Forecasting I

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42586 – Decisions under Uncertainty – 5 March 2024

Learning Objectives

At the end of this lecture (and related hands-on session), a student should be able to:

- Define a **time series** and its various characteristics
- Describe various **applications** of times series
- Understand the different types of **forecasting models**
- Use different **measures** to evaluate forecasting models

Outline

1. Times series forecasting applications
2. Characteristics of time series
3. Trends and Seasonality
4. Evaluating forecasting models

Applications: Transport

You're the logistics manager for a taxicab company in New York, and you need to decide how many drivers you will ask to come in this coming Thursday.

Applications: Energy

You are a manager at a utility company and you need to decide how much energy to produce for the next day.

- Electric Load Forecasting
- Electricity Price Forecasting
- Renewable Energy Forecasting (wind, solar)
- Solar Power Forecasting

Applications: Retail

You are a manager at a big retail store and you need to forecast the demand to decide how much to order of Nutella for next month.

The Importance of Forecasting

- **Governments** forecast unemployment rates, interest rates, and expected revenues from income taxes for **policy** purposes.
- Marketing **executives** forecast demand, sales, and consumer preferences for **strategic planning**.
- College **administrators** forecast enrollments to **plan for facilities** and for faculty **recruitment**.
- **Retail** stores forecast demand to **control inventory** levels, **hire** employees and provide **training**.
- **Hospitals** forecast a certain class of patients to **estimate recourses**.

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- Time series **analysis** extracts meaningful statistics and characteristics from the time series data (trends, seasonality, etc.).
- Time series **forecasting** is the use of a model to predict future values based on time and previously observed values.

Time Series: Data

- Types of time series
  - continuous
  - discrete
- **Discrete** means that observations are recorded in discrete times - it says nothing about the nature of the observed variable
- The time intervals can be annually, quarterly, monthly, weekly, daily, hourly, etc.
- **Continuous** means that observations are recorded continuously -e.g. temperature and/or humidity in some laboratory

Time Series Forecasting

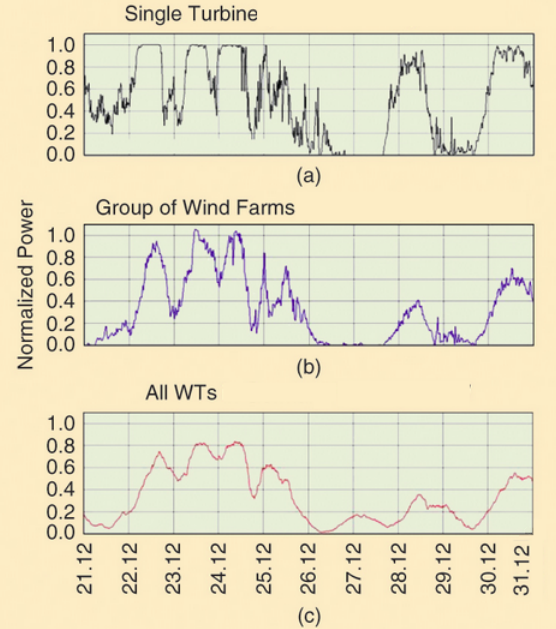
Horizon:

- Short-term (days, weeks, months) $\rightarrow$ operations
- Medium-term (1 or 2 years) $\rightarrow$ budgeting and R&D
- Long-term (many years) $\rightarrow$ strategic planning

A time series plot is a two-dimensional plot of the time series data

- the vertical axis measures the variable of interest
- the horizontal axis corresponds to the time periods

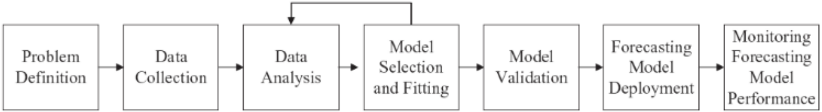
Time Series: Example



Time Series: Uses

Objectives of time series analysis:

- description: summary statistics, plots and graphs
- analysis and interpretation: find a model to describe the time dependence in the data, can we interpret the model?
- forecasting or prediction: given a sample from a time series, forecast the next value, or the next few values



Time Series: Type of Variation

Trend: long-term change in the mean level.

