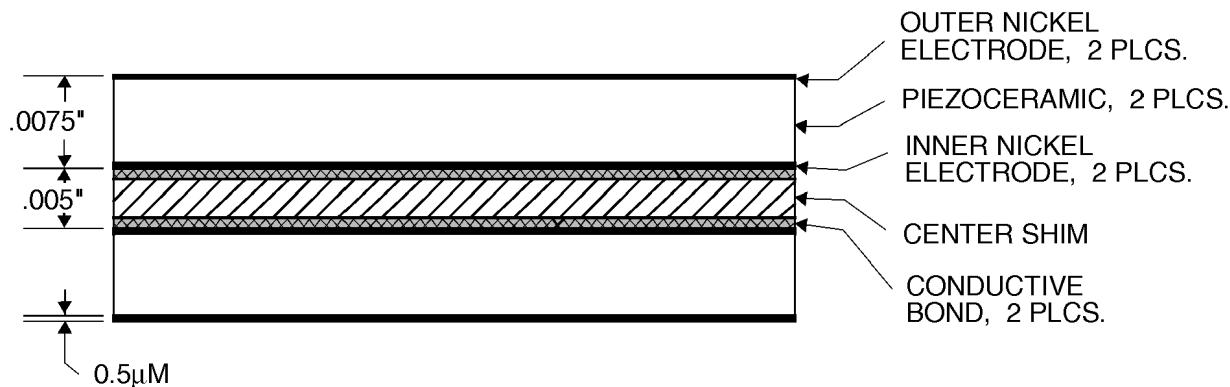
when possible, although this can add leakage and capacitance to the circuit. Motion of the cable is another source of noise. The cable should be held down firmly to eliminate any movement or vibration. Polar plastic materials, used for cable insulation, can generate charge. Teflon is a good choice of material to minimize this problem

## 5. Piezoelectric Bending Generators

Although the mechanical stress and electric field are uniformly distributed for most transducers, this is not the case for bending transducers. In addition, there are various combinations with regard to their construction and operation. For this reason, they are discussed in greater detail.

### 5.1 Principles of Operation

The most common type of piezoelectric bending generator is composed of two layers of piezoceramic bonded to a thin metal shim sandwiched in the middle. This is sometimes referred to as a "bimorph" element. The construction and typical dimensions of 2-layer elements are shown in *Figure 21*.

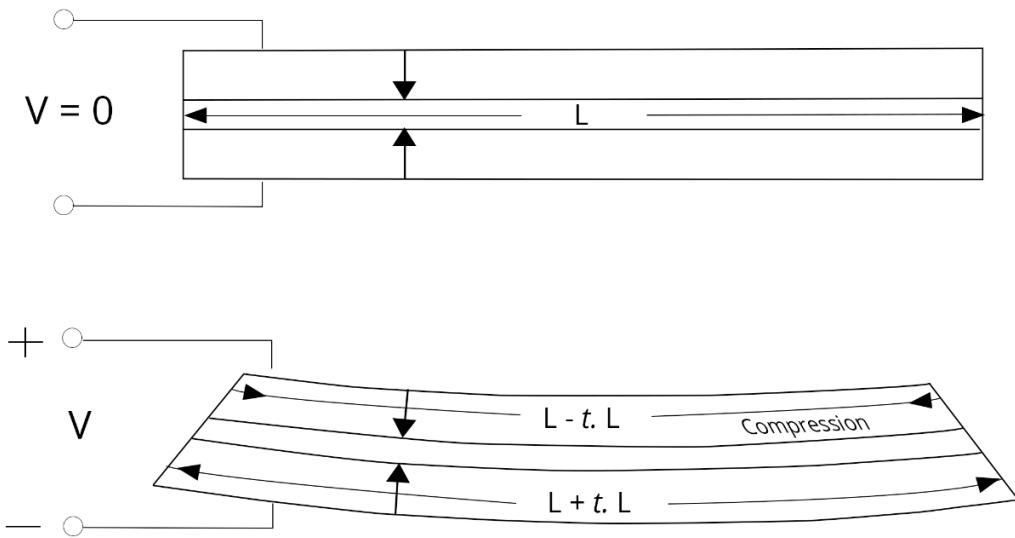


*Figure 21. Construction of laminated 2-layer generator element*

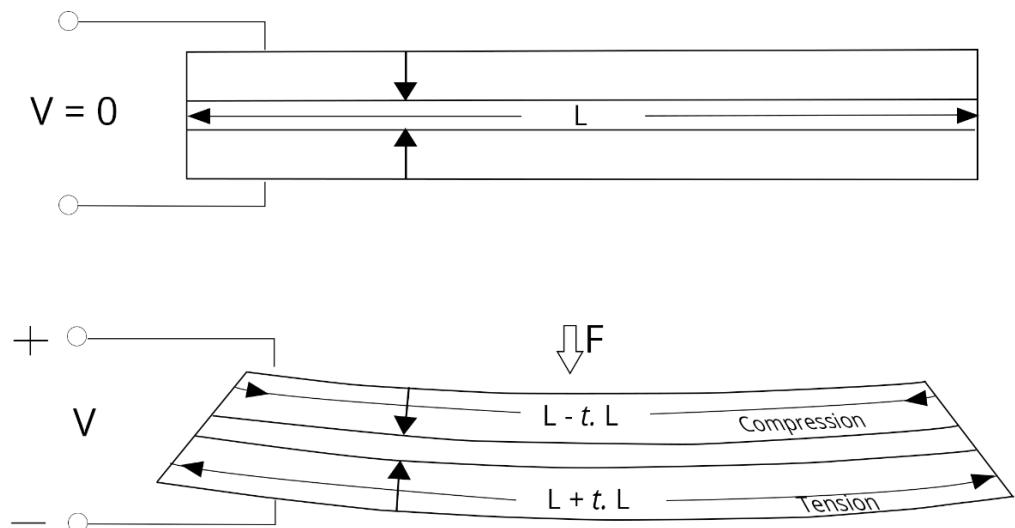
As an actuator, the application of voltage across the bender element forces one layer to expand, while the other contracts, as depicted in *Figure 22*. The result of these physical changes is a strong curvature and large deflection at the tip when the other end is clamped. The tip deflection is much greater than the change in length of either ceramic layer. A similar effect, to a lesser extent, may be obtained by bonding one piezoceramic layer to a passive non-piezoceramic layer. This is sometimes referred to as a "monomorph" construction.

As a generator, when the bender is forced to flex, one layer will be in tension while the other is in compression. The stresses in each layer will produce electrical outputs. Based on the orientation of polarization discussed next (series or parallel), the electrical output of the bender will then be the summation of the outputs of each layer. This is depicted in *Figure 23*.

Bending generators exhibit unique properties. They require no outside energy source to produce a signal. They may be operated over billions of cycles without wear or deterioration. Their low profile allows their use in very restricted locations.



