DTU Danmarks Tekniske Universitet

Written exam, 22 May, 2024. Page 1 of 6 pages
Course name: Decisions under uncertainty
Course number: 42586
Allowed aids: All aids allowed (closed internet).
Duration: 4 hours
Weights of questions: The exam consists of **6** Problems. The points allocated to each problem and question are indicated where relevant. The maximum number of points is 100.

NOTE:

(A) Please submit a pdf document with all the answers included there.

(B) If you think a question is ambiguous, then point out the ambiguity in your answer and explain how you have chosen to interpret the question.

(C) Remember to explain your calculations and answers. *A correct computation without any explanation does not give many points.*

Problem 1: Friends Party Ride Options (Weight: 20 points)

Responsible: Bissan Ghaddar

It is Saturday morning and you realize that the sky is cloudy and it is not clear if it will rain or not. You are organizing a party with your friends on this day as it is the end of the exam period and you want to celebrate. It is important to be on time as much as possible as you are in charge of the drinks and setting up the location.

As the weather changes rapidly in Copenhagen, there is a probability p that the weather gets better and it is sunny, and $1 - p$ that the weather actually gets worse and it rains. You have 3 different options to go to the party location. They all imply a certain cost, as well as how late you will be at the party for setting up the location. These are

- use your bike. It does not cost you anything. If the weather gets better, you should be able to get there on time. However, if the weather gets worse, you will be 25 minutes late;
- take the bus 2A. The bus ticket will cost you 24 DKK, but it should give you a better opportunity to be there on time. You have to walk to the bus stop first though, and this was not planned. If the weather gets better, you should be able to be 5 minutes late only. However, if the weather gets worse, buses may also be delayed, and you will be 15 minutes late;
- use a ShareNow electric car. There, the price is much higher (65 DKK), but at least, the situation should be more under control. As it takes time to walk around to find a car, even if the weather gets better, you will be 2 minutes late. And, if the weather gets worse, you should only be 5 minutes late.

Putting these cost figures into perspective, the cost of being late for the party would be equivalent to 6 DKK per minute you are late.

Question 1.1. (Weight: 3 points) Describe the decision-making problem, i.e.,

- who is the decision-maker and what decisions can be made?
- what is the exogenous variable (from the considered environment) that affects the system, and what are the potential outcomes?

Question 1.2. (Weight: 4 points) Calculate the pay-off table for all mode choices and weather outcomes (rain or no rain).

Question 1.3. (Weight: 4 points) What decisions will you obtain if you apply the *Maximax* and the *Maximin* strategies?

Question 1.4. (Weight: 4 points) Deduce the regret table and find the best decision if following a *Minimax regret* strategy.

Question 1.5. (Weight: 5 points) Formulate your *Expected Utility Maximization* (EUM) strategy, as a function of p. Solve it for the case of $p = 0.3$. What is the decision you obtain with that EUM strategy for this value of p?

Problem 2: Snacks (Weight: 22 points)

Responsible: Evelien van der Hurk

The cafe in 358 considers to add a new snack alternative to their assortment. Everytime the inventory is 0, 4 new snacks are ordered and arrive the next day at the opening of the cafe (before demand for that day arrives). Let's assume the snacks sell often enough to ignore the good-by-date. The available inventory of snacks at the cafe at the end of the day can then be modelled as a Markov process. The demand for snacks can be described as:

demand for snacks	probability
0	0.1
1	0.2
2	0.2
3	0.2
4	0.2
5	0.1

Question 2.1. (Weight: 4 points) Is the Markov process describing the available inventory of snacks per day a discrete or continuous time Markov chain? Motivate your answer.

Question 2.2. (Weight: 4 points) Next we would like to calculate the one-step transition matrix P. Write out the formula, including numbers belonging to the snack inventory case, for the calculations for $p_{00}, p_{02}, p_{20}, p_{24}$. Explain in words what p_{02} represents.

The full 1-step transition matrix P is described as:

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.3 & 0.2 & 0.2 & 0.2 & 0.1 \\ 0.9 & 0.1 & 0 & 0 & 0 \\ 0.7 & 0.2 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0.1 & 0 \\ 0.3 & 0.2 & 0.2 & 0.2 & 0.1 \end{bmatrix} \end{array}$$

Question 2.3. (Weight: 4 points) Calculate the steady-state probabilities using P.

Question 2.4. (Weight: 4 points) Calculate the expected first passage time to go from no inventory, to no inventory again.

Question 2.5. (Weight: 6 points) Calculate the expected costs of the current inventory policy provided the following costs:

- a fixed costs of 100DKK per order of additional snacks
- a loss of possible sales fee of 40DKK for every unit of unsatisfied demand (so when demand was higher than inventory).

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Problem 3: Maple Syrup Stall (Weight: 9 points)

Responsible: Evelien van der Hurk.

The owner of a Canadian maple syrup stall asks for your advice as a queuing expert. They would like your help analyzing their service.
They observed that the previous weekends there were, on average, 20 customers in the queue, and customers spent on average 3.5 minutes in the system. It takes about 2 minutes to serve a customer.

Question 3.1. (Weight: 6 points) Calculate the arrival rate λ in customers per hour, the average time spent in the queue W_q and the expected number of customers in the system L, for the Maple Syrup Stall.

Question 3.2. (Weight: 3 points) How many servers does the above system need (as a minimum) to ensure a stable queuing system, given that a single server can serve a customer in 2 minutes? Motivate your answer.