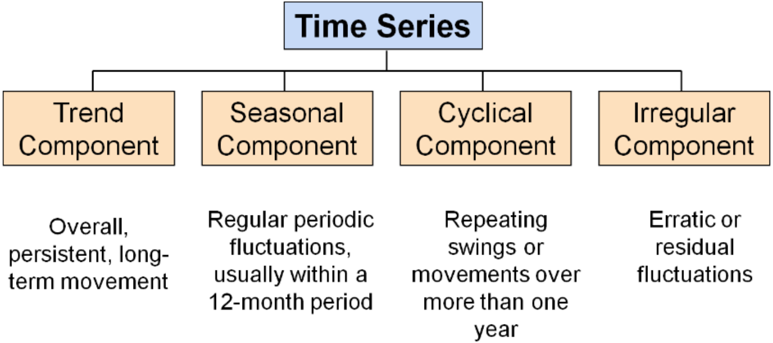
Seasonal variation: regular and predictable patterns that repeat over a specific period, such as a day, week, month, or season.

Cyclic variation: fluctuations occurring at irregular intervals longer than seasonal effects, often related to economic or business cycles.

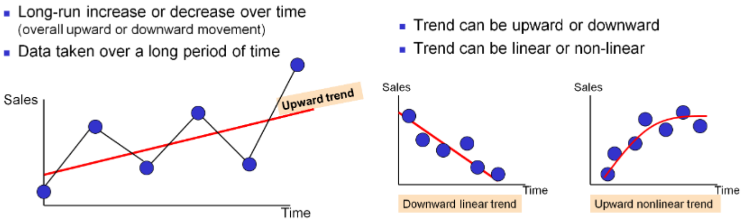
Examples

- Sales of a given item increase due to population increase.
- Swimsuit sales are high in the summer and low in the winter.
- Daily variation in temperature is high at noon and low at night.
- Economic data is affected by business cycles.

Time Series: Components



Time Series: Trend

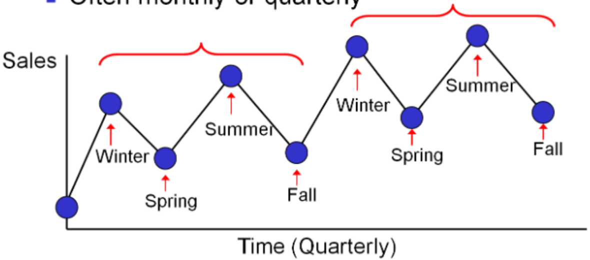


Does The Time-Series have a Trend Component?

- A time-series plot should help answer this question.
- Often it helps if you smooth the time series data to help answer this question.
- a popular smoothing method is the moving average.

Time Series: Seasonal

- Short-term regular wave-like patterns
- Observed within 1 year
- Often monthly or quarterly



Time Series: Irregular/Random

- Unpredictable, random, residual fluctuations.
- Noise in the time series.
- Stochastic factors.

Time Series: Characteristics

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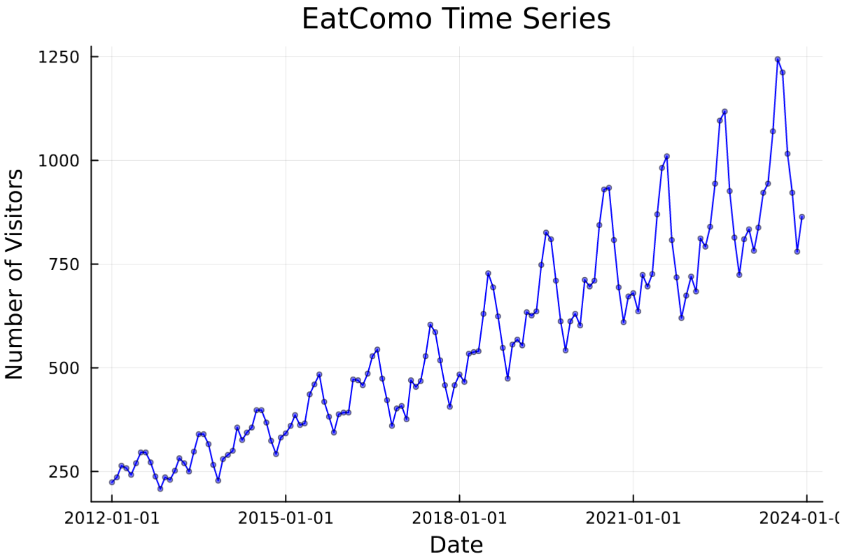
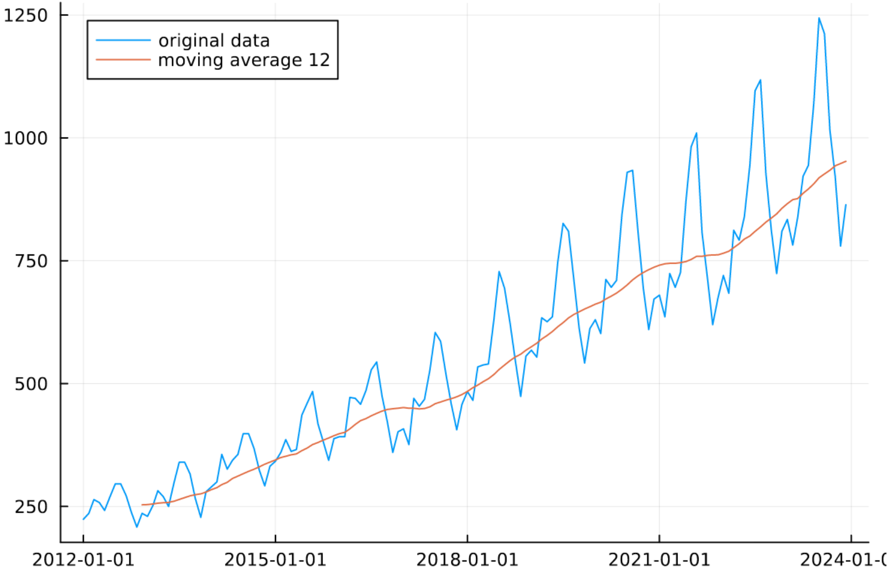
Plotting the time series to visualize trends, seasonality, and outliers.