**Figure 22.** The curvature of a bending motor is due to the expansion of one layer and contraction of the other



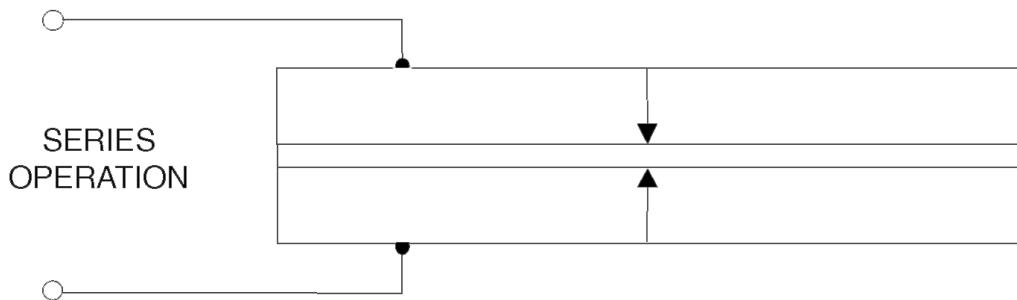
**Figure 23.** Charge is produced by a bending generator due to the expansion of one layer and contraction of the other when forced to flex

## 5.2 Standard Polarization Configurations

There are two standard polarization configurations for the two-layer bending generator construction: series and parallel. These are illustrated in *Figure 17*. Both the series and parallel elements are limited to electric fields below the coercive field (~ 475 V/mm for PZT-5A).

### 5.2.1 SERIES OPERATION

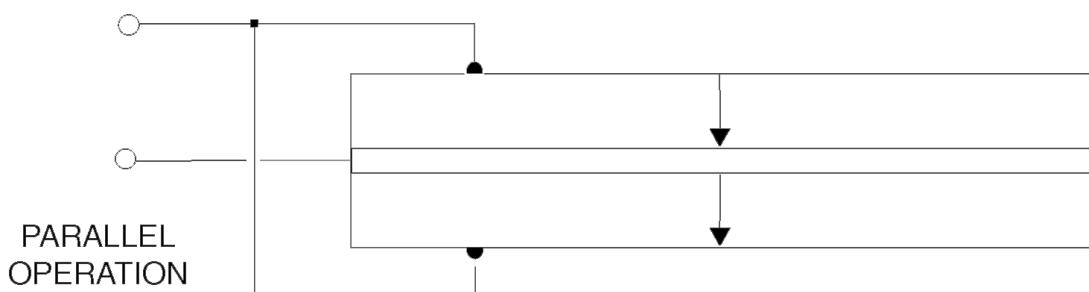
The bender poled for series operation (which we refer to as “X poled”) is the simplest and most economical. As depicted in *Figure 24*, it requires two connections to the outside surfaces of the piezoceramic layers which are electrically in series. Thus, their voltages add. It is characterized by a lower capacitance, lower current, and higher voltage.



*Figure 24. Bending generator poled for series operation*

### 5.2.2 PARALLEL OPERATION

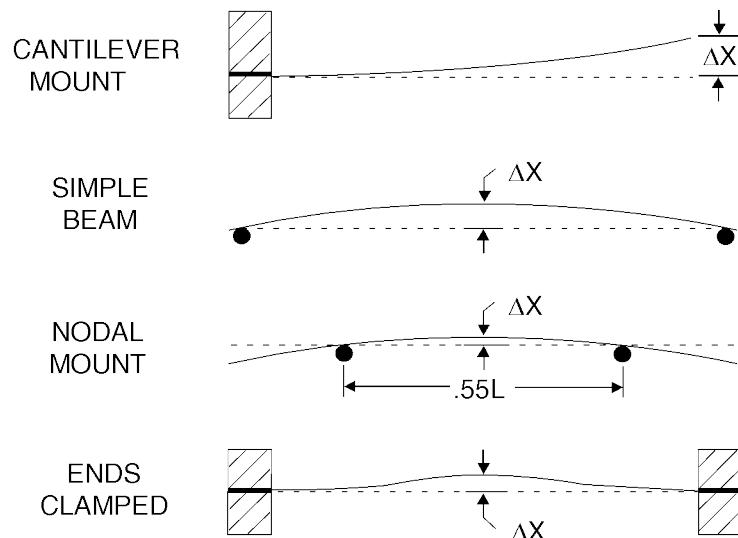
As depicted in *Figure 25*, the bender poled for parallel operation (which we refer to as “Y poled”) requires three electrical connections. The third connection accesses the center shim of the bender, requires an extra manufacturing step and has a somewhat higher production cost. Voltage is developed across the individual layers, and since they are wired electrically in parallel their charges (or currents) add. The parallel bender is characterized by higher capacitance, higher current, and lower voltage.



*Figure 25. Bending generator poled for parallel operation*

### 5.3 Standard Mounts for Bending Generators

Standard mounts for bending generators are illustrated in *Figure 26* and fall into two general categories. The first category has power input at one end and is mounted at the other. Known as the cantilever mount, it provides maximum compliance. The second category, known as the simple beam mount, has power input at the center and is mounted at the ends. The simple beam mount allows the ends to move in and out as well as rotate but fixes their vertical position. Compared to the cantilever mount, the simple beam mount provides increased stiffness and frequency. For high frequency-resonant applications, power dissipation at the mounts can be minimized by using nodal mounts. The nodes are evenly spaced, .55L apart, where L is the length of the beam. Ridgid clamping at both ends is often considered by novice designers for expedience (e.g. by bonding to the edges of a cutout or clamping as illustrated in *Figure 26*). In theory if the end clamps are perfectly rigid the motion at the center of the beam will be exactly zero. In practice no material is perfectly rigid, so typically the designers are left to puzzle out why the actuator is only making 1/10 the expected excursion.



*Figure 26. Standard mounting techniques for bending generators*

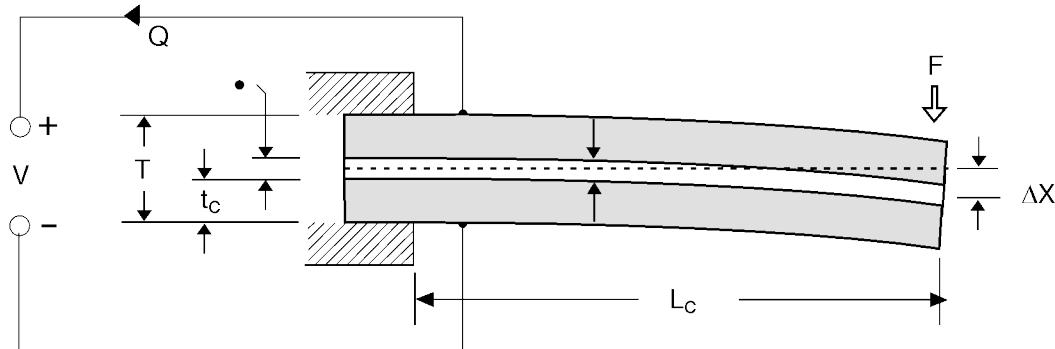
## 5.4 Bending Generator Equations

The following equations for bending generators are more accurate than those in Table 4 in so far as they take into account the center shim. They can be used to: 1.) verify that the bending generator is operating properly, and 2.) scale the dimensions of the experimental generator to define the dimensions of the final design, or vice versa. The relations below were taken from an article titled "Flexure Mode Piezoelectric Transducers" by Carmen Germano, published in IEEE Transactions on Audio and Electroacoustics, Volume AU-19, No.1, March 1971.

