DTU Danmarks Tekniske Universitet

Written exam, 23 May, 2023. Page 1 of 9 pages
Course name: Decisions under uncertainty
Course number: 42586
Allowed aids: All aids allowed (closed internet).
Duration: 4 hours
Weights of questions: The exam consists of **6** Problems, with a total of **21** questions. The points allocated to each problem and question are indicated where relevant. The maximum number of points is 100.

NOTE:

(A) If you think a question is ambiguous, then point out the ambiguity in your answer and explain how you have chosen to interpret the question.

(B) Remember to explain your calculations and answers. *A correct computation without any explanation does not give many points.*

Problem 1: Planning a first date (Weight: 20 points)

Responsible: Evelien van der Hurk

A friend of yours has recently met someone new and now needs to decide on the activity for the first date. The options are:

(A) Going out for a coffee

(B) Going out for dinner

(C) Going to a movie

To help make this decision, you propose that your friend assigns a utility score between 0 and 100 to each possible activity. However, the utility depends on how good the date will actually turn out.

(A) **coffee:** When the date is good, this is an excellent option, as one could easily plan a secondary activity afterward (utility = 80). If the date is bad, at least one can make it short (utility= 60).

(B) **dinner:** In case of a good date, this can be a great option with a high probability for a next date (utility = 100). However, in case the date is bad it can become very awkward and actually is also relatively expensive (utility = 10)

(C) **movie**: As you get to select a movie you like anyways, the date being good has a limited impact: utility=70 if the date is good, utility=65 when the date is bad.

Let's assume that the probability of a date going well or not, e.g. there being a match, is independent on the activity.

Question 1.1. (Weight: 3 points) Describe the decision-making problem, i.e., (i) who is the decision-maker and what decisions can be made? (ii) what is/are the exogenous variable(s) from the environment that affects the system, and what are the potential states of nature (i.e., values for the exogenous variable)?

Question 1.2. (Weight: 3 points) Deduce the payoff table, i.e., calculate the utility for all potential combinations of decisions and states of nature.

Question 1.3. (Weight: 4 points) What decisions will you obtain if following *Maximax* and *Maximin* strategies? Shortly motivate the answer.

Question 1.4. (Weight: 4 points) Deduce the regret table and find the best decision if following a *Minimax regret* strategy.

Question 1.5. (Weight: 6 points) Formulate the *Expected Utility Maximization* (EUM) strategy, as a function of p being the probability the date goes well (and thus, 1-p for the date not going well). Solve it for the case of $p = 0.6$. What are the decisions you obtain with that EUM strategy for this value of p? For what value(s) of p would you select the same strategy as the Maximax strategy in this example?

Problem 2: KleenHub Cups (Weight: 18 points)

Responsible: Evelien van der Hurk

The cafe in 358 participates in the KleenHub cups initiative, where users get to borrow a sustainable reusable cup in which their coffee is served instead of a single use cup. The Kleenhub cup needs to be returned to any participating cafe within 10 days of first use. Let's assume we can model the available inventory of KleenHub cups per day at the cafe as a Markov process, where per day the net balance of the sum of total returns minus total demand follows the below distribution:

Net cup-demand balance	probability
-3	0.05
-2	0.05
-1	0.1
0	0.35
1	0.2
2	0.15
3	0.1

So, a negative value indicates higher demand than returns, and a positive value indicates higher returns than demand.

When the inventory reaches 0 cups at the end of the day, 3 new cups are delivered to the cafe by the start of the next day. When more than 3 cups are in inventory at the end of the day, the surplus cups are sent back to the warehouse at the end of the day. Let the state denote the number of cups in inventory at the cafe after sending the surplus to the warehouse, and before the arrival of the new cups in case the inventory at the end of the day was zero.

Question 2.1. (Weight: 3 points) Is this a discrete or continuous Markov chain? Motivate your answer.

Question 2.2. (Weight: 6 points) Calculate the 1-step transition matrix, and show/motivate your calculations.

Question 2.3. (Weight: 4 points) Calculate the steady-state probabilities

Question 2.4. (Weight: 5 points) Assume that not having enough cups represents a loss of possible coffee sales of 30DKK per cup the cafe is short. Calculate the expected costs of the current inventory policy.

Problem 3: At the coffee bar (Weight: 12 points)

Responsible: Evelien van der Hurk.