Time Series: Example

EatComo is a small restaurant in Lake Como, Italy. The owner has been collecting data since 2012 about the number of customers visiting each month, capturing the essence of how customer traffic can vary over time due to various factors such as seasons, holidays, etc. We want to analyze this data to understand any patterns and obtain some insights.

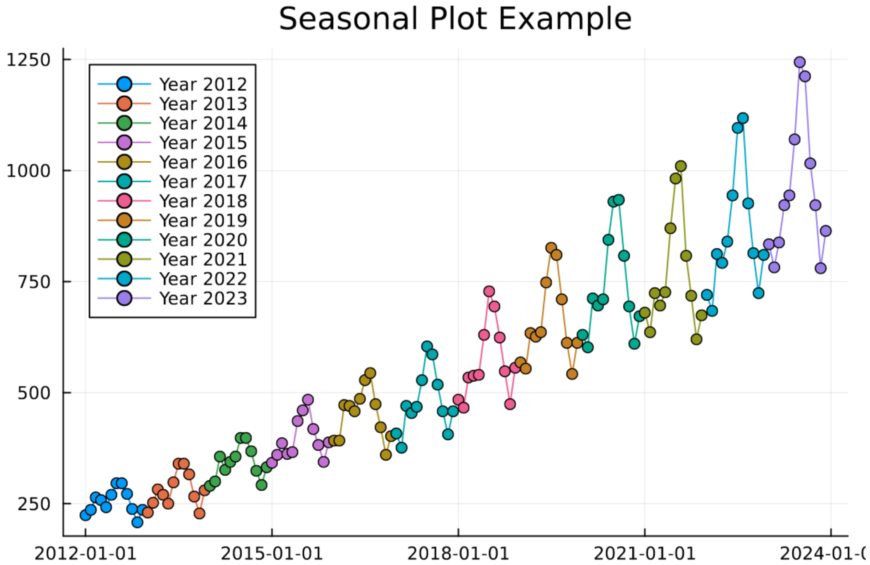


Time Series: Example Trend

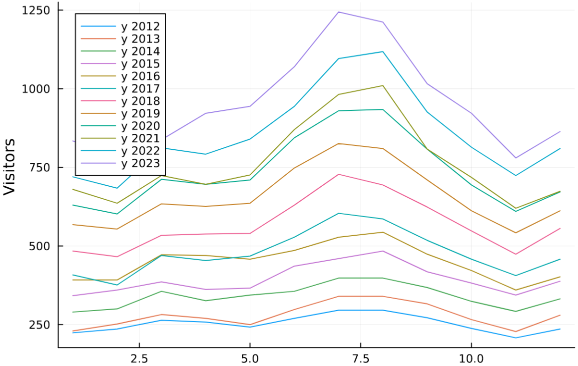


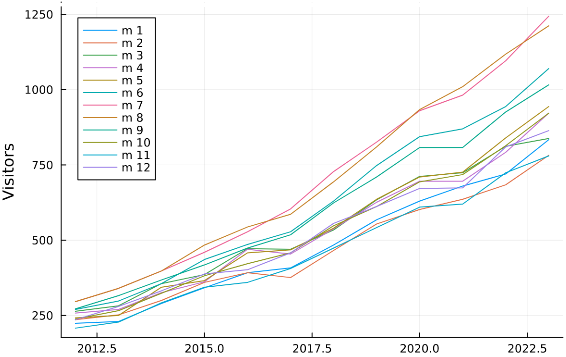
Time Series: Example Seasonality

:



Time Series: Example Trend and Seasonality





Time Series: Model Selection

1. Regression models that use time indices as x-variables and find a linear relationships between time and the time series variable. These can be helpful for an initial description of the data and form the basis of several simple forecasting methods.

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3. ARIMA (Autoregressive Integrated Moving Average) models that relate the present value of a series to past values and past prediction errors.
4. Deep Learning models that are capable of capturing long-term dependencies in time series data.

Time Series: Evaluation

No method is superior to any other method in every context. The performance of a forecasting technique can be measured by the error produced over time.

- Splitting Data: The dataset is divided into training and testing sets to evaluate the model's performance (usually 80% training and 20% testing).
- Error Metrics: Common metrics include Mean Absolute Error (MAE), Mean Squared Error (MSE), and Root Mean Squared Error (RMSE).