### Definition of terms:

- 
- L = Total Length of the bending generator (m)
  - $L_c$  = Active length of the bending generator (m)
  - T = Total thickness of bending generator (m)
  - $\delta$  = Thickness of center shim and adhesive layers (m)
  - $t_c$  = Thickness of a single layer of piezoceramic (m)
  - b = Width of bending generator (m)
  - $d_{31}$  = Piezoelectric transverse charge coefficient (m/V)
  - $g_{31}$  = Piezoelectric transverse voltage coefficient (Volt Meters / Newton)
  - Y = Average Young's modulus of elasticity (N/m<sup>2</sup>)
  - $K_{33}$  = Relative dielectric constant of piezoceramic
  - $\epsilon_0$  = Permittivity of free space ( $8.85 \times 10^{-12}$  Farads/Meter)
  
  - $\rho$  = Average density of bending generator (Kilograms / Meter<sup>3</sup>)
  - V = Voltage output from the piezo generator (Volts)
  - Q = Charge output from the piezo generator (Coulombs)
  - F = A force applied to the piezo generator (Newtons)
  - $\Delta x$  = A displacement applied to the piezo generator (Meters)

#### 5.4.1 BENDING GENERATOR MOUNTED AS A CANTILEVER



**Figure 27.** Terminology used for cantilevered bending generator equations

#### SHORT CIRCUIT CHARGE

The short-circuit charge,  $Q_s$ , is usually twice the required operating charge.

$$Q_s(F) = \left(\frac{3}{2}\right) \left(1 + \frac{\delta}{T}\right) \left(\frac{L_c^2}{T^2}\right) d_{31} F \quad (21)$$

$$Q_s(X) = \left(\frac{3}{8}\right) \left(1 + \frac{\delta}{T}\right) \left(\frac{bT}{L_c}\right) Y d_{31} \Delta X \quad (22)$$

where  $0 < \delta < t_c$

#### OPEN CIRCUIT VOLTAGE

The open circuit voltage,  $V_0$ , is usually twice the required operating voltage.

$$V_0(F) = \left(\frac{3}{2}\right) \left(1 - \frac{\delta^2}{T^2}\right) \left(\frac{L_c}{bT}\right) g_{31} F \quad (23)$$

$$V_0(X) = \left(\frac{3}{8}\right) \left(1 - \frac{\delta^2}{T^2}\right) \left(\frac{T^2}{L_c^2}\right) Y g_{31} \Delta X \quad (24)$$

where  $0 < \delta < T_c$

## RESONANT FREQUENCY

$$F_r = \left( \frac{0.16T}{L_c^2} \right) \left( \frac{Y_{11}}{\rho} \right)^{\frac{1}{2}} \quad (25)$$

## MAXIMUM SURFACE STRAIN

$$S_{max} \sim \left( \frac{T}{L_c^2} \right) \Delta X \quad (26)$$

where  $\sim 500 \times 10^{-6}$  is the maximum recommended strain limit in tension

## CAPACITANCE

$$C \text{ (series operation)} = \frac{K_3 \varepsilon_0 L b}{2 t_c} \quad (27)$$

$$C \text{ (parallel operation)} = 4 \times C \text{ (series operation)} \quad (28)$$

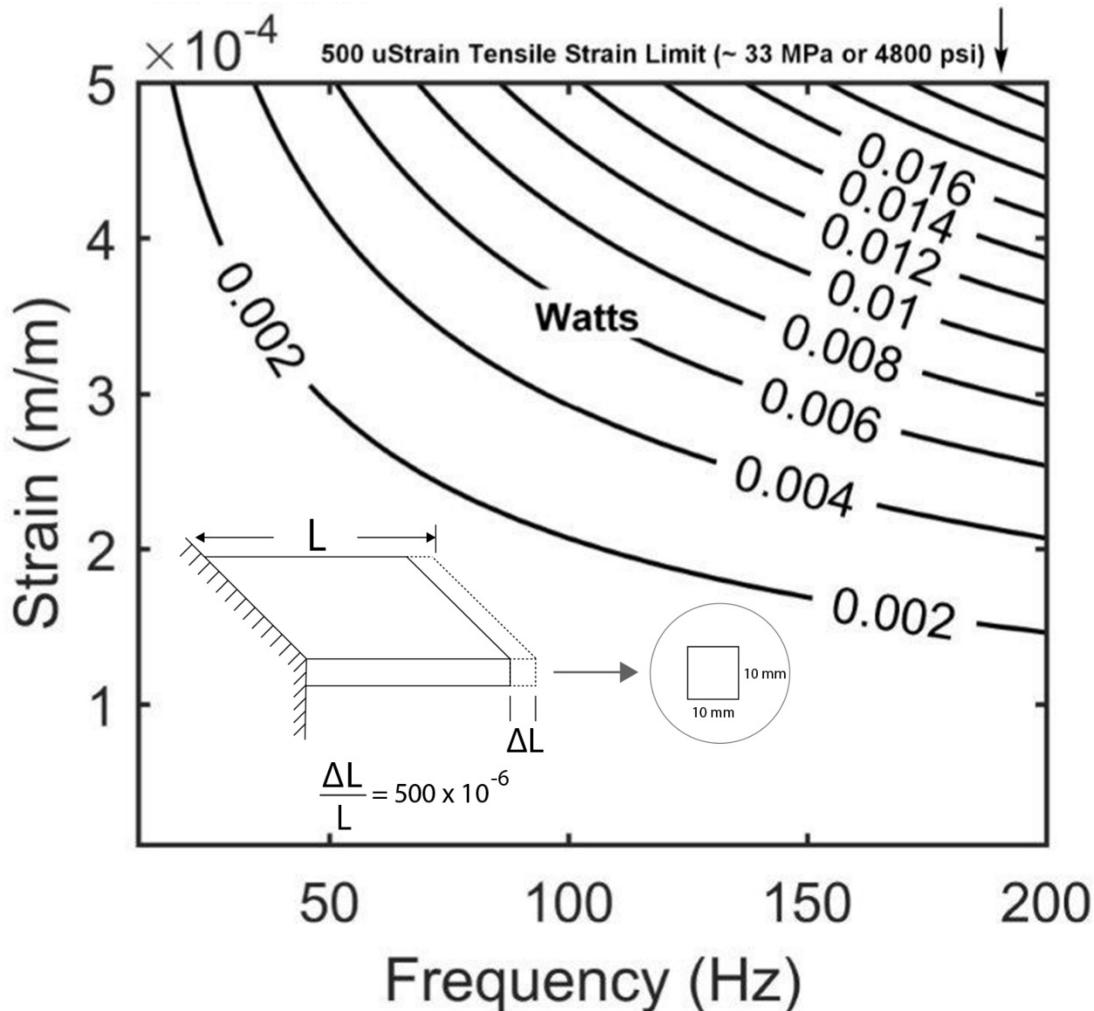
## 6. Calculating Generator Outputs

### 6.1 The Spectrum of Piezoelectric Generator Transducers

Transducers which convert mechanical energy to electrical energy (i.e. generators) come in a wide range of shapes and sizes, each having their own characteristic voltage-charge output capabilities as well as input force - displacement characteristics.

