Problem 3.4.4 (1)

Define T as the time to absorption, and $p_i = \mathbb{P}(X_1 \neq 2, \ldots, X_{T-1} \neq 2 \mid X_0 = i)$ for $i = 0, 1, 2, 3$. Using FSA, we establish the system

$$p_1 = 0.1 + 0.2\,p_1 + 0.2\,p_3$$
$$p_3 = 0.2 + 0.2\,p_1 + 0.3\,p_3$$

The solution is $(p_1, p_3) = (11/52, 9/26)$.

ALTERNATIVELY:

Consider it as a problem with two absorbing states and calculate the prob. of being absorbed in state 0.

Problem 3.4.4 (2)

Define A as the event that the process is absorbed without visiting state 2.

Furthermore, let $p_i = \mathbb{P}(A \mid X_0 = i)$ for $i = 0, 1, 2, 3$. Then we can establish the system

$$p_1 = 0.1 + 0.2 p_1 + 0.2 p_3,$$
$$p_3 = 0.2 + 0.2 p_1 + 0.3 p_3,$$

which has the solution $(p_1, p_3) = \left(\frac{11}{52}, \frac{9}{26} \right)$