- Model Selection: Based on performance metrics, complexity, domain knowledge, and the model's ability to capture the dynamics of the time series.

Time Series: Evaluation

Define the one step ahead error

$$e_t = y_t - \hat{y}_t$$

where $\hat{y}_t$ is the forecast of y_t that was made one period prior.

- Mean Error:

$$ME = \frac{1}{T}\sum_{t=1}^{T} e_t$$

- Mean absolute error:

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |e_t|$$

- Mean squared error:

$$MSE = \frac{1}{T}\sum_{t=1}^{T} [e_t]^2$$

- Root mean squared error:

$$RMSE = \sqrt{MSE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} [e_t]^2}$$

- Moving average method uses the mean of the data, spreading the weight equally across all N time points.
- Exponential smoothing is based on the idea that forecasts are best made using decreasing weights some distance back in time. Because the weights are exponentially decreasing with a parameter α, they eventually become negligible if we go back far enough in time.

Time Series: Moving Average Method

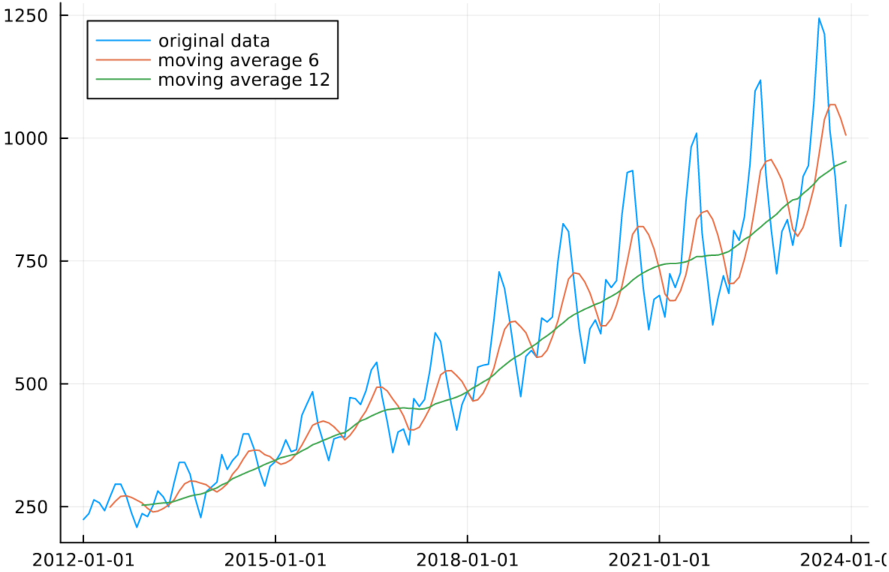
Moving Average Method

- This method averages the data for only the last N periods as the forecast for the next period

$$\hat{y}_{t+1} = \sum_{i=t-N+1}^{t} \frac{y_i}{N}$$

- A moving average of order N is simply the arithmetic average of the most recent N observations. For 3-week moving averages $N = 3$; for 6-week moving averages $N = 6 \ldots$
- Note that the use of moving average is not appropriate when there is a trend in the series. Why?