On the mechanical input side, the design process involves correctly matching the volume and stiffness of the generator to the target input force. The stiffness of the transducer determines how much mechanical energy gets transferred into the transducer from the source force. The piezoceramic material properties determine what percentage of this energy will be available as electrical energy. The total volume of piezoceramic sets an upper limit on amount electrical energy available for use.

While there is no simple way to estimate the electrical power that can be extracted from any given piezo harvester construction, some intuition of the maximum extractable power in the “transverse mode” from a 10mm square patch of ceramic itself can be gained by inspection of *Figure 28*.



**Figure 28.** Electrical power available from 1 cm x 1 cm PZT5A patch in transverse mode for applied sinusoidally varying uniform strain

The basis for this graph is as follows:

*Piezo material:* PZT5A

*Size of harvester:* 10mm x 10mm x 0.25mm

*Electrodes:* full coverage of both 10mm x 10mm faces

*Polarization:* Through the 0.25mm thickness

*Mechanical input:* dynamic (i.e. +/-) uniform strain applied on a 10mm x .025mm edge over a range of frequencies

*Electrical output:* taken between the two electrodes with imaginary perfect circuitry

Strain rather than stress is utilized as the independent applied mechanical input for two reasons:

1. Piezoceramics' mechanical dynamic excitation limits can be easily approximated as +/- 500E-06 meters/meter (i.e. 500 microstrain). This figure is a commonly used as a rule of thumb arising from the tensile limit of the materials and is incorporated into the graph as an upper limit.
2. It is far easier experimentally to measure surface strain on various constructions and infer the interior stresses than to measure stress directly.

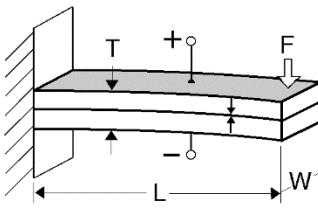
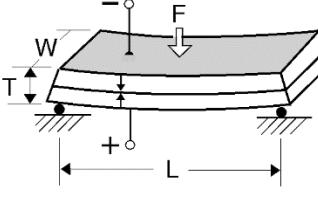
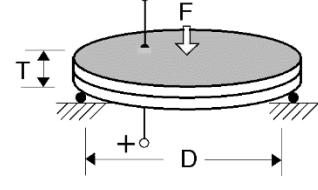
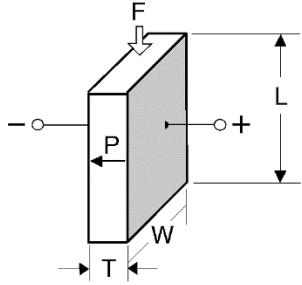
The Y axis of this graph shows the applied +/- uniform strain, the X axis shows the frequency of sinusoidal strain application. The lines are "contours of constant electrical power output" showing the trade-off between applied strain and applied frequency for this small sample of ceramic. For the same size of ceramic, low frequency vibrational environments will require devices designed to apply higher stresses than high frequency vibrational environments.

No practical design will achieve this upper limit. Strains/stresses are seldom uniform, never applied without loss, and electronic circuits are never perfect. Harvester design work consists mainly of exploring various compromises. Consider as an example the cantilevered bimorph beam. When the beam is bent, the average strain in the ceramic layers is exactly half the surface strain, so when the surface is at its limit the voltage on the plates will only be 1/2 the achievable max, and the energy stored in the capacitance of the layer will therefore be only 1/4. But it gets worse! During vibration the distribution of strain on the surface is not uniform along the length of the beam - it's high near the cantilever point and drops off going out to the tip, reducing the limiting output by perhaps another 1/4 to 1/3. So, the price of the convenience of the bimorph is that its electrical output for every 10mm square of device will really be only about 1/16 what is on our graph. **WARNING: power outputs may be smaller than they appear!**

On the electrical output side, the design process centers on delivering the total electrical energy to the load at a specific voltage-current combination (e.g. 5V @ .05 mA rms). In principle this determination is independent of the input design and consists primarily of dividing up the piezoceramic into a number of layers which get wired in parallel.

As a general-purpose reference guide, *Table 4* shows the spectrum of generator transducers commonly considered in piezoelectric applications. This can be accomplished after the energy design by subdividing the volume of ceramic into layers (with parallel wiring). The result is a multi-layer design capable of delivering the same energy at a lower voltage and a higher current.

**TABLE 4. SPECTRUM OF COMMON PIEZOELECTRIC GENERATORS**

PIEZOELECTRIC CONFIGURATION	SHORT CIRCUIT CHARGE	OPEN CIRCUIT VOLTAGE	RESONANT FREQUENCY
CANTILEVERED BENDING ( D31 ) GENERATOR			
	$\frac{3 L^2}{2 T^2} d_{31} F$ $\frac{3 T W}{8 L} Y d_{31} \Delta X$	$\frac{3 L}{2 W T} g_{31} F$ $\frac{3 T^2}{8 L^2} Y g_{31} \Delta X$	$\frac{.16 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$
SIMPLY SUPPORTED BENDING ( D31 ) GENERATOR			
	$\frac{3 L^2}{8 T^2} d_{31} F$ $\frac{3 T W}{2 L} Y d_{31} \Delta X$	$\frac{3 L}{8 W T} g_{31} F$ $\frac{3 T^2}{2 L^2} Y g_{31} \Delta X$	$\frac{.48 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$
SIMPLY SUPPORTED DISK BENDING (D31) GENERATOR			
	$\frac{.42 D^2}{T^2} d_{31} F$ $3.1 T Y d_{31} \Delta X$	$\frac{.56}{T} g_{31} F$ $\frac{4.1 T^2}{D^2} Y g_{31} \Delta X$	$\frac{T}{D^2} \sqrt{\frac{Y_{11}}{\rho}}$
TRANSVERSE ( D31 ) GENERATOR			
	$\frac{L}{T} d_{31} F$ $W Y d_{31} \Delta X$	$\frac{1}{W} g_{31} F$ $\frac{T}{L} Y g_{31} \Delta X$	$\frac{1}{2 L} \sqrt{\frac{Y_{11}}{\rho}}$

