Exercise 4.32

We consider a DTMC $\{X_n\}_{n \in \mathbb{N}_0}$ governed by

$$P = \begin{bmatrix} 1/3 & 0 & 1/3 & 0 & 0 & 1/3 \\ 1/2 & 1/4 & 1/4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1/4 & 1/4 & 1/4 & 0 & 0 & 1/4 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

We classify the states as follows.

State 5 is absorbing. (Hence recurring)

Once the process enters the class (2,4) it can never leave. Since it will constantly switch between 2 and 4, states 2 and 4 are recurrent.

~~States 0, 1, and 3 can all eventually~~ /////

From states 0, 1, and 3, the process can always transition to one of states (2,4,5) and consequently ~~for~~ $f_{00}, f_{11}, f_{33} < 1$.

Thus, states 0, 1, and 3, are nonrecurrent and therefore transient, cf. first paragraph on p. 199.