Time Series: Moving Average Method



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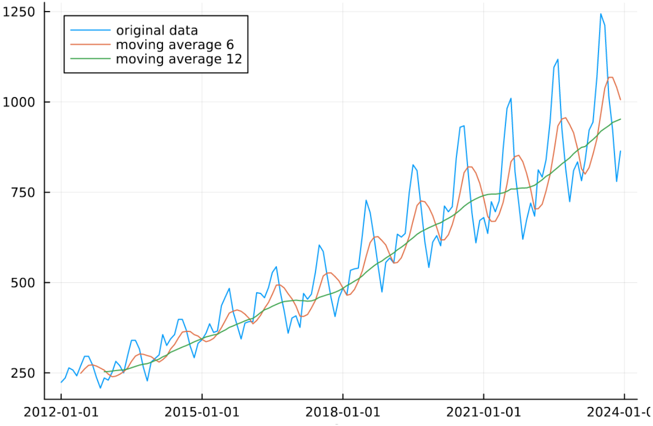


- Higher values of N make the model more stable, but less responsive to the changes in the process.
- Increased sensitivity may be good if the model is tracking a real trend in the data, or bad if overreacting to an unusual observation.
- The model underestimates demand with an increasing trend, and overestimates demand with a decreasing trend.

Time Series: Moving Average

In contrast to modeling in terms of a mathematical equation, the moving average merely smooths the fluctuations in the data. A moving average works well when the data has

- a fairly linear trend
- a definite rhythmic pattern of fluctuations

Time Series: Exponential Smoothing Method

- Exponential smoothing method computes a forecast value which is the weighted average of the most recent observation and the forecast values.
- It gives greater weight to more recent values, and the weights decrease exponentially as the series goes farther back in time.
- The weight assigned to the most recent observation is called the smoothing constant, α and the weight assigned to the most recent forecast is $(1 - \alpha)$.
- The method requires an initial forecast value.
- If the smoothing constant, α is large, the forecast values fluctuate with the actual data. If α is small, the fluctuation is less.

Time Series: Exponential Smoothing Method

$$\hat{y}_t = \alpha y_{t-1} + (1-\alpha)\hat{y}_{t-1}$$

Notice that,

$$\begin{aligned}
\hat{y}_t &= \alpha y_{t-1} + (1-\alpha)(\alpha y_{t-2} + (1-\alpha)\hat{y}_{t-2}) \\
&= \alpha y_{t-1} + \alpha(1-\alpha)Y_{t-2} + (1-\alpha)^2\hat{y}_{t-2} \\
&= \alpha y_{t-1} + \alpha(1-\alpha)y_{t-2} + \alpha(1-\alpha)^2 y_{t-3} + (1-\alpha)^3\hat{y}_{t-3} \\
&= \ldots
\end{aligned}$$

with further expansion, it can be seen that the forecast for period t depends on all previous data!!!

Time Series: Exponential Smoothing Method

$$\hat{y}_t = \alpha y_{t-1} + (1 - \alpha)\hat{y}_{t-1}$$

So far we have given forecast equations for only one step ahead. Simple exponential smoothing has a "flat" forecast function, and therefore for longer forecast horizons,

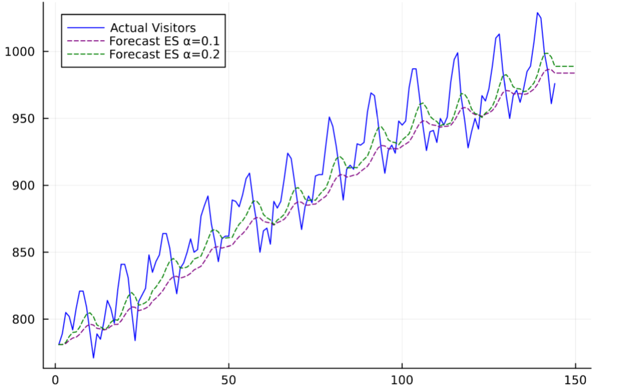
$$\hat{y}_{T+\tau} = \hat{y}_{T+1}$$

Remember these forecasts will only be suitable if the time series has no trend or seasonal component.