The owner of a coffee bar in London asks for your advice as a queuing expert. They would like your help analyzing their service.
They observed that the previous weekends there were, on average, 5.59 customers in the queue, and customers spent on average 8.4 minutes in the system. It takes about 2 minutes to serve a customer.

Question 3.1. (Weight: 6 points) Calculate the arrival rate λ in customers per hour, the average time spent in the queue W_q and the expected number of customers in the system L.

Question 3.2. (Weight: 3 points) How many servers does the above system need (as a minimum) to ensure a stable queuing system, given that a single server can serve a customer in 2 minutes?

Question 3.3. (Weight: 3 points) On observation, the manager indicates that during busy times customers may join the queue and leave again before being served. What is the name for this phenomenon (in the queuing literature)? Describe how the fact that customers may leave before being served when the queue is long, impacts state-dependent arrival rate and/or departure rate (e.g. as in: lower, higher, or the same). Motivate your answer.

General formulas:

$$
\begin{aligned}
W_q &= \frac{L_q}{\lambda} \\
W &= W_q + \frac{1}{\mu} \\
L = \lambda W = \lambda \left(W_q + \frac{1}{\mu} \right) &= L_q + \frac{\lambda}{\mu}
\end{aligned}
$$

Problem 4: Simulation of a supermarket (Weight: 15 points)

Responsible: Evelien van der Hurk

To analyze the service level of the current self-service counter set-up at the local supermarket, the students of course 42586 have been asked to program a simulation. The two main aspects the manager of the supermarket cares about are

- waiting times for their clients, and
- revenues (since to decrease waiting times, they may have to spend more money on the check-out design)

You perform a pilot study for a proposal setup over a representative day of operations of the supermarket. The resulting waiting times and revenues from 20 replicates are shown in the following table.

Table 1: Waiting times (in minutes) and revenues (in 1000 DKK) for 20 replicates of the operations of a local supermarket over a single representative day.

Replicate	Waiting time [mins]	Revenue [kDKK]
1	1.2	197
2	3.5	830
3	5.7	140
4	6.8	523
5	4.2	115
6	5.8	421
7	8.9	150
8	9.0	801
9	5.2	195
10	6.5	185
11	1.2	197
12	4.5	143
13	6.7	540
14	4.1	240
15	5.2	110
16	5.1	1210
17	10.9	150
18	9.4	1010
19	7.2	955
20	2.5	285

When presenting the results to the management of the local supermarket, they inform you they are interested in expected revenues and waiting time. Though, they will only trust your outcome if you can promise that the precision of your estimate of the expectation is within

- ± 0.5 minutes for the waiting time, and

- ±100 DKK for the revenues (note table values are in 1000DKK, so 100DKK is 0.1 in the table),

in both cases with 95% confidence.

Question 4.1. (Weight: 5 points) Assuming you can use a Gaussian assumption for the calculation of confidence intervals (based on 20 replicates), what are the expected waiting time and revenue, as well as associated 95% confidence intervals for these estimates?

Question 4.2. (Weight: 5 points) In order to respect the precision and confidence required by the supermarket manager, also assuming you can use a Gaussian assumption, what is the minimum number of replicates you would need to use? (note that the number of replicates may be different when focusing on waiting time and revenues)

Question 4.3. (Weight: 5 points) The manager would like you to simulate two different types of lay-outs and draw a conclusion on which of these lay-outs would be best. The smartest way would be to make comparable runs for the two lay-out settings, so that a difference in waiting time and revenue *per run* indicates the one lay-out being better than the other. To this end, where should one set the seed? At the start of the full simulation framework, or at the start of the simulation per day? Motivate your answer.

Problem 5: TVs-vendor dilemma (Weight: 15 points)

Responsible: Dario Pacino

An electronics store needs to make its quarterly order of products. The manager is a bit concerned about the sales of TVs. Online stores are taking over the market; hence it is important to find the correct amount of TVs to order. Each TV set can be purchased for 1200 DKK. The store can then sell a TV set for 3000 DKK.

If the manager orders more TVs than the actual demand, the store will be forced to sell them at a discounted price of 1000 DKK. When the store runs out of TVs, the cost is equivalent to the lost revenue associated with the potential sale.

Question 5.1. (Weight: 3 points) Assuming that the quarterly demand of TVs follows a normal distribution (with $\mu = 30$ and $\sigma = 10$), what are the overage and underage penalties, and what is the uncertain parameter?