General formulas:

$$W_q = \frac{L_q}{\lambda}$$
$$W = W_q + \frac{1}{\mu}$$
$$L = \lambda W = \lambda \left(W_q + \frac{1}{\mu} \right) = L_q + \frac{\lambda}{\mu}$$

Problem 4: Optimal seats for a Theatre (Weight: 17 points)

Responsible: Bissan Ghaddar

I am the manager of a Theatre in Copenhagen, Giulia Production (GP), preparing to host an event on May 24th. Basically, the main point is to decide on the number of chairs to put in the theatre so that the audience fit, given potential demand and revenues. There are two types of chairs: fancy and regular. The aim of Giulia Production is to make a decision in advance on the number of chairs (fancy and regular) to have in the Theatre, in order to maximize revenue in expectation.

To solve this problem, analysts are slightly simplifying the pricing and cost setup. Important figures include:

- operating costs for GP are 100 DKK per seat, for both fancy and regular chairs,
- the price at which a single seat is sold is 900 DKK for regular chairs, and 1300 DKK for fancy ones;
- in case of overbooking (i.e., the number of people having bought tickets is more than the available chairs), GP has to give a voucher with 1200 DKK per client who did not get a regular seat, and 2000 DKK for fancy seats;
- for each non-occupied seat, the theatre obviously does not get any revenue.

In parallel, an analytics consultancy provides GP with probabilistic demand forecasts for these two sets of seats

- for regular seats, the demand ω follows a Normal distribution, $\omega \sim \mathcal{N}(90, 3)$ (mean of 90, and variance of 3);
- for fancy seats, the demand ω also follows a Normal distribution, but with different characteristics, $\omega \sim \mathcal{N}(115, 6)$ (mean of 115, and variance of 6).

Question 4.1. (Weight: 3 points) Is that a newsvendor problem? How do you recognize that?

Question 4.2. (Weight: 4 points) If so, what are the underage and overage penalties (for both fancy and regular seats)?

Question 4.3. (Weight: 5 points) What is the optimal number of seats for both regular and fancy seats? (in this case it is fine to round it to the nearest integer values since seats are indivisible).

Question 4.4. (Weight: 5 points) Assuming that on May 24th, we received a booking of 80 regular and 120 fancy seats. What is the Total Profit for GP in this case.

Problem 5: Relocation of Carsharing vehicles from Copenhagen to Lyngby (Weight: 20 points)

Responsible: Bissan Ghaddar

You manage a relocation company that handles the repositioning of empty cars for different carsharing companies. Your business model is hybrid, since you have trucks that can be used to move cars, but you also subcontract a company that can do that for you. The subcontractor might offer a good price, *but you can not be sure in advance.* Just now, ShareNow has asked you to relocate at least 20 cars from Copenhagen to Lyngby. You have several options to deliver,

- do that yourself: it will only cost you the transportation costs
- get the subcontractor to transport the cars for you

You have to decide on the number x_1 of cars to transport now. You can only transport up to 12 cars though, and it costs you $c = 35$ DKK/car. This means you will also have to decide on the number x_2 of cars to get transported by the subcontractor later on, when the corresponding transportation costs are known.

Even though you cannot be sure of which price you will get, you know there are $K = 4$ scenarios (denoted ω_k) for transportation costs, with corresponding probabilities p_k ($k = 1, \ldots, K$). These are summarized in the following table.

Question 5.1. (Weight: 4 points) Identify both the here-and-now and the recourse decisions for this stochastic linear program. Also, what is the uncertain parameter in this car transportation problem?

Question 5.2. (Weight: 6 points) Formulate (mathematically) the problem as a stochastic linear program with recourse. The model should minimize expected costs. Directly write it in its deterministic equivalent form. Is that a stochastic linear program with fixed recourse (remember to justify your answer)? *You do not have to solve it.*

Question 5.3. (Weight: 5 points) Similarly, formulate mathematically the expected value (EV) problem. *You do not have to solve it.*

Table 1: Transportation costs for the $K = 4$ scenarios ($k = 1, \ldots, K$).

Scenario k	Probability p_k	transportation Costs ω_k [DKK/car]
1	0.35	15
2	0.3	31
3	0.1	200
4	0.25	30

Question 5.4. (Weight: 5 points) When considering a stochastic linear program like the one we have here, what relationship do you expect between (i) the objective function value of the stochastic linear program (call it RP), and (ii) the expected costs over all scenarios (call it EEV) of the expected value (EV) solution?

Problem 6: Street Food Venture (Weight: 12 points)

Responsible: Bissan Ghaddar

You are starting a small street food stand of hotdogs and would like to satisfy as much customers as possible. You estimate the number of customers follows a normal distribution with mean μ= 40 and standard deviation $\sigma = 12$. To keep things simple, assume that you are planning to buy two types of hotdogs (Type 1 and 2). The unit cost for buying the hotdogs is 40 DKK and 65 DKK for types 1 and 2 respectively. Now, assume that each customer will consume either two units of hotdog type 1 or one unit of hotdog type 2. You want to minimize your costs, but at the same time make sure that you have enough food for all customers with a certainty level of 95%.

Question 6.1. (Weight: 5 points) Write the resulting chance-constrained problem.

Question 6.2. (Weight: 7 points) Write the deterministic linear reformulation of the above problem. Show your work. *You do not have to solve the linear optimization problem.*