Time Series: Exponential Smoothing Method

- Initial $\hat{y}_0$:
  - Set $\hat{y}_0$ to be equal to y_1.
  - Set $\hat{y}_0$ to be equal to $\bar{y}$.
- Choice of α:
  - A value between 0.1 and 0.6 is commonly recommended

Time Series: Exponential Smoothing Method



- ES(0.1) line is smoother and more responsive to changes in the data than the moving averages.
- ES(0.2) is more responsive to recent changes in the data compared to the one with alpha 0.1.
- These exponential smoothing lines provide a way to understand and forecast the trend in demand with different levels of responsiveness to recent changes.
- The higher the alpha, the more weight is given to recent observations, making the smoothed line follow the actual demand more closely.

Time Series: Exponential Smoothing Method

- Lower values of α make the model more stable, but less responsive to changes in the process.
- Increased sensitivity may be good if the model is tracking a real trend in the data, or bad if overreacting to unusual observations.
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Time Series: Comparison of ES and MA Methods

Similarities

- Both methods are appropriate for stationary series
- Both methods depend on a single parameter
- Both methods lag behind a trend

Differences

- ES carries all past history. MA uses only last N.
- MA requires saving all N past data points while ES only requires saving last forecast and last observation. Consider the amount of data storage required for forecasting demand for $30,000$ inventory items.

Time Series: Trend Based Forecasting Methods

Models such as MA and ES lag behind a trend. This can be accounted for using:

- Regression Analysis
- Double Exponential Smoothing using Holt's Method

Time Series: Double ES using Holt's Method

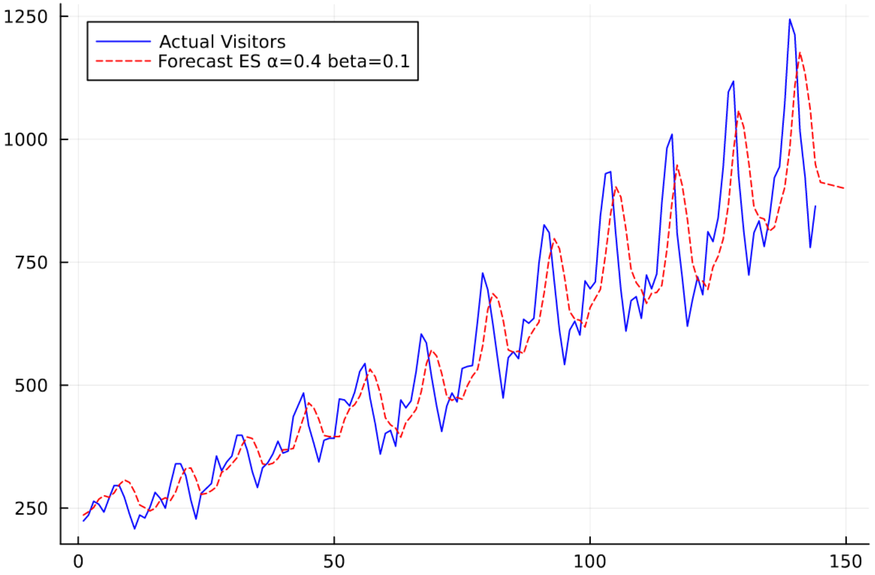
- Another method for forecasting linear trend in time series.
- The method uses two smoothing constants α and β and two smoothing equations:

$$L_t = \alpha Y_t + (1-\alpha)(L_{t-1} + B_{t-1}) \qquad \rightarrow \text{level equation}$$
$$B_t = \beta(L_t - L_{t-1}) + (1-\beta)B_{t-1} \qquad \rightarrow \text{trend equation}$$

- where L_t denotes an estimate of the level of the series at time t, B_t denotes an estimate of the trend (slope) of the series at time t, α is the smoothing parameter for the level and β is the smoothing parameter for the trend.
- α and β are between 0 and 1.
- The method iteratively computes L_t and B_t in period t.
- Initial L_0 and B_0 are needed to start the computation: $L_0 = y_1$ and $B_0 = y_2 - y_1$.
- Finally, L_t and B_t can be used to obtain a τ-step ahead forecast, $\hat{Y}_{t+\tau,t}$ from period t: $\hat{Y}_{t+\tau,t} = L_t + \tau B_t$

- The forecast function is no longer flat but trending.
- The τ-step-ahead forecast is equal to the last estimated level plus τ times the last estimated trend value.
- The forecasts are a linear function of τ.

Time Series: Double ES using Holt's Method Example



Wrapping up

Thanks for your attention!