Question 5.2. (Weight: 6 points) How can this problem be solved? show the mathematical problem you are trying to solve, the analytical solution, and the optimal number of TVs the manager should order (you do not need to provide an integer value).

Note: You are welcome to use the distribution tables made available in the appendix to solve this problem. Alternatively, code it in Julia.

Question 5.3. (Weight: 6 points) The marketing department has collected historical data and argues that TV demand does not follow a normal distribution. They now provide you with a forecast-based set of K scenarios. The parameter ω_k represents the demand, and p_k is the probability for scenario $k \in K$.

Model the problem as a stochastic linear model. Explain the variable, parameters, objective, and constraints of the model.

Problem 6: Workforce planning (Weight: 20 points)

Responsible: Dario Pacino

A container terminal needs to plan next month's work assignments. The manager can choose to hire full-time employees or part-time employees. Full-time employees are hired for a total of $h = 132$ hours at a fixed cost of $c = 45.000$ DKK. Part-time employees are paid by the hour and cost $q = 500$ DKK/hour.

The marketing department has estimated that the demand for work-hours for next month is $D = 2500$. The optimal solution to this problem can be solved by the following mathematical model, where the variables represent: x the number of full-time employees, z the number of part-time employees, and y the total number of hours the part-time employees are working.

$$\begin{aligned}
\min_{x,y,z} \quad & cx + qy && (1)\\
s.t. \quad & hx + y \geq D && (2)\\
& y \geq hz && (3)\\
& x, y, z \in \mathbb{Z}_+ && (4)
\end{aligned}$$

The objective function (1) minimizes the total cost given by the fixed cost per full-time employee and the hourly cost for the part-time employees. Constraint (2) ensures that the total number of hours employed fulfills the demand. Constraint (3) ensures that the total number of part-time hours cannot exceed what the number of part-time employees can work (one employee can work at most h hours a month). Finally, constraint (4) imposes the variables to be positive integers.

Question 6.1. (Weight: 8 points) The manager knows that the demand estimate from the marketing department is very conservative, and asks for a better estimate. The marketing department responds that they looked at the historical data and fitted the demand to a normal distribution with $\mu = 2400$ and $\sigma = 300$. The manager asks you to relax the demand constraint (2) so that it holds at least 95% of the time.

Reformulated the problem to follow the wishes of the manager. Comment on the reformulation, and show it using chance constraint and in its deterministic equivalent.

Question 6.2. (Weight: 4 points) You talk to the manager and find out that part-time employees can be employed on a daily basis, hence you can postpone the decision to a later stage. You realize the problem can be modeled as a two-stage stochastic problem with fixed recourse.

What are the here-and-now decisions? what are the recourse decisions, and what is the stochastic parameter?

Question 6.3. (Weight: 8 points) Based on your answer to Question 6.2, present a two-stage stochastic model with fixed recourse in deterministic equivalent form. Describe the model, and how you handled the stochastic parameter.

Appendix A CDF Tables for Normal distribution

$\mu = 60$ and $\sigma = 10$	
x	CDF
0	0,0013499
2	0,00255513
4	0,00466119
6	0,00819754
8	0,0139034
10	0,0227501
12	0,0359303
14	0,0547993
16	0,0807567
18	0,11507
20	0,158655
22	0,211855
24	0,274253
26	0,344578
28	0,42074
30	0,5
32	0,57926
34	0,655422
36	0,725747
38	0,788145
40	0,841345
42	0,88493
44	0,919243
46	0,945201
48	0,96407
50	0,97725
52	0,986097
54	0,991802
56	0,995339
58	0,997445
60	0,99865

$\mu = 2400$ and $\sigma = 300$	
x	CDF
1500	0,0013499
1572	0,00289007
1644	0,00586774
1716	0,0113038
1788	0,0206752
1860	0,0359303
1932	0,0593799
2004	0,0934175
2076	0,140071
2148	0,200454
2220	0,274253
2292	0,359424
2364	0,452242
2436	0,547758
2508	0,640576
2580	0,725747
2652	0,799546
2724	0,859929
2796	0,906582
2868	0,94062
2940	0,96407
3012	0,979325
3084	0,988696
3156	0,994132
3228	0,99711
3300	0,99865
3372	0,999402
3444	0,999749
3516	0,9999
3588	0,999963
3660	0,999987