**TABLE 4. SPECTRUM OF COMMON PIEZOELECTRIC GENERATORS, CONTINUED**

PIEZOELECTRIC CONFIGURATION	SHORT CIRCUIT CHARGE	OPEN CIRCUIT VOLTAGE	RESONANT FREQUENCY
LONGITUDINAL ( D33 ) GENERATOR			
	$d_{33} F$	$\frac{T}{LW} g_{33} F$	$\frac{1}{2T} \sqrt{\frac{Y_{33}}{\rho}}$
	$\frac{LW}{T} Y d_{33} \Delta X$	$Y g_{31} \Delta X$	
MULTI-LAYER LONGITUDINAL ( D33 ) GENERATOR			
	$n d_{33} F$	$\frac{T}{nLW} g_{33} F$	$\frac{1}{2T} \sqrt{\frac{Y_{33}}{\rho}}$
	$\frac{LW}{T} Y d_{33} \Delta X$	$\frac{Y g_{31} \Delta X}{n^2}$	
PARALLEL SHEAR ( D15 ) GENERATOR			
	$d_{15} F$	$\frac{T}{LW} g_{15} F$	$\frac{1}{2T} \sqrt{\frac{Y_{55}}{\rho}}$
	$\frac{LW}{T} G d_{15} \Delta X$	$LW G g_{15} \Delta X$	
TRANSVERSE SHEAR ( D15 ) GENERATOR			
	$\frac{L}{T} d_{15} F$	$\frac{g_{15} F}{W}$	$\frac{1}{2L} \sqrt{\frac{Y_{55}}{\rho}}$
	$W G d_{15} \Delta X$	$\frac{T}{L} G g_{15} \Delta X$	

## 6.2 Charge and Voltage Calculations

The equations for charge and voltage as functions of either input force or displacement shown in Table 4 are based on linear relationships and low signal values for their piezoelectric coefficients.

## 6.3 Resonant Frequency Calculations

*Table 4* also provides the equations for determining the fundamental resonant frequency. The expression:

$$\sqrt{\frac{Y}{\rho}} \quad (29)$$

common to all calculations of frequency represents the velocity of sound in piezoceramic along the associated axis of interest. The time it takes for an element to actuate is related to how quickly a pressure wave can travel through the medium.

For devices which may be constructed of multiple material layers (such as benders), the modulus, Y, and density,  $\rho$ , are determined by the following relations:

$$Y = \frac{T_1 Y_1 + T_2 Y_2}{T_{total}} \quad (30)$$

$$\rho = \frac{T_1 \rho_1 + T_2 \rho_2}{T_{total}} \quad (31)$$

## 6.4 Basic Energy Calculations

### STORED MECHANICAL ENERGY

The mechanical energy stored by a piezo device acted upon by a force is:

$$E_m = \frac{1}{2} Y S^2 V \quad (32)$$

where  $E_m$  is the stored mechanical energy in Joules,  $Y$  is Young's modulus in N / m<sup>2</sup>,  $S$  is the induced strain in m / m, and  $V$  is the volume of piezoceramic in m<sup>3</sup>.

### STORED ELECTRICAL ENERGY

The electrical energy stored by a piezo device is:

$$E_e = \frac{1}{2} C V_0^2 \quad (33)$$

where  $E_e$  is the stored electrical energy in Joules,  $C$  is the capacitance in Farads, and  $V_0$  is the open circuit voltage.

# Building Piezoelectric Transducers

## 7. Building Piezoelectric Transducers

### 7.1 Working with Piezoceramic

#### 7.1.1 THIN-SHEET (SINGLE LAYER) PIEZOCERAMIC STOCK

**PLEASE NOTE THAT THE PRIMARY COMPONENT OF PIEZOCERAMIC IS LEAD OXIDE (AS IN LEAD PAINT). WHEN HANDLING, USE THE PRECAUTIONS RECOMMENDED FOR HANDLING LEAD PAINT.** For more, see “What You Need to Know About Working with LEAD PAINT” <https://www.health.ny.gov/publications/2502.pdf> or similar.

Unlike laminated bending motor stock, thin sheet piezoceramic is extremely fragile and difficult to handle. With proper care and practice it can be handled and manipulated quite easily.

Clean cutting of piezoceramic is best achieved by sending parts to diamond dicing saw or a waterjet cutting service. Some laser cutting services will cut piezoceramic as well.

With some risk to the part, rough cutting can be accomplished by placing parts on a flat surface such as a glass plate, and then lightly scribing a ruled line along its surface with a sharp razor blade until the piece cracks and separates. To retrieve the part, one edge should be lifted up by inserting the razor blade underneath and lifting until it can easily be grasped with one’s fingers (or plastic tweezers).

If the intent is to use the piece itself, then all jagged edges should be gently sanded down using 220 and/or 400 grit sandpaper. This will reduce the probability of subsequent crack propagation.

If the intent is to bond the part to make piezo-composite stock, the edges do not necessarily need to be cleaned up prior to bonding. Subsequent diamond saw or waterjet machining operations can be performed on the bonded composite.

#### 7.1.2 2-LAYER GENERATOR STOCK

### DURABILITY

The two-layer motor stock and generator stock are much more rugged than generally assumed. It can be handled without special care and oftentimes dropped without damage. The ceramic is nonporous and is impervious to moisture as well as chemically inert with acids and solvents. The adhesives used for lamination, the center shim, and the nickel electrodes, however, are susceptible to particular solvents and acids.

### CUTTING AND SHAPING

For prototyping purposes, the generator stock can be rough cut on a band saw (having ~14 teeth/inch or more) as long as it is supported underneath by a back-up plate (plexiglass, metal,

etc.). This is not recommended for dimensions less than 1/4". Rough cutting usually produces burrs at the center shim which may make electrical contact to one of the outer electrodes. The burrs can be removed by filing or sanding the edge. Chipping will occur along the edge, but this is seldom great enough to affect performance. With some practice, the generator stock can be trimmed with scissors when one wants to remove thin slivers.

High quality cuts, necessary for long term stable performance, require the use of a high-speed diamond wheel saw or waterjet.

### ACCESSING THE CENTER SHIM

A milling machine can be used to remove ceramic in order to access the center shim electrode for soldering an electrical lead. Removal of ~1 mil per pass is recommended. A handheld grinding tool (i.e. Dremel Tool) is suitable for quick center shim access. Alternatively, on rectangular parts a sharp three-corner file ( $\frac{1}{4}$  inch on a side or less) can be used to grind away the ceramic from one corner, exposing the brass center shim for soldering.

To make temporary contact to the center layer (in order to measure capacitance of a single layer or perhaps to repolarize it) the center shim can be contacted with a razor blade or push pin,

#### 7.1.3 BONDING AND ATTACHING TO PIEZOCERAMIC

Attachments for power input or mechanical grounding are usually accomplished by bonding to the piezoceramic at its ends or middle. Holes or fasteners are put in these secondary members. Almost any adhesive bonds well to the piezoceramic nickel surface. These include epoxies, anaerobics, silicones, and cyanoacrylates. For quick and/or reversible mounting, the bending element is often clamped between two surfaces

#### 7.1.4 SOLDERING & ATTACHING LEADS TO THE ELECTRODES & CENTER SHIM

Piezoceramic electrodes will be either fired silver or nickel. Silver electrodes are flat white in color while nickel electrodes are grey. Electrical connections are usually made to these electrodes by soldering, but one may also use conductive adhesive, or clips to attach wires.

Silver electrodes are not recommended for high electric field DC applications because silver ions are likely to migrate forming a bridge between the two electrodes, which is a resistive short circuit. Silver electrode piezoceramic is, however, often used in AC applications. Silver is applied to piezoceramic in the form of flakes suspended in a glass frit paste which is generally screened onto the ceramic and fired. The glass makes the bond between the ceramic and the silver particles. Silver is soluble in tin and a silver loaded solder should be used to prevent scavenging of silver in the electrode. Good solder joints can be made to the silver electrodes on piezoceramics with resin-core type solders with technique similar to that used for attaching components to printed circuit boards.

Nickel has good corrosion resistance and is a good choice for both AC and DC applications. It is applied to the piezoceramic either by vacuum deposition or electroless nickel process. It can usually be soldered to easily using an organic acid flux and RoHS compliant lead-free solder.

**NOTE:** Vacuum deposited nickel electrodes are usually very thin, making soldering somewhat tricky. Use of low wattage soldering irons with small tips and choice of the correct flux (to remove surface oxidation) makes soldering to electrode surfaces easy even under adverse conditions.

To prepare a poled for parallel operation (i.e. 'Y' poled) piezo bimorph for use, a wire must be attached to the center shim. Generally, the center shim layer of a 2-layer piezoelectric bending elements is either .004" (.1mm) thick brass or stainless steel. Shims are soldered to in the same way as the nickel electrode.

#### Tools & Materials for Soldering

- Soldering iron set – 550°- 650° F
- Lead-free Solder
- Supersafe #67 DSA Liquid Flux
- Wires (preferably #30 gauge or smaller)
- Pencil eraser and paper clip

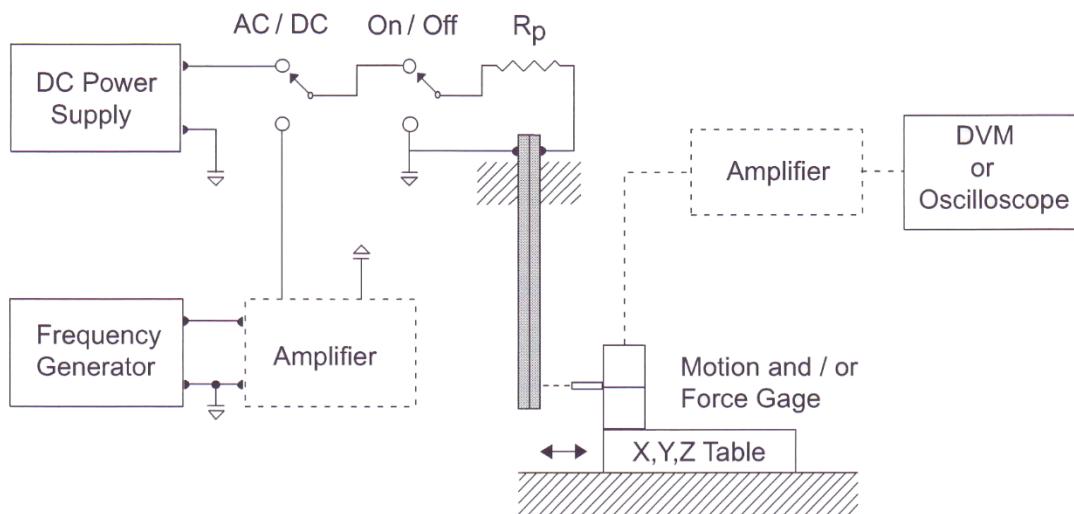
#### Procedure for Soldering

- Clean surface to be soldered with an abrasive (pencil eraser) and wipe with alcohol. This step can usually be skipped when using the proper flux.
- Dip the tip of a paper clip into the flux and apply a small dot of Supersafe Liquid Flux to the electrode area to be soldered.
- Apply small amount of solder to iron tip and transfer solder to the piezoceramic electrode by touching iron tip to flux dot. A good solder joint should flow rapidly ( $\leq 1$  second) and look shiny. Metal shims take longer due to the increased thermal mass ( $\sim 2$  seconds).
- Apply another small dot of Supersafe Liquid Flux to the solder dot on electrode.
- Position pre-tinned wire on solder dot and apply soldering iron to the wire until the solder melts. Remove iron quickly after the solder melts and hold the wire still until the solder solidifies. A 30-gauge wire or smaller is recommended to minimize strain on the solder joint during wire handling.
- Remove Supersafe Liquid Flux residue with clean running water. This flux residue is electrically conductive and must be removed for proper functioning of a piezo device. Any rosin residue may be removed with alcohol.
- Wherever feasible, the wire-solder joint should be strain relieved with a drop of adhesive.

## 7.2 Performance Testing

Generally, measurements of free deflection, blocked force, resonant frequency, and capacitance are easy to make and should be recorded for each design configuration explored. Capacitance is a good measure of the health of the element. If the capacitance of the piece decreases during operation or testing, from that of its initial value, then the element has probably depolarized, cracked, or lost electrical contact during operation.

For fundamental performance testing, a power supply and switching box with an adequate series protection resistor are suggested. Any frequency generator can be used to measure resonant frequency. The resonance patterns may be observed by sprinkling sugar on the piece and sweeping through the frequency band. At resonance, sugar will be tossed off the anti-nodes and collect at the nodes. Suitable means for measuring and deflection, such as an X-Y table and high compliance force gage, are needed. Such a system is shown in *Figure 29*.



**Figure 29.** System for measuring output performance (free deflection, blocked force, and resonant frequency) of a bending motor element

## Resources

The following pages contain all the tables from the manual and a list of symbols.

**TABLE 1. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5A CERAMIC**

<b>PIEZOELECTRIC</b>				
Composition		Lead Zirconate Titanate, Navy Type-II		
Material Designation		PZT-5A		
Relative Dielectric Constant (@1KHz)	K <sub>T<sub>3</sub></sub> K <sub>T<sub>1</sub></sub>	1800 1800		
Piezoelectric Strain Coefficient	d <sub>33</sub> d <sub>31</sub> d <sub>15</sub>	390 x 10 <sup>-12</sup> -190 x 10 <sup>-12</sup> ~550 x 10 <sup>-12</sup>	Meters / Volt Meters / Volt Meters / Volt	
Piezoelectric Voltage Coefficient	g <sub>33</sub> g <sub>31</sub> g <sub>15</sub>	24.0 x 10 <sup>-3</sup> -11.8 x 10 <sup>-3</sup> ~26.0 x 10 <sup>-3</sup>	Volt Meters / Newton Volt Meters / Newton Volt Meters / Newton	
Coupling Coefficient	k <sub>33</sub> k <sub>31</sub> k <sub>15</sub>	0.72 0.32 0.59		
Polarization Field	Ep	2 x 10 <sup>6</sup>	Volts / Meter	
Coercive Field (DC) (@ 60 Hz)	Ec	5 x 10 <sup>5</sup> 6 x 10 <sup>5</sup>	Volts / Meter Volts / Meter	
<b>MECHANICAL</b>				
Density	ρ	7750	Kg / Meter <sup>3</sup>	
Elastic Modulus	Y <sub>E<sub>33</sub></sub> Y <sub>E<sub>11</sub></sub>	4.9 x 10 <sup>10</sup> 6.2 x 10 <sup>10</sup>	Newtons / Meter <sup>2</sup> Newtons / Meter <sup>2</sup>	
Poisson' Ratio	ν	0.31		
Compressive Strength		5.2 x 10 <sup>8</sup>	Newtons / Meter <sup>2</sup>	
Tensile Strength (Static) (Dynamic)		7.5 x 10 <sup>7</sup> 2.0 x 10 <sup>7</sup>	Newtons / Meter <sup>2</sup> Newtons / Meter <sup>2</sup>	
Mechanical Q		80		
<b>THERMAL</b>				
Curie Temperature		350	°C	
Pyroelectric Coefficient		~420 x 10 <sup>-6</sup>	Coulombs / Meter <sup>2</sup> °C	
Thermal Expansion Coefficient		~4 x 10 <sup>-6</sup>	Meters / Meter °C	
Specific Heat	C <sub>p</sub>	440	Joules / Kg °C	

**TABLE 2. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5H, PZT-5A, and PZT-5J PIEZOCERAMIC**

Property	Symbol	Units	Material Type		
			PZT-5H 3203HD	PZT-5A 3195HD	PZT-5J 3222HD
Dielectric Constant (1kHz)	K <sup>T</sup> <sub>3</sub>		3200	1900	2650
Dielectric Loss Factor (1kHz)	Tanδ <sub>e</sub>	%	2.0	0.02	0.02
Dielectric Constant (1kHz)	K <sup>T</sup> <sub>1</sub>			1600	2948
Clamped Dielectric Constant	K <sup>S</sup> <sub>3</sub>		1200	900	800
Density	ρ	g/cm <sup>3</sup>	7.87	7.95	7.90
Curie Point	T <sub>c</sub>	°C	225	350	270
Mechanical Quality Factor	Q <sub>m</sub>		30	80	80
Coercive Field (Measured < 1Hz)	E <sub>c</sub>	kV/cm	8.0	12.0	
Remanent Polarization	P <sub>r</sub>	μCoul/cm <sup>2</sup>	39.0	39.0	
	k <sub>p</sub>		0.75	0.68	0.72
	k <sub>33</sub>		0.75	0.72	0.74
Coupling Coefficients	k <sub>31</sub>		0.43	0.40	0.45
	k <sub>t</sub>		0.55	0.49	0.53
	k <sub>15</sub>		0.78	0.61	0.77
	d <sub>31</sub>	Coul/N x 10 <sup>-12</sup>	-320	-190	-270
Piezoelectric Charge (Displacement Coefficient)	d <sub>33</sub>	or	650	390	485
	d <sub>15</sub>	m/V x 10 <sup>-12</sup>	1000	460	850
	g <sub>31</sub>		-9.5	-11.3	-11.5
Piezoelectric Voltage Coefficient (Voltage Coefficient)	g <sub>33</sub>	V · m/N x 10 <sup>-3</sup>	19.0	23.2	21.3
	g <sub>15</sub>		35.3	32.4	32.6
Frequency Constants Radial	N <sub>r</sub>	kHz · cm			191
Resonant Thickness	N <sub>tr</sub>	kHz · cm	202	211	205
Anti-Resonant Thickness	N <sub>ta</sub>	kHz · cm	236	236	235

**TABLE 2. PIEZOELECTRIC AND MATERIAL PROPERTIES FOR PZT-5H, PZT-5A, and PZT-5J PIEZOCERAMIC, CONTINUED**

Property	Symbol	Units	Material Type		
			PZT-5H 3203HD	PZT-5A 3195HD	PZT-5J 3222HD
Thermal Expansion (Perpendicular to Poling)	$\alpha$	ppm/ $^{\circ}$ C	3.5	3.0	
Specific Heat	$C_p$	J/kg $\cdot$ $^{\circ}$ C J/mol $\cdot$ $^{\circ}$ C	420 138	440 145	
Thermal Conductivity with Au Electrodes	$K_d$	W/cm $\cdot$ $^{\circ}$ C W/m $\cdot$ $^{\circ}$ K W/m $\cdot$ $^{\circ}$ K	1.9-2.3 1.2 1.45	1.9-2.3 1.2 1.45	
Poisson's Ratio	$\nu$		0.31	0.34	0.31
Elastic Constants Short Circuit	$S_{11}^E$ $S_{33}^E$ $S_{12}^E$ $S_{13}^E$ $S_{55}^E$		16.6 21.0 $\times 10^{-12}$ m $^2$ /N -4.8 52.4	15.1 18.6 -7.6 40.0 47.0	15.8 18.8 -5.0 -7.7 12.6
Elastic Constants Open Circuit	$S_{11}^D$ $S_{33}^D$ $S_{55}^D$		13.9 $\times 10^{-12}$ m $^2$ /N 8.8 20.5	12.7 9.0 25.1	12.6 8.5 19.1
Elastic Constants Short Circuit	$Y_{11}^E$ $Y_{33}^E$		6.2 $\times 10^{10}$ N/m $^2$ 4.9	6.6 5.4	6.4 5.3
Elastic Constants Open Circuit	$Y_{11}^D$ $Y_{33}^D$		7.0 $\times 10^{10}$ N/m $^2$ 11.0	7.9 11.1	7.9 11.7

**TABLE 3. SPECTRUM OF COMMON PIEZOELECTRIC TRANSDUCERS**

PIEZOELECTRIC CONFIGURATION	FREE DEFLECTION	BLOCKED FORCE	RESONANT FREQUENCY	GENERAL FEATURES
	CANTILEVER BENDING MOTOR			
	$\frac{3 d_{31} L^2 E}{2 T}$	$\frac{3 d_{31} Y b T^2 E}{8 L}$	$.16 T \sqrt{\frac{Y_{11}}{\rho}}$	\$1 - \$100 10 - 500 Hz 10 - 500 grams 5 mm
	SIMPLE BENDING MOTOR			
	$\frac{3 d_{31} L^2 E}{8 T}$	$\frac{3 d_{31} Y b T^2 E}{2 L}$	$.48 T \sqrt{\frac{Y_{11}}{\rho}}$	
	$d_{31} L E$	$d_{31} Y A E$ where $A = b T$	$\frac{1}{2 L} \sqrt{\frac{Y_{11}}{\rho}}$	INCREASING COST INCREASING RESONANT FREQUENCY INCREASING FORCE INCREASING DISPLACEMENT
	$d_{33} L E$	$d_{33} Y A E$ where $A = a b$	$\frac{1}{2 L} \sqrt{\frac{Y_{33}}{\rho}}$	
	$d_{15} T E$	$d_{15} G A E$ where $A = b L$	$\frac{1}{2 T} \sqrt{\frac{Y_{55}}{\rho}}$	\$100 1 MHz $10^3$ kg $\mu M$

**TABLE 4. SPECTRUM OF COMMON PIEZOELECTRIC GENERATORS**

PIEZOELECTRIC CONFIGURATION	SHORT CIRCUIT CHARGE	OPEN CIRCUIT VOLTAGE	RESONANT FREQUENCY
CANTILEVERED BENDING ( D31 ) GENERATOR			
	$\frac{3 L^2}{2 T^2} d_{31} F$ $\frac{3 T W}{8 L} Y d_{31} \Delta X$	$\frac{3 L}{2 W T} g_{31} F$ $\frac{3 T^2}{8 L^2} Y g_{31} \Delta X$	$\frac{.16 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$
SIMPLY SUPPORTED BENDING ( D31 ) GENERATOR			
	$\frac{3 L^2}{8 T^2} d_{31} F$ $\frac{3 T W}{2 L} Y d_{31} \Delta X$	$\frac{3 L}{8 W T} g_{31} F$ $\frac{3 T^2}{2 L^2} Y g_{31} \Delta X$	$\frac{.48 T}{L^2} \sqrt{\frac{Y_{11}}{\rho}}$
SIMPLY SUPPORTED DISK BENDING (D31) GENERATOR			
	$\frac{.42 D^2}{T^2} d_{31} F$ $3.1 T Y d_{31} \Delta X$	$\frac{.56}{T} g_{31} F$ $\frac{4.1 T^2}{D^2} Y g_{31} \Delta X$	$\frac{T}{D^2} \sqrt{\frac{Y_{11}}{\rho}}$
TRANSVERSE ( D31 ) GENERATOR			
	$\frac{L}{T} d_{31} F$ $W Y d_{31} \Delta X$	$\frac{1}{W} g_{31} F$ $\frac{T}{L} Y g_{31} \Delta X$	$\frac{1}{2 L} \sqrt{\frac{Y_{11}}{\rho}}$

**TABLE 4. SPECTRUM OF COMMON PIEZOELECTRIC GENERATORS, CONTINUED**

PIEZOELECTRIC CONFIGURATION	SHORT CIRCUIT CHARGE	OPEN CIRCUIT VOLTAGE	RESONANT FREQUENCY
LONGITUDINAL ( D33 ) GENERATOR			
	$d_{33} F$	$\frac{T}{L W} g_{33} F$	$\frac{1}{2 T} \sqrt{\frac{Y_{33}}{\rho}}$
	$\frac{L W}{T} Y d_{33} \Delta X$	$Y g_{31} \Delta X$	
MULTI-LAYER LONGITUDINAL ( D33 ) GENERATOR			
	$n d_{33} F$	$\frac{T}{n L W} g_{33} F$	$\frac{1}{2 T} \sqrt{\frac{Y_{33}}{\rho}}$
	$\frac{L W}{T} Y d_{33} \Delta X$	$\frac{Y g_{31} \Delta X}{n^2}$	
PARALLEL SHEAR ( D15 ) GENERATOR			
	$d_{15} F$	$\frac{T}{L W} g_{15} F$	$\frac{1}{2 T} \sqrt{\frac{Y_{55}}{\rho}}$
	$\frac{L W}{T} G d_{15} \Delta X$	$L W G g_{15} \Delta X$	
TRANSVERSE SHEAR ( D15 ) GENERATOR			
	$\frac{L}{T} d_{15} F$	$\frac{g_{15} F}{W}$	$\frac{1}{2 L} \sqrt{\frac{Y_{55}}{\rho}}$
	$W G d_{15} \Delta X$	$\frac{T}{L} G g_{15} \Delta X$	

**TABLE 5. LIST OF SYMBOLS**

Symbol	Name	Unit
A	Area	m <sup>2</sup>
C	Capacitance	F
D	Diameter	m
D <sub>i</sub>	(i=1 to 3) Dielectric displacement	C / m <sup>2</sup>
d <sub>ij</sub>	(i=1 to 3) (j=1 to 6) Piezoelectric charge constants	C / N
E <sub>i</sub>	(i=1 to 3) Electric field components	V / m
E <sub>c</sub>	Coercive field	V / m
F <sub>r</sub>	Resonant frequency	kHz
F	Force	N
F <sub>b</sub>	Blocking Force	N
g <sub>ij</sub>	(i=1 to 3) (j=1 to 6) Piezoelectric voltage constants	V m / N
G	Shear modulus	N / m <sup>2</sup>
k	Electromechanical coupling coefficient	
k <sub>33</sub>	Longitudinal coupling coefficient	
k <sub>31</sub>	Transverse coupling coefficient	
k <sub>15</sub>	Shear coupling coefficient	
k <sub>p</sub>	Planar coupling coefficient	
k <sub>t</sub>	Thickness coupling coefficient	
k <sub>eff</sub>	Effective coupling coefficient	
K <sub>S</sub>	(i=1 to 3) (j=1 to 3) Relative dielectric constant at constant strain	
K <sub>T</sub>	(i=1 to 3) (j=1 to 3) Relative dielectric constant at constant stress	
L	Length	m
p	Pressure	N / m <sup>2</sup>
p	Pyroelectric coefficient	C / m <sup>2</sup> K
P <sub>i</sub>	(i=1 to 3) Polarization components	C / m <sup>2</sup>
P	Power	W
Q	Mechanical quality factor	
Q	Electric charge	C

**TABLE 5. LIST OF SYMBOLS, CONTINUED**

Symbol	Name	Unit
$Q_s$	Short circuit charge	C
R	Electrical resistance	$\Omega$
$s_{ij}E$ (i=1 to 6) (j=1 to 6)	Elastic compliance at constant E	$m^2 / N$
$s_{ij}D$ (i=1 to 6) (j=1 to 6)	Elastic compliance at constant D	$m^2 / N$
$S_i$ (i=1 to 6)	Strain components	
$S_{max}$	Maximum recommended strain	
t	Time	s
t	Response time	s
t	Thickness	m
r		
$T_i$ (i=1 to 6)	Stress components	$N / m^2$
V	Volume	$m^3$
$V_o$	Open circuit voltage	V
$v_s$	Velocity of sound	$m / s$
x	Deflection	m
$x_0$	Free deflection	m
	Thermal expansion coefficient	$1 / K$
$\epsilon_0$	Dielectric constant of free space	$F / m$
	Density	$kg / m^3$
	Electrical bulk resistivity	$\Omega/m$