Problem 3.6.7

We consider a DTMC model with

$$P = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3/10 & 0 & 7/10 & 0 \\ 0 & 1/10 & 0 & 9/10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First we use eq. (3.24)

$$V_1 = 1 + \frac{7}{10} V_2,$$
$$V_2 = 1 + \frac{1}{10} V_1.$$

which yields $(V_1, V_2) = (170/93, \; 110/93)$.

Next, we use the setup for eq. (3.66).

$$V_1 = (\phi_1 + \phi_2) / (1 + \rho_1 + \rho_2) - 0.$$

Here: $\rho_1 = \frac{q_1}{p_1} = 3/7$, $\rho_2 = \frac{q_1 q_2}{p_1 p_2} = \frac{3}{7} \cdot \frac{1}{9} = \frac{1}{21}$.

$$\phi_1 = \frac{1}{p_1} = \frac{10}{7}, \quad \phi_2 = \frac{1}{p_2} + \frac{q_2}{p_1 p_2} = \frac{10}{9} + \frac{1/10}{63/100}$$

$$\ne \frac{10}{9} \cdot \frac{63}{10} = \frac{667}{90}.$$

$$= \frac{10}{9} + \frac{10}{63} = \frac{80}{63}$$

Hence: $V_1 = \left(\frac{10}{7} + \frac{667}{90}\right) / \left(1 + \frac{3}{7} + \frac{1}{21}\right)$

$$V_1 = \left(\frac{10}{7} + \frac{80}{63}\right) / \left(1 + 3/7 + 1/21\right) = \frac{170}{